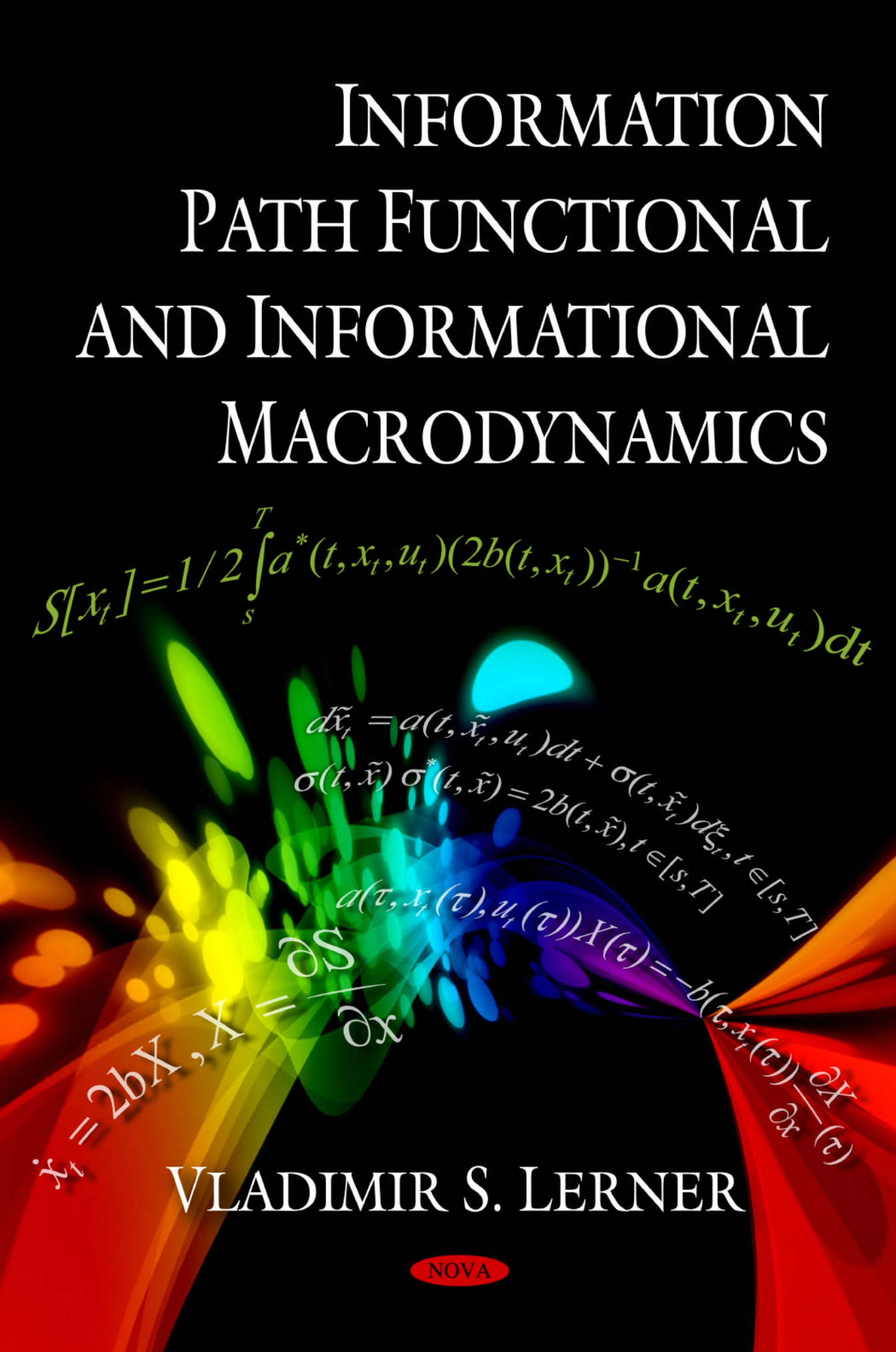
Information Path Functional and Informational Macrodynamics



Vladimir S. Lerner

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INFORMATION PATH FUNCTIONAL AND INFORMATIONAL MACRODYNAMICS

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VLADIMIR S. LERNER

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CONTENTS

Abstract **xi**

Preface **xiii**

Introduction **xix**

PART 1.

THE INFORMATION PATH FUNCTIONAL'S FOUNDATION **1**

1.0. Introduction **3**

1.1. The Initial Mathematical Models **7**

1.1.1. Model of the Microlevel Process 7

1.1.2. Model of the Macrolevel Process 9

1.1.3. The Feedback Equation-Control Law 9

1.1.4. Model of the Programmable Trajectories (as a task) at Microlevel 10

1.1.5. Model of the Programmable Trajectories (as a task) at the Macrolevel 11

1.1.6. The Equations in Deviations 11

1.1.7. Model of Disturbances 13

1.1.8. The Microlevel Process' Functional 13

1.1.9.The Jensen Inequality for Entropy Functional 18

1.2. Dynamic Approximation of a Random Information Functional and the Path Functional **23**

1.2.1. The Extremal Principle and the Problem Formulation 23

1.2.2. The Problem Solution. Information Path Functional 24

1.2.3. The Estimation of an Accuracy of the Probability's Approximation 42

1.3. Variation Problem for the Information Path Functional and Its Solution **45**

1.3.1. The Problem Statements 45

1.3.2. Solution to the Variation Problem 46
1.3.3. A Minimum Condition for the Microlevel's Functional 60
1.3.4. The Optimal Control Synthesis 63
1.3.5. A Summary of the Information Path Functional Approach and IMD. The Information Invariants 77
1.4. The IMD Information Space Distributed Macromodels 103
1.4.1. Introduction 103
1.4.2. The Variation Problem for Space Distributed Macromodel 105
1.4.3. The Invariant Conditions at the Transformation of the Space Coordinates 107
1.4.4. The Parameters of the Space Transformation and the Distributed Macromodels 114
1.4.5. The IMD Macromodel's Singular Points and the Singular Trajectories 120
1.4.6. The IPF Natural Variation Problem, Singular Trajectories, and the Field's Invariants 128
1.5. The Cooperative Information Macromodels and Information Network 141
1.5.1. Introduction 141
1.5.2. The Time-Space Movement toward the Macromodel's Cooperation 142
1.5.3. The Consolidation And Aggregation of the Model Processes in a Cooperative Information Network (IN) 164
1.5.4. The IN Dynamic Structure 173
1.5.5. Geometrical Structure of the Optimal Space Distributed Cooperative Macromodel (OPMC). The IN Geometric Structure 182
1.6. The IMD Model's Phenomena and an Information Code 195
1.6.1. The model Time Course and the Evaluation of the Information Contributions into IN. The Triplet's Genetic Code 195
1.6.2. The Model Information Geometry (IG), Its Specific, and the Structure 203
1.6.3. The Model Uncertainty Zone and Its Evaluation 211
1.6.4. Creation of the IN Geometry and a Genetic Code of the Information Cellular Geometry 212
1.6.5. The Minimal Admissible Uncertainty and Its Connection to Physics 217
1.6.6. Information Structure of the Model Double Spiral's (DSS) Control Mechanism 221

1.6.7. Examples of the DSS Codes 224
1.6.8. A system's Harmony, Regularities, and the VP 226
1.7. The Macrodynamic and Cooperative Complexities 229
1.7.1. Introduction 229
1.7.2. The Notion of Interactive and Cooperative Complexities and Their Information Measures 231
1.7.3. The Information Indicator of a Cooperative Complexity 233
1.7.4. Illustration of Arising of the Information Cooperative Complexity at Discrete Points of Applied Controls 239
1.7.5. The Complexity Invariant Measure in a Cooperative Dynamic Process 245
1.7.6. The IN Cooperative Mechanism with the Complexity's Measures 251
1.7.7. The Equations of the Spatial Information Cooperative Dynamics. Information Attraction and Complexity 259
1.7.8. Connection to Kolmogorov's Complexity 269
1.8. The Regularities of Evolutionary Dynamics and the Information Law of Evolution 271
1.8.1. Introduction 271
1.8.2. The Equations Regularities and the Evolutionary Law 272
1.8.3. A mechanism of an Enhancement of the Acceptable Mutations 280
1.8.4. The Conditions of the Model's Self-control, Adaptation, and Self-organization 286
1.8.5. The Evolution of the Model Invariants and a Potential Macroprocess' Cyclicity 291
1.8.6. Requirements for the Model Self–organization. The Results of Computer Simulations 297
1.8.7. Evaluation of Some Prognosis Parameters of the Evolutionary Dynamics. Example 301
1.8.8. Information Mechanism of Assembling the Node's Frequencies and Automatic Selection 302
1.8.9. The Functional Schema of the Evolutionary Informational Mechanisms. A Summary 307

1.9. The Physical Analogies Related to the Information Path Functional 311

1.9.1. The Connection between the Information Path Functional (IPF) and the Kolmogorov's (K) Entropy of a Dynamic System, and the Relations to Physics 311

1.9.2. An IPF Analogy with the Feynman Path Functional in Quantum Mechanics and Information form of Schrödinger Equation 314

1.9.3. About the Invariant Transformation of the Model Imaginary Eigenvalues and Information 321

1.9.4. The Superimposing Processes, Control and Asymmetry. The IMD Relation to Nonequilibrium Thermodynamics (NT) 323

1.9.5. About the Notion of a System 330

PART 2.

THE INFORMATION PATH FUNCTIONAL'S AND IMD'S APPLICATIONS 335

2.1. Solution of the Control Problems for a Complex Object 335

2.1.1. The Control Problems for a Complex Object 335

2.1.2. Solving the Identification Problem 338

2.1.2.1. The Identification of the Concentrated Object's Models 338

2.1.2.2.The Identification of the Space Distributed Object's Models 342

2.1.3. Solving the Optimal Control Problem 349

2.1.3.1. A Joint Solution of the Optimal Control and Identification Problems. The Basic Results 349

2.1.3.2. The Procedure of the Joint Identification, Optimal Control, and Consolidation 353

2.1.3.3. Building the Object's Cooperative Information Network 370

2.2. The Information Modeling of the Encoding-Decoding Processes at Transformation of Biological and Cognitive Information 373

2.2.1. The Objective and Methodology 373

2.2.2. An Inner Information Structure of the IN with the Ranged and Nonranged Sequences of the Starting Eigenvalues. The DSS Code 374

2.2.3 Mathematical Model of the IN with an Arbitrary Sequence of the Starting Eigenvalues 378

2.2.4. The Procedure of Encoding, Compression, Synthesis, and Decoding the IN Information 388
2.2.5. Summarized Results 394
2.2.6. About Other Related Applications 399
2.2.7. The Connections between Some Physical Neuronal Functions and Mechanisms with Their IMD Information Analogies 402
2.3. Information Modeling and Control of Some Industrial Technology Processes with Complex Superimposing Phenomena 409
2.3.1. Process Solidification and Its Application in Casting Technology 409
2.3.2. Some Electrotechnological Processes 420
A.2.3. Appendix: Additional Practical Results 421
2.4. An Elementary Information Macrodynamic Model of a Market Economic System 427
2.4.1. About Information Systems Modeling of a Modern Economy. The Objectives 427
2.4.2. An Elementary Local Production System (LP) 430
2.4.3. An Information Model of a Local Market 432
2.4.4. Managing the LP. A Bank and a Stock Market 437
2.4.5. Other Information Markets 441
2.4.6. Example 442
2.4.7. Summary 443
2.5. An Outline of the Computer Based Methodology 445
2.5.1. The Hierarchy of the Model's Micro-and Macrovariables and Their Identification 445
2.5.2. The Computer Restoration of the IMD Model 446
2.5.3. The Structure of the IMD Software Package 449
Conclusion 451
References 455
About the Author 471
Index 473

ABSTRACT

The book, introduces entropy *functional*, defined on a controllable random process, which serves as the random process' integral information measure, while Shannon's information measure is applicable for the process's selected states.

The random process' *information dynamics* is studied by establishing an *information path functional* (IPF), which, using a variation principle (VP), allows both approximating the entropy functional with a maximal functional probability and finding dynamic trajectories of the random *microprocess* as its *macroprocess*.

The IPF mathematical formalism connects the controlled system's randomness and its dynamic regularities by the informational macrodynamics' (IMD) equations for both concentrated and distributed systems, which describe a *system* of the information dynamic macroprocesses, generated by random observations.

The IMD *provides modeling* of a controlled random *system* by its dynamic macromodel for a disclosure of the system's *information regularities*, expressed by the VP.

The modeling includes the discrete intervals of the process' observation with the identification of the macromodel VP extremal's sequence (segments), and a potential model's renovation (between the VP extremal segments).

The VP information invariants present both dynamic and information measures of each IPF extremal segment.

The invariants allow the process' simple encoding by applying the the IPF or Shannon information measure.

The optimal controls, implementing the VP for the process' integral functional, connect the process exremal segments into a dynamic chain, which is encoded into the process' information network.

Connecting the extremal segment of multi-dimensional process into a *cooperative* chain is accompanied by its states consolidation, which reduces the number of the model independent states.

These allow grouping the cooperative macroparameters and aggregating their equivalent dimensions in an *ordered hierarchical* information network (IN), built on a multidimensional spectrum of the systems operator, which is identified during the VP minimax principle a real-time optimal motion.

The IN synthesis, based on the VP minimax principle, includes the operator eigenvector's ordered arrangement and their aggregation in the elementary cooperative units.

An optimal aggregation, satisfying both a minimal IPF *local* path to each cooperating unit and a stability of the formed cooperative unit, leads to assembling the extremal segments' spectrum sequentially into the IN elementary nodes, composed by a pair of eigenvectors (doublets) and/or by their triple (triplets)-as a maximal *elementary* unit.

The triple cooperation is also able to deliver an elementary quantity of the information contribution, measured by the above information invariants, as the IN elementary triplet's code word.

The IN is formed by ordering and adjoining of the cooperated nodes, which finally move to a single IN ending node, being encoded by the same elementary code word.

This allows both encoding the initial random process in the IN ordered hierarchical macrostructure and compressing the process' encoded information into the IN final node.

The computer procedure, based on the above mathematical formalism, includes the object's identification, combined with optimal control's synthesis, process' consolidation in the cooperative dynamics, and building the IN.

The introduced concept and measure of macrocomplexity (MC) arises in an irreversible macrodynamic cooperative process and determines the process components' ability to assemble into an integrated system.

MC serves as a common indicator of the origin of the *cooperative* complexity, defined by the *invariant information* measure, allowing for both analytical formulation and computer evaluation.

The MC of the IN optimal cooperative triplets' structure is measured by the triplet *code, which* provides the MC *hierarchical* invariant information measure by both its quantity and quality. MC presents a *computable* complexity measure of a *cooperative dynamic irreversible process*, as an opposite to the Kolmogorov complexity's *incomputability*.

The bi-levels renovated macromodel embraces the regularities of the *evolutionary dynamics*, such as creation of an order from stochastics, evolutionary hierarchy, stability, adaptive self-controls and a self-organization with copying information and a genetic code. The equations' regularities follow from the informational path functional's VP as a *mathematical law of evolution*, which is capable of a prognosis of the evolutionary dynamics and its specific components: evolution potentials, diversity, speed, and genetic code.

Book also studies some *physical analogies* related to the information path functional, including its connection to Feynman's path functional, Schrödinger's equation, and Irreversible Thermodynamics.

Process of Cognition, formalized by a minimization of observed process' uncertainty, is described by a piece-wise sequence of the VP dynamic macromodel's externals, identified during observation and built to maximize a reception of information.

The developed computer-based methodology and software were practically used for systems modeling, identification, and control of a diversity of information interactions in some physical (technological) and non-physical (economical, information-biological) objects.

The book on Information Path functional is published for the first time.

PREFACE

"Mathematics Is More Than Just A Language- It Is Language Plus Logic"
R. Feynman

"A new logical basis for information theory as well as probability theory is proposed, based on computing complexity".
A.Kolmogorov (the theory author)

This book introduces an *information path functional* as a basic information measure of a random *process*, whereas Shannon's information measure is applicable to the process' *states.*

The book's mathematical formalism uses a *variation principle* (VP) for the path functional to find the process' *dynamic* trajectories as the VP extremals and to get the process' dynamic equations in the informational macrodynamics (IMD).

The information path functional, defined on a *controlled* random process, extends the obtained dynamic macromodels on a wide class of the *control information systems*. This allows *modeling* (identification) of a controlled random *system* by its dynamic macromodel for a disclosure of the system's *information regularities*, expressed by the VP.

The developed mathematical formalism connects the controlled system's randomnesses and dynamic regularities by the IMD equations, which describe a system of the information dynamic macroprocesses, generated by random observations.

The book's path functional, its VP and the IMD present a *new approach to dynamic information modeling*, applied to a functional on trajectories of a random process and its relation to the random process' entropy functional.

This book focuses on the *mathematical roots* of informational macrodynamics and the recent applications; while broader IMD basics were considered in the author's other books and research papers (please see the References).

The book consists of two parts.

Part1 contains *the foundation of the information path functional and the variation principle with the IMD dynamic model.*

In ch.1.1 we introduce the *initial mathematical models* of the controlled random (microlevel) processes in the forms of a controlled stochastic equation and a random

information (entropy) functional, defined on the microlevel process, and present a class of the considered dynamic macroprocesses.

In ch.1.2 we provide the probabilistic evaluation of the micro- and macrolevel processes, using the probabilities of the processes' proximity via the process trajectories' metrical space distances.

Applying these probabilities, we formulate the problem of *dynamic approximation of the random information functional by the information path functional (IPF),* determined through the parameters of the initial stochastic equation. This leads to a variation problem as an extreme of the path functional with a *dynamic constraint,* defined by a maximal closeness of the microprocess' conditional entropy functional to the path functional at the macrotrajectories. The constraint establishes a connection between the micro- and macroprocesses. The functional's structure and the constraint bring the nontraditional solutions of both the extremal's and the control synthesis' problems.

Using both Pontryagin's maximum principle and Lagrange's methods of eliminating constraints, we find in ch.1.3 the *solution to the variation problem* in the form of a dynamic macromodel and the specified equation of constraint, binding the dynamics and stochastics. The solution determines the piece-wise extremal segments, where the macrodynamics act, and the "windows" between the segments, where the microlevel's random information affects the macrolevel dynamics. We use the connection of the micro- and macroprocesses to identify the macromodel operator via the observed random processes, in particular, by measuring and computing the corresponding covariation (correlation) functions.

We also solve the corresponding Bolza problem for *optimal control synthesis*.

We obtain a discrete function for the optimal regular control, applied at each extremal segment, and the optimal "jump" function for the optimal "needle" control, applied between the segments and thereby connecting them.

These controls allow us to build a procedure for optimal control synthesis *combined* with macromodel identification during the optimal control's action along each extremal. Because the above controls also stick the extremals, they sequentially *consolidate* the extremals (of an initially multi-dimensional process) into a process' *cooperative* structure. By this chapter's end we summarize the formal results of chs.1.1-1.3, establishing the IMD math foundation, applied for information modeling of a random *concentrated* system.

Ch. 1.4 introduces the space-time distributed entropy's functional in a random field and the basic path functional's distributed dynamic models (with fixed space coordinates) by the solution of Euler-Ostrogradsky equations for the functional's extremals. These models define the primary macroequations in partial deviations (PDE) of a *distributed* system. Then we consider a family of the space coordinates' *transformations* with the *invariant condition* imposed on the IPF. Searching for the VP's natural limitations, we obtain the IMD extreme model, defined on the admissible space coordinates' variations, which satisfies these transformations. Applying the Noether theorem, we get sufficient conditions for the invariance of the functional at the above transformations and the general forms of the PDE models in a mobile evolving space coordinate system. The invariant conditions bring an *additional* differential constraint to that, imposed by the IPF's VP on the distributed macromodel.

We obtain the IMD controllable distributed macromodel with the optimal controls, operating on space-time discrete intervals, which are found from Erdman-Weierstrass' conditions. We study the IMD macromodel's singular points and the singular trajectories; the

IPF natural variation problem, singular trajectories' control functions, and the field's invariants for the IPF.

In Ch.1.5, analyzing the time-space movement toward the macromodel's cooperation, we use the obtained results for building the *information cooperative network* (IN). The IN nodes integrate the information time-space macromovement along a sequence of extremal segments. The IN dynamic and geometrical structures are studied by applying the VP *information invariants*, following from the solution of the variation problem. The *optimal IN nodes'* cooperation, satisfying to the VP, leads to forming a *set* of triplet's nodes substructures, which are sequentially *enclosed* according to the IN hierarchy.

In ch.1.6 we examine the model *phenomena* and the process' information contributions into the IN triplet's hierarchy, evaluate both the model's reversible and irreversible time courses. We analyze the model information *cellular geometry* and its genetic information *code*, generated by the control processes and the IN triplet's hierarchy. This code, being a specific for each system, is memorized on a *double spiral* cellular time-space trajectory, generated by the IN triplets during the optimal control process.

The optimal *triple digital code* encloses the IN time-space hierarchy of the macromodel's cooperating nodes. The chapter results specify the macromodel's information geometry and its connection to macrodynamics. The path functional's integrated information is revealed through a decoded finite set of the extremal segments, assembled into the IN, which had been encoded by the triplet code.

In ch.1.7 we study the cooperative macrodynamic *complexities* (MC), which determine the process components' (in particular, the segments') ability to assemble into an integrated system in the cooperative informational dynamics.

We introduce the MC notion and the *invariant information measures*, allowing for both the complexities' analytical formulation and computer evaluations. Exploring the formal multi-cooperative mechanism of the IN, we establish the MC *hierarchical* invariant information measure by the *quantity and quality* in the triplet code.

We also consider the MC complexity's connections to Kolmogorov's complexity measures. Applying the *information geometry's space equations*, we determine an *intensity of information attraction* in the cooperative dynamics and its connection to the MC complexity.

Ch.1.8 studies the *regularities of evolutionary dynamics* and the mathematical law of evolution, following from the application of the variation principle for the informational path functional.

The law, applied to Darwinian evolutionary theory, embraces the following bio-regularities' information forms: creation of an order from stochastics via the evolutionary dynamics, described through the gradient of dynamic potential, local evolutionary speeds, and the evolutionary conditions of a fitness; evolutionary hierarchy, stability, potential of evolution; adaptive self-controls and a self-organization with a copying of information and a genetic code. We show that the VP single form of information law, which defines the above regularities, is capable of a prognosis of the evolutionary dynamics and its specific components: potentials, diversity, speed, and genetic code.

In ch.1.9 we study some *physical analogies* related to the information path functional, including its connection to Kolmogorov entropy, Feynman path functional, Schrödinger equation, and Irreversible Thermodynamics.

We also analyze information description for both *superimposition and the controls* by revealing the *information mechanism of the cross phenomena*.

Part 1 is aimed at establishing the IPF *mathematical formalism, describing information regularities, the IMD explicit dynamic features and information mechanisms,* following from the regularities' general VP form.

In part 2, the *formalism is applied* to the information modeling of complex systems with examples, studying the information regularities of *control systems* and the processes' regularities from *biology, technology, and economics*.

The *first goal* of part 2 is to show how these regularities, features, and the information mechanisms work in different areas of applications.

The *second goal* is to show that most of the applications' *specific* regularities (being known and unknown), studied in the particular branches of science, *actually are the concrete realizations* of the *general information regularities* and the IMD mechanisms of their revealing. These results are important for the regularities' understanding and practical use.

Ch.2.1 provides the IMD solution of the *control problems* for a complex system, applied for the system's *joint* identification, optimal control, and consolidation.

The studied in chs. 2.2-2.4 *information complex systems* embrace the case pattern from biology, technology, and economics, which include:

- the information modeling of the encoding-decoding processes, the structure of the information network and its code, applied to *biological and cognitive* systems;
- information modeling and control of some *industrial technology* processes with complex *superimposing phenomena*; and
- the application of the information modeling approach to an elementary *market macroeconomic* system.

All applications focus on disclosing of the information regularities of a studied object using the IMD equations and exploring their specific features for each considered object.

The model's formalism provides a *tool* for developing a *computer-based methodology,* and the *programs* (ch.2.5), which have been applied toward the solutions to the problems of information modeling, identification, optimal control, and consolidation for a wide diversity of complex systems.

The book's *essentials* consist of not only presenting a new theory of the information path functional, but also bringing this math theory up to very practical implementations, allowing the solution of actual problems in biology, technology, economics, and other considered applications.

The book *style* is directed on the initiation and keeping of the audience interest in this new interdisciplinary information science, which enhances both theory and computer applications.

The book starts with basic mathematical statements, followed by their examination and assessments in the comments and in subsequent conceptual reviews.

The book's mathematical results are motivated by means of their potential applications.

The author developed this approach between the years of 1960-1980 using physical models and applying them to technological systems (the References [R], published in Russian, reflect a history of the approach).

The book utilizes the author's scientific and practical experience of more than 40 years. The main research and academic results have been published in more than 250 scientific

articles and 6 books, which contain different parts of the developed formalism and its applications.

The author has taught the course “Information Modeling” for graduate students at the University of California at Los Angeles (UCLA) and at West Coast University. Based on the formalism applications, some new advanced courses were given at UCLA and WCU, such as “Information Systems Analysis”, ”Advanced Artificial Intellect”, and “Dynamic Cognitive Modeling”.

This book would be interesting for scholars, researchers, and students in applied mathematics, mathematical modeling and control, information and computer sciences, engineering, biology, and economics, as well as other interdisciplinary fields.

Specifically, part 1(chs.1.1-1.2, 1.3-1.4) would be interesting for a reader who wants to understand the basic *mathematical* formalism, which is an essential attribute of dynamic information modeling mechanisms and their correctness.

The reader interested only in an *essence* of the formalism and its conceptual understanding may start reading the book from sec.1.3.5 (by omitting the proofs in chs.1.2-1.3) and then continue with the practical applications in part 2.

The author addresses this book not only to scholars and scientists, but also to curious and searching minds trying to understand a modern World of Information and Uncertainty.

This book provides new ideas for both the theory of the novel path information functional and its applications, which allow the solution of actual practical problems in many areas where other related methods do *not* work.

The results are practically implemented on real objects and demonstrated by the book examples.

Regarding the book’s *formulas and references*: Only the book's formulas, cited outside of the part and/or chapter's references, start with part numbers, for example 1.4 for part 1, chapter 4. Within each chapter we use a sequential numbering of formulas starting with the chapter number, and apply them only for the inside references. The same principle we apply also to all figures; the cited references are also related to the corresponding chapters.

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INTRODUCTION

The main subject of this book is mathematical formalism, describing the creation of the dynamic and information regularities from stochastics.

The formalism is based on the introduction of an informational path functional (IPF) via a dynamic approximation of the entropy functional (EF), defined on trajectories of a controlled random process.

Using a *variation principle* with its *dynamic constraint*, connecting both EF and the IPF, we find the IPF extemals, allowing a dynamic approximation of the random *process*.

The solution provides both the *information dynamic model* of a random process and the model of *optimal control* in the forms of differential equations of informational macrodynamics (IMD).

This allows building of a *two-level information model* with a random process at the microlevel and a dynamic process at macrolevel.

Considering a variation principle (VP) as a mathematical form that expresses some regularity, it is assumed that the VP extremals, represented by the solutions of the above dynamic model, describe a movement possessing these regularities.

Such an approach has been used by R. P. Feynman, who introduced the functional on trajectories of an electron's movement and applied the variation principle for this *path functional* to obtain the equations of quantum mechanics.

The same way, we use the IPF and its VP to obtain the IMD equations.

Feynman's path functional is defined on the *dynamic* trajectories and has *not* been applied to *random* trajectories of a *controlled* process.

For an observed multi-dimensional *random controllable process,* affected stochastic perturbations, the mathematical results related to the information path functional (IPF), as well as the variation principle and the following dynamic model have been *unknown*.

For the IPF we use *Shannon's* definition of quantity of information applied to the *functional* probability on the process' *trajectories.*

The path functional's connection to information theory allows bringing a *common information language* for modeling the micro-macro level's processes and their regularities in diverse *interdisciplinary* systems.

For a wide class of random systems, modeled by the Markov diffusion process, and a common structure of the process's information path functional, this approach leads to a

broad-spectrum information structure of the dynamic macromodel, which we *specify and identify* on a particular system with the applied optimal control functions.

Unlike the existing literature on mathematical modeling, this book introduces a *unified* mathematical-information formalism for *information modeling*, which includes a common computer-based modeling *methodology, algorithms, information code, and computer software.*

According to the model dynamic regularities, the code arises from stochastics in a form of optimal double and/or triple digits, which compose an information network, formed by a hierarchy of the macromodel cooperating nodes.

The formalism has been *applied* to the information modeling of complex systems, and allows revealing their information regularities, particularly, demonstrated in the book examples from *biology, technology, and economics*.

The book specifics include the following key topics:

The information path functional is found as a *dynamic approximation* of a *random information functional*, defined on the process trajectories, using the probabilistic evaluation of the Markov diffusion process by a functional of action, whose Lagrangian is determined by the parameters of a controlled stochastic equation.

The optimal approximation of the system's random processes by the model's dynamic process leads to the *minimization* of their information difference, evaluated by the information path functional's measure of uncertainty. This problem automatically includes the solution of the *variation* problem (VP) for the system's mathematical model, the *synthesis* of the optimal control, and the solution of an *identification* problem.

The VP solution selects the functional's *piece-wise extremal segments*, where the macrodynamics act, and the *windows* between the segments, where the microlevel random information affects the macrolevel dynamics, and the stochastic-dynamic connection takes place. The VP solution also automatically introduces the model *piece-wise controls*, which operate by both joining the extremals' segments and acting along them.

The model's piece-wise *dependency* upon the observed information allow *identification* of both the controllable stochastic's and dynamic model's operators in *real time under* the optimal control action.

These controls, applied to the random process and its dynamic model, stick the extremals' segments (of an initially multi-dimensional process), sequentially *consolidating* these segments into a *cooperative* process.

The model segments' consolidation leads toward the model's *compressed* representation by the segments' *cooperative macrodynamics with the collective macrostates*. This creates a sequence of the *aggregated macrodynamic processes and the ordered* macrostates, *consolidated* in an *informational network* (IN) of hierarchical macrostructures, which organizes the system's mathematical model. These macrostates, memorized by the controls, create the model's *genetic information code*, which is able to encode the whole macromodel and its IN, allowing the eventual restoration of the system's model.

The microlevel's ability to discretely affect the macrodynamics at the segment's windows brings the model's piece-wise dependency on the randomness, being a source of the model's *renovation and evolution.* The consolidating extremals create the *cooperative complexity of the evolutionary macrodynamics*, initiated by the VP.

The book contains the following key features:

Revealing the dynamic regularities of a random process by applying VP to the process' information functional (as a universal attribute for any natural process) *automatically brings the dynamic constraint*, imposed *discretely* on the random process, which allows selecting the process *quantum states that represent both the process' dynamics and randomness.*

The informational macrodynamics (IMD), resulting from the VP solution for the information path functional, create the Lagrange-Hamiltonian *macromechanics of uncertainty,* which represent an information analog of physical *irreversible* thermodynamics.

The macrolevel's *function of action* portraits a dynamic eqiuvalent of the microlevel's *entropy functional*, which, being sufficient in the theory of dynamic systems, communication theory, and computer science, has not been used before.

A "deterministic impact" of microlevel stochastics on the Hamiltonian mechanics changes the structure and value of the dynamic macromodel operator, carrying its *dynamic piece-wise dependency* upon observed data. The macrodynamic process is characterized by the discrete of time-space intervals (defined by the extremals' segments), which are selected from the Hamilton's solutions and determined by the VP *invariants*. The discrete renovated operator and macrostates (at the window's discrete points (DP)) are sources of new information, new properties, and the macrostructures created by the *state consolidation.*

The Hamilton equations determine the *reversible* dynamic solution within the extremals' time intervals and the *irreversible* solutions that emerge out of these intervals.

The macromodel interacts with the environment at the segments' widows when the model is *open* for external influence. The open system does *not* satisfy the preservation *laws* at the moments of interaction when the external environment is able to change the model structure.

Optimal control functions are an intrinsic part of the model as an inner model's feedback mechanism, which is synthesized by the *duplication* of macrostates at the DPs during the optimal model's movement. These discrete controls are memorized and stored along with the corresponding ordered macrostates.

The *specifics* of the path functional's *controls*, applied at the beginning of the extremal segment, allows the proceeding of the segment's operator identification *under* the optimal control *action*, and, as a result, leads to a joint solution of the object's optimal identification and consolidation problems.

The IPF optimum predicts each extremal's segments movement not only in terms of a *total functional path goal*, but also by setting at each following segment the renovated values of this functional, identified *during the optimal movement*, which currently correct this goal. The *concurrently synthesized optima*l control's actions provide a maximal Markov's probability and optimal filtering of the random process. This optimal dual strategy at each current movement cannot be improved (even theoretically) because it defined by an extremal of a *total path* to a terminal state, which is updated at each optimal control's action.

The IMD system model contains the following main layers: *microlevel stochastics, macrolevel dynamics, and a hierarchical dynamic network of information structures with an optimal code language for the communication and model restoration.*

The system's complex dynamics initiate unique *information geometry and evolution* of the model equations, using the *functional's information mechanisms of ordering, cooperation, mutation, stability, adaptation, and the genetic code.*

In the formalism's applications, the information modeling's *unified language* and systemic categories, such as information entropy, the VP information Hamiltonian and

invariants, quantity and quality of information, information flows, forces, and information complexity, *are translated* into the corresponding categories of energy, entropy, temperature, and other physical and/or computer analogies of specific systems.

A computer implementation works directly with the real system's information model that produces its information network and a code. This builds a *bridge* connecting mathematical modeling formalism to the world of information, intelligence, and information technologies, allowing the revelation of the common systemic information regularities across a variety of modeling objects, including a specific information code and complexity for each system, and applying the universal computer-based methodologies, algorithms and programs.

Finally, applying the information path functional allows us to evaluate the *information content* of a random process, build its dynamic macromodel, the corresponding information network, and disclose the process's information code and macrodynamic complexity.

PART 1.
THE INFORMATION PATH FUNCTIONAL'S FOUNDATION

Chapter 1.0

INTRODUCTION

Uncovering unknown information regularities of a random process is an actual problem in the theory of dynamic and stochastic systems, control and system theories, theory of information, and physics.

Physics studies natural processes in order to reveal their regularities, using different principles and methods, applied to the measurement and estimation of observed processes.

By R. P. Feynman's ideology [1], a measured process has to reflect some regularity of the observed systems. Numerous examples from physics show that regularities of deterministic and stochastic systems are formulated and formalized by some extremal principles: *a real movement of a physical system represents trajectory-extremals of some functional.*

This assumes that an extremal principle can express regularities of an observed system.

Using this approach, R. P. Feynman introduced the functional on trajectories of an electron's movement and applied the variation principle for this path functional to obtain the equations of quantum mechanics [2]. This functional is defined on the dynamic trajectories and has not been applied to trajectories of a random process.

M. Kac [3] joins the Wiener integral measure [4] with Feynman's path integral in the theory of stochastic processes, providing the Feynman-Kac formula for solving the Schrödinger equation in statistical mechanics. Latter publications [5-12] summarize and extend the applications of Feynman-Kac's results to different problems in physics and mathematics.

An observed system is usually represented by a *random controllable process*, affected by stochastic perturbations.

The mathematical results, related to stochastic control theory and theory of controlled Markov processes are studied by E.B. Dynkin [13], I.I.Gihman, A.V. Scorochod [14], and W. H. Fleming and H.M. Soner [15].

R.L Stratonovich developed theory of the conditional Markov processes, applied to optimal control problems [16] and to information theory [17].

M. I. Freidlin and A.D. Wentzell, considering the stochastic behavior of perturbed Hamiltonian systems [18], introduced a functional of action.

Other related to the path functional's results, such as the stochastic Hamiltonian and an approximation of the components of diffusion processes, were published in [19, 20] accordingly.

N.V. Krylov [21] used a normalized Bellman's equation for an estimation of optimal stopping for the controlled diffusion process.

The path functional's applications to the controllable Markov processes and a process' *dynamic* model is still *unknown*.

Our aim is modeling (identification) a controlled random system by its information dynamic model to disclose the system's *information regularities*.

We assume that a *measured* system's process might reveal the system regularities, serving for the *identification* of an *extremal* problem, expressed by the system functional.

Because most of the unknown regularities are covered by a random environment, their revelation and identification involve a *probabilistic* evaluation of a measured control process.

This brings an essence of the extremal principle as *an approximation of the process' extremals using the most probable system's trajectories.*

Identification of such a *natural* system's functional (providing the exremals) theoretically allows us to restore the system's model, reflecting the system's regularities, by applying variation methods to this functional.

Because modeling is based on disclosing regularities by *minimizing uncertainties* in the observed object, we may suppose that an unknown functional covers uncertainty, associated with the object's probabilistic description and/or with the information necessary for the object's measurement and identification.

This leads to the functional's connection to *information theory* and to an information model, obtained by a minimization of such an *information* functional, which becomes the object's own (eigen) information functional.

That is why, introducing the controllable random process (as a *microlevel* process), and an information (*entropy) functional* on its trajectories, the *objectives* consist of:

(1) *finding a dynamic approximation of this functional*, considered as *an information path functional* of the controllable random process;
(2) *solving the variation (extremal) problem for this path functional to find a dynamic model* of the controllable random process (as a *macrolevel* process), which, we suppose, will describe the process's *dynamic regularities;*
(3) *solving the control's synthesis problem* for the path functional;
(4) *solving the problem of the identification of the dynamic model using an observable random process;*
(5) *finding the optimal controls* solving the problem *identification* of the model's dynamic operator *under the* optimal control *action*.

Problem (1) is solved in the book chs. 1.1-1.2, solutions to problems (2-5) are considered in chs.1.3, 1.4 and then applied in chs.2.1, 2.2.

Due to a lack of specific references in known literature, related to the path functional's applications for the *controllable random* processes, the key sources for this approach are

provided by the Feynman-Kac results [1-3], the Freidlin functional of action [18], and the Stratonovich entropy functional for a diffusion Markov process [17].

Foundation of the mathematical formalisms, describing the information regularities of the creation of dynamics from stochastics, requires the proofs of many problematic mathematical statements, which use the results from stochastic theory, the theory of dynamic systems, calculus of variations, thermodynamics, and information theory.

The development and proofs of this complicated formalism took many years (see [R]), with a brief publication in 1989 [31].

Unlike the following publications [32-34], containing the separate findings, this book brings a comprehensive and new mathematical results to the entropy functional of controlled Markov process, problem of dynamic modeling of a random system, and the solution of the system's identification, combined with optimal control's synthesis, *based on the variation principle for information path functional*.

The chapter is organized as followings.

In ch.1.1, we introduce the model of microlevel processes, including a controlled random process (as a solution of the stochastic equation with a specified class of control functions), the models of the disturbances, and a given (programmable) process.

Along with the problem statement, we define a class of macroprocesses, followed by an extremal probabilistic approximation of the microlevel process by a macrolevel process with the aid of the applied control.

We introduce an entropy functional for a controllable Markov diffusion process, describe the functional's representation through the process' additive functional and the parameters of corresponding stochastic differential equation.

Then we apply the results of ch.1.1 for establishing the Jensen inequality for the entropy functional (sec.1.1.9), which is used for the functional estimation.

In ch.1.2 we provide the probabilistic evaluation of the micro- and macrolevel processes. We define the probabilities of the processes' proximity, using the process trajectories' metrical space distances to determine the trajectories' closeness (in C) and a distance (in L^2) accordingly.

Applying the "functional action" approach, we express the extremal problem using the macroprocess' (path) functional, determined by the parameters of the stochastic equation, and evaluate the above probabilities' closeness in terms of the microprocess' conditional entropy functional.

This allows us to formulate in ch.1.3 the variation problem as an extreme of the macroprocess functional with the constraint, defined by a maximal closeness of the microprocess' conditional entropy to the path functional at the macrotrajectories.

This constraint establishes a connection between the micro- and macroprocesses. It is shown that a macrolevel process, which approximate the microlvel's process with a maximal functional probability, enables minimize the entropy functional, while the extremal of the path functional defines a macroprocess.

The functional's structure and the constraint lead to the nontraditional solutions to both the extreme and the control's synthesis problems.

Using both Pontryagin's minimum principle and Lagrange's methods of eliminating constraints, we find the solution to the variation problem in the form of a dynamic macromodel and a specified equation of *constraint, connecting the dynamics and stochastics.* The solution determines the piece-wise extremal segments, where the macrodynamics act, and the "windows" between the segments, where the microlevel's random information affects the macrolevel.

We use the connection of the micro- and macroprocesses to identify the macromodel's operator via the observed random processes, in particular, by measuring and computing the corresponding covariation (correlation) functions.

In sec.1.3.4 we synthesize the model optimal controls by solving the corresponding Bolza problem.

We obtain a discrete function for the optimal *regular* control, applied at each extremal segment, and the optimal "jump" function for the optimal "*needle*" control, applied between the segments and connecting them.

Existence of the widows between segments leads to dynamic model's possibility of forming an optimal piece-wise control, during a real time movement at each segment, which is applied to the diffusion process at each of such widow.

These controls allow the optimal control synthesis *combined* with macromodel identification, while the identification proceeds during the optimal control's action along each extremal.

Because the above controls also stick the extremals, they sequentially consolidate the extremals into a process *cooperative* structure.

In sec.1.3.5 we summarize the chapter results and introduce the model's information invariants.

Chapter 1.1

THE INITIAL MATHEMATICAL MODELS*

1.1.1. Model of Microlevel Process

We consider controllable random processes, as the trajectories in a Gilbert space, defined by the solutions of n-dimensional controlled stochastic differential equation Ito [15]:

$$d\tilde{x}_t = a(t,\tilde{x}_t,u_t)dt + \sigma(t,\tilde{x}_t)d\xi_t, \tilde{x}_s = \eta, t \in [s,T] = \Delta, s \in [0,T] \subset R^1_+, \quad (1.1)$$

where $\xi_t = \xi(t,\omega)$ is an increment of a Wiener process $\upsilon_t = \upsilon(t,\omega)$ during the time $(t-s)$ on the probability space (Ω,Ψ,P_o), $\omega \in \Omega$ with the variables located in R^n; Ω is a space of random events, Ψ is a σ-algebra on Ω; $P_o = P_o(B)$ is a probability measure on Ψ, $B \subset \Psi$, β is a Borel algebra in R^n; function $\xi(\bullet,\omega)$ is continuous on Δ; $\xi(t,\bullet):(\Omega,\Psi,P_o) \to (C,\upsilon,\mu_o)$, $C = C(\Delta,R^n)$ is a space of the n-dimensional, continuous on Δ vector functions, υ is a σ-algebra in C, generated by all possible opened sets (in C metric), μ_o is a probability measure on υ:

$$\mu_o(A) = P_o\{\xi_t \in A\} = P_o\{\omega : \xi(t,\bullet) \in A\}, A \subset \upsilon, \{\xi_t \in A\} \subset \Psi, \ \mu_o(C) = 1; \quad (1.1a)$$

$\eta(\omega^1)$ is random vector on the probability space (Ω^1,Ψ^1,P^1), $\omega^1 \in \Omega^1$ with the variables in R^n:

$$\eta:(\Omega^1,\Psi^1,\mathrm{P}^1) \to (R^n,\beta,P_s);$$

$$P^1 = P^1(B^1), B^1 \subset \Psi^1, P_s = P_s(D) = P^1\{\omega^1 : \eta(\omega^1) \subset D\} = P^1\{\eta \subset D\},,$$

$$\omega^1 \in \Omega^1, D \subset \beta, \{\eta \in D\} \subset \Psi^1, P_s(x) = P^1\{\eta(\omega^1) = x\}, x \in R^n; \quad (1.1b)$$

* All details of the initial models are essentially important for the use in the following proves and results.

$\tilde{x}_t = \tilde{x}(t,x,\eta)$ is a random process, considered on a family of the probability spaces (Ω,Ψ,P_x), or on the probability space $(\Omega^{11},\Psi^{11},P)$ with the variables in R^n; $P_x = P_x(B)$ is a is a family of probability measures on $\Psi, B \subset \Psi$, depending on $x \in R^n$ with the probability $P_s(x)$; $P_o = P_{x=0}$,

$$\omega^{11} = (\omega, x), \omega^{11} \in \Omega^{11}, \Omega^{11} = \Omega \times R^n, \Psi^{11} = \Psi \times \beta, B \times D \subset \Psi^{11},$$

$P(B \times D)$ is a probability measure on Ψ^{11}, which, following the Markovian property, satisfies the equation

$$P(B \times D) = \int_D P_s(dx) P_x(B). \tag{1.1c}$$

For (Ω,Ψ,P_x) and $(\Omega^{11},\Psi^{11},P)$ we have accordingly:

$$\tilde{x}(t,\bullet,\eta): (\Omega,\Psi,P_x) \to (C,\upsilon,\mu_x), \tilde{x}(t,x,\bullet): (\Omega^{11},\Psi^{11},P) \to (C,\upsilon,\mu), \tag{1.1d}$$

where $\mu_x = \mu_x(A), \mu = \mu(A)$ are the probability measures on υ, $A \subset \upsilon$, which, for the process $\tilde{x}_t$, in both cases, correspond to the equalities:

$$\mu_x = \mu_x(A) = P_x\{\tilde{x}_t \in A\} = P_x\{\omega : \tilde{x}(t,\bullet,\eta) \in A\}, \{\tilde{x}_t \in A) \subset \Psi, \tag{1.1e}$$
$$\mu = \mu(A) = P\{\tilde{x}_t \in A\} = P_x\{\omega^{11} : \tilde{x}(t,x,\bullet) \in A\}, \{\tilde{x}_t \in A) \subset \Psi^{11}\}, \mu_x(C) = \mu(C) = 1.$$

Function of diffusion $\sigma(t,x) = (\sigma_{ij}(t,x))_{ij=1}^n$ is a nonsingular operator, defined on $\Delta \times R^n$ with the values from space $L(R^n,R^n)$ of linear operators in R^n:

$$\begin{aligned} &\sigma(t,x): \Delta \times R^n \to L(R^n,R^n), \det \sigma(t,x) \neq 0, \forall (t,x) \in \Delta \times R^n, \\ &\sigma_{ij}(t,\bullet) \in C^1(R^n,R^1), \end{aligned} \tag{1.2}$$

where function $\sigma_{ij}(\bullet,x) \in C^1(\Delta^o,R^1)$ is a continuous differentiable everywhere on Δ, possibly excluding my be the set $\{\tau_k\}_{k=1}^m$: $\Delta^o = \Delta \,|\, \{\tau_k\}_{k=1}^m$.

Function of shift: $a(t,x,u) = a^u(t,x)$ is a controllable vector, defined on $\Delta \times R^n \times U$, $U \subset R^r, r \le n$ with variables in R^n, function $a^u : \Delta \times R^n \times U \to R^n$ is a continuous differentiable by $x \in R^n$, $u \in \hat{U}$, $\hat{U} = \operatorname{int} U$ and has the bounded second derivatives by

each of the indicated variables; at fixed $(x,u) \in \Delta \times U$, function $a(\bullet, x, u)$ is a continuous on Δ and is a continuous differentiable function on Δ^o:

$$a\ (t,\bullet,u) \in C^1(R^n, R^n), a(t,x,\bullet) \in C^1(\hat{U}, R^n),$$
$$a(\bullet, x, u) \in C(\Delta, R^n) \bigcap C^1(\Delta^o, R^n) = KC^1(\Delta, R^n). \quad (1.2a)$$

1.1.2. Model of the Macrolevel Process

Model of the macrolevel process is a set of trajectories $\overline{x}_t \overset{def}{=} \overline{x}(t, \overline{\eta})$ in the space state with the initial conditions $\overline{x}_s = \overline{\eta}$ being averaged by (Ω, Ψ, P_o) and therefore independent on $\omega \in \Omega$:

$$\overline{x}_t : \Delta \times (R^n, \beta, P_s) \to (R^n, \beta), \overline{x}(\bullet, x) \overset{def}{\in} KC^1(\Delta, R^n) \subset C(\Delta, R^n)(\mathrm{mod}\, P^1), \quad (1.3)$$

where KC^1 is a space of continuous piece-wise differentiable on Δ, n-dimensional vector-functions.

1.1.3. The Feedback Equation-Control Law

Control (u_t) is formed as a function of time and macrovariables $(\overline{x}_t)$, which had averaged by (Ω, Ψ, P_x), becoming the nonrandoms with respect to set Ω[11].

This control, acting on the object, moves $\tilde{x}_t(u)$ toward $\overline{x}_t$.

The control law is defined by the following feedback equation:

$$u_t \overset{def}{=} u(t, \overline{x}_t), u : (\Delta \times R^r) \to U, U \in \beta(R^r), \quad (1.4)$$

where $\overline{x}_t$ satisfies (1.3), $\beta(R^r)$ is a Borel algebra in R^r, $r \le n$.

At a fixed $x \in R^n$, function $u(\bullet, x)$ is a piece-wise continuous by $t \in \Delta$, and at fixed $t \in \Delta$, function $u(t, \bullet)$ is a continuous differentiable and has the limited second derivatives by $x \in R^n$:

$$\forall x \in R^n, u(\bullet, x) \in KC(\Delta, U), u(\bullet, x) \in C^1(\Delta^o, U); \forall t \in \Delta, u(t, \bullet) \in C^1(R^n, U). \quad (1.4a)$$

The fulfillment of equations (1.3), (1.4), (1.4.a) defines u_t as a piece-wise continuous function:

$$u_t \in KC(\Delta,U), u_+ \overset{def}{=} \lim_{t\to\tau_k+o} u(t,\overline{x}_{\tau_k}), u_- \overset{def}{=} \lim_{t\to\tau_k-o} u(t,\overline{x}_{\tau_k}), k=0,...,m,$$
$$\tau_k \in \Delta, \tau_o = 0, \tau_m = T(\mathrm{mod}\, P^1), \tag{1.5}$$

where $KC(\Delta,U)$ is a space of the piece-wise continuous on Δ functions, defined with probability P^1=1.

From (1.2), (1.2a) it follows that the Lipschitz conditions, the linear growth by $x \in R^n$, and the uniformity with respect to $(t,u) \in \Delta \times \mathrm{U}$ are satisfied with necessity, and limitations (1.4a) are correct.

Therefore, according to [22], solution (1.1) exists, is unique on (Ω,Ψ,P_x) and $(\Omega^{11},\Psi^{11},P)$, and the moments of different orders at these spaces exist.

Vector $a(t,\overline{x},u) = \overline{a}^u(t,\overline{x})$ of the stochastic equation in physical problems defines a macroscopic speed of a medium (with the diffusing Brownian particles), which has the meaning of a regular flow:

$$d\overline{x}_t / dt = \overline{a}^u(t,\overline{x}_t), \overline{x}_s = \overline{\eta}. \tag{1.6}$$

Matrix $\sigma = \sigma(t,x)$ characterizes the peculiarities of a medium to conduct flow (1.6). Functions $\tilde{x}_t, \overline{x}_t$ define the micro-and macrolevel's processes. Their values $\tilde{x}$, $\overline{x}$ at the fixed moments of time define the vectors of micro-and macrostates of object (1.1).

The variables, measured by some physical instruments, are represented by vector $\overline{x}$.

1.1.4. Model of Programmable Trajectories (as a Task) at Microlevel

Model of programmable trajectories (as a task) at microlevel is given by process $\tilde{x}_t^1$ that satisfies the corresponding stochastic equation

$$d\tilde{x}_t^1 = a^1(t,\tilde{x}_t^1)dt + \sigma(t,\tilde{x}_t^1)d\xi_t, \tilde{x}_s^1 = \eta^1, \tilde{x}_t^1 = \tilde{x}^1(t,\omega,\eta^1), \tag{1.7}$$

and the relations analogous to (1.1a-1.2):

$$\eta^1 = \eta^1(\omega^1) : (\Omega^1,\Psi^1,\mathrm{P}^1) \to (R^n,\beta,P_s);$$
$$P_s^1 = P_s^1(D) = P^1(\eta^1 \in D) = P^1\{\omega^1 : \eta^1(\omega^1) \in D\}, P_s^1(x) = P^1\{\eta^1(\omega^1) = x\},$$

$$\tilde{x}^1(t,\bullet,\eta^1):(\Omega,\Psi,P_x)\to(C,\boldsymbol{\upsilon},\mu_x^1),\tilde{x}^1(t,\omega,\bullet):(\Omega^{11},\Psi^{11},P)\to(C,\boldsymbol{\upsilon},\mu^1),$$

$$\mu_x^1=\mu_x^1(A)=P_x\{\tilde{x}_t^1\in A\},\{\tilde{x}_t^1\in A\}\subset\Psi,\mu^1=\mu^1(A)=P\{\tilde{x}_t^1\in A\},\{\tilde{x}_t^1\in A\}\subset\Psi^{11},$$

$$\mu_x^1(C)=\mu^1(C)=1;$$

$$\forall t\in\Delta,a^1(t,\bullet)\in C^1(R^n,R^n),\forall x\in R^n,a^1(\bullet,x)\in C^1(\Delta,R^n). \quad (1.7a)$$

1.1.5. Model of Programmable Trajectories (as a Task) at the Macrolevel

Model of programmable trajectories (as a task) at the macrolevel is defined by process

$$\bar{x}_t^1=\bar{x}_t^1(t,\bar{\eta}^1),\bar{x}_t^1:\Delta\in(R^n,\beta,P_s)\to(R^n,\beta),\bar{x}_t^1(\bullet,\bar{\eta}^1)\overset{def}{\in}C^1(\Delta,R^n),(\mathrm{mod}\,P^1), \quad (1.8)$$

where $C^1(\Delta,R^n)$ is the space of the continuous differentiable on Δ n-dimensional vector-functions.

The corresponding regular flow at the macrolevel is defined by an equation similar to (1.6) in the form:

$$d\bar{x}_t^1/dt=a^1(t,\bar{x}_t^1),\bar{x}_s^1=\bar{\eta}^1. \quad (1.9)$$

The difference in distributions for the macroprocesses $\bar{x}_t$ and $\bar{x}_t^1$ has a simple physical interpretation: the object and control tasks are measured by different instruments.

1.1.6. The Equations in Deviations

The micro-and macrotrajectories we consider in the deviations from the corresponding programmable movements, which are given for the two-level model by the appropriate tasks:

$$\tilde{x}_t^*=\tilde{x}_t-\tilde{x}_t^1,x_t=\bar{x}_t-\bar{x}_t^1,x_s=\bar{\eta}-\bar{\eta}^1=\bar{\eta}^*. \quad (1.10)$$

The selection of u_t is limited by several conditions:

1)-each process' $\tilde{x}_t^*$, $\tilde{x}_t$, measured on $(C,\boldsymbol{\upsilon})$, is absolutely continuous with respect to the other;

2)-the measure of x_t coincides with the measure of $\bar{x}_t$.

According to the first one, $\tilde{x}_t^*$ will be found as a solution of the stochastic equation with the same function diffusion $\sigma(t,x)$, $(t,x)\in\Delta\times R^n$ as the one in (1.1) and with unknown drift $a^*(t,\tilde{x}_t^*,u_t)$, i.e. from the equation

$$d\tilde{x}_t^* = a^*(t,\tilde{x}_t^*,u_t)dt+\sigma(t,\tilde{x}_t^*)d\xi_t, \tilde{x}_s^*=\eta^*, \tilde{x}_t^*=\tilde{x}^*(t,\omega,\eta^*), \tag{1.11}$$

where $\eta^* = \eta - \eta^1 = \eta^*(\omega^1)$, $\eta^*:(\Omega^1,\Psi^1,\mathrm{P}^1)\to(R^n,\beta,P_s^*)$;

$$P_s^* = P_s^*(D) = P^1\{\omega^1:\eta^*(\omega^1)\in D\} = P^1\{\eta^*\in D\}, D\subset\beta, \{\eta^*\in D\}\subset\Psi^1,$$

$$P_s^*(x) = P^1\{\eta^*=x\} = P^1\{\omega^1:\eta^*(\omega^1)=x\}, x\in R^n.$$

According to [22], the following equations are true:

$$P_s^*(x) = \int_{R^n} P_s(x+y)dP_s^1(y), E_s^*[\bullet] = \int_{R^n}[\bullet]P_s^*(dx),$$
$$\tilde{x}^*(t,\bullet,\eta^*):(\Omega,\Psi,P_x)\to(C,\upsilon,\mu_x^*),$$

$$\mu_x^* = \mu_x^*(A) = P_s\{\tilde{x}_t^*\in A\}, A\subset\upsilon, \{\tilde{x}_t^*\in A\}\subset\Psi, \{\tilde{x}_t^*\in A\}\subset\Psi,$$

$$\tilde{x}^*(t,\omega,\bullet):(\Omega^{11},\Psi^{11},P)\to(C,\upsilon,\mu^*), \{\tilde{x}_t^*\in A\}\subset\Psi^{11},$$
$$\mu^* = \mu^*(A) = P\{\tilde{x}_t^*\in A\}, A\subset\upsilon, \{\tilde{x}_t^*\in A\}\subset\Psi^{11}, \tag{1.11a}$$

where $E_s^*[\bullet]$ is a corresponding conditional mathematical expectation and $\mu_x^*(C)=\mu^*(C)=1.$

Function $a^*(t,x,u_t)=a^u(t,x)$ with control $u_t=u(t,x_t+\bar{x}_t^1)$ at $(t,x,u)\in\Delta\times R^n\times\mathrm{U}$ satisfies the same relations, which are applied to function $a(t,x,u)$.

According to (1.10), we have

$$x_t = x(t,\bar{\eta}^*),\ x_t:\Delta\times(R^n,\beta,P_s^*)\to(R^n,\beta), x(\bullet,\bar{\eta}^*)\in KC^1(\Delta,R^n)\ (\mathrm{mod}\,P^1). \tag{1.12}$$

The points of discontinuity of the vector-functions $\bar{x}(\bullet,\bar{\eta})$ and $x(\bullet,\bar{\eta}^*)$ are defined by the set $\{\tau_k\}$, $k=1,\ldots,m$ following from the points of the control switching.

At these points, we consider the one-sided derivatives:

$$\dot{x}_{-} \overset{def}{=} \lim_{t \to \tau_k - o} \dot{x}(\bullet, \overline{\eta}^*), \dot{x}_{+} \overset{def}{=} \lim_{t \to \tau_k + o} \dot{x}(\bullet, \overline{\eta}^*). \quad (1.12a)$$

The lack of an explicit macrolevel description brings to consideration a wide class of dynamic macroprocesses as a subset of the piece-wise differentiable n-dimensional vector–functions $x_t \in KC^1(\Delta \times R^n)$ on $\Delta = [s,T)$, $x \in R^n$, where the dynamic process x_t is characterized by the initial conditions $\overline{\eta}^*$ at Δ.

1.1.7. Model of Disturbances

Model of disturbances is considered as an auxiliary random process (chosen as a standard process):

$$\zeta_t = \zeta(t,\omega), \zeta_t : \Delta \times (\Omega, \Psi, P_x) \to (R^n, \beta), \quad (1.13)$$

which models perturbations $\zeta_t = \int_s^t \sigma(v, \zeta_v) d\zeta_v$ at the following conditions for mathematical expectations:

$$E_x[\zeta_t] = E[\zeta_t] = \mathrm{O} = (0_i)_{i=1}^n, \quad (1.13a)$$

where

$$E_x[\bullet] = E[\bullet] = \int_{\Omega''} [\bullet] P(d\omega^{11}). \quad (1.13b)$$

1.1.8. The Microlevel Process' Functional

Let us have a diffusion process $\tilde{x}_t$ with transition probabilities $P(s, \tilde{x}, t, B)$ and have a σ-algebra $\Psi(s,t)$ created by the events $\{ \tilde{x}(\tau) \in B \}$ at $s \le \tau \le t$; $P_{s,x} = P_{s,x}(A)$ are the corresponding conditional probability distributions on an extended $\Psi(s,\infty)$, $E_{s,x}[\bullet]$ are the related mathematical expectations.

A family of the real or complex random values $\varphi_s^t = \varphi_s^t(\omega)$ depending on $s \le t$ defines an *additive functional* of process $\tilde{x}_t = \tilde{x}(t)$ [27, 28], if each $\varphi_s^t = \varphi_s^t(\omega)$ is measured regarding the related σ-algebra $\Psi(s,t)$ at any $s \le \tau \le t$ with probability 1 at $\varphi_s^t = \varphi_s^\tau + \varphi_\tau^t$; and $E_{s,x}[exp(-\varphi_s^t(\omega))] < \infty$.

Then the transformation with the additive functional:

$$\tilde{P}(s,\varsigma,t,B) = \int\limits_{\tilde{x}(t)\in B} \exp\{-\varphi_s^t(\omega)\} P_{s,x}(d\omega), \tag{1.14}$$

defines the transitional probabilities of a transformed diffusion process ς_t.

At this transformation, the transitional probability functions determine the corresponding family of the extensive distributions $\tilde{P}_{s,x} = \tilde{P}_{s,x}(A)$ on $\Psi(s,\infty)$ with the density measure

$$p(\omega) = \frac{\tilde{P}_{s,x}(d\omega)}{P_{s,x}(d\omega)} = \exp\{-\varphi_s^t(\omega)\}. \tag{1.14a}$$

Applying the definition of a conditional entropy [17] to the logarithmic probability *functional density measure on trajectories* (1.14a) for process $\tilde{x}_t$ regarding process ς_t we have

$$S(\tilde{x}_t / \varsigma_t) = \int\limits_{\tilde{x}(t)\in B} -\ln[p(\omega)] P_{s,x}(d\omega) = E_{s,x}\{-\ln[p(\omega)]\}, \tag{1.15}$$

where $E_{s,x}$ is a is a mathematical expectation for functionals, taken along the trajectories by the probability measure.

Using (1.3) we get the following equality for the entropy functional expressed via the additive functional on the trajectories of the considered diffusion processes:

$$S(\tilde{x}_t / \varsigma_t) = E_{s,x}\{\varphi_s^t(\omega)\}. \tag{1.16}$$

Let the transformed process be $\varsigma_t = \int\limits_s^t \sigma(v,\zeta_v) d\zeta_v$ having the same diffusion matrix as the initial process, but the zero drift.

Then the above additive functional at its fixed upper limit T acquires the form [17, 28]:

$$\varphi_s^T = 1/2 \int\limits_s^T a^u(t,\tilde{x}_t)^T (2b(t,\tilde{x}_t))^{-1} a^u(t,\tilde{x}_t) dt + \int\limits_s^T (\sigma(t,\tilde{x}_t))^{-1} a^u(t,\tilde{x}_t) d\xi(t),$$

$$2b(t,\tilde{x}) = \sigma(t,\tilde{x})\sigma^T(t,\tilde{x}) > 0, \tag{1.17}$$

where

$$E_{s,x}\{\int\limits_s^T (\sigma(t,\tilde{x}_t)^{-1} a^u(t,\tilde{x}_t) d\xi(t)\} = 0. \tag{1.18}$$

Finally we get the information entropy functional (EF) expressed via parameters of the initial controllable stochastic equation (1.1):

$$S(\tilde{x}_t / \varsigma_t) = 1/2E_{s,x}\{\int_s^T a^u(t,\tilde{x}_t)^T (2b(t,\tilde{x}_t))^{-1} a^u(t,\tilde{x}_t)dt\}, \qquad (1.19)$$

For a positive quadratic form in (1.19), the above information entropy is a positive.

Comments 1.1.

Functional (1.14), integrated by the above probability measure, represents the functional probability on the trajectories of a Markov diffusion process, which is connected to Feynman's path functional (ch.1.9).

It's seen that the EF (1.19), which is built on the functional probability (1.14) at the measured trajectories using its conditional relations for the considered measures (1.14a) and definition (1.15), is also determined at these trajectories.

The EF arises at a *transformation* of the functional probability measures, representing a specific form of more *general relationships* connecting the sets, characterized by these measures.

The EF connection to the additive functional (1.16), (1.17) allows both establishing the EF as a functional on the measured trajectories and expressing the functional's integrand (1.19) via the functions of shift and diffusion of a standard Ito's stochastic equation, leading to the EF constructive use.

It will be demonstrated that solving the IPF variation problem (ch.1.3) brings the above EF to the form of a *regular integral functional* that simplifies the practical path functional's applications.

Example. Let us have a single dimensional equation (1.1) with the shift function $a^u = u(t)\tilde{x}(t)$ at the given control function $u_t = u(t)$, and the diffusion $\sigma = \sigma(t)$.

Then the entropy functional has the form

$$S(\tilde{x}_t / \varsigma_t) = 1/2\int_s^T E_{s,x}[u^2(t)\tilde{x}^2(t)\sigma^{-2}(t)]dt, \qquad (1.20)$$

from which at the nonrandom $u(t), \sigma(t)$ we get

$$S(\tilde{x}_t / \varsigma_t) = 1/2\int_s^T [u^2(t)\sigma^{-2}(t)E_{s,x}[\tilde{x}^2(t)]]dt = 1/2\int_s^T u_t^2 \dot{r}_t^{-1} r_s dt, \qquad (1.20a)$$

where for the diffusion process, the following relations hold true:

$$2b(t) = \sigma(t)^2 = dr/dt = \dot{r}_t,\ E_{s,x}[\tilde{x}^2(t)] = r_s,$$

and the functional (1.20a) is expressed via the process' covariation functions r_s, r_t and known u_t.

This allows us to *identify* the entropy functional on an observed controlled process $\tilde{x}_t = \tilde{x}(t)$ by measuring the above covariation (correlation) functions.

The *n*-dimensional form of functional (1.20a) follows directly from using the related *n*-dimensional covariations and the control. •

The jump of the control function u_- (1.5) from τ_{k-o} to τ_k, acting on a diffusion process $\tilde{x} = \tilde{x}(t)$, might "cut off" this process after moment τ_{k-o}. The "cut off" diffusion process has the same drift vector and the diffusion matrix as the initial diffusion process.

The additive functional related to this "cut off" has the form [28]:

$$\varphi_s^{t-} = \begin{cases} 0, t \le \tau_{k-o} \\ \infty, t > \tau_k \end{cases}. \tag{1.21}$$

The jump of the control function u_+ (1.5) from τ_k to τ_{k+o} might cut off the diffusion process *after* moment τ_k with the related additive functional

$$\varphi_s^{t+} = \begin{cases} \infty, t > \tau_k \\ 0, t \le \tau_{k+o} \end{cases}. \tag{1.22a}$$

At the moment τ_k, between the jump of control u_- and the jump of control u_+, we consider a control impulse

$$\delta u_{\tau_k}^{\mp} = u_-(\tau_{k-o}) + u_+(\tau_{k+o}). \tag{1.23}$$

The related additive functional at a vicinity of $t = \tau_k$ acquires the form of an impulse function

$$\varphi_s^{t-} + \varphi_s^{t+} = \delta\varphi_s^{\mp}. \tag{1.23a}$$

The entropy functional at the localities of the control's switching moments (1.5) takes the values

$$S_- = E[\varphi_s^{t-}] = \begin{cases} 0, t \le \tau_{k-o} \\ \infty, t > \tau_k \end{cases}, \; S_+ = E[\varphi_s^{t+}] = \begin{cases} \infty, t > \tau_k \\ 0, t \le \tau_{k+o} \end{cases}, \tag{1.24}$$

changing from 0 to ∞ and back from ∞ to 0 and acquiring an *absolute maximum* at $t > \tau_k$, between τ_{k-o} and τ_{k+o}.

The related multiplicative functionals (1.14a) are:

$$p_s^{t-}=\begin{cases}0, t\le\tau_{k-o}\\ 1, t>\tau_k\end{cases},\ \ p_s^{t+}=\begin{cases}1, t>\tau_k\\ 0, t\le\tau_{k+o}\end{cases}, \tag{1.24a}$$

which determine $\tilde{P}_{s,x}(d\omega)=0$ at $t\le\tau_{k-o}, t\le\tau_{k+o}$ and $\tilde{P}_{s,x}(d\omega)=P_{s,x}(d\omega)$ at $t>\tau_k$.

For the "cut-off" diffusion process, transitional probability (at $t\le\tau_{k-o}$, $t\le\tau_{k+o}$) turns to zero, then the states $\tilde{x}(\tau-o), \tilde{x}(\tau+o)$ become independent, and the mutual time correlations are dissolved:

$$r_{\tau-o,\tau+o}=E[\tilde{x}(\tau-o)\tilde{x}(\tau+o)]\to 0. \tag{1.24b}$$

The entropy $\delta S_-^+(\tau_k)$ of the additive functional $\delta\varphi_s^{\mp}$, produced within or at a border of the control impulse (1.23), is define by the equality

$$E[\varphi_s^{t-}+\varphi_s^{t+}]=E[\delta\varphi_s^{\mp}]=\int_{\tau_{k-o}}^{\tau_{k+o}}\delta\varphi_s^{\mp}P_\delta(d\omega), \tag{1.25}$$

where $P_\delta(d\omega)$ is a probability evaluation of impulse $\delta\varphi_s^{\mp}$.

Taking integral of the δ-function $\delta\varphi_s^{\mp}$ between the above time interval, we get on the border: $E[\delta\varphi_s^{\mp}]=1/2P_\delta(\tau_k)$ at $\tau_k=\tau_{k-o}$, or $\tau_k=\tau_{k+o}$ [70].

The impulse, produced by the controls, is a non random with $P_\delta(\tau_k)$ =1, which brings the EF estimation at $t=\tau_k$:

$$S_{\tau_k}^{\delta u}=E[\varphi_s^{\mp}]=1/2. \tag{1.26}$$

This entropy increment evaluates an information contribution from the controls (1.23) at a vicinity of the above discrete moments.

Since that, each information contribution from the step-wise functions

$$E[\varphi_s^{t-}]_{\tau_k}=S_{\tau_k}^{u_-},\ \ E[\varphi_s^{t+}]_{\tau_k}=S_{\tau_k}^{u_+}$$

at a vicinity of $t=\tau_k$, produced by the corresponding controls' step functions $u_-(\tau_k), u_+(\tau_k)$ (1.5) in (1.23) accordingly can be estimated by

$$S_{\tau_k}^{u_-}=1/4,\ u_-=u_-(\tau_k),\ \tau_{k-o}\to\tau_k; S_{\tau_k}^{u_+}=1/4,\ u_+=u_+(\tau_k),\ \tau_k\to\tau_{k+o}, \tag{1.26a}$$

where the entropy, according to its definition (1.15), is measured in the units of Nat (1 Nat ≅ 1.44bits).

Estimations (1.26), (1.26a) determine the entropy functional's cut-off values at the above time's borders under actions of these controls.

1.1.9. The Jensen's Inequality for the Entropy Functional

Let us have $g(t,x), g:\Delta \times R^n \to R^1$ as a measured and limited on $\Delta \times R^n$ function, convex down by the argument

$$x \in R^n, \forall (t,x^1) \in \Delta \times R^n, \exists \lambda_1(t,x^1), \lambda_1 : \Delta \times R^n \to R^n : \forall x \in R^n,$$

and consider the Jensen's inequality [25, 26] in a simple form:

$$g(t,x) \geq g(t,x^1) + (x - x^1)\lambda_1(t,x^1). \tag{1.27}$$

Proposition 1.1

Assume $x = \tilde{x}(t,\omega)$, and $\tilde{x}(t,\bullet)$ is measured function of argument $\omega \in \beta(C)$ at $\forall t \in \Delta$; and at a fixed $\omega \in \beta(C)$, $\tilde{x}(\bullet,\omega)$ is a continuous function $\tilde{x}_t$ on Δ.

According to [25], function $g(t,\omega) = g(t,\tilde{x}(t,\omega))$ is measured, limited, and therefore is summable by measure $\mu_\Delta = mes\Delta \times P_{s,\tilde{x}_s}$ on the set

$$\Delta \times \beta(C) : g(t,\omega) \in L_1(\Delta \times C, \beta(\Delta) \times \beta(C), \mu_\Delta), \tag{1.27a}$$

where

$$\beta(\Delta) = \{t : B^1 \cap \Delta, B^1 \subset \beta(R^1)\},$$

$mes\Delta$ is a Lebeg's measure on $\beta(\Delta)$, and $\times$ is the index of a direct multiplication of the sets of the β-algebra's, and the measures are finite.

Then the *Jensen's inequality* is in the form as follows

$$\int_s^T E_{\tilde{x}_s,\tilde{B}}[g(t,\tilde{x}_t]dt \geq \int_s^T g(t,\overline{x}_t)dt, \tag{1.28}$$

where $\overline{x}_t = E_{\tilde{x}_s,\tilde{B}}[\tilde{x}_t]$ is a macroprocess in (1.3).

For function $g(t,x)$, which is convex up by argument $x \in R^n$, we come to the *Jensen's inequality*

$$\int_s^T E_{\tilde{x}_s,\tilde{B}}[g(t,\tilde{x}_t]dt \leq \int_s^T g(t,\overline{x}_t)dt. \tag{1.28a}$$

Proof. Relation (1.27) and the Fubini theorem [25, 26] lead to equation

$$\int_s^T (\int_{\tilde{B}} g(t,\tilde{x}(t,\omega))P_{s,\tilde{x}_s}(d\omega))dt = \int_{\tilde{B}} (\int_s^T g(t,\tilde{x}(t,\omega))dt)\ P_{s,\tilde{x}_s}(d\omega),\ \tilde{B}\subset\beta(C),$$

which can be directly written via the conditional mathematical expectations:

$$\int_s^T E_{\tilde{x}_s,\tilde{B}}[g(t,\tilde{x}_t]dt = E_{\tilde{x}_s,\tilde{B}}[\int_s^T g(t,\tilde{x}_t)dt\ ]. \tag{1.28b}$$

Let us take $x=\tilde{x}(t,\omega)$, $x^1 = E_{\tilde{x}_s,\tilde{B}}[\,\tilde{x}_t\,]$ in (1.27), and integrate both sides of (1.27) by measure μ_Δ on the set $\Delta\times\tilde{B}$, $\tilde{B}\subset\beta$(C).

Then, applying (1.28b), we come to the *Jensen's inequality* in the form (1.28) (for function $g(t,x)$, which is convex down by argument $x\in R^n$).

For function $g(t,x)$, which is convex up by argument $x\in R^n$, we get by analogy the inequality (1.28a). •

Proposition 1.2.

Let us consider function $\hat{L}^u(t,x)$ on $\Delta\times R^n$:

$$\hat{L}^u(t,x) = 1/2\sum_{i,j=1}^n (2b(t,x))^{-1}_{ij}\hat{a}^u_i(t,x)\hat{a}^u_j(t,x), \tag{1.29}$$

where functions $b_{ij}(t,x),\hat{a}^u_i(t,x)$, $i,j=1,..,n$ are measured and limited on the set $\Delta\times R^n$.

And let function $x=\tilde{x}(t,\omega)$ in (1.29) holds the limitation for Proposition 1.1.

Because the class of the measured functions is closed regarding the arithmetic operations, function $\hat{L}^u(t,x)$ is limited.

This means that $\hat{L}^u(t,\omega)=\hat{L}^u(t,\tilde{x}\ (t,\omega))$ is a measured, limited, and therefore is a summable function by the measure μ_Δ on the set $\Delta\times\beta(C)$.

Assuming $g=\hat{L}^u$ and following the relations (1.14a),(1.9),(1.28), (1.28b),(1.29), (1.1-1.3) we get the Jensen inequalities for the *entropy functional convex down* by the arguments:

$$S(s,T,\tilde{x}(\bullet))\ge S(s,T,\overline{x}(\bullet)), \tag{1.29a}$$

and for the *entropy functional convex up* by the arguments:

$$S(s,T,\tilde{x}(\bullet))\le S(s,T,\overline{x}(\bullet)), \tag{1.29b}$$

where

$$S(s,T,\overline{x}(\bullet)) \overset{def}{=} \int_s^T L^u(t,\overline{x}_t)dt\,, \tag{1.30}$$

and the relation

$$E[1/2a^u(t,\tilde{x}_t)^T(2b(t,\tilde{x}_t))^{-1}a^u(t,\tilde{x}_t)] = E[\hat{L}^u(t,\tilde{x}_t)] = L^u(s,\overline{x}_t) \tag{1.30a}$$

plays a role of a Lagrangian L^u.

For the functionals with the arguments from (1.30a),(1.10) we obtain by the analogy accordingly:

$$S(s,T,\tilde{x}^*(\bullet)) \geq S(s,T,x(\bullet))\,, \tag{1.31a}$$

$$S(s,T,\tilde{x}^*(\bullet)) \leq S(s,T,x(\bullet))\,, \tag{1.31b}$$

where

$$S(s,T,x(\bullet)) \overset{def}{=} \int_s^T L^u(t,x_t)dt\,,\ L^u(t,x_t)=1/2(a^u)^T(2b)^{-1}a^u\,.\ \bullet \tag{1.32}$$

Comments 1.2.

Applying (1.28b) and (1.19) we get the following equality for the integrals

$$E_x[\int_s^T \tilde{L}dt] = \int_s^T E_x[\tilde{L}dt], \tilde{L} = 1/2\sum_{i,j=1}^n (2b(t,\tilde{x}_t^*))_{ij}^{-1}a_i^u(t,\tilde{x}_t^*)a_j^u(t,\tilde{x}_t^*), \tag{1.33}$$

$$2b = \sigma\sigma^T\,,\ a^u(t,\tilde{x}_t^*) = a^*(t,\tilde{x}_t^*,u_t)\,,\ u_t = u(t,x_t,\overline{\tilde{x}}_t^1)\,,$$

where functions

$$\overline{a}^u(t,x) = a(t,x,u)\,,\ a^u(t,\tilde{x}_t^*) = a^*(t,\tilde{x}_t^*,u_t)\,,$$

and $\sigma_{ij}(t,x)$, $a_i^u(t,x)$ are measured and limited on $\Delta \times R^n$ and $\Delta \times R^n \times U$ accordingly.

From this, using a closeness of the class of the measured and limited functions with respect to arithmetic operations [25], it follows that the function

$$L^u(t,x) = 1/2\sum_{i,j=1}^n (2b(t,x))_{ij}^{-1}a_i^u(t,x)a_j^u(t,x)\,, \tag{1.34}$$

which corresponds to $\tilde{L} = L^u(t,\tilde{x}_t^*)$ in (1.33), belongs to the same class of functions on $\Delta \times R^n \times U$.

Since $\tilde{x}_t^*$ is a function, measured on $\omega \in \Omega$ with P_x and is a continuous for $\forall t \in \Delta$, we conclude that the function $\tilde{L} = L^u(t, \tilde{x}_t^*)$ is also measured, limited, and therefore is a summable function by measure $mes\Delta \times P_x$, with the Lebeg measure $mes\Delta$ on $\Delta \times \Omega$.

Then using the Fubini theorem, we come to a possibility of the reordering for the integrals (1.33) on Δ and Ω.

Chapter 1.2

DYNAMIC APPROXIMATION OF A RANDOM INFORMATION FUNCTIONAL AND THE PATH FUNCTIONAL

1.2.1. The Extremal Principle and the Problem Formulation

We formulate the considered extremal principle as a probability problem of approximating the microlevel processes ($\tilde{x}_t$) by the macrolevel processes ($\bar{x}_t$) with an accuracy δ>0:

$$P_1 = P\{\rho_{L^2}(\tilde{x}_t, \bar{x}_t) < \delta\} \to \underset{\bar{x}_t}{Sup}, \tag{2.1}$$

where

$$\rho_{L^2}(\varphi, \psi) = \left(\int_0^T |\varphi - \psi|^2 \, dt\right)^{1/2}, \; |\varphi - \psi|^2 = \sum_{i=1}^{n} (\varphi_i - \psi_i)^2,$$

is a metrical distance between some functions $(\varphi, \psi) \in L^2$;

$$\bar{x}_t \overset{def}{=} \bar{x}_t(t, \bar{\eta}), \; \bar{x}_t \overset{def}{\in} KC^1(\Delta, R^n)(\mathrm{mod}\, P^1), \; u_t \in KC(\Delta, U), \tag{2.1a}$$

and $L^2(\Delta, R^n)$ is the Gilbert space; $KC^1(\Delta, R^n)$ and $KC(\Delta, U)$ are the spaces of the piece-wise differentiable, piece-wise continuous at $t \in \Delta$ functions in (R^n, U) accordingly, with ($\mathrm{mod}\, P^1$) and the probability P_1=1.

The probability problem of approximating the programmable process $\tilde{x}_t^1$ by the programmable macrotrajectories $\bar{x}_t^1$ has a form, analogous to (2.1):

$$P_2 = P\{\rho_{L^2}(\tilde{x}_t^1, \bar{x}_t^1) < \delta\} \to \underset{\bar{x}_t^1}{Sup}. \tag{2.2}$$

The probabilities, approximating other considered processes, satisfy the following requirements:

$$P_3 = P\{\rho_{L^2}(\tilde{x}_t, \tilde{x}_t^1) < \delta\} \to \underset{\bar{x}_t^1}{Sup}, \ \underline{m}(P_3) \to \underline{m}(P_4), \ P_4 = P\{\rho_{L^2}(\bar{x}_t^1 + \zeta_t, \bar{x}_t) < \delta\}, \quad (2.2a)$$

where $\underline{m}(P_i)$ is the lowest limit of the probabilities P_i, i=1– 4.

The corresponding probability equation for the terminate states: $P^1\{|\bar{x}_T - \bar{x}_T^1| > \varepsilon\}$, ε>0 can be joined to (2.1)– (2.2a) as an additional condition, depending on the requirements for the object.

Relation (2.2a) expresses the closeness of the object to the task at the macrolevel, and (2.1), (2.2) establish the relations between the deviations from the tasks for the micro- and macrolevel processes.

An essence of the probabilities' P_3 and P_4 nearness consists of connecting the micro- and macrolevel processes. With some limitations (see sec.1.2.2), the fulfillment of (2.2a) for P_3 leads to the following maximal conditions for probabilities

$$P_4 = P\{\rho_{L^2}(\tilde{x}_t^*, x_t) < \delta\} \to \underset{x_t}{Sup}, \quad (2.3)$$

$$P_5 = P\{\rho_{L^2}(\zeta_t, x_t) < \delta\} \to \underset{x_t}{Sup}. \quad (2.3a)$$

1.2.2. The Problem Solution. Information Path Functional

Definition. Trajectory φ_t passes a locality of trajectory ψ_t with a maximal probability if the lowest probability limit of their closeness reaches the maximum:

$$P\{\rho_\Delta(\psi_t, \varphi_t) < \delta\} \to \underset{\varphi_t}{Sup}, \ \forall\ \delta > 0, \ \varphi_t \in KC^1(\Delta, R^n), \ \psi_t \in C(\Delta, R^n) \quad (2.4)$$

and the upper probability limit of their distance reaches the minimum:

$$P\{\rho_\Delta(\psi_t, \varphi_t) \geq \delta\} \to \underset{\varphi_t}{Inf}, \ \forall\ \delta > 0, \ \rho_\Delta(\psi_t, \varphi_t) = \|\psi_t - \varphi_t\|_{(\bullet)} = \rho_{(\bullet)}(\psi_t, \varphi_t). \quad (2.4a)$$

Depending on the considered distance in the C- or L^2-metrics (at $\rho_\Delta = \rho_C$, or $\rho_\Delta = \rho_{L^2}$), the evaluation of (2.4) has a meaning of the C- or L^2-closeness, and the evaluation of (2.4a) has a meaning of the C- or L^2-distance.

The following relations determine the connections between the considered probabilistic evaluations:

$$\rho_{L^2}(\psi_t, \varphi_t) = [\int_s^T \sum_{i=1}^n (\psi_i(t) - \varphi_i(t))^2 dt]^{1/2} = [\int_s^T (\sum_{i=1}^n (\psi_i(t) - \varphi_i(t))^2)^{1/2}$$

$$\times (\sum_{i=1}^n (\psi_i(t) - \varphi_i(t))^2)^{1/2} dt]^{1/2}$$

$$\le [\int_s^T \max_{t\in\Delta} (\sum_{i=1}^n (\psi_i(t)-\varphi_i(t))^2)^{1/2} (\sum_{i=1}^n (\psi_i(t)-\varphi_i(t))^2)^{1/2} dt]^{1/2}$$

$$\le [\int_s^T \max_{t\in\Delta} (\sum_{i=1}^n (\psi_i(t)-\varphi_i(t))^2)^{1/2} (\sum_{i=1}^n (\psi_i(t)-\varphi_i(t))^2)^{1/2} dt]^{1/2}$$

$$= \max_{t\in\Delta} (\sum_{i=1}^n (\psi_i(t)-\varphi_i(t))^2)^{1/2} (T-s)^{1/2} = \rho_C(\psi_t,\varphi_t)(T-s)^{1/2};$$

$$\rho_C(\psi_t,\varphi_t) \ge \rho_{L^2}(\psi_t,\varphi_t)(T-s)^{-1/2},$$

$$\{\rho_C(\psi_t,\varphi_t) < \delta\} \subseteq \{\rho_{L^2}(\psi_t,\varphi_t)(T-s)^{-1/2} < \delta\},$$

$$P\{\rho_{L^2}(\psi_t,\varphi_t) < \delta\} \ge P\{\rho_C(\psi_t,\varphi_t)(T-s)^{1/2} < \delta\}, \tag{2.4b}$$

where (2.4a) evaluates how far a potential probabilistic deviation of φ_t is located from ψ_t.

Because the C-closeness is stronger than L^2-closeness, we consider the C-closeness for the evaluation of (2.4), and the L^2-distance for the evaluation of (2.4a).

Lemma 2.1. (L1)

The evaluation of the lowest probability limit of the closeness of the controlled process ($\tilde{x}_t(u)$) to the standard process (ζ_t) at the microlevel is reduced to the following evaluations, connecting the micro- and macrolevel processes to each other:

$$P_1 = P\{\rho_\Delta(\tilde{x}_t, \overline{x}_t) < \delta\} \to \underset{\overline{x}_t}{Sup}, \tag{2.5}$$

$$P_2 = P\{\rho_\Delta(\tilde{x}_t^1, \overline{x}_t^1) < \delta\} \to \underset{\overline{x}_t^1}{Sup}, \tag{2.5a}$$

$$P_3 = P\{\rho_\Delta(\tilde{x}_t, \tilde{x}_t^1) < \delta\} \to \underset{\overline{x}_t}{Sup}. \tag{2.5b}$$

Proof. Using (1.10), (1.13) (from ch.1.1) and following the triangle inequality [25], we come to relations

$$\rho_\Delta(\tilde{x}_t, \tilde{x}_t^1) = \rho_\Delta(\tilde{x}_t^*, \mathrm{O}) \le (\rho_\Delta(\tilde{x}_t^*, \zeta_t) + \rho_\Delta(\zeta_t, \mathrm{O})) \le (\rho_\Delta(\zeta_t, \mathrm{O}) + \rho_\Delta(\tilde{x}_t^*, x_t) + \rho_\Delta(x_t, \zeta_t));$$

$$\{\rho_\Delta(\tilde{x}_t^*, \mathrm{O}) < \delta\} \supseteq \{(\rho_\Delta(\zeta_t, \mathrm{O}) + \rho_\Delta(\tilde{x}_t^*, x_t) + \rho_\Delta(\zeta_t, x_t)) < \delta\}$$

$$\supseteq \{\rho_\Delta(\zeta_t, \mathrm{O}) < \delta/3\}\{\rho_\Delta(\tilde{x}_t^*, x_t) < \delta/3\}\{\rho_\Delta(\zeta_t, x_t) < \delta/3\};$$

$$P\{\rho_\Delta(\tilde{x}_t, \tilde{x}_t^1) < \delta\} = P\{\rho_\Delta(\tilde{x}_t^*, \mathrm{O}) < \delta\}$$

$$\le P\{\{\rho_\Delta(\zeta_t, \mathrm{O}) < \delta/3\}\{\rho_\Delta(\tilde{x}_t^*, x_t) < \delta/3\}\{\rho_\Delta(\zeta_t, x_t) < \delta/3\}\},$$

where

$P\{\rho_\Delta(\tilde{x}_t^*, \mathrm{O}) < \delta\}$ is the probability of the process' $\tilde{x}_t^*$ deviation from a null-vector O.

The events: $\{\rho_\Delta(\zeta_t, \mathrm{O}) < \delta/3\}, \{\rho_\Delta(\tilde{x}_t^*, x_t) < \delta/3\}$, and $\{\rho_\Delta(\zeta_t, x_t) < \delta/3\}$ assume independence, provided by the chosen (x_t, u_t, δ).

Then, because of an arbitrariness of $\delta > 0$ we get relations

$$P\{\rho_\Delta(\tilde{x}_t, \tilde{x}_t^1) < \delta\} = P\{\rho_\Delta(\tilde{x}_t^*, \mathrm{O}) < \delta\} \geq P\{\rho_\Delta(\zeta_t, \mathrm{O}) < \delta\} P_4 P_5,$$
$$P_4 = P\{\rho_\Delta(\tilde{x}_t^*, x_t) < \delta\}, P_5 = P\{\rho_\Delta(\zeta_t, x_t).$$

From this, we conclude that the following conditions are equivalent:

$$P_3 = P\{\rho_\Delta(\tilde{x}_t^*, \mathrm{O}) < \delta\} \to \sup_{x_t}, P_4 = P\{\rho_\Delta(\tilde{x}_t^*, x_t) < \delta\} \to \sup_{x_t},$$
$$P_5 = P\{\rho_\Delta(\zeta_t, x_t) < \delta\} \to \sup_{x_t}. \quad (2.5c)$$

This reduces the probabilities' conditions (2.1)-(2.3b) to (2.5)-(2.5b) and proves Lemma 2.1. •

Remark. Transformation $\bar{x}_t \to x_t$ in (1.10), at a fixed $\bar{x}_t^1$, is a linear transformation. Therefore, for any twice differentiable (by Frechet [25]) functional $F(\bullet)$, its extreme values $F(\bar{x}_t)$ and $F(x_t + \bar{x}_t^1)$ exist or not exist simultaneously.

Indeed, according to the principle of superposition, the first two differentials $dF(\bullet), d^2F(\bullet)$, defined on $\bar{x}_t$ and x_t are equal consequently:

$$dF(\bar{x}_t) = dF(x_t + \bar{x}_t^1), d^2F(\bar{x}_t) = d^2F(x_t + \bar{x}_t^1).$$

Because these two differentials determine the necessary and sufficient extreme conditions, both extremal problems for these functionals are equivalent, and from the existence or nonexistence of one of them follows that of the other. Probabilities P_3, P_4, P_5 are similar functionals (having the first and the second variations). This follows from the application of the Radon-Nikodim theorem for these probability's evaluations.

Lemma 2.2 (L2).
Let us *introduce* the functions

$$y_t = y(t,x): \Delta \times (R^n, \beta, \bar{P}_s) \to (R^n, \beta),\ y(\bullet, x) \in KC^1(\Delta, R^n), (\mathrm{mod}\, P^1),$$
$$y_s = y_s(\omega^1): (\Omega^1, \Psi^1, P^1) \to (R^n, \beta, \bar{P}_s);\ \omega^1 \in \Omega^1, \quad (2.6)$$

which satisfy the equation

$$\tilde{y}_t = \tilde{y}_s + \int_s^t q(\upsilon, \tilde{y}_\upsilon, u_\upsilon) d\ \upsilon + \int_s^t \sigma(\upsilon, \tilde{y}_\upsilon) d\ \zeta_\upsilon, \quad (2.6a)$$

where

$$\tilde{y}_s = \tilde{y}_s(\omega^1):(\Omega^1,\Psi^1,P^1)\to(R^n,\beta,\hat{P}_s),$$

$$\tilde{y}_t = \tilde{y}(t,\omega,x),\ \tilde{y}_t = \tilde{y}(t,\bullet,\bullet):(\Omega,\Psi,P_x)\to(C,\ \upsilon,\mu_x^3),\ \mu_x^3(C)=\mu^3(C)=1,$$

$$\tilde{y}(t,\bullet,x):(\Omega^{11},\Psi^{11},P)\to(C,\ \upsilon,\mu_x^3),\omega^{11}\in\Omega^{11};$$

and the function $q(t,x,u)$, $q:\Delta\times R^n\times \mathrm{U}\to R^n$, which satisfies the conditions analogous to (1.2) for $a^u=a^u(t,x)$.

Then the following probabilistic evaluations are fulfilled:

$$P\{\rho_\Delta(\tilde{y}_t,y_t)<\delta\}=P_x\{\rho_\Delta(\tilde{y}_t,y_t)<\delta\},\ y_s=\tilde{y}_s, \tag{2.7}$$

$$P\{\rho_\Delta(\tilde{y}_t,y_t)<\delta\}=\hat{P}_s^*\ P_x\{\rho_\Delta(\tilde{y}_t-y_t,\tilde{y}_s-y_s)<\delta\},\ y_s\neq\tilde{y}_s, \tag{2.7a}$$

$$\hat{P}_s^*=\hat{P}_s^*(D_\delta)\int\limits_{D_\delta}\Big(\int\limits_{R^n}\overline{P}_s(y)\hat{P}_s(x+y)dy\Big)dx;\ D_\delta\subset\beta,$$

$$D_\delta=K(\mathrm{O},\delta)=\{x\in R^n:\|x\|<\delta\}, \tag{2.8}$$

where $K(x,\delta)$ is a ball, opened in R^n with a center in a point $x\in R^n$:

$$K(x,\delta)=\{y\in R^n:\|y-x\|<\delta\},\ \text{at}\ \|y-x\|=\Big(\sum_{i=1}^n(y_i-x_i)^2\Big)^{1/2}, \tag{2.8a}$$

and $\hat{P}_s^*$ is the initial probability for process x_t^*, while $\hat{P}_s^*(D_\delta)$ is probability of set D_δ, defined by the ball.

Proof. Using the Markov property [22] for $y_s=\tilde{y}_s$, we come to relations

$$\begin{aligned}P\{\rho_\Delta(\tilde{y}_t,y_t)<\delta\}&=\int\limits_{K(y_s,\delta)}dP_s(\tilde{y}_s)P_{\tilde{y}_s}(\rho_\Delta(\tilde{y}_t,y_t)<\delta)\\&=\int\limits_{K(x,\delta)}\hat{P}_s(y)P_y\{\rho_\Delta(\tilde{y}_t,y_t)<\delta\}dy=\\&=\int\limits_{K(x,\delta)}\hat{\delta}(y-x)P_y\{\rho_\Delta(\tilde{y}_t,y_t)<\delta\}dy=P_x\{\rho_\Delta(\tilde{y}_t,y_t)<\delta\};(x,\ y\ )\in R^n,\end{aligned}$$

where $\hat{\delta}(x)$ is the n-dimensional delta-function.

For $y_s\neq\tilde{y}_s$, $\overline{P}_s\neq\hat{P}_s$, we apply the triangle inequality:

$$\begin{aligned}\rho_\Delta(\tilde{y}_t,y_t)&\le(\rho_\Delta(\tilde{y}_t,y_t+\tilde{y}_s-y_s)+\rho_\Delta(y_t,y_t+\tilde{y}_s-y_s))\\&=\rho_\Delta(\tilde{y}_t,y_t+\tilde{y}_s-y_s)+\rho_\Delta(\tilde{y}_s-y_s,\mathrm{O}),\ \mathrm{O}=(0_i)_{i=1}^n,\end{aligned}$$

$$\{\rho_\Delta(\tilde{y}_s - y_s, \mathrm{O}) = \|y - x\|_{R^n} = (\sum_{i=1}^{n} (\tilde{y}_{is} - y_{is})^2)^{1/2},$$

$$\{\rho_\Delta(\tilde{y}_t, y_t\} < \delta\} \supseteq \{(\rho_\Delta(\tilde{y}_t, y_t + \tilde{y}_s - y_s) + \rho_\Delta(\tilde{y}_s - y_s, \mathrm{O})) < \delta\}$$
$$\supseteq \{\{\rho_\Delta(\tilde{y}_t, y_t + \tilde{y}_s - y_s) < \delta/2\} \times \{\rho_\Delta(\tilde{y}_s - y_s, \mathrm{O}) < \delta/2\}\};$$

$$P\{\rho_\Delta(\tilde{y}_t, y_t\} < \delta\} \geq P\{\{\rho_\Delta(\tilde{y}_t, y_t + \tilde{y}_s - y_s) < \delta/2\} \times \{\rho_\Delta(\tilde{y}_s - y_s, \mathrm{O}) < \delta/2\}\};$$

The events and $\{\rho_\Delta(\tilde{y}_s - y_s, \mathrm{O}) < \delta/2\}$ are independent because the first one does not depend on β, and the second one has been defined on β.

Then, because δ>0 is chosen arbitrary, we get the following relation:

$$P\{\rho_\Delta(\tilde{y}_t, y_t\} < \delta\} \geq P^1(B'_\delta) P_x\{\rho_\Delta(\tilde{y}_t - y_t, \tilde{y}_s - y_s) < \delta\}; y_s \neq \tilde{y}_s,$$

where $B'_\delta = \{\omega^1 : \rho_\Delta(\tilde{y}_s - y_s, \mathrm{O}) < \delta\} \subset \Psi^1$.

According to [22] we have equality $P^1(B'_\delta) = \hat{P}^*_s(D_\delta)$, where the last probability satisfies (2.8). Finally we get (2.7). •

The lowest probability evaluations (2.7),(2.7a) distinguish only by the multiplier, responsible for the probabilistic closeness of the initial conditions.

We need to evaluate the right-hand side of (2.7), (2.7a).

Theorem 2.1.

The probability of the evaluation of a closeness φ_t to ξ_t is defined by relations

$$P\{\rho_\Delta(\xi_t, \varphi_t) < \delta\} = P_{x=0}\{\rho_\Delta(\xi_t, \varphi_t) < \delta\} = P_0\{\rho_\Delta(\xi_t, \varphi_t) < \delta\}$$
$$\geq P_0(B_\delta)\, \varepsilon \exp\{-[(S(\varphi_t) + (2S(\varphi_t)(1-\varepsilon)^{-1})^{1/2}]\},$$
$$\varphi_t \in KC^1(\Delta, R^n), \xi_t \in \mathrm{C}(\Delta, R^n), \tag{2.9}$$

$$S(\varphi_t) = 1/2 \int_s^T \dot{\varphi}_t^T \dot{\varphi}_t \, dt = 1/2 \int_s^T |\dot{\varphi}_t|^2 \, dt, \; \varphi_s = 0, \; \xi_s = 0, \tag{2.10}$$

$$|\dot{\varphi}_t|^2 = \sum_{i=1}^{n} \dot{\varphi}_i^2(t), \; \varepsilon \in (0,1), \; B_\delta = \{\omega : \rho_\Delta(\xi_t, \mathrm{O}) < \delta\} \subset \Psi.$$

Proof. Let $\tilde{\varphi}_t = -\varphi_t + \xi_t$ and assume that the measures (μ_0, μ_0^4) of the corresponding functions $(\tilde{\varphi}_t, \xi_s)$ on (C, U) are absolutely continuous with respect each to other.

Then, according to (2.7), we get the following relations

$$P_0\{\rho_\Delta(\varphi_t, \xi_t) < \delta\} = P_{x=0}\{\rho_\Delta(\varphi_t, \xi_t) < \delta\} = P_{x=0}\{\rho_\Delta(\tilde{\varphi}_t, \mathrm{O}) < \delta\} = P_0\{\rho_\Delta(\tilde{\varphi}_t, \mathrm{O}) < \delta\}.$$

Using the Radon-Nikodim theorem [22] and the relation for the density measures [17], we come to the equality for the considered probability

$$P_0\{\rho_\Delta(\varphi_t,\mathrm{O})<\delta\}=\int\limits_{B_\delta}\frac{d\mu_0^4}{d\mu_0}(\xi(t,\bullet))P_0(d\omega)=\int\limits_{B_\delta}\exp[-(S(\varphi_t)+\int\limits_s^T\dot{\varphi}_t\bullet d\xi_t)]P_0(d\omega).$$

The last relation equals to the following expression

$$\exp[-S(\varphi_t)]\int\limits_{B_\delta}\exp[-\sum_{i=1}^n\int\limits_s^T\dot{\varphi}_i(t)d\xi_i(t,\omega)]P_0(d\omega),S(\varphi_t)=1/2\int\limits_s^T|\dot{\varphi}_t|^2dt, \qquad (2.11)$$

where $B_\delta=\{\omega:\rho_\Delta(\xi_s,\mathrm{O})<\delta\}\subset\Psi$, and μ_o^4, μ_0 are the measures for $\tilde{\varphi}_t,\xi_t$ on $(C,\mathcal{U})$.

We evaluate the second co-multiplier in (2.11) by Chebyshev's inequality [22] in the form

$$P_0\{\eta(\omega)\le a\}\le E_0[f(\eta)]/f(a), \qquad (2.12)$$

where $\eta(\omega)$ is a nonnegative random variable,

$$\omega\in\Omega,\ a\in R_+^1, E_0[\bullet]=E_{x=0}[\bullet]=\int\limits_\Omega[\bullet]\,P_0(d\omega),$$

$f(s)$ is a monotonous increasing on R_+^1 function, s $\in R_+^1$.

Let us assume

$$\eta(\omega)\overset{def}{=}\exp[-\int\limits_s^T(\dot{\varphi}_t\bullet d\xi_t)]\lambda(B_\delta),\ a=\exp[-(2S(\varphi_t))^{1/2}(1-\varepsilon)^{-1/2}],$$

$$f(\eta)=\eta\,\lambda(B_\delta),\lambda(B_\delta)=\begin{cases}1,\omega\in B_\delta\\0,\omega\notin B_\delta\end{cases}. \qquad (2.13)$$

Remark. Using relations

$$P_o\{\rho_\Delta(\tilde{\varphi}_t,0)<\delta\}\overset{def}{=}\int\limits_{B_\delta^{'}}P_o^4(d\omega),B_\delta^{'}=\{\rho_\Delta(\tilde{\varphi}_t,0)<\delta\},$$

$$\mu_o^4(A)=P_o^4\{\omega:\tilde{\varphi}_t(\bullet,\omega)\in A\},A\subset\Psi, \qquad (2.13a)$$

and the formula of changing the measure in integrals [22], we get the following representation of (2.13a)

$$\int\limits_{B_\delta^{'}}P_0^4(d\omega)=\int\limits_\Omega\lambda(B_\delta^{'})P_0^4(d\omega)=\int\limits_\Omega\lambda(B_\delta^{'})\frac{d\mu_0^4}{d\mu_0}(\tilde{\varphi}_t(\bullet,\omega)).\bullet$$

Assuming (2.13), we have $f(a)=a$, and according to relations (2.12), we come to

$$\int\limits_{B_\delta^{'}}\eta(\omega)\,P_0(d\omega)\ge aP_0\{\lambda(B_\delta)\eta(\omega)\ge a\},$$

$$\{\lambda(B_\delta)\ \eta(\omega)\ge a\}\overset{def}{=}\{\omega:\omega\ \in B_\delta,\ \eta(\omega)\ge a\}$$
$$=\{\omega:\omega\ \in B_\delta\}\cap\{\omega:\eta(\omega)\ge a\},$$
$$P_0\{\lambda(B_\delta)\ \eta(\omega)\ge a\}\overset{def}{=}P_0(B_\delta)P_0\{\eta(\omega)\ge a\,|\,B_\delta\},\qquad(2.13b)$$

where $P_0\{\eta(\omega)\ge a\,|\,B_\delta\}$ is the conditional probability of the event $\eta(\omega)\ge a$ at the condition for B_δ (the independence of events $\{\omega:\omega\ \in B_\delta\}$ and $\{\omega:\eta(\omega)\ge a\}$ is not supposed).

Then, taking into account the relation (2.13), we have:

$$\int_{B_\delta}\exp[-\int_s^T\dot\varphi_t(t)d\zeta_t\,]P_0d\omega\ge P_0(B_\delta)\exp[-(2S(\varphi_t))^{1/2}(1-\varepsilon)^{-1/2}]$$
$$\times P_0\{\exp[-\int_s^T\dot\varphi_t(t)d\xi_t]\ge\exp[-(2S(\varphi_t))^{1/2}(1-\varepsilon)^{-1/2}]\}\,|\,B_\delta\}.\qquad(2.14)$$

Since $\forall B\subset\Psi, P_0(B\,|\,B_\delta)=1-P_0(\overline{B}\,|\,B_\delta),\overline{B}=\Omega\setminus B,$
we assume

$$B=\{\exp[-\int_s^T\dot\varphi_t(t)d\xi_t]\ge\exp[-(2S(\varphi_t))^{1/2}(1-\varepsilon)^{-1/2}]\}$$
$$=\{-\int_s^T\dot\varphi_t(t)d\xi_t\ge-(2S(\varphi_t))^{1/2}(1-\varepsilon)^{-1/2}\}.$$

After that, we obtain the relations

$$\overline{B}=\{\exp[-\int_s^T\dot\varphi_t(t)d\xi_t]\le\exp[-(2S(\varphi_t))^{1/2}(1-\varepsilon)^{-1/2}]\}$$
$$=\{-\int_s^T\dot\varphi_t(t)d\xi_t\le-(2S(\varphi_t))^{1/2}(1-\varepsilon)^{-1/2}\};$$
$$P_0(B\,|\,B_\delta)=1-P_0\{[-\int_s^T\dot\varphi_t(t)d\zeta_t]\le[-(2S(\varphi_t))^{1/2}(1-\varepsilon)^{-1/2}]\,|\,B_\delta.\qquad(2.15)$$

For the evaluation of $P_0(\overline{B}\,|\,B_\delta)$ we are using a generalized Kolmogorov's inequality for the martingals [25], and also the peculiarities of the stochastic integral [22]:

$$P_0(\overline{B}\,|\,B_\delta)\le P_0\{[|\int_s^T\dot\varphi_t(t)d\zeta_t\,|]\ge[-(2S(\varphi_t))^{1/2}(1-\varepsilon)^{-1/2}]\,|\,B_\delta$$
$$\le(1-\varepsilon)(2S(\varphi_t))^{-1}E_0\{[\int_s^T\dot\varphi_t(t)d\zeta_t]^2\,|\,B_\delta\}$$
$$=(1-\varepsilon)(S(\varphi_t))^{-1})E_0\{1/2[\int_s^T|\dot\varphi_t(t)|^2\,dt\,|B_\delta\}=(1-\varepsilon);E\{1\,|\,B_\delta\}=(1-\varepsilon).\qquad(2.16)$$

From that and using equations (2.11), (2.15) we obtain

$$P_0\{\exp[-\int_s^T \dot{\varphi}_t(t)d\zeta_t] \geq \exp[-(2S(\varphi_t))^{1/2})(1-\varepsilon)^{-1/2}]\} \geq \varepsilon, \tag{2.17}$$

$$P_0\{\rho_\Delta(\tilde{\varphi}_t, \mathrm{O}) < \delta\} \geq P_0(B_\delta)\varepsilon \exp\{-[S(\varphi_t) + (2S(\varphi_t)(1-\varepsilon)^{-1})^{1/2}]\}, \tag{2.17a}$$

and by the substitutions of the obtained relations we come to (2.9), (2.10). •

Using equations (2.9), (2.10) for the evaluation of the lowest probabilities limit (2.6), (2.7), we have

$$P_x\{\rho_\Delta(y_t, \tilde{y}_t) < \delta\} \geq P_0\{\rho_\Delta(Q_x y_t, \xi_t) < \delta\}, \varphi_t = Q_x y_t, \tag{2.18a}$$

where Q_x is a sought operator depending on $x \in R^n$ at $Q_x: KC^1(\Delta, R^n) \to KC^1(\Delta, R^n)$.

Assume that we might construct an operator (or a family of operators):

$$G_x : C(\Delta, R^n) \to C(\Delta, R^n), G_x : KC^1(\Delta, R^n) \to KC^1(\Delta, R^n), \tag{2.18b}$$

which reflects ξ_t on $\tilde{y}_t$ (one-to-one in a probabilistic meaning) and φ_t on y_t (one-to-one in a regular meaning) accordingly and satisfies the following relations

$$G_x : \xi_t \to \tilde{y}_t, P_x\{G_x \xi_t = \tilde{y}_t\} = 1, \tag{2.18}$$

$$G_x : \varphi_t \to y_t, , G_x \varphi_t = y_t, \; G_x^{-1} x : y_t \to \varphi_t, \; G_x^{-1} y_t = \varphi_t, \tag{2.19}$$

where $G_x^{-1} = Q_x$ is an inverse operator on $KC^1(\Delta, R^n)$ at $(G_x^{-1} y_t)_{t=s} = \varphi_s$.

Then, the following proposition holds true:

Lemma 2.3.
If the transformation (2.18) exists, then the lowest probabilities limit (2.7), (2.7a) can be evaluated by the relations

$$P\{\rho_\Delta(y_t, \tilde{y}_t) < \delta\} \geq P_0(B_\delta)\varepsilon \exp\{-S(y_t) + [2S(y_t)(1-\varepsilon_1)^{-1}]^{1/2}\}, y_s = \tilde{y}_s; \tag{2.20}$$

$$P\{\rho_\Delta(y_t, \tilde{y}_t) < \delta\} \geq P_s^*(D_\delta)\varepsilon \exp\{-S(y_t) + [2S(y_t)(1-\varepsilon_1)^{-1}]^{1/2}\}, y_s \neq \tilde{y}_s; \tag{2.21}$$

$$S(y_t) = 1/2 \int_s^T \frac{d}{dt}(G_x^{-1} y_t)^T \frac{d}{dt}(G_x^{-1} y_t) dt. \tag{2.22}$$

Proof. Using relations (2.18), (2.19) we may write (2.7) in the form

$$P_x\{\rho_\Delta(\tilde{y}_t, y_t\} < \delta\} = P_x\{\rho_\Delta(G_x \varphi_t, G_x \xi_t) < \delta\},$$

$$\rho_\Delta(G_x \varphi_t, G_x \xi_t) \leq \| G_x \|_c \; \rho_\Delta(\varphi_t, \xi_t) = \| G_x \|_c \; \rho_\Delta(G_x^{-1} y_t, \xi_t),$$

$$\{\rho_\Delta(G_x \varphi_t, G_x \xi_t) < \delta\} \subseteq \{\| G_x \|_c \; \rho_\Delta(G_x^{-1} y_t, \xi_t) < \delta\} = \{\rho_\Delta(G_x^{-1} y_t, \xi_t) < \delta\} / \| G_x \|_c\},$$

where $\| G_x \|_c$ is the norm of G_x in a subspace of $C = C(\Delta, R^n)$.

Because δ>0 is chosen arbitrary, we have

$$\{\rho_\Delta(G_x^{-1} y_t, \xi_t) < \delta\} / \| G_x \|_c\} \subset \{\rho_\Delta(G_x^{-1} y_t, \xi_t) < \delta\}, \tag{2.23a}$$

$$P_x\{\rho_\Delta(\tilde{y}_t, y_t\} < \delta\} \geq P_x\{\rho_\Delta(G_x^{-1} y_t, \xi_t) < \delta\} = P_0\{\rho_\Delta(G_x^{-1} y_t, \xi_t) < \delta\}. \tag{2.23}$$

Applying relations (2.9), (2.10) (for the evaluation of the right-hand side of equation (2.23)), and taking into account relations (2.7)-(2.8), we come to (2.20)-(2.22) •.

Because the *C*-closeness is stronger than L^2-closeness, the obtained lowest probability limits (2.20), (2.22) are also satisfied for the evaluation of L^2-closeness.

Lemma 2.4.

The operator, created by the solution of (2.6a), satisfies relations (2.18), (2.19), and function (2.22) has a view:

$$S(y_t) = 1/2 \int_s^T (\dot{y}_t - q(t, y_t, u_t))^T (2b(t, y_t))^{-1} (\dot{y}_t - q(t, y_t, u_t)) dt, 2b = \sigma\sigma^T. \tag{2.24}$$

Proof. It is natural to choose G_x as an operator created by solution (2.6a), which is continuous with probability 1, exists, and is unique.

Then G_x reflects $C(\Delta, R^n)$ on itself with probability 1, and relation (2.18) is fulfilled (because any two solutions of (2.6a) at the same initial conditions coincide with probability 1).

Operator G_x on subspace $C(\Delta, R^n)$ of space $KC^1(\Delta, R^n)$ defines a reflection $\varphi_t \in KC^1$ to $y_t \in KC^1$ as a solution of the Volterra second order integral equation [24]:

$$y_t = y_s + \int_s^t [q(\upsilon, y_\upsilon, u_\upsilon) + \sigma(\upsilon, y_\upsilon)\dot{\varphi}_\upsilon] d\upsilon, \tag{2.25}$$

which exists and is unique on KC^1 at the limitations (ch.1.1) for the considered functions of drift and diffusion; G_x has an inverse operator on KC^1, its explicit form follows from (2.25):

$$G_x^{-1} y_t = \varphi_t = \int_s^t (\sigma(\upsilon, y_\upsilon))^{-1} (\dot{y}_\upsilon - q(t, y_\upsilon, u_\upsilon)) d\upsilon. \tag{2.26}$$

Therefore, the relations (2.19) are satisfied for the initial object's stochastic equation, and from (2.26), (2.22), and (2.24) follows (2.25). •

Assuming the sequential fulfillment of

$$y_t = \overline{x}_t, \tilde{y}_t = \tilde{x}_t; y_t = \overline{x}_t^1, \tilde{y}_t = \tilde{x}_t^1; y_t = x_t, \ \tilde{y}_t = \overline{x}_t^1 + \zeta_t; \ y_t = x_t, \tilde{y}_t = \tilde{x}_t^*,$$

we obtain from relations (2.20), (2.21), (2.22) the following estimators for (2.5),(2.5a),(2.3), (2.3a):

$$P_i \geq P_0(B_\delta)\varepsilon \exp[(-S_i + (2S_i(1-\varepsilon)^{-1})]^{1/2}],\ S_i = \int_s^T L_i dt\ ,\ i = 1,2,5\ , \qquad (2.27)$$

$$L_1 = 1/2(\dot{\bar{x}} - \bar{a}^u(t,\bar{x}))^T (2b(t,\bar{x}))^{-1}(\dot{\bar{x}} - a^u(t,\bar{x}))\ ,$$

$$L_2 = 1/2(\dot{\bar{x}}^1 - a^1(t,\bar{x}^1))^T (2b(t,\bar{x}^1))^{-1}(\dot{\bar{x}}^1 - a^1(t,\bar{x}^1))\ , \qquad (2.27a)$$

$$L_5 = 1/2(\dot{x} - a^u(t,x))^T (2b(t,x))^{-1}(\dot{x} - a^u(t,x))\ , \qquad (2.27b)$$

and for

$$P_4 \geq P_s^*(D_\delta)P_0(B_\delta)\varepsilon \exp[(-S_4 + (2S_4(1-\varepsilon)^{-1})]^{1/2}], \qquad (2.28)$$

we get

$$S_4 = \int_s^T L_4 dt,\ L_4 = 1/2\,\dot{x}^T (2b(t,x))^{-1}\dot{x}\ . \qquad (2.28a)$$

Theorem 2.2.

The lowest probability limit of the evaluation of probability (2.5b) is defined by relations:

$$P_3 \geq P_s^*(D_\delta)P_0(B_\delta)\varepsilon \exp[(-S_3 + (2S_3(1-\varepsilon)^{-1})]^{1/2}], \qquad (2.29)$$

$$S_3 = E_x[\int_s^T \tilde{L}_3\ dt] = \int_s^T E_x[L_3]\,dt,\ \tilde{L}_3 = 1/2\, a^u(t,\tilde{x})^T\,(2b(t,\tilde{x}))^{-1}\,a^u(t,\tilde{x})\ ,\ 2b = \sigma\sigma^T\ , \qquad (2.30)$$

where equation (2.30) coincides with the conditional entropy of the processes $\tilde{x}_t$* regarding ζ_t (or with the controlled processes' entropy, defined with respect to a standard process by the transformation $\tilde{x}_t \to \zeta_t$):

$$S(\tilde{x}_t^*/\zeta_t) = E_x\{[\ln[\frac{d\mu_0^2}{d\mu_x^*}(\tilde{x}_s(t,\bullet,\bullet))]^{-1}\} = S_3\ . \qquad (2.31)$$

Proof. By analogy with the relations for Lemma 2.1, we may write

$$P_3 = P\{\rho_\Delta(\tilde{x}_t,\tilde{x}_t^1) < \delta\} = P\{\rho_\Delta\ (\tilde{x}_t^*, O) < \delta\} \geq P_s^*(D_\delta)P_x\{\rho_\Delta(\tilde{x}_t^* - \tilde{x}_s^*), O) < \delta\}, \qquad (2.32)$$

where $\tilde{x}_t^* = \tilde{x}_t - \tilde{x}_t^1$.

Because the processes' $(\tilde{x}_t^*, \zeta_t)$ measures μ_x^* and μ_0^2 are absolutely continuous, one related to other, we may apply the Radon-Nikodim theorem to the last multiplier in (2.32):

$$P_x\{\rho_\Delta(\tilde{x}_t^* - \tilde{x}_s^*, O) < \delta\} = \int_{B_{2\delta}} \frac{d\mu_0^2}{d\mu_x^*}(\tilde{x}^*(t,\bullet,\bullet))\ P_x(d\,\omega);\ B_{2\delta} = \{\omega : \rho_\Delta(\zeta_t, O) < \delta\}. \qquad (2.33)$$

According to [22, 25] and the previous results, we come to the following relations:

$$\frac{d\mu_0^2}{d\mu_x^*} = \exp[-(\tilde{S}_3(\tilde{x}_t^*) + \int_s^t (\sigma(t,\tilde{x}_t^*))^{-1} a^u(t,\tilde{x}_t^*) d\xi_t)], \tag{2.34}$$

$$\tilde{S}_3 = 1/2 \int_s^T a^u(t,\tilde{x}_t^*)^T (2b(t,\tilde{x}_t^*))^{-1} a^u(t,\tilde{x}_t^*)\, dt\,, 2b = \sigma\sigma^T, \tag{2.35}$$

For the evaluation of (2.33), (2.34) we use relations (2.12), (2.13) by exchanging the symbols P_0 and E_0 with P_x and E_x accordingly and assuming

$$\eta(\omega) = \frac{d\mu_0^2}{d\mu_x^*}(\tilde{x}^*(t,\bullet,\bullet))\,,\ a = \exp[-(S_3(\tilde{x}_t^*) + (2S_3(\tilde{x}_t^*)(1-\varepsilon)^{-1}))^{1/2}],$$
$$S_3 = E_x[\tilde{S}_3], f(\eta) = \eta\,. \tag{2.36}$$

Then by analogy with (2.14) we obtain for (2.36) the following inequalities

$$P_x\{\rho_\Delta(\tilde{x}_t^* - \tilde{x}_s^*, \mathrm{O}) < \delta\} \geq P_x(B_{2\delta}) \exp[-(S_3(\tilde{x}_t^*) + (2S_3(\tilde{x}_t^*)(1-\varepsilon)^{-1}))^{1/2}]$$
$$\times P_x \exp[-(\tilde{S}_3(\tilde{x}_t^*) + \int_s^t (\sigma(t,\tilde{x}_t^*))^{-1} a^u(t,\tilde{x}_t^*) d\xi_t)] \geq \exp[-(S_3(\tilde{x}_t^*) + (2S_3(\tilde{x}_t^*)(1-\varepsilon)^{-1}))^{1/2}]. \tag{2.37}$$

For the evaluation of the first co-multiplier in (2.37) we are using equalities:

$$P_x(B_{2\delta}) = P_x\{\rho_\Delta(\zeta_t, \mathrm{O}) < \delta\} = P\{\rho_\Delta(\zeta_t, \mathrm{O}) < \delta\}. \tag{2.38}$$

Suppose relation $G_{x=0} = G_0$ is created by the transformation $G_0\ \xi_t = \zeta_t$.

Then, because of $G_0\ \mathrm{O} = \mathrm{O}$ and an arbitrary $\delta > 0$, we get the following relations

$$\rho_\Delta(\zeta_t, \mathrm{O}) = \rho_\Delta(G_0\zeta_t, G_0\mathrm{O}) \geq \| G_0 \|_c\ \rho_\Delta(\xi_t, \mathrm{O}),$$
$$\{\rho_\Delta(\xi_t, \mathrm{O}) < \delta\} \subseteq \{\| G_0 \|_c\ \rho_\Delta(\xi_t, \mathrm{O}) < \delta\}$$
$$= \{\rho_\Delta(\xi_t, \mathrm{O}) < \delta / \| G_0 \|_c\} \supseteq \{\rho_\Delta(\xi_t, \mathrm{O}\}, \tag{2.38a}$$

$$P(B_{2\delta}) = P\{\rho_\Delta(\zeta_t, \mathrm{O}) < \delta\} = P_0\{\rho_\Delta(\zeta_t, \mathrm{O}) < \delta\} = P_0(B_\delta). \tag{2.39}$$

For the evaluation of the last co-multiplier in (2.37) we apply the following inequalities

$$D = \{[-(\tilde{S}_3(\tilde{x}_t^*) + \int_s^T (\sigma(t,\tilde{x}_t^*))^{-1} a^u(t,\tilde{x}_t^*) d\xi_t)]$$
$$\geq \exp[-(S_3(\tilde{x}_t^*) + (2S_3(\tilde{x}_t^*)(1-\varepsilon)^{-1}))^{1/2}]\}$$

$$= \{-[(\tilde{S}_3(\tilde{x}_t^*) + \int_s^T (\sigma(t,\tilde{x}_t^*))^{-1} a^u(t,\tilde{x}_t^*) d\xi_t)] \geq -[S_3(\tilde{x}_t^*) + (2S_3(\tilde{x}_t^*)(1-\varepsilon)^{-1})^{1/2}]\}$$

$$\supseteq \{-(\tilde{S}_3(\tilde{x}_t^*) \geq S_3(\tilde{x}_t^*)\} \cap \{(-\int_s^T (\sigma(t,\tilde{x}_t^*))^{-1} a^u(t,\tilde{x}_t^*) d\xi_t))\}$$

$$\geq -[2S_3(\tilde{x}_t^*)(1-\varepsilon)^{-1})]^{1/2}\} = AB, \tag{2.40}$$

using them for

$$P_x(D) \geq P_x(AB) = 1 - P_x(\overline{AB}), \tag{2.41}$$

where $(A,B,D) \subset \Psi$, $\overline{AB} = \Omega \setminus AB$, $AB = \{\omega : \omega \in A\} \cap \{\omega : \omega \in B\}$.
Applying the duality principle in the set theory [25] to the events:

$$\overline{A} = \{-(\tilde{S}_3(\tilde{x}_t^*) \geq S_3(\tilde{x}_t^*)\},$$

$$\overline{B} = \{(-\int_s^T (\sigma(t,\tilde{x}_t^*))^{-1} a^u(t,\tilde{x}_t^*) d\xi_t)) \leq -[2S_3(\tilde{x}_t^*)(1-\varepsilon)^{-1}]^{1/2}\} \tag{2.42}$$

in the forms

$$\overline{AB} = \overline{\{\omega \in A\} \cap \{\omega \in B\}} = \overline{\{\omega \in A\}} \cup \overline{\{\omega \in B\}} = \overline{A} + \overline{B}\ , \tag{2.43}$$

$$P_x(\overline{A} + \overline{B}) = P_x(\overline{A}) + P_x(\overline{B}) - P_x(\overline{A}\overline{B}), \tag{2.43a}$$

we get from (2.41),(2.43) and (2.43a) the inequality

$$(P_x(D) \geq 1 - [P_x(\overline{A} + \overline{B}) = P_x(\overline{A}) + P_x(\overline{B}) - P_x(\overline{A}\overline{B})].$$

Using the initial equations (1.11) and (2.35), (2.42), we obtain

$$\int_s^T (\sigma(t,\tilde{x}_t^*))^{-1} d\xi_t$$

$$= \int_s^T (\sigma(t,\tilde{x}_t^*))^{-1} a^u(t,\tilde{x}_t^*)) [(\sigma(t,\tilde{x}_t^*))^{-1} d\tilde{y}_t - (\sigma(t,\tilde{x}_t^*))^{-1} a^u(t,\tilde{x}_t^*) dt]$$

$$= \int_s^T (2b(t,\tilde{x}_t^*))^{-1} a^u(t,\tilde{x}_t^*)\ d\tilde{y}_t - 2\tilde{S}_3(\tilde{x}_t^*);$$

$\sigma(\cdot\ ,x)$, $a^u = a^u(\cdot,x), b = b(\cdot,x) = 1/2\,\sigma(\cdot,x)\,\sigma^T(\cdot,x)$,

$$\overline{B} = \{(-\int_s^T (2b(t,\tilde{x}_t^*))^{-1} a^u(t,\tilde{x}_t^*) d\tilde{y}_t + 2S_3(\tilde{x}_t^*)) \leq -[2S_3(\tilde{x}_t^*)(1-\varepsilon)^{-1}]^{1/2}\}$$

$$\{\overline{N} \mid \overline{A}\} \supseteq \overline{A}, \tag{2.45}$$

where

$$\bar{N} = \{(-\int_s^T (2b(t,\tilde{x}_t^*))^{-1} a^u(t,\tilde{x}_t^*) d\tilde{y}_t \le -(2S_3(\tilde{x}_t^*)) + [2S_3(\tilde{x}_t^*) = (1-\varepsilon)^{-1}]^{1/2}\},$$

$$\bar{N} \subset \Psi \ . \tag{2.45a}$$

From that we get the following relations

$$\bar{A}\bar{B} = \{\omega \in \bar{A}\} \cap \{\omega \in \bar{B}\} = \{\omega \in \bar{A}\} = \bar{A}, P_x(D) = 1 - P_x(\bar{B}). \tag{2.46}$$

For the evaluation of an upper probability $P_x(\bar{B})$ limit we are using (2.43a), (2.36), (2.35) and Chebychev's inequality:

$$P_x(\bar{B}) \le P_x\{|\int_s^T (\sigma(t,\tilde{x}_t^*))^{-1} a^u(t,\tilde{x}_t^*)\, d\xi_t| \ge [2S_3(\tilde{x}_t^*)(1-\varepsilon)^{-1}]^{1/2}\}$$

$$\le (1-\varepsilon)\ E_x[|\int_s^T (\sigma(t,\tilde{x}_t^*))^{-1} a^u(t,\tilde{x}_t^*)\, d\zeta_t|^2](2S_3(\tilde{x}_t^*))^{-1}$$

$$= (1-\varepsilon)\ E_x[|\int_s^T |(\sigma(t,\tilde{x}_t^*))^{-1} a^u(t,\tilde{x}_t^*)|^2\, dt\,](2S_3(\tilde{x}_t^*))^{-1} = (1-\varepsilon). \tag{2.47}$$

After the substitution of (2.47) in (2.46) we obtain $P_x(\mathrm{D}) \ge \varepsilon$.

From that and according to (2.32)-(2.35), (2.38)-(2.40) we get (2.29)-(2.30).

Functional (2.30) coincides with the definition of the conditional entropy (2.31) in [17] and directly follows from (1.15c) (in sec. (1.1.8)).

Finally, using (2.34), (2.35) and

$$E_x[\int_s^T ((\sigma(t,\tilde{x}_t^*)^{-1} a^u(t,\tilde{x}_t^*)\, d\zeta_t)] = 0,$$

we get $S(\tilde{x}_t^* / \zeta_t) = S_3, .\bullet$

Theorem 2.3.

If transformations (2.18), (2.19) satisfy relation (2.23), then the absolute minimum of the upper limit of the probabilistic evaluation in the L^2-distance:

$$P\{\rho_{L^2}(\tilde{y}_t, y_t) \ge \delta\} \to \underset{y_t}{Inf}\ ,\ \forall\ \delta > 0, \tag{2.48}$$

is reached on the solution of equation

$$\dot{y}_t = q(t, y, u)\ . \tag{2.49}$$

Proof. To evaluate relation (2.48) we apply inequality (2.12) and transformations (2.18), (2.19):

$$P\{\rho_{L^2}(\tilde{y}_t, y_t) \geq \delta\} \leq 1/\delta^2 E[\rho^2_{L^2}(\tilde{y}_t, y_t)] = 1/\delta^2 E[\rho^2_{L^2}(G_y\,\zeta_t, G_y\,\varphi_t)]$$
$$\leq 1/\delta^2\; E_s[K_y^2]\, E_0[\rho^2_{L^2}(\zeta_t, \varphi_t)],$$
$$t \in \Delta = (s,T); \delta > 0,\; G_y = G_{y=0},\; y_s = \tilde{y}_s = 0, \tag{2.50}$$

where K_y is the Lipschitz constant for operator G_y,which, at the limitation imposed on the stochastic equation, can be expressed via the Lipschitz constants for q, σ, and

$$(T-s) = mes(\Delta).$$

Here $E_0[\bullet] = E_{y=0}[\bullet]$ is operator of mathematical expectation, corresponding to probability measure P_0 on Ψ; $E_s[\bullet]$ is operator, corresponding to probability measure $\hat{P}_s$ on σ-algebra Ψ^1, created by deviations $(\tilde{y}_s = y_s)(\mathrm{mod}\,\hat{P}_s)$;

$$\zeta_s = \lim_{t \downarrow s}(\upsilon_t - \upsilon_s) = 0(\mathrm{mod}\,P_0);\; \varphi_s = 0.$$

The fulfillment of (2.48) at conditions (2.18), (2.19) leads to the problem

$$E_0[\rho^2_{L^2}(\tilde{y}_t, y_t)] \to \inf_{y_t}. \tag{2.51}$$

For the problem solution we are using the following relations:

$$E_{\zeta_s}[\bullet] = \lim_{h \downarrow 0} E_{\xi_{s+h}}[\bullet],\; (s+h) \in \Delta,\; E_{\zeta_s}[\rho^2_{L^2}(\zeta_t, \varphi_t)] = \int_\Omega \rho^2_{L^2}(\zeta_t, \varphi_t)\, P_{\xi_s}(d\,\omega)$$
$$= \int_\Omega \rho^2_{L^2}(\zeta_t, \varphi_t)\, P_0(d\,\omega) = E_0[\rho^2_{L^2}(\zeta_t, \varphi_t)] = \lim_{h \downarrow 0} E_{\xi_{s+h}}[\rho^2_{L^2}(\zeta_\upsilon, \varphi_\upsilon)],\; \upsilon \in [s+h, T]. \tag{2.52}$$

Then, the problem (2.51) consists of minimizing of the right-hand side in the last equality in (2.52).
Since h is arbitrarily chosen, let us assume $(s+h) = t \in \Delta$; and let us estimate

$$E_{\xi_t}[\rho^2_{L^2}(\zeta_\Theta, \varphi_\Theta)] = E_\xi[\rho^2_{L^2}(\zeta_\Theta, \varphi_\Theta)]$$
$$= E_\xi[\int_s^T |\zeta_\Theta - \varphi_\Theta|^2\, d\,\Theta] = \tilde{u}(\zeta, t);\; \Theta \in [t,T],\; \zeta = \zeta_t,$$

where function $\tilde{u}(\zeta_t, t)$ satisfies the equation following from [24] (see its solution in *Comments* 2.1):

$$-\partial\tilde{u}/\partial t = 1/2\;\nabla\,\tilde{u} + |\varphi_t - \zeta|^2;\; \nabla = \sum_{i=1}^n \partial^2/\partial\zeta_i^2, \tag{2.53}$$

$$\lim_{t \uparrow T} \tilde{u}(\zeta, t) = 0,\;\; (\zeta, t) \in (R^n \times \Delta),\; |\bullet|^2 = ||\bullet||^2_{R^n}. \tag{2.53a}$$

This equation for function $\tilde{u}(\zeta, t)$ is connected to (2.51), (2.52) by the relation

$$E_0[\rho^2_{L^2}(\zeta_\Theta,\varphi_\Theta)]=\lim_{t\downarrow s}\tilde{u}(\zeta,t)=\tilde{u}(s,0)\to \underset{\varphi_t}{Inf};\ \zeta(t=s)=0. \tag{2.54}$$

The fulfillment of (2.54) follows if we write the condition (2.54) in the form

$$\lim_{t\downarrow s} E_0[\rho^2_{L^2}(\zeta_t,\varphi_t)]=E_0[\|\varphi_t\|^2_{L_2}]=\|\varphi_t\|^2_{L_2}. \tag{2.55}$$

We will show that at $y_s\neq\tilde{y}_s$ we obtain the same result.

Indeed, using the triangle inequality we have

$$\{\rho_{L^2}(\tilde{y}_t,y_t)<\delta\}\supseteq\{\rho_{L^2}(\tilde{y}_t-\tilde{y}_s,y_t-y_s)<\delta/2\}$$
$$\times\{\rho_{L^2}(\tilde{y}_s-y_s,\mathrm{O})<\delta/2\}=\{A\}\{B\};$$
$$\{A\}=\{\rho_{L^2}(\tilde{y}_t-\tilde{y}_s,y_t-y_s)<\delta/2\}\subset\Psi,\{B\}=\{\rho_{L^2}(\tilde{y}_s-y_s,\mathrm{O})<\delta/2\}\subset\Psi_1,$$
$$\{\rho_{L^2}(\tilde{y}_t,y_t)<\delta\}\supseteq\overline{\{A\}\{B\}};\ \overline{\{A\}\{B\}}\overset{def}{=}\Omega\times\Omega^1\setminus\{A\}\{B\}, \tag{2.56}$$
$$\{\bar{A}\}=\{\rho_{L^2}(\tilde{y}_t-\tilde{y}_s,y_t-y_s)>\delta/2\}\subset\Psi,\{\bar{B}\}=\{\rho_{L^2}(\tilde{y}_s-y_s,\mathrm{O})>\delta/2\}\subset\Psi_1.$$

Applying relations (2.42), (2.43a) to (2.56) at $P_x=P$, we obtain

$$P\{\rho_{L^2}(\tilde{y}_t,y_t)\geq\delta\}\leq P\{\bar{A}+\bar{B}\}=P\{\bar{A}\}+P\{\bar{B}\}-P\,\overline{\{A,B\}}. \tag{2.57}$$

From that and because Ψ and Ψ^1 are independent, it follows

$$P\{\rho_{L^2}(\tilde{y}_t,y_t)\geq\delta\}\leq P\{\bar{A}\}+P\{\bar{B}\}-P\{\bar{A}\}P\{\bar{B}\}=P\{\bar{B}\}+(1-P\{\bar{B}\})P\{\bar{A}\} \tag{2.58}$$

where $P\{\bar{A}\}$ is the probability of the considered event ($\tilde{y}_s=y_s$=0, mod $\hat{P}_s$).

Since the function (2.57) is increasing monotonously with respect to $P\{\bar{A}\}$, its upper estimator is defined by the upper estimator for $P\{\bar{A}\}$, and this estimator can be found from (2.58).

Beside this, and because only y_t is covered by $P\{\bar{A}\}$, finding the $\underset{y_t}{Inf}$ of the upper limit of probability (2.50) is reduced to $\underset{y_t}{Inf}$ of the upper limit of $P\{\bar{A}\}$.

(At this case, $G_y=G_{y=0}$, $K_y=K_{y=0}$).

Therefore, the upper limit of probability (2.50) has a minimum on the solutions (2.49) independently on the values $\tilde{y}_s, y_s$. (The numerical values of these limitations are different in the cases of $y_s\neq\tilde{y}_s$, and $y_s=\tilde{y}_s$ accordingly). •

Corollary 2.1.

The solutions of equation (2.49) remain at a locality of function $\tilde{y}_t\in C(\Delta,R^n)$ with a maximal probability (2.27) applied to $\rho_{L^2}(\tilde{y}_t,y_t)$.

Indeed, the problem of maximizing the estimator (2.27) is reduced to condition

$$\min_{y_t} S_i(y_t) = \min_{y_t} \int_s^T L_i dt\,,\ L_i = 1/2(\dot{y}-q)^T (2b)^{-1}(\dot{y}-q), i = 1,2,5,, \qquad (2.59)$$

which is fulfilled at the solution of equation $\dot{y} = q(t, y, u)$.

As a result, we obtain relations

$$P\{\rho_{L^2}(\tilde{y}_t, y_t) < \delta\} \geq \underline{m} \to \underset{\dot{y}=q}{Sup}; P\{\rho_{L^2}(\tilde{y}_t, y_t) \geq \delta\} \geq \overline{m} \to \underset{\dot{y}=q}{Inf}, \qquad (2.60)$$

i.e., solutions (2.49) become the nearest to the most probable solutions (in the L^2-metric) by upper evaluation ($\overline{m}$) as well as by lower ($\underline{m}$) evaluation of the above probabilities. •

Comments 2.1. The solution of equations (2.53), (2.53a).
Let us apply to (2.53) the Fourier transformation

$$F[\bullet] = \int_{R^n} [\bullet]\exp(i(\xi,\lambda))d\xi, \lambda = (\lambda_k)_{k=1}^n, (\xi,\lambda) = \sum_{k=1}^n \xi_k\lambda_k\ ; \qquad (2.61)$$

$$-\frac{d\overline{u}}{dt} = -\frac{1}{2}|\lambda|^2\,\overline{u} + \overline{f}; \qquad (2.62)$$

at
$$\overline{u}(T,\lambda) = 0\ , \qquad (2.63)$$

where

$$\overline{u} = F[\tilde{u}],\ \overline{f} = F[f],\ f = |\varphi_t - \xi|^2. \qquad (2.64)$$

A general solution of (2.62) will be built using the method of variations of an arbitrary constant in

$$\overline{u}(t,\lambda) = (-\int \overline{f}(t,\lambda)\exp(-\frac{1}{2}|\lambda|^2\,t)dt + C)\exp(\frac{1}{2}|\lambda|^2\,t)\,. \qquad (2.65)$$

This function should satisfy (2.62).
Let's check it:

$$\frac{d\overline{u}}{dt} = -\overline{f}(t,\lambda) + \int_t^T \overline{f}(\tau,\lambda)\exp(-\frac{1}{2}|\lambda|^2\,(\tau-t))\frac{1}{2}|\lambda|^2\,d\tau;$$

$$-\frac{d\overline{u}}{dt} = \overline{f}(t,\lambda) - \frac{1}{2}|\lambda|^2 \int_t^T \overline{f}(\tau,\lambda)\exp(-\frac{1}{2}|\lambda|^2\,(\tau-t))d\tau = \overline{f}(t,\lambda) - \frac{1}{2}|\lambda|^2\,\overline{u}.$$

It seen that function (2.65) satisfies (2.62) identically.
According to (2.65),(2.63) a partial solution of (2.62),(2.64) has a form

$$\bar{u}(t,\lambda)=\int_t^T \bar{f}(\tau,\lambda)\exp(-\frac{1}{2}|\lambda|^2\ (\tau-t))d\tau\ , \tag{2.66}$$

which also satisfies (2.62):

$$\frac{d\bar{u}}{dt}=-\bar{f}(t,\lambda)+\frac{1}{2}|\lambda|^2\int_t^T \bar{f}(\tau,\lambda)\exp(-\frac{1}{2}|\lambda|^2\ (\tau-t))d\tau;$$

$$-\frac{d\bar{u}}{dt}=\bar{f}(t,\lambda)-\frac{1}{2}|\lambda|^2\ \bar{u}.$$

Then taking into account (2.66) we have

$$\tilde{u}(\xi,t)=F^{-1}[\bar{u}]=(2\pi)^{-n}\int_{R^n}\bar{u}(t,\lambda)\exp(-i(\xi,\lambda))d\lambda=\int_t^T \bar{f}(\xi,\tau)*F^{-1}[\exp(-\frac{1}{2}|\lambda|^2\ (\tau-t))]d\tau\ ; \tag{2.67}$$

$$F^{-1}[\exp(-\frac{1}{2}|\lambda|^2\ (\tau-t))]=\prod_{k=1}^n F^{-1}[\exp(-\frac{1}{2}{\lambda_k}^2(\tau-t))]=(2\pi(\tau-t))^{-n/2}\exp(-\frac{|\xi|^2}{2(\tau-t)}). \tag{2.68}$$

And according to (2.67), (2.68) the following relations hold true:

$$\tilde{u}(\xi,t)=\int_t^T\int_{R^n} f(x,t)(2\pi(\tau-t))^{-n/2}\exp(-\frac{|\xi-x|^2}{2(\tau-t)})dxd\tau$$

$$=\int_t^T\int_{R^n}|\varphi(\tau)-x|^2\ (2\pi(\tau-t))^{-n/2}\exp(-\frac{|\xi-x|^2}{2(\tau-t)})dxd\tau$$

$$=\int_t^T(2\pi(\tau-t))^{-n/2}\int_{R^n}\sum_{i=1}^n(\varphi_i(\tau)-x_i)^2\exp(-\frac{|\xi-x|^2}{2(\tau-t)})dxd\tau$$

$$=\int_t^T(2\pi(\tau-t))^{-n/2}\sum_{i=1}^n\int_{R^n}(\varphi_i(\tau)-x_i)^2\exp(-\frac{\sum_{k=1}^n(\xi_k-x_k)^2}{2(\tau-t)})dxd\tau$$

$$=\int_t^T(2\pi(\tau-t))^{-n/2}\sum_{i=1}^n\int_{R^1}((\varphi_i(\tau)-\xi_i)+(\xi_i-x_i))^2\exp(-\frac{(\xi_i-x_i)^2}{2(\tau-t)}))dx_i\ ;$$

$$\int_{R^{n-1}}\exp(-\frac{\sum_{k=1,k\neq i}^n(\xi_i-x_i)^2}{2(\tau-t)})dx_1...dx_{i-1}dx_{i+1}...dx_n d\tau$$

$$= \int_t^T (2\pi(\tau-t))^{-n/2}(2\pi(\tau-t))^{(n-1)/2}\sum_{i=1}^n \int_{R^1}[(\varphi_i(\tau)-\xi_i)^2 + 2(\varphi_i(\tau)-\xi_i)(\xi_i - x_i) + (\xi_i - x_i)^2]\exp(-\frac{(\xi_i - x_i)^2}{2(\tau-t)}))dx_i d\tau$$

$$= \int_t^T (2\pi(\tau-t))^{-1/2}\sum_{i=1}^n [(\varphi_i(\tau)-\xi_i)^2 \int_{R^1} \exp(-\frac{(\xi_i - x_i)^2}{2(\tau-t)}))dx_i + 0 + \int_{R^1}(\xi_i - x_i)^2 \exp(-\frac{(\xi_i - x_i)^2}{2(\tau-t)})dx_i]d\tau$$

$$= \int_t^T (2\pi(\tau-t))^{-1/2}\sum_{i=1}^n (\varphi_i(\tau)-\xi_i)^2 (2\pi(\tau-t))^{1/2} d\tau$$

$$+\int_t^T (2\pi(\tau-t))^{-1/2}\sum_{i=1}^n \int_{R^1}(\xi_i - x_i)^2 \exp(-\frac{(\xi_i - x_i)^2}{2(\tau-t)})dx_i d\tau = \tilde{u}_1 + \tilde{u}_2;$$

$$\tilde{u}_1 = \int_t^T \sum_{i=1}^n (\varphi_i(\tau)-\xi_i)^2 d\tau,\ \tilde{u}_2 = \int_t^T (2\pi(\tau-t))^{-1/2}\sum_{i=1}^n \int_{R^1}(\xi_i - x_i)^2 \exp(-\frac{(\xi_i - x_i)^2}{2(\tau-t)})dx_i d\tau .$$

Let us find $\tilde{u}_2$ assuming $I_2^i = \int_{R^1}(\xi_i - x_i)^2 \exp(-\frac{(\xi_i - x_i)^2}{2(\tau-t)})dx_i, \xi_i - x_i = -y_i, a = \tau - t.$

We have

$$I_2^i = \int_{R^1} y_i^2 \exp(-\frac{y_i^2}{2a})dy_i;\ I_2^i = y_i \int y_i \exp(-\frac{y_i^2}{2a})dy_i\,|_{-\infty}^{+\infty} - [\int_{R^1}[y_i \exp(-\frac{y_i^2}{2a})dy_i]dy_i;$$

$$z_i = y_i(2a)^{-1/2}; (2a)^{1/2}2^{-1}\int 2y_i(2a)^{-1/2}\exp(-\frac{y_i^2}{2a})(2a)^{1/2}d(y_i(2a)^{-1/2})$$

$$= a\int 2z_i \exp(-z_i^2)dz_i = a\int \exp(-z_i^2)d(z_i^2) = a(-1)\exp(-z_i^2) = -a\exp(-\frac{y_i^2}{2a});$$

$$I_2^i = a\int_{R^1}\exp(-\frac{y_i^2}{2a})dy_i = a(2a\pi)^{1/2} = (2\pi)^{1/2}a^{3/2} = (2\pi)^{1/2}(\tau-t)^{3/2};$$

$$\tilde{u}_2 = \int_t^T \sum_{i=1}^n I_2^i (2\pi)^{-1/2}(\tau-t)^{-1/2}d\tau = n\int_t^T (\tau-t)d\tau = n/2(T-t)^2 .$$

And as a result we get

$$\tilde{u}(\xi,t) = \int_t^T \sum_{i=1}^n (\varphi_i(\tau)-\xi_i)^2 d\tau + n/2(T-t)^2 . \qquad (2.69)$$

The problem essence.

Evaluator (2.2a) reaches a maximum on the solutions of equation $\dot{x} = a^u$, and because of that, the problems (2.5),(2.5a,b),(2.28), (2.28a) lead to the fulfillment of the condition

$$\min S_3(x_t) \to \min S_4(x_t), S_4(x_t) = \int_s^T L_4 dt,\ L_4 = 1/2\dot{x}^T(2b)^{-1}\dot{x},\ \dot{x} = a^u(t,x) \neq 0, \qquad (2.70)$$

from which are found macroprocess x_t and the corresponding control function u_t (ch.1.3),

[30, 33].

This leads to the problem of minimizing of the entropy functional, defined on the macroprocess.

The essence of the functionals (2.70) nearness (for the probabilities P_3 and P_4) consists of connecting the micro-and macrolevel processes by their abilities to approximate the disturbance (ζ_t).

This is achieved by their probabilistic closeness to some lower limits.

As a result, the macrolevel process, which approximates the microlevel process with a maximal probability, following from (2.70), enables us to minimize the entropy functional for the bi-level (micro-macro) structure of the object's processes.

The path functional (2.70)(defined on the macroprocess) provides a dynamic approximation of the entropy functional (defined on the microprocess) with a maximal probability.

1.2.3. The Estimation of an Accuracy of the Probability's Approximation

At the given macroprocesses and the corresponding functionals (2.27)-(2.29), let us find such $\varepsilon_i \in (0,1), i = 1,2,3$ at which the conditions for the maximums of these probabilities: $\max_{\varepsilon_i (0,1)}\{\varepsilon_i \exp[-(2S_i/(1-\varepsilon_i))^{1/2}]\}$ are satisfied.

Proposition 2.1.

A maximum of the function

$$f(x=\varepsilon)=\varepsilon\exp[-(\frac{2S_i}{1-\varepsilon})^{1/2}] \to \max_{\varepsilon\in(0,1)} S_i>0,\ i=1,2,4 \tag{2.71}$$

is reached on the solutions of equation

$$(1-x)^3=\frac{Sx^2}{2} \text{ at } x\in(0,1), S>0,\ S\in S_i. \tag{2.72}$$

Proof. Introducing function

$$f(x)=x\exp[-(2S(1-x))^{1/2}], \tag{2.72a}$$

we will analyze its maximum considering its first derivative:

$$f'(x)=\exp[-(\frac{2S}{1-x})^{1/2}]+x\exp[-(\frac{2S}{1-x})^{1/2}](-1)$$
$$-(2S)^{1/2}(-\frac{1}{2})\times\frac{1}{(1-x)^{3/2}}(-1)=0,1=(2S)^{1/2}(\frac{1}{2})\frac{x}{(1-x)^{3/2}}.$$

We come to equation $(1-x)^3=\frac{Sx^2}{2}$, $x\in(0,1)$, S>0, which has a unique real root.

Let us determine a second derivative f"(x) at the point $x \in (0,1)$, defined by the solution of the equation (2.72a):

$$f''(x) = \exp[-(\frac{2S}{1-x})^{1/2}]\frac{(2S)^{1/2}}{2}(-1)\frac{1}{(1-x)^{3/2}} - \frac{(2S)^{1/2}}{2}\exp[-(\frac{2S}{1-x})^{1/2}];$$

$$\frac{1}{(1-x)^{3/2}} - \frac{(2S)^{1/2}}{2}\, x\frac{d}{dx}\{\frac{\exp[-(\frac{2S}{1-x})^{1/2}]}{(1-x)^{3/2}}\}$$

$$= -\frac{(2S)^{1/2}}{(1-x)^{3/2}}\exp[-(\frac{2S}{1-x})^{1/2}] - \frac{(2S)^{1/2}}{2}\, x\frac{d}{dx}\{\frac{\exp[-(\frac{2S}{1-x})^{1/2}]}{(1-x)^{3/2}}\};$$

$$\frac{d}{dx}\{\frac{\exp[-(\frac{2S}{1-x})^{1/2}]}{(1-x)^{3/2}}\} = \frac{\exp[-(\frac{2S}{1-x})^{1/2}]}{2(1-x)^{3}}(-(2S)^{1/2} + 3(1-x)^{1/2}).$$

At the extremal point of the solution, we come to relation

$$f''(x) = -\frac{(2S)^{1/2}\exp[-(\frac{2S}{1-x})^{1/2}]}{(1-x)^{3/2}}(1+\frac{3(1-x)^{1/2} - (2S)^{1/2}}{2(2S)^{1/2}})$$

$$= -\frac{(2S)^{1/2}\exp[-(\frac{2S}{1-x})^{1/2}]}{(1-x)^{3/2}}(\frac{1}{2}+\frac{3(1-x)^{1/2}}{2(2S)^{1/2}}) < 0,$$

that satisfies the maximum of (2.72a).

Therefore, the fulfillment of (2.71) at $x = \varepsilon$ leads to (2.72), with the following roots of the equation:

$$x = \varepsilon_i \text{ at } S = S_i,\ i = 1,2,4. \bullet$$

Proposition 2.2.

A lower limit of $P_0(B_\delta)$ or $P_s^*(D_\delta)$ in (2.20)-(2.21) is estimated by formula

$$P_0(B_\delta) \geq K_n \int_{\delta_o/(T)^{1/2}}^{\infty} \rho^{n-1}\exp(-\rho^2/2)d\rho\ ,\delta_o < \delta\ ,$$

$$K_n^{-1} = \Gamma(n/2)2^{(n-2)/2}\ ,\ \Gamma(\alpha), \alpha > 0\ , \tag{2.73}$$

where $\Gamma(\alpha)$ is an Euler's gamma function.

Proof. The estimation $P_0(B_\delta)$ in (2.29), according to [22] acquires the form:

$$P_o\{\xi_t \in \overline{A}\} \geq 1 - 2\ P_o\{\xi_T \notin \overline{A}\}, \tag{2.73a}$$

where $\overline{\mathrm{A}} \subset U$ is an arbitrary closed concave set.

Let us introduce the balls

$\overline{\mathrm{A}} = \overline{\mathrm{A}}(0,\delta_o)$, $\bar{K}(0,\delta_o)$ in (C,U) and (R^n,β) accordingly, with a fixed value $\delta_o > \delta$:

$$\overline{\mathrm{A}} = \overline{\mathrm{A}}(0,\delta_o) = \{\varphi_t \in C : \rho_\Delta(\varphi_t, 0) \le \delta_o\},$$

$$A(0,\delta) = \{\varphi_t \in C : \rho_\Delta(\varphi_t,0) < \delta\}, \bar{K}(0,\delta_o) = \{x \in R^n : (\sum_{i=1}^{n} x_i^2 \le \delta_o\}.$$

And let us assume $\overline{\mathrm{A}} = \overline{\mathrm{A}}(0,\delta_o)$, and use the following relations

$$B_\delta \overset{def}{=} \{\xi_t \in \overline{\mathrm{A}}(0,\delta)\} \supset \{\xi_t \in \overline{\mathrm{A}}(0,\delta_o)\},$$

$$\{\xi_T \notin \overline{\mathrm{A}}(0,\delta_o)\} = \{\xi_T \notin \bar{K}(0,\delta_o)\} = \{|\xi_T| > \delta\},\ |\xi_T| = (\sum_{i=1}^{n} \xi_i^2(T,\omega))^{1/2}.$$

Then, applying the above relations and the representation of probability in [22], we get

$$P_o\{|\xi_T| > \delta\} = P_o\{|\xi_T| > \delta\} = K_n \int_{\delta_o/(T)^{1/2}} r^{n-1} \exp(-r^2/2) dr,$$

where $K_n^{-1} = \Gamma(\frac{n}{2}) \times 2^{\frac{n-2}{2}}$, $\Gamma(\alpha)$, $\alpha > 0$ is the Euler gamma-function.

Finally, we come to the estimation of these probability by (2.73). •

Comments 2.2.

At given trajectories of (1.11) and corresponding functionals (2.30) on them, it is found such $\varepsilon_i \in (0,1)$, $i = 1-3$ in (2.27)-(2.29) at which the relations (2.71), (2.72) are fulfilled.

This conditions satisfy to the maximums of these probabilities.

Then, using the evaluation of the lowest limit of the probabilities $P_0(B_\delta)$ and P_s^* in (2.27)-(2.29) by (2.73), we obtain the numerical evaluations for the probabilities.

Actually, solving the variation problem (2.70) (in ch.1.3) allows approximating the random information functional by the path functional with the maximal probabilities (2.5)-(2.5b).

Chapter 1.3

THE VARIATION PROBLEM FOR THE INFORMATION PATH FUNCTIONAL AND ITS SOLUTION

In this chapter, the found solution to the path functional's variation problem provides both a dynamic model of a random process and the model's optimal control synthesis, which allows us to build a two-level information model with a random process at the microlevel and a dynamic process at the macrolevel.

1.3.1. The Problem Formulation

Let us *formulate* the variation problem (VP) using the Lagrange method of eliminating constraints [23] and the Pontryagin maximum principle [24], applied to the information path functional in forms (2.59),(2.70):

$$S_p(x_t) = \int_0^T L_p^o(t, x_t, \dot{x}_t, u_t)dt \to extr; u_t = u(t, x_t), u \in \hat{U}, \hat{U} = intU, \tag{3.1}$$

$$S_p = S_p(s, T, x_t(\bullet)),\ x_t(\bullet, x) \in KC^1(\Delta, R^n),$$

$$\|x_t\|_{KC^1} \overset{def}{=} \|x_t\|_C = \underset{t\in\Delta}{Sup}(\sum_{i=1}^{n} x_i^2(t,\bullet))^{1/2}, x_T = \|0_i\|_{i=1}^n. \tag{3.1a}$$

$$L_p^o = L_p = \lambda_o L(t, x_t, \dot{x}_t) + p_t^T(\dot{x}_t - a^u(t, x_t)), \tag{3.2a}$$

or

$$L_p^o = L_p^u = \lambda_o L(t, x_t, a^u(t, x_t)) + p_t^T(\dot{x}_t - a^u(t, x_t)), \tag{3.2b}$$

$$L(t, x_t, \dot{x}_t) = 1/2\dot{x}_t^T(2b(t, x_t))^{-1}\dot{x}_t, \tag{3.3}$$

$$L(t, x_t, a^u(t, x_t)) = 1/2(a^u(t, x_t))^T(2b(t, x_t))^{-1}a^u(t, x_t), a^u(t, x_t) \neq 0, \tag{3.4}$$

$$\psi_j(\tau, x_\tau, u(\tau, x_\tau)) = 0, j = 1, ..., N, \tau \in \Gamma_\psi \subset \Delta; \| \psi_j \| = \psi : (\Gamma_\psi, R^n, U) \to R^1, \tag{3.5}$$

where Lagrangian L_p^o is written in both the traditional (3.2a) (for Calculus in Variations) and the Pontryagin's (3.2b) forms, and λ_o, p_t are the Lagrange multipliers: $\lambda_o \in R_+^1, p_t = p(\bullet, x) \in KC^1(\Delta, R^n)$.

The conditions (3.5) are determined by reaching the equalization of $S_2(x_t)$ with $S_3(\tilde{x}_t)$ in (2.70) and represent the constraint's equation, imposed by stochastics; Γ_ψ is a discrete set of points $t \in \Delta$ defined by the function $\psi_j = \psi_j(\tau, x_\tau, u(\tau, x_\tau))$, Γ_ψ and N will be found based on (2.70) and the applied controls (1.5).

The differential constraint below, implementing (2.70), brings an *important specific* of the VP.

1.3.2. Solution to the Variation Problem

Lemma 3.1.

The equations for the *field* of the functional S_p on the set

$$\tilde{Q} = (\Delta \setminus \bigcup_{k=1}^{m} \tau_k \bigcup \Gamma_\psi) \times R^n; \Delta^o = \Delta \setminus \bigcup_{k=1}^{m} \tau_k, \tag{3.5a}$$

where τ_k is the point of the control's discontinuity, follow from the application of the functional's field variation conditions [23] imposed on the conjugate vectors $X_p(t)$, p_t and the corresponding Hamiltonians H_p, H_p^u accordingly.

Indeed, the field's equations brings the following relations, which are satisfied at $\forall t \in \Delta^o \setminus \Gamma_\psi$:

$$X_p(t) = \frac{\partial L_p}{\partial \dot{x}_t} = \lambda_o X_t + p_t, X_t = \frac{\partial L(t, x_t, \dot{x}_t)}{\partial \dot{x}_t} = (2b(t, x_t))^{-1} \dot{x}_t, \tag{3.6}$$

$$\lambda_o X_t + p_t = \lambda_o (2b(t, x_t))^{-1} \dot{x}_t + p_t, \dot{x}_t = a^u(t, x_t) = 2b(t, x_t) X_t, \tag{3.7}$$

$$H_p(t) = \dot{x}_t^T X_p(t) - L_p(t)$$

$$= \lambda_o \dot{x}_t^T (2b(t, x_t))^{-1} \dot{x}_t + \dot{x}_t^T p_t - \frac{\lambda_o}{2} \dot{x}_t^T (2b(t, x_t))^{-1} \dot{x}_t - p_t^T \dot{x}_t + p_t^T a^u(t, x_t), \tag{3.8}$$

$$H_p(t, x, X_p) = \lambda_o (X_p - p_t)^T b(t, x)(X_p - p_t) + p_t^T a^u(t, x_t), \tag{3.8a}$$

$$H_p(t, x, X) = \lambda_o X^T b(t, x) X + p_t^T a^u(t, x_t). \tag{3.8b}$$

We come to the differential equations of the VP's extremals:

$$\dot{x}_t = \frac{\partial H_p(t, x_t, X_p(t))}{\partial X_p} = \lambda_o 2b(t, x_t) X_t = \lambda_o a^u(t, x_t), \tag{3.9}$$

$$\frac{\partial H_p}{\partial X_p} = \frac{\partial X}{\partial X_p}\frac{\partial H_p}{\partial X} = I\frac{\partial H_p}{\partial X} = \frac{\partial H_p(t,x,X)}{\partial X}, \tag{3.9a}$$

where I is the identity matrix and

$$p_t = \frac{\partial L_p^u}{\partial \dot{x}_t},$$

$$H_p^u(t) = \dot{x}_t^T p_t - L_p^u(t) = \dot{x}_t^T p_t - \frac{\lambda_o}{2} a^u(t,x_t))^T (2b(t,x_t))^{-1} a^u(t,x_t)) + p_t^T a^u(t,x_t)$$

$$- p_t^T \dot{x}_t . \tag{3.10}$$

From the comparison of (3.7) and (3.9) follows $\lambda_o = 1$ and

$$H_p^u(t) = -\frac{1}{2} a^u(t,x_t))^T (2b(t,x_t))^{-1} a^u(t,x_t) + p_t^T a^u(t,x_t) . \tag{3.10a}$$

The functional's field on $Q = (\Delta^o \setminus \Gamma_\psi) \times R^n$ is determined by the tranversality's conditions

$$\frac{\partial X_i(t,x,a^u(t,x))}{\partial x_j} = \frac{\partial X_j(t,x,a^u(t,x))}{\partial x_i}, i,j = 1,...,n, \tag{3.11}$$

$$\frac{dX_p(t,x_t,a^u(t,x_t))}{dt} = -\frac{\partial H_p(t,x_t,X_p(t))}{\partial x_t}, \tag{3.12}$$

or is expressed by both relation (3.11) and equation

$$\frac{dp_t}{dt} = -\frac{\partial H_p^u}{\partial x_t} = (\frac{\partial a^u(t,x_t)}{\partial x_t})^T p_t + \frac{\partial}{\partial x_t}(\frac{1}{2} a^u(t,x_t))^T (2b(t,x_t))^{-1} a^u(t,x_t)) , \tag{3.13}$$

corresponding to the forms (3.2a) and (3.2b).

Equation (3.12) for the conjugate vector in (3.6) relates to the Lagrangian in form (3.2a), while the equation for the conjugate vector in (3.10) relates to the Lagrangian in form (3.2b).

Equation (3.13) is a stationary condition for the maximum principle.

These equalities for the functional's S_p field should be equivalent on $(\Delta^o \setminus \Gamma_\psi) \times R^n$ because both of them are a consequence of the same variation principle, represented in the two forms. •

Corollary 3.1.

Function of action $S_p(t,x)$, defined on extremals, satisfies the Hamilton-Jacobi (HJ) equation at $(t,x) \in Q$ in the forms

$$-\frac{\partial S_p(t,x)}{\partial t} = H_p^u(t,x,X_p), \frac{\partial S_p(t,x)}{\partial x} = X_p(t,x), \tag{3.14}$$

$$-\frac{\partial S_p(t,x)}{\partial t}=\frac{1}{2}a^u(t,x))^T(2b(t,x))^{-1}a^u(t,x))+p_t^Ta^u(t,x)\,. \tag{3.15}$$

Lemma 3.2 (L2).

Let us consider the distribution of functional (3.1) on $\tilde{Q}=\Delta^o\times R^n$ as a *function* of current variables: $\tilde{S}=\tilde{S}(t,x)$, which satisfies the Kolmogorov equation (K) [25], applied to equation (3.1) in the form

$$-\frac{\partial\tilde{S}}{\partial t}=(a^u)^T\frac{\partial\tilde{S}}{\partial x}+\sum_{i,j=1}^{n}b_{ij}\frac{\partial^2\tilde{S}}{\partial x_i\partial x_j}+\frac{1}{2}(a^u)^T(2b)^{-1}a^u\ ,\ a^u=a^u(t,x),\ b=b(t,x), \tag{3.16}$$

and let us have a function $\tilde{S}_p=\tilde{S}_p(t,x)$, which satisfies equation (3.16) at each point $(t,x)\in Q$ of the extremal's field:

$$-\frac{\partial\tilde{S}_p}{\partial t}=(a^u)^T\frac{\partial\tilde{S}_p}{\partial x}+\sum_{i,j=1}^{n}b_{ij}\frac{\partial^2\tilde{S}_p}{\partial x_i\partial x_j}+\frac{1}{2}(a^u)^T(2b)^{-1}a^u \tag{3.16a}$$

and satisfies the HJ equations (3.14), (3.15) on a certain set $Q^o\subset Q$, where holds true the equation

$$(a^u)^T\frac{\partial\tilde{S}_p}{\partial x}+\sum_{i,j=1}^{n}b_{ij}\frac{\partial^2\tilde{S}_p}{\partial x_i\partial x_j}=p^Ta^u\ ,\ a^u=a^u(t,x),\ b=b(t,x). \tag{3.17}$$

Then, the above function $\tilde{S}_p=\tilde{S}_p(t,x)$ exists and satisfies the equation

$$\frac{\partial X_p}{\partial x}=\frac{\partial X}{\partial x}=-2XX^T, \tag{3.18}$$

which determines $N=n^2$ equations of constraint in (3.5), imposed on the Hamilton equations (at the "punched" discretely selected points (DP) (3.5a)).

Proof. Applying equations (3.6)-(3.12) and the principle of superposition for the continuous and differentiable transformations, we come to the existence of equation (3.17) and the fulfillment of (3.15) for $\tilde{S}_p=\tilde{S}_p(t,x)$.

Because of that, on $Q^o\subset Q$ are satisfied the equations

$$\frac{\partial S_p}{\partial x}=X_p,-\frac{\partial S_p}{\partial x}=H_p\,. \tag{3.18a}$$

Since (3.17) is a linear elliptic equation solvable under the given boundary conditions (on the right-hand side of (3.17)), the function $\tilde{S}_p(t,x)$ exists.

According to (3.17), (3.18a) and (3.6)-(3.8) we get on $Q^o\subset Q$ the following equations

$$(a^u)^T X + (a^u)^T p + \sum_{i,j=1}^{n} b_{ij} \frac{\partial X_{pi}}{\partial x_j} = p^T a^u,$$

$$X^T 2bX + \sum_{i,j=1}^{n} b_{ij} \frac{\partial X_{pi}}{\partial x_j} = 0, \sum_{i,j=1}^{n} b_{ij} (\frac{\partial X_{pi}}{\partial x_j} + 2X_i X_j) = 0, \tag{3.19}$$

$$\psi = \sum_{i,j=1}^{n} b_{ij} (\frac{\partial X_{ip}}{\partial x_j} + 2X_i X_j) = 0, b > 0, \det b \neq 0 \tag{3.19a}$$

$$\frac{\partial X_p}{\partial x} = \frac{\partial X}{\partial x} = -2XX^T. \tag{3.20}$$

Relation (3.19a) is fulfilled if, in particular, equation (3.18) is satisfied, which determines $N = n^2$ equations of the differential constraints in (3.5).

The condition (3.18), being applied, should not contradict to the VP (particularly, in forms (3.14) and (3.11)).

Indeed, from (3.14, 3.18) we have

$$\frac{\partial X_p}{\partial t} = -\frac{\partial H_p}{\partial x} - \frac{\partial X_p}{\partial x} a^u, \frac{\partial}{\partial x}(\frac{\partial X_p}{\partial t}) = -2(\frac{\partial X}{\partial t} X^T + X \frac{\partial X^T}{\partial t}), (t,x) \in Q^o, \tag{3.20a}$$

or in other form:

$$\frac{\partial}{\partial t}(\frac{\partial X}{\partial x} + 2XX^T) = (o_{ij})_{i,j=1}^{n}.$$

This proves the correctness of (3.18), which satisfies (3.20a). •

Comment 3.1. For the implementation of (3.18), (3.20), the distributions for both functionals $\tilde{S}_p(t,x)$ and $\tilde{S}(t,x)$ on $\tilde{Q} = \Delta^o \times R^n$, as well as condition $\tilde{S}_p(t,x) = extrS(t,x)$ at $(t,x) \in \tilde{Q}$, should be considered in the same region by letting $Q = \tilde{Q}$ for the above HJ and K equations. From that follows $Q = \Delta^o \times R^n$, $\Gamma_\psi = \bigcup_{k=1}^{m} \tau_k = \Delta^o$, i.e., the set $\Gamma_\psi \subset R^1_+$, where the constraint (3.5),(3.18) holds true, coincides with the discrete moments, imposed at a locality of the control's discontinuity $\{\tau_{k\mp 0}\}_{k=0}^{m}$.

For fulfillment of equation (3.18), (3.20) by applying the controls, let us assume

$$Q^0 = Q^0_- \bigcup Q^0_+, Q^0_{\pm} \overset{def}{=} \bigcup_{k=1}^{m} \tau_{k\mp 0} \times R^n, \tag{3.21}$$

i.e. the constraint (3.5), (3.18) at (3.21) is imposed at a vicinity of the hyperplanes of space $R^1_+ \times R^n$, defined by the set $\Gamma_\psi = \bigcup_{k=1}^{m} \tau_k$, which will be selected constructively later on. •

Let us represent vector a^u in (1.6) in a traditional form $a^u = A(t,x)x + u$, where $A(t,x)$ is a macromodel's differential operator, while the control can be written in the form $u = A(t,x)v$, where v is a control vector reduced to a state vector *x*.

Then we formulate the following

Theorem 3.1(T1).

The equations for the functional field (3.6)-(3.7), (3.15), (3.16) and for differential constraint (3.5), (3.18) of the VP are satisfied jointly at a limited set Δ^o when the following equations for the macromodel and controls hold true:

$$a(t, x_t, u_t) = a^u,\ a^u = A(t)(x + v),\ A(t) = A_t,$$

$$A_t(t) \in KC(\Delta, L(R^n)) \cap C^1(\Delta^o, L(R^n)), \tag{3.22}$$

$$(t, x) \in (\Delta \times R^n), \Delta^o = \Delta \setminus \bigcup_{k=1}^{m} \tau_k, v_t \in KC(\Delta, V) \cap C^1(\Delta^o, L(R^n)), V \subset R^n, \tag{3.23}$$

where $v = A^{-1}u$ is the control vector u, reduced to a state vector x, with $rank[v] = rank[x] = n$; $A = A(t, x)$ is a nonsingular macromodel matrix, and C^1, KC are the spaces of continuous differentiable and the piece wise differentiable n-dimensional vector functions on Δ, respectively, Δ^o is a set of discrete moments $\tau \in \Gamma_\psi$ in Δ.

Proof. The equalities (3.6) and (3.14), (3.15) are the consequences of the same variation principle with the Lagrangians in two forms (3.2a,b).

Therefore, either (3.6), (3.7) or (3.6), (3.14) must be equivalent at $Q = \tilde{Q} = \Delta^o \times R^n, (t, x) \in \tilde{Q}$.

This involves a joint consideration of the following equations: the field equations (3.6), (3.7), the differential equation for the conjugate vector in the field for Haminltonian (3.10):

$$\frac{\partial X_p}{\partial t} = -\frac{\partial H_p}{\partial x} - \frac{\partial X_p}{\partial x} a^u,\ X_p = X(t, x) + p(t, x),$$

$$\frac{\partial X_p}{\partial x} = \left(\frac{\partial X_p}{\partial x}\right)^T = \frac{\partial X}{\partial x} = \left(\frac{\partial X}{\partial x}\right)^T, \tag{3.24}$$

$$H_p = (X_p - p)^T b (X_p - p) + p^T a^u = X^T b X + p^T a^u,\ a^u = 2bX, \tag{3.24a}$$

the equation of differential constraint in the form (3.18), and the equation

$$\partial(\partial X_p / \partial t) / \partial x = -2[(\partial X / \partial t) X^T + X(\partial X / \partial t)^T]. \tag{3.25}$$

The right side of (3.25), after applying H_p from (3.12), (3.24a) and substituting (3.18), acquires the forms:

$$\frac{\partial H_p}{\partial x} = \left(\frac{\partial}{\partial x}(X_p - p)^T b\right) X - \left(\frac{\partial X}{\partial x} b\right)^T X + \left(\frac{\partial}{\partial x} a^u\right)^T p, \tag{3.25a}$$

$$\partial H_p / \partial x = 1/2 (\partial a^u / \partial x)^T X + 2 X X^T b X + (\partial a^u / \partial x)^T p. \tag{3.25b}$$

The equations (3.25) and (3.25b) can be written in the following two forms:

$$\partial X_p / \partial t = -1/2(\partial a^u / \partial x)^T X + XX^T a^u - (\partial a^u / \partial x)^T p,$$
$$\partial[1/2(\partial a^u / \partial x)^T X - XX^T a^u + (\partial a^u / \partial x)^T p] / \partial x = 2[(\partial X / \partial t)X^T + X(\partial X / \partial t)^T],$$
$$\partial[1/2(\partial a^u / \partial x)^T - XX^T a^u + (\partial a^u / \partial x)^T p] / \partial x = 1/2\partial(\partial a^u / \partial x)^T / \partial x) : (X + 2p) \quad (3.26)$$
$$-(\partial a^u / \partial x)^T XX^T - XX^T \partial a^u / \partial x + 4XX^T (X^T a^u).$$

Since functional (3.1) reaches its extreme on the solutions of equations (3.8,3.9), the variation conditions of a coordination in the functional field [23] are satisfied in the form:

$$\partial X(t, x, a^u(t, x)) / \partial t = -\partial H(t, x, a^u(t, x)) / \partial x, H = 1/2(a^u)^T (2b)^{-1} a^u = 1/2 X^T a^u \ . \quad (3.26a)$$

Using the last equation, we obtain

$$-\partial H / \partial x = -1/2[(\partial X / \partial x)^T a^u + (\partial a^u / \partial x)^T X] = -1/2[-2XX^T + (\partial a^u / \partial x)^T X], \quad (3.27)$$

and equality (3.25) takes the form

$$2[(\partial X / \partial t)X^T + X(\partial X / \partial t)^T] = 2XX^T a^u X^T - (\partial a^u / \partial x)^T XX^T + 2X(a^u)^T XX^T$$
$$-XX^T \partial a^u / \partial x = 4X(X^T a^u)X^T - (\partial a^u / \partial x)^T XX^T - XX^T \partial a^u / \partial x. \quad (3.28)$$

From the joint consideration of (3.26) and (3.28) we get equality

$$1/2\partial(\partial a^u / \partial x)^T / \partial x : (X + 2p) = \left\| o_{ij} \right\|_{i,j=1}^n = \mathrm{O}, \quad (3.29)$$

which is satisfied identically on the set

$$Q = (\Delta \setminus \bigcup \tau_k \times R^n), \text{ if } \partial^2 a_k^u(t, x) / \partial x_i \partial x_j \equiv \mathrm{O}; i, j, k = 1, ..., n, (t, x) \in Q^0. \quad (3.30)$$

From that, the equations (3.22), (3.23) follow.

Operator $A = A(t)$ does not depend on the microlevel's randomness ($\omega \in \Omega$) by its definition. •

Comments 3.2. The path functional on the extremals, expressed through the parameters of the object equations (1.11), which satisfies the VP, we call an eigenfunctional.

This functional, according to (3.13), meets the conditions of the model's stability.

The Lagrangian of the eigenfunctional:

$$L_p = -1/2[(dx/dt)^T (2b)^{-1} (dx/dt) + p^T (dx/dt - a^u)], \quad (3.31)$$

with the equations (3.16),(3.18):

$$p^T a^u = (X + p)^T a^u + b\partial X / \partial x - (2bX)^T X, dx/dt = a^u, 2bX = dx/dt \quad (3.32)$$

takes the form

$$L_p = -1/2[(a^u)^T (2b)^{-1}(a^u) - X^T dx/dt + (dx/dt)^T X]. \quad (3.33)$$

Corollary 3.3.

From (3.8), (3.22), (3.23) follow the explicit relations for vector X and the differential constraint (3.18) in the forms

$$X(t,x) = (2b(t,x)^{-1} A(t)(x+v), A = A^T, b = b^T, (2b)^{-1} A = A(2b)^{-1}, b = b(t,x), \quad (3.34)$$

$$\partial X / \partial x_j = -(2b)^{-1} \partial (2b)^{-1} / \partial x_j (2b)^{-1} a^u + (2b)^{-1} \partial a^u / \partial x_j = 2XX_j = 2(2b)^{-1} a^u X_j, \quad (3.35)$$

$$-\partial (2b) / \partial x_j (2b)^{-1} a^u + \partial a^u / \partial x_j = 2a^u X_j,$$

$$-(\partial (2b) / \partial x_j X)_i + \partial a_i^u / \partial x_j = 2a_i^u X_j, \; i,j = 1,\ldots,n, \quad (3.36)$$

$$-\sum_{k,m=1}^{n} \frac{\partial (2b_{ij})}{\partial x_j} (2b)^{-1}_{km} a^u_m + \frac{\partial a_i^u}{\partial x_j} = 2a_i^u \sum_{k=1}^{n} (2b)^{-1}_{jk} a^u_k,$$

$$2b_{ij}(t,x_j) = \sigma_{ij}(t,x_j)\sigma_{ji}(t,x_j) = L^{ij}_{kin}, a_i^u = a_i^u(t,x_j), \quad (3.37)$$

$$-\sum_{k,\nu=1}^{n} \partial (2b_{ik}) / \partial x_j (2b)_{k\nu}^{-1} a_\nu^u + A_{ij} = 2a_i^u \sum_{k=1}^{n} (2b)^{-1}_{jk} a_k^u, i,j = 1,\ldots,n,,$$

$$i,j = 1,\ldots,n, (t,x) \in (\Delta^o \times R^n). \quad (3.38)$$

Both (3.17), (3.37) [at $b = b(t,x)$] and (3.22) [at $b = b(t)$] define the dynamic model of random object satisfying the VP.

The macroprocess' extremals provide the prognosis of an evolution (for the (1.11) solutions) using the microstates' initial math expectations.

Form (3.9),(3.34) coincides with the equations of Nonequilibrium Thermodynamics [37-38], where function $b = b(t,x)$ defines the nonlinear kinetic operator in L_{kin} in (3.37), while Lagrangian (3.33) corresponds to the Onsager-Machlup Lagrangian [39,40].

Considering an information speed $\dot{x}_t$ as an analog of information flow $I \sim \dot{x}_t$, we get a basic information relation between the flow and an the information force X that cause the flow:

$$I = L(b(x_t)X, \quad (3.38a)$$

on the extremals of the path functional with Lagrangian (3.33). •

Comments 3.3.The informational macromodel (3.22), (3.34), (3.38) includes control (3.23), which, unlike the traditional maximum principle, appears in the conjugate vector's expression, and therefore, participates in the equation of constraint (3.20), (3.36), (3.37).

This connects the functions of drift and diffusion and represents a basic equation for identification of the macromodel operator with an unknown structure by measuring a nonlinear matrix of diffusion.

However, at $b_t = b(t)$ the equation of differential constraint (3.18) acquires the form

$$A_t[E+2(x+v)(x+v)^T A_t(2b_t)^{-1}]=(o_{ij})_{i,j=1}^n=\mathrm{O},(t,x)\in Q^0,, \tag{3.39}$$

which can be satisfied only when $A_t=(o_{ij})_{i,j=1}^n=\mathrm{O}$ because

$$\det[(x+v)(x+v)^T A_t(2b_t)^{-1}]\equiv \mathrm{O}. \tag{3.39a}$$

This relation leads to the fulfillment of $dx/dt\,(t\in\Delta)=0$ for (3.22), which contradicts the initial condition (3.4). This means that $b=b(t)=b_t$ satisfies (3.38) at the "punched" points of set $\tilde{Q}$: $Q^0=(\bigcup \tau_k\times R^n)$, and function $b=b(t,x)$ satisfies (3.37) within a "coupled region" of $\tilde{Q}$.

The condition

$$\sum_{i,j=1}^n (2b_{ij})^{-1}x_i x_j\ge 0, x_t\ne 0, x\in R^n, \tag{3.40}$$

following from a positivity of both Langrangian and the functional at $L_2\ne 0, S_2\ne 0$ in (2.27), (2.27a), requires a *nonsingularity* of matrix b, which according to (3.37),(3.38) leads also to the *nonsingularity* of matrix A.

Equations (3.23)-(3.25),(3.38) allow the restoration of the macromodel's eigenfunctional, Lagrangian and Hamiltonian letting us to find the macromodel's equation directly from the solution of the VP problem.

Corollary 3.4.

Because of the above (3.18) limitations, let us consider the implementation of (3.5) using relation

$$\frac{\partial X}{\partial x}+2XX^T=\varepsilon,\ \rho(\varepsilon)\to\min, \tag{3.41}$$

where an accuracy $\rho(\varepsilon)$ of approximating function $\varepsilon=\varepsilon(t,x)$ can have the following forms

$$\rho(\varepsilon)=(\sum_{i,j=1}^n \varepsilon_{ij}^2)^{1/2} \tag{3.42a}$$

$$\rho(\varepsilon)=(\sum_{i,j=1}^n E[\varepsilon_{ij}^2])^{1/2}, \tag{3.42b}$$

$$\rho(\varepsilon)=\sum_{i,j=1}^n (E[\varepsilon_{ij}^2])^{1/2}. \tag{3.42c}$$

Taking into account (3.41), the coordination of relations (3.17) or (3.20) with the variation principle in the forms (3.14), (3.11) accordingly and equation (3.24), leads to equalities

$$\frac{\partial X_p}{\partial t}=-\frac{\partial H_p}{\partial x}-\frac{\partial X_p}{\partial x}a^u,\ \frac{\partial}{\partial t}(\frac{\partial X}{\partial x}+2XX^T-\varepsilon)=\|o\|_{i,j=1}^n=\mathrm{O}, \tag{3.43}$$

$$\frac{\partial X}{\partial x}(\frac{\partial X}{\partial t})=[\frac{\partial X}{\partial x}X^T+X\frac{\partial X}{\partial x}^T]+\frac{\partial \varepsilon}{\partial t}, \tag{3.43a}$$

$$\frac{d}{dt}\varepsilon_{ij}(\tau,x_\tau)=0,\ \frac{\partial}{\partial x_\nu}\varepsilon_{ij}(\tau,x_\tau)=0,\ i,j,\nu=1,..,n,\ \tau\in\bigcup_{k=0}^m \tau_k, \tag{3.44}$$

from which we get the equation of constraint (3.39) in the form

$$1/2I + (x+v)X^T = A^{-1}b\varepsilon \ . \tag{3.45}$$

Corollary 3.5.

Applying estimation (3.42c) to (3.18), we come to equality

$$E[\frac{\partial X}{\partial x} + 2XX^T] = \| o \|_{i,j=1}^{n} = \mathrm{O} \ . \tag{3.46}$$

Considering (3.46) jointly with (3.43), we obtain the relation for the identification of A_t at $t \in \Delta^o$:

$$A_t = -r_v^{-1} b = -b\, r_v^{-1}, 2b(t,x_t) = \sigma(t,x_t)\sigma(t,x_t)^T, r_v(t) = E[(x_t+v_t)(x_t+v_t)^T],$$
$$r_1(t) = E[\dot{x}_t(x_t+v_t)^T], \tag{3.47}$$

(with the aid of control, applied at $t = \tau$), using the corresponding covariation functions $r_v(t)$ and $r_1(t)$, which satisfy relations

$$b(t) = 1/2\dot{r}_v(t),\ \dot{r}_v(t) = r_1(t) + r_1^T(t) + E[\dot{v}_t(x_t+v_t)^T + (x_t+v_t)\dot{v}_t^T]. \tag{3.48}$$

It is seen that the dispersion matrix b is expressed through the derivation of covariation matrix $r_v(t)$, defined via the observed macrovariables, while matrices r_v^{-1} and b mutually commutate, satisfying the equations for functional's field (3.7) and for constraint (3.46). •

Theorem 3.2 (T2).

Equations of the functional's field (3.6), (3.7) and the VP's differential constraints (3.41), (3.45), (3.46) are consistent *if* equations (3.22), (3.23) are fulfilled, and the equation for the identification of the model's operator on $Q^0 = (\bigcup \tau_k \times R^n)$ (including the $\tau_{k\pm o} = \tau_k(\pm o)$ locality) has the forms

$$A(\tau_{k\pm o}) = A_{\pm} = r_{1\pm} r_{v\pm}^{-1} = r_{v\pm}^{-1} r_{1\pm}, r_{1\pm} = r_1(\tau_{k\pm o}) = r_1^T(\tau_{k\pm o}), r_{v\pm} = r_1(\tau_{k\pm o}), r_1 = M[dx/dt(x+v)^T], \tag{3.49}$$

$$\dot{r}_{v\pm}(\tau_{k\pm o}) = 2r_1(\tau_{k\pm o}) = 2r_1^T(\tau_{k\pm o}),$$
$$A(\tau_{k\pm o}) = 1/2\dot{r}_{v\pm}(\tau_{k\pm o}) r_{v\pm}(\tau_{k\pm o}) = 1/2 r_{v\pm}(\tau_{k\pm o})\dot{r}_{v\pm}(\tau_{k\pm o}) \ . \tag{3.49a}$$

To *prove* T2 we use jointly equations (3.24a, 3.41) and (3.42c), (3.43), (3.44). Following the methodology of proving T1, we write down the equations

$$\partial H_p / \partial x = 1/2[(\partial a^u / \partial x)^T X + XX^T a^u - 1/2\varepsilon^T a^u + (\partial a^u / \partial x)^T p], \tag{3.50}$$

$$\partial X_p(t,x)/\partial t = -\partial H_p / \partial x + 2XX^T a^u - \varepsilon^T a^u$$
$$= -1/2(\partial a^u / \partial x)^T X + XX^T a^u - 1/2\varepsilon^T a^u + (\partial a^u / \partial x)^T p \ . \tag{3.50a}$$

From those and after substituting (3.50a) into (3.43a) we obtain

$$\partial(1/2(\partial a^u / \partial x)^T)(X + 2p) - XX^T a^u + 1/2\varepsilon a^u)/\partial x = 2[\frac{\partial X}{\partial t} X^T + X(\frac{\partial X}{\partial t})^T]. \tag{3.50b}$$

Using (3.41) and (3.50b) we have

$$\partial(1/2(\partial a^u/\partial x)^T)/\partial x:(X+2p)-(\partial a^u/\partial x)^T XX^T+$$
$$1/2(\partial a^u/\partial x)^T\varepsilon - XX^T\partial a^u/\partial x$$
$$+4XX^T(X^Ta^u)-2\varepsilon(X^Ta^u)+1/2(\partial\varepsilon/\partial x)^T:a^u+1/2\varepsilon^T\partial a^u/\partial x, \quad (3.51)$$

and after applying (3.26a) to (3.41) we get

$$\frac{\partial X}{\partial t}=-1/2[\frac{\partial X}{\partial x}a^u+(\frac{\partial a^u}{\partial x})^T X]=XX^Ta^u-1/2\varepsilon a^u-1/2(\frac{\partial a^u}{\partial x})^T X \quad (3.51a)$$

Considering jointly (3.43a), (3.50b), and (3.51a) we come to

$$2[\frac{\partial X}{\partial x}X^T+X(\frac{\partial X}{\partial x})^T]+\frac{\partial\varepsilon}{\partial t}=2X(X^Ta^u)-\varepsilon a^uX^T-(\frac{\partial a^u}{\partial x})^T XX^T$$
$$+2XX^T(X^Ta^u)-X(a^u)^T\varepsilon^T-XX^T\frac{\partial a^u}{\partial x}+\frac{\partial\varepsilon}{\partial t} \quad (3.51b)$$

From (3.50b), (3.51), (3.51b), and (3.44), we get

$$\partial(1/2(\partial a^u/\partial x)^T)/\partial x:(X+2p)+1/2\varepsilon^T\partial a^u/\partial x+1/2(\partial a^u/\partial x)^T-2\varepsilon(X^Ta^u)$$
$$=-\varepsilon a^uX^T-(\varepsilon\, a^uX^T)^T. \quad (3.52)$$

The last relation has to be identical on $Q^0\subset Q$ irrespectively of the explicit form for function $\varepsilon(t,x)$. This condition is fulfilled if both the representation of (3.22) and relations

$$\varepsilon=\varepsilon^T,\ \det\varepsilon\neq 0;\varepsilon\partial a^u/\partial x=(\partial a^u/\partial x)^T\varepsilon,\varepsilon\, a^uX^T=(\varepsilon\, a^uX^T)^T \quad (3.53)$$

hold true on Q. From equations (3.22),(3.52), (3.53) we obtain the following relations

$$2\varepsilon a^uX^T+\varepsilon\partial a^u/\partial x=2\varepsilon X^T, A+2a^uX^T=2X^Ta^uI,\varepsilon\varepsilon^{-1}=I, \quad (3.54)$$
$$(2b)^{-1}A+2XX^T=(2b)^{-1}2X^Ta^u,\partial X/\partial x+2XX^T=(2b)^{-1}2Sp(a^uX^T),\varepsilon=b^{-1}Sp(a^uX^T), \quad (3.54a)$$
$$A(2b)^{-1}A^{-1}+2a^u(x+v)^T=2b2A^{-1}(X^Ta^u),X^Ta^u=Sp(a^uX^T), \quad (3.54b)$$
$$A^{-1}2bA+2(x+v)(a^u)^T=2A^{-1}2b(X^Ta^u),b=b(t,x),(t,x)\in Q. \quad (3.54c)$$

Relations (3.54a,b), (3.22), and (3.36) lead to equations

$$\ldots\ldots a^u(x+v)^T=(x+v)(a^u)^T,\ldots.$$
$$\ldots.\ (dx/dt)_{\pm}(x+v_{\pm})^T=(x+v_{\pm})(dx/dt)_{\pm}^T, t\in\tau_{k\pm o},k=1,\ldots,n. \quad (3.55)$$

After taking the math expectation and applying (3.48) we arrive at (3.49), (3.49a) for the identification of $A_{\pm}$ on $Q^0=(\bigcup\tau_k\times R^n)$ by the

covariance function and its derivative at the moments $\tau_{k\pm o}$ of applying control. •

Comments 3.4. From equations (3.34), (3.48) we get $X = -1/2r_v^{-1}(x+v)$ and Lagrangian (3.33) in the form

$$L_p = 1/2(a^u)^T(2b)^{-1}a^u - 1/2(dx/dt)^T r_v^{-1}(x+v) - (x+v)^T(dx/dt). \quad (3.56)$$

Comments 3.5. Solution of the equation for differential constraint (3.20).

Let us consider the equation of constraint (3.21) in a more general form:

$$\frac{\partial A}{\partial x} + A^T F A = 0 \text{ at } A = A(x), A = A^T, F = F(x), x = \{x_i\}, i = 1,..,n.$$

Multiplying both sides of this equation on matrix A^{-1} and using the symmetry of A, we get

$$A^{-1}\frac{\partial A}{\partial x}A^{-1} + F = 0.$$

After differentiating matrix $A^{-1}A = I$ and then multiplying result on A^{-1} we get

$$A^{-1}\frac{\partial A}{\partial x}A^{-1} = -\frac{\partial A^{-1}}{\partial x},$$

which after substitution to the previous equation brings

$$-\frac{\partial A^{-1}}{\partial x} + F = 0.$$

By integrating the last equation with respect to all x components, we obtain the solution

$$A^{-1} = \int F dx_i, A^{-1}(x_1,...,x_{i-1},0,x_{i+1},...).$$

In particular, considering the simplified equation for a single-dimensional x:

$$F = 2I, \frac{\partial A}{\partial x} + 2A^T A = 0, \text{ where } A^T(x) = A(x) = \begin{bmatrix} a(x), b(x) \\ b(x), c(x) \end{bmatrix}$$

and substituting it into the previous matrix equation, we get the solutions

$$\int F dx = 2Ix = \begin{bmatrix} 2x, 0 \\ 0, 2x \end{bmatrix}, \; A^{-1}(O) = \begin{bmatrix} \frac{c}{ac-b^2}, -\frac{b}{ac-b^2} \\ -\frac{b}{ac-b^2}, \frac{a}{ac-b^2} \end{bmatrix},$$

$$A = \begin{bmatrix} \frac{2x(ac-b^2)+a}{4x^2(ac-b^2)+2x(a+c)+1}, \frac{b}{4x^2(ac-b^2)+2x(a+c)+1} \\ \frac{b}{4x^2(ac-b^2)+2x(a+c)+1}, \frac{2x(ac-b^2)+c}{4x^2(ac-b^2)+2x(a+c)+1} \end{bmatrix}.$$

The last solution can be directly applied to the matrix equation (3.20) for

$$A = X = \begin{bmatrix} X_{11}, X_{12} \\ X_{21}, X_{22} \end{bmatrix} = \begin{bmatrix} a(x), b(x) \\ b(x), c(x) \end{bmatrix}, \; A(0) = X(x=0) = \begin{bmatrix} X_{11}(0), X_{12}(0) \\ X_{21}(0), X_{22}(0) \end{bmatrix}.$$

The above solutions can be equal at the state coordinate points…

$$x^* = \left\lceil \frac{a(0) - b(0)}{2(a(0)c(0) - b^2(0))} \right\rceil$$

with the following equalities for the conjugate coordinates, determined by x^*:

$$X_{11}(0) \neq X_{12}(0),\ X_{11}(x^*) = X_{12}(x^*) = X_{21}(x^*) = \frac{b(0)}{4x^{*2}(a(0)c(0) - b^2(0)) + 2x^*(a(0) + c(0)) + 1}.$$

There is also a possibility of the equalization of solutions

$$X_{12}(x) = X_{21}(x), X_{22}(x),$$

or all solutions

$$X^o(x) = X_{ij}(x), i, j = 1,2$$

for the considered initial variables

$$X_{11}(0) = X_{22}(0) \neq 0, X_{12}(0) \neq 0.$$

The constraint in the form (3.20) for each pair of variables X_i, X_k holds *four* equations

$$\partial X_i / \partial x_k = -2X_i X_k, \partial X_i / \partial x_i = -2X_i^2, \partial X_k / \partial x_i = -2X_i X_k, \partial X_k / \partial x_k = -2X_k^2, \quad (3.56a)$$

from which after substituting in

$$\partial X_i / \partial x_k = \partial X_k / \partial x_i = -2X_i X_k$$

the derivations from (3.34),(3.35) we get

$$b_i(\tau) = b_k(\tau), A_i(\tau) = A_k(\tau), x_i(\tau) = x_k(\tau),$$

and finally come to

$$X_i(\tau)X_k(\tau) = (X_i(\tau))^2, X_k(\tau)X_i(\tau) = (X_k)^2, X_k(\tau) = X_i(\tau). \quad (3.56b)$$

This brings a symmetrization of the matrix' solutions, binding the solutions, which, working jointly with variation equations (3.26a); others, leads to their potential integration in cooperative dynamics.

At

$$X_i = X_i^\alpha \pm jX_i^\beta, X_k = X_k^\alpha \pm jX_k^\beta,$$

because

$$X_i^\beta(\tau) = X_k^\beta(\tau) = 0,$$

these equations lead to

$$|X_i(\tau)|^2 = |X_i(\tau)X_k^*(\tau)|, |X_k(\tau)|^2 = |X_k(\tau)X_i^*(\tau)|, \quad (3.56c)$$

where $|X_i(\tau)X_k^*(\tau)| = X_i^\alpha X_k^\alpha,$

for $|X_i|^2 = (X_i^\alpha)^2 + (X_i^\beta)^2;$

it brings

$$|X_i(\tau)|^2=(X_i^{\alpha})^2=|X_i(\tau)X_k^*(\tau)|,$$

which holds true for both parts of (3.56c). These equations connect the conjugated variables (modeling the entangled physical forces, ch.1.9).

Comments 3.6. Condition $\tilde{S}_p=\min S_p$ is implemented by control $u_t=u(x_t(\tau))$, applied within (or at the border) of region $Q=\Delta^o\times R^n$, where extremals $x_t(\tau)$ exist.

Out of this region, the functional extreme: $\min S_p=S_p^o(x_t)$ is not defined, but the microlevel's functional $\tilde{S}_p(\tilde{x}_t)$ has a meaning. The question is: How to formulate the extremal principle using $\tilde{S}_p(\tilde{x}_t)$, which depends on microlevel process $\tilde{x}_t$?

Considering $\min S_p=S_p^o(x_\tau)$, where $S_p^o(x_\tau)$ takes a fixed ("frozen") value on the region's border (at $x_t(\tau)=x_\tau$), the difference: $\tilde{S}_p-\min S_p=\Delta\tilde{S}_p>0$ starts at $\tilde{x}_t(\tau+o)$ with $\Delta\tilde{S}_p(\tilde{x}_t(\tau+o))$. For its derivation:

$$-\frac{\partial\Delta\tilde{S}}{\partial t}=(a^u)^T\frac{\partial\Delta\tilde{S}}{\partial x}+\sum_{i,j=1}^{n}b_{ij}\frac{\partial^2\Delta\tilde{S}}{\partial x_i\partial x_j}+\frac{1}{2}(a^u)^T(2b)^{-1}a^u \tag{3.57}$$

the condition

$$(a^u)^T\frac{\partial\Delta\tilde{S}}{\partial x}+\sum_{i,j=1}^{n}b_{ij}\frac{\partial^2\Delta\tilde{S}}{\partial x_i\partial x_j}=0,\ \text{at}\ x=x(\tau+o)$$

for the constraint (3.19) is not fulfilled.

Let us express each components of (3.57) using the related functions on the external. We have

$$(a^u)^T\frac{\partial\Delta\tilde{S}}{\partial x}+\sum_{i,j=1}^{n}b_{ij}\frac{\partial^2\Delta\tilde{S}}{\partial x_i\partial x_j}\Rightarrow(a^u)^TX+b\frac{\partial X}{\partial x},$$

where according to (3.34) at $b=b(t)$ we get

$$\frac{\partial X}{\partial x}=(2b)^{-1}A\ \text{and}\ b(\tau)\frac{\partial X}{\partial x}(\tau)=1/2A(\tau),\ \text{while}$$

$$(a^u(\tau))^TX(\tau)=(a^u(\tau))^T(2b(\tau))^{-1}a^u(\tau). \tag{3.57a}$$

Applying mathematical expectations $E[\bullet]$ to (3.57a):

$$E[(a^u(\tau))^T(2b(\tau))^{-1}a^u(\tau)]=E[A^T(\tau)(2b(\tau))^{-1}(x(\tau)+v(\tau))(x(\tau)+v(\tau))^TA(\tau)]$$
$$=Tr[A(\tau)^T(2b(\tau))^{-1}r_v(\tau)A(\tau)]=-1/2Tr[A(\tau)],\ A(\tau)=-r_v(\tau)^{-1}b(\tau)$$

we get the constraint equation on the extremal in the form:

$$E[(a^u(\tau))^T(2b(\tau))^{-1}a^u(\tau)+1/2A(\tau)]=1/2Tr[-A(\tau)+A(\tau)]=0, \tag{3.57b}$$

where on a stable extremal's segment, $A(t)<0$ at any moment $t\in(s,\tau)$ belonging to the segment. At the points $\tau,\tau+\varepsilon$, the control's (1.5) sign is changed affecting the sign of

$a^u(\tau+\varepsilon)$ and the matrix' $A(\tau+\varepsilon)=-A(\tau)>0$, but it will not change the sign of a quadratic form in (3.57a). Therefore, out of the extremal we have

$$E[(a^u(\tau+\varepsilon))^T(2b(\tau+\varepsilon))^{-1}a^u(\tau+\varepsilon)-1/2A(\tau+\varepsilon)]$$

$$=-1/2Tr[A(\tau+\varepsilon)+A(\tau+\varepsilon)]=-Tr[A(\tau+\varepsilon)],$$

and get a total derivation's math expectation in the forms

$$E[-\frac{\partial\Delta\tilde{S}(\tilde{x}_t(\tau+\varepsilon))}{\partial t}]=Tr[-A(\tau+\varepsilon)+1/4A(\tau+\varepsilon)],$$

or $E[\frac{\partial\Delta\tilde{S}(\tilde{x}_t(\tau+\varepsilon))}{\partial t}]=3/4Tr[A(\tau+\varepsilon)]>0$,

where on the extremal:

$$E[-\frac{\partial\Delta\tilde{S}(x_t(\tau))}{\partial t}]\Rightarrow 1/4Tr[A(\tau)],A(\tau)<0;$$

$$E[\frac{\partial\Delta\tilde{S}(x_t(\tau))}{\partial t}]\Rightarrow 1/4Tr\,|[A(\tau)]|>0. \qquad (3.58)$$

This means that at transferring from $x_t(\tau)$ to $\tilde{x}_t(\tau+o)$, the average entropy's derivation gets a jump, changing the above derivation on the extremal in three times.

For the process, transferred to the following extremal segment, the applied control should compensate this jump. Then, an analog of the initial variation equation

$$\min E[-\frac{\partial\Delta\tilde{S}(x_t(\tau)))}{\partial t}]=\min E[\frac{1}{2}(a^u)^T(2b)^{-1}a^u]>0 \qquad (3.58a)$$

acquires the form

$$\min E[-\frac{\partial\Delta\tilde{S}(\tilde{x}_t(\tau+\varepsilon))}{\partial t}]=\max E[\frac{\partial\Delta\tilde{S}(\tilde{x}_t(\tau+\varepsilon))}{\partial t}]=\max Tr[A(\tau+\varepsilon)]>0. \qquad (3.58b)$$

In this case, a special control u_t^δ (or v_t^δ) is needed to maximize the entropy derivation (3.58b) between the extremal's segments. Therefore, the controls should do both the selection of each extremal's segments (by the fulfillment of (3.57b),(3.58a)), acting at each segment's DP punched points (where the identification of operator (3.47) takes place), *and* connecting the extremal's segments between these points.

Example 3.1. Let us *illustrate* the results on a simple and rather formal example for the identification of unknown nonrandom symmetric matrix $A(t)$ by observing the solution of equation $\dot{x}=A(t)x$ via a measurement of the correlation function $r=E(x(t)x^T(t))$.

Substituting the solutions into identification equation (3.49a) (at $v(\tau)=0$) in form: $R_{in}=1/2\dot{r}r^{-1}$ with unknown R_{in} and using relations $\dot{r}=E(\dot{x}(t)x^T(t))+E(x(t)\dot{x}^T(t))$ $=E(A(t)x(t)x^T(t))+E(x(t)x^T(t)A^T(t))=Ar+rA^T$, at $A=A^T$, we obtain from this identification equation $R_{in}=A$. It seen that matrix A is identified precisely.

For a real object, the procedure involves statistical methods and a numerical computation. •

Alongside with the implementation of the functional's extreme by solving the VP problem (3.1)-(3.5), it is possible to show (at the fulfillment of (3.39) at a $\tilde{Q}$ boundary) that a minimum condition for the entropy functional (1.15b) at the segment's boundary is also satisfied.

1.3.3. The Minimum Condition for the Microlevel Functional

Theorem 3.3.

Let us have a closed measured set $\overline{B}_\tau = [s, \tau] \times \overline{B}$, where $[s, \tau] \subset \Delta, \overline{B} \subset \beta(\Omega)$, $B = \operatorname{int} \overline{B}$, $\Gamma \overset{def}{=} \overline{B} \setminus B$, $B_\tau = (s, \tau) \times B$, $\Gamma_\tau \overset{def}{=} \overline{B}_\tau \setminus B_\tau$ and consider the distribution of the entropy functional $S(\tilde{x}(\bullet))$ (1.15b), satisfying (3.16) on the set B_τ, as a function of time $t \in (s, \tau)$ and a current random $x \in B$:

$$-\frac{\partial S(t,x)}{\partial t} = \sum_{i=1}^{n} a_i^u(t,x)\,\frac{\partial S(t,x)}{\partial x_i} + \sum_{i,j=1}^{n} b_{ij}(t,x)\,\frac{\partial^2 S(t,x)}{\partial x_i \partial x_j} + W(t,x),\ (t,x) \in B_\tau\ ,$$

$$W(t,x) \geq 0, \tag{3.59}$$

and *solve* the following *boundary value problem*

$$S(s,x) = f_1(x),\ f_1(x) \in C(B, R_+^1), \tag{3.60}$$

or

$$S(\tau,x) = f_2(x),\ f_2(x) \in C(B, R_+^1)\ , \tag{3.60a}$$

and

$$S(t,y) = f_3(t,y), y \in \Gamma, f_3(t) \in C([s,\tau], R_+^1)\ . \tag{3.60b}$$

Then, the solution of problem (3.60), 3.60a,b) reaches its minimal value on the border of the set $\overline{B}_\tau$: e.g., at t=s, or $t = \tau$; or on the border Γ of the set $\overline{B}$.

This means that for $\exists (t_o, x_o) \subset B_\tau$, the following inequality is satisfied:

$$S(t_o, x_o) \geq \inf_{x \in B, y \in \Gamma} \min_{t \in (s,\ \tau)} [f_1(x),\ f_2(x), f_3(t,y)]. \tag{3.61}$$

Proof. Using the concept of proving the maximum principle (for the heat transfer problem [35]), let us assume the opposite by considering such moments $\exists (t_o, x_o) \subset B_\tau$ when (3.61) is satisfied in the form:

$$S(t_o, x_o) - \inf_{x \in B, y \in \Gamma} \min_{t \in (s,\ \tau)} [f_1(x), f_2(x), f_3(t,y)] \leq -\varepsilon, \varepsilon > 0\ . \tag{3.62}$$

Let us form an auxiliary function

$$V(t,x) = S(t,x) + \frac{\varepsilon(t_o - t)}{2\tau} \tag{3.63}$$

and show that it takes a minimal value on the set B_τ.

Because the set $\overline{B}_\tau$ is closed and limited, and the second Weierstrass' theorem for continuous functions [25] is fulfilled on $\overline{B}_\tau$, function $V(t,x)$ reaches a precise lower limit on $\overline{B}_\tau$.

Using inequality (3.62) and function (3.63) we have

$$V(s,x) \geq f_1(x) - \frac{\varepsilon t}{2\tau} \geq f_1(x) - \frac{\varepsilon}{2} \geq \inf_{x \in B} f_1(x) - \frac{\varepsilon}{2}, \tag{3.64a}$$

$$V(t,x) \geq f_2(x) - \frac{\varepsilon t}{2\tau} \geq f_2(x) - \frac{\varepsilon}{2} \geq \inf_{x \in B} f_2(x) - \frac{\varepsilon}{2}, \tag{3.64b}$$

$$V(t,x) \geq f_3(t,y) - \frac{\varepsilon t}{2\tau} \geq f_3(t,y) - \frac{\varepsilon}{2} \geq \inf_{y \in \Gamma} \min_{t \in (s,\tau)} f_3(t,y) - \frac{\varepsilon}{2}. \tag{3.64c}$$

From (3.59), (3.60), (3.60a,b), (3.64a-c) it follows relations:

$$\inf_{x \in B} f_1(x) - S(t_o, x_o) \geq \varepsilon, \tag{3.65a}$$

$$\inf_{x \in B} f_2(x) - S(t_o, x_o) \geq \varepsilon, \tag{3.65b}$$

$$\inf_{y \in \Gamma} \min_{t \in (s,\tau)} f_3(t,y) - S(t_o, x_o) \geq \varepsilon. \tag{3.65c}$$

Because (3.63) leads to $V(t_o, x_o) = S(t_o, x_o)$, after the joint solution of systems (3.64a-c) and (3.65a-c), we get

$$V(s,x) \geq V(t_o, x_o) + \frac{\varepsilon}{2}, \tag{3.66a}$$

$$V(\tau,x) \geq V(t_o, x_o) + \frac{\varepsilon}{2}, \tag{3.66b}$$

$$V(t,y) \geq V(t_o, x_o) + \frac{\varepsilon}{2}. \tag{3.66c}$$

From this system of the inequalities, it follows that the function $V(t,x)$ does not get a minimal value on Γ_τ. Because a minimum on $\overline{B}_\tau$ does exist, it means that the minimum can be reached at some inner points of $\overline{B}_\tau$, and therefore, on the set B_τ.

For function $V(t,x)$ at the points (t^*, x^*) of its minimum, the following relations hold true:

$$\frac{\partial V}{\partial t}(t^*, x^*) = 0; \; \frac{\partial V}{\partial x_i}(t^*, x^*) = 0, \; i = 1, \ldots, n; \tag{3.67}$$

$$\frac{1}{2} \sum_{i,j=1}^{n} h_{ij} \Delta x_i \, \Delta x_j \geq 0, \; h_{ij} = \frac{\partial^2 V}{\partial x_i \partial x_j}(t^*, x^*), \; \Delta x_i = x_i - x_i^*. \tag{3.67a}$$

A current state x of the diffusion process for any $\delta > 0$ satisfies the equality [22]:

$$\int_{|x - x^*| \leq \delta} \Delta x_i \, \Delta x_j \, P(t^*, x^*, t^* + \Delta t, dy) = 2 b_{ij}(t^*, x^*) \, \Delta t + o(\Delta t) \tag{3.68}$$

Because δ is an arbitrary, we may choose it from the condition

$$B_\delta = \{ |x - x^*| \geq \delta \} \subset B.$$

Let us integrate (3.68) by the probability measure $P(t^*, x^*, t^* + \Delta t, B_\delta)$ on the set B_δ and then divide both of the integral's sides on $\Delta t \to 0$.

Using the condition

$$\lim_{\Delta t \to 0} \frac{o(\Delta t)}{\Delta t} = 0 \ , \forall t^* \in (s, \tau),$$

we get from (3.67a), (3.68)

$$\sum_{i,j=1}^{n} h_{ij}(t^*, x^*) b_{ij}(t^*, x^*) = \sum_{i,j=1}^{n} \frac{\partial^2 V}{\partial x_i \partial x_j}(t^*, x^*) b_{ij}(t^*, x^*) \geq 0. \tag{3.69}$$

Applying jointly equalities (3.67) and (3.69), we write the following inequality

$$\frac{\partial V}{\partial t}(t^*, x^*) + \sum_{i=1}^{n} \frac{\partial V}{\partial x_i}(t^*, x^*) a_i^u(t^*, x^*) + \sum_{i,j=1}^{n} \frac{\partial^2 V}{\partial x_i \partial x_j}(t^*, x^*) b_{ij}(t^*, x^*) \geq 0 . \tag{3.69a}$$

Using relations (3.66a-c), (3.69a) we have

$$\frac{\partial V}{\partial t}(t, x) = \frac{\partial S(t, x)}{\partial t} - \frac{\varepsilon}{2\tau}, \frac{\partial V}{\partial x_i}(t, x) = \frac{\partial S(t, x)}{\partial x_i}, \frac{\partial^2 V}{\partial x_i \partial x_j}(t, x) = \frac{\partial^2 S(t, x)}{\partial x_i \partial x_j},$$
$$i, j = 1, \ldots, n . \tag{3.69b}$$

After substituting (3.69a) into (3.69b) at $t = t^*$, $x = x^*$ we get

$$\frac{\partial S(t^*, x^*)}{\partial t} + \sum_{i=1}^{n} \frac{\partial S}{\partial x_i}(t^*, x^*) a_i^u(t^*, x^*) + \sum_{i,j=1}^{n} \frac{\partial^2 S}{\partial x_i \partial x_j}(t^*, x^*) b_{ij}(t^*, x^*) \geq \frac{\varepsilon}{2\tau} . \tag{3.70}$$

Equations (3.60) and (3.70) lead to the inequalities

$$- W(t^*, x^*) \geq \frac{\varepsilon}{2\tau} > 0, \ W(t^*, x^*) < 0. \tag{3.70a}$$

Because the inequality $W(t, x) \geq 0$ is correct by definition, we come to a contradiction that proves the initial statement. •

Corollary 3.3a.

Let us compare the IPF minimum, reached within (and at the borders) of an IPF extremal's segment $x_t^i = x^i(t)$, whose both ends are fixed at some moments $t \in (s, \tau)$ of the initial random process $\tilde{x}_t$ (at $x^i = \tilde{x}^i(s)$ and $x^i = \tilde{x}^i(\tau)$ accordingly), *with* a minimum of the entropy functional (3.61), reached at the same $t \in (s, \tau)$ (according to T3.3).

Then we conclude:

(i). The T3.3 results show that both minimum *coincides at the same* $x^i = \tilde{x}^i(s)$ and $x^i = \tilde{x}^i(\tau)$.

(ii). To fix these extremal segment's ends, the random process $\tilde{x}_t$ should be cut off at each moment, following the first exit from the process' border on the above set. (Here we are

using the random process' cut off as an effective method for the process' fixing at its exit from some interval [22]).

In particular, considering $\tau = T$ in (3.60), (3.60a, b) and taking the $t = T$ as a moment of cutting off the process, we have

$$f_2(x) = f_3(t, y) = 0. \tag{3.70b}$$

(iii). Thus, following (3.70b) and T3.3, the *absolute minimum* of $S(t, x)$-function will be reached at the moment of the process' $\tilde{x}_t$ exit of the bordered moments $t \in (s, \tau)$ by the process' *cut off* at these moments.

The operation of cutting off the random process is performed by the controls, which are able to keep the random process within a given set.

The *n*-dimensional Markov process generally might have n such the first exits on Γ_τ.

(iv). According to the Jensen inequality for both IPF (1.21a-b),(1.22) and the entropy functional (1.15c), the entropy functional's minimum estimates *an upper limit* for the IPF minimum, considered also *out* of the above $t \in (s, \tau)$. •

In the *Example* (ch.1.1) with the entropy functional (1.16),(1.16a), (1.16d), the "cut off" operation is performed by the control $u = u(s, \tau) \in \Gamma_u$ at a moment $\tau + o$ of the exit from Γ_u. This provides both the entropy functional minimum and an upper limit for the IPF minimum.

The dynamic process represents a prognostic movement among the probable random trajectories.

1.3.4. The Optimal Control Synthesis

The problem of optimal synthesis for controls (3.23) is solved in the following sequence.

Using the Lagrange principle of eliminating constraints, we formulate the corresponding Bolza problem to find both regular and special controls.

Then, we find the class of an admissible control function v_t from relations (3.11)-(3.13), (3.22), (3.47), and implement the transversality conditions to determine the jump's value for functions v_t, A_t, and get the moments $\{\tau_k\}_{k=1}^m$ of their occurrences.

Finally, we will find the solution of the optimal control problem for v_t, combined with the problem of identification for A_t (3.49) under this control.

The above problems are solved at $b = b_t$, $V = R^n$, using the constraint's equations in the forms (3.41),(3.42c).

Applying the method of eliminating the constraints for problems (3.1)-(3.5), (3.45), we have

$$\min_{v \in V} S_{pl}(s, T, x_t(x_s, \mathrm{v})) = \min_{v \in V} \{\int_0^T L_p^o dt + l(x, \mathrm{v})\} = S_{pl}^o, \; x^s = x_t^s, x_T = (o_i)_{i=1}^n, \tag{3.71}$$

where L_p^o is defined by eqs. (3.2a,b), and $l = l(x, \mathrm{v})$ is the functional's terminal part that uses equation of constraint (3.45) at $Q^0 = (\bigcup \tau_k \times R^n)$ in the form

$$l=\sum_{k=0}^{m} l_k^{\pm},\ \ l_k^{\pm}=\sum_{i,j=1}^{n}\lambda_{ijk}^{\pm}\psi_{ijk}^{\pm}, l_k^{\pm}\in Q_{\pm}^{o},\ \ \Lambda_k^{\pm}=\{\lambda_{ijk}^{\pm}\}\in L(R^n,R^n)\ , \tag{3.72}$$

$$\psi_{ijk}^{\pm}=1/2\delta_{ij}+(x_i(\tau_k)+v_i(\tau_k\pm o))X_j(\tau_k\pm o, x\ (\tau_k))$$

$$-\sum_{\nu=1}^{n}(A^{-1}(\tau_k\pm o)b(\tau_k\pm o))_{i\nu}\varepsilon_{i\nu}(\tau_k\pm o)\,, \tag{3.73}$$

where $\Lambda_k^{\pm}$ are the matrices of Lagrange's multipliers, indexes $\pm$ correspond to the matrice's values at $t=\tau_k\pm o$, and

$$\delta_{ij}=\begin{cases}1, i=j\\ 0, i\neq j\end{cases}. \tag{3.73a}$$

Lemma 4.1.

The joint solution of the constraint equations (3.72-3.73a) and (3.44) leads to the following constraint's forms:

$$l_k^{\pm}=\{\sum_{i,\nu=1}^{n}(\Lambda_k^{\pm}(2b)^{-1}A)_{i\nu}(x+v)_i(x+v)_\nu\}\,|_{t=\tau_k\pm o}+1/2Tr(\Lambda_k^{\pm})\,|_{t=\tau_k\pm o}-(\sum_{i,j=1}^{n}\Lambda_k^{\pm}\sum_{\nu=1}^{n}(A^{-1}b)_{i\nu}\varepsilon_{\nu j})\,|_{t=\tau_k\pm o}\,, \tag{3.74}$$

$$\frac{\partial l_k^{\pm}}{\partial x_q}=2(\sum_{i,\nu=1}^{n}(\Lambda_k^{\pm}(2b)^{-1}A)_{i\nu}(x+v)_\nu\delta_{iq})\,|_{t=\tau_k\pm o}-(\sum_{i,j=1}^{n}\Lambda_k^{\pm}\sum_{\nu=1}^{n}(A^{-1}b)_{i\nu}\partial\varepsilon_{\nu j}/\partial x_q)\,|_{t=\tau_k\pm o}=$$

$$=2(\Lambda_k^{\pm}(2b)^{-1}A(x+v))_q\,|_{t=\tau_k\pm o}, q=1,....,n, k=0,...,m \tag{3.75}$$

$$\frac{\partial l_k^{\pm}}{\partial x_{\tau_k}}=2\Lambda_k^{\pm}X^{\pm},\ X^{\pm}=X(\tau_k\pm o),\ \frac{\partial l_k^{\pm}}{\partial\tau_k}=0,\ \ k=0,...,m\,. \tag{3.76}$$

The result follows directly after transforming (3.72)-(3.73a) to equation

$$\sum_{i,j=1}^{n}\Lambda_k^{\pm}(x+v_{\pm})_i\sum_{\nu=1}^{n}((2b_{\pm})^{-1}A_{\pm})_{j\nu}(x+v_{\pm})_\nu=$$

$$\sum_{i,j=1}^{n}(\sum_{j=1}^{n}\Lambda_k^{\pm}((2b_{\pm})^{-1}A_{\pm})_{j\nu})(x+v_{\pm})_i(x+v_{\pm})_\nu=\sum_{i,\nu=1}^{n}(\Lambda_k^{\pm}(2b_{\pm})^{-1}A_{\pm})_{i\nu}(x+v_{\pm})_i(x+v_{\pm})_\nu \tag{3.76a}$$

and substituting (3.76a) into (3.44).

Equations (3.75),(3.75a), (3.76) define the constraint as a function of the control for the solution of Bolza's problem.

Theorem 4.1. (T4.1)

Problem (3.71) with constraint (3.74)-(3.76) has a solution under

(1)-the class of the piece-wise controls (3.23);

(2)-the controls, which are switched at the moments $\tau \in \bigcup_{k=1}^{m} \tau_k$, defined by the conditions of equalization of the dynamic model's relative phase speeds:

$$|dx_i / dt\ (\tau_k - o)\ x_i^{-1}(\tau_k)| = |\ dx_j / dt(\tau_k - o) x_j^{-1}(\tau_k)|,$$

$$x_i(\tau_k) \neq 0, x_j(\tau_k) \neq 0,\ i, j = 1,\ldots,n\ ; \tag{3.77}$$

(3)-the controls, which at moments (3.77) change the model's matrix from $A_- = A(\tau_{k-o})$ to its renovated form $A_+ = A(\tau_{k+o})$ (at a subsequent extremal segment), while both matrices are identifiable by the following relations for the conditional covariance (correlation) functions:

$$A_- = 1/2\, \dot{r}_- r^{-1} = 1/2\, r^{-1} \dot{r}_-, \dot{r}_- = \dot{r}(\tau_k - o), r = E_-(xx^T), E_- = I_{\tau_k - o}, \tag{3.78}$$

$$A_+ = \pm\ A_-(1 + \mu_v^1)^{-1} = \pm (1 + \mu_v^1)^{-1}\ A_-,\ \mu_v^1 \in R^1,\ \mu_v^1 \neq -1, \tag{3.78a}$$

or

$$A_+ = \pm\ 1/2\, r^{-1} \dot{r}_- (1 + \mu_v^2)^{-1} = \pm\ 1/2\, \dot{r}_- r^{-1} (1 + \mu_v^2)^{-1},\ \mu_v^2 \in R^1\ ,\ \mu_v^2 \neq -1\ ; \tag{3.78b}$$

(4)-*the control function*:

$$v_- = \angle_v^1 x_-, v_- = v(\tau_k - o), x_- = x(\tau_k - o), \angle_v^1 = \mu_v^1 I \neq 0\ , \tag{3.79a}$$

which changes matrix A_- to A_+ (according to (3.78a),

and *the control function*:

$$v_+ = \angle_v^2 x_+, \angle_v^2 = \mu_v^2 I \neq 0, x_+ = x(\tau_k)\ , \tag{3.79b}$$

which changes matrix A_+ to $A(\tau_k + o)$ (at the next extremal segment), where $A(\tau_k + o) = \pm A(\tau_k)(I + \angle_v^2)$,

or the control function:

$$v_+ - v_- = \angle_v^1 (x_- + v_-), v_+ = v(\tau_k), \tag{3.79c}$$

which changes the above matrix according to $A(\tau_k + o) = \pm A(\tau_k)(I + \angle_v^1)$.

Proof of Theorem T4.1(1) uses the representation of the equation for the conjugate vector X in two forms, one of them is

$$dX\ (t, x, a^u(t,x)) / dt = -\ A\ (2b)^{-1}\ A\ (x + v);\ b = b_t\ , \tag{3.80}$$

which follows from the Hamiltonian (3.10a) and from equation (3.12),(3.13), (3.15) at

$$dX_p / \partial t = dp / dt + dX / dt = -\partial H_p / \partial x = -(\partial a^u / \partial x)^T\ p\ ,$$

$$dX / dt = -\partial((1/2a^u)^T (2b)^{-1} a^u) / \partial x\ , \tag{3.80a}$$

where the equivalence of conditions (3.12),(3.13) will be reached by the control, applied at $t \in (\tau_{k-1}, \tau_k), k = 1,...,m, \tau_o = 0, \tau_m = T.$

The other form we derive by differentiating the right–hand side of relation

$$X = (2b)^{-1}\ r_1 r_v^{-1}(x+v) = E[(2b)^{-1}\ \dot{x}(x+v)^T]r_v^{-1}(x+v),$$

obtained after substitution r_1 from (3.49).

We get

$$\begin{aligned} &dX\ /\ dt = d\{E(2b)^{-1})\dot{x}(x+v)^T]\ /\ dt\}r_v^{-1}(x+v)^T\} \\ &+E[(2b)^{-1}\dot{x}(x+v)^T](d(r_v^{-1})/dt)(x+v) + E[(2b)^{-1}\dot{x}(x+v)^T]r_v^{-1}d(x+v)/dt. \end{aligned} \tag{3.80b}$$

The first components of the right–hand side of (3.80b) lead to:

$$\begin{aligned} &d\{E[((2b)^{-1}\dot{x}(x+v)^T)\}/dt)]r_v^{-1}(x+v) \\ &= \{E[(d(2b)^{-1}\dot{x})/dt)(x+v)^T] + E[(2b)^{-1})\dot{x}(\dot{x}+\dot{v})^T]\}r_v^{-1}(x+v) \\ &= \{E[(dX\ /\ dt)(x+v)^T] + E[X\dot{x}^T] + E[X\dot{v}^T]\}r_v^{-1}(x+v) \\ &= \{E[(dX\ /\ dt)(x+v)^T] + E[(2b)^{-1}A(x+v)(x+v)^T A] \\ &+E[(2b)^{-1}A(x+v)\dot{v}^T]\}r_v^{-1}(x+v)^T. \end{aligned} \tag{3.81}$$

By substituting (3.81) into the expression, following from (3.80):

$$E[dX\ /\ dt = -A(2b)^{-1}A(x+v)^T)] + (2b)^{-1}Ar_v, \tag{3.81a}$$

we have

$$\begin{aligned} &d\{E[(2b)^{-1}\dot{x}(x+v)^T]/dt\}r_v^{-1}(x+v) = -A(2b)^{-1}A(x+v) \\ &+(2b)^{-1}Ar_vAr_v^{-1}(x+v) + (2b)^{-1}AE[(x+v)\dot{v}^T]r_v^{-1}(x+v). \end{aligned} \tag{3.82}$$

For the second and third components of (3.80b) we come to the related equalities:

$$\begin{aligned} &\{E[(2b)^{-1}\dot{x}(x+v)^T]d(r_v^{-1})/dt\}(x+v) \\ &= E[(2b)^{-1}\dot{x}(x+v)^T](-1)r_v^{-1}\dot{r}_v r_v^{-1}(x+v) = (2b)^{-1}A(-1)\dot{r}_v r_v^{-1}(x+v) \\ &= (2b)^{-1}A(-1)\dot{r}_v r_v^{-1}(x+v) \\ &= -(2b)^{-1}A\{r_1 + E[\dot{v}(x+v)^T] + E[(x+v)\dot{v}^T] + r_1^T\}r_v^{-1}(x+v); \end{aligned} \tag{3.82a}$$

$$E[(2b)^{-1}\dot{x}(x+v)^T]r_v^{-1}d(x+v)/dt = (2b)^{-1}AA(x+v) + (2b)^{-1}A\dot{v}^T. \tag{3.82b}$$

By the substitution of the obtained relations into (3.80b) we reduce it to the form

$$dX\ /\ dt = -A(2b)^{-1}A(x+v) + (2b)^{-1}A(\dot{v} - E[\dot{v}(x+v)^T])r_v^{-1}(x+v), \tag{3.83}$$

using relations

$$dX / dt = -A(2b)^{-1}A(x+v) + (2b)^{-1}Ar_v Ar_v^{-1}(x+v) +$$
$$(2b)^{-1}Ar_v Ar_v^{-1}(x+v) + (2b)^{-1}A(E[(x+v)\dot{v}^T])r_v^{-1}(x+v) - (2b)^{-1}A(r_1 + r_1^T)r_v^{-1}(x+v)$$
$$-(2b)^{-1}AE[\dot{v}(x+v)^T]r_v^{-1}(x+v)^T$$
$$+E[(x+v)\dot{v}^T]r_v^{-1}(x+v)^T + (2b)^{-1}AA(x+v) + (2b)^{-1}A\dot{v}^T$$
$$= -A(2b)^{-1}A(x+v) + (2b)^{-1}A(r_v Ar_v^{-1} - (r_1 + r_1^T)r_v^{-1} + A)(x+v)$$
$$+(2b)^{-1}A\{E[(x+v)\dot{v}^T]\}r_v^{-1} - E[(x+v)\dot{v}^T]r_v^{-1} - E\{\dot{v}(x+v)^T\}r_v^{-1}[(x+v)+\dot{v}]$$
$$= -A(2b)^{-1}A(x+v) + (2b)^{-1}A(r_1^T r_v^{-1} - (r_1 + r_1^T)r_v^{-1} + r_1 r_v^{-1})(x+v)$$
$$+(2b)^{-1}A\{\dot{v} - E[\dot{v}(x+v)^T]\}r_v^{-1}(x+v)$$
$$= -A(2b)^{-1}A(x+v) + (2b)^{-1}A\{\dot{v} - E[\dot{v}(x+v)^T]\}r_v^{-1}[(x+v].$$

From that, due to the validity of (3.80), we get from (3.83):

$$(2b)^{-1}A(\dot{v} - E[\dot{v}(x+v)^T])r_v^{-1}(x+v) = (0_{ij})_{i,j=1}^n = \mathrm{O}. \tag{3.83a}$$

At the fulfillment of inequalities

$$2b \neq (0_{ij})_{i,j=1}^n = \mathrm{O}, A \neq a^u = A(x+v) \neq (0_{ij})_{i,j=1}^n = \mathrm{O},$$

when

$$a^u = A(x+v) \neq (0_{ij})_{i,j=1}^n = \mathrm{O},$$

it follows

$$\dot{v} = E[\dot{v}(x+v)^T]r_v^{-1}(x+v). \tag{3.83b}$$

Equality (3.83b) can be identically true on $(\tau_k, \tau_{k-1}) \times R^n$ only if the control satisfies

$$dv / dt = (0_{ij})_{i,j=1}^n = \mathrm{O}.$$

Due to an arbitrariness of the chosen control's intervals $(\tau_k, \tau_{k-1}) \in \Delta^0$, we come to the result, which proves T4.1(1):

$$\dot{v}_t = (o_{ij})_{i,j=1}^n = \mathrm{O}, \ \forall (\tau_k, \tau_{k-1}) \in \Delta^0, \ k = 1,\ldots,m, \ v_t \in KC(\Delta, R^n). \tag{3.83c}$$

To prove T4.1(2) let us apply method [24] for formulating Erdmann-Weierstrass' condition at the *points* of control's *discontinuity* $\tau = \{\tau_k\}, k = 0,....,m$, using equations for the conjugate vector and Hamiltonian:

$$X_p(\tau - o) - \frac{\partial l}{\partial x}(\tau - o) = X_p(\tau + o) + \frac{\partial l}{\partial x}(\tau + o), \tag{3.84}$$

$$H_p(\tau - o) + \frac{\partial l}{\partial \tau}(\tau - o) = H_p(\tau + o) - \frac{\partial l}{\partial \tau}(\tau + o). \tag{3.85}$$

According to (3.75), (3.76) and because of the arbitrariness of index *k* in $\tau = \{\tau_k\}$, we represent relations (3.84), (3.85) in the forms

$$X^- + p^- - 2\Lambda^- X^- = X^+ + p^+ + 2\Lambda^+ X^+,$$
$$X^- = X_p(\tau - o), X^+ = X_p(\tau + o), p^- = p(\tau - o), p^+ = p(\tau + o), \tag{3.86}$$

$$H_p(\tau - o) = H_p(\tau + o). \tag{3.87}$$

By introducing auxiliary matrices $D^{\pm} \in L(R^n, R^n)$, $\forall X^{\pm}, p^{\pm} \exists D^{\pm}$, we assume

$$p^{\pm} = D^{\pm} X^{\pm}. \tag{3.88}$$

Applying (3.88) for the Hamiltonian and using equation $dX / dt = -AX$ and (3.49), we get at $t \in \{\tau_{k-o}\}_{k=0}^{m}$:

$$\begin{aligned} H_p &= X^T bX + (a^u)^T p = (x+v)^T A(2b)^{-1} b(2b)^{-1} A(x+v) + (x+v)^T AD(2b)^{-1} A(x+v) \\ &= 1/2 \sum_{i,j=1}^{n} [(A(2b)^{-1} A)_{ij} (x+v)_i (x+v)_j] + 1/2 \sum_{i,j=1}^{n} [(AD(2b)^{-1} A)_{ij} (x+v)_i (x+v)_j] \\ &= 1/2 \sum_{i,j=1}^{n} [(A(2b)^{-1} A + 2AD(2b)^{-1} A)_{ij} (x+v)_i (x+v)_j] \\ &= 1/2 \sum_{i,j=1}^{n} [(A(I+2D)(2b)^{-1} A)_{ij} (x+v)_i (x+v)_j], \end{aligned} \tag{3.88a}$$

where the mathematical expectation of equation's (3.88a) first component acquires the form

$$\begin{aligned} &E[X^T bX] = 1/2 \sum_{i,j=1}^{n} [(A(2b)^{-1} A)_{ij} r_{vij} = \\ &1/4 \sum_{i,j=1}^{n} (A(2b)^{-1} \dot{r}_v r_v^{-1})_{ij} r_{vij} = 1/4 \sum_{i,j=1}^{n} \sum_{k=1}^{n} (A(2b)^{-1} \dot{r}_v)_{ik} r_{vij}^{-1}) r_{vij} \\ &= 1/4 \sum_{i,j=1}^{n} (A(2b)^{-1} \dot{r}_v)_{ik} \sum_{k=1}^{n} r_{vij}^{-1} r_{vij} = 1/4 \sum_{i,j=1}^{n} (A(2b)^{-1} \dot{r}_v)_{ik} (\dot{r}_v r_v^{-1})_{ki} \\ &= 1/4 \sum_{i,k=1}^{n} (A(2b)^{-1} \dot{r}_v)_{ik} \delta_{ik} = 1/4 \sum_{i=1}^{n} (A(2b)^{-1} \dot{r}_v)_{ii} = 1/4 Tr[A(2b)^{-1} \dot{r}_v], \end{aligned} \tag{3.88b}$$

or applying $(2b)^{-1} = \dot{r}_v$ from (3.48) we obtain

$$E[X^T bX] = 1/4 TrA. \tag{3.88c}$$

Following the same procedure, we get the math expectations for both the second component and the Hamiltonian:

$$E[p^T a^u] = 1/2 Tr[AD(2b)^{-1} \dot{r}_v], \tag{3.89a}$$

$$E[H_p] = 1/4 Tr[A(I+2D)(2b)^{-1} \dot{r}_v]. \tag{3.89b}$$

From that and applying (3.49),(3.88a,b), (3.89a,b) to (3.87) we come to the following eqs

$$H_p(\tau + o) - H_p(\tau - o) = 1/4 \sum_{i,j=1}^{n} [(\Theta_+ \dot{r}_{v+})_{ij} (r_{v+}^{-1} (x + v_+)(x + v_+)^T)_{ij}$$

$$-\Theta_+^{-1}\dot{r}_{v+})_{ij}(r_{v-}^{-1}(x+v_-)(x+v_-)^T)_{ij}],\Theta_\pm = A_\pm(I+2D^\pm)(2b_\pm)^{-1}, \quad (3.89c)$$

at

$$r_{v+} = E_-(x+v_+)(x+v_+)^T, r_{v-} = E_-(x+v_-)(x+v_-)^T,$$
$$E_-[\bullet] = E_{x_{\tau-1}}[\bullet] = \int_{R^n}[\bullet]P^*_{\tau_{k-1}}(y)dy, y = x_{\tau_{k-1}}, \quad (3.89d)$$

where $E_-[\bullet]$ is an operator of the math expectation taken by the starting conditions $x_{\tau-1}$ at the moment $t=\tau_{k-1}, k=1,...,m-1$.

Substituting the obtained relations into equations (3.86), (3.87), we get

$$(I+2\Delta^+ + D^+)X^+ = (I+2\Delta^- + D^-)X^-, \quad (3.90)$$

$$\sum_{i,j=1}^{n}[(\Theta_+\dot{r}_{v+})_{ij}(r_{v+}^{-1}(x+v_+)(x+v_+)^T)_{ij} - (\Theta_-\dot{r}_{v-})_{ij}(r_{v-}^{-1}(x+v_-)(x+v_-)^T)_{ij}] = 0. \quad (3.90a)$$

Applying operator (3.89d) to (3.90a)) and taking into account (3.89b,c), we come to

$$Tr(\Theta_+\dot{r}_{v+}) - Tr(\Theta_-\dot{r}_{v-}) = 0,\ \Theta_\pm = A_\pm(I+2D^\pm)(2b_\pm)^{-1}. \quad (3.90b)$$

Since a matrix trace (Tr) and a matrix continuity are invariant under the linear transformations, equality (3.90b) must be satisfiable independently on a selected coordinate system.

This is possible at the fulfillment of equation

$$A_+(I+2D^+)(2b_+)^{-1}\dot{r}_{v+} = A_-(I+2D^-)(2b_-)^{-1}\dot{r}_{v-}. \quad (3.91)$$

Relation (3.90b) is a condition of the H_p continuity in equations (3.87), (3.89b)-(3.90b) by the probability measure, consistent with operator (3.89d).

Since $D^\pm \in L(R^n, R^n)$ are auxiliary matrices, which are not imposed by the variation principle, it is expedient to eliminate them from the subsequent analysis by selecting the Lagrange's multipliers in equations (3.74),(3.86) according to relations

$$(I+2\Delta^+ + D^+) \overset{def}{=} \pm(I+2\Delta^+), (I-2\Delta^- + D^-)\Delta^- = 1/2D^- \overset{def}{=} \pm(I+2\Delta^-). \quad (3.92)$$

From that we get equalities

$$\Delta^+ = 1/2D^+, or\Delta^+ = -I-3/2D^+; \Delta^- = 1/2D^-, or\Delta^- = -I-3/2D^-, \quad (3.92a)$$

and condition (3.90) acquires the form

$$(I+2D^+)X^+ = \pm(I+2D^-)X^-. \quad (3.93)$$

From equations (3.91), (3.93) we get the following relations

$$I+2D^+ = (A_+)^{-1}A_-(I+2D^-)(2b_-)^{-1}((2b_+)^{-1}\dot{r}_{v+})^{-1}, \quad (3.94)$$

$$X^+ = 1/2(2b_+)^{-1}\dot{r}_{v+}r_{v+}^{-1}(x+v_+), X^- = 1/2(2b_-)^{-1}\dot{r}_{v-}r_{v-}^{-1}(x+v_-), \quad (3.94a)$$

$$(A_+)^{-1}A_-(I+2D^-)(2b_-)^{-1}\dot r_{v-}r_{v+}^{-1}(x+v_+)=\pm(I+2D^-)(2b_-)^{-1}\dot r_{v-}r_{v-}^{-1}(x+v_-). \quad (3.95)$$

By multiplying equality (3.95) on $(x+v_+)^T$ and applying operator (3.89d) we obtain

$$\underline{(A_+)^{-1}A_-}\underline{(I+2D^-)(2b_-)^{-1}\dot r_{v-}}=\pm(I+2D^-)(2b_-)^{-1}\dot r_{v-}r_{v-}^{-1}E_-[(x+v_-)(x+v_+)^T]. \quad (3.96)$$

If we eliminate D^-, the equality (3.96) will serve for the identification of matrix A_+, being a result of the matrix's A_- transformation, or its renovation at the moment of applying the above controls.

This requires the fulfillment of the following related to $D^\pm$ equalities

$$D^\pm \overset{def}{=}(D^\pm)^T,(I+2D^\pm)(2b_\pm)^{-1}\dot r_{v\pm}\overset{def}{=}((I+2D^\pm)(2b_\pm)^{-1}\dot r_{v\pm})^T, \quad (3.97)$$

$$(I+2D^+)(2b_+)^{-1}\dot r_{v+}[(I+2D^-)(2b_-)^{-1}\dot r_{v-}]^{-1}$$

$$\overset{def}{=}\{(I+2D^+)(2b_+)^{-1}\dot r_{v+}[(I+2D^-)(2b_-)^{-1}\dot r_{v-}]^{-1}\}^T \quad (3.97a)$$

According to equations (3.91), (3.97), and (3.97a), the left side of (3.96) represents a symmetric matrix, which is equal to the product of the underlined symmetric matrices.

That is why we come to equality

$$(A_+)^{-1}A_-(I+2D^-)(2b_-)^{-1}\dot r_{v-}=(I+2D^-)(2b_-)^{-1}\dot r_{v-}(A_+)^{-1}A_-. \quad (3.97b)$$

From equalities (3.96), (3.95) and (3.97b) it follows the identification equation

$$A_+=\pm A_-E_-[(x+v_-)(x+v_+)^T]r_{v-} \quad (3.98)$$

as well as the relation for (3.95) in the form

$$E_-[(x+v_-)(x+v_+)^T]r_{v+}^{-1}(x+v_+)=(x+v_-). \quad (3.99)$$

Let us find a structure of controls (v_+,v_-), satisfying the accepted assumptions, i.e. (3.99), (3.90a), (3.91) and the identification equation (3.98) for each of them.

The following control functions with coefficients

$$\angle_v^1=\mu_v^1 I,\ \mu_v^1\in R^1: \quad (3.99a)$$

$$v_+=v_-+\angle_v^1(x+v_-),\ (x+v_+)=(\angle_v^1+I)(x+v_-),\angle_v^1\in L(R^n,R^n), \quad (3.100)$$

$$r_{v+}=(\angle_v^1+I)r_{v-}(\angle_v^1+I)^T,\ r_{v+}^{-1}=(\angle_v^1+I)^{-1}r_{v-}^{-1}((\angle_v^1+I)^{-1})^T,\ \angle_v^1\overset{def}{=}(\angle_v^1) \quad (3.100a)$$

$$(x+v_-)(x+v_-)^T(\angle_v^1+I)\overset{def}{=}(\angle_v^1+I)(x+v_-)(x+v_-)^T \quad (3.100b)$$

satisfy identically to (3.99), (3.90a) and are verifiable by a direct substitution, while condition (3.100b) is fulfilled identically at (3.99a).

Indeed, we have

$$E_-[(x+v_-)(x+v_-)^T](\angle_v^1+I)(\angle_v^1+I)^{-1}r_{v-}^{-1}(\angle_v^1+I)^{-1}(\angle_v^1+I)(x+v_-)$$

$$=E_-[(x+v_-)(x+v_-)^T]r_{v-}^{-1}(x+v_-)\equiv(x+v_-);$$

$$H_p(\tau+o)-H_p(\tau-o)=1/4\sum_{i,j=1}^{n}[(\Theta_+\dot{r}_{v+})_{ij}((\angle_v^1+I)^{-1}r_{v-}^{-1}(\angle_v^1+I)^{-1}(\angle_v^1+I)$$
$$\times(x+v_-)(x+v_-)^T(\angle_v^1+I))_{ij}-(\Theta_-\dot{r}_{v-})_{ij}(r_{v-}^{-1}(x+v_-)(x+v_-)^T)_{ij}]$$
$$=1/4\sum_{i,j=1}^{n}(\Theta_+\dot{r}_{v+})_{ij}((\angle_v^1+I)^{-1}(x+v_-)(x+v_-)^T(\angle_v^1+I)-r_{v-}^{-1}(x+v_-)(x+v_-)^T)_{ij}$$
$$=1/4\sum_{i,j=1}^{n}[(\Theta_+\dot{r}_{v+})_{ij}\left((0_{i,j})_{i,j=1}^{n}\right)]\equiv \mathrm{O}, \tag{3.100c}$$

which holds true with respect to (3.100b) and (3.99a).

By applying the last two equalities (3.100a,b), the equation (3.98) acquires the form

$$A_+=\pm\, A_-(\angle_v^1+I)^{-1},\ \text{or}\ A_+=\pm\, A_-(1+\boldsymbol{\mu}_v^1)^{-1}\ . \tag{3.101}$$

Considering the following relations with coefficients $\angle_v^2=\boldsymbol{\mu}_v^2 I$:

$$v_+=\angle_v^2 x,\ v_+-v_-=\angle_v^2 x-v_-,\ \angle_v^2\in L(R^n,R^n),\ \angle_v^2\overset{def}{=}(\angle_v^2)^T, \tag{3.102}$$
$$(x+v_+)=(\angle_v^2+I)x,\quad (x\,x^T)(\angle_v^2+I)\overset{def}{=}(\angle_v^2+I)(x\,x^T),$$
$$r_{v+}=(\angle_v^2+I)r(\angle_v^2+I),\ r=E_-(x\,x^T), \tag{3.102a}$$

we find the second form of the controls (3.102) with equalities (3.102a), which should also satisfy (3.99).

After the substitution we get relations

$$E_-[(x+v_-)x^T](\angle_v^2+I)(\angle_v^2+I)^{-1}r^{-1}(\angle_v^2+I)^{-1}(\angle_v^2+I)x$$
$$=E_-[(x+v_-)x^T]r^{-1}x=x+v_-\ ; \tag{3.103}$$

$$r^{-1}x=(E_-[(x+v_-)x^T])^{-1}(x+v_-);\ r_{v-}^{-1}(x+v_-)=r_{v-}^{-1}(E_-[(x+v_-)x^T])r^{-1}x\,, \tag{3.103a}$$

$$(E_-[(x+v_-)x^T])^{-1}r_{v-}=r^{-1}E_-[(x+v_-)x^T]. \tag{3.103b}$$

From that we obtain the following equation

$$(dx/dt)_-=E_-[(dx/dt)_-\,x^T]r^{-1}x. \tag{3.104}$$

Equalities (3.103), (3.103a,b), and (3.104) are equivalent. By writing the model in the form

$$dx/dt=A^v(t,x)x\ ,\ A^v=A[I+(\frac{v_j(\tau,.)}{x_i(t,.)}\delta_{ij})_{i,j=1}^{n}],\ x_i(t)\neq 0\ , \tag{3.105}$$

(where δ_{ij} is the Kronnecker delta) and comparing (3.104) with (3.105), we arrive at equation

$$E_-[A_-^v(x\,x^T)]=[E_-(A_-^v)]r\,, \tag{3.106}$$

which is satisfied identically if equality $\angle_v^2=(\angle_{vij}^2\delta_{ij})_{i,j=1}^{n}$ is fulfilled.

In this case, matrix $A^{v}(\tau-0)=A^{v}(\tau_k-0)$ is independent on the initial random conditions x_{τ_k-1}, $k=1,\ldots,(m-1)$.

According to (3.89d), the matrix gets averaged by these variables.

From the above relations and (3.106) it follows

$$E_{-}(A_{-}^{v})=A_{-}^{v}=E_{-}[(dx/dt)_{-}x^{T}]r^{-1}, \tag{3.107}$$

and (3.104) is satisfied identically.

Therefore (3.103), (3.103a,b) are also true.

From that follows the fulfillment of (3.95)-(3.96).

After substituting relations (3.91),(3.100a,b) and (3.103a,b) into equation (3.89c), we arrive at

$$\{E_{-}[(x+v_{-})x^{T}]\}^{-1}(x+v_{-})x^{T}=\{[E_{-}[x(x+v_{-})^{T}]\}^{-1}x(x+v_{-})^{T}, \tag{3.108}$$

which fulfills (3.90a).

From that we get the condition which the considered controls should satisfy:

$$\begin{aligned}&[E_{-}(xx^{T})](v_{-}x^{T}+xv_{-}^{T})+[E_{-}(xv_{-}^{T})-E_{-}(v_{-}x^{T})](xx^{T})\\&+[E_{-}(xv_{-}^{T})](v_{-}x^{T})-[E_{-}(v_{-}x^{T})](xv_{-}^{T})=(0_{ij})_{i,j=1}^{n}=\mathrm{O}.\end{aligned} \tag{3.109}$$

Both equations (3.108) and (3.109) are satisfied identically if the following equality is valid at the moments (3.77):

$$v_{-}x^{T}=xv_{-}^{T},\ \ \tau\in\{\tau_k\},\ k=1,\ldots,m\ . \tag{3.110}$$

According to (3.55) and from the above equations we get relation

$$(dx/dt)_{-}(x+v_{-})^{T}=(x+v_{-})((dx/dt)_{-})^{T}, \tag{3.111}$$

which at $x_i(\tau_k)\neq 0,\ i=1,\ldots,n$ can be written in the form

$$(dx_i/dt)_{-}x_j(1+v_{j-}/x_j)=(dx_j/dt)_{-}x_i(1+v_{i-}/x_i),i,j=1,\ldots,n. \tag{3.112}$$

This equation with respect to condition (3.110) assumes the form

$$(dx_i/dt)_{-}x_j=(dx_j/dt)_{-}x_i,\ \ \tau\in\{\tau_k\},\ k=1,\ldots,m. \tag{3.113}$$

The validity of (3.113) (or (3.110) is provided by the corresponding selection of the moments $\{\tau_k\}$, $k=1,\ldots,m$ of the applied control. This *proves* T4.1(2).

Let us write equality (3.107), according to its connection to (3.113), in the form

$$A_{-}^{v}=1/2(dx/dt)_{-}r^{-1}=1/2r^{-1}(dx/dt)_{-}. \tag{3.114}$$

Because equality (3.102a) is satisfied identically if the following relation is true

$$\angle_{v}^{2}=\mu_{v}^{2}I,\ \mu_{v}^{2}\in R^{1}, \tag{3.115}$$

and taking into account equalities (3.102, 3.102a), the equation (3.98) acquires the form

$$A_+ = \pm\, 1/2(\angle_v^2 + I)^{-1}\, r^{-1}\, (dx/dt)_- = \pm\, 1/2(\angle_v^2 + I)^{-1}\, (dx/dt)_-\, r^{-1}\,, \qquad (3.116)$$

or it can be represented in other form, consistent with relation (3.115):

$$A_+ = \pm\, 1/2(1+\boldsymbol{\mu}_v^2)^{-1}\, r^{-1}\, (dx/dt)_- = \pm\, 1/2(1+\boldsymbol{\mu}_v^2)^{-1}\, (dx/dt)_-\, r^{-1}\,. \qquad (3.117)$$

Using (3.98) at the moment $\tau = \tau_k$ and under the control $v_+ = v(\tau_k)$ (3.102) when $v_- = 0$, we have

$$A(\tau_k + o) = \pm A(\tau_k)[E_{\tau_k}(x(x+v_+)^T)]^{-1} r_{v+}, \qquad (3.117a)$$

where

$$r_{v+} = E_{\tau_k}(x+v_+)(x+v_+)^T = r_v(\tau_k),$$

$$r_v(\tau_k) = E_{\tau_k}(x+\angle_v^2 x)(x+\angle_v^2 x)^T = (I+\angle_v^2)E_{\tau_k}(xx^T)(I+\angle_v^2)^T,$$

$$E_{\tau_k}(x(x+v_+)^T) = E_{\tau_k}(xx^T)(I+\angle_v^2)^T = (I+\angle_v^2)^{-1}(I+\angle_v^2)E_{\tau_k}(xx^T)(I+\angle_v^2)^T$$
$$= (I+\angle_v^2)^{-1} r_{v+};$$

$$[E_{\tau_k}(x(x+v_+)^T)]^{-1} r_{v+} = (I+\angle_v^2) r_{v+}^{-1} r_{v+};\; A(\tau_k + o) = \pm A(\tau_k)(I+\angle_v^2)\,. \qquad (3.117b)$$

The composite control (3.79c) represents both v_+ and v_-. This control, being applied to (3.98) as $v_- = v(\tau_k - o)$, while using relations (3.100), brings

$$\ldots\ldots A(\tau_k) = \pm A(\tau_k - o)(I+\angle_v^1)^{-1},$$

which corresponds to (3.78a),(3.101); and being applied to (3.98) as $v_+ = v(\tau_k)$, while using relations (3.100), leads to

$$A(\tau_k + o) = \pm A(\tau_k)(I+\angle_v^1)\,. \qquad (3.117c)$$

The above relations prove both T4.1(3) and T4.1(4). •

Corollary 4.1

The values of $\boldsymbol{\mu}_v^2, \boldsymbol{\mu}_v^1$ for controls (3.100) or (3.102) are obtainable from the additional conditions (e.g., $\boldsymbol{\mu}_v^1$ from the condition $x(T) = x_T = (0_{ij})_{i,j=1}^n = \mathrm{O}$).

At the same A_- in all (3.78),(3.78a, b) we have $\boldsymbol{\mu}_v^1 = \boldsymbol{\mu}_v^2$. Since the feedback control is applied to a closed loop system, it is natural to assume the fulfillment of equality $A_+ = A_-^v$, which, according to (3.114),(3.117, (3.105), is satisfied at $\boldsymbol{\mu}_v^2 = (0, -2)$.

The first part of this option ($\boldsymbol{\mu}_v^2 = 0$) is inconsistent with applying new controls.

Hence, the remaining part: $\boldsymbol{\mu}_v^2 = -2$ brings us to the resulting equalities for the synthesized control:

$$v_+ = -2\, x(\tau), \qquad (3.118)$$

$$\delta v = v_+ - v_- = -2x(\tau) - v_-,\ \delta v = v_t^{\delta}. \tag{3.119}$$

The last equation determines the control jump (a "needle" control's action), as the control, applied to the closed loop system.

The model's operator is identified at the moment τ of applied control according to formulas

$$A_+ = (A_+)^T = 1/2\,\dot{r}_-\, r^{-1}(\tau),\ \dot{r}_- = dr/dt\,|_{t=\tau-o},\ r(t) = E(x_t x_t^T). \tag{3.120}$$

By choosing $\mu_v^1 = -2$, we get from (3.78a), (3.79a) :

$$A_+ = \mp A_- \text{ and } v_+ = -2x(\tau) - v_-, \tag{3.120a}$$

being an equivalent of control (3.119).

Substituting $A_+ = -A_- = A_-^v$ and the control (3.120a) into (3.105) we get $v_- = 0$ and the control (3.118). At $A_+ = A_- = A_-^v$ we have $v_- = -2x$.

Sequentially applying of the both controls at the time interval $(\tau - o, \tau), \delta\tau = o(\tau)$ brings $v_- = v(\tau - o) = -2x(\tau - o)$, $v_+ = v(\tau) = -2x(\tau)$ (representing jointly the $\delta v(\tau)$ control), and sequentially changes the sign of the above matrices.

For an extremal's segment with a stable process, we have

$$A_- < 0 \text{ and } A_+(v(\tau - o)) > 0, A_- < 0.$$

Then $A_+(v(\tau)) < 0$, and therefore $\delta v(\tau)$ control connects the segments with the stable processes closing the $o(\tau)$ -window between them. •

Comments 4.1. Relations (3.11), (3.119) determine the values of the discrete controls, which initiate both the process of the optimal motion and the identification of unknown dynamic operator (3.120).

This is an essence of the *joined process of the optimal control and identification* (ch.1.1). The time of the identification is a part of the optimal process. As a distinction from the known discrete systems, the macroprocess' discrete intervals are determined by the dynamic constraint and the controls action with the macromodels operator, identified in the process of optimal motion; the macromodel operator is not given a priory.

Condition (3.114) corresponds to the equalization of the model's operator eigenvalues $A = (\lambda_i)_{i=1}^n$: $\lambda_i(\tau - o) = \lambda_j(\tau - o)$ at the moments, preceding to the applied controls actions (for the model, written in the diagonal form):

$$\dot{x}_i = \lambda_i(x_i + v_i), v_i = -2x_i(\tau), i = 1,...,n, \tag{3.121}$$

where v_i is an optimal control, reduced to a state vector x_i. Each such control v_i selects the corresponding extremal's segment $x_i(t, \tau - o)$.

The reduced control presents a projection of control u_t on each of the state macrocoordinates, which is consistent with the object's controllability and identifiability [41].

This control specifies the structure of the controllable drift-vector $a^u = A(x + v)$ and corresponding identifiable operator (3.78); it provides also the fulfillment of equalities (3.83a),(3.103).

The reduced controls in (3.128), (3.129), as the VP solution, are an important part of the macrosystem's structure, reflecting a mechanism of a self-control synthesis, being a function of the macrocoordinates.

These controls are also useful for a direct programming and prognosis of the macromovement.

At the known reduced control (3.100) or (3.102) and the corresponding identified operator (3.101) or (3.127), (for example at $x_T = 0$), the initial control (1.4) is found by solving equations with regard to u_t:

$$\dot{\bar{x}}_t = \dot{x}_t + \dot{\bar{x}}_t^1, \bar{a}^u(t, x_t) = A_t(x_t + v_t) + \dot{\bar{x}}_t^1, A_t(x_t + v_t) + \dot{\bar{x}}_t^1 = a(t, x_t + \bar{x}_t^1, u_t), \quad (3.122)$$

where $\bar{x}_t^1$ is a solution of (1.9), x_t is the solution of (3.22) at known A_t, v_t. •

Corollary 4.2.

Let us determine the constant D in (3.88) using the equivalent equations:

$$\dot{x} = 2bX, \dot{x} = br^{-1}x.$$

We get

$$X = 1/2hx, h = E[xx^T]^{-1}. \quad (3.123)$$

Following (3.88c) we have

$$E[H] = 1/2E[X^T\dot{x}] = 1/4TrA, \text{ at } \dot{x} = Ax, \quad (3.124)$$

where we may hold $X = \mu X_p = \mu hx$.

By substituting, we get

$$E[X^T\dot{x}] = E[x^T h\mu Ax] = \mu TrA = 1/2TrA. \quad (3.125)$$

From that we have $\mu = 1/2, X = 1/2X_p$, and at $X_p = X + p$, we get $p = 1/2X_p = X$.

Using (3.88): $p = DX$, we get $D = I$. This allows us to determine

$$E[H_p] = 1/4TrA + 1/2Tr[AD] = 3/4TrA \quad (3.126)$$

from (3.89c), while (3.124) coincides with

$$X = 1/2X_p \text{ in } E[H] = 1/2M[X^T\dot{x}] = 1/4E[X_p^T\dot{x}] = 1/4TrA, \quad (3.127)$$

and we have

$$E[X_p^T\dot{x}] = TrA,\ E[H_p] = 3/4E[X_p^T\dot{x}], E[p^T a^u] = E[H_p^u] = 1/2TrA. \quad (3.128)$$

Following equations

$$H = -\frac{\partial S}{\partial t}, H_p^u = -\frac{\partial S_p^u}{\partial t}, H_p = -\frac{\partial S_p}{\partial t}, \quad (3.128a)$$

we get relation

$$E[H]\Delta t = -\Delta\hat{S} = -E[\Delta S] \tag{3.129}$$

that defines an increment of the averaged *internal* entropy, which an object consumes, while relation

$$E[H_p^u]\Delta t = -\Delta\hat{S}_p^u \tag{3.129a}$$

defines an increment of the *external* object's average entropy, delivered by the controls (3.118), (3.119), whereas

$$E[H_p]\Delta t = -\Delta\hat{S}_p \tag{3.130}$$

defines the increment of a *total* average entropy.

We may conclude that the external entropy exceeds the internal entropy in *three times*, while the entropy's increment of the object's controls exceeds the internal entropy in *two* times. ●

Comments 4.2. At the extremals with a stable process we have

$$signA(\tau - o) = signA(\tau + o) < 0\,,\ A_- = A(\tau - o), A_+(v(\tau)) = A(\tau + o)\,,$$

while at applying the control $v(\tau - o)$ we get

$$signA(\tau) > 0\,, A(\tau) = A_+(v(\tau - o))\,.$$

This brings

$$\hat{H}_p = 3/4Tr[A(\tau - o)] < 0, \hat{H}_p^u = 1/2Tr[A(\tau)] > 0$$

with a total Hamiltonian increment

$$\hat{H}(\tau) = 1/2Tr[A(\tau)] - 3/4Tr\,|[A(\tau - o)]|$$
$$= -1/4Tr[A(\tau)], A(\tau) = -A(\tau - o)\,. \tag{3.130a}$$

Considering the entropy derivation's increment at the extremal under the control action we have

$$\frac{\partial\Delta S_p^u}{\partial t} = \frac{\partial S_p^u}{\partial t}(\tau - o) - \frac{\partial S_p^u}{\partial t}(\tau) = -\hat{H}_p^u(\tau - o) + \hat{H}_p^u(\tau)$$
$$= -\hat{H}_p^u(\tau - o) + \hat{H}_p^u(\tau) = -1/2Tr[A(\tau - o)] + 1/2Tr[A(\tau)] = Tr[A(\tau)]\,. \tag{3.130b}$$

This means, the above control's action increases the entropy increment, which the microlevel provides, compared with that on the extremal.

The result is a consequence of applying the total entropy derivation (3.16) and removing the constraint (3.19) at transferring the process to the microlevel.

Therefore, compared with a minimum entropy at the extremals, the control $v(\tau - o)$ brings a maximum entropy derivation at the moments τ (between the extremal segments) delivering it to each segment.

This is an essence of the considered *minimax* VP, which also corresponds to *Comments* 3.6.

1.3.5. A Summary of the Information Path Functional Approach. The IPF invariants

In this sec. we summarize an *essence* of the IPF approach, *illustrate the theorems results and some limitations.*

The initial assumptions and the problem.

Let us have two random processes, one of them $\tilde{x}_t = \tilde{x}_t(u)$ is a controlled process, being a solution of equation (1.1.1), another one $\tilde{x}_t^1$ (as a solution of (1.1.7)) is given as a programmed process**;** the task for the controlled process consists of moving $\tilde{x}_t = \tilde{x}_t(u)$ close to $\tilde{x}_t^1$ by applying control u, or moving the difference $\tilde{x}_t(u) - \tilde{x}_t^1 = \tilde{x}_t^*(u)$ to its minimum.

The problem formalization.

The control task can be formalized by the evaluation of these processes $\delta-$closeness via a probability measure $P[\rho_\Delta(\tilde{x}_t, \tilde{x}_t^1) < \delta]$, where $\rho_\Delta(\tilde{x}_t, \tilde{x}_t^1) = \rho_\Delta(\bullet, \bullet)$ is a metric distance in a Banach space (estimated with an accuracy of δ>0), with the requirement of this probability's maximum

$$\underset{\tilde{x}_t(u)}{Sup}\, P[\rho_\Delta(\tilde{x}_t, \tilde{x}_t^1) < \delta].$$

Process $\tilde{x}_t^*(u)$ is modelled by the solutions of a control stochastic differential equation Ito:

$$d\tilde{x}_t^* = a^u(t, \tilde{x}_t^*, u_t)dt + \sigma(t, \tilde{x}_t^*)d\xi_t,$$
$$\tilde{x}_{t=s}^* = \tilde{x}_s^*, t \in [s, T] = \Delta, s \in [0, T], \tilde{x}^* \in R^n, (t, \tilde{x}^*) \in (\Delta \times R^n), \quad (3.131)$$

whose function of shift $a^u = a^u(t, \bullet, \bullet)$ depends on control, and the diffusion component of the solutions

$$\zeta_t = \int_s^t \sigma(t, \xi_\upsilon)d\xi_\upsilon, \; E[\zeta_t] = \mathrm{O} \quad (3.132)$$

models an uncontrollable noise.

In this case, the control task can be *reduced* to these processes' maximum probability of $\delta-$closeness for difference $\tilde{x}_t(u) - \tilde{x}_t^1 = \tilde{x}_t^*(u)$ to ζ_t, *or* to null-vector O:

$$\underset{\tilde{x}_t(u)}{Sup}\, P[\rho_\Delta(\tilde{x}_t, \tilde{x}_t^1) < \delta] \to \underset{\tilde{x}_t^*(u)}{Sup}\, P[\rho_\Delta(\tilde{x}_t^*, \zeta_t) < \delta] \to \underset{\tilde{x}_t^*(u)}{Sup}\, P[\rho_\Delta(\tilde{x}_t^*, \mathrm{O}) < \delta]. \quad (3.133)$$

This control task we will express via a functional, defined on the process' trajectories using:

Proposition 5.1.

The probability in (3.133) in the logarithmic form is defined by relation

$$\ln P[\rho_\Delta(\tilde{x}_t^*, \zeta_t) < \delta] = -S(x_t^* / \zeta_t), \quad (3.134)$$

where $S(x_t^* / \zeta_t)$ is a conditional entropy functional of processes $\tilde{x}_t^*$ regarding ζ_t.

Proof follows from the representation of (3.134)(left) by the Radom-Nikodim density measure [17, 28] $\frac{d\mu^{\zeta}}{d\mu^{\tilde{x}_t^*}}$ of a transformation $\tilde{x}_t^*$ to ζ_t on a set B_δ:

$$\ln P[\rho_\Delta(\tilde{x}_t^*,\zeta_t)<\delta]=\int\limits_{B_\delta}\ln\frac{d\mu^{\zeta}}{d\mu^{\tilde{x}_t^*}}(\tilde{x}_t^*)P(d\tilde{x}_t^*)=E[\ln\frac{d\mu^{\zeta}}{d\mu^{\tilde{x}_t^*}}],\ B_\delta=\{\rho_\Delta(\tilde{x}_t^*,\zeta_t)<\delta\},\quad (3.135)$$

where

$$E[\ln\frac{d\mu^{\zeta}}{d\mu^{\tilde{x}_t^*}}]=E[-\ln\frac{d\mu^{\tilde{x}_t^*}}{d\mu^{\zeta}}]=-S(x_t^*/\zeta_t) \quad (3.135a)$$

and its left side is a math expectation of logarithmic probability's *functional density measure on trajectories* for process $\tilde{x}_t^*$ regarding process ζ_t, taken along the process' trajectories. •

Proposition 5.2.

For the considered Markov diffusion process (3.131), (3.132), the entropy functional (3.134):

$$S(\tilde{x}_t^*/\varsigma_t)=E_{s,x}[\varphi_s^T]=\tilde{S}\ , \quad (3.136)$$

expressed through an additive functional [28], (ch.1.1):

$$\varphi_s^T=1/2\int\limits_s^T a^u(t,\tilde{x}_t^*)^T(2b(t,\tilde{x}_t^*))^{-1}a^u(t,\tilde{x}_t^*)dt+\int\limits_s^T(\sigma(t,\tilde{x}_t^*))^{-1}a^u(t,\tilde{x}_t^*)d\xi(t)\,, \quad (3.136a)$$

acquires the form

$$\tilde{S}=1/2E_{s,x}[\int\limits_s^T a^u(t,\tilde{x}_t^*)^T(2b(t,\tilde{x}_t^*))^{-1}a^u(t,\tilde{x}_t^*)dt]\ ,\ b=1/2\sigma\sigma^T\,. \quad (3.137)$$

Proof follows from the definition of the density measure in (3.135) through an additive functional φ_s^T of the above diffusion processes ($\tilde{x}_t^*,\zeta_t$) (ch.1.1):

$$\frac{d\mu^{\zeta}}{d\mu^{\tilde{x}_t^*}}=\exp(-\varphi_s^T)\,. \quad (3.138)$$

At

$$E_{s,x}\{\int\limits_s^T(\sigma(t,\tilde{x}_t^*)^{-1}a^u(t,\tilde{x}_t^*)d\xi(t)\}=0\,, \quad (3.138a)$$

the fulfillment of equations (3.136), (3.138) lead to (3.137), which determines the entropy functional via the *integrand of the Ito equation's* functions of controllable shift and diffusion. •

This allows us to *reformulate* the problem (3.133) using the entropy functional:

$$\underset{\tilde{x}_t^*(u)}{Inf}\ \tilde{S}(\tilde{x}_t^*/\zeta_t)=\underset{\tilde{x}_t^*(u)}{Inf}\ \tilde{S}[\tilde{x}_t^*],\ \tilde{S}=E[\int\limits_s^T 1/2a^u(t,\tilde{x}_t^*)^T(2b(t,\tilde{x}_t^*))^{-1}a^u(t,\tilde{x}_t^*)dt]\ , \quad (3.139)$$

which connects this stochastic variation problem to the given initial Eq. (3.131).

A direct solution of this variation problem is complicated, because functional (3.137), integrated by the above probability measure, represents a *path functional* on the trajectories of a Markov diffusion process, which is connected to Feynman's path functional [2, 3].

That is why an essential are

The secondary level's problem.

We will approximate $\tilde{x}_t^*$ by a dynamic process x_t, which we call a *macroprocess*, defined in the space KC^1 (of the piece–wise differentiable, continuous functions), considering $\tilde{x}_t, \tilde{x}_t^1, \tilde{x}_t^*$ as the corresponding *microlevel* processes and ζ_t as a model of an irremovable disturbance for macrolevel, where $x_t = \overline{x}_t - \overline{x}_t^1$ is a difference between a current macrolevel process $\overline{x}_t$ and its given target $\overline{x}_t^1$.

Control u_t (acting on both Eqs. (1.1.1) and (3.131)) is formed as a function of dynamic variables (x_t), defined by a sought feedback equation: $u_t \overset{def}{=} u(x_t)$ at the limitation ch.1.1.

Then by analogy of problem (3.133), we require a maximum probability of $\delta-$*closeness* of x_t to ζ_t:

$$\underset{x_t}{Sup}\, P[\rho_\Delta(\zeta_t, x_t) < \delta]\ , x_t \in KC^1(\Delta, R^n), t \in [s,T] = \Delta, \tag{3.140a}$$

and impose on x_t the requirement to be a dynamic analog of $\tilde{x}_t^*$, while both requirements we formalize through the evaluation of these process' probabilities closeness by analogy with problem (3.133):

$$\underset{\tilde{x}_t^*}{Sup}\, P[\rho_\Delta(\tilde{x}_t^*, \zeta_t) < \delta] \Leftrightarrow \underset{x_t}{Sup}\, P[\rho_\Delta(\zeta_t, x_t) < \delta]\ , x_t \in KC^1(\Delta, R^n), t \in [s,T] = \Delta\ , \tag{3.140}$$

which also connects the processes by their mutual ability to approximate ζ_t (considered as a standard process for a comparison of both $\tilde{x}_t^*$ and x_t). Condition (3.140) formally connects both problems. According to this condition, the *control* task of moving the difference $\tilde{x}_t^*$ close to ζ_t, also approximates x_t through ζ_t and, therefore, leads to the *approximation* of $\tilde{x}_t^*$ by x_t, which *redefines the initial control problem*.

We *formulate* the secondary problem using

Proposition 5.3.

The probability condition (3.140) (right) by an analogy with (3.134) we define by relation

$$\underset{x_t}{Sup} \ln P[\rho_\Delta(\zeta_t, x_t) < \delta] = -\underset{x_t}{Inf}\ S[x_t],\ S = \int_s^T L(t, x, \dot{x})dt\ , \tag{3.141}$$

where x_t is an extremal of integral functional $S[x_t]$, which, as well as its integrand (in (3.141)), will be found using condition (3.140) in the form

$$\underset{\tilde{x}_t^*(u)}{Inf}\ \tilde{S}[\tilde{x}_t^*] = \underset{x_t}{Inf}\ S[x_t]\ . \tag{3.142}$$

Solving this variation problem (VP) will determine x_t as an *extremal* of the variation problemand identify the random process $\tilde{x}_t^*$ via this extremal. •

The relations (3.139)-(3.142) allow for the *dynamic* approximation of the entropy functional of the diffusion process by the integral functional $S[x_t]$ with a *maximal* probability, and also *connect* both the random process $\tilde{x}_t^*$ to this functional's extremal x_t, as its macroprocess, and this functional to *information theory*.

In this connection, $S[x_t]$ becomes an *information path functional* (IPF).

We illustrate the *specifics of the solution* of the above variation problem (VP) using condition (3.142) in the form:

$$\min_{x_t^*(u)} \tilde{S}[\tilde{x}_t^*] = \min_{x_t} S[x_t],, \tag{3.143}$$

where the condition's left hand side is fulfilled by applying the control, while the right hand functional is defined on its extremals, which provide its minimum.

The considered here and below functional (3.137) is the *conditional* entropy functional related to the model's noise.

Proposition 5.4.

The *solution* of variation problem (3.143) for the entropy functional brings the following equations of extremals for a vector x and a conjugate vector X accordingly:

$$\dot{x} = a^u,\ (t,x) \in Q\ , \tag{3.144a}$$

$$X = (2b)^{-1} a^u; \tag{3.144b}$$

and the *constraint*

$$a^u(\tau)X(\tau) + b(\tau)\frac{\partial X}{\partial x}(\tau) = 0\,, \tag{3.144c}$$

imposed on the solutions (3.144a), (3.144b) at some set

$$Q^o \subset Q\,,\ Q^o = R^n \times \Delta^o, \Delta^o = [0,\tau], \tau = \{\tau_k\}, k = 1,...,m\ ; \tag{3.144d}$$

where the controls (1.5)(ch.1.1) are applied at the above discrete moments (τ_k) (1.5a) .

Proof. Using the Jacobi-Hamilton (JH) equation for a function of action $S = S(t,x)$, defined on the extremals $x_t = x(t)$ of functional $S[x_t]$ at $(t,x) \in Q$, we have

$$-\frac{\partial S}{\partial t} = H, H = \dot{x}^T X - L, \tag{3.145}$$

where X is a conjugate vector for x and H is a Hamiltonian for this functional.

(All derivations here and below have vector form).

Let us consider the distribution of functional (3.137) on $(t,x) \in Q$ as a function of current variables $\tilde{S} = \tilde{S}(t,x)$, which satisfies the Kolmogorov (K) equation [25, 26, others], applied to the math expectation of functional (3.137) in the form:

$$-\frac{\partial \tilde{S}}{\partial t} = (a^u)^T \frac{\partial \tilde{S}}{\partial x} + b\frac{\partial^2 \tilde{S}}{\partial x^2} + 1/2(a^u)^T (2b)^{-1} a^u\ . \tag{3.146}$$

From condition (3.142) it follows

$$\frac{\partial S}{\partial t} = \frac{\partial \tilde{S}}{\partial t}, \frac{\partial \tilde{S}}{\partial x} = \frac{\partial S}{\partial x}, \tag{3.146a}$$

where $\frac{\partial S}{\partial x} = X, -\frac{\partial S}{\partial t} = H$. This allows us to join (3.146) and (3.146a) in the form

$$-\frac{\partial \tilde{S}}{\partial t} = (a^u)^T X + b\frac{\partial X}{\partial x} + 1/2 a^u (2b)^{-1} a^u = -\frac{\partial S}{\partial t} = H . \qquad (3.146b)$$

Applying to (3.146) the Hamilton equation $\frac{\partial H}{\partial X} = \dot{x}$ we get (3.144a), and after substituting it to (3.145) (at the fulfillment of (3.146a)) we come to Lagrangian

$$L = -b\frac{\partial X}{\partial x} - 1/2 \dot{x}^T (2b)^{-1} \dot{x} . \qquad (3.146c)$$

On the extremals $x_t = x(t)$, both a^u and b are nonrandom. After their substitution to (3.137) we get the integral functional $\tilde{S}$ on the extremals:

$$\tilde{S}_e[x(t)] = 1/2 \int_s^T (a^u)^T (2b)^{-1} a^u dt , \qquad (3.147)$$

which should satisfy the variation condition (3.143), or

$$\tilde{S}_e[x(t)] = S[x(t)] , \qquad (3.147a)$$

where both integrals are determined on the same extremals.

From (3.147a) it follows

$$L = 1/2 (a^u)^T (2b)^{-1} a^u , \text{ or } L = 1/2 \dot{x}^T (2b)^{-1} \dot{x} . \qquad (3.147b)$$

Both expressions for Lagrangian (3.146c) and (3.147b) coincide at

$$b\frac{\partial X}{\partial x} + \dot{x}^T (2b)^{-1} \dot{x} = 0 . \qquad (3.147c)$$

Applying to (3.147b) (at the fulfillment of (3.144a)) Lagrange's equation $\frac{\partial L}{\partial \dot{x}} = X$, we get the equations for the conjugate vector $X = \dot{x}^T (2b)^{-1}$ and $\dot{x} = L_k X, L_k = 2b$, which prove (3.144b).

The fulfillment of (3.146) by controls (1.5),(1.5a) is possible if equation (3.147c) is satisfied at each point $(t, x) \in Q$ of the functional field $\tilde{S}(x(t))$, except a certain set (3.144d):

$$Q^o \subset Q, Q^o = R^n \times \Delta^o, \Delta^o = [0, \tau], \tau = \{\tau_k\}, k = 1, ..., m ,$$

where the following relations hold true:

$$E_{x,\tau}[(a^u)^T \frac{\partial \tilde{S}}{\partial x} + b\frac{\partial^2 \tilde{S}}{\partial x^2}] = 0, a^u = a^u(t, x), b = b(t, x) . \qquad (3.147d)$$

Substituting to (3.147d) the relations from (3.146a), (3.144a) we come to (3.144c), which also follows from (2.13),(2.2).

This equation determines the constraint, which JH imposes on K to fulfill (3.143). •

The constraint allocates a set of the states $(x, \tau) \in Q^o$ for which the information path functional (IPF) coincides with the entropy functional. The IPF becomes a *dynamic*

equivalent of the EF (defined on trajectories of diffusion process), while the IPF extremals form the *dynamic equivalents* of the diffusion process' trajectories.

Such a trajectory, as a *most probable* solution of (1.249) (or (3.144a)) (in the Theorem 2.3' probability measure), starting with math expectations of the diffusion process' initial conditions (ch.1.1), is also *an equivalent of the process' macrotrajectory.*

Imposing the constraint (3.147d), implementing (3.143), is *initiated* by the controls (1.5), applied at the states' set (3.144d).

Remark. To prove (3.144a, b) we can also use directly an integral functional $S = S[x_t]$, defined on its extremals $x_t = x(t)$ in a form (3.141)(right), and connect it to the entropy functional (3.137) by the variation conditions (3.146a), with following after that the relations (3.146b)-(3.147d). (This proof will omit the probability condition (3.141)).

Corollary 5.1.

The control action on equation (3.146b), which implements the variation conditions (3.143) at the set of discrete moments (3.144d), requires *turning* the constraint (3.144c) *on* with changing the process function $-\partial\tilde{S}/\partial t$ from its *maximum* to a *minimum.*

Proof. With the fulfillment of (3.144c), Hamiltonian (3.146b) acquires the form

$$H = 1/2(a^u)^T (2b)^{-1} a^u , \tag{3.148}$$

which, after substituting (3.144a) to (3.146b), corresponds to the function's $-\partial\tilde{S}/\partial t$ minimum on the extremals when it coincides with the Hamiltonian:

$$\min_{x(t)}(-\partial\tilde{S}/\partial t) = H = 1/2(\dot{x})^T (2b)^{-1}\dot{x} , \tag{3.149}$$

whereas this function in (3.146) reaches its *maximum* when the constraint is not imposed .

Both minimum and maximum are conditional regarding the constraint imposition.

The constraint's imposition follows from the fulfillment of the variation conditions (3.143) that requires the satisfaction of JH equations.

The constraint's implementation is carried by the control switch, turning the constraint *on* (at some starting moment $\tau_k = \tau_k^o$) for the extremal movement and turning it *off* while transferring the dynamics to the process' stochastics (where the variation conditions are not satisfied).

Therefore, an extremal is determined by imposing the constraint during the control actions, which select an extremal *segment* from the initial random process. •

Below we find the limitations on the fulfillment of constraint equation (3.144c), which also restrict the controls action, specifically when it should be turn off.

Proposition 5.5.

The constraint in the form

$$\frac{\partial\Delta\tilde{S}}{\partial t}|_{x_\tau} = [(a^u)^T \frac{\partial\tilde{S}}{\partial x} + b\frac{\partial^2\tilde{S}}{\partial x^2}]|_{x_\tau} = 0 \tag{3.150}$$

corresponds to the operator equation [28]:

$$\tilde{L}S[x_\tau] = 0, \tilde{L} = a^u \frac{\partial}{\partial x} + b\frac{\partial^2}{\partial x^2},\ x_\tau = x(\tau)\ , (x,\tau) \in Q^o, \tau = \{\tau_k\}, \tag{3.150a}$$

whose solutions allow classifying the states $x(\tau) = \{x(\tau_k)\}, k = 1,...,m$, considered to be the *boundary* points of a diffusion process at $\lim_{t \to \tau} \tilde{x}^*(t) = x(\tau)$.

A boundary point $x_\tau = x(\tau)$ *attracts* only if the function

$$R(x) = \exp\{-\int_{x_o}^{x} a^u(y) b^{-1}(y) dy\}, \tag{3.150b}$$

defining the general solutions of (3.150a), is integrable at a locality of $x = x_\tau$, satisfying the condition

$$|\int_{x_o}^{x_\tau} R(x)dx| < \infty. \tag{3.150c}$$

From the above reference [28] and (3.150), (3.150a) it *follows* that a boundary point *repels* if equation (3.150c) does not have the limited solutions at this locality**;** it means that the above (3.150b) is not integrable. •

Comments 5.1. Therefore, the constraint equation (3.144c), or (3.147d) establishes a connection between the microlevel's diffusion and macrolevel's dynamics only at some "punched" *points* of the *space* $Q^o = R^n \times \Delta^o$, while macroequations (3.144a,b) act along each extremal, except the related discrete *points* (DP)

$$\Gamma_\tau = \bigcup_{k=1}^{m} \tau_k, \Gamma_\tau \in \Delta^o \tag{3.151}$$

with the states

$$x_\tau = x(\tau) = \{x(\tau_k)\},\ x(\tau) = E_\tau[\tilde{x}(\tau)], \tag{3.151a}$$

for which the information path functional (IPF) coincides with the entropy functiona (EF).
Thus, both the constraint imposition and the control action are limited by the punched points, $x(\tau_k), x(\tau_{k+1})$ when the control, starting at the moment $\tau_k^o = \tau_k + o$, should be *turned off* at a moment $\tau_k^1 = \tau_{k+1} - o$ preceding τ_{k+1}, which corresponds to the solution of (3.150a) for the extremal, approaching the punched point τ_{k+1}.

To continue the extremal movement, the control should be turned *on* again at the moment $\tau_{k+1}^o = \tau_{k+1} + o$ following τ_{k+1} to start a next extremal segment.
This determines a discrete action of control, imposing the constraint during the extremal movement within each interval

$$t_k = \tau_k^1 - \tau_k^o, t_{k+1} = \tau_{k+1}^1 - \tau_{k+1}^o, k = 1,...,m$$

by a step-wise control (m is the number of applied controls), which limits a time length of the extremal segment between the punched localities. (A very first control is applied at the moment $\tau_o^o = s + o$, following the process' initial condition (1.1.1) at $t = s$.)

The process continuation requires connecting the extremal segments between the punched localities by the joint step-wise control action: with a k-control *turning off* at the

moment τ_k^1 while transferring to τ_{k+1} locality, and a next $k+1$-control which, transferring from τ_{k+1} locality, is *turning on* at the moment τ_{k+1}^o in the following extremal segment.

Both step-wise controls form an impulse control function $\delta(x(\tau_k^1), x(\tau_{k+1}), x(\tau_{k+1}^o))$ acting between moments $\tau_k^1, \tau_{k+1}, \tau_{k+1}^o$ (or $\tau_{k-1}^1, \tau_k, \tau_k^o$) and implementing (1.23, ch.1.2).

As it follows from (3.150), (3.150a) the functional's (3.137) increments at each t_k, generated by the constraint between the punched points, is preserved:

$$\Delta S_c[x(t_k)] = inv,\ k = 1,...,m. \tag{3.152}$$

Equality (3.152), which turns to zero equation (3.150a) at each moment τ_k, indeed joins (3.150a) with the VP, becoming an information invariant that depends on both functions a^u, b. Condition (3.152) will be used to find the interval of applying the control t_k.

Comments 5.2. Equations (3.144c), (3.147d) are the necessary and sufficient conditions for the existence of the solutions (3.150a),(3.150b),(3.150c), which define the set of states $x_\tau = x(\tau)$, where the macrodynamics arise from stochastics and determine some *boundary* conditions, limiting the above set.

The necessary condition for the punched points to be attractive include: $b(y) \neq 0$, which corresponds to the existence of a regular diffusion process [28], and $a^u(y) \neq 0$-being necessary for the creation of the dynamics.

At $b(y) = 0$ both the entropy functional and the related IPF are degenerated: $\tilde{S} \to \infty$, $S \to \infty$; at $a^u(y) = 0$ the process' dynamics vanishes.

Thus, the fulfillment of (3.150b), (3.150c) guarantees that Eq. (3.150a) is integrable, the punched points, where the dynamics can start, exist and are attractive.

This *brings a quantum character of generation of the macrostates carrying the macrodynamic information at the VP fulfillment.*

The total information, *originated by the macrodynamics*, (following from (3.147**),** (3.151)), is evaluated by:

$$S(\Gamma_\tau) = \int_{\Gamma_\tau} a^u(\tau_k)^T (2b(\tau_k))^{-1} a^u(\tau_k) d\tau_k, \tag{3.153}$$

where the operator shift and the diffusion matrix are limited by (3.150b), (3.150c), Γ_τ is the union of a total number of τ_k time instants (3.151) for *n*-dimensional model (3.131).

Equations (3.150a), (3.153) also satisfy a *stationary* condition at τ_k-locality. •

The DP (3.151) divides the macrotrajectory into a *sequence* of the extremal segments limited by the punched localities, where the model's *randomness and regularities are connected*, and therefore the model's identification is possible.

At these points, the constraint (3.147d) is applicable for the *identification* of the random states and the related DP in the form

$$E_{x,\tau}[\frac{\partial X}{\partial x}(\tau) + XX^T(\tau)] = 0. \tag{3.153a}$$

Constraint (3.147c),(3.153a), imposed on the random process (through applying control (1.23)), selects also the process' most informative states (satisfying to the EF maximum, ch.

1.2). The constraint action between the punched localities *limits* the segment's length and determines the control's functions, which, acting along each segments, also connect them through a *random window* between the segments.

The constraint equation is the *main mathematical structure* that distinguishes the equations of diffusion stochastics from related *dynamic* equations: at the constraint imposition, both equation's solutions coincide (at the process's boundary points).

This corresponds to *transferring* the stochastics into dynamics, and vice versa, at the constraint termination. With the constraint aid, the path integral on random trajectories (EF) is expressed through the integral functional on dynamic trajectories (IPF).

Comments 5.3. Writing the equation of extremals $\dot{x} = a^u$ in a dynamic model's the traditional form

$$\dot{x} = Ax + u, u = Av, \dot{x} = A(x + v), \tag{3.154}$$

where v is a control, reduced to the state vector x, we will identify matrix A and find the control v that solves the initial problem.

After substituting $a^u = 2bX$ (following from (3.144b) at $b \neq 0$) to (3.144c), the *constraint* also acquires the forms

$$\frac{\partial X}{\partial x}(\tau) = -2XX^T(\tau) \ , \tag{3.155a}$$

$$a^u = \sigma(\tau)\sigma(\tau)^T X(\tau) \ . \tag{3.155b}$$

Substituting

$$X = (2b)^{-1}A(x+v), X^T = (x+v)^T A^T (2b)^{-1}, \frac{\partial X}{\partial x} = (2b)^{-1}A \tag{3.156}$$

to (3.154) and (3.155) we get

$$(2b)^{-1}A = -2E[(2b)^{-1}A(x+v)(x+v)^T A^T (2b)^{-1}], \tag{3.157}$$

from which, at a nonrandom A and $E[b] = b$, we obtain the equations for the identification of matrix

$$A(\tau) = -b(\tau)r_v^{-1}(\tau), r_v = E[(x+v)(x+v)^T], b = 1/2\dot{r}, r = E[\tilde{x}\tilde{x}^T] = r_v(\tau) \tag{3.150b}$$

via the above correlation functions, or directly, by the dispersion matrix b from (3.131), (3.137), (3.157a):

$$|A(\tau)| = b(\tau)(2\int_{\tau-o}^{\tau} b(t)dt)^{-1} > 0 \ , \tag{3.157b}$$

where $\tau - o = (\tau_k - o), k = 1..., m.$

The fulfillment of relations (3.154),(3.157),(3.157a) is reached with the aid of the control's action, which we find using (3.157) in the form

$$A(\tau)E[(x(\tau)+v(\tau))(x(\tau)+v(\tau))^T] = -E[\dot{x}(\tau)x(\tau)^T], \text{at } \dot{r} = 2E[\dot{x}(\tau)x(\tau)^T]. \tag{3.158}$$

This relation after substituting (3.149) leads to

$$A(\tau)E[(x(\tau)+v(\tau))(x(\tau)+v(\tau))^T] = -A(\tau)E[(x(\tau)+v(\tau))x(\tau)^T],$$

and then to

$$\ldots E[(x(\tau)+v(\tau))(x(\tau)+v(\tau))^T+(x(\tau)+v(\tau))x(\tau)^T]=0,$$

which is satisfied at applying control

$$v(\tau)=-2x(\tau)\ . \tag{3.158a}$$

Because $x(\tau)$ is a discrete set of states, satisfying (3.150a), (3.151a), the control has a discrete form. This control imposes the constraint (in the form (3.157), (3.157a)), which follows from the variation conditions (3.143), and, therefore it implements this condition.

This control, applied to both the random process and the extremal segment, initiates the identification of matrix $A(\tau)$. When the control turns off: $v(\tau)=0$, equations (3.158) and below, following from the constraint, change the sign, which leads to the constraint termination. Finding this control here just *simplifies* some results of Theorem *T4*, sec.1.3.4.

We also *illustrate* the theorem's results, considering alongside with model (3.154) and control (3.158a), the model of a closed system with a feedback in the form

$$\dot{x}(t)=A^v x(t)\,, \tag{3.159}$$

where matrix A^v is a subject of both the object's characterization and identification.

Using both (3.154) and (3.159):

$$a^u(\tau,x(\tau,t))=A(\tau,t)(x(\tau,t)+v(\tau));\ A(\tau)(x(\tau)+v(\tau))=A^v(\tau)x(\tau) \tag{3.159a}$$

at applying control (3.158a), we get

$$A^v(\tau)=-A(\tau)=1/2b(\tau)r_v^{-1}(\tau),b(\tau)=1/2\dot{r}_v(\tau)\,. \tag{3.159b}$$

In particular, by applying control $v(\tau_k^o)=-2x(\tau_k^o)$, which imposes the constraint during time interval $t_k=\tau_k^1-\tau_k^o$ (starting it at τ_k^o and terminating at τ_k^1) to both (3.154) and (3.159), we get the same solutions by the *end* of this interval:

$$x(\tau_k^1)=x(\tau_k^o)[2-\exp(A(\tau_k^o)\tau_k^1)]\,. \tag{3.160}$$

Substituting this solution to $\dot{x}(\tau_k^1)=A^v(\tau_k^1)x(\tau_k^1)$ we come to

$$-x(\tau_k^o)A(\tau_k^o)\exp(A(\tau_k^o)\tau_k^1)=A^v(\tau_k^1)x(\tau_k^o)[2-\exp(A(\tau_k^o)\tau_k^1)], \tag{3.160a}$$

or to the connection of both matrixes $A^v(\tau_k^{1`})$ and $A(\tau_k^1)$ (at the interval end) with the matrix $A(\tau_k^o)$ (at the interval beginning) in the forms:

$$A^v(\tau_k^1)=-A(\tau_k^o)\exp(A(\tau_k^o)\tau_k^1)[2-\exp(A(\tau_k^o)\tau_k^1)]^{-1}$$

and

$$A(\tau_k^1)=A(\tau_k^o)\exp(A(\tau_k^o)\tau_k^1)[2-\exp(A(\tau_k^o)\tau_k^1)]^{-1}\,. \tag{3.160b}$$

Following (3.159a),(3.159b) and (3.160b) we come to changing both matrixes signs:

$$A^v[v(\tau_k^{1`})]=-A[v(\tau_k^{1`})],A^v(\tau_{k+1}^{`})_{v=0}=A(\tau_{k+1}^{`})_{v=0},A^v[v(\tau_{k+1}^{o`})]=-A(\tau_{k+1}^{`})_{v=0} \tag{3.160c}$$

at $(\tau_k^1,\tau_k,\tau_{k+1}^o)$-locality , where the nearest segments are connected through the microlevel.

The fulfillment of (3.159a) allows identifying matrix A as a component of the drift vector a^u in (3.159b) via the dispersion (3.157b) at the moments τ .

Specifically, this moment, following the end of the time interval $t_{k-1} = \tau_{k-1}^1 - \tau_{k-1}^o$, is $\tau = \tau_k$, when relations $b(\tau_k) = 1/2\dot{r}_v(\tau_k)$, or $r(\tau_k) = r_v(\tau_k)$ (in (3.157a), (3.159b)) are satisfied, indicates that the solution of the constraint equation reaches the boundary points of the diffusion process.

At this moment, the control, acting on both Ito's equation (1.1.1) and its dynamic model (3.154), should turn the constraint *off*, transferring the extremal movement to a random process.

The control, turning the constraint *on*, creates a dynamic model, satisfying Hamilton equations (3.149). Such a dynamic model possesses a matrix A with the complex (imaginary) eigenvalues (Example 3.3).

After the constraint's termination (at the diffusion process' boundary point), the control transforms this matrix to its real form (3.157b) (being a component of the drift vector in (1.1.1)and (3.160)).

Controlling the dynamic model's current $r(t)$ allows (according to (3.157a)) identifying the model current $A(t) = 1/2r^{-1}(t)\dot{r}(t)$ (see Example 3.1).

Comments 5.4. Specifying the constraint equations (3.156), (3.157) by the end at any interval $t_k, k = 1,...,m$ (at the moment τ_k^1) and using (3.56a-c), we have

$$X_i = A_i(x_k + v_k)(2b_i)^{-1}, X_k = A_k(x_i + v_i)(2b_k)^{-1}, r_{ik} = E[(x_i + v_i)(x_k + v_k)] = r_{ki} \ ;$$

$$\frac{\partial X_i}{\partial x_k} = 2b_i A_i = -2E[(2b_i)^{-1} A_i(x_k + v_k)(x_i + v_i)A_k(2b_k)^{-1}]; A_k = -(r_{ki})^{-1} b_k$$

$$\frac{\partial X_k}{\partial x_i} = 2b_k A_k = -2E[(2b_k)^{-1} A_k(x_i + v_i)(x_k + v_k)A_i(2b_k)^{-1}]; A_i = -(r_{ik})^{-1} b_i,$$

and

$$\frac{\partial X_i}{\partial x_k} = -2X_i X_k = \frac{\partial X_k}{\partial x_i}.$$

This leads to $b_i A_i = b_k A_k$, and $b_i (r_{ik})^{-1} b_i = b_k (r_{ki})^{-1} b_k$, which brings

$$b_i = b_k \text{ and } A_i = A_k, \tag{3.161}$$

or for the matrix eigenvalue i, k , considered at the same moment τ_k^1 , we get

$$\lambda_i(\tau_k^1) = \lambda_k(\tau_k^1) \ . \tag{3.161a}$$

Applying(3.161) for complex conjugated eigenvalue

$$\lambda_i(\tau_k^1) = \alpha_i(\tau_k^1) + j\beta_i(\tau_k^1), \lambda_i^*(\tau_k^1) = \lambda_i(\tau_k^1) = \alpha_i(\tau_k^1) - j\beta_i(\tau_k^1),$$

at the moment τ_k^1 of fulfilling the constraint Eq, we come to

$$\mathrm{Im}\,\lambda_i(\tau_k^1) = \beta_i(\tau_k^1) = \mathrm{Im}[\lambda_i^*(\tau_k^1)] = -\beta_i(\tau_k^1),\ 2\beta_i(\tau_k^1)=0, \mathrm{Im}\,\lambda_i(\tau_k^1) = 0 \qquad (3.161b)$$

and

$$\mathrm{Re}\,\lambda_i(\tau_k^1) = \mathrm{Re}\,\lambda_i^*(\tau_k^1). \qquad (3.161c)$$

It seen that fulfillment of any (3.161),(3.161a-c) actually follow from imposing the constraint (starting at τ_k^o) , which by the moment τ_k^1 reaches the $(\tau_{k+1} - o)$ locality.

Indeed. After applying the control at the moment τ_k^o (to both object and the model), according to (3.155a) and (3.146b):

$$b(t)\frac{\partial X(t)}{\partial x(t)} + A(\tau_k^o, t)x(\tau_k^o, t)(2 - \exp(A(\tau_k^o)t))^T X^T(t)$$

$$+1/2a^u(\tau_k, \tau_k^o, t)(2b(t)^{-1}(a^u(\tau_k, \tau_k^o, t)))^T = -\frac{\partial S}{\partial t}(t),$$

the model moves to fulfill (3.155a) at moment τ_k^1 when the constraint reaches the form

$$\frac{\partial X(\tau_k^1)}{\partial x(\tau_k^1)} + 2A(\tau_k^o, \tau_k^1)x(\tau_k^o, \tau_k^1)(2 - \exp A(\tau_k^o)\tau_k^1)X^T(\tau_k^1) = 0\ . \qquad (3.161d)$$

Thus, the fulfillment of (3.161d) *indicates* the moment τ_k^1 when the solution of the constraint equation approaches the process' punched locality, and control $v(\tau_k^o)$ should be turned off, transferring the extremal movement to a random process at the following moment τ_{k+1}.

As it follows from Prop. 3.5, the state $x_t(\tau)$ is a *dynamic* analog of the random $\tilde{x}_t(\tau)$ as its boundary point.

Therefore this dynamic state can also be used to form the control (3.158) starting a next extremal segment, while each of these controls connects the segments between the boundary points.

Both random process and its dynamic model proceed under the same control (imposing the constraint during each t_k), which brings the same dynamic trends of functions $a^u(\tau_k, \tau_k^o, t), b(\tau_k, \tau_k^o, t), t \le \tau_k^1$, in both (1.1.1) and the extremal equations (3.144a,b), (3.161b).

These functions are renovated, being identified at each following moment τ_{k+1} , and reach $a^u(\tau_k, \tau_{k+1}), b(\tau_k, \tau_{k+1})$.

This provides a dynamic equivalence for the macroprocess and the diffusion process.

Proposition 5.6.

Operator (3.150), (3.153a) and the matrix's $A(\tau)$ eigenvalues for each of the model dimension $i = 1,...,n,$ are connected with the functionals (3.137) and (3.141) by equations

$$E[\frac{\partial \tilde{S}}{\partial t}(\tau)] = 1/4Tr[A(\tau)]\ , \qquad (3.162)$$

$$A(\tau)=(\lambda_i(\tau)),\lambda_i(\tau_k)=4E[\frac{\partial\tilde{S}_i}{\partial t}(\tau_k)],i=1,...n,k=1,...m\,. \tag{3.162a}$$

The *proof* follows from substituting the constraint (3.153a) and the relations (3.154), (3.157a) to (3.146) in the forms

$$E[-\frac{\partial\tilde{S}}{\partial t}(\tau)]=1/2E[(x(\tau)+v(\tau))^T A(\tau)^T(2b)^{-1}A(\tau)(x(\tau)+v(\tau))],$$
$$A(\tau)=-b(\tau)r_v^{-1}(\tau)\,, \tag{3.162b}$$

from which we get (3.162), where for each *i*-dimensional model's eigenvalue with $\tau=\tau_k$ we come to (3.162a). •

<u>*Comments* 5.5.</u> Substituting matrix $A(\tau)$ from (3.157b) to (3.162), we come to a direct expression of differential information entropy (negentropy) via the dispersion at the τ_k locality, where the macrolevel's dynamics receive information from the microlevel:

$$E[\frac{\partial\tilde{S}}{\partial t}(\tau_k)]=1/4Tr[b(\tau_k)(2\int_{\tau_k}b(t)dt)^{-1}]>0\,, \tag{3.162c}$$

and a total macrodynamic information is collected according to (3.153).

This information along with the information, delivered by the controls, is spent between each $\tau_k\,,(\tau_{k+1}-o)$ -localities during each time interval $t_k=\tau_k^1-\tau_k^o$.

The constraint's differential entropy (3.146) is

$$\frac{\partial\tilde{S}_c}{\partial t}=2bX^TX=(a^u)^T(2b)^{-1}a^u\,, \tag{3.163}$$

which we get after substituting X from (3.144b).

Comparing this information entropy, reduced to the state $x(\tau_k):\Delta S_c[x(\tau_k)]$, with that from the path functional $\Delta S[x(\tau_k)]$ in (3.147), we come to

$$\Delta S_c[x(\tau_k)]=2\ \Delta S[x(\tau_k)]$$

with a total model's entropy, starting at $x(\tau_k)$:

$$\Delta S_o[x(\tau_k)]=3\ \Delta S[x(\tau_k)]\,.$$

Evaluating integral of (3.162a) for each *i*-model's dimension by

$$E\int_{\tau_k}\frac{\partial\tilde{S}_i}{\partial t}(t)dt=E[\Delta\tilde{S}_i[x(\tau_k)]]=1/4\lambda_i(\tau_k)\tau_k\,,i=1,...n,k=1,...m,$$

we get the related constraint's average information

$$\Delta S_{ic}=1/2E[\Delta S_{ic}[x(\tau_k)]=1/2\lambda_i(\tau_k)\tau_k$$

with the total information

…. $E[\Delta S_{io}[x(\tau_k)]]=3/4\lambda_i(\tau_k)\tau_k\,,$

where both $\lambda_i(\tau_k)$ and τ_k are unknown before their measurement at τ_k locality.

From (3.152) we come to invariant

$$E[\Delta S_i[x(\tau_k)]]=1/2\lambda_i(\tau_k)\tau_k=inv,i=1,...n,k=1,...m. \tag{3.163a}$$

This entropy should be compensated by a full amount of control action, evaluated by information

$$E[\Delta S_i^u] = 1/2\lambda_i(\tau_k)\tau_k . \quad (3.163b)$$

From that we get a balance equation

$$E[\Delta S_{io}[x(\tau_k)]] - E[\Delta S_i^u] = E[\Delta \tilde{S}_i[x(\tau_k)] \quad (3.163c)$$

with the ratio

$$E[\Delta S_i^u] / E[\Delta S_{io}[x(\tau_k)] = 2/3 . \quad (3.164)$$

Information delivered by the control consists of two step-wise control's contributions

$$2E[\Delta S_i^v] = E[\Delta S_i^u],$$

while turning the control on at τ_k^o and off at τ_k^1.

This leads to the evaluation each of these control information contributions by

….. $E[\Delta S_i^v] = 1/4\lambda_i(\tau_k)\tau_k$,

which compensates for the related constraint's information consumption $E[\Delta S_{ic}^v]$, satisfying the balance relation

$$E[\Delta S_{ic}^v] = E[\Delta S_{ic}] - E[\Delta S_i^v], \quad (3.164a)$$

at the segment's end, prior a second step-wise control turns the constraint off.

According to (3.160c), we get the related eigenvalues' signs at each $(\tau_k^1, \tau_{k+1}, \tau_{k+1}^o)$-locality

$$-sign\lambda_i(\tau_k^1) = sign\lambda_i(\tau_k) = -\lambda_i(\tau_{k+1}^o) . \quad (3.164b)$$

Those relations lead to changing signs for the entropy derivations at this locality:

$$\ldots\ldots sign\frac{\partial S_i}{\partial t}(\tau_k^1) = sign\frac{\partial S_i}{\partial t}(\tau_{k+1}^o) < 0 . \quad (3.164c)$$

And also to changing the signs in the related information contributions in (3.163b).

Because within this locality (at τ_{k+1}) the entropy derivation at the microlevel is positive: $\frac{\partial \tilde{S}_i}{\partial t}(\tau_{k+1}) > 0$, we come to the negative entropy derivations at the macrolevel in (3.164c) .

Based on these, and using conditions (3.146), (3.149) for an extremal in the form

$$\max(-\frac{\partial S_i}{\partial t}(t_k)) = \min\frac{\partial S_i}{\partial t}(t_k),$$

we can reformulated this condition as reaching a minimum for the entropy derivations (by the moment τ_k^1) at the macrolevel through its maximum at the microlevel (at $\tau_k^1 + o = \tau_{k+1}$):

$$\max\frac{\partial \tilde{S}_i}{\partial t}(\tau_{k+1}) = \min\frac{\partial S_i}{\partial t}(\tau_k^1) = \min\frac{\partial S_i}{\partial t}(t_k) , \quad (3.164c)$$

where the maximum corresponds to the second thermodynamic law.

Just departing from (3.164c), implies that the considered above *variation conditions satisfy to this law*.

Because of the dynamic equivalence of both model and controllable process (within each t_k), reaching $\min\frac{\partial S_i}{\partial t}(\tau_k^1)$ means also reaching $\max\frac{\partial \tilde{S}_i}{\partial t}(\tau_{k+1})$ at the microlevel.

Thus, the dynamic model allows selecting the microstates (micro-distributions) having a *maximum* information entropy's derivation as the most informative (at the microlevel), which are used for the process control and identification.

Proposition 5.8.

The constraint's information consumption $E[\Delta S_{ic}^{v}]$ (3.164a) during interval t_k of applying controls $v(\tau_k^o) = -2x(\tau_k^o)$ brings the following two invariants:

$$\lambda_i(\tau_k^o)\tau_k^1 = inv,\ i = 1,..,n, k = 1,...,m\ , \tag{3.165}$$

$$\lambda_i(\tau_k^1)\tau_k^1 = inv\ , \tag{3.165a}$$

where $\lambda_i(\tau_k^o)$ and $\lambda_i(\tau_k^1)$ are the model eigenvalues taken at the moment τ_k^o and τ_k^1 of a segment's time interval t_k accordingly; whereas relation (3.163a) brings the information invariant

$$\lambda_i(\tau_k)\tau_k = inv\,. \tag{3.165b}$$

The *proof* of (3.165a,b) follows from both writing the constraint's information consumption $E[\Delta S_{ic}^{v}]$ during interval t_k of applying $v(\tau_k^o) = -2x(\tau_k^o)$ and using (3.163a) :

$$E[\Delta S_{ic}^{v}[x(t_k)] = E[\int_{\tau_k^o}^{\tau_k^1} \frac{\partial \tilde{S}_{ic}}{\partial t}(t)dt] = 1/2\int_{\tau_k^o}^{\tau_k^1} \lambda_i(t)dt$$

$$= 1/2[\lambda_i(\tau_k^1)\tau_k^1 - \lambda_i(\tau_k^o)\tau_k^o] = 1/4\lambda_i(\tau_k)\tau_k = inv\ , \tag{3.166}$$

where eigenvalue $\lambda_i(\tau_k)$ (identified at the moment τ_k) under the control action (at the moment τ_k^o) is transformed to $-\lambda_i(\tau_k^o)$, satisfying

$\lambda_i(\tau_k) = -\lambda_i(\tau_k^o)$ and $-\lambda_i(\tau_k^o)\tau_k^o = \lambda_i(\tau_k)\tau_k = inv$.

This brings the right side of (3.166) to the form

$$\lambda_i(\tau_k^1)\tau_k^1 - \lambda_i(\tau_k^o)\tau_k^o = -1/2\lambda_i(\tau_k^o)\tau_k^o, \lambda_i(\tau_k^1)\tau_k^1 = 1/2\lambda_i(\tau_k^o)\tau_k^o = inv\,. \tag{3.166a}$$

To prove (3.165) we use the eigenvalues' function

$$\lambda_i(\tau_k^1) = \lambda_i(\tau_k^o)\exp(\lambda_i(\tau_k^o)\tau_k^1)[(2 - \exp(\lambda_i(\tau_k^o)\tau_k^1)]^{-1} \tag{3.166b}$$

following from (3.161) after applying the control $v_i(\tau_k^o) = -2x_i(\tau_k^o)$ to equation (3.154) at $A = (\lambda_i), i = 1,...,k,...,n$.

Substituting (3.166b) to (3.166a) we obtain

$$\lambda_i(\tau_k^1)\tau_k^1 = \lambda_i(\tau_k^o)\tau_k^1 \exp(\lambda_i(\tau_k^o)\tau_k^1)[(2 - \exp(\lambda_i(\tau_k^o)\tau_k^1)]^{-1} = inv\ . \tag{3.166c}$$

This invariant condition leads to the filfullment of both (3.165) and (3.165a).

These two invariants define both the necessary (3.166a) and sufficient conditions (3.166) and therefore (3.152) for $E[\Delta S_{ic}^{v}]$. •

Corollary 5.2.

Invariants (3.165), (3.165a-b) have the following information forms:….

$$E[\Delta S_{io}(x(t_k)] = -1/2\lambda_i(\tau_k^o)\tau_k^1\ , \tag{3.167a}$$

$$E[\Delta S_{ic}^{v}(x(t_k)] = -1/2\lambda_i(\tau_k^1)\tau_k^1\,, \tag{3.167b}$$

which connect them to a total segment entropy and the control's information contribution (in (3.163a-c)) accordingly.

To prove that, we multiply (3.166c) on a constant *c*, aiming to link a segments' total entropy with invariant (3.165) by relation

$$E[\Delta S_{io}(x(t_k))] = c\lambda_i(\tau_k^o)\tau_k^1 \ , \qquad (3.168)$$

and the entropy contribution (from the constraint) *with* invariant (3.165a) by relation

$$E[\Delta S_{ic}^{v}(x(t_k))] = c\lambda_i(\tau_k^1)\tau_k^1 . \qquad (3.168a)$$

The constant $\boldsymbol{c}$ we find from (3.166a) following the relation

$$E[\Delta S_{ic}^{v}(x(t_k))] = c\lambda_i(\tau_k^1)\tau_k^1 = 1/4\lambda_i(\tau_k)\tau_k = -1/2\lambda_i(\tau_k^1)\tau_k^1 , \ c = -1/2 , \qquad (3.168b)$$

This allows writing (3.167) and (3.167a) in the forms (3.167a,b). •

By knowing the above invariant and the identified $\lambda_i(\tau_k^o) = \mid \lambda_i(\tau_k) \mid$ one can find the interval t_k of the control action, using condition (3.161a) (the details are in chs.2.1,2.2), while control $v_i(\tau_k^o) = -2x_i(\tau_k^o)$ is applied before finding t_k; at the interval's end $\tau_k^1 + o$ the identification of a next $\lambda_i(\tau_{k+1})$ takes place.

Comments 5.6. At the existence of random window between a pair of extremal segments (with a time interval $\Delta\tau_k = (\tau_k^1, \tau_{k+1}, \tau_{k+1}^o)$), the step–wise control

$$v_- = -2x(\tau_k^1) \ , \qquad (3.169)$$

applied at the moment τ_k^1 to a current extremal segment, turns the constraint off at the moment τ_k, when $x(\tau_k)$ is identified, which serves to form the second step-wise control

$$v_+ = -2x_i(\tau_{k+1}) \ , \qquad (3.169a)$$

being applied to a next extremal segment at the moment τ_{k+1}^o, starting the dynamics at a next segment similar to

$$v_k = -2x(\tau_{k+1}^o) .$$

Joint action of both step-wise controls forms an impulse control, which crosses the window.

Building the process' continuous dynamic trajectory (satisfying the VP) requires minimal random windows between the extremal segments.

A needle control v_t^δ, formed by a pair of the above step–wise controls and applied at a small $\delta\tau \to o(\tau)$ interval, provides a closest the segments connection:

$$\delta v(\tau_k^1, \tau_{k+1}, \tau_{k+1}^o) = v_+ - v_- = -2x(\tau_{k+1}^o) + 2x(\tau_k^1), \delta v(o(\tau)) = v_t^\delta, k = 1,...,m . \qquad (3.169b)$$

Controls (3.158a), (3.169),(3.169a), and (3.169b), solving the VP (specifically implementing a minimum in (3.149)), we call the *optimal* controls, which start at the beginning of each segment, act along the segment, and connect the segments in the macrodynamic optimal process.

The impulse needle δ-control (acting between the moments $(\tau_k^1, \tau_{k+1}, \tau_{k+1}^o)$) also performs a decoupling (a "decorrelation") of the pair correlations at *these* moments (ch.1.1, sec.1.1.8). The δ-control provides also the fulfillment of equality

$$\mid \frac{dx_i}{dt}(\tau_k^1)x_i^{-1}(\tau_k^1) \mid = \mid \frac{dx_j}{dt}(\tau_{k+1}^o)x_j^{-1}(\tau_{k+1}^o) \mid , \qquad (3.170)$$

which connects the current extremal segment's eigenvalue $\mid \lambda_i(\tau_k^1) \mid = \mid \frac{dx_i}{dt}(\tau_k^1)x_i^{-1}(\tau_k^1) \mid$ with

eigenvalue $|\lambda_j(\tau_{k+1})| = \left|\frac{dx_j}{dt}(\tau_{k+1})x_j^{-1}(\tau_{k+1})\right|$, identified at τ_{k+1}, and then to the eigenvalue $|\lambda_j(\tau_{k+1}^o)| = \left|\frac{dx_j}{dt}(\tau_{k+1}^o)x_j^{-1}(\tau_{k+1}^o)\right|$ at a beginning of next the extremal segments.

(These connections can also be used for the identification of the eigenvalues via observed states $x_i(\tau_{k+1})$ and their derivations).

If the eigenvalue, identified at τ_{k+1}, is renovating, compared to $\lambda_i(\tau_k^1)$, then the unequal eigenvalues $\lambda_i(\tau_k^1)$ and $\lambda_j(\tau_{k+1})$ are joint at a first connection, and at the second connection, the transformation $\lambda_j(\tau_{k+1})$ to $\lambda_i(\tau_{k+1}^o)$ at the moment τ_{k+1}^o takes place.

At this transformation, the step-wise control v_+ also serves for changing the eigenvalue $\lambda_i(\tau_{k+1})$ sign.

The reduced controls, built by the macrostates *memorized* at (τ_k^1, τ_{k+1}^o), according to (3.169) and (3.169a), presents a projection of control

$$u_t = Av_t = -2Ax(\tau)$$

on each of the state macrocoordinates.

The above results summarize and simplify the *object's dynamic macromodel* and the synthesized *optimal controls*, following from *Theorems (T1, T2)*, sec.1.3.2, and *T4*, sec.1.3.4.

The DP divide the macrotrajectory into a sequence of the extremal segments, defined by the solutions of macromodel (3.144a,b), (3.154), while the discrete controls (3.158a), (3.169) applied at each segment, allow a piece-wise approximation of the initial entropy functional, the functions of shift and diffusion as well.

For the IPF, defined by started functions of shift and diffusion, the IPF optimum predicts each extremal segment's movement not only in terms of a total functional path goal, but also by setting at each following segment the renovated values of these functions, identified during the optimal movement (via diffusion and correlations (3.157a), or directly by diffusion (3.157b)), which currently correct this goal.

In this optimal dual strategy, concurrently synthesized optimal control provides a maximal Markov's probability (ch.1.1) at each punch locality of the random process and, therefore, the considered identification with a maximal probability.

This optimal strategy for each current movement cannot be improved because it's defined by an extremal of a *total path* to a terminal state, which is renovated at each control's action.

The IPF, as an *entropy functional measure* of a priory given performance criterion, can be applied to any specific performance criterion [34, others].

Comments 5.7. Using the equivalent equations:

$$a^u = 2bX, a^u = A^v x = br^{-1}x \tag{3.171}$$

we get the expressions for the conjugate vector

$$X = -1/2hx, h = r^{-1}, \tag{3.172}$$

$$X(\tau) = -1/2(\int_{\tau-0}^{\tau} \sigma\sigma^{\mathrm{T}} dt)^{-1} x(\tau). \qquad (3.172a)$$

A potential, corresponding to this conjugate vector in (3.144b), which satisfies (3.146a,b) at the DP, loses its *deterministic* dependency on the shift vector in (3.131), becoming the function of *diffusion* and a state vector at the DP vicinity (3.172a).
The gradient of (3.172a) at the punched point depends only on the diffusion:

$$gradX(\tau) = \frac{\partial X(\tau)}{\partial x(\tau)} = -1/2(\int_{\tau-0}^{\tau} \sigma\sigma^{T} dt)^{-1} = -2X(\tau)X^{T}(\tau), \qquad (3.173)$$

and at a vicinity of the boundary, where $\sigma\sigma^{\mathrm{T}} \to 0$, it acquires a form of the δ-function. Out of the DP, the gradient (3.173) does not exist, as well as the potential function in the form (3.172a). (At the moment $(\tau - o)$, preceding the control action, the conjugate vector and its gradient have the opposite signs:

$$X(\tau - o) = -X(\tau), gradX(\tau - o) = -gradX(\tau)$$

according to (3.156)).

Following (3.164b) ,we have the following signs for these functions at $(\tau_k^1, \tau_{k+1}, \tau_{k+1}^o)$-locality (limited by the moments $\tau_{k+1}^o - \tau_k^1 = \delta\tau_{k+1}$):

$$-signX(\tau_k^1) = signX(\tau_{k+1}) = -signX(\tau_{k+1}^o);$$
$$-sign[gradX(\tau_k^1)] = sign[gradX(\tau_{k+1})] = -sign[gradX(\tau_{k+1}^o)]. \qquad (3.173a)$$

Those functions, satisfying (3.173a), limit a *macrodynamic* border of the locality, where the maximum of entropy derivation (3.164b) takes place.

The equalities (3.155a), (3.155b), and (3.172a), (3.173), (3.173a) (following from (3.142),(3.143)) define a set of states $X(\tau), x(\tau)$ on the extremal trajectory, which are used for an access to the random process, specifically by forming the control functions (3.169), (3.169a,b) and the operator that is identified by (3.157a,b).

From other consideration, using (3.172),(3.155b), we get a direct connection of the shift vector and diffusion matrix at DP in the form

$$a^u(\tau)(a^u(\tau))^T = b(\tau)(\int_{\tau-0}^{\tau} 2b(t)dt)^{-1} b(\tau)^T. \qquad (3.174)$$

For $a^u(\tau) = -A(\tau)x(\tau) = b(\tau)r^{-1}(\tau)x(\tau)$, we get (3.150b) in the form

$$R(x) = \exp\{-\int_{x_o}^{x} r^{-1}(y)ydy\} < \infty, y = x(\tau) \neq 0 \text{ if } r(y) \neq 0, \qquad (3.175)$$

which is satisfied for a regular diffusion process.

Comments 5.8. Using (3.161b) and (3.167a,b) for each

$$\lambda_i(\tau_k^o) = \alpha_{io} + j\beta_{io}, \lambda_i^*(\tau_k^o) = \alpha_{io} - j\beta_{io},$$

we get the entropy increment by the segment's end, evaluated by the real eigenvalue's invariant :

$$\mathrm{Re}\, E[\Delta S_{io}[x(t_k)]] = -\alpha_{io}\tau_k^1 = \mathbf{a}_{oi} = inv, \qquad (3.176)$$

where in a stable process $\alpha_{io} < 0$ within the segment's time interval, as well as $\alpha_{it} = \alpha_i(\tau_k^1) < 0.$

Because α_{io} starts at a boundary point and measures a fixed entropy derivation for each segment, invariant $\mathbf{a}_{io}$ evaluates a segment's information contribution between a punched localities.

Following (3.167a) we get the related entropy increments

$$\mathrm{Re}\,E[\Delta S_{ic}^{v}] = \alpha_i(\tau_k^1)\tau_k^1 = \mathbf{a}_i = inv\,, \tag{3.176a}$$

$$\mathrm{Re}\,E[\Delta S_{ic}^{u}] = 2\,\mathbf{a}_i = inv\,, \tag{3.177}$$

which evaluate the information contribution of both the constraint and the controls by invariant $\mathbf{a}_i$.

The invariants (3.165), (3.165a) for the above complex eigenvalues also bring the invariant relations for the eigenvalues' imaginary parts:

$$\mathrm{Im}\,E[\Delta S_{io}[x(t_k)] = -\beta_{io}\tau_k^1 = \mathbf{b}_{oi} = inv\,, \tag{3.178}$$

and

$$1/2E[\Delta S_{ic}] = \beta_i(\tau_k^1)\tau_k^1 = \mathbf{b}_i = inv\ , \tag{3.178a}$$

where at a fixed $\gamma_i = \beta_{io} / a_{io}$, the invariants are connected :

$$\mathbf{b}_o(\gamma_i) = \gamma_i\,\mathbf{a}_o(\gamma_i)\,,\ \mathbf{b}(\gamma_i) = \gamma_i\,\mathbf{a}(\gamma_i)\,.$$

From these relations follow that the invariants $\mathbf{a}_o$**,** $\mathbf{b}_o$ measure the quantity of real and a potentially imaginary information, produced *during* the interval by its end; invariants **a, b** measure the quantity of real and potentially imaginary information, produced by the control *at the ending moment* the interval.

These invariants evaluate the corresponding information contributions, equivalent to the constraint and control's actions, while a total information, generated by an information quant at a punched locality, is evaluated by $\mathbf{a}_o$**.**

The real value of invariant in (3.163a) is evaluated by

$$\mathrm{Re}[\Delta S_{ic}] = \mathrm{Re}[1/2\lambda_i(\tau_k)\tau_k] = a_\tau\ , \tag{3.179}$$

where according to (3.166c) and (3.177) we get

$$\mathrm{Re}[\Delta S_{ic}] = a_\tau = 2\,\mathbf{a}(\gamma_i)\,.$$

With the fulfillment of (3.177), this brings

$$\mathrm{Re}[\Delta S_i^{v}] = 1/2a_\tau = \mathbf{a}(\gamma_i)$$

and

$$\mathrm{Re}[\Delta S_{io}] = 3/2a_\tau = 3\,\mathbf{a}(\gamma_i) = \mathbf{a}_o(\gamma_i)\,. \tag{3.179a}$$

Therefore, to produce an elementary information quant (at τ_k locality), the required actions are evaluated by the invariants $\mathbf{a}(\gamma_i)$ and a_τ.

Corollary 5.3.

(i)-The filfullment of (3.161b) imposes on the model's complex eigenvalue $\lambda_i(\tau_k^o) = \alpha_{io} + j\beta_{io}, \lambda_i^*(\tau_k^o) = \alpha_{io} - j\beta_{io}$ condition $\mathrm{Im}\,\lambda_i(\tau_k^1) = 0$, for which the solution of (3.166b) brings equation

$$2\sin(\gamma_i\mathbf{a}_o) + \gamma_i\sin(\gamma_i\mathbf{a}_o) - \gamma_i\exp(-2\mathbf{a}_o) = 0 \tag{3.180}$$

at

$$\mathrm{Im}\,\lambda_i(\tau_k^1) = 0\,,\ \alpha_{io} \neq 0\,,\ \beta_{io} \neq 0\,. \tag{3.180a}$$

This equation allows us to determine *function* $\mathbf{a}_o = \mathbf{a}_o(\gamma_i)$ and find interval t_k for each starting α_{io}, where for a stable process within a time interval $\mathbf{a}_o < 0$;
(ii)-Condition (3.180a),imposed on the solution (3.166c), allows determine the corresponding *function* $\mathbf{b}_o(\gamma_i)$ and a time interval t_k for each known starting imaginary β_{io}.

Indeed. Following (3.165c) we can write the solutions by the end of interval t_k for starting only imaginary eigenvalue

$\lambda_i(\tau_i^o) = j\beta_{io}$, $\lambda_i^*(\tau_k^o) = -j\beta_{io}$:

$$\lambda_i(\tau_i^1) = j\beta_{io}\exp(j\beta_{io}\tau_k^1)[2-\exp(-j\beta_{io}\tau_k^1)]^{-1},$$

$$\lambda_i^*(\tau_k^1) = -j\beta_{io}\exp(-j\beta_{io}\tau_k^1)[2-\exp(-j\beta_{io}\tau_k^1)]^{-1},$$

and select the solution's real and imaginary and component at τ_k^1:

$$\mathrm{Re}\,\lambda_i(\tau_k^1) = -2\beta_{io}\sin(\beta_{io}\tau_k^1)[5-4\cos(\beta_{io}\tau_k^1)]^{-1} = \mathrm{Re}\,\lambda_i^*(\tau_k^1),$$

$$\mathrm{Im}\,\lambda_i(\tau_k^1) = \beta_{io}(2\cos(\beta_{io}\tau_k^1)-1)[5-4\cos(\beta_{io}\tau_k^1)]^{-1} = -\mathrm{Im}\,\lambda_i^*(\tau_k^1),$$

which at $\mathrm{Im}\,\lambda_i(\tau_k^1) = 0$ determines an invariant $\mathbf{b}_o(\gamma_i) = \beta_{io}\tau_k^1$. •

For a pure imaginary staring eigenvalue, this invariant can be used as an indicator of the control's turning off.

Example 3.2. Applying condition $\mathrm{Im}\,\lambda_i(\tau_k^1) = 0$ to the above solutions, we get $2\cos(\beta_{io}\tau_k^1) - 1 = 0$ and $\beta_{io}\tau_k^1 = \pi/6 = inv \cong 0.5235$, which corresponds to the specific invariant $\mathbf{b}_o$ in (3.176); at given β_{io}, $\mathbf{b}_o$ defines $\tau_k^1 = (\beta_{io})^{-1}\pi/6$.

This also allows finding $\mathrm{Re}\,\lambda_i(\tau_k^1) = -0.577\beta_{io}$ and the macrostate at this moment: $x_i(\tau_k^1) = x_i(\tau_k^o)[2-\exp(-0.577\beta_{io}\tau_k^1)] \cong 1.26\tau_k^1$. •

Remark. Solving (3.165c) at condition $\mathrm{Re}\,\lambda_i(\tau_k^1) = 0$ brings for invariant $\mathbf{b}_0' = \beta_{io}\tau'^1_k$ the equation

$$2\cos(\gamma\mathbf{b}_0') - \gamma\sin\gamma(\mathbf{b}_0') - \exp(\mathbf{b}_0') = 0, \qquad (3.181)$$

which identifies an interval t_k',when a control, applied at the interval beginning $\tau_k'^o$, theoretically, is able turning to zero the matrix' *real* components by the interval end τ'^1_k.

Comments 5.9. A potential continuation of a dynamic movement within the punched locality is limited by two points: the control turn's off at τ_k^1 and the *boundary* point of process' stopping at τ_{k+1}.

Such a *dynamic* movement is a finite, localized in a potential hall between these points and formed an *oscillation* with conjugated imaginary eigenvalues [71,67]. The related eigenfunction at moment τ_{k+1} is identified by its real value $\alpha_i(\tau_{k+1})$ according to (3.157a).

Following the moment τ_{k+1}, at the moment τ_{k+1}^o, the control starts a dynamic Hamiltonian model with an imaginary eigenvalue $\lambda_i(\tau_{k+1}^o) = \pm j\beta_{io}(\tau_{k+1}^o) = \pm j\alpha_i(\tau_{k+1})$.

By the end of t_{k+1} interval, the eigenfunction $\lambda_i(\tau^1_{k+2})$ reaches its real value, which is renovated at the moment τ_{k+2}, becoming $\alpha_i(\tau_{k+2})$.

To start a next Hamiltonian segment with the renovated eigenvalue, the control, switching random process to the constraint (3.155a), changes $\alpha_i(\tau_{k+2})$ to $\lambda_i(\tau^o_{k+2})$ and then by the interval's end, the equation $\mathrm{Im}\,\lambda_i(\tau^o_{k+2}) = 0$ is fulfilled.

This condition (following the Example 3.2) allows finding τ^1_{k+2} and identifying $\alpha_i(\tau_{k+3})$ at a next τ_{k+3}, and so on.

In a more general case, when the model's complex eigenvalues include real parts, the invariant relations (3.176), (3.176a),(3.180) and condition (3.161b) are used.

Let us show that a control jump, changing the sign of some matrix elements (specifically performing $a_{ii}(v_i) \to -a_{ii}(-v_i), a_{ki}(v_k) \to -a_{ki}(-v_k)$ for matrix $A = \begin{pmatrix} a_{ii}, a_{ik} \\ a_{ki}, a_{kk} \end{pmatrix}$, leads to appearance of complex eigenvalues in this matrix.

Example 3.3. Let us have a real matrix $A = \begin{pmatrix} +3, -2 \\ -4, +1 \end{pmatrix}$ with eigenvalues $\lambda_{1,2} = 2 \pm 3$.

After a single dimensional control changes the corresponding matrix elements we get $A = \begin{pmatrix} -3, +2 \\ -4, +1 \end{pmatrix}$, and the matrix eigenvalue become complex $\lambda_{1,2} = -1 \pm j2$.

More commonly, considering the dynamic model at the moment τ^1_k of the equalization of some matrix' eigenvalue $\lambda_i(\tau^1_k) = \alpha_i(\tau^1_k) + j\beta_i(\tau^1_k),\ \lambda^*_i(\tau^o_k) = \alpha_i(\tau^1_k) - j\beta_i(\tau^1_k)$,

we have $\lambda_i(\tau^1_k) = \lambda^*_i(\tau^1_k) = \alpha_i(\tau^1_k),\ \beta_i(\tau^1_k) = 0$, and after reaching a punched locality at $\tau_{k+1} = \tau^1_k + o$ we get $\lambda_i(\tau_{k+1}) = \lambda^*_i(\tau_{k+1}) = \alpha_i(\tau_{k+1})$; then at the moment τ^o_{k+1}, the control changes the sign of model's matrix leading to $\lambda_i(\tau^o_{k+1}) = -\lambda_i\,(\tau_{k+1}) = j\beta_i(\tau^o_{k+1})$, with a possibility that the model's eigenvalues become imaginary by analogy with the above example. •

The invariants locate the DP when both starting controls (3.169a) and needle control (3.169b) are applied. The limited time intervals restrict both discrete actions of the constraint and the controls. The *time intervals* between the DP $(\tau^o_{k+1} - \tau^1_k = \delta\tau_{k+1})$ are connected by the conditions (3.170), (3.164c), (3.173a) for each cooperating i, j eigenvalues.

Solving (3.154) by applying (3.170) with the above controls determines the model macrotrajectory as a *chain* of the extremal segments, joined at each DP, where, through the access to the microlevel, the identification of the model's renovated operator is possible using the identification equations (3.157a), (3.157b).

Under control (3.169) action we get the matrix equation (3.161), where matrix $A(\tau^1_k)$ is renovated each DP moment τ_{k+1} becoming $A(\tau_{k+1})$, and with applying control $v(\tau_{k+1}) = -2x(\tau^o_{k+1})$ it's transformed to $A(\tau^o_{k+1})$, starting

the matrix equation at a next segment. Interval t_k is used for the matrix's computation from the data had obtained at τ_{k+1}.

Invariants (3.176),(3.176a) and condition (3.161a) allow computing any t_k, t_{k+1}, which identify both each DP of the model's renovation and the control's action for a sequence of the extremal segments.

In an *n*-dimensional model, at each $(\tau_k^1, \tau_{k+1}, \tau_{k+1}^o)$-locality, the equalization of an eigevalues' pair $\lambda_k(\tau_k^1) = \lambda_i(\tau_k^1)$ takes place at the moment τ_k^1 (according to (3.161a,c)) and, at the moment τ_{k+1}, the joint real's eigenvalues pair $\mathrm{Re}\,\lambda_i(\tau_k^1)$ is connected to $\mathrm{Re}\,\lambda_j(\tau_{k+1})$, and then, at τ_{k+1}^o, it leads to eigenvalue $\lambda_j(\tau_{k+1}^o)$ on the following segment (according to (3.170)), forming a *triple* eigenvalue's connection.

Thus, the segments' triple $(\lambda_i(\tau_k^1), \lambda_j(\tau_{k+1}), \lambda_j(\tau_{k+1}^o))$ *cooperate* under the needle control' actions, reducing the model's dimension at each of this locality, with a possibility of forming the model's cooperative information network (IN).

Relations (3.162a), (3.163a,b), (3.164) and (3.164a,b) connect the identified matrix's elements to the initial variation conditions (3.142) and the above information equations.

Corollary 5.6.

Balance ratio (3.164) is satisfied at the following relations between invariants

$$\mathbf{a}_o(\gamma_i)/\mathbf{a}(\gamma_i) = 3, \tag{3.182}$$

which corresponds $\gamma_i^o \cong 0.5$.

The result follows from both (3.179a) and (3.179):

$$\mathbf{a}(\gamma_i)/\mathbf{a}_o(\gamma_i) = \exp(\mathbf{a}_o(\gamma_i))\,[2 - \exp[\exp(\mathbf{a}_o(\gamma_i))]^{-1} = 1/3. \tag{3.183}$$

Solution of this equation brings $\mathbf{a}_o(\gamma_i) \cong \ln 2$, which corresponds to $\gamma_i^o \cong 0.5$ and $\mathbf{a}(\gamma_i^o) \cong 0.23$.

Therefore, this γ_i^o and the above invariants' parameters provide the fulfillement of the balance ratio (3.164), which satisfies to balance equation (3.163c). •

From these it follows that the balance equation imposes the additional constraint on the invariant connection, which allows selecting the model invariants satisfying this balance.

The balance relations might not be fulfilled when

$$\gamma_i \neq \gamma_i^o. \tag{3.184}$$

Invariant $\mathbf{a}=\mathbf{a}(\gamma_i \to 0)$, at the model's only *real* eigenvalues, evaluates the information contribution (ch.1.2) (1.1.26a) by a step-wise control (3.169); while the information contribution of needle control (3.169b) (that includes two step-wise controls with the additive information contributions) is evaluated by $2\mathbf{a}(\gamma_i \to 0)$, which is related to (1.1.26).

The computation using the model's invariant equations (3.176),(3.177) and the invariant's connection to γ_i in (3.180) (see also ch.1.4)) brings $\mathbf{a}_o(\gamma_i \to 0) \cong 0.75$ and $\mathbf{a}(\gamma_i \to 0) \cong 0.25$ that leads to the evaluation of (3.167a,b) at the real eigenvalues by the above invariants.

The information values of the above invariants $\mathbf{a}_o$ and a correspond to 1 bit and 0.35 bits accordingly; and the elementary information quantum can be evaluated by $a_{io} \cong 1$ bit.

(These results, following from the IPF, do not apply classical information theory.)

The information invariants connect the IPF VP with information specific of the constrain's imposing.

Because both $\mathbf{a}_o(\gamma_i \to 0) \cong 0.75$ and $\mathbf{b}_o = \beta_{io}\tau_k^1 = \pi/6$ are determined at the imaginary eigenvalue $\beta_i(\tau_k^1) = 0$ their relation $\mathbf{b}_o = \gamma_i \mathbf{a}_o$ at the same t_k leads to finding $\gamma_i \cong 0.7$ in this non regular case.

Comments 5.10. The model dynamics is initiated by applying a starting step-wise control in the form

$$v(\tau_o^o) = -2E_{\tau_o^o}[\tilde{x}_t(s)], \tag{3.185}$$

at $\tau_o^o = s + o$, where τ_o^o is the moment of the control's starts, $\tilde{x}_t(s)$ are the object's initial conditions, which also include given correlations

$r(s) = E[\tilde{x}_t(s)\tilde{x}_t(s)^T]$ and/or $b(s) = 1/2\dot{r}(s)$.

These initial conditions also determine a starting *external* control

$$u(\tau_o^o) = b(\tau_o^o)r(\tau_o^o)^{-1}v(\tau_o^o), \tag{3.185a}$$

where $v(\tau_o^o) = -2x(\tau_o^o)$, and a nonrandom state can be defined via

$$x(\tau_o^o) \cong |\, r^{1/2}(\tau_o^o)\,| \,. \tag{3.185b}$$

This control imposes the constraint (3.155a), in the form (3.152), thereby starting the dynamic process. The above initial conditions determine matrix

$$A(\tau_o^o) = b(\tau_o^o)r(\tau_o^o)^{-1}, A(\tau_o^o) = (\lambda_i(\tau_o^o)), i = 1,...,n, \tag{3.185c}$$

and the condition $\operatorname{Im}\lambda_i(\tau_o^1) = 0$ (or (3.161a)) is used to find a first time interval between the punched points, where the next invariant should be identified, and so on.

Each following $\alpha_i(\tau_k)$ $k = 1,...,m-1$, identified at a fixed moment τ_k, also identifies invariants $a_\tau = \alpha_i(\tau_k)\tau_k$ and a related invariant $\mathbf{a}(\gamma_i)$, which serves to determine $\mathbf{a}_o(\gamma_i)$ according to (3.179a).

Equation (3.178) allows finding γ_i for each $\mathbf{a}_o(\gamma_i)$ with a starting complex $\lambda_i(\tau_k^o)$, and then using this invariant to compute t_k for each identified $\alpha_{io} = \alpha_i(\tau_k)$ in the following segment time interval, which determines τ_k locality, where the next $\alpha_i(\tau_{k+1})$ should be measured, and so on.

From (3.179a):

$$3\alpha_i(\tau_k^o)\tau_k^o = \alpha_i(\tau_k^o)\tau_k^1 = -3/2\alpha_i(\tau_k)\tau_k,\ \alpha_i(\tau_k) = -\alpha_i(\tau_k^o)$$

it follows $\tau_k \cong \tau_k^o \cong 2/3\tau_k^1$. This allows us to predict τ_k^1 and therefore t_k by knowing τ_k.

The moment $\tau_k^1 + o$ estimates a punch point where $x_i(\tau_{k+1})$ should serve for forming a second step-wise control and the identification (measurement) takes place.

If the model segment's $\lambda_i(\tau_k^o) = \alpha_i(\tau_k^o) \pm \beta_i(\tau_k^o)$ is currently identified, *then* γ_i is known, and using (3.180) can be found $\mathbf{a}_o(\gamma_i)$ which determines t_k for $\alpha_i(\tau_k^o)$.

Because during the process, only a real component of the model's complex eigenvalue is changing, its imaginary component can be fixed or set up as the model's initial condition; still, each renovated real component changes the current γ_i.

Specific of the considered optimal process *consists of the computation of each following time interval* (where the identification of the object's operator takes place and the next optimal control is applied) *during the optimal movement under the current optimal control, formed by a simple function of dynamic states.*

Comments 5.11. An elementary entropy increment ΔS_i^{δ} between the nearest segments, evaluated via correlation (3.157a) using (3.165) and (3.163b) is:

$$\Delta S_i^{\delta} = 1/2 \int_{\tau-o}^{\tau+o} (a_i^u)^2 (2b_i)^{-1} dt = 1/2 \int_{\tau-o}^{\tau+o} \lambda_i^2 x_i^2 (2b_i)^{-1} dt,\ i = 1,....m, \tag{3.186}$$

$$E[\Delta S_i^{\delta}] = 1/4 \int_{\tau-o}^{\tau+o} \lambda_i dt = 1/8 \int_{\tau-o}^{\tau+o} r_i^{-1} \dot{r}_i dt = 1/8[\ln r_i(\tau+o) - \ln r_i(\tau-o)]. \tag{3.186a}$$

At a fixed $0 < r_i(\tau-o) \le 1$, the entropy increment depends on the correlation $r_i(\tau+o)$, which can take its border's values $r_i(\tau+o) = 1$, or $r_i(\tau+o) = 0$.

At the maximal correlation $r_i(\tau+o) = 1$, the increment $E[\Delta S_i^{\delta}] = -1/8 \ln r_i(\tau-o) \ge 0$.

At a decoupling (dessolving) the correlation $r_i(\tau+o) \to 0$, we get $E[\Delta S_i^{\delta}] \to \infty$ independently on the value of $r_i(\tau-o)$; this corresponds to breaking off the segment's connection.

Connection to Shannon's entropy.

Considering a set of discrete states $\tilde{x}(\tau) = \{\tilde{x}(\tau_k)\}, k = 1,..., m$ at each moments $\tau = \tau_k$ along the random process, and using definition of the entropy functional (1.4), (3.136a), we get the entropy *function* for these conditional probabilities $p_k = p_k[\tilde{x}(\tau_k)]$ (corresponding to (1.3)) for all moments $\tau_k, k = 1,...m$:

$$\tilde{S}_{\tau}(\tilde{x}/\varsigma) = -\sum_{k=1}^{m} \ln p_k[\tilde{x}(\tau_k)] p_k[\tilde{x}(\tau_k)], \tag{3.187}$$

which *coincides with the Shannon entropy* for the probability distributions $p_k = p_k[\tilde{x}(\tau_k)]$.

Function (3.187) holds all characteristics of Shannon's entropy, following from the initial Markov process and its additive functional.

The entropy for the set $\tilde{x}(\tau) = \{\tilde{x}(\tau_k)\}, k = 1,..., m$ is

$$\tilde{S}_{\tau m}(\tilde{x}/\varsigma) = \sum_{k=1}^{m} \Delta S_k[\tilde{x}(\tau_k)], \tag{3.188}$$

where $\Delta S_k[\tilde{x}(\tau_k)]$ is the entropy at each $\tau_k, k = 1,...m$.

Both entropy measures (3.187) and (3.188) (for (3.136)) and their invariant's form (3.179a) coincide for the set $\tilde{x}(\tau) = \{\tilde{x}(\tau_k)\}, k = 1,..., m$.

Distribution $p_k = p_k[\tilde{x}(\tau_k)]$ is selected by variation condition (3.143) as an extremal probability distribution, where a macrostate $x(\tau_k)$ estimates a random state $\tilde{x}(\tau_k)$ with a maximum probability $p_k = p_k[\tilde{x}(\tau_k)]$.

In other point of view, at a τ_k -vicinity, the information entropy (ch.1.2), (1.24) reaches a maximum, which, for the considered variation problem, is associated with turning the constraint (3.144c) off (by the controls) when function (3.146) takes its maximum (3.164c) (at switching to the random process).

Thus, $x(\tau_k)$ emerges as the *most informative* state's evaluation of the random process at the τ_k -vicinity.

Selecting these states (with an aid of the dynamic model) allows an optimal discrete filtration of random process at all $\tau_k, k = 1,...m$, where the macromodel is identified.

Computing the above invariant allows to predict each τ_k -vicinity where entropy (3.187) should be measured and a sum of process's entropies (3.188) determines the IPF entropy with a maximal process' probability.

The conditions (3.164c), (3.173a), impose a macrodynamic limit on the vicinity's time interval $\delta\tau_k = \tau_{k+1}^o - \tau_k^1$, where the maximum can be reached.

Following (3.176), it's seen that the invariant's information measure $\mathbf{a}_o(\gamma_i)$ evaluates each segment's total information contribution delivered at the segment's τ_k -locality: $\Delta S_k[\tilde{x}(\tau_k)]$, while $\tilde{S}_{\tau m}(\tilde{x}/\varsigma)$ is measured by the sum of the invariants:

$$\tilde{S}_{\tau m}(\tilde{x}/\varsigma) = \sum_{k=1}^{m}\sum_{i=1}^{n} k\,\mathbf{a}_o(\gamma_i), \tag{3.188a}$$

(where $m = n$, if the number of the segments equals to the model's dimension n, assuming each segment has a single τ_k -locality).

Computing the above invariants a_τ, and $\mathbf{a}_o(\gamma_i)$ allows predicting each τ_k -vicinity where entropy (3.187) should be measured, and a sum of process's entropies (3.188), (3.188a) determines the process entropy with a maximal process' probability.

Knowing this entropy allows encoding the random process using the Shannon formula for an average optimal code-word length:

$$l_{cm} \geq S_{\tau m} / \ln D\,, \tag{3.189}$$

where D is the number of letters of the code's alphabet, which encode $S_{\tau m}$.

An elementary code-word to encode the process' segment is

$$l_{cs} \geq \mathbf{a}_o(\gamma_i) / \ln D_o\,, \tag{3.190}$$

where D_o is a segment's code alphabet.

At $\mathbf{a}_o(\gamma_i \to 0) \cong 0.75$, D_o=2, we get $l_{cs} \geq 1$, or a bit per the letter.

Therefore, the invariant $\mathbf{a}_o$=$\mathbf{a}_o(\gamma_i)$, or $\mathbf{a}(\gamma_i)$ for each *i*-segment (with the identified eigenvalues, or a_τ) allows us to encode the *process* using (3.189),(3.190) without counting each related entropy (3.187).

The assigned code also encodes and evaluates both constraint and controls' actions.

The code sequence allows building a *hierarchical information network* (IN), which, along with encoding a sequence of the process extremals in a *real* time, also ranges the segments by the values of their local entropies–invariants.

The space distributed hierarchical (IN) (chs.1.5, 1.6) is built by the cooperating segments' sequence during a real time process, which possesses the specific IN structure with an individual genetic information code for each process, described by equation 1.1.1. •

Comments 5.12. The specific solutions of equations (3.147d) or their macrodynamic analogies (3.155a,b) determine the actual number (*m*) of the discrete instances (DP), and the particular *sequence* of the equations's both roots: eigenvectors and the related time intervals. This means that under the constraint action, *each stochastic equation* (1.1.1) with specific functions of the drift-vector and diffusion components *encloses a potential number of the considered discrete intervals and their sequential logics.*

Comments 5.13. The bi-level model's controls, as a part of solution the VP problem for IPF, perform the following functions: allows *selecting* the dynamic equivalents of the random states; *allocate* the dynamic process' discrete extremal segments with a window between them (where the identification takes place), and then *connect* the segments in a chain, which carry on the IPF *path dynamics*; allows the process' filtering with an optimal Shannon's encoding (by selecting the random process' most informative state); join the set of the ranged segments into *cooperative IN*, allows modeling the initial process' *cooperative dynamics.*

These controls compose an *inner mechanism* of the model's structure.

An external control, chosen according to a given programmable process, or performance criterion, is synthesized on the basic of the internal control (sec. 1.3.4).

Finally, the IPF, the variation equations, and the obtained macromodels form a *mathematical foundation of informational macrodynamics*, which provide the transformation of a random process' eigenfunctional EF to the corresponding dynamic process' eigenfunctional IPF, having the invariant information measure in terms of the Hamiltonian, eigenfunctions of model's matrix, and a code.

IPF models the random object's trajectory by the related dynamic trajectory, consisting of the set of extremal segments, whose punched localities allow the object identification under the optimal control action.

In this optimal dual strategy, the IPF optimum predicts each extremal's segments movement not only in terms of a *total functional path goal,* but also by setting at each following segment the renovated values of this functional, identified *during the optimal movement*, which currently correct this goal.

The *concurrently synthesized optima*l control provides a maximal Markov's probability and optimal filtering of the random process at each of its identified punched localities.

Chapter 1.4

THE INFORMATION SPACE DISTRIBUTED MACROMODELS DEVELOPED BY THE SOLUTIONS OF THE IPF VARIATION PROBLEM

1.4.1. Introduction

Studying a random space *distributed* object, we intend to reveal its *information dynamics* through the modeling (identification) of a corresponding random *information field*.

Important problem's aspect includes a transfer from a random field, modeled by a stochastic differential equation with a random time's and the non random space's arguments, to its *macromodel* in the form of a conventional differential equations in partial derivatives (PDE). Such a transfer by itself represents a very complicated problem, which includes finding of the structure, parameters, and a specific class of PDE based on the observation of the random field. The restored dynamic space model could be stationary or nonstationary, linear or nonlinear, depending on corresponding characteristics of the random field.

The known identification methods [46], including the recent results [47, others], are based on approximation of an observed process by some class of equations (operators).

The choice of an approximating *operator* is more often defined not by the equations of the related physical law [48], but typically is determined subjectively.

Applied approximating *criteria*, as a measure of closeness between both an observation process and a solution of identification equation, usually are not associated with the field's physical regularities, the cooperative phenomena of complex systems, and, in general, are *arbitrary* [49]. At such a formal approximation, the quantitative effect of the above field's phenomena could be insignificant in terms of the accepted PDE operator and the approximating criteria, but is very important from the physical and application viewpoints [50,others]. Applied mathematical models are mostly reversible for the irreversible observed processes, and have been developed basically for "simple objects" without taking into account the superimposing and cooperative phenomena.

This problem acquires a significant importance for a complex object with a wide diversity of superimposing process's interactions of a *distinct* nature, which all can be

modeled by *information* interactions, making the information model's description a *universal language* for dynamic modeling.

Both the identification with the approximation criteria and the optimization with a performance criteria require the formulation and solution of the specific variation problems for each of them [51, 52].

We use the informational path functional (IPF), defined on a random field, to find the dynamic PDF macromodel from a solution of the IPF variation principle (VP), considering the VP as a mathematical form to convey the model's regularities.

This formalism extends the results (ch.1.3), obtained for the information concentrated controllable objects, on the space distributed information systems; uses the invariant transformation of space coordinates, following from the PF VP, and also applies the identification methods [53] and results [54-55].

Thus the objective includes: application of the PDF for the controllable informational dynamic model, being able to describe the complex object's regularities; identification of this model in a random field, using the VP for an extreme approximation of both the model's dynamics and the identified object's random process.

Section 4.2 introduces the space-distributed entropy's functional in a random field and the basic path functional's models (with a fixed space coordinates) by the solution of Euler-Ostrogradsky equations for the functional's extremals. These basic models describe a complex object at both the micro-and macrolevels.

Section 4.3 presents the family of the space coordinates' transformations with the invariant condition imposed on the (entropy) path functional. Searching for the VP's natural limitations, we obtain an extreme model, defined on the admissible space coordinates' variations, which satisfies these transformations. Applying the Noether theorem, we get the sufficient conditions for the invariance of the functional at the above transformations and the general forms of the PDE models in a mobile evolving space coordinate system. The invariant conditions bring an *additional* differential constraint to that, imposed by the functional's VP on the distributed macromodel.

Section 4.4 applies the equations of extremals and the differential constraints to find the space transformation**'**s parameters and the specific model's structure. As a result, we obtain the controllable distributed macromodel with the optimal controls, applied on the space-time discrete intervals, which had been found from the Erdman-Weierstrass' conditions.

The dynamic model's operator is identified during the observation of random filed by the actions of the optimal controls. We show the macromodel's direct connection to the equation of the nonequibrium irreversible thermodynamics and physical kinetics.

Section 4.5 studies the IPF macromodel's singular points and the singular trajectories, and the invariants following from their connections. The singularities arise at the DP-windows with shortening the initial model's dimension and the potential chaotic dynamics' bifurcations.

Section 4.6 analyzes the solutions of a natural border problem for the IPF under the control actions. It is shown that both the model extremals and the model's singular trajectories belong to these solutions, if the segment's controls are bound by the found limitations. We also establish the invariant conditions, as the model's field's functions, being the analogies of the information *conservations laws.*

1.4.2. The Variation Problem for Space Distributed Macromodel

The controlled bi-level model of distributed object contains the distributed random process $\tilde{x}_t(\tilde{x}_s, u_t, t, \overline{l}, \xi_t)$ at the microlevel, which depends on a nonrandom space parameter $\overline{l} \in R^{\nu}$, ν=1,2, 3, the control $u_t = u(\overline{l}, \tilde{x}_s)$, random initial conditions $\tilde{x}_s$, and a Wiener process ξ_t.

The microlevel process is considered to be a solution of the controlled stochastic equation

$$d\tilde{x}_t(\overline{l}) = a(t, \tilde{x}_t(\overline{l}), u_t(\overline{l}))dt + \sigma(t, \tilde{x}_t(\overline{l}))d\xi_t, \tilde{x}_s = \eta(\omega^1, \overline{l}), t \in [s, T] = \Delta, s \in [0, T]$$

, (4.1a)

where $\xi_t = \xi(t, \omega)$ is a random process, defined on probability space (Ω, Ψ, P_o), $\omega \in \Omega$ with the variables located in R^n; Ω is a space of random events, Ψ is a σ-algebra on Ω; $P_o = P_o(B)$ is a probability measure on Ψ, $B \subset \Psi$, β is a Borel's algebra in R^n;

$\eta(t, \overline{l}, \omega^1)$ is random distributed process on the probability space (Ω^1, Ψ^1, P^1), $\omega^1 \in \Omega^1$ with the variables in $C^2(R^n, R^{\nu})$; $\tilde{x}_t(\overline{l}) = \tilde{x}(\omega^{11}, \tilde{x}_s, t, \overline{l})$ is a random distributed process, considered on a family of the probability spaces (Ω, Ψ, P_x), or on the probability space $(\Omega^{11}, \Psi^{11}, P)$ with the variables in $C^2(R^n, R^{\nu})$; $P_x = P_x(B)$ is a family of probability measure on Ψ, $B \subset \Psi$ depending on $x \in R^n$ with the probability $P_s(x)$; $P_o = P_{x=0}$, $\omega^{11} = (\omega, x)$, $\omega^{11} \in \Omega^{11}$, $\Omega^{11} = \Omega \times C^2(R^n, R^{\nu})$, $\Psi^{11} = \Psi \times \beta$, $P = P(B \times D)$ is a probability measure on $(B \times D) \subset \Psi^{11}$, which satisfies the Markovian property for the P_x and $P_s(x) = P^1(\eta(\omega^1, \overline{l}))$.

The solution of (4.1a) satisfies the theory of Ito's stochastic integral [13,14] and the requirements for the concentrated model (ch.1.1) with the following functions and the mathematical expectations

$$x(\bullet, \bullet, \overline{l}) \in C^2(R^{\nu}, R^n), a(t, \bullet, \bullet), \sigma(t, \bullet), u(t, x, \bullet), \sigma\sigma^T = 2b,$$

$$\overline{x}_t(\overline{l}) = E^1[\tilde{x}_t] = \overline{x}(\omega^1, t, \overline{l}), \eta(\omega^1, \bullet), E^1[\bullet] = \int_{\Omega^1} [\bullet] P^1(d\omega^1), \omega^1 \in \Omega^1 \quad . \quad (4.1b)$$

Both the drift (a) (as a regular distributed flow) and the diffusion conductivity of a medium (b) depend parametrically on the nonrandom the ν-dimensional vector $\overline{l} = \overline{e}l$, $l = \{l_1, l_2, l_3\}$ that characterizes the geometrical coordinates of a point in some selected affine space system.

The Ito equation of a *distributed diffusion* (directly connected to physical diffusion and conductivity) does not require applying a special δ-correlation needed in the Stratonovich equation's symmetrical form, which would affect an object's (natural) *eigen path functional***.**

The controllable microlevel process $\tilde{x}_t(\bar{l}) = \tilde{x}(\omega'', t, \bar{l})$ is considered in a deviation of some given (programmed) process $\tilde{x}_t^{1}(\bar{l}) = \tilde{x}^{1}(\omega'', t, \bar{l})$, $\hat{x}_t(\bar{l}) = \tilde{x}_t(\bar{l}) - \tilde{x}_t^{1}(\bar{l})$, where the $\hat{x}_t(\bar{l})$ characterizes an *object*'s process in the above deviations.

An entropy random functional $\Delta\tilde{S}[\hat{x}_t(\bar{l})]$, which evaluates an *information closeness* of the above deviation's processes, is defined on the object's random trajectories $\hat{x}_t(\bar{l})$.

Following this, an *information path functional* $S^{l}[x_t]$, as a *dynamic* approximation of the entropy functional (ch.1.2), is built by analogy with functional of action [18], using approximation (sec.1.3.5):

$$Sup\Delta\tilde{S}[\hat{x}_t] \le Inf S^{l}[x_t], \tag{4.1c}$$

where an evaluator $S^{l}[x_t]$ is defined on a dynamic (macro) process x_t to be found, and the structure of both $\Delta\tilde{S}[\hat{x}_t]$ and $S^{l}[x_t]$ functionals depend on the parameters of stochastic equation (4.1a).

Solution of variation problem (VP) for the path functional:

$$S^{l} = \int_{G^4} L(x_i(t,\bar{l}), \frac{\partial x_i(t,\bar{l})}{\partial t}) d\bar{\upsilon}, d\bar{\upsilon} = d\upsilon dt, d\upsilon = dl_1 dl_2 dl_3, \tag{4.2}$$

$$L = 1/2 \sum_{i,j=1}^{n} (2b)_{ij}^{-1} \frac{\partial x_i(t,\bar{l})}{\partial t} \frac{\partial x_j(t,\bar{l})}{\partial t}; x : \Delta \times R^3 \to R^n, \Delta \subset R_+^1, \tag{4.3}$$

determines the *macrotrajectories* $x_t(\bar{l})$, which approximate the process $\hat{x}_t(\bar{l})$ with a minimum uncertainty; where L is a Lagrangian, defined in a four-dimensional space-time region $G^4(\bar{\upsilon})$, with an elementary volume $\bar{\upsilon} = \bar{\upsilon}(\bar{l}, t)$.

The variation problem of finding the extremals of the functional (4.2) includes the constraint's equation in partial derivations (ch.1.3) with applying control to eq. (4.1).

This variation problem differs from ch.1.2 only by existence of parameter $\bar{l}$, which is fixed for each $x_t(t,\bar{l})$.

This allows us to apply the Euler-Ostrogradsky equation [24] to get the extremals for the Lagrangian (4.3) and then use the same methodology to obtain the distributed macromodel.

We received the following macromodel's equation

$$\sum_{j=1}^{n} \{(2b)^{-1}{}_{kj} \frac{\partial^2 x_i}{\partial t^2} + \frac{\partial x_j(t,\bar{l})}{\partial t} [\frac{\partial (2b)^{-1}{}_{kj}}{\partial t} - 1/2 \sum_{i=1}^{n} \frac{\partial (2b)^{-1}{}_{kj}}{\partial x_i} \frac{\partial x_i(t,\bar{l})}{\partial t}]\} = 0; \tag{4.4}$$

$$\frac{\partial x_m}{\partial t} = a_m(t,x,u), a_m = A(x + v(\tau,\bar{l})), A = A^T, m = 1,...,n;$$

$$b = b(t,x), b, A : \Delta \times R^3 \to \angle(R^n), u = Av(\tau,\bar{l}), \tau \in \Delta, r \le n. \tag{4.4a}$$

In particular, considering the model with a diagonalized matrix of diffusion and the Lagrangian

$$L = 1/2\sum_{i=1}^{n}\sigma_{ii}^{-1}(\frac{\partial x_i}{\partial t})^2, \quad (4.4b)$$

we obtain the equation of extremals in the form

$$\frac{\partial x_i^2}{\partial t^2} = \sigma_{ii}^{-1}\frac{\partial \sigma_{ii}}{\partial t}\frac{\partial x_i}{\partial t}, i = 1,...,n. \quad (4.5)$$

The admissible transformations of the space parameters will determine a potential spatial macromovement and impose the additional constrains on the VP solution.

Thus, the VP leads to the macroequations, which according to the approximation (4.1c), carry out the *information closeness* to the object and, as a result, an ability for *the identification of the random object*.

1.4.3. The Invariant Conditions at the Transformation of the Space Coordinates

The space motion in a field is presented by the transformation of coordinates $\bar{l}^o = \{l_k^o\}$ that are assumed to be given initially in an immobile system, connected with an observer and then are transformed into the mobile coordinate system $\bar{l} = \{l_k\}$, k =1, 2, 3 according to relations

$$\bar{l} = \bar{A}\bar{l}^o + \bar{L}, \bar{L} = \bar{L}(t), \quad (4.6)$$

with an orthogonal matrix of rotation $\bar{A} = (\bar{a}_{ij})_{i,j=1}^3$ and a vector $\bar{L} = \{\bar{L}_k\}$, k =1,2,3, reflecting an appropriate space *shift* of the origin of the initial coordinate system.

Relation (4.6) represents a continuous, simple, and a single-parametrical transformation for a family of Euclid's spaces in the theory of solid-state matter [56, 57], which allows changing its scale at each of the space points.

The extreme principle provides an invariance of functional (4.2) on the family of transformations (4.6), which enables a system to preserve the macrodynamic laws in all observed mobile systems at deterministic movements of the object (4.4a).

The process, defined by Lagrangian (4.3) and the above transformation, determines the object's natural space coordinate system.

The problem consists of application of Noether's theorem [23,24] to find the parameters of the transformation.

Lemma 2.1**.**

The invariant conditions for the functional (4.2) at the transformation (4.6) in the form:

$$Z_0 = \begin{cases} t = t, \\ \bar{l}^o = \bar{A}(t)^{-1}(\bar{l} - \bar{L}) \end{cases} \quad (4.7)$$

is an equivalent to the invariant conditions for the functional's transformation at a time increment δt:

$$Z_* = \begin{cases} t^* = t^*, t^* = t + \delta t, \\ \bar{l}^{\,o} = \bar{A}(t^*)^{-1}(\bar{l}^{\,*} - \bar{L}(t^*)) \end{cases}. \tag{4.8a}$$

Indeed. Let us consider a vector $\bar{l}^{\,*}(t^*, \bar{l}^{\,o}) = \bar{l}\,(t + \delta t) = \bar{A}(t + \delta t)\bar{l}^{\,o} + \bar{L}(t + \delta t)$ at fixed t^*, defined by a space shift $\delta\bar{l}^{\,o} = \bar{l}^{\,*} - \bar{l}^{\,o}$.

According to Noether's theorem for a functional, invariant at transformations (4.7), the conservation laws hold true at small deviations of parameter δt. It defines the relations:

$$Z_o S^l = S^l_o = S^l, Z_* S^l = S^l_* = S^l. \tag{4.8b}$$

From that we get $S^l_o = S^l_*$. Besides this, for the space shift $\delta\bar{l}^{\,o}$, which can be observed only in an immobile system, the invariant conditions must be preserved during a time t.

Theorem 2.1. (T1).

The fulfillment of model's equations in the form

$$\frac{\partial x_j}{\partial t}(t, \bar{l}\,) = -1/2 \frac{\partial x_i}{\partial \bar{l}}\bar{y}\,, \tag{4.9}$$

$$\bar{y} = \lim_{\delta t \to 0} \frac{\bar{l}\,(t + \delta t) - \bar{l}\,(t)}{\delta t} = \frac{\partial \bar{l}}{\partial t}; \frac{\partial \bar{l}}{\partial t} = \dot{\bar{A}}(\bar{A})^{-1}(\bar{l} - \bar{L}) + \dot{\bar{L}}; \bar{y} = \{y_k\}, \frac{\partial \bar{y}}{\partial t} = 0, k = 1, 2, 3, \tag{4.10}$$

represents the sufficient condition of the invariance for the functional (4.2) at the transformation (4.8a) for a twice continuous differentiable function $x_i = x_i(t, \bar{l}\,)$ with the nonzero derivatives $\frac{\partial x_i}{\partial t} \neq 0$.

Proof. Functional (4.2), (4.3) holds an invariant at transformation (4.6) if the following equality is true:

$$\int_{G^4} L(x_i, \frac{\partial x_i}{\partial t}) d\bar{\upsilon} + \int_{G^4} L^1(\frac{\partial x_i}{\partial t}, \frac{\partial x_i}{\partial \bar{l}}) d\bar{\upsilon} = \int_{G^4} L(x_i^{\,o}, \frac{\partial x_i^{\,o}}{\partial t}) d\bar{\upsilon}_o, L^1 = 0, d\bar{\upsilon} = d\upsilon dt, \tag{4.11}$$

where the states $(x_i^{\,o}, x_i)$, given in the immobile and in the mobile coordinate systems accordingly, are connected by the following relations (with a symbol of scalar multiplication<>):

$$x_i^o = x_i(t, \bar{A}\bar{l}^{\,o} + \bar{L}), \tag{4.12}$$

$$\frac{\partial x_i^{\,o}}{\partial t} = \frac{\partial x_i}{\partial t}(t, \bar{A}\bar{l}^{\,o} + \bar{L}) + < \frac{\partial x_i}{\partial \bar{l}}, \frac{\partial \bar{l}}{\partial t} >$$

$$= \frac{\partial x_i}{\partial t}(t, \bar{A}\bar{l}^{\,o} + \bar{L}) + < (\frac{\partial x_i}{\partial \bar{l}}, \dot{\bar{A}}(\bar{A})^{-1}(\bar{l} - \bar{L}) + \dot{\bar{L}} > . \tag{4.13}$$

The last equation follows directly from the gradient form of the equation (considering below), which is determined in the coordinate systems, connected by the transformation (4.8a):

$$\frac{\partial x_i}{\partial l_j} = \sum_{k=1}^{3} (\frac{\partial x_i}{\partial l_k^o}) \bar{a}_{kj}, (\bar{a}_{kj}) = \bar{A}^{-1};\ k, j = 1,2,3;\ grad x_i = \bar{A} grad^o x_i . \qquad (4.13a)$$

From this equation, at the fulfillment of $\bar{A}^{-1} = \bar{A}^T$ we obtain

$$< \frac{\partial x_i}{\partial \bar{l}}, \frac{\partial \bar{l}}{\partial t} >= (\frac{\partial x_i}{\partial \bar{l}})^T (\frac{\partial \bar{l}}{\partial t}) = (\frac{\partial x_i}{\partial \bar{l}^o})^T \bar{A}^T (\frac{\partial \bar{l}}{\partial t}) = \sum_{p,q} \frac{\partial x_i}{\partial l^{op}} \bar{a}_{qp} \frac{\partial l^{oq}}{\partial t} . \qquad (4.14)$$

Condition (4.11), after substituting (4.12), (4.13), leads to Lagrangian in the form

$$L^1 = 1/2 \sum_{i,j=1}^{n} (2b)_{ij}^{-1} \{ < (\frac{\partial x_j}{\partial \bar{l}}, \dot{\bar{A}} (\bar{A})^{-1} (\bar{l} - \bar{L}) + \dot{\bar{L}}) >$$

$$\times [\frac{\partial x_i}{\partial t} + 1/2 < (\frac{\partial x_i}{\partial \bar{l}}, \dot{\bar{A}} (\bar{A})^{-1} (\bar{l} - \bar{L}) + \dot{\bar{L}}) >]$$

$$+ < (\frac{\partial x_i}{\partial \bar{l}}, \dot{\bar{A}} (\bar{A})^{-1} (\bar{l} - \bar{L}) + \dot{\bar{L}}) >$$

$$\times [\frac{\partial x_j}{\partial t} + 1/2 < (\frac{\partial x_j}{\partial \bar{l}})^T, \dot{\bar{A}} (\bar{A})^{-1} (\bar{l} - \bar{L}) + \dot{\bar{L}}) >] \} = 0, \qquad (4.15)$$

which is fulfilled if one of the following equations is true:

$$\frac{\partial x_i(t, \bar{l})}{\partial t} + 1/2 < (\frac{\partial x_i}{\partial \bar{l}})^T, \dot{\bar{A}} (\bar{A})^{-1} (\bar{l} - \bar{L}) + \dot{\bar{L}}) >= 0, i = 1, ..., n; \qquad (4.16a)$$

$$< (\frac{\partial x_i}{\partial \bar{l}})^T, \dot{\bar{A}} (\bar{A})^{-1} (\bar{l} - \bar{L}) + \dot{\bar{L}} >= 0, i = 1, ..., n. \qquad (4.16b)$$

At $\frac{\partial x_i}{\partial t} \neq 0$, only the equation (4.16a) is left.

Besides (4.16a), the sufficient condition of the functional's (4.2) invariance is the equality for its first variation: $\delta S^l = 0$, which, for integrals $\delta S^l = \delta S_1^l + \delta S_2^l$, within a variable domain G^4, has a view:

$$\delta S_1^l = \int_{G^4} (\sum_{i=1}^{n} [\frac{\partial L}{\partial x_i} - \sum_{k=1}^{4} \frac{\partial}{\partial l_k} (\frac{\partial L}{\partial (\partial x_i / \partial l_k)}] \bar{\delta} x_i) d\bar{\upsilon},\ \bar{\delta} x_i = \delta x_i - \sum_{k=1}^{4} \frac{\partial x_i}{\partial l_k} y_k . \qquad (4.17a)$$

$$\delta S_2^l = \int_{G^4} \sum_{i,k=1}^{n,4} [\frac{\partial}{\partial l_k} (\frac{\partial L}{\partial(\partial x_i / \partial l_k)} \bar{\delta} x_i) + \frac{\partial (L y_k)}{\partial l_k}] d\bar{\upsilon}\,,\ \{y_k\} = \bar{y}_o\,,\ k = 1,..,4, \quad (4.17b)$$

where the last component in δS_2^l, which represents the integral of divergence, can be reduced to an integral at the border of the domain G^4.

Parameters y_k, according to [23], are defined by the transformation of the coordinate system (4.8a), which after the decomposition in Maclaurin's series acquires the form

$$\bar{l}(t+\delta t) \cong \bar{l}(t) + \delta t[\dot{\bar{A}}(\bar{A})^{-1}(\bar{l} - \bar{L}) + \dot{\bar{L}}]. \quad (4.18a)$$

From this equation follows the equality for the auxiliary function

$$\bar{y} = \lim_{\delta t \to 0} \frac{\bar{l}(t+\delta t) - \bar{l}(t)}{\delta t} = \frac{\partial \bar{l}}{\partial t} = \dot{\bar{A}}(\bar{A})^{-1}(\bar{l} - \bar{L}) + \dot{\bar{L}},\ y_4 = 1. \quad (4.18b)$$

The condition δS_1^l=0 defines the equation of the extremal in the form (4.4a).

The remained equation is δS_2^l=0, from which (at an arbitrary G^4) the relation follows:

$$\sum_{k=1}^{4} \frac{\partial}{\partial l_k} (\sum_{i=1}^{n} \frac{\partial L}{\partial(\partial x_i / \partial l_k)} (-\sum_{k=1}^{4} \frac{\partial x_i}{\partial l_k} y_k)) + \sum_{k=1}^{4} \frac{\partial (L y_k)}{\partial l_k} = 0, i = 1,..., n. \quad (4.19a)$$

Because L does not depend on $\frac{\partial x_i}{\partial l_k}$, k =1,2,3 we arrive at equation

$$\frac{\partial}{\partial t} (\sum_{i=1}^{n} \frac{\partial L}{\partial(\partial x_i / \partial t)} (-\sum_{k=1}^{4} \frac{\partial x_i}{\partial l_k} y_k)) + \sum_{k=1}^{4} \frac{\partial (L y_k)}{\partial l_k} = 0. \quad (4.19b)$$

From that, taking into account the relations

$$\frac{\partial L}{\partial(\partial x_i / \partial t)} = X_i^l, \frac{\partial L}{\partial x_i} = \frac{\partial X_i^l}{\partial t}\,, \quad (4.20a)$$

we come to equations

$$\frac{\partial}{\partial t} \sum_{i=1}^{n} X_i^l (-\sum_{k=1}^{4} \frac{\partial x_i}{\partial l_k} y_k) = \sum_{i=1}^{n} [\frac{\partial X_i^l}{\partial t} (-\sum_{k=1}^{4} \frac{\partial x_i}{\partial l_k} y_k) + X_i^l (-\sum_{k=1}^{4} \frac{\partial^2 x_i}{\partial l_k \partial t} y_k) + X_i^l (-\sum_{k=1}^{3} \frac{\partial x_i}{\partial l_k} \frac{\partial y_k}{\partial t})], \quad (4.20b)$$

$$\frac{\partial L}{\partial l_k} = \sum_{i=1}^{n} (\frac{\partial L}{\partial x_i} \frac{\partial x_i}{\partial l_k} + \frac{\partial L}{\partial(\partial x_i / \partial t)} \frac{\partial^2 x_i}{\partial l_k \partial t}) = \sum_{i=1}^{n} (\frac{\partial X_i^l}{\partial t} \frac{\partial x_i}{\partial l_k} + X_i^l \frac{\partial^2 x_i}{\partial l_k \partial t}), \quad (4.21a)$$

$$\sum_{k=1}^{4} \frac{\partial (L y_k)}{\partial l_k} = < gradL, \bar{y}_o > = \sum_{i=1}^{n} (\frac{\partial X_i^l}{\partial t} (\sum_{k=1}^{4} \frac{\partial x_i}{\partial l_k} y_k) + X_i^l (\sum_{k=1}^{4} \frac{\partial^2 x_i}{\partial l_k \partial t} y_k). \quad (4.21b)$$

After substituting the equations (4.20a)-(4.21b) into the condition (4.19b) we obtain

$$\sum_{i=1}^{n}[\frac{\partial X_i^l}{\partial t}(-\sum_{k=1}^{4}\frac{\partial x_i}{\partial l_k}y_k)+X_i^l(-\sum_{k=1}^{4}\frac{\partial^2 x_i}{\partial l_k\partial t}y_k)+X_i^l(-\sum_{k=1}^{3}\frac{\partial x_i}{\partial l_k}\frac{\partial y_k}{\partial t})$$
$$+\frac{\partial X_i^l}{\partial t}(\sum_{k=1}^{4}\frac{\partial x_i}{\partial l_k}y_k)+X_i^l(\sum_{k=1}^{4}\frac{\partial^2 x_i}{\partial l_k\partial t}y_k)+Ldiv\bar{y}=0,$$

$$\bar{y}=(y_1,y_{2,}y_3), div\bar{y}_o=div\bar{y}. \tag{4.22}$$

For a twice differentiable function $x_i=x_i(t,\bar{l})$ of the arguments $(t,\bar{l})$ on $\Delta\times R^3$ the relation

$$\frac{\partial^2 x_i}{\partial l_k\partial t}=\frac{\partial^2 x_i}{\partial t\partial l_k};i=1,...,n,k=1,2,3,4 \tag{4.23a}$$

is correct.
Then, the equality (4.22) leads to the equation

$$\sum_{i=1}^{n}X_i^l(-\sum_{k=1}^{3}\frac{\partial x_i}{\partial l_k}\frac{\partial y_k}{\partial t})+Ldiv\bar{y}=0, \tag{4.23b}$$

from which at

$$L=1/2\sum_{i=1}^{n}(\sigma_{ii}{}^{-1}\frac{\partial x_i}{\partial t})^2=1/2\sum_{i=1}^{n}X_i^l\frac{\partial x_i}{\partial t} \tag{4.23c}$$

we arrive at the equation

$$[\sum_{i=1}^{n}X_i^l(-\sum_{k=1}^{3}\frac{\partial x_i}{\partial l_k}\frac{\partial y_k}{\partial t})+1/2div\bar{y}\frac{\partial x_i}{\partial t}]=0. \tag{4.24}$$

Because the matrix $\dot{\bar{A}}(\bar{A})^{-1}=C$ is a skew-symmetric, the relation $div\bar{y}$ =0 holds true. The remaining equation is

$$\sum_{i=1}^{n}X_i^l(-\sum_{k=1}^{3}\frac{\partial x_i}{\partial l_k}\frac{\partial y_k}{\partial t})=0, \tag{4.25}$$

which is fulfilled at condition

$$\frac{\partial\bar{y}}{\partial t}=\frac{\partial(y_1,y_2,y_3)}{\partial t}=0. \tag{4.26}$$

The last result is also true for the general Lagrangian form (4.3), because the equations (4.16b), (4.20a) are satisfied for the functional (4.2), (4.3).

If a functional is invariant, then the first of its variation turns to zero automatically on the functional's extremals. Because of that, the equation (4.25) must be satisfied.

This is an additional requirement for the fulfillment of (4.26).

The identical implementation of (4.25) by using condition (4.26) (that is fulfilled by an appropriate choice of functions $\overline{A} = \overline{A}(t), \overline{L} = \overline{L}(t)$), corresponds to the condition of the conservation for the informational laws.

Considering the fulfillment of the Noether theorem at an arbitrary (nonextremal) surface, we may write the first variation of functional δS^l in the form

$$\delta S^l = \int_{G^4} \{\sum_{i=1}^{n} (\frac{\partial L}{\partial x_i} \overline{\delta} x_i + \sum_{k=1}^{4} \frac{\partial L}{\partial(\partial x_i / \partial l_k)} \frac{\partial}{\partial l_k} \overline{\delta} x_i) + \sum_{k=1}^{4} \frac{\partial (Ly_k)}{\partial l_k} d\overline{\upsilon}\} =$$

$$\int_{G^4} \{\sum_{i=1}^{n} (\frac{\partial L}{\partial x_i} \overline{\delta} x_i + \frac{\partial L}{\partial(\partial x_i / \partial l_k)} \frac{\partial}{\partial t} (-\sum_{k=1}^{4} \frac{\partial x_i}{\partial l_k} y_k) + \sum_{k=1}^{4} \frac{\partial (Ly_k)}{\partial l_k}\} d\overline{\upsilon};$$

$$\overline{\delta} x_i = (-\sum_{k=1}^{4} \frac{\partial x_i}{\partial l_k} y_k). \tag{4.27}$$

At the condition of an arbitrariness of the integrating domain we obtain

$$\sum_{i=1}^{n} (\frac{\partial L}{\partial x_i} \overline{\delta} x_i + \frac{\partial L}{\partial(\partial x_i / \partial l_k)} \frac{\partial}{\partial t} (-\sum_{k=1}^{4} \frac{\partial x_i}{\partial l_k} y_k)) + \sum_{k=1}^{4} \frac{\partial (Ly_k)}{\partial l_k} = 0. \tag{4.28}$$

The last component of equation (4.28), taking into account (4.20), can be reduced to the form

$$\sum_{k=1}^{4} \frac{\partial (Ly_k)}{\partial l_k} = < gradL, \overline{y}_o > + Ldiv\overline{y}_o = < gradL, \overline{y}_o >$$

$$= 2\sum_{i=1}^{n} \frac{\partial x_i}{\partial t} \sigma_{ii}^{-3} (\sum_{k=1}^{4} \frac{\partial^2 x_i}{\partial t \partial l_k} y_k \sigma_{ii} - \sum_{k=1}^{4} \frac{\partial x_i}{\partial l_k} y_k \frac{\partial \sigma_{ii}}{\partial t}). \tag{4.29}$$

By substituting equations (4.20), (4.29) into equation (4.28) we get relation

$$\sum_{i=1}^{n} [(-2\frac{\partial x_i}{\partial t} \sigma_{ii}^{-3} \frac{\partial \sigma_{ii}}{\partial t})(-\sum_{k=1}^{4} \frac{\partial x_i}{\partial l_k} y_k) + 2\frac{\partial x_i}{\partial t} \sigma_{ii}^{-2} \frac{\partial}{\partial t} (-\sum_{k=1}^{4} \frac{\partial x_i}{\partial l_k} y_k)]$$

$$+ \sum_{i=1}^{n} 2\frac{\partial x_i}{\partial t} \sigma_{ii}^{-3} (\sum_{k=1}^{4} \frac{\partial^2 x_i}{\partial t \partial l_k} y_k \sigma_{ii} - \sum_{k=1}^{4} \frac{\partial x_i}{\partial l_k} \frac{\partial \sigma_{ii}}{\partial t})$$

$$= \sum_{i=1}^{n} 2\frac{\partial x_i}{\partial t} \sigma_{ii}^{-3} [\sum_{k=1}^{4} (\frac{\partial x_i}{\partial l_k} y_k \frac{\partial \sigma_{ii}}{\partial t} - \frac{\partial^2 x_i}{\partial t \partial l_k} y_k \sigma_{ii}$$

$$- \frac{\partial x_i}{\partial l_k} \frac{\partial y_k}{\partial t} \sigma_{ii} + \frac{\partial^2 x_i}{\partial t \partial l_k} y_k \sigma_{ii} - \frac{\partial x_i}{\partial l_k} \frac{\partial \sigma_{ii}}{\partial t})] = 0. \tag{4.30a}$$

From that, taking into account relation (4.20a), we arrive at equation

$$\sum_{i=1}^{n} 2\frac{\partial x_i}{\partial t}\sigma_{ii}^{-2}\left(\sum_{k=1}^{3} -\frac{\partial x_i}{\partial l_k}\frac{\partial y_k}{\partial t}\right) = \sum_{i=1}^{n} 2X_i^l\left(\sum_{k=1}^{3} -\frac{\partial x_i}{\partial l_k}\frac{\partial y_k}{\partial t}\right) = 0, \tag{4.30b}$$

which coincides with the considered equation (4.25) on the extremals (4.4a).

This means that the identity of $\frac{\partial y_k}{\partial t} \equiv 0$ is fulfilled, and the condition (4.26) is satisfied.

Therefore, if the functional is invariant, its first variation turns to zero by the transformation that fulfills the condition (4.10). •

Corollary 2.1. (C1).

The sufficient condition of the functional invariance requires the satisfaction of the equation (4.10), i.e. the fulfillment of the equation

$$\frac{1}{2}\sum_{k=1}^{4}\frac{\partial x_i}{\partial l_k} y_k = -\frac{\partial x_i}{\partial t}, \tag{4.31}$$

Equations (4.9), (4.10), and (4.31) are the *differential constraints* imposed by the variation principle on the field equations.

Proof of C1. By substituting the equations (4.31) and (4.20a), (4.29) into (4.28) we obtain

$$\sum_{i=1}^{n}\left[\left(-2\frac{\partial x_i}{\partial t}\sigma_{ii}^{-3}\frac{\partial \sigma_{ii}}{\partial t}\right)\frac{\partial x_i}{\partial t} + 2\frac{\partial x_i}{\partial t}\sigma_{ii}^{-2}\frac{\partial^2 x_i}{\partial t^2}\right] + \sum_{i=1}^{n} 2\frac{\partial x_i}{\partial t}\sigma_{ii}^{-3}$$

$$\times\left(\sum_{k=1}^{4}\frac{\partial^2 x_i}{\partial t \partial l_k} y_k\sigma_{ii} + \frac{\partial x_i}{\partial t}\frac{\partial \sigma_{ii}}{\partial t}\right) = \sum_{i=1}^{n} 2\frac{\partial x_i}{\partial t}\sigma_{ii}^{-3}\left(\frac{\partial^2 x_i}{\partial t^2}\sigma_{ii} + \sum_{k=1}^{4}\frac{\partial^2 x_i}{\partial t \partial l_k} y_k\sigma_{ii}\right) = 0 \tag{4.31a}$$

.

$$\sum_{i=1}^{n} 2\frac{\partial x_i}{\partial t}\sigma_{ii}^{-2}\left(\frac{\partial^2 x_i}{\partial t^2} + \frac{\partial}{\partial t}\left(\sum_{k=1}^{4}\frac{\partial x_i}{\partial l_k} y_k\right) - \sum_{k=1}^{3}\frac{\partial x_i}{\partial l_k}\frac{\partial y_k}{\partial t}\right)$$

$$= \sum_{i=1}^{n} 2\frac{\partial x_i}{\partial t}\sigma_{ii}^{-2}\left(\frac{\partial^2 x_i}{\partial t^2} - \frac{\partial^2 x_i}{\partial t^2} - \sum_{k=1}^{4}\frac{\partial x_i}{\partial l_k} y_k\right)$$

$$= \sum_{i=1}^{n} 2\frac{\partial x_i}{\partial t}\sigma_{ii}^{-2}\left(-\sum_{k=1}^{3}\frac{\partial x_i}{\partial l_k}\frac{\partial y_k}{\partial t}\right) = \sum_{i=1}^{n} 2X_i^l\left(-\sum_{k=1}^{3}\frac{\partial x_i}{\partial l_k}\frac{\partial y_k}{\partial t}\right) = 0. \tag{4.31b}$$

Thus, relation (4.25) has been obtained without using condition (4.23a) and the extremal equations. This is the result of the fulfillment of (4.31a,b) that guarantees the zero equality for the first variation of the functional.

From this fact, however, it is not possible to get directly the sought transformation (4.7). Only after transforming the first variation into the form (4.31b), (4.11), it becomes clear that the identical equalization of this variation to zero must be reached by the fulfillment of relation (4.26), which is a consequence of the initial variation principle.

Moreover, the result (4.26) represents a condition of a constant space velocity in an arbitrary coordinate system. •

1.4.4. The Parameters of the Space Transformation and the Distributed Macromodels

The equation (4.10) determines the parameters of the space coordinates' transformation at the following conditions

$$\bar{y} = \bar{y}(t, \bar{l}) = \dot{\bar{A}}(\bar{A})^{-1}(\bar{l} - \bar{L}) + \dot{\bar{L}} = C(\bar{l} - \bar{L}) + \dot{\bar{L}}; \frac{\partial \bar{y}}{\partial t} = \dot{C}(\bar{l} - \bar{L}) - C\dot{\bar{L}} + \ddot{\bar{L}} = 0. \quad (4.32)$$

The last equality has to be true identically at any $\bar{l}$. From which the equation for an angular of rotation follows:

$$\dot{C}(t) = 0, C = (C_{ij}), C_{ij} = Const, C_{ij} = -C_{ji}, C_{jj} = 0, i \neq j, \quad (4.33)$$

and the equality (4.32) is divided on two equations:

$$\dot{\bar{A}}(\bar{A})^{-1} = C, \dot{\bar{A}} = C\bar{A}; C = 1/2(C - C^T); C^T = (C_{ij}); \quad (4.34)$$

$$\ddot{\bar{L}} = C\dot{\bar{L}}, \quad (4.35)$$

where C is the angular's constant of the speed rotation.

By integrating (4.34) we obtain the equation for the transformation of the space coordinates

$$\bar{A}(t) = \exp[(t - t_o)C]\bar{A}(t_o), t_o = s. \quad (4.36)$$

For the considered plan movement with the transformation matrix:

$$\bar{A} = \begin{pmatrix} \cos\phi, \sin\phi \\ -\sin\phi, \cos\phi \end{pmatrix}; \bar{A}^{-1} = \bar{A}^T, \quad (4.37)$$

the equality (4.34) is satisfied at the fulfillment of the equations:

$$\frac{d\phi}{dt}\begin{pmatrix} \cos\phi, -\sin\phi \\ \sin\phi, \cos\phi \end{pmatrix}\begin{pmatrix} -\sin\phi, \cos\phi \\ -\cos\phi, -\sin\phi \end{pmatrix} = C, \frac{d\phi}{dt} = Const. \quad (4.38)$$

Thus, the relation (4.38) is the necessary and sufficient condition for the fulfillment (4.33) at a plan movement. From whence, taking into account the requirement

$$\bar{y} = \frac{d\bar{l}}{dt} = \frac{\partial \bar{y}}{\partial \phi}\frac{d\phi}{dt} = C, \bar{l} = \bar{l}(\phi(t)), \quad (4.39)$$

we arrive at the condition

$$\frac{\partial \bar{y}}{\partial \phi} = Const. \quad (4.40)$$

However, the fulfillment of this condition is not enough for the case of a general space movement.

Equation (4.35) admits reducing its order, and it is represented by the solution of a system of the ordinary differential equations with constant coefficients at the given initial conditions:

$$\dot{R} = C\ R;\ \dot{\bar{L}} = R;\ \bar{L}(t_o) = \bar{L}_o;\ \dot{\bar{L}}\ (t_o) = R_o\ . \tag{4.41}$$

The fulfillment of the extremal principle leads to the equations

$$\frac{\partial x}{\partial t} = -1/2 < \frac{\partial x}{\partial \bar{l}} \dot{\bar{A}}(\bar{A})^{-1}(\bar{l} - \bar{L}) + \dot{\bar{L}} >= 0, \dot{\bar{A}}(\bar{A})^{-1} = C,$$

$$\ddot{\bar{L}} - \dot{\bar{L}} = 0\ , C = 1/2(C - C^T)\ . \tag{4.42}$$

The joint consideration of the equations for extremals (1.4a) and differential constraint (4.10) brings the following forms of the distributed macromodels.

By substituting equation (4.31) into equality (4.5) we arrive at the model

$$\sigma_{ii} \frac{\partial^2 x_i}{\partial t^2} + 1/2(\sum_{k=1}^{3} \frac{\partial x_i}{\partial l_k} y_k) \frac{\partial \sigma_{ii}}{\partial t} = 0, i = 1, 2, ..., n. \tag{4.43}$$

(See also the proof of Theorem 2.1 and C.1).

After differentiating equality (4.31) we come to equation

$$\frac{\partial^2 x_i}{\partial t^2} + \sum_{k=1}^{3} \frac{\partial^2 x_i}{\partial l_k \partial t} + \sum_{k=1}^{3} \frac{\partial x_i}{\partial l_k} \frac{\partial y_k}{\partial t} = 0; \frac{\partial^2 x_i}{\partial t^2} = -1/2 \sum_{k=1}^{3} \frac{\partial^2 x_i}{\partial l_k \partial t} y_k, i = 1, 2, ..., n. \tag{4.44}$$

By substituting the last one into (4.5) we get

$$\frac{\partial \sigma_{ii}}{\partial t} \frac{\partial x_i}{\partial t} + 1/2 \sigma_{ii} \sum_{k=1}^{3} \frac{\partial^2 x_i}{\partial l_k \partial t} y_k = 0, i = 1, 2, ..., n. \tag{4.45}$$

And finally, after substituting equation (4.31) into (4.44) we obtain the model

$$\sum_{k=1}^{3} (\frac{\partial^2 x_i}{\partial l_k \partial t} \sigma_{ii} - \frac{\partial x_i}{\partial l_k} \frac{\partial \sigma_{ii}}{\partial t}) y_k = 0; \frac{\partial x_i}{\partial l_k} \frac{\partial \sigma_{ii}}{\sigma_{ii} \partial t} = \frac{\partial (\partial x_i / \partial t)}{\partial l_k}; y_k \neq 0. \tag{4.46}$$

The controllable distributed macromodel with the reduced controls v, satisfying $u = A(t, \bar{l})v$, gets the form

$$\frac{\partial x}{\partial t} = A(t, \bar{l})(x + v); C \frac{\partial x}{\partial l} = A(t, \bar{l})(x + v). \tag{4.47}$$

If each of the phase vector's component $x_i = x$ performs a single-dimensional movement in the direction of the space axis $l_i = l$ with a local speed c, then (4.47) acquires the diagonal form

$$\frac{\partial x}{\partial t} = \lambda(x+v); c\frac{\partial x}{\partial l} = \lambda(x+v); x = x(t,l), v = v_s(l) = -2x(s,l), \lambda = \lambda(t,l) \tag{4.48}$$

at the above reduced optimal control, applied at the initial point.

For this system, the parameter $\lambda = \lambda(t,l)$, which defines the matrix A, is identified by an analogy with the concentrated model, using the corresponding correlation's matrices r_v, $\dot{r}_v$ (measured at the microlevel):

$$\lambda(t,l) = cE[\frac{\partial x}{\partial l}(x+v)^T]r_v^{-1}; r_v = E[(\tilde{x}+v)(\tilde{x}+v)^T], \lambda_i = 1/2\dot{r}_{vi}r_{vi}^{-1}. \tag{4.49}$$

The needle control's action (ch.1.3), applied at the "punched" fields' locations, jumps the model operator:

$$\delta v = v(\tau+o,l) - v(\tau-o,l), \delta A = A(\tau+o,l) - A(\tau-o,l). \tag{4.50}$$

At these locations, each path functional's extremal, approximates the microlevel process with a maximal probability (chs.1.2,1.3).

The total extremal is *divided on the segments* limited by these locations, while the set of discrete moments: $\tau = \bigcup_k \tau_k \subset \Delta$ between the locations is determined from the Erdman-Weierstrass' conditions [23,24].

As a result, we obtain the *optimal control, fixed along an extremal segment*, particularly, in the form

$$v(\tau+o,l) = -2x(\tau+o,l) \tag{4.51}$$

and the following equation for the *matrix A identification* by the correlation matrix r and its derivative:

$$A(\tau+o,l) = 1/2\frac{\partial r}{\partial t}(\tau-o,l)\, r^{-1}(\tau,l),\ r(\tau,l) = E[(\tilde{x}+v)(\tilde{x}+v)^T], E = E_{x,\tau} \tag{4.52}$$

at the moments of the equalization of the relative phase speeds of the state vector (between the segments):

$$\frac{\partial z_i}{\partial t}(\tau-o,l)z_i^{-1}(\tau,l) = \frac{\partial z_j}{\partial t}(\tau-o,l)z_j^{-1}(\tau,l); z = Gx, G: \Delta \times R^3 \to \angle(R^n), \tag{4.53}$$

as a part of the optimal synthesis, where G is the diagonal form of matrix A; z is the corresponding state vector.

The relation (4.53) identifies the starting time-space instants of the applied controls and the matrix A *renovation*.

The renovated matrix is identified *during the optimal control action*, applied during each segment and at a *current* punch locality (the given initial conditions determine the first of the applied control).

The renovated matrix's divergence (4.50) is a result of both identification and needle control action.

The condition of the state's equalization allows characterizing each pair of the state vectors by a *single joint* vector. The consequent realization of this process leads to the state's *cooperation during the optimal control and identification*.

Thus, the equalization (4.53) and cooperation follow from the solution of the optimal problem for the path functional approximating the entropy functional (4.1c).

The Lagrangian, in a more general case than (4.3), admits the representation [39, 58]:

$$L = -\sum_{i,j=1}^{n} B_{ij}\hat{y}_i\hat{y}_j^{\,*} - 1/2\sum_{i=1}^{n}\left(x_i^{\,*}\frac{\partial x_i}{\partial t} - x_i\frac{\partial x_i^{\,*}}{\partial l_k}\right), \hat{y}_i = \sum_{k=1}^{3} G_{ik}\frac{\partial x_i}{\partial l_k},\ \hat{y}_i^{\,*} = \sum_{k=1}^{3} G_{ik}^{*}\frac{\partial x_i^{\,*}}{\partial l_k}, \tag{4.54}$$

where vector $\hat{y}$ is proportional to the gradient of the state vector, and it is analogous to a general force, or to the conjugate vector X; and the vector x is analogous of generalized flow [58]. (The complex conjugated variables are indicated by*).

The corresponding Euler-Ostrogradsky equations lead to the equation of extremals for the main $(x, \hat{y})$ and the conjugate variables $(x^*, \hat{y}^*)$ in the above equations:

$$\frac{\partial x_i}{\partial t} = \sum_{j=1}^{n} B_{ij}\left(\sum_{k,m=1}^{3} G_{jm}G_{ik}^{*}\frac{\partial^2 x_j}{\partial l_m \partial l_k}\right), i = 1,2,...,n. \tag{4.55}$$

In a particular, at the conditions

$$G_{jm} = g_j G_m, G_{ik}^{*} = g_i^{*} G_k,\ G_k G_m = G_{km} = \delta_{km}, k, m = 1,2,3, \tag{4.56}$$

the equations of the extremals acquire a form

$$\frac{\partial x_i}{\partial t} = \sum_{j=1}^{n} D_{ij}\sum_{m=1}^{3}\frac{\partial^2 x_j}{\partial l_m^{\,2}};\ i, j = 1,2,\ldots,n,\ \left(D_{ij}\right)_{i,j=1}^{n} = D; D_{ij} = B_{ij}g_i g_i^{*}. \tag{4.57}$$

The last equation leads to a diffusion equation with the diffusion matrix D and the Laplace operator:

$$\frac{\partial x}{\partial t} = D\Delta x, \Delta = \nabla^2. \tag{4.58}$$

The macromodel in the form of the nonequilibrium thermodynamic equation follows from (4.55), using the definition of generalized flow $I = \frac{\partial x}{\partial t}$, and, thus, connecting the flows I and forces $\overline{X}$:

$$I_i = \sum_{j=1}^{n} \overline{l}_{ij} \overline{X}_j, \overline{l}_{ij} = B_{ij} g_i, \overline{X}_i = g_i \sum_{m=1}^{3} \frac{\partial^2 x_j}{\partial l_m^{\,2}}, i, j = 1, 2, ..., n. \tag{4.59}$$

The equation of the extremals takes the nonequilibrium thermodynamics' form [56-58]:

$$\frac{\partial x}{\partial t} = 2b\overline{X}^l, \overline{X}^l = \frac{\partial \Delta \overline{S}^l}{\partial \nabla x}, \nabla x = grad x, (\overline{l}_{ij}) = \sigma\sigma^T = 2b, \tag{4.60}$$

and it follows directly from the entropy form of the macromodel's functional for the local homogenous fields: $\Delta \overline{S}^l = 1/2 \nabla x^T g \nabla x$, where the kinetic operator $(\overline{l}_{ij})$ in our approach is *identified by the operator of diffusion* in (4.1a).

The macromodels of the distributed systems, described by the integral equations

$$\frac{\partial x_i}{\partial t}(t, \overline{l}) = \sum_{j=1}^{n} \int_{\tilde{v}} \hat{l}_{ij}(\overline{l}, \tilde{\overline{l}}, t) \overline{X}_i(\tilde{\overline{l}}, t) d\tilde{\overline{l}}, \tag{4.61}$$

depend on the kinetic operator $\hat{l}\ (\overline{l}, \tilde{\overline{l}}, t) = 1/2 \frac{\partial \hat{r}}{\partial t}\ (\overline{l}, \tilde{\overline{l}}, t)$, which is identifiable by the correlation vector-function $\hat{r}$ of the initial random field.

These equations are also used in physical kinetics and theory of dissipative systems [38, 58, other].

The optimization problem is formulated as an extreme condition for the Hamiltonian

$$H = \int_v \frac{\partial x}{\partial t}(t, \bar{l}) * X(t, \bar{l}) d\bar{l}. \tag{4.62}$$

For the space distributed system, the generalized flow $I = a^u$ depends on gradients and a function $g_o(x, t, v)$ of the coordinates, and control:

$$a^u = g_o \frac{\partial x}{\partial \overline{l}}; g_o = g_o(x, t, v). \tag{4.63}$$

In this case, the random field is averaged with respect to some piece-wise space curves, defined from equation (4.1a) with the shift-vector depending on the generalized flow.

Among these curves, the variation principle is able to select the extremals of the path functional.

For this model, the functional and its Lagrangian acquire the form:

$$S = \iint L dt dl; l = (l_x, l_y, l_z);$$

$$L = 1/2 (a^u)^T (2b)^{-1} a^u - 1/2 (\frac{\partial x}{\partial t}^T r_v^{-1} (x + v) - (x + v)^T r_v^{-1} \frac{\partial x}{\partial t}). \tag{4.64}$$

The corresponding Euler-Ostrogradsky equation has a form

$$\frac{\partial L}{\partial x^T} - \frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{x}_t} - \frac{\partial}{\partial l}\frac{\partial L}{\partial \dot{x}_l} = 1/2[r_v^{-1}\frac{\partial x}{\partial t} + \frac{\partial}{\partial t}(r_v^{-1}(x+v)) - (2b)^{-1}\frac{\partial}{\partial l}(g_o\frac{\partial x}{\partial l})] = 0, \tag{4.65}$$

from which the equation of the extremals with the coefficients, represented by the correlation matrix, follows:

$$r_v^{-1}\frac{\partial x}{\partial t} + 1/2\frac{\partial}{\partial t}(r_v^{-1}(x+v)) = 1/2(2b)^{-1}g_o\frac{\partial^2 x}{\partial l^2}. \tag{4.66}$$

We get the controlled macroequation, whose parameters identified at the microlevel

$$\frac{\partial x}{\partial t} = 1/2(r_v(2b)^{-1}g_o\frac{\partial^2 x}{\partial l^2} + \frac{\partial r_v}{\partial t}r_v^{-1}(x+v)^T; r_v = E[(\tilde{x}+v)(\tilde{x}+v)^T], \tag{4.67}$$

including the function

$$g_o = E[\frac{\partial x}{\partial t}(x+v)^T]\times\{E[\frac{\partial x}{\partial l}(x+v)^T]\}^{-1}; E = E_{x,\tau}, \tag{4.68}$$

which corresponds to a known form of the diffusion equation:

$$\frac{\partial x}{\partial t} = f(t,b(t))\frac{\partial^2 x}{\partial l^2} + \dot{f}(t)(x+v); \tag{4.69}$$

with the controls $v(\tau,l)$ and the relations for the identification of the auxiliary functions:

$$f = 1/2r_v(2b)^{-1}g_o; \dot{f} = 1/2\frac{\partial r_v}{\partial t}r_v^{-1}; g_o = r_1(r_1^l)^{-1};$$

$$r_1 = E[\frac{\partial x}{\partial t}(x+v)^T)], r_1^l = E[\frac{\partial x}{\partial l}(x+v)^T]. \tag{4.69a}$$

Finally, the macroequation for a space distributed system acquires a general *operator's* form

$$\frac{\partial x}{\partial t} = A(t,l,v,)\Delta x,\ \Delta = \nabla^2,\ \nabla x = gradx,\ x = \{x_i\}_{i=1}^n,\ l = \{l_i\}_{i=1}^m,\ l_i = \{l_x,l_y,l_z\}, \tag{4.70}$$

$$v = -2x(\tau),\ v = \{v_i\}_{i=1}^n,\ \tau = \{\tau_{imk}\}_{k=1}^{n,m},\ m \le n, \tag{4.70a}$$

where $A(t,l,v,)$ is the model's operator, Δ is Laplacian of the macrovector x; v is the reduced control, switched at discrete points (DP) τ.

The *macromodel processes* $x = x(t,l,u,\xi)$ describe the *evolution dynamics* in the time-space coordinates (t,l).

Actually, Theory of Markov random process [60] includes building of the PDE (in Fourier's, Fick's and other forms) based on Fokker-Plank-Kolmogorov equation for the probability function, defined on Brownian motion.

In particular, the condition (4.53) follows from the Markovian property (sec.1.2.1).

The models operator depends on the microlevel's diffusion that allows the model's statistical identification.

Specific examples of the model's identification, based on the microlevel's correlation and dispersion functions, are given in [53, 54] and in sec.2.1.2.2.

For a real object, the procedure involves both statistical methods and a numerical computation.

The concrete object's path functional determines the macromodel's specific *structure*, whose *parameters* are identified on the object along with the path functional's parameters and the controls.

The identified macroequations reflect the object's *informational regularities*, in particular, representing the *information analogies of the irreversible thermodynamics equations*.

1.4.5. The IPF Macromodel's Singular Points and the Singular Trajectories

For the considered concentrated and distributed macromodels, the analysis of the existence and uniqueness of singular points and/or the singular trajectories represents the principal and sufficient interest.

The pair equalization of the relative phase speeds (3.77), (3.164b) at each discrete point for the concentrated model (3.22) leads to singularities of the dynamic operator, which follow from the consideration below.

We will show that the dynamics and geometry at each of the singular points of the spatial macromodel (sec. 4.3) are bound, and the singularities are associated with the model's cooperative phenomena.

Such analysis we provide for the model in partial derivations of the first order:

$$A_1 \frac{\partial x}{\partial t} = B_1 \frac{\partial x}{\partial l} + v_1; A_2 \frac{\partial y}{\partial t} = B_2 \frac{\partial y}{\partial l} + v_2 \qquad (4.71)$$

with variables (x, y), spatial coordinate l, time t, controls $v_1(t,l,x), v_2(t,l,y)$, and coefficients

$$A_1 = A_1(t,l,x),\ B_1 = B_1(t,l,x),\ A_2 = A_2(t,l,y), B_2 = B_2(t,l,y).$$

The initial conditions are given by the following distributions

$$x|_{t=s} = \varphi_1^l(l) = \tilde{x}(s, l_o, \tau, l), \text{ or } x|_{l=l_o} = \varphi_1^t(t); y|_{t=s} = \varphi_2^l(l) = \tilde{y}(s, l_o, \tau, l),$$
$$\text{or } y|_{l=l_o} = \varphi_2^t(t). \tag{4.71a}$$

Equations (4.71), (4.71a) characterize a reflection of some region of plane (t, l) on a region of space $(\Delta S, x, y)$, where the peculiarities and class of the surface $\Delta S = \Delta S(x, y)$ are completely defined by a specific equation of the reflection.

At the known solution of problem (4.71), (4.71a), this surface's equation can be defined in a parametrical form:

$$x = x(t, l), y = y(t, l), \Delta S = \Delta S[x(t, l), y(t, l)]. \tag{4.72}$$

For the given system, a singular point of the second order of the considered surface is determined by the condition of decreasing the following matrix's rank:

$$rank \begin{vmatrix} \frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial \Delta S}{\partial t} \\ \frac{\partial x}{\partial l}, \frac{\partial y}{\partial l}, \frac{\partial \Delta S}{\partial l} \end{vmatrix} \neq 2, \tag{4.73}$$

which corresponds to decreasing the above matrix's rank, it means turning to zero all minors of the second order of the above matrix.

By introducing the radius-vector $\vec{r} = x\vec{e}_1 + y\vec{e}_2 + \Delta S\vec{e}_3$ with the orths of basic vectors $\{\vec{e}_i\}_i^3$ and the derivatives

$$\vec{r}_t = \frac{\partial \vec{r}}{\partial t}, \vec{r}_l = \frac{\partial \vec{r}}{\partial l},$$

we write (4.73) in the form

$$[\vec{r}_t \times \vec{r}_l] = 0. \tag{4.73a}$$

The equation of a normal $\vec{N}$ to surface (4.72) has view:

$$\vec{N} = \begin{vmatrix} \vec{e}_1, \vec{e}_2, \vec{e}_3 \\ \frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial \Delta S}{\partial t} \\ \frac{\partial x}{\partial l}, \frac{\partial y}{\partial l}, \frac{\partial \Delta S}{\partial l} \end{vmatrix}, \tag{4.74}$$

where $\vec{n} = \frac{\vec{N}}{N}$ is an orth of the normal $\vec{N}$.

Because $\vec{r}_t$, $\vec{r}_l$ are the tangent vectors to the coordinate lines, the fulfillment of (4.72) or (4.73) is an equivalent of the condition of a *nonexistence of the normal* (4.74) at the given singular point.

Since a normal to a surface is determined independently on a method of the surface parameterization, we come to the following conditions of the existence of the singular point:

$$\vec{N} = M_1\vec{e}_1 + M_2\vec{e}_2 + M_2\vec{e}_2 = 0\,, \tag{4.75}$$

or

$$M_1 = \det\begin{vmatrix} x_t, y_t \\ x_l, y_l \end{vmatrix} = 0\,,\ \Delta S_l = \frac{\partial \Delta S}{\partial l}\,,\ x_t = \frac{\partial x}{\partial t}, y_t = \frac{\partial y}{\partial t}\,,$$

$$x_l = \frac{\partial x}{\partial l}, y_l = \frac{\partial y}{\partial l}\,; \tag{4.75a}$$

$$M_2 = \det\begin{vmatrix} x_t, \Delta S_t \\ x_l, \Delta S_l \end{vmatrix} = 0\,,\ \Delta S_t = \frac{\partial \Delta S}{\partial t}\,,\ \Delta S_l = \frac{\partial \Delta S}{\partial l}\,; \tag{4.75b}$$

$$M_3 = \det\begin{vmatrix} y_t, \Delta S_t \\ y_l, \Delta S_l \end{vmatrix} = 0\ . \tag{4.75c}$$

According to (4.72) we write a complete derivation

$$\frac{\partial \Delta S}{\partial t} = \frac{\partial \Delta S}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial \Delta S}{\partial y}\frac{\partial y}{\partial t}; \frac{\partial \Delta S}{\partial l} = \frac{\partial \Delta S}{\partial x}\frac{\partial x}{\partial l} + \frac{\partial \Delta S}{\partial y}\frac{\partial y}{\partial l}\,. \tag{4.76}$$

That is why relations (4.75a-c) are fulfilled automatically if (4.75) holds true.

Indeed, using (4.76) for (4.75b), we get

$$\frac{\partial x}{\partial t}\left(\frac{\partial \Delta S}{\partial x}\frac{\partial x}{\partial l} + \frac{\partial \Delta S}{\partial y}\frac{\partial y}{\partial l}\right) - \frac{\partial x}{\partial l}\left(\frac{\partial \Delta S}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial \Delta S}{\partial y}\frac{\partial y}{\partial t}\right) = \frac{\partial \Delta S}{\partial x}\left(\frac{\partial x}{\partial t}\frac{\partial y}{\partial l} - \frac{\partial x}{\partial l}\frac{\partial y}{\partial t}\right) = \frac{\partial \Delta S}{\partial x}J\,, \tag{4.77}$$

where Jakobian for this system satisfies (4.75a):

$$J = \frac{D(x,y)}{D(t,l)} = \det\begin{vmatrix} x_t, y_t \\ x_l, y_l \end{vmatrix} = M_1 = 0\,. \tag{4.77a}$$

This brings the strong connection of the system's geometrical coordinates $(l, \bar{e})$ with the dynamics of (x, y).

Therefore, at a chosen representation (4.72), the singular points correspond also a degeneracy of Jacobean J, or the fulfillment of condition

$$\frac{\partial x}{\partial t}\frac{\partial y}{\partial l}=\frac{\partial y}{\partial t}\frac{\partial x}{\partial l}\,, \tag{4.77b}$$

which for the distributed model is an analog of the equalization of the relative phase speeds (3.77), (3.160).

Indeed. The analog of the relation (4.72) for a corresponding matrix of the concentrated system is

$$rank\begin{vmatrix}\frac{dx}{dt},\frac{d\Delta S}{dt}\\ \frac{dy}{dt},\frac{d\Delta S}{dt}\end{vmatrix}\neq 2\,,$$

which leads to

$$\det\begin{vmatrix}\frac{dx}{dt},\frac{d\Delta S}{dt}\\ \frac{dy}{dt},\frac{d\Delta S}{dt}\end{vmatrix}=0\,,$$

or

$$\frac{d\Delta S}{dt}(\frac{dx}{dt}-\frac{dy}{dt})=0\,,\ \frac{d\Delta S}{dt}\neq 0\,,\ \frac{dx}{dt}=\frac{dy}{dt}\,. \tag{4.77c}$$

The last relation at $x(\tau)\cong x(\tau+o)\neq 0$ coincides with (3.77) at $t=\tau$.

Let us apply (4.77b) to the system (4.71), written in the diagonal form:

$$(\lambda_1^t)^{-1}\frac{\partial x}{\partial t}=(\lambda_1^l)^{-1}\frac{\partial x}{\partial l}+v_1(t,l)\,;\ (\lambda_2^t)^{-1}\frac{\partial y}{\partial t}=(\lambda_2^l)^{-1}\frac{\partial y}{\partial l}+v_2(t,l)\,, \tag{4.78}$$

where $(\lambda_1^t,\lambda_1^l),(\lambda_2^t,\lambda_2^l)$ are the corresponding eigenvalues.

For the diagonalized equations, it is possible to build the system of the regular differential equations in a symmetric form, generally

$$\frac{dt}{(\lambda_i^t)^{-1}}=-\frac{dl}{(\lambda_i^l)^{-1}}=\frac{dx_i}{v_i},\ i=1,\ldots,n \tag{4.79}$$

with its common integrals

$$\Phi^i=(\phi_1^i,\phi_2^i)=0, \tag{4.80}$$

where the first integrals:

$$\phi_1^i=\int\lambda_i^l(l)dl+\int\lambda_i^t(t)dt\,,\ \phi_2^i=x_i+\int\lambda_i^l(l)v_i dl \tag{4.80a}$$

are the solutions of (4.79).

The concrete form of the common integral is defined by (4.71a) and (4.78):

$$\Phi = \phi_2 - \int \lambda^l(l)[f(\phi)]v[f(\phi)]\frac{\partial f}{\partial \phi}d\phi + \varphi^l[f(\phi)], \tag{4.81}$$

where

$$\phi = \phi_1 - \int \lambda^\tau(\tau)d\tau = \int \lambda^l(l)dl + \int \lambda^t(t)dt - \int \lambda^\tau(\tau)d\tau; \int \lambda^\tau(\tau)d\tau = \int \lambda^t(t)dt\,|_{t=\tau},$$
$$\lambda^t = (\lambda_1^t, \lambda_2^t), \lambda^l = (\lambda_1^l, \lambda_2^l),$$
$$x = (x_1, x_2), \Phi = (\Phi^1, \Phi^2), \phi_1 = (\phi_1^1, \phi_1^2), \phi_2 = (\phi_2^1, \phi_2^2), \tag{4.81a}$$

and f is a root of equation $f = l(\phi_1(\tau), \tau)$ solved for l and a fixed $t = \tau$:

$$\int \lambda^l(l)dl = \bar{\phi}_1 - \int \lambda^\tau(\tau)d\tau, \ \bar{\phi}_1 = \phi_1\,|_{t=\tau}. \tag{4.82}$$

A partial solution of (4.79, (4.81)-(4.82) acquires the form:

$$x = -\int \lambda^l(l)vdl + \int \lambda^l(l)[f(\phi))]v[f(\phi))]\frac{\partial f}{\partial \phi}d\phi + \varphi^l[f(\phi)]. \tag{4.83}$$

Then, the corresponding partial derivations have the view:

$$\frac{\partial x}{\partial t} = -\int \lambda_1^l \frac{\partial v_1}{\partial t}dl + \Phi_1\lambda_1^t, \ \frac{\partial y}{\partial t} = -\int \lambda_2^l \frac{\partial v_2}{\partial t}dl + \Phi_2\lambda_2^t, \tag{4.84}$$

$$\frac{\partial x}{\partial l} = -\lambda_1^l v_1 + \Phi_1\lambda_1^l, \ \frac{\partial y}{\partial l} = -\lambda_2^l v_2 + \Phi_2\lambda_2^l, \tag{4.84a}$$

$$\Phi_1 = [\frac{\partial \varphi_1^l}{\partial f_1}(\phi_1) + \lambda_1^l v_1(\phi_1)]\frac{\partial f_1}{\partial \phi_1} \tag{4.85}$$

By imposing the condition (4.77b) on the systems (4.84)-(4.85), we come to equation

$$(\int \lambda_1^l \frac{\partial v_1}{\partial t}dl - \Phi_1\lambda_1^t)(\lambda_2^l v_2 - \Phi_2\lambda_2^l) = (\int \lambda_2^l \frac{\partial v_2}{\partial t}dl - \Phi_2\lambda_2^t)(\lambda_1^l v_1 - \Phi_1\lambda_1^l), \tag{4.86}$$

which is fulfilled at the following cases:

$$\lambda_1^l = 0, \text{ or } \lambda_2^l = 0, \tag{4.86a}$$

and at the different combinations of the following pairs of the relations:

$$v_1 = \Phi_1, \text{ or } v_2 = \Phi_2; \tag{4.86b}$$

$$(\int \lambda_1^l \frac{\partial v_1}{\partial t}dl - \Phi_1\lambda_1^t) = 0, \text{ or } (\int \lambda_2^l \frac{\partial v_2}{\partial t}dl - \Phi_2\lambda_2^t) = 0; \tag{4.86c}$$

where the last two relations are correct if $\lambda_1^t \neq 0, \lambda_1^l \neq 0$, (4.86d)

or, in particular, at the fulfillment of any of these relations:

$$\lambda_1^t = 0, \Phi_1 = 0, \frac{\partial v_1}{\partial t} = 0, \tag{4.86e}$$

$$\lambda_2^l = 0, \Phi_2 = 0, \frac{\partial v_2}{\partial t} = 0. \tag{4.86f}$$

Finally, we come to the condition:

$$\frac{(\int \lambda_1^l \frac{\partial v_1}{\partial t} dl - \Phi_1 \lambda_1^t)}{\lambda_1^l (v_1 - \Phi_1)} = \frac{(\int \lambda_2^l \frac{\partial v_2}{\partial t} dl - \Phi_2 \lambda_2^t)}{\lambda_2^l (v_2 - \Phi_2)} = \text{Inv}. \tag{4.87}$$

It means, that for the *n*-dimensional PDE model (4.78) could exist an invariant condition (4.87) on the solution of (4.84), (4.85), which is not dependable on the indexes in (4.87), or Inv could take a constant value for some pair of the indexes.

If we omit the trivial conditions (4.86a)–(4.86f) and the invariant (4.87), then (4.86) leads to the following relations:

$$\frac{\partial x}{\partial l} = \frac{\partial x}{\partial l} = 0$$

and

$$\frac{\partial y}{\partial l} = \frac{\partial y}{\partial t} = 0,$$

or

$$\frac{\partial x}{\partial l} = \frac{\partial y}{\partial t} = 0$$

and (4. 77a).

The conditions (4.86) define the different equations of the singular points, or the singuar trajectories, created by any of the separated processes $x(t,l)$, or $y(t,l)$, while (4.87) defines the singular trajectory, created by the process' interactions.

At such singularities, the rank of extended matrix (4.73) decreases that declines the number of independent equations in a system; and a normal to a surface ΔS at a singular point does not exist.

Because of the equations' (3.22) and (4.77b) connections, these conditions of singularities are applied also to the considered in chs.1.2,1.3 concentrated models.

Therefore, the singular points, defined by the conditions (4.86) and (4.87) do exist, and they are not singles.

The geometrical locations of the singular points could be the isolated states of the system (4.71), as well as the singular *trajectories*.

The invariant Inv corresponds to the equalization of the local subsystems relative speeds (at the phase trajectories) at the process of transferring via the singular curve, which is an analog of the condition (3.77), (3.164b).

At these points, relation (4.86b) gets the form

$$\Phi_1 = [\frac{\partial \varphi_1^l}{\partial f_1}(\phi_1) + \lambda_1^l v_1(\phi_1)]\frac{\partial f_1}{\partial \phi_1} = v_1, \tag{4.88}$$

and it is fulfilled along the singular trajectory, in particular, at

$$\lambda_1^l = \mathrm{const}, \phi_1 = \lambda_1^t t + \lambda_1^l l - \lambda_1^\tau \tau,\ f_1 = \phi_1 (\lambda_1^l)^{-1} = l + (\lambda_1^t / \lambda_1^l)(t - \tau),$$
$$\frac{\partial (f_1)}{\partial \phi_1} = (\lambda_1^l)^{-1}, \tag{4.88a}$$

which is satisfied at

$$\frac{\partial \varphi_1^l}{\partial f_1} = \frac{\partial \varphi_1^l}{\partial l}|_{l=f_1} = \lambda_1^l (v_1(t,l) - v_1(t, f_1)). \tag{4.88b}$$

This condition binds the automatic fulfillment of (4.76b) (at the macrotrajectories' singlular points) *with* the initial distribution (4.71a) (which depends on the model's microlevel).

That is why relations (4.86b-f), (4.87) might be considered as the *limitations* imposed on the class of the model's random processes, for example, applicable for Markov fields.

At a given random field, which does not satisfies these limitations, the conditions (4.86b-c) could be fulfilled by choosing the corresponding controls.

At $\lambda_1^1 = \mathrm{var}$, in particular at $\lambda_1^1(v_1)$, it's a possibility of the Jacobean degeneracy, as it follows from (4.88), which is also covered by relations (4.88b).

From that it follows that the model's singular phenomena could be implemented by the controls.

Therefore, the singular points and trajectories carry out the *additional information* about a connection of the micro- and macroprocesses, the model's geometry, dynamics, and control. Because relations (3.77), (4.53), and (4.77a) are the conditions *connecting* the extremal's segments at the *o*-window, the singularities are related also to the model's cooperative phenomena.

The state consolidation at the singular points is possible.

The detail analysis of the singular points is provided in [34] for a two dimensional concentrated model, where it's shown that before the consolidation, the model has a *saddle* singular point, and after the consolidation its singular point becomes an *attractor*.

More generally, the equalization of the subsystem's eigen frequencies (connected to the eigenvalues) (in (4.53), (4.77)) is an indicator of arising *oscillations*, which, at the superposition of the *diffusion* at the *o*-window, are faded into an *attractor*.

Actually, applying just a regular control (as a first part of the needle control actions) at the model's *o*-window transfers the dynamic trajectories at the macrolevel to the random trajectories at the microlevel, while both of them are unstable.

Applying a second regular control (being a second part of the needle control) brings stability to both of them.

Generally, the model undergoes a global bifurcation at the *o*-window between the segments, under the control actions and by transferring from kinetics to a diffusion and then back from the diffusion to kinetics.

Indeed. At the segment extremal's ending moment we have (sec.1.3.5)

$$a^u(\tau - o) = b(\tau - o)r^{-1}(\tau - o)x(\tau - o), \tag{4.89}$$

where

$$b(\tau - o)r^{-1}(\tau - o) = D_x(\tau - o) \tag{4.89a}$$

is the diffusion component of stochastic equation, which is compensated by the kinetic part, delivered with the regular control.

The needle control, applied between the segments at the moments $(\tau, \tau + o)$, brings the increment

$$\delta a^u = -a^u(\tau) + a^u(\tau + o) = -\lambda(\tau)x(\tau) + \lambda(\tau + o)x(\tau + o),$$

where

$$\lambda(\tau - o) < 0, sign\lambda(\tau) = -sign\lambda(\tau - o), \tag{4.90}$$

which at

$$x(\tau + o) \approx x(\tau), \lambda(\tau + o) \approx -\lambda(\tau)$$

determines

$$\delta a^u = -2\lambda(\tau)x(\tau).$$

Thus, the needle control *decreases* the initial diffusion part (sec.1.3.5)

$$D_x(\tau - o) = \lambda(\tau - o)$$

according to relation

$$b(\tau + o)r^{-1}(\tau + o) = D_x(\tau + o) \approx D_x(\tau) - 2\lambda(\tau) \approx -\lambda(\tau),$$

transferring the diffusion into kinetics.

This means that applying of the needle controls to a sequence of the extremal segments increases an influence of the kinetics on the model, decreasing the diffusion components.

The phenomenon is discussed in chs.1.7, 1.8.

1.4.6. The IPF Natural Variation Problem, Singular Trajectories, and The Field's Invariants for the IPF

The information functional (IPF) of the distributed model in the form:

$$\Delta S = \iint_{\bar{G}} L dl dt, \tag{4.91}$$

$$L = \frac{\partial x}{\partial t} X + \frac{\partial x}{\partial l} X, \tag{4. 91a}$$

is defined on the controlled processes $x = \{x_i\}$, which are determined by the solutions of Euler-Ostrogradsky's equations for this functional and the natural border conditions, connected with the initial distributions (4.71a).

Using the equations (4.64) – (4.69a) we will apply the following expressions for

$$X = 1/2hx, x = \{x_i\},\ h = r^{-1}, r = E_{\tilde{x},s,l_o}[\tilde{x}\tilde{x}^T], \tilde{x} = \{\tilde{x}_i\}, \tag{4.91b}$$

in the Lagrangian (4.91a).

The considered problem consists of the synthesis of a control law in a form $v = v(t, l, \dot{x}_t, \dot{x}_l)$ that carries out the fulfillment of extremal principle for the functional ΔS at the natural border conditions.

This problem, which is called the *natural variation* problem, we solve for the equations having the structure (4.71) at the Lagrangian *L* in form (4.91a).

The aim of this problem's solution is establishing its formal connection to an appearance of a singular curve (sec.1.4.4).

Writing the functional's variation at a variant domain G (under a control's action) according to [35] and sec.1.4.4, we have

$$\begin{aligned}
\delta\Delta S = \delta\Delta S_1 + \delta\Delta S_2 = \iint_{\bar{G}} \{\sum_{i=1}^{2} [\frac{\partial L}{\partial x_i} - \frac{\partial}{\partial t}(\frac{\partial L}{\partial \dot{x}_{it}}) - \frac{\partial}{\partial l}(\frac{\partial L}{\partial \dot{x}_{il}})]\delta\overline{x}_i\} dl dt + \\
\iint_{\bar{G}} \{\sum_{i=1}^{2} [\frac{\partial}{\partial t}(\frac{\partial L}{\partial \dot{x}_{it}}) + \frac{\partial}{\partial l}(\frac{\partial L}{\partial \dot{x}_{il}})]\delta\overline{x}_i + \frac{\partial}{\partial t}(L\delta t) + \frac{\partial}{\partial l}(L\delta l)\} dl dt = 0; \\
\delta\overline{x}_i = \delta x_i + \sum_{j=1}^{2} \frac{\partial x_i}{\partial l_j} \delta l_j; l_1 = t, l_2 = l.
\end{aligned} \tag{4.92}$$

Condition $\delta\Delta S_1 = 0$ is fulfilled by the implementation of Euler-Ostrogradsky's equation

$$\frac{\partial L}{\partial x_i} - \frac{\partial}{\partial t}(\frac{\partial L}{\partial \dot{x}_{it}}) - \frac{\partial}{\partial l}(\frac{\partial L}{\partial \dot{x}_{il}}) = 0, \tag{4.93}$$

which for (4.71) and (4.91a,b),(4.72) acquires the forms

$$\frac{\partial x_i}{\partial t}h_{ii}+\frac{\partial x_i}{\partial t}x_i\frac{\partial h_{ii}}{\partial x_i}+\frac{\partial x_i}{\partial l}h_{ii}+\frac{\partial x_i}{\partial l}x_i\frac{\partial h_{ii}}{\partial x_i}-\frac{\partial}{\partial t}(h_{ii}x_i)-\frac{\partial}{\partial l}(h_{ii}x_i)=0;$$

$$\frac{\partial h_{ii}}{\partial x_i}\frac{\partial x_i}{\partial t}+\frac{\partial h_{ii}}{\partial x_i}\frac{\partial x_i}{\partial l}=\frac{\partial h_{ii}}{\partial t}+\frac{\partial h_{ii}}{\partial l}\ (\forall x_i\neq 0,\forall(l,t)\in G). \tag{4.94}$$

We get the equation of extremals in the form

$$\frac{\partial h_{ii}}{\partial t}+\frac{\partial h_{ii}}{\partial l}=0;\frac{\partial r_{ii}}{\partial t}+\frac{\partial r_{ii}}{\partial l}=0; \tag{4.95}$$

At the solutions of this equation, the following relation holds true

$$E_i[L_i]=E_i[h_{ii}x_i(\dot{x}_{it}+\dot{x}_{il})]\equiv 0. \tag{4.96}$$

The condition $\frac{\partial^2 L}{\partial x_i^2}\neq 0$ on an extremal determines the regular, *not* the singular extremal's points.

For the linear form (4.91a), regarding variables $(x,\dot{x}_t,\dot{x}_l)$, it's fulfilled $\frac{\partial^2 L}{\partial x_i^2}=0\ \forall(l,t)\in G$, and the obtained extremals are a non regular.

At these extremals, the differential equations (4.73) turn into the parametrical equations for the functions h_{ii} (4.94), (4.95), and (4.96) determined via $x(t,l)=\{x_i(t,l)\}$ in (4.91a).

Applying the differential equation with the control processes $\{x_i=x_i(t,l)\}$, the piece-wised controls $\{v_i\}$, and random initial conditions, let us find the control in (4.71) for which the solutions of equations (4.91) satisfy to (4.95).

Using (4.95) as the initial condition for the control synthesis, we get

$$L_i=h_{ii}x_i(\frac{\lambda_i^t}{\lambda_i^l}\dot{x}_{il}+u_i^t+\dot{x}_{il})];\ E_i[L_i]=0\text{ at }u_i^t=v_i\lambda_i^t=-\frac{\dot{r}_{iit}}{E_i[x_i]}[\frac{\lambda_i^t}{\lambda_i^l}+1]. \tag{4.97}$$

The same way we find

$$u_i^l=v_i\lambda_i^l=-\frac{\dot{r}_{iit}}{E_i[x_i]}[\frac{\lambda_i^l}{\lambda_i^t}+1]. \tag{4.98}$$

From these relations also it follows the representation of the control function $v_i=v_i(t,l),i=1,2$, which corresponds to the control's form in the initial equations (4.71), (4.71a).

Let us specialize the above controls, which act both within the controls definition's domain G and at its border ∂G, for example, a square.

For these controls, in domain G might exist a geometrical set of points, where the partial derivations of eq. (4.71) get a first kind of the discontinues. For a simplicity, let us consider a monotonous smooth curve γ_5 (Fig.4.1) as such a set.

Generally, such a curve does not cross the above border, and we can prolong this curve by two auxiliary curves γ_2, γ_4 up to ∂G in such a way that the obtained $\gamma_2 \bigcup \gamma_4 \bigcup \gamma_5$ will be a monotonous curve (at an arbitrary the method of the continuation).

As a result of these formations, the initial of two bound domain split on the two single subdomains G_1, G_2 with the borders $\partial G_1, \partial G_2$ accordingly (Fig.4.1).

Because the curve γ_5 is priory unknown, the above subdomains are variable.

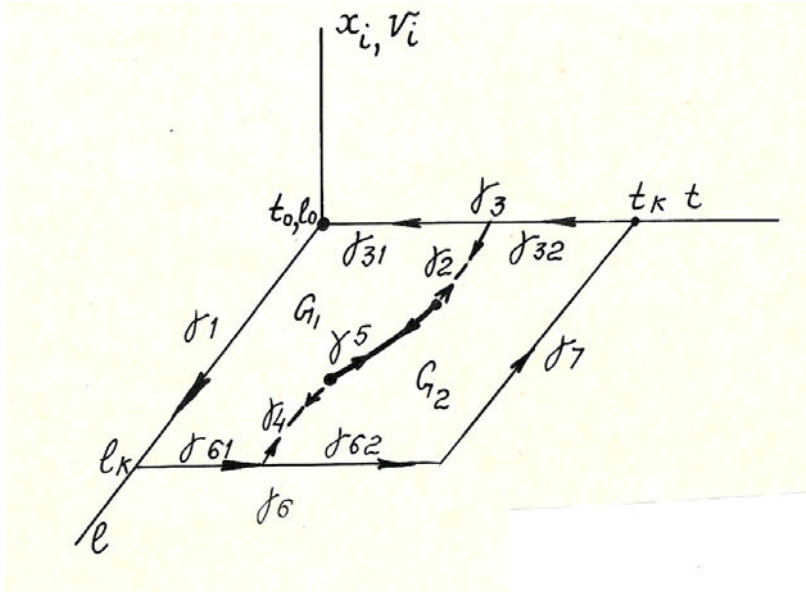

Figure 4.1. An illustration of the control's domain and the auxiliary curves.

The following relations formalize the considered domain's and subdomains' descriptions:

$$\bar{G} = G \bigcup \partial G \bigcup \gamma_5 ; G = G_1 \bigcup G_2 \bigcup \gamma_2 \bigcup \gamma_4 ; \partial G = \gamma_1 \bigcup \gamma_3 \bigcup \gamma_6 \bigcup \gamma_7 ;$$
$$\bar{G}_1 = G_1 \bigcup \partial G_1 ; \partial G_1 = \gamma_1 \bigcup \gamma_{31} \bigcup \gamma_2 \bigcup \gamma_5 \bigcup \gamma_4 \bigcup \gamma_{61} \; \bar{G}_2 = G_2 \bigcup \partial G_2 ; \qquad (4.99)$$
$$\gamma : \begin{cases} t = t_o = s = const \\ l_o \le l \le l_k \end{cases} ; \; \gamma_3 = \gamma_{31} \bigcup \gamma_{32} ; \gamma_{31} : \begin{cases} l = l_o = const \\ t_o = s \le t \le t_3^{12} \end{cases} ;$$

where at $t_3^{12} : \begin{cases} F_2(l,t) = 0 \\ l = l_o \end{cases}$,

the equations $F_m(l,t)=0, m=2,4$ describe any curve in the domains considered below for

$$\gamma_{32}:\begin{cases} l=l_o=const \\ t_3^{12}\le t\le t_k \end{cases};\ \gamma_6=\gamma_{61}\bigcup\gamma_{62};\ \gamma_{61}:\begin{cases} l=l_k=const \\ t_o=s\le t\le t_6^{12} \end{cases};$$

$$\text{at } t_6^{12}:\begin{cases} F_4(l,t)=0 \\ l=l_k \end{cases};\ \text{and for } \gamma_{62}:\begin{cases} l=l_k=const \\ t_6^{12}\le t\le t_k \end{cases};\ \gamma_7:\begin{cases} t=t_k=const \\ l_o\le l\le l_k \end{cases}. \qquad (4.100a)$$

The border domain has the form

$$\Gamma=\partial G_1\bigcup\partial G_2=\partial G_2\bigcup[(\gamma_2^+\bigcup\gamma_5^+\bigcup\gamma_4^+)\bigcup(\gamma_2^-\bigcup\gamma_5^-\bigcup\gamma_4^-)]=\partial G\bigcup\Gamma_{\text{int}}^+\bigcup\Gamma_{\text{int}}^-, \qquad (4.100b)$$

where Γ_{int} is an internal part of domain Γ, + and – mean the particular curve's movement along the above domains accordingly.

Let us implement the border condition $\delta\Delta S_2=0$ using Green's form [35] and the above relations:

$$\iint\limits_{\bar G}(\frac{\partial P_1}{\partial l_1}+\frac{\partial P_2}{\partial l_2})dl_1dl_2=\int\limits_{\Gamma}-P_1dl_2+P_2dl_1\ , \qquad (4.101)$$

$$P_1=\sum_{i=1}^{2}\frac{\partial L}{\partial\dot x_{il_1}}\delta x_i-\sum_{i=1}^{2}\frac{\partial L}{\partial\dot x_{il_1}}\sum_{i=1}^{2}\frac{\partial x_j}{\partial l_j}\delta l_j+L\delta l_1;\ P_2=\sum_{i=1}^{2}\frac{\partial L}{\partial\dot x_{il_2}}\delta x_i-\sum_{i=1}^{2}\frac{\partial L}{\partial\dot x_{il_2}}\sum_{i=1}^{2}\frac{\partial x_j}{\partial l_j}\delta l_j+L\delta l_2;$$

Applying relations (4.101) to functional (4.91), (4.91a), we come to

$$\delta\Delta S_2=\iint\limits_{\bar G}[\sum_{k=1}^{2}\frac{\partial L}{\partial l_k}\sum_{i=1}^{2}(\frac{\partial L}{\partial\dot x_{il_k}}\delta x_i-\sum_{i=1}^{2}\frac{\partial L}{\partial\dot x_{il_k}}\sum_{j=1}^{2}\frac{\partial x_j}{\partial l_j}\delta l_j+L\delta l_k)]dl_1dl_2=\int\limits_{\Gamma}-P_1dl_2+P_2dl_1\ .$$

Because of the δx_i, δl_i arbitrariness we get

$$\int\limits_{\partial G}-P_1dl_2+P_2dl_1=0\ , \qquad (4.102a)$$

$$\int\limits_{\Gamma_{\text{int}}^+\bigcup\Gamma_{\text{int}}^-}-P_1dl_2+P_2dl_1=0\ . \qquad (4.102b)$$

The first of them (4.102a) leads to the natural border conditions at the external border ∂G, for example, in the following forms

$$\frac{\partial L}{\partial \dot{x}_{it}}|_{t=s}=0 \Rightarrow h_{ii}(s,l)=0\,,\, x_i\neq 0\,;\frac{\partial L}{\partial \dot{x}_{it}}|_{t=t_k}=0\,,\Rightarrow h_{ii}(t_k,l)=0\,,\, x_i\neq 0\,;$$
$$\frac{\partial L}{\partial \dot{x}_{il}}|_{l=l_o}=0\,,\Rightarrow h_{ii}(l_o,t)=0\,,\, x_i\neq 0\,;\frac{\partial L}{\partial \dot{x}_{it}}|_{t=s}=0\,,\Rightarrow h_{ii}(l_k,t)=0\,,\, x_i\neq 0\,. \quad (4.103)$$

The second relation (4.102b) leads to an analogy of the Erdman-Weierstrass' conditions [24] at the curve γ_5.

Indeed. Because γ_2, γ_4 are the arbitrary (being the virtual) curves, at crossing them, the partial derivations are continuous, and the integral, taken along the opposite directions, is equal to zero.

From that for (4.102b) is fulfilled

$$\int\limits_{\Gamma^+_{\text{int}}\cup\Gamma^-_{\text{int}}} -P_1dl_2+P_2dl_1=\int\limits_{\gamma_5^+\cup\gamma_5^-} -P_1dl_2+P_2dl_1\,. \quad (4.104)$$

Suppose, the curve γ_5 can be defined by equation $l=l^*(t)$.

Then integral (4.104), written in a single (arbitrary) direction, acquires the forms

$$\int\limits_{\gamma_5} -P_1dl_2+P_2dl_1=\int\limits_{\tau_1}^{\tau_2}(-P_{11}\dot{l}^*+P_2)dt=0\,;$$
$$\int\limits_{\tau_1}^{\tau_2}\{\sum_{i=1}^{2}(\frac{\partial L}{\partial \dot{x}_{il_2}}\frac{\partial x_i}{\partial l_2}-\dot{l}^*\frac{\partial L}{\partial \dot{x}_{il_1}})\delta x_i+[L-\sum_{i=1}^{2}\frac{\partial L}{\partial \dot{x}_{il_2}}\frac{\partial x_i}{\partial l_2}+\dot{l}^*\sum_{i=1}^{2}\frac{\partial L}{\partial \dot{x}_{il_1}}\sum_{i=1}^{2}\frac{\partial x_i}{\partial l_2}]\delta l_2$$
$$+[-L+\sum_{i=1}^{2}\frac{\partial L}{\partial \dot{x}_{il_1}}\frac{\partial x_i}{\partial l_1}\dot{l}^*-\sum_{i=1}^{2}\frac{\partial L}{\partial \dot{x}_{il_2}}\sum_{i=1}^{2}\frac{\partial x_i}{\partial l_1}]\delta l_1\}dt=0. \quad (4.105)$$

Writing the integral in the opposite directions at the arbitrariness of δx_i, δl_i, we get the system of four equations along γ_5:

$$h^-_{ii}x_i-\dot{l}^*h^-_{ii}x_i=h^+_{ii}x_i-\dot{l}^*h^+_{ii}x_i\,,h^-_{ii}=h^+_{ii}\,,i=1,2\,,\ \dot{l}^*=1\,,(i=1,2)\,, \quad (4.105a)$$

$$L^--\sum_{i=1}^{2}h^-_{ii}x_i\frac{\partial x^-_i}{\partial l}+\dot{l}^*(h^-_{11}x\frac{\partial x^-}{\partial l}+h^-_{22}y\frac{\partial y^-}{\partial l})=L^+-\sum_{i=1}^{2}h^+_{ii}x_i\frac{\partial x^+_i}{\partial l}+\dot{l}^*(h^+_{11}x\frac{\partial x^+}{\partial l}+h^+_{22}y\frac{\partial y^+}{\partial l})\,, \quad (4.105b)$$

where the indexes +and– indicate the above functions for the domains G_1 and G_2 accordingly.

Substituting (4.105a) into (4.105b) we come to the system of equalities, which determine a jump of the Lagrangian on γ_5 :

$$L^- - L^+ = x(h_{11}^- - \dot{l}^* h_{11}^-)(\frac{\partial x^-}{\partial l} - \frac{\partial x^+}{\partial l}) + y(h_{22}^- - \dot{l}^* h_{22}^-)(\frac{\partial y^-}{\partial l} - \frac{\partial y^+}{\partial l}),$$
$$(L^- - L^+)\dot{l}^* = x(h_{11}^- - \dot{l}^* h_{11}^-)(\dot{x}_t^+ - \dot{x}_t^-) + y(h_{22}^- - \dot{l}^* h_{22}^-)(\dot{y}_t^+ - \dot{y}_t^-). \quad (4.106)$$

The obtained relations are equivalent along the curves

$$\frac{\dot{x}_t^+}{\dot{x}_l^+} = \frac{\dot{x}_t^-}{\dot{x}_l^-} = \frac{\dot{y}_t^+}{\dot{y}_l^+} = \frac{\dot{y}_t^-}{\dot{y}_l^-} = \dot{l}^*; \quad (4.106a)$$

at the corresponding Jakobians' forms

$$\frac{D^+(x,y)}{D^+(t,l)} = \frac{D^-(x,y)}{D^-(t,l)} = 0. \quad (4.106b).$$

According to (4.106a) the controls at the singular curve become bound:

$$\frac{u_1^t}{u_2^t} = \frac{\dot{r}_{11l}}{\dot{r}_{22l}} \frac{E[x_2]}{E[x_1]} \frac{[\frac{\lambda_1^t}{\lambda_1^l} + 1]}{[\frac{\lambda_2^t}{\lambda_2^l} + 1]}, \frac{v_1}{v_2} = \frac{\lambda_2^t}{\lambda_1^t} \frac{\lambda_2^l}{\lambda_1^l} [\frac{\lambda_1^t + \lambda_1^l}{\lambda_2^t + \lambda_2^l}] \frac{E[x_2]}{E[x_1]} \dot{l}^*. \quad (4.107)$$

Let us assume that along the singular curve the conditional probability density is defined by a δ -distribution.

Then, according to the features of δ -function, we get the equivalent relations:

$$\frac{\dot{x}_{it}^{\pm}}{\dot{x}_{il}^{\pm}} = -\dot{l}^*$$

and

$$\frac{\dot{r}_{11t}^{\pm}}{\dot{r}_{22l}^{\pm}} = \dot{l}^*,$$

and can write the relation for the Lagrangian's jump in the form

$$\Delta L = xh_{11}^-(1+\dot{l}^*)(\dot{x}_l^- - \dot{x}_l^+) + yh_{22}^-(1+\dot{l}^*)(\dot{y}_l^- - \dot{y}_l^+)$$
$$= xh_{11}^-(1 - \frac{\dot{x}_t^-}{\dot{x}_l^-})(\dot{x}_l^- - \dot{x}_l^+) + yh_{22}^-(1 + \frac{\dot{y}_t^-}{\dot{y}_l^-})(\dot{y}_l^- - \dot{y}_l^+), \quad (4.107a)$$

at $h_{ii}^- = h_{ii}^+, i = 1,2.$

Because on γ_5 holds true

$$x_i = E_i[x_i], \dot{x}_{it}^{-} = E_i[\dot{x}_{it}^{-}],\ \dot{x}_{il} = E[\dot{x}_{il}],$$

the following equality is correct:

$$\Delta L = (1+\frac{\dot{r}_{11t}^{\ -}}{\dot{r}_{11l}^{\ -}})r_{11}^{-1}(\dot{r}_{11l}^{\ -} - \dot{r}_{11l}^{\ +}) + (1+\frac{\dot{r}_{22t}^{\ -}}{\dot{r}_{22l}^{\ -}})r_{22}^{-1}(\dot{r}_{22l}^{\ -} - \dot{r}_{22l}^{\ +}). \qquad (4.108)$$

According to (4.106a) and (4.108) we get

$$\frac{\dot{r}_{11t}^{\ \pm}}{\dot{r}_{11l}^{\ \pm}} = \frac{\dot{r}_{22t}^{\ \pm}}{\dot{r}_{22l}^{\ \pm}}; \qquad (4.108a)$$

$$\Delta L = [1+(\frac{\dot{r}_{11t}}{\dot{r}_{11l}})^{\pm}]r_{11}^{-1}(\dot{r}_{11l}^{\ -} - \dot{r}_{11l}^{\ +}) + r_{22}^{-1}(\dot{r}_{22l}^{\ -} - \dot{r}_{22l}^{\ +}). \qquad (4.108b).$$

Now we can determine the functional's value at the extremals of equation (4.71):

$$\Delta S = \iint\limits_{\bar{G}} L dl dt = \iint\limits_{\bar{G}} -(x\dot{X}_t + x\dot{X}_l + y\dot{Y}_t + y\dot{Y}_l) dl dt$$
$$+ \iint\limits_{\partial G \cup \gamma_5} (-xh_{11}x - yh_{22}y)dl + (xh_{11}x - yh_{22}y)dt = \Delta S_{\text{int}} + \Delta S_{\Gamma}, \qquad (4.109)$$

where $\dot{X}_t, \dot{X}_l, \dot{Y}_t, \dot{Y}_l$ are the corresponding covariant functions, ΔS_{int} is an internal functional's increment, ΔS_{Γ} is a border's increment, and the Lagrangian is represented by the sum

$$L = L_1 + L_2 = \dot{x}_t h_{11} x + \dot{x}_l h_{11} x + \dot{y}_t h_{22} y + \dot{y}_l h_{22} y = \dot{x}_t X + \dot{x}_l X + \dot{y}_t Y + \dot{y}_l Y$$
$$= \frac{\partial}{\partial t}(xX + yY) + \frac{\partial}{\partial t}(xX + yY) - x\dot{X}_t - y\dot{Y}_t - x\dot{X}_l - y\dot{Y}_l.$$

According to (4.96) we have

$$\Delta S_{\text{int}} = -\iint\limits_{\bar{G}} [\sum_{i=1}^{2}(x_i h_{ii} \dot{x}_t + x_i h_{il} \dot{x}_{tl})] dl dt$$

and

$$\vec{E}[\Delta S_{\text{int}}] = -\iint_{\tilde{G}} \{\sum_{i=1}^{2} \dot{r}_{iit} + \dot{r}_{iil}\}(\dot{r}_{ii})^{-1}\}dldt \equiv 0 \ , \tag{4.109a}$$

where $\vec{E} = \{E_1, E_2\}$ is a symbol of mathematical expectation acting additively on the Lagrangian;
and

$$\Delta\hat{S}_{\Gamma} = \vec{E}[\Delta S_{\Gamma}] = 2 \iint_{\partial G \cup \gamma_5} -dl + dt = 2\iint_{\gamma_5} -dl + dt = 2\int_{\tau_1}^{\tau_2} (1 - \dot{l}^*)dt = 2[(\tau_2 - \tau_1) + \dot{l}^*(\tau_1) - \dot{l}^*(\tau_2)]. \tag{4.109b}$$

At $\dot{l}^* = 1$ we get $\Delta S_{\Gamma} = 0$, which brings to a total entropy's increment in the optimal process equals to zero, where $\dot{l}^* = 1$ corresponds, in particular, the fulfillment of $\dot{x}_t = \dot{x}_l, \dot{y}_t = \dot{y}_l$, i.e. appearance (according to (4.77b,c) a singular curve by the equalization of the above phase speeds.

At $\dot{l}^* > 1$ the entropy's increment is positive, at $\dot{l}^* < 1$ the increment is negative
Let us build at an ε-locality of the singular curve a domain

$$\tilde{G} = \tilde{G}_1 \bigcup \tilde{G}_2 \, ; \tilde{G} : \begin{cases} 0 < |\, l^* - \tilde{l}^* \,| < \varepsilon \\ t_6^{12} - o(\Delta t) \le t \le t_3^{12} + o(\Delta t) \end{cases}.$$

Then the relations (4.97), (4.98) in $\tilde{G}_1$ and $\tilde{G}_2$ holds true, specifically in the forms:

$$\tilde{G}_1 : u_{i1}^t = -\frac{\dot{r}_{ii1l}}{E_i[x_i]}[\frac{\lambda_i^t}{\lambda_i^l} + 1], u_{i1}^l = -\frac{\dot{r}_{ii1t}}{E_i[x_i]}[\frac{\lambda_i^l}{\lambda_i^t} + 1]; \tag{4.110a}$$

$$\tilde{G}_2 : u_{i2}^t = -\frac{\dot{r}_{ii2l}}{E_i[x_i]}[\frac{\lambda_i^t}{\lambda_i^l} + 1], u_{i2}^l = -\frac{\dot{r}_{ii2t}}{E_i[x_i]}[\frac{\lambda_i^l}{\lambda_i^t} + 1], \tag{4.110b}$$

where the lover indexes 1,2, at the above $\dot{r}_{iit}, \dot{r}_{iil}, u_i^t, u_i^l$, indicate that these functions belong to the $\tilde{G}_1$ and $\tilde{G}_2$ accordingly.
Using these relations we find the control's jumps:

$$\Delta u_i^t = u_{i1}^t - u_{i2}^t = \lambda_i^t(v_{i1} - v_{i2}) = \frac{(\dot{r}_{ii2l} - \dot{r}_{ii1l})}{E_i[x_i]}[\frac{\lambda_i^t}{\lambda_i^l} + 1], \tag{4.111a}$$

$$\Delta u_i^l = u_{i1}^l - u_{i2}^l = \lambda_i^l(v_{i1} - v_{i2}) = \frac{(\dot{r}_{ii2t} - \dot{r}_{ii1t})}{E_i[x_i]}[\frac{\lambda_i^l}{\lambda_i^t} + 1] \ . \tag{4.111b}$$

Therefore, in a general case, there exists the jumps for both the controls and Lagrangian (according to (4.108b)) at crossing the singular curve.

These jumps can be found if the derivatives of the corresponding correlation functions are known.

The conditions $\dot{r}^-_{iit} = \dot{r}^-_{iil}, \dot{r}^+_{iit} = \dot{r}^+_{iil}$ in particular, for the concentrated systems (at $\dot{r}_{iil} = 0$) acquire the forms

$$\dot{r}^-_{iit} = E[\dot{x}(\tau)x^T(\tau + o)], \dot{r}^+_{iit} = E[\dot{x}(\tau_1)x^T(\tau_1 + o)], \tau_1 = \tau + o; \quad (4.112)$$
$$\dot{r}^-_{iit} = \dot{r}^+_{iit}, E_i[x_i(\tau)\dot{x}_i(\tau + o)] + E_i[\dot{x}_i(\tau)x_i(\tau + o)] = 0,$$

at

$$x(\tau) = x^-, x(\tau + o) = x^+; x^- = x^+. \quad (4.112a)$$

From that we have

$$x(\tau) = x(\tau + o)$$

and

$$\dot{x}(\tau) = -\dot{x}(\tau + o).$$

Thus, at crossing the singular curve, or a singular point, $\dot{x}(\tau)$ changes the sign.

If

$$\dot{x}(\tau) = \lambda(\tau)(x(\tau) + v(\tau)), \ \dot{x}(\tau + o) = \lambda(\tau)(x(\tau) + v(\tau))$$

then the control, at crossing the singularity, is found from relation

$$\lambda(\tau)(x(\tau) + v(\tau)) = -\lambda(\tau)(x(\tau) + v(\tau)),$$

or

$$v(\tau) = -2x(\tau)$$

and

$$v(\tau_1) = -2x(\tau_1),$$

which determines the needle control (secs.1.3.4, 1.3.5):

$$v(\tau) - v(\tau_1) = -\delta v(\tau).$$

The control's *strategy that solves the natural border problem* consists of:

-the movement along an extremal (4.95) by applying controls (4.97),(4.98) (being the functions of the initial distribution (4.71a)) that proceed up to the moment of time, when the conditions (4.86a) are fulfilled and the controls become bound by (4.107);

-the movement along a singular curve (at the control's jump) until the condition (4.106a) is violated;

-the movement's continuation along the above extremals with the controls (4.97), (4.98).

The following *statement* summarizes the results:

The natural border problem's solutions for the path functional with the model (4.71),(4.71a) *are both the extremal (4.95) and the singular curve of this equation, for which (4.96) holds true, and the controls are bound according to (4.110a,b).*

Along the singular curve (and/or the singular points) the initial model's dimension is decreased and the state's cooperation takes place.

All these results include from the solution of variation problem for the information path functional (ch.1.3).

According to the initial VP, the IPF's extremals hold the principle of stationary action. This allows us to find the invariant conditions, as the model field's functions, being the analogies of the information form of *conservations laws.*

Following to the Noether theorem [24] and the results ch.1.4.2 we come to

$$\vec{Q} = [\sum_{i=1}^{n} \frac{\partial L}{\partial(\partial x_i / \partial l_k)} (\sum_{m=1}^{4} -\frac{\partial x_i}{\partial l_m} y_m) + L y_k]_{k=1}^{4} = 0 \, , \, y_k = \frac{\partial l_k}{\partial t} , l_4 = t \ . \qquad (4.113)$$

Let us have a four dimensional volume Ω limited by a surface Σ^4:

$$\Sigma^4 = \Sigma \cup \Sigma_1 \cup \Sigma_2 \, ; \; \Sigma = \Sigma^3 \cap (l_4 > a) \cap (l_4 < b) \, ;$$

$$\Sigma_1 = (F(l_1, l_2, l_3) \le 0) \cap (l_4 = a); \Sigma_2 = (F(l_1, l_2, l_3) \le 0) \cap (l_4 = b) \, , \qquad (4.114)$$

where *a,b* are the auxiliary fixed moments of time; Σ^3 is a non self-crossing surface defined by equation

$$F(l_1, l_2, l_3) = 0;$$

Σ^4 is a four dimensional cylindrical surface limited by two parallel planes

$$l_4 = a \, , l_4 = b \, ,$$

where the cylinder's vertical line is in parallel to the time axis, and the basis is a geometrical space of points Σ^3.

After integrating (4.114) by Ω, applying the Ostrogradsky-Gauss theorem [23], we get

$$\int_{\Omega} div \vec{Q} dv^4 = \int_{\Sigma^4} (\vec{Q}, \vec{n}^+) d\sigma^4 = 0 \ , \qquad (4.115)$$

where $\vec{n}^+$ is a positive oriented external normal to the surface Σ^4; $d\sigma^4$ is an infinite small element of Σ^4.

Integral (4.115) is represented by the sum of the following integrals, taken by the cylinder's two bottom parts Σ_1, Σ_2 and its sidelong part Σ of Σ^4:

$$\int_{\Sigma^4}(\vec{Q},\vec{n}^{+})d\sigma^4=\int_{\Sigma_1}(\vec{Q},\vec{n}_1^{-})d\sigma_1+\int_{\Sigma_2}(\vec{Q},\vec{n}_1^{+})d\sigma_2+\int_{\Sigma}(\vec{Q},\vec{n}_2^{+})d\sigma=\int_{G^3}[(\vec{Q},\vec{n}_1^{+})|_{l_4=b}-(\vec{Q},\vec{n}_1^{-})|_{l_4=a}]dv^3$$

$$+\int_{\Sigma}(\vec{Q},\vec{n}_2^{+})d\sigma=\int_{G^3}[(\vec{Q},\vec{n}_1^{+})|_{l_4=b}-(\vec{Q},\vec{n}_1^{+})|_{l_4=a}]dv^3+\int_{\Sigma}(\vec{Q},\vec{n}_2^{+})d\sigma\,, \qquad (4.116)$$

where $\vec{n}_1^{+}=(0,0,0,1)$ is a positive oriented external normal to the bottom part Σ_2 of the surface Σ^4;

$l_4=t$; $\vec{n}_1^{-}=(0,0,0,-1)=-\vec{n}_1^{+}$
is a negative (internal) normal to the bottom part Σ_1 of Σ^4;

$\vec{n}_2^{+}=(\vec{n}_{21}^{+},\vec{n}_{22}^{+},\vec{n}_{23}^{+},0)$
is a positive external normal to Σ;

$G^3=(F(l_1,l_2,l_3)\le 0)\cap(l_4=0)$
is a part of space being a projection of (Σ_1,Σ_2) on a hyper plane $l_4=0$;
dv^3 is an infinite small element of volume G^3;
$d\sigma$, $d\sigma_1, d\sigma_2$ are the infinite small elements of the surfaces $\Sigma, \Sigma_1, \Sigma_2$ accordingly.

Let us implement (4.116) at the typical physical assumptions, supposing that both function

$$F(l_1,l_2,l_3)=l_1^2+l_1^2+l_1^2-R_o^2$$

and the field are decreasing rapidly at approaching the infinity.

This means that at $R_o\to\infty$ and $d\sigma\sim R_o^2$, the integral by Σ in (4.116) can be excluded.

Then (4.116) according to (4.115) acquires the form

$$\int(\vec{Q},\vec{n}_1^{+})|_{l_4=b}\,dv^3=\int(\vec{Q},\vec{n}_1^{-})|_{l_4=a}\,dv^3\,, \qquad (4.117)$$

where both integrals are taken by an infinite domain.

Because of the auxiliary *a* and *b*, the equality (4.117) means the preservation in time the values

$$\int(\vec{Q},\vec{n}^{+})dv^3=\int[\sum_{i=1}^{n}\frac{\partial L}{\partial(\partial x_i/\partial l_k)}(\sum_{m=1}^{4}-\frac{\partial x_i}{\partial l_m}y_m)+L]dv^3\,,\ y_m=\frac{\partial l_m}{\partial t},l_4=t\,. \qquad (4.118)$$

Applying the Lagrange-Hamilton equations we get invariant

$$\int [\sum_{i=1}^{n} (\frac{\partial L}{\partial(\partial x_i / \partial t)} \frac{\partial x_i}{\partial t}) + L] dv^3 = \int \sum_{i=1}^{n} (X_i \frac{\partial x_i}{\partial t} + L) dv^3 = \int (H + 2L) dv^3 = inv,$$

$$dv^3 = dl_1 dl_2 dl_3, \tag{4.119}$$

which at

$$-H = L - \dot{x}^T X, L = 1/2 \dot{x}^T X \text{ (ch.1.3)}$$

leads to the invariant

$$\int H dv^3 = inv, H = 1/2 \sum_{i=1}^{n} (\frac{\partial x_i}{\partial \vec{l}}, \frac{\partial \vec{l}}{\partial t}) X_i, \tag{4.120}$$

preserving the volume's Hamiltonian of the information path functional.

Chapter 1.5

THE COOPERATIVE INFORMATION MACROMODELS AND INFORMATION NETWORK

1.5.1. Introduction

This chapter focuses on *forming* the IPF *cooperative* information macromodels and information *networks*.

In section 5.1 we synthesize the cooperative distributed model, built during the optimal time-spaced movement directed toward the equalization and collectivization of the model operators' eigenvectors, by applying the PDE models in the moving space coordinates' system. The *optimal* time-space *model movement* with the discrete optimal control, implementing the variation principle, generates the states' consolidation as an auxiliary mechanism at this movement. Both the invariant space transformation and the PDE cooperative process follow from the VP.

The chapter examples provide a detailed procedure for building of the undistinguished collective states and the cooperative models. In the process of the eigenvectors' collectivization, the chaotic bifurcations and resonance arise. This process is associated with an essential irreversibility, breaking determinism and the time's symmetry.

We also consider the transformation of imaginary into real information with transforming the imaginary eigenvectors into the real eigenvectors at the discrete points. It is shown that a time-space metric is a result of the above *transformation of information*.

The structural symmetry follows from the VP, and the consolidation leads to a dynamic asymmetry of the forming macrostructures.

In section 5.2. we consider the consolidation and aggregation of the model’s processes in an information netwok (IN). The IN aggregations of the cooperative macroparameters are formed during the joint process of optimal control, identification, and consolidation.

The solution is achieved by applying the invariant transformation preserving the PDE (built in an immobile coordinate system) along a family of the moving space coordinate.

This allows the equalization and consolidation for some of the space *state vectors* at a certain mobile coordinate system, reducing a number of the independent state’s vectors, with the following aggregation of the consolidated macrostates into the IN.

Sections 5.3, 5.4 introduce both the IN's dynamic and geometrical structures, considering the optimal space-time distributed model with the consolidated states, defined by the complex spectrum of the model's eigenvectors. The IN model brings a concurrent solution to the consolidation problem of modeling the object's hierarchy with a sequentially reduced number of independent variables. The optimal cooperative strategy is accomplished by the consolidation of the eigenvalues in threes, which determines a *triplet*, or in, a particular, a doublet (with a subsequent joining it to a third eigenvalue), as an elementary optimal dynamic collective structure. In the ranged sequence of the optimal model's processes, each previous process can sequentially control each following process, performing the optimal control functions, which enable generating the superimposing phenomena and cooperative processes.

The found IN geometrical structure is determined by the VP's dynamical and geometrical invariants. The optimal time-space trajectory represents a spiral, located on conic surfaces. The geometrical structures, formed by the cones' consolidations into the enclosed triplets, compose a *hierarch*y of the space distributed information network.

The obtained relations define the procedure for the restoration of a specific IN and its node's formation, using the above identified invariants and the three given model's basic values (dimension n, parameters of structural uncertainty γ and of space curvature k). The procedure's details and the computer applications to various objects in technology, biology, and economy are considered in part 2.

Here we extend the results of distributed modeling [53-55, 59, 60, 69], and ch.1.4 on a comprehensive math solution the problem of consolidation, combined with optimal control's synthesis and process' identification.

1.5.2. The Time-Space Movement Toward the Macromodel's Cooperation

Let us have $3n$ dimensional space distributed macromodel

$$\frac{\partial x}{\partial t} = Ax, x = \{x_i(\bar{l}^i)\}_{i=1}^{i=n}, \bar{l}^i = (l_1^i, l_2^i, l_3^i),\ A = (a_{ik}),\ i,k = 1,\dots,n, \tag{5.1}$$

whose each of three-dimensional space coordinates vector $\bar{l}^i$ is built on the local coordinate system, created by the corresponding eigenvector $\bar{X}_i$ of the matrix A.

The considered cooperative process includes the equalization of the matrix A eigenvalues, which requires the matrix diagonalization.

During the cooperative optimal movement, the equation (5.1) will be transformed into relations

$$\dot{z} = \Lambda^v z, \Lambda^v = -(\frac{\lambda_{io}\exp(\lambda_{io}t)}{2-\exp(\lambda_{io}t)})_{i=1}^{n},\ T^{-1}A(0)T = \Lambda(0),$$

$$\Lambda(0) = (\lambda_{io})_{i=1}^{n}, T^{-1}A(\tau)T = \Lambda(\tau), \tag{5.2a}$$

$$z(\tau) = Tx(\tau), \tau = (t_i)_{i=o}^{n}, -T^{-1}A^{v}(\tau)T = \Lambda(\tau) = -\Lambda^{v}(\tau), T = \overline{A} \qquad (5.2b)$$

$$-T^{-1}A(0)\exp(A(0)\tau)(2E - \exp(A(0)\tau))^{-1}T = \Lambda^{v}(\tau), \qquad (5.2c)$$

where T is a transformation, reducing the matrix $A^{v}(\tau)$ to diagonal form $\Lambda^{v}(\tau)$ by the considered in ch.14.2 space transformation $\overline{A}$ of the matrix A eigenvectors' coordinate system with the aid of applied control $v = v(\tau)$.

At each discrete moment (DP) $\tau = (t_i)_{i=o}^{n}$, the applied control transforms a complex eigenvalue of the matrix $\Lambda^{v}(\tau)$: $\lambda_i(t_i - o) = \alpha_i(t_i - o) \pm j\beta_i(t_i - o)$ into a real eigenvalue $\lambda_i(t_i) \to \alpha_i(t_i), \beta_i(t_i) = 0$.

For each fixed t_i, this operation changes an initial $\Lambda^{v}(t_{i-1})$, bringing a new set of the eigenvectors for the eigenvalues $\Lambda^{v}(t_i)$ and

$\Lambda^{v}(t_1 - o) = \begin{pmatrix} \lambda_1(t_1 - o) \\ \lambda_2(t_1 - o) \\ \dots \\ \lambda_n(t_1 - o) \end{pmatrix}$ with the eigenvectors $\overline{X}(t_1 - o)$.

At the moment t_1 this matrix acquires the form $\Lambda^{v}(t_1) = \begin{pmatrix} \alpha_1(t_1) \\ \lambda_2(t_1) \\ \dots \\ \lambda_n(t_1) \end{pmatrix}$, which corresponds to the new eigenvectors $\overline{X}(t_1)$.

Transformation $T(t_1 - o, t_1)$, applied to $\Lambda^{v}(t_1 - o)$, is able to create both $\Lambda^{v}(t_1)$ and $\overline{X}(t_1)$. During the time-interval $(t_1, t_2 - o)$, the transformation $T(t_1, t_2 - o)$, applied to $\Lambda^{v}(t_1)$, is able to produce $\Lambda^{v}(t_2 - o) = \begin{pmatrix} \alpha_1(t_2 - o) \\ \lambda_2(t_2 - o) \\ \dots \\ \lambda_n(t_2 - o) \end{pmatrix}$ with eigenvector $\overline{X}(t_2 - o)$,

where the equation for

$\alpha_1(t_2 - o) = -\alpha_1(t_1)\exp\alpha_1(t_1)((t_2 - o) - t_1)[2 - \exp((\alpha_1(t_1)((t_2 - o) - t_1)]^{-1}$

follows from (5.2c). At the moment t_2, this matrix acquires the form

$\Lambda^{v}(t_2) = \begin{pmatrix} \alpha_1(t_2) \\ \alpha_2(t_2) \\ \dots \\ \lambda_n(t_2) \end{pmatrix}$ with the eigenvector $\overline{X}(t_2)$ and $\alpha_2(t_2) = \mathrm{Re}\,\lambda_2(t_2)$.

During the next time-interval, transformation $T(t_2, t_3 - o)$ generates $\Lambda^{v}(t_3 - o) = \begin{pmatrix} \alpha_1(t_3 - o) \\ \alpha_2(t_3 - o) \\ \lambda_3(t_3 - o) \\ \dots \\ \lambda_n(t_3 - o) \end{pmatrix}$ with $\overline{X}(t_3 - o)$ and the eigenvalues satisfying (5.2b,c).

At the moment t_3, using $T(t_3 - o, t_3)$, we get $\Lambda^{v}(t_3) = \begin{pmatrix} \alpha_1(t_3) \\ \alpha_2(t_3) \\ \alpha_3(t_3) \\ \dots \\ \lambda_n(t_3) \end{pmatrix}$ with $\overline{X}(t_3)$ and the eigenvalues satisfying (5.2a,b).

The DP moments are chosen such a way that the equalization of each triple real eigenvalues' occurs at the moment $(t_3 + o)$:

$$\alpha_1(t_3 + o) = \alpha_2(t_3 + o) = \alpha_3(t_3 + o) \tag{5.3}$$

with joining all of them into a single common $\alpha_3(t_3 + o)$.

The correspondent matrix $\Lambda^{v}(t_3 + o) = \begin{pmatrix} \alpha_3(t_3 + o) \\ \lambda_4(t_3 + o) \\ \dots \\ \lambda_n(t_3 + o) \end{pmatrix}$ has the $(n-2)$ dimensions comparing with the n-dimensional matrixes at the previous time-intervals.

Creation of this matrix and the associated $(n-2)$-dimensional eigenvector $\overline{X}(t_3 + o)$ requires to apply the transformation $T(t_3, t_3 + o)$.

To continue this procedure, the analogous transformations should be applied at the subsequent time-intervals $T(t_3 + o, t_4 - o)$, $T(t_4 - o, t_4)$, $T(t_4, t_5 - o), T(t_5 - o, t_5)$ until the transformation $T(t_5, t_5 + o)$ joins the second eigenvalues' triple with the first triple into a common triplet, which cooperates all previous eigenvalues into a single unit.

We suppose that the considered sequence of transformations $T(\tau)$, applied to the matrix $A(\tau)$ according to the equations (5.2a-c), is performed by the sequential space shifts and rotations of a mobile coordinate system built on the matrix eigenvalues.

To execute this movement, let us associate the transformation $T(\tau)$ with the matrix of rotation $\overline{A}(\tau)$, whose increment of angle of rotation $\varphi_i = \varphi_i(t_i)$ is defined by the considered local transformation at each DP, and use the vector $\overline{L} = (\overline{L}_j^i)_{j=1,2,3}^{i=1,\dots,n}$, related to $\overline{A}(\tau)$, which describes the appropriate shifts of the origin of the local mobile coordinate systems.

We assume that assigning of the matrix $\overline{A} \to T$ and the vector $\overline{L}$ takes place automatically, along with assigning of the space coordinates to each of the eigenvector's three dimensional components.

The identification of $\overline{A}(\tau)$ and $\overline{L}(\tau)$ is based on the known $T(\tau)$.

The macrovariables, changing according to (5.2a-c), at the moments of the equalization (5.3), satisfy the equation

$$\frac{\dot{z}_1}{z_1}(t_3+o) = \frac{\dot{z}_2}{z_2}(t_3+o) = \frac{\dot{z}_3}{z_3}(t_3+o). \tag{5.4}$$

The condition of an indistinguishability of the corresponding macrovariables:

$$\hat{z}_1(t_3+o+\delta) = \hat{z}_2(t_3+o+\delta) = \hat{z}_3(t_3+o+\delta) \tag{5.5}$$

requires applying the transformation $\hat{T}(\delta) = \hat{T}(\psi_5)$ in addition to $T(t_5, t_5+o) = T(\varphi_5)$, with the increment of the angle ψ_5, determined by the fulfillment of (5.5), which is automatically joint to (5.4) at the δ-moment of the applied control.

The transformation $\hat{T}(\psi_k)$ takes place at each DP t_k, $k = 3,5,7,\dots$

The operators T, $\hat{T}$ are connected with the method of choosing $\overline{A}(\tau)$, $\overline{L}(\tau)$, satisfying to the path functional invariant's condition during the optimal spatial movement. The local rotations and shifts of the moving coordinate system are defined by the considered time-interval's transformations. In a particular, the matrix's $\overline{A}(\tau)$ space parameter φ_i is connected to the dynamic parameters $(t_i, \overline{c}_i)$ by the formula following from (4.36),(4.37):

$$\overline{A}(t_i, \overline{c}_i) = \overline{A}(0)\exp(t_i\overline{c}_i) \text{ at } \overline{A}(\varphi_i) = \overline{A}(t_i, \overline{c}_i), \tag{5.6}$$

where $\overline{c}_i$ is the vector of a space speed. Because at the initial moment t=0, the transformation $\overline{A}(0)$=*I*, we can find the elements of matrix $\overline{c}_i$ at the known t_i and φ_i using the relations (4.38)-(4.41) and (5.3)-(5.6).

Choosing the three-dimensional matrix $\overline{A}$ of rotation

$$\overline{A}(\varphi_i) = \begin{pmatrix} \cos\varphi_i, \sin\varphi_i, 0 \\ -\sin\varphi_i, \cos\varphi_i, 0 \\ 0, 0, \pm 1 \end{pmatrix}, \tag{5.7}$$

for the matrix of transformation T, we can employ the results [61], based on a redefinition of quadratic form, given in

R^n: $\Phi = \sum_{i,j=1}^{n} g_{ij}x_i x_j$ into a subspace R^k:$\Phi^k = \sum_{i=1}^{k} \mu_i y_i^2$ of a lesser dimension $k < n$.

The orthogonal and normal canonical basis in R^k is determined by the corresponding egenvectors $(\overline{\mu}_1, \overline{\mu}_2, .., \overline{\mu}_k)$, whose eigenvalues are connected with the corresponding ranged values of the quadratic form with $(|\overline{\lambda}_i|)_{i=1}^n$: $\overline{\lambda}_1 \geq \overline{\lambda}_2 \geq ... \overline{\lambda}_k ... \geq \overline{\lambda}_n$, defined on R^n. It is proven that the maximal value μ_1 of quadratic form Φ^k, defined on the sphere R^k of radius one, is bigger or equals to $\overline{\lambda}_{n-k+1}$, leading to the inequalities

$$\overline{\lambda}_1 \geq \overline{\mu}_1 \geq \overline{\lambda}_{n-k+1},\ \overline{\lambda}_2 \geq \overline{\mu}_2 \geq \overline{\lambda}_{n-k+2},\ \overline{\lambda}_3 \geq \overline{\mu}_3 \geq \overline{\lambda}_{n-k+3}, ..., \overline{\lambda}_k \geq \overline{\mu}_k \geq \overline{\lambda}_n. \quad (5.8)$$

In particular, the method defines the three-dimensional canonical basis of matrix $\overline{A}$, corresponding to $(\overline{\mu}_1, \overline{\mu}_2, \overline{\mu}_3)$ by the given eigenvalues of matrix T. The matrix A eigenvalues should be ranged the same way. Their decreasing values determine the sequence of the increasing time-intervals, defined by the macromodel's invariants (sec.1.3.5).

Let us show how to build the orthogonal matrix of rotation

$$\overline{A}(\varphi_i) = \begin{pmatrix} \cos\varphi_i + j\sin\varphi_i, 0, 0 \\ 0, \cos\varphi_i - j\sin\varphi_i, 0 \\ 0, 0, 1 \end{pmatrix} = \begin{pmatrix} \exp(j\varphi_i), 0, 0 \\ 0, \exp(-j\varphi_i), 0 \\ 0, 0, 1 \end{pmatrix} = \begin{pmatrix} a_i^1, 0, 0 \\ 0, a_i^2, 0 \\ 0, 0, a_i^3 \end{pmatrix} \quad (5.9)$$

using the known $(\overline{\mu}_1, \overline{\mu}_2, \overline{\mu}_3)$. The condition $\det|a_i^1 a_i^2 a_i^3| = 1$ is satisfied by the fulfillment of the following equations for the corresponding coefficients:

$$a_i^1 = (\overline{\mu}_1)^2 / \overline{\mu}_2 \overline{\mu}_3,\ a_i^2 = (\overline{\mu}_2)^2 / \overline{\mu}_1 \overline{\mu}_3,\ a_i^3 = (\overline{\mu}_3)^2 / \overline{\mu}_1 \overline{\mu}_2. \quad (5.10)$$

The angle of rotation around the direction of the eigenvector, associated with the eigenvalue +1 of $\overline{A}(\varphi_i)$, equals $\varphi_i = \arccos(a_i^1 + a_i^2 + a_i^3)$.

Macroequation (5.1) acquires a most simple form in the direction of the main normal to the plane of symmetry of the second order's surface

$$\Phi_l^3 = \sum_{i=1}^{3} \overline{a}_{ij}^l l_i l_j, \quad (5.11)$$

whose orth-vector coincides with the eigenvector of the matrix A.

The related quadratic form: $\Phi^n = \sum_{i=1}^{n} a_{ij} x_i x_j$ has a diagonal form

$$\Phi^n = \sum_{i=1}^{n} \lambda_i z_i^2 \ \text{ at } \ A = (a_{ik}) = (\lambda_i),\ i, k = 1, ..., n.$$

Let us bring each of the matrices' A n-eigenvectors (with the corresponding n directional main normals) to the n-quadratic forms (5.11), building the n vectors' space coordinates $\bar{l}_i^N$ for each of the spatial increments of the macrocoordinate $z_i(\bar{l}_i^N)$ $i = 1,\dots,n$.

Then the differential equation for the time-space movement splits on the n differential equations of the first order for each $z_i(\bar{l}_i^N)$, being independent (within each discrete interval $t_k, k = 3,5,7,\dots$):

$$\frac{\partial z_i}{\partial t} = c_i \frac{\partial z_i}{\partial \bar{l}_i^N}, \ \frac{\partial z_i}{\partial t} = \lambda_i z_i, \ \frac{\partial z_i}{\partial \bar{l}_i^N} = \underset{\bar{l}_i^N}{grad}\, z_i, \tag{5.12}$$

where c_i is a local speed of rotation for each normal $\bar{l}_i^N$ in the process of the matrix A diagonalization.

At the moment $(t_k + o)$ of the consolidation of each three eigenvectors, the three orth-vectors of the normals form a *common three-dimensional* coordinate system for the joint triple. The rotations and shifts of the local coordinate systems constitute a part of the optimal time-space cooperative movement directed toward the equalization of the n macromodel's eigenvalues. This process is governed by applying the optimal control, which initiates also the movement of the space coordinate systems, assigned to the matrix eigenvectors.

Under the applying control, the matrix eigenvalues, defined on the eigenvectors' space coordinate systems, move toward their cooperation in a joint coordinate system, while both matrices A, $\overline{A}$ are diagonalized.

Thus, the movement toward coinciding of the eigenvectors' space coordinate systems is also a part of macromodel's cooperative process. By the end of this process, the sequential equalization of the matrix eigenvalues completes at the discrete moment $(t_{n-1} + o)$.

At every t_k vicinity, each three coinciding eigenvectors acquire a unique common three-dimensional coordinate system, whose dimension changes to one at $(t_k + o)$ for a joint common eigenvector. This procedure sequentially involves each subsequent triple's eigenvalues including such one that had been cooperated before.

By the end of optimal time-space optimal movement, the matrix A gets the diagonalized form and the equal eigenvalues.

The considered transformations are summarized in the relation

$$A\overline{A} = \Lambda, \Lambda U^v U^\delta = \overline{\alpha}(\bar{l}), \tag{5.13}$$

where $\Lambda = (\lambda_i)$, i=1,..., n is the matrix of eigenvalues λ_i, U^v is the transformation, corresponding to the applied optimal control $v = v(t_i)$, which also transforms the complex eigenvalues λ_i into the real eigenvalues $\alpha_i(t_i)$, U^δ is the impulse $\delta(t_i + o)$-control, which joins the equal real eigenvalues (α_i) and changes both the dynamic and space model's dimensions at the moment $(t_k + o + \delta)$.

After applying (n-1)-th transformation by the end of $(t_{n-1} + o + \delta)$ interval, the matrix A acquires a single real $\overline{\alpha}$ eigenvector with the assigned space vector $\bar{l}$.

If each of the initial eigenvectors, before joining in the consolidation process, has a three-dimensional space, defined in each local moving coordinate systems, then the final space dimension of the eigenvector $\overline{\alpha}(\bar{l})$ will also be equal to three at (t_{n-1}).

During the *process* of the every triple's sequential diagonalization (before joining by the δ-control), the first triplet could have an intermediate space dimension equals 9, or 6, and for each following triplet the intermediate dimensions could be 12, 9, 6.

Actual forming of the subsequent triplets is accomplished by the consolidation of a previous created triplet with a following doublet. If the local eigenvectors have a single initial dimension of spatial coordinates, then by joining a following doublet, the intermediate spatial dimension still holds three (before the consolidation of its all three equal eigenvectors).

The final eigenvector also has the three dimensional space coordinates before its complete consolidation in a single dimension.

And a *final three dimensional space is created* as a result of the transformation of imaginary *information* with forming the real eigenvalue (5.13) and starting a real time.

Physical meaning of the considered $\delta(t_i + o) = v(t_i) - v(o)$-control's action is the following:

- At the moment $(t_i)$of the equalization of the pair (i,k) eigenvalues, related to frequencies $\omega_i(t_i) = \omega_k(t_i)$, (when a local equilibrium takes place and the pair correlation arises), the control $v(t_i)$ disolves the correlation, changes the sign of the eigenvalues; this leads to a local instability and initiates the attraction of the pair's trajectories in the process of couple's resonance and chaotic bifurcations [42-45], which generate the diffusion and entropy.
- At the moment $(t_i + o)$of the resonance, the control $-v(o)$ is applied, which finally binds the frequencies (that belong to different macrotrajectory's dimensions), stabilizing the cooperative bifurcations at the negative sign of the joint eigenvalue. The triplet is formed by a sequential consolidation of two eigenvalues (for example, dimensions *m-1, m*) at each $(t_k + o)$ with ajoining to them to a third eigenvalue (dimension *m-2*) that had been cooperated before. The sequence of the above dissipative resonances leads to a finale resonance, which joins all initial eigenvalues into a common single dimensional dynamic and space optimal movements.

These cooperative actions model the macroprocess' superimposition, revealing its phenomena and singularities at the discrete moments (DP) of applying controls. The bifurcation's singularities at the DP's locality reflect the instabilities at the matrix renovation. The process is associated with an essential irreversibility, breaking the determinism (within interval $(t_{i-1} + o, t_i)$),and the time's symmetry, and is ended with a local stability.

Example 5.1. Let us illustrate the above procedure considering the time-space dynamics of two-dimensional matrix $A^v = \begin{pmatrix} a_{11}, a_{12} \\ a_{21}, a_{22} \end{pmatrix}$ for the controlled system (5.12).

Using the matrix eigenvalues λ_1^v, λ_2^v and the matrix eigenvectors $\bar{x}_1^v = (\bar{x}_1^1, \bar{x}_1^2), \bar{x}_2^v = (\bar{x}_2^1, \bar{x}_2^2)$, determined by the equations for their components:

$$(a_{11} - \lambda_1^v)\bar{x}_1^1 + a_{12}\bar{x}_2^1 = 0,\ a_{21}\bar{x}_1^2 + (a_{22} - \lambda_2^v)\bar{x}_2^2 = 0, \tag{5.14a}$$

at the fixed $\bar{x}_1^2 = \bar{x}_2^1 = 1$, we get the solutions

$$\bar{x}_1^1 = -a_{12}(a_{11} - \lambda_1^v)^{-1},\ \bar{x}_2^2 = -a_{21}(a_{22} - \lambda_2^v)^{-1}.$$

With respect to the basis $(0 e_1 e_2)$, the eigenvectors have a view:

$$\bar{x}_1^1 = -a_{12}(a_{11} - \lambda_1^v)^{-1} e_1 + e_2,\ \bar{x}_2^2 = e_1 - a_{21}(a_{22} - \lambda_2^v)^{-1} e_2 \tag{5.14b}$$

and acquire the lengths:

$$|\bar{x}_1^v| = \{1 + [a_{12}(a_{11} - \lambda_1^v)^{-1}]^2\}^{1/2},\ |\bar{x}_2^v| = \{1 + [a_{21}(a_{22} - \lambda_2^v)^{-1}]^2\}^{1/2}. \tag{5.14c}$$

To find the orths (e_1', e_2') for the coordinate system, built on the first $\bar{x}_1^v$ and the second $\bar{x}_2^v$ eigenvectors, we normalize them and get the orths

$$e_1' = (a_{22} - \lambda_2^v)\{a_{21}^2 + [(a_{22} - \lambda_2^v)^2]^2\}^{-1/2} e_1 + (-1) a_{21} sign(a_{22} - \lambda_2^v) \times \{a_{21}^2 + [(a_{22} - \lambda_2^v)^2]^2\}^{-1/2} e_2; \tag{5.15a}$$

$$e_2' = (-1) a_{12} sign(a_{11} - \lambda_1^v)\{a_{12}^2 + [(a_{11} - \lambda_1^v)^2]^2\}^{-1/2} e_1 + (a_{11} - \lambda_1^v) \times \{a_{12}^2 + [(a_{11} - \lambda_1^v)^2]^2\}^{-1/2} e_2. \tag{5.15b}$$

The sought matrix' T elements $T = \begin{pmatrix} T_{11}, T_{12} \\ T_{21}, T_{22} \end{pmatrix}$, which perform the transformation $\begin{pmatrix} e_1' \\ e_2' \end{pmatrix} = \begin{pmatrix} T_{11}, T_{12} \\ T_{21}, T_{22} \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$ and satisfy to the equations $e_1' = T_{11} e_1 + T_{12} e_2$, $e_2' = T_{21} e_1 + T_{22} e_2$, are determined by the above normalized relations for the orths (e_1', e_2').

At $a_{12} = a_{21} = a_{12}$, directly from (5.14a) follow the relations:

$$(a_{11} - \lambda_1^v)^2 (\bar{x}_1^1)^2 + a_{12}^2 (\bar{x}_2^1)^2 = 0,\ (a_{22} - \lambda_2^v)^2 (\bar{x}_1^2)^2 - a_{12}^2 (\bar{x}_2^2) = 0,$$

$$(a_{12}^2 + (a_{11} - \lambda_1^v)^2)(\bar{x}_1^1)^2 = a_{12}^2,\ ((a_{11} - \lambda_1^v)^2 + a_{12}^2)(\bar{x}_1^2)^2 = (a_{11} - \lambda_1^v)^2,$$

$$(a_{12}^2 + (a_{22} - \lambda_2^v)^2)(\bar{x}_2^1)^2 = (a_{22} - \lambda_2^v)^2,\ ((a_{22} - \lambda_2^v)^2 + a_{12}^2)(\bar{x}_2^2)^2 = a_{12}^2;$$

$$\bar{x}_1^1 = \pm a_{12}((a_{11}-\lambda_1^v)^2 + a_{12}^2)^{-1/2},\ \bar{x}_2^1 = \mp(a_{11}-\lambda_1^v)\ ((a_{11}-\lambda_1^v)^2 + a_{12}^2)^{-1/2},$$

$$\bar{x}_1^2 = \pm(a_{22}-\lambda_2^v)((a_{22}-\lambda_2^v)^2 + a_{12}^2)^{-1/2}, \bar{x}_2^2 = \mp a_{12}((a_{22}-\lambda_2^v)^2 + a_{12}^2)^{-1/2}. \quad (5.16)$$

Let us order the vectors $\bar{x}^1(\bar{x}_1^1, \bar{x}_2^1), \bar{x}^2(\bar{x}_1^2, \bar{x}_2^2)$ such a way that they will be oriented and coordinated with the initial basis orths (e_1, e_2) and select the orthogonal matrix $T=I$ at

$$\Lambda^v = \begin{pmatrix} \lambda_1^v \\ \lambda_2^v \end{pmatrix}.$$

To satisfy these requirements, we will choose the vector's pairs with the alternating signs $\pm$ and $\mp$.

At $T = \bar{A}$, we get two possible transformations:

$$T = ((a_{11}-\lambda_1^v)^2 + a_{12}^2)^{-1/2} \begin{pmatrix} \mp(a_{11}-\lambda_1^v), \mp a_{12} \\ \pm a_{12}, \mp(a_{11}-\lambda_1^v) \end{pmatrix} = \begin{pmatrix} T_{11}, T_{12} \\ T_{21}, T_{22} \end{pmatrix} = \begin{pmatrix} \cos\varphi, \sin\varphi \\ -\sin\varphi, \cos\varphi \end{pmatrix}, \quad (5.17a)$$

$$T' = ((a_{22}-\lambda_2^v)^2 + a_{12}^2)^{-1/2} \begin{pmatrix} \mp(a_{22}-\lambda_2^v), \mp a_{12} \\ \pm a_{12}, \mp(a_{22}-\lambda_2^v) \end{pmatrix} = \begin{pmatrix} T_{11}', T_{12}' \\ T_{21}', T_{22}' \end{pmatrix} \begin{pmatrix} \cos\varphi', \sin\varphi' \\ -\sin\varphi', \cos\varphi' \end{pmatrix}. \quad (5.17b)$$

At the moment of equalization of $\lambda_1^v = \lambda_2^v$, $T = T'$, the angle $\varphi = \varphi'$ is found from the relations

$$\cos\varphi = \mp(a_{11}-\lambda_1^v)((a_{11}-\lambda_1^v)^2 + a_{12}^2)^{-1/2},\ \sin\varphi = \mp a_{12}((a_{11}-\lambda_1^v)^2 + a_{12}^2)^{-1/2},$$

$$\varphi = arctg(a_{12}(a_{11}-\lambda_1^v)^{-1}) + k\pi, k = \pm 1, 0. \quad (5.17c)$$

If $T_{11} > 0$, then $\varphi \in (-\pi/2, \pi/2)$, $k = 0$. *If* $T_{11} = 0$ and $T_{12} > 0$, then $\varphi = \pm\pi/2$;

if $T_{11} = 0$ and $T_{12} < 0$, then $\varphi = -\pi/2$.

At $T_{11} < 0$ and $T_{12} > 0$, we obtain $\varphi = arctg(a_{12}(a_{11}-\lambda_1^v)^{-1}) - \pi$.

At $T_{11} < 0$ and $T_{12} < 0$, we get $\varphi = arctg(a_{12}(a_{11}-\lambda_1^v)^{-1}) + \pi$.

For the given initial matrix $A(0) = \begin{pmatrix} 2,3 \\ 3,10 \end{pmatrix}$, by applying of the optimal control, we get the following relations for the matrix $A^v(\tau_1)$ elements at the first DP's moment $t_1 = \tau_1$:

$$a_{11}(\tau_1) = \frac{2\exp(12\tau_1) - 2.2\exp(11\tau_1) - 1.8\exp(\tau_1)}{\exp(12\tau_1) - 2\exp(11\tau_1) - 2\exp(\tau_1) + 4},$$

$$a_{22}(\tau_1)=\frac{10\exp(12\tau_1)-19.8\exp(11\tau_1)-0.2\exp(\tau_1)}{\exp(12\tau_1)-2\exp(11\tau_1)-2\exp(\tau_1)+4},$$

$$a_{12}(\tau_1)=a_{21}(\tau_1)=\frac{3(\exp(12\tau_1)-2.2\exp(11\tau_1)+0.2\exp(\tau_1))}{\exp(12\tau_1)-2\exp(11\tau_1)-2\exp(\tau_1)+4}, \qquad (5.18)$$

where the moment $t=\tau_1$ is found from equation (5.14a).

For this example, we get $\tau_1 \cong 0.7884$, whose substitution into the relations for the matrix elements leads to $a_{12}(\tau_1)=a_{21}(\tau_1)=0$, $a_{11}(\tau_1)=a_{22}(\tau_1)$.

Therefore, the simultaneous diagonalization $A^{v}(\tau_1)$ and the equalization of the matrix elements take place at the DP by applying optimal control.

The maximal angle of rotation $\varphi=\varphi(\tau-o)$ on a plane is determined by the above relations:

$$\sin\varphi(\tau_1-o)=\frac{a_{12}sign(a_{12})}{(a_{12}^2+(a_{11}-a_{22})^2)^{1/2}}\Big|_{t=\tau_1-o},$$

which for our example gives $\varphi(\tau_1-o)\approx -3.45$.

Transformation $\hat{T}=\begin{pmatrix}\cos\psi,\sin\psi\\ -\sin\psi,\cos\psi\end{pmatrix}$, which reduces (5.14a) toward the undistinguished states (5.16), for this example, is determined by the angle $\psi=0.147\,\pi$. The resulting angle of the rotation is $\varphi+\psi\cong -3$ and the corresponding transformation is

$$\overline{T}=T(\varphi)\hat{T}(\psi)=\begin{pmatrix}\cos(\varphi+\psi),\sin(\varphi+\psi)\\ -\sin(\varphi+\psi),\cos(\varphi+\psi)\end{pmatrix}=\begin{pmatrix}0.998,0.0471\\ -0.0471,0.998\end{pmatrix}. \qquad (5.19)$$

The initial orth's coordinates $e_1=(1,0), e_2=(0,1)$ are transformed into $e_1^{'}(\overline{e}_1^1,\overline{e}_2^1), e_2^{'}(\overline{e}_1^2,\overline{e}_2^2)$ according to relations $e_1^{'}=\overline{T}e_1, e_2^{'}=\overline{T}e_2$ from which follows $e_1^{'}=(0.998,-0.0471)$, $e_2^{'}=(0.0471,0.998)$.

The initial vector $\overline{L}^o=\begin{pmatrix}l_1^o\\ l_2^o\end{pmatrix}$ gets increment $\Delta\overline{L}^o=(\overline{T}-E)\overline{L}^o$, which at $l_1^o=1$, $l_2^o=1$ equals to $\Delta\overline{L}^o=\begin{pmatrix}0.451\\ -0.491\end{pmatrix}$.

This space vector can be used to fix and store the undistinguished macrovariables.

The rotation speed's vector $\overline{c}$ is determined from relation (5.16):

$$\overline{T}=\exp(\overline{c}\,\tau_1),\ \text{where}\ \overline{c}=\begin{pmatrix}0,c\\ -c,o\end{pmatrix}.$$

Using formula

$$\exp(\bar{c}t) = \frac{\exp(\lambda_1^c t)}{\lambda_1^c - \lambda_2^c}(\bar{c} - \lambda_2^c E) + \frac{\exp(\lambda_2^c t)}{\lambda_2^c - \lambda_1^c}(\bar{c} - \lambda_1^c E),$$

where $\bar{c} = \begin{pmatrix} \lambda_1^c, 0 \\ 0, \lambda_2^c \end{pmatrix}$, $\lambda_1^c = j$, $\lambda_2^c = -j$,

we get

$$\exp(\bar{c}\tau_1) = \begin{pmatrix} 1/2[\exp(jc\tau_1) + \exp(-jc\tau_1)], 1/j2[\exp(jc\tau_1) - \exp(-jc\tau_1)] \\ -1/j2[\exp(jc\tau_1) - \exp(-jc\tau_1)], 1/2[\exp(jc\tau_1) - \exp(-jc\tau_1)] \end{pmatrix}$$

and

$$\begin{pmatrix} \cos(\varphi+\psi), \sin(\varphi+\psi) \\ -\sin(\varphi+\psi), \cos(\varphi+\psi) \end{pmatrix} = \begin{pmatrix} \cos(c\tau_1), \sin(c\tau_1) \\ -\sin(c\tau_1), \cos(c\tau_1) \end{pmatrix}. \quad (5.20)$$

We obtain $c\tau_1 = \varphi + \psi$, or for the above $\tau_1 \cong 0.7884$ and $\varphi + \psi \cong -3$ we get $|c| \cong 3.805$.

This example provides a detail methodology for the calculation of the matrices $A^v(\tau), T(\tau), \bar{A}(\tau)$, the rotation speed's vector $\bar{c}$ =|C| and the shift 's vector $\bar{L}$.

The analogous examples for three-dimensional matrix are considered in [33, 34].

Example.5.2. *Illustration of the cooperative process in PDE.*

To illustrate the cooperative process in PDE let us consider an example of the space movement described by the second order differential equation:

$$\frac{\partial x_i}{\partial t} = -c(l_2 \frac{\partial x_i}{\partial l_1} - l_1 \frac{\partial x_i}{\partial l_2}); \; \frac{\partial x_i}{\partial t} = -\lambda_i x_i, \; i = 1,2,$$

$$x_i = z_i, \; \lambda_i x_i = c(l_2 \frac{\partial x_i}{\partial l_1} - l_1 \frac{\partial x_i}{\partial l_2}), \quad (5.21)$$

$$\bar{l} = \bar{A}(t)\bar{l}^o, \; \bar{l}^o = \begin{pmatrix} l_1^o \\ l_2^o \end{pmatrix}, \; \bar{A}(t) = \bar{A}(0)\exp(t\bar{c}), \; \bar{A}(0) = I,$$

with the applied starting control $v_i(s) = -2z_i(s)$ and a given initial distribution $z(l_s, s) = z(l_{1s}, l_{2s})$, which at $l_{1s} = l_1, l_{2s} = l_2$ and for the fixed $l_1 = \hat{l}_1$ we write in the form

$$\varphi_i = x_i(\hat{l}_1, l_2, s) = k_i(2 - \exp\frac{\lambda_i}{c} l_2), \lambda_i = \lambda_i(s), \lambda_i(s) = 1,$$

$$\lambda_2(s) = 11, s = 0, k_i = k_i(l_1, s), k_1 = 1, k_2 = 11 \bullet 10^{-6}.$$

The problem consists of finding the geometric region of the points $l_2^o = l_2^o(l_1^o)$, where at the moment τ of the eigenvalues' equalization (4.53), the distributions $x_1(l_1^o, l_2^o, \tau) = x_2(l_1^o, l_2^o, \tau)$ will coincide.

For this purpose, using the equations (5.21), we determine their solutions at the moment s: $x_i(\hat{l}_1, l_1, l_2, s)$, $i = 1,2$ first, and then, we find their solutions at the moment τ.

Getting the solutions, we come to the relations

$$\frac{dl_1}{l_2} = -\frac{dl_2}{l_1} = \frac{c}{\lambda_i}\frac{dx_i}{x_i},\ \psi_1 = l_1^2 + l_2^2,\ \psi_2 = \frac{l_1}{l_2} + \frac{c}{\lambda_i}\ln x_i,\ \hat{\psi}_1 = \hat{l}_1^2 + l_2^2,$$

$$\hat{\psi}_2 = \frac{\hat{l}_1}{l_2} + \frac{c}{\lambda_i}\ln x_i,\ l_2 = \pm(\hat{\psi}_1^2 - l_1^2)^{1/2};\ x_i = \exp(\frac{\lambda_i}{c}(\hat{\psi}_2 - \frac{\hat{l}_1}{l_2})),$$

$$x_i = \exp(\frac{\lambda_i}{c}(\hat{\psi}_2 - +\hat{l}_1(\hat{\psi}_1^2 - l_1^2)^{-1/2})),$$

$$\exp(\frac{\lambda_i}{c}(\frac{l_1}{l_2} + \frac{c}{\lambda_i}\ln x_i) \mp \hat{l}_1(l_2^2)^{-1/2}) - \varphi_i(\pm(l_1^2 - \hat{l}_1^2 + l_2^2)^{1/2}) = 0,\ l_1^2 + l_2^2 > \hat{l}_1^2,$$

$$x_i = x_i(l_1, l_2, s) = -\exp(\frac{\lambda_i}{cl_2}(l_1 - \hat{l}_1)) + \varphi_i(\pm(l_1^2 - \hat{l}_1^2 + l_2^2)^{1/2}),$$

$$x_i = -\exp(\frac{\lambda_i}{cl_2}(l_1 - \hat{l}_1)) + k_i(2 - \exp(\pm\frac{\lambda_i}{c}(l_1^2 - \hat{l}_1^2 + l_2^2)^{1/2})).$$

Solutions $x_i(l_1, l_2)$, $i = 1,2$ have a meaning on the set $\bar{\Omega}_1 \setminus \Omega^o$, where $\bar{\Omega}_1 = \Omega_1 \cup \Gamma_1$, $\Omega_1: (l_1^2 + l_2^2) > R_1^2$, $R_1 = \hat{l}_1$; $\Gamma_1: l_1^2 + l_2^2 = R_1$; $\Omega^o: |l_2| < \delta_1$, $\delta_1 > 0$, (Fig. 5.1).

Let us introduce the set $\bar{\Omega}_2: (l_1^2 + l_2^2) \geq R_2^2$, $R_2 = R_1 + \delta$, $\delta > 0$, $\Omega = \Omega_2 \setminus \Omega_1 \setminus \Omega^o$. At a small $\delta > 0$, we may restrict the solutions x_i, $i = 1,2$ by a linear approximations on Ω considering only the first two parts of the function's $x_i(l_1, l_2)$ decomposition in Taylor's series at a δ-locality of $l^* = (l_1^*, l_2^*)$, $l_1^* = \hat{l}_1$, $l^* \in \Gamma_1$.

We get the following relations

$$-\exp(\frac{\lambda_i}{cl_2}(l_1 - \hat{l}_1)) \cong -(1 + \frac{\lambda_i}{cl_2^*}(l - l_1^*) - \frac{\lambda_i}{c(l_2^*)^2}(l - l_2^*)),$$

$$-k_i \exp(\pm\frac{\lambda_i}{c}(l_1^2 - (\hat{l}_1^*)^2 + l_2^2)^{1/2})) \cong -k_i(\exp(\frac{\lambda_i l_2^*}{c}) + \exp(\frac{\lambda_i l_2^*}{c})\frac{\lambda_i}{c}\frac{l_1^*}{|l_2^*|}(l_1 - l_1^*)$$

$$+ \exp(\frac{\lambda_i l_2^*}{c})\frac{\lambda_i}{c}(l_2 - l_2^*))$$

$$= -k_i(\exp(\frac{\lambda_i l_2^*}{c})(1 + \frac{\lambda_i}{c}\frac{l_1^*}{|l_2^*|}(l_1 - l_1^*) + \frac{\lambda_i}{c(l_2^*)^2}(l_2 - l_2^*));$$

$$x_i \cong (2k_i - 1 - k_i \exp(\frac{\lambda_i l_2^*}{c})) - (\frac{\lambda_i}{c l_2^*} + k_i \exp(\frac{\lambda_i l_2^*}{c})\frac{\lambda_i}{c}\frac{l_1^*}{|l_2^*|})(l_1 - l_1^*)$$

$$+(\frac{\lambda_i}{c(l_2^*)^2} - k_i \exp(\frac{\lambda_i l_2^*}{c})\frac{\lambda_i}{c})(l_2 - l_2^*)) = k_i(2 - \exp(\frac{\lambda_i l_2^*}{c})) - 1)$$

$$-\frac{\lambda_i}{c}((l_2^*)^{-1} + k_i \exp(\frac{\lambda_i l_2^*}{c})\frac{l_1^*}{|l_2^*|}))(l_1 - l_1^*) + \frac{\lambda_i}{c}((l_2^*)^{-2} - k_i \exp(\frac{\lambda_i l_2^*}{c}))(l_2 - l_2^*)).$$

Let us implement the equality $x_1 = x_2$ at the moment s for the obtained linearized functions. This leads to equations

$$B_o + B_1(l_1 - l_1^*) + B_2(l_2 - l_2^*) = 0,$$

where

$$B_o = -k_1(2 - \exp(\frac{\lambda_1 l_2^*}{c})) + k_2(2 - \exp(\frac{\lambda_2 l_2^*}{c})),$$

$$B_1 = -\frac{\lambda_2}{c}((l_2^*)^{-1} + k_2 \exp(\frac{\lambda_2 l_2^*}{c})\frac{l_1^*}{l_2^*}) - \frac{\lambda_1}{c}((l_2^*)^{-1} + k_1 \exp(\frac{\lambda_1 l_2^*}{c})\frac{l_1^*}{l_2^*}),$$

$$B_2 = \frac{\lambda_1}{c}((l_2^*)^{-2} - k_1 \exp(\frac{\lambda_1 l_2^*}{c})) + \frac{\lambda_2}{c}((l_2^*)^{-2} - k_2 \exp(\frac{\lambda_2 l_2^*}{c})).$$

We receive the equation of a strait line:

$$l_2 = B_3 + \frac{B_1}{B_2} l_1,\ B_3 = l_2^* + \frac{B_o}{B_2} - \frac{B_1}{B_2},$$

at which the equalization of the considered distributions is reached at the initial moment s. Computing the above coefficients at given $\lambda_1, \lambda_2, k_1, k_2$, we get $\frac{B_1}{B_2} = 1.827, B_3 = -0.816$, and the equation $l_2^o = -0.816 + 1.827 l_1^o$, which is indicated by (a) on Fig.5.2a.

The moment τ_1, satisfying (4.53) is found from the transcendent equation

$$\eta^\alpha - \frac{\alpha}{2}\eta + \alpha - 1 = 0, \qquad (5.22a)$$

where $\alpha = \frac{\gamma_{ij} - 1}{\gamma_{ij}}$, $\gamma_{ij} = \frac{\lambda_{is}}{\lambda_{js}}$, $\eta = \exp(\lambda_{is}\tau_1)$, which for $\lambda_{is} = 11, \lambda_{js} = 1$, $\alpha = \frac{10}{11}$, $\eta = \exp 11\,\tau_1$ determines $\tau_1 = 0.7884$ that also corresponds to

$$\lambda_1(\tau_1) = \lambda_2(\tau_1) = -11. \qquad (5.22)$$

To get the analogous equations at the moment τ_1, we use the transformation

$$l_1 = \overline{A}_{11} l_1^o + \overline{A}_{12} l_2^o,\ l_2 = \overline{A}_{21} l_1^o + \overline{A}_{22} l_2^o \text{ at } \overline{A} = \begin{pmatrix} \overline{A}_{11}, \overline{A}_{12} \\ \overline{A}_{21}, \overline{A}_{22} \end{pmatrix},$$

which satisfies the equations $\overline{A}(t) = \overline{A}(0)\exp(t\overline{c}) = I(I + t\overline{c} + \frac{t}{2!}\overline{c}^2 + ... + \frac{t^n}{n!}\overline{c}^n)$

$$\cong I + tc\begin{pmatrix} 0,1 \\ -1,0 \end{pmatrix} + \frac{(tc)^2}{2}(-1)I + \frac{(tc)^3}{6}\begin{pmatrix} 0,1 \\ -1,0 \end{pmatrix} + \frac{(tc)^4}{24} I + \frac{(tc)^5}{120}\begin{pmatrix} 0,1 \\ -1,0 \end{pmatrix}.$$

We come to relations

$$\overline{A}_{21} l_1^o + \overline{A}_{22} l_2^o = B_3 + \frac{B_1}{B_2}\overline{A}_{11} l_1^o + \overline{A}_{12} l_2^o,\ (\overline{A}_{22} - \frac{B_1}{B_2}\overline{A}_{12}) l_2^o = B_3 + (\frac{B_1}{B_2}\overline{A}_{11} - \overline{A}_{21}) l_1^o, \quad (5.23)$$

which determine the equation of the strait line, where hold true the equalities

$$l_2^o = (\overline{A}_{22} - \frac{B_1}{B_2}\overline{A}_{12})^{-1} B_3 + (\frac{B_1}{B_2}\overline{A}_{11} - \overline{A}_{21})(\overline{A}_{22} - \frac{B_1}{B_2}\overline{A}_{12})^{-1} l_1^o,$$

$$\overline{A}_{11} = \overline{A}_{11}(t) \cong 1 - \frac{(tc)^2}{2} + \frac{(tc)^4}{24}, \quad (5.24)$$

$$\overline{A}_{12} = \overline{A}_{12}(t) \cong ct - \frac{(tc)^3}{6} + \frac{(tc)^5}{120},\ \overline{A}_{21} = \overline{A}_{21}(t) \cong -ct + \frac{(tc)^3}{6} - \frac{(tc)^5}{120},$$

$$\overline{A}_{22} = \overline{A}_{22}(t) \cong 1 - \frac{(tc)^2}{2} + \frac{(tc)^4}{24}.$$

At $t = \tau_1$ the above coefficients take the values $\overline{A}_{11}(\tau_1) \cong 0.7$, $\overline{A}_{12}(\tau_1) \cong -0.06, \overline{A}_{21}(\tau_1) \cong 0.06$, $\overline{A}_{22}(\tau_1) \cong 0.7$, and the seeking equation acquires the form $l_2^o = -1 + 1.5 l_1^o$. This strait line is indicated by (b) on (Fig. 5.2a).

The example's results correspond to the transformation of the strait line (a), considered at the initial moment s, into the strait line (b), considered at the moment $\tau_1 = 0.7884$.

This transformation corresponds to moving the initial model's distributions $z_1(l_s^*, s)$, $z_2(l_s^*, s)$ toward the distribution's cooperation into a joint distribution $z(l_\tau^o, \tau)$, or $z(l_{1\tau}^o, l_{1\tau}^o)$ on the line (b), (Figs 5.2a and 5.2b.). Both distributions coincide because the model's eigenvalues (5.22) are also equal at the moment τ_1.

The consolidation, along with forming an information macrostructure, is an *additional mechanism* for the path functional minimization that reduces the initial minimax principle to the principle of a minimum of uncertainty, responsible for the informational space-time cooperations. The regularities of these processes are defined by the movement along the path functional's extremals and by the proceedings directed toward the state consolidation.

The results illustrate the possibility of forming the *cooperative model* based on the *regular equations* in partial derivations without a direct use of the stochastic equations.

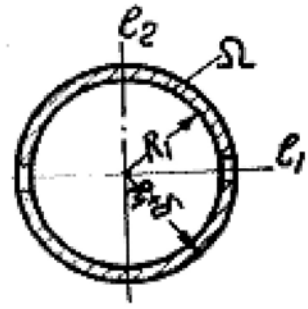

Figure 5.1. The distribution's geometrical locations at the moment s (a) and the moment τ_1 -(b).

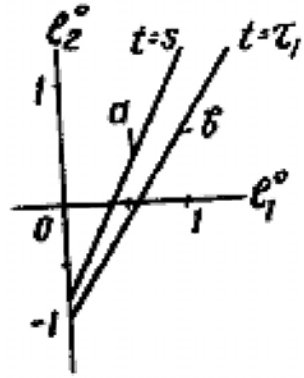

Figure 5.2.a. The potential geometrical locations of the model's distributions.

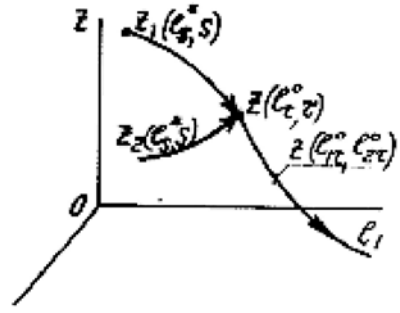

Figure 5.2.b. The process of cooperation for the model's distributions in three dimensional space.

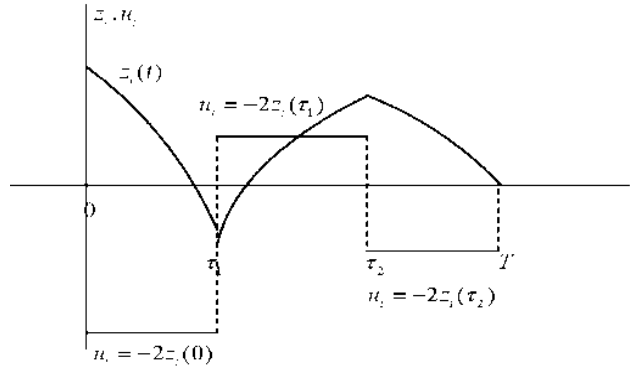

Figure 5.3.

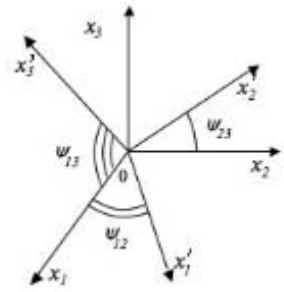

Figure 5.3.a.

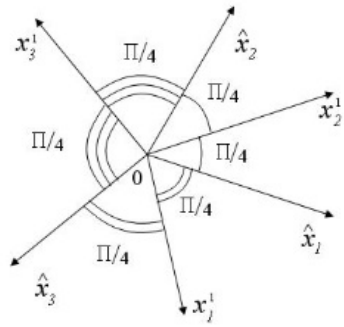

Figure 5.3.b.

Example 5.2a. *Model of a third order*.

Let us have a system model of a third order which is given by equations:

$$\dot{x} = A(x+v), v = -2x(\tau) \text{ with the initial matrix } A(0) = \begin{pmatrix} 3,-1,1 \\ -1,5,-1 \\ 1,-1,3 \end{pmatrix}. \qquad (5.25)$$

The calculated initial eigenvalues λ_{io}, $i=1,2,3$ and the eigenvector's components ϕ^i_{jo}, $j=1,2,3$ are:

$$\lambda_{1o}=2;\phi^1_{1o}=\phi^1_{3o}=\pm(2)^{-1/2},\phi^1_{2o}=0,\phi^1_{1o}=-\phi^1_{3o};$$
$$\lambda_{2o}=3;\phi^2_{1o}=\phi^2_{2o}=\phi^2_{3o}=\pm(3)^{-1/2};$$
$$\lambda_{3o}=6;\phi^3_{1o}=\phi^3_{3o}=\pm(6)^{-1/2},\phi^3_{2o}=\mp 2(6)^{-1/2}. \quad (5.26)$$

The corresponding orths e^1_i of a coordinate system that is built on the related eigenvectors (with regard to the orths of the initial coordinate system e_i) are

$$e^1_1=\pm(2)^{-1/2}e_1\pm 0e_2\mp(2)^{-1/2}e_3,$$
$$e^1_2=\pm(3)^{-1/2}(e_1+e_2+e_3),$$
$$e^1_3=\pm(6)^{-1/2}(e_1-2e_2+e_3).$$

From that, the possible transformations, converting matrix $A(0)$ to a diagonal form, have view:

$$T_1=\pm\begin{pmatrix}(2)^{-1/2},0,-(2)^{-1/2}\\(3)^{-1/2},(3)^{-1/2},(3)^{-1/2}\\(6)^{-1/2},-2(6)^{-1/2},(6)^{-1/2}\end{pmatrix},\ T_2=\pm\begin{pmatrix}(2)^{-1/2},0,-(2)^{-1/2}\\(6)^{-1/2},-2(6)^{-1/2},(6)^{-1/2}\\(3)^{-1/2},(3)^{-1/2},(3)^{-1/2}\end{pmatrix},$$

$$T_3=\pm\begin{pmatrix}(3)^{-1/2},(3)^{-1/2},(3)^{-1/2}\\(2)^{-1/2},0,-(2)^{-1/2}\\(6)^{-1/2},-2(6)^{-1/2},(6)^{-1/2}\end{pmatrix}$$

with a total of 24 combinations of the matrix's transformations T_i, chosen for the similar oriented orths (without taking into account the operation of their inversion).

Let us select such of them, which correspond, for example, to a right triple of the eigenvectors that determine the matrix of a right rotation. These matrixes satisfy the condition $\det T_k=+1, k=1,...,6$.

We illustrate the choice on the example for the matrix T_1, considering a different order of this matrix's elements at $\det T_1$=1. We get the following matrices

$$T_{11}=\begin{pmatrix}(2)^{-1/2},0,-(2)^{-1/2}\\(3)^{-1/2},(3)^{-1/2},(3)^{-1/2}\\(6)^{-1/2},-2(6)^{-1/2},(6)^{-1/2}\end{pmatrix},\ T_{12}=\begin{pmatrix}(2)^{-1/2},0,-(2)^{-1/2}\\-(3)^{-1/2},-(3)^{-1/2},-(3)^{-1/2}\\-(6)^{-1/2},2(6)^{-1/2},-(6)^{-1/2}\end{pmatrix},$$

$$T_{13} = \begin{pmatrix} -(2)^{-1/2}, 0, (2)^{-1/2} \\ (3)^{-1/2}, (3)^{-1/2}, (3)^{-1/2} \\ -(6)^{-1/2}, 2(6)^{-1/2}, -(6)^{-1/2} \end{pmatrix}, \; T_{14} = \begin{pmatrix} -(2)^{-1/2}, 0, (2)^{-1/2} \\ -(3)^{-1/2}, -(3)^{-1/2}, -(3)^{-1/2} \\ (6)^{-1/2}, -2(6)^{-1/2}, (6)^{-1/2} \end{pmatrix}.$$

In the subsequent calculations, we use just one of potential 24 matrices, for example, $T_{14} = G$ with

$$G^{-1} = G^T = \begin{pmatrix} (2)^{-1/2}, (3)^{-1/2}, (6)^{-1/2} \\ 0, (3)^{-1/2}, -2(6)^{-1/2}, \\ -(2)^{-1/2}, (3)^{-1/2}, (6)^{-1/2} \end{pmatrix}. \tag{5.27}$$

The solutions of the dynamic model's equation in a diagonal form:

$$\dot{z} = \Lambda(z+u), \Lambda(0) = \begin{pmatrix} \lambda_{1o}, 0, 0 \\ 0, \lambda_{2o}, 0 \\ 0, 0, \lambda_{3o} \end{pmatrix}, u = -2z(0) \tag{5.28}$$

can be ordered, for example, by the increasing of λ_{io} (in(5.28)):

$$z_1(\tau_1) = z_1(0)(2 - \exp(\lambda_1^1 \tau_1)); z_1(\tau_2) = z_1(\tau_1)(2 - \exp(\lambda_1^2 \tau_2));$$
$$z_2(\tau_1) = z_2(0)(2 - \exp(\lambda_2^1 \tau_1)); z_2(\tau_2) = z_2(\tau_1)(2 - \exp(\lambda_2^2 \tau_2));$$
$$z_3(\tau_1) = z_3(0)(2 - \exp(\lambda_3^1 \tau_1)); z_3(\tau_2) = z_3(\tau_1)(2 - \exp(\lambda_3^2 \tau_2)).$$

The discrete moment τ_1 of the equalizations we find from the fulfillment of one of the equalities

$\lambda_1^1(\tau_1) = \lambda_2^1(\tau_1), \lambda_1^1(\tau_1) = \lambda_3^1(\tau_1), \lambda_2^1(\tau_1) = \lambda_3^1(\tau_1),$

at

$\lambda_1^1(\tau_1) = -2\exp(2\tau_1)(2 - \exp(2\tau_1))^{-1}$;

$\lambda_2^1(\tau_1) = -3\exp(\tau_1)(2 - \exp(3\tau_1))^{-1}; \lambda_3^1(\tau_1) = -6\exp(\tau_1)(2 - \exp(6\tau_1))^{-1};$

from which we get the equation

$$6\exp(\tau_1) - \exp(3\tau_1) - 4 = 0; 4\exp(3\tau_1) - \exp(6\tau_1) - 6 = 0;$$
$$3\exp(4\tau_1) - \exp(6\tau_1) - 3 = 0.$$

A positive solution $\tau_1 > 0$ has only the first equation.

We get $\tau_1 = 0.693177, \lambda_1^1(\tau_1) = \lambda_2^1(\tau_1) = \lambda_2(\tau_1) = 4$.

To find τ_2 we solve jointly the equations

$$\lambda_1^2(\tau_2) = \lambda_1^1(\tau_1)\exp(\lambda_1^1(\tau_1)\tau_2))(2-\exp(\lambda_1^1(\tau_1)\tau_2))^{-1};$$
$$\lambda_3^2(\tau_2) = \lambda_3^1(\tau_1)\exp(\lambda_3^1(\tau_1)\tau_2))(2-\exp(\lambda_3^1(\tau_1)\tau_2))^{-1}$$

at

$$\lambda_2^2(\tau_2) = \lambda_2^2(\tau_2) = \lambda_3^2(\tau_2) = \lambda_3(\tau_2).$$

We come to

$$\exp(\lambda_3^1(\tau_1)\tau_2)[\lambda_3^1(\tau_1)(\lambda_2^1(\tau_1))^{-1}-1]-2(\lambda_3^1(\tau_1)\lambda_2^1(\tau_1))^{-1}\exp[(\lambda_2^1(\tau_1)\tau_2))-(\lambda_1^1(\tau_1)\tau_2)]-1)=0$$

and get $\tau_2 = 0.341761, \lambda_3^2(\tau_2) = \lambda_3(\tau_2) = 8.158562$.

Thus, at the moment τ_1, the operators of the processes' $z_1(t), z_2(t)$ are equalized, and at the moment τ_2, these operators are equalized with $\lambda_3^2(\tau_2)$. At $z_1(0)=10^{-3}, z_2(0)=0.96\times10^{-3}, z_3(0)=10^{-5}$ we have $z_1(\tau_2)=0.36, z_2(\tau_2)=0.35, z_3(\tau_2)=0.375$. A total time of optimal movement is

$T=\tau_1+\tau_2+\tau_3$, at $\tau_3=\ln 2(\lambda_3^2(\tau_2))^{-1}\cong 0.77811$, $z_1(T)=z_2(T)=z_3(T)=0$, $x(T)=0$, and T =1.181305.

The resulting optimal process is shown on Fig. 5.3.

The optimal controls are switched at the moments of the model operators' equalization.

Because matrix $\Lambda(\tau_2)=\begin{pmatrix}\lambda_1(\tau_2)\\0,\lambda_2(\tau_2),0\\0,0,\lambda_3(\tau_2)\end{pmatrix}$ satisfies the relation $\lambda_1(\tau_2)=\lambda_2(\tau_2)=\lambda_3(\tau_2)$, we have $A(\tau_2)=G\lambda_1(\tau_2)IG^{-1}=\lambda_1(\tau_2)I$, i.e., the system's renovated matrix $A(\tau_2)$ is diagonalzed at moment τ_2.The same is true for the moment τ_1: $\lambda_1(\tau_1)=\dot z_1/z_1=\dot z_2/z_2=\lambda_2(\tau_1)$ *if* the transformation G is implemented by the rotation around one of the coordinate axis ($0z_3$).

Indeed, writing this matrix in the form $G=\begin{pmatrix}\cos\varphi,\sin\varphi,0\\-\sin\varphi,\cos\varphi,0\\0,0,1\end{pmatrix}$, we get the system's matrix

$$A(\tau_1)=\begin{pmatrix}\cos\varphi,\sin\varphi,0\\-\sin\varphi,\cos\varphi,0\\0,0,1\end{pmatrix}\begin{pmatrix}\lambda_1(\tau_1)\\0,\lambda_2(\tau_1),0\\0,0,\lambda_3(\tau_1)\end{pmatrix}\begin{pmatrix}\cos\varphi,-\sin\varphi,0\\\sin\varphi,\cos\varphi,0\\0,0,1\end{pmatrix}=$$

$$\begin{pmatrix}(\lambda_1\cos^2\varphi+\lambda_2\sin^2\varphi),(-\lambda_1\sin\varphi\cos\varphi+\lambda_2\sin\varphi\cos\varphi),0\\(-\lambda_1\sin\varphi\cos\varphi+\lambda_2\sin\varphi\cos\varphi),(\lambda_1\cos^2\varphi+\lambda_2\sin^2\varphi),0\\0,0,\lambda_3\end{pmatrix}=\begin{pmatrix}\lambda_1(\tau_2)\\0,\lambda_2(\tau_2),0\\0,0,\lambda_3(\tau_2)\end{pmatrix}. \quad (5.29)$$

The condition of the equalization of all phase speeds holds true at the moment τ_2; this condition, taking into account the diagonalization of $A(\tau_2)$, acquires the form $\dot{z}_1 / z_1 = \dot{z}_2 / z_2 = \dot{z}_3 / z_3 = \dot{x}_1 / x_1 = \dot{x}_2 / x_2 = \dot{x}_3 / x_3$.

The phase picture of the control system at $\tau_2 < t \le T$ is portrayed by three straight lines, crossing a beginning of coordinate system in the planes $0x_1x_2, 0x_2x_3, 0x_3x_2$ (Fig.5.3a), which are turned on the angles $\psi_{12}, \psi_{23}, \psi_{31}$ with regard the coordinate axis's $0x_1, 0x_2x_3, 0x_3$ accordingly. A transformation of the initial coordinate system $0x_1x_2x_3$ to the affine system $0x_1^1x_2^1x_3^1$, built on these strait lines as the vectors, has a view

$$\overline{T}_1 = \begin{pmatrix} \cos\psi_{12}, \sin\psi_{12}, 0 \\ 0, \sin\psi_{23}, \cos\psi_{23} \\ \cos\psi_{31}, 0, \sin\psi_{31} \end{pmatrix}, tg\psi_{ij} = \frac{x_i(\tau_2)}{x_j(\tau_2)}, \psi_{ij} = \psi_{ij}^o \pm \pi k.$$

To find the above angles let us transform $z_i(\tau_2)$ to $x_i(\tau_2)$:

$$x_1(\tau_2) = G_{11}^T z_1(\tau_2) + G_{12}^T z_2(\tau_2) + G_{13}^T z_3(\tau_2),$$
$$x_2(\tau_2) = G_{21}^T z_1(\tau_2) + G_{22}^T z_2(\tau_2) + G_{23}^T z_3(\tau_2),$$
$$x_3(\tau_2) = G_{31}^T z_1(\tau_2) + G_{32}^T z_2(\tau_2) + G_{33}^T z_3(\tau_2).$$

After substitution of the calculated $z_i(\tau_2)$ and G_{ij}^T, we have

$$x_1(\tau_2) = 0.61,\ x_2(\tau_2) = -0.101,\ x_3(\tau_2) = 0.101$$

and

$$\overline{T}_1 = \begin{pmatrix} 0.985, -0.171, 0 \\ 0, 0.564, -0.826 \\ 0.986, 0, 0.165 \end{pmatrix};$$

with

$$\det \overline{T}_1 = 0.985(0.564 \times 0.165) + 0.171(0.386 \times 0.826) > 0.$$

We get

$$tg\psi_{12} = -0.172, tg\psi_{23} = -1.463, tg\psi_{13} = 0.167,$$

and

$$\psi_{12}^o = -0.172, \psi_{23}^o = -0.972, \psi_{13}^o = 0.166.$$

Forming the indistinctive states $\hat{x}(\hat{x}_1, \hat{x}_2, \hat{x}_3)$ is associated with turning the coordinate system $0x_1^1x_2^1x_3^1$ in such a way that the axis's $0x_1^1$, $0x_2^1, 0x_3^1$ will become the bisectrices of the coordinate planes $(0\hat{x}_1\hat{x}_3, 0\hat{x}_1\hat{x}_2, 0\hat{x}_2\hat{x}_3)$. This corresponds to applying the transformation

$$\hat{T} = \begin{pmatrix} (2)^{1/2}2^{-1}, 0, (2)^{1/2}2^{-1} \\ (2)^{1/2}2^{-1}, (2)^{1/2}2^{-1}, 0 \\ 0, (2)^{1/2}2^{-1}, (2)^{1/2}2^{-1} \end{pmatrix},$$

which is not an orthogonal transformation:

$$\hat{T}^{-1} = (2)^{-1/2} \begin{pmatrix} 1,1,-1 \\ -1,1,1 \\ 1,-1,1 \end{pmatrix} \neq \hat{T}^{T}.$$

The indistinctive states' vectors are connected with the initial vector by relation $\hat{x} = \overline{T}_1 \hat{T}^{-1} x = \overline{T} x$, where $\overline{T} = (2)^{-1/2} \begin{pmatrix} -0.001, 0.394, -0.991 \\ 0.001, 0.735, -0.661 \\ 1.971, -0.735, 0.991 \end{pmatrix}$.

Let have a vector $x^1 = (1,1,1)$, given in $0x_1^1 x_2^1 x_3^1$, then a length of this vector in $0\hat{x}_1\hat{x}_2\hat{x}_3$ (equals to the square root of the sum of the coordinate's squares) is

$$\hat{l}_x = (2)^{-1/2}[(-0.001 + 0.394 - 0.991)^2 + (0.001 + 0.735 - 0.661)^2 + (1.971 - 0.735 + 0.991)^2]^{1/2} \cong 2.3(2)^{-1/2} \cong 1.62.$$

The length's increment, determined by the transformation $\overline{T}$: $\Delta\hat{l}_x = |\hat{l}_x - (3)^{-1/2}| \cong 0.1$, is caused by a non- orthogonal transformation $\hat{T}$.

A length of the corresponding vector's difference we find from relation

$$\overline{T} = \left| \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right| = \left| (\overline{T} - I) \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right| = (2)^{-1/2} \begin{pmatrix} -1.413, 0.394, -0.991 \\ 0.001, -0.679, -0.661 \\ 1.971, -0.735, -0.423 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} =$$

$$(2)^{-1/2}[(-1.413 + 0.394 - 0.991)^2 + (0.001 - 0.679 - 0.661)^2]^{1/2} = 1.8.$$

As a result, the above increments are associated with reaching the model coordinates' pair indistinctiveness.

Let us also find the discretization moments from the conditions of equalization of phase speeds for a non diagonal system with

$$\dot{x}_1 / x_1(\tau_{12}) = \dot{x}_2 / x_2(\tau_{12}), \dot{x}_2 / x_2(\tau_{23}) = \dot{x}_3 / x_3(\tau_{23}), \dot{x}_1 / x_1(\tau_{13}) = \dot{x}_1 / x_1(\tau_{13}).$$

The solutions of the initial system at optimal control have the form

$$x(t) = \begin{pmatrix} -(2^{-1}\exp 2t + 3^{-1}\exp 3t + 6^{-1}\exp 6t - 2), 3^{-1}(\exp 6t - \exp 3t), (2^{-1}\exp 2t - 3^{-1}\exp 3t - 6^{-1}\exp 6t) \\ 3^{-1}(\exp 6t - \exp 3t), -3^{-1}(\exp 3t + 2\exp 6t - 6), 3^{-1}(\exp 6t - \exp 3t) \\ (2^{-1}\exp 2t - 3^{-1}\exp 3t - 6^{-1}\exp 6t), 3^{-1}(\exp 6t - \exp 3t), -(2^{-1}\exp 2t + 3^{-1}\exp 3t + 6^{-1}\exp 6t - 2) \end{pmatrix}$$

$$\times \begin{pmatrix} x_{10} \\ x_{20} \\ x_{30} \end{pmatrix};$$

$$\dot{x}_1 / x_1 = \frac{-x_{10}(\exp 2t + \exp 3t + \exp 6t) + x_{20}(2\exp 6t - \exp 3t) + }{-x_{10}(2^{-1}\exp 2t + 3^{-1}\exp 3t + 6^{-1}\exp 6t - 2) + x_{20}3^{-1}(2\exp 6t - \exp 3t) + }$$
$$\frac{+x_{30}(\exp 2t - \exp 3t - \exp 6t)}{+x_{30}(2^{-1}\exp 2t - 3^{-1}\exp 3t - 6^{-1}\exp 6t)},$$

$$\dot{x}_2 / x_2 = \frac{x_{10}(2\exp 6t - exp3t) - x_{20}(\exp 3t + 4\exp 6t) + x_{30}(2\exp 6t - exp3t)}{3^{-1}[x_{10}(\exp 6t - \exp 3t) - x_{20}(\exp 3t + 2\exp 6t - 6) + x_{30}(\exp 6t - \exp 3t)]},$$

$$\dot{x}_3 / x_3 = \frac{x_{10}(\exp 2t - \exp 3t - \exp 6t) + x_{20}(2\exp 6t - \exp 3t) + }{x_{10}(2^{-1}\exp 2t - 3^{-1}\exp 3t - 6^{-1}\exp 3t) + x_{20}3^{-1}(\exp 6t + \exp 3t) + }$$
$$\ldots\ldots\ldots \frac{-x_{30}(\exp 2t + \exp 3t + \exp 6t)}{-x_{30}(2^{-1}\exp 2t + 3^{-1}\exp 6t + 6^{-1}\exp 3t - 2)}.$$

From that we get the equations for finding the discrete moments:

$$\tau_{12} : A_7 y^7 + A_6 y^6 + A_5 y^5 + A_4 y^4 + A_3 y^3 + A_1 y^7 + A_o = 0,$$
$$\tau_{13} : B_6 y^6 + B_4 y^4 + B_3 y^3 + B_1 y + B_o = 0,$$
$$\tau_{23} : C_7 y^7 + C_6 y^6 + C_4 y^4 + C_3 y^3 + C_1 y + C_o = 0\,,\ y = \exp t\,,$$

where the above coefficients are expressed through the initial conditions

$$A_7 = (2)^{-1}x_{10}^2 - x_{20}^2 + (2)^{-1}x_{30}^2 - (2)^{-1}x_{10}x_{20} - (2)^{-1}x_{20}x_{30} + x_{10}x_{30};$$
$$A_6 = 2(3)^{-1}(x_{10}^2 - x_{30}^2 - 2x_{10}x_{20} + 2x_{20}x_{30});$$
$$A_4 = 2(2x_{20}^2 - 2x_{10}^2 - 2x_{10}x_{30} + 3x_{10}x_{20} - x_{20}x_{30});$$
$$A_3 = (6)^{-1}(x_{30}^2 - x_{10}^2 - x_{10}x_{20} + x_{20}x_{30});\ A_1 = 2(x_{10}^2 - x_{20}^2 + x_{10}x_{30} - x_{20}x_{30});$$
$$A_0 = 2(x_{20}x_{30} - x_{10}x_{30});$$
$$B_6 = 2(3)^{-1}(x_{10} - 2x_{20} + x_{30});\ B_4 = 2(2x_{20} - x_{10} - x_{30});\ B_3 = (3)^{-1}(x_{10} + x_{20} + x_{30});$$
$$B_1 = -2(x_{10} + x_{20} + x_{30});\ B_0 = -2(x_{10} + x_{30});$$
$$C_7 = x_{20}^2 - (2)^{-1}x_{10}^2 - (2)^{-1}x_{30}^2 + (2)^{-1}x_{10}x_{20} - x_{10}x_{30} + (2)^{-1}x_{20}x_{30}\,;$$
$$C_6 = 2(3)^{-1}(x_{10}^2 - x_{30}^2 - 2x_{10}x_{20} + 2x_{20}x_{30})\,;$$
$$C_4 = 2(2x_{30}^2 - 2x_{20}^2 + x_{10}x_{20} + 2x_{10}x_{30} - 3x_{20}x_{30})\,;$$
$$C_3 = (6)^{-1}(x_{30}^2 - x_{10}^2 - x_{10}x_{20} + x_{20}x_{30})\,;\ C_1 = 2(x_{20}^2 - x_{30}^2 - x_{10}x_{30} + x_{10}x_{20})\,;$$
$$C_0 = 2(x_{20}x_{30} - x_{10}x_{20}). \qquad (5.30)$$

From the above relations it follows that the discrete moments, determined by these relations, depend on the random initial conditions and therefore are random; whereas the discrete moments found for the diagonal system, are not random, determined exclusively by a given matrix $A(0)$ that characterizes an inner systems peculiarities.

1.5.3. The Consolidation and Aggregation of the Model Processes. Forming an Information Network (IN)

The fulfillment of condition (5.4) connects the extremal segments of a multi-dimensional process leading to the segment's cooperation and the corresponding state's consolidation, which reduces a number of the model independent states.

These allow grouping the cooperative macroparameters and aggregating their equivalent dimensions in an ordered hierarchical information network (IN), built on a multi-dimensional spectrum of the system operator, which is identified during the optimal motion.

The IN organization includes the following steps: arranging the extremal segments in an ordered sequence; finding an optimal mechanism of connecting the arranged segments into a sequence of their consolidating states, whose joint dimensions would be sequentially deducted from the initial model's dimension; and forming an hierarchy of the adjoining cooperating dimensions.

Below we consider the formal relations and procedure of implementing these steps, which are based on the variation and invariant conditions following from the initial VP (sec. 1.3.5).

The relations that we illustrate use the *n*-dimensional spectrum of the model starting matrix $A(t_o)$ with the different complex eigenvalues $\lambda_{io} = \alpha_{io} \pm j\beta_{io}$, characterized by the ratio $\gamma_{io} = |\beta_{io} / \alpha_{io}|, \alpha_{io} \neq 0$, $i = 1, ..., n$.

The segments' cooperation produces a chain of the matrix eigenvalues $A(t_k, t_k + o) = (\lambda_{it}^k, \lambda_{it}^k + o)$ with $\lambda_{it}^k = \alpha_{it}^k \pm j\beta_{it}^k$ and $(\lambda_{it}^k + o) = (\alpha_{it}^k + o) \pm j(\beta_{it}^k + o)$ at each segment's end and the beginning of a following segment accordingly; where $k = 1, ..., N$ is the number of the DP ($t_k + o$) where cooperation of λ_{it}^k and $\lambda_{it}^k + o$ takes place. This cooperative chain includes a sequence of eigenvalues $\lambda_{it}^k + o$ *renovated* regarding λ_{it}^k. The procedure includes the following sequence.

The cooperative grouping.

A feasible IN unites of the multiple nodes, while each its node joins a group of equal eigenvalues, gained in the cooperative process. The optimal condition (3.49), (3.160c),(3.164b) for such groups of the eigenvalues, considered at a moment of cooperation $t_k + o$, acquires the form

$$\min Tr[\lambda_{it}^k(t_k + o)] = \min[\sum_{r=1}^{m} g_r \lambda_r^g], \tag{5.31}$$

where g_r is a r-th group number with its joint eigenvalue λ_r^g, m is a total number of groups-the IN's nodes. Cooperation of the corresponding states' variables is carried by transforming the related λ_r^g according to the model equation.

Applying the invariant relations.

For building the cooperating chain of the eigenvalues, satisfying (3.170), (4.53), and (5.31), we implement Propostion 3.5.8 using both the invariant relation (3.165),(3.165a) and the condition (3.180a).

Thus, we apply the following relations:

$$|\lambda_i(t_k)| = |\lambda_j(t_k + o)| \tag{5.32}$$

for each cooperating i, j segment, whose eigenvalues, at the time interval between DP: $\Delta t_k = t_k - t_{k-1}$, satisfy the solutions

$$\lambda_i(t_k) = \lambda_i(t_{k-1})\exp(\lambda_i(t_{k-1}))\Delta t_k)[(2-\exp(\lambda_i(t_{k-1})\Delta t_k)]^{-1}, i = 1,..,k,...,n \tag{5.33}$$

(under the control (4.51) action), and fulfill the condition

$$\operatorname{Im}\lambda_i(t_k) = 0, \tag{5.34}$$

which holds true also by the moment $(t_k + o)$ of segment's cooperation;

the invariants $\mathbf{a}_o$ and $\gamma_{io} = |\beta_{io}/\alpha_{io}| = \gamma$ are connected by the equation

$$2(\sin(\gamma\mathbf{a}_o) + \gamma\cos(\gamma\mathbf{a}_o)) - \gamma\exp(\mathbf{a}_o) = 0. \tag{5.35}$$

We also use the connection of invariants $\mathbf{a}_o$, $\mathbf{a}$:

$$\mathbf{a} = \mathbf{a}_o\exp(-\mathbf{a}_o)(1+\gamma^2)^{1/2}(4 - 4\exp(-\mathbf{a}_o)\cos(\gamma\mathbf{a}_o) + \exp(-2\mathbf{a}_o))^{-1/2}, \tag{5.36}$$

following from relation $|\lambda_i^t(t_i)t_i|^2 = \mathbf{a}^2$ and the representation (5.33) via the invariants.

This allows us evaluate both invariants.

From (5.35) we get at $\gamma \to 0$ the equation

$$\exp(\mathbf{a}_o(\gamma \to 0)) = \lim_{\gamma\to 0} 2[\sin(\gamma\mathbf{a}_o)/\gamma + \cos(\gamma\mathbf{a}_o)] = 2[\mathbf{a}_o(\gamma \to 0) + 1], \tag{5.36a}$$

whose solution brings the maximal $|\mathbf{a}_o(\gamma = 0)| = 0.768$.

From the solution of (5.35) at $\gamma \to 1$ we get a minimal $\mathbf{a}_o(\gamma = 1) \to 0$, which brings also the minimal $\mathbf{a}(\gamma = 1) = 0$ from (5.36). The first one at $\gamma \to 0$ limits a maximal quantity of a real information produced at each segment; the second one at $\gamma \to 1$ restricts to a minimum the information contribution necessary for cooperation and, therefore, puts a limit on the information cooperation. It's also seen that relation (5.36c) as the function of γ reaches its extreme at $\gamma = 0$, which at $|\mathbf{a}_o(\gamma = 0)| = 0.768$, brings $\mathbf{a}(\gamma = 0) \cong 0.23193$.

Actually, a feasible admissible diapason of $\gamma_{io} = \gamma$, following from the model simulation [33], is $0.00718 \le \gamma_{io} \le 0.8$ with the condition of a model equilibrium at $\gamma = 0.5$, $\mathbf{a}_o(\gamma = 0.5) \cong \ln 2$, $\mathbf{a}(\gamma = 0.5) \cong 0.25$.

Cooperation of the model's real eigenvalues.

The cooperation of the real eigenvalues, according to (5.34), reduces the condition (5.31) to the form

$$\min[\sum_{r=1}^{R} g_r \mid \lambda_r^g \mid] = \min[\sum_{r=1}^{R} g_r \alpha_r^g], \tag{5.37}$$

where $\alpha_r^g > 0$ is a joint real eigenvalue for each group, satisfying the requirement of a positive eigenvalue α_r^g at applying the optimal control (4.50), (4.51); *R* is the total number of the cooperating real eigenvalues.

We find the number of the joint eigenvalues in a group g_r starting with a *doublet* as a minimal cooperative unit (Fig.5.4). The minimal α_r^g for the doublet with two starting real eigenvalues at $|\alpha_{io}| < |\alpha_{ko}|$ can be reached, if by the end of interval t_i when at $\alpha_{it} = \mathbf{a}_o / t_i$, the initial eigenvalue α_{ko} brings the equation (5.33) to the form

$$\alpha_k(t_i) = \alpha_{ko} \exp(\alpha_{ko}(t_i - t_{ko}))[2 - \exp(\alpha_{ko}(t_i - t_{ko}))]^{-1}, \alpha_{ko} = \alpha_{ko}(t_{ko}), \tag{5.37a}$$

whose solution $\alpha_k(t_i)$ will coincide with α_{it} by the end of the t_i duration, getting $\alpha_r^g = 2\alpha_{it}$.

The fulfillment of $|\alpha_{it}| / |\alpha_{io}| = |\mathbf{a}/\mathbf{a}_o|$ at the admissible γ and $|\mathbf{a}| < |\mathbf{a}_o|$ guarantees the decrease of both α_{it} and $\alpha_k(t_i)$ that leads to the inequalities

$$|\alpha_{it}| < |\alpha_{io}|, \; |\alpha_k(t_i)| < |\alpha_{ko}|. \tag{5.38}$$

Using a triplet as an elementary cooperative unit.

Let us consider also a triplet as an elementary group of the cooperating three segments with the initial eigenvalues $|\alpha_{jo}| < |\alpha_{io}| < |\alpha_{ko}|$, where we add the minimal eigenvalue $|\alpha_{jo}|$ of a third segment to the previous doublet (for a convenience of the comparison).

Then the minimal α_r^g can be reached (at other equal conditions) if the moment for joining the first two eigenvalues (with the initials $|\alpha_{io}| < |\alpha_{ko}|$) coincides with the moment t_j of forming the minimal $\alpha_{jt} = |\mathbf{a}_o| / t_j$ for the third eigenvalue.

In that case, the additional discrete interval is not required.

Compared with the related doublet, we have $|\alpha_{jt}| < |\alpha_{it}|$, where each minimal eigenvalue is limited by a given ordered initial spectrum.

Therefore, a minimal optimal cooperative group is a triplet with $\alpha_r^g = 3\alpha_{jt}$ (more details are in sec.5.3).

For a space distributed macromodel (ch.1.4), the minimal number of cooperating segments is three, each one for every space dimension. This also limits a total number of the cooperating segments of the *n*-dimensional state system, distributed in space.

Each of such a cooperation starts with the rotation of a dimension's eigenvalue (initiated by the dimension's information interactions) and finishes by the eiegenvalues' joining at DP locality (within each local three-dimensional system).

Arranging the model initial eigenvalues.

The selection of the triplet sequence and their arrangement into the IN is possible after ordering the initial $(\alpha_{io})_{i=1}^{n}$ in their decreasing values:

$$|\alpha_{1o}|>|\alpha_{2o}|>,....|\alpha_{io}|>|\alpha_{i+1o}|,.....>|\alpha_{no}|. \tag{5.39}$$

Applying the needle controls at the moment of cooperation (for example at $(t_i + o)$ for the doublet) takes place when, in addition to the real eigenvalue equalization: $\alpha_i(t_i + o)|=|\alpha_k(t_i + o)|$ and reaching a minimum among the sum of the egenvalues, prior the cooperation:

$$|\alpha_i(t_i + o)|+|\alpha_k(t_i + o)|=2|\alpha_i(t_i + o)|=|\alpha_i^g|=\min(|\alpha_i(t_i)|+|\alpha_k(t_k)|), \tag{5.40}$$

the cooperated sum also satisfies a maximum condition regarding any sum of the following two eigenvalues:

$$2|\alpha_k(t_i + o)|=|\alpha_i^g|=\max[|\alpha_{i+1}(t_{i+1})|+|\alpha_{i+2}(t_{i+2})|], \tag{5.41}$$

because each joint eigenvalue should bring a decrease to the ordered sequence of the eigenvalues that has already been formed.

Since for the ranged $(\alpha_{io})_{i=1}^{n}$ the conditions (5.40), (5.41) are satisfied, also the following relations are fulfilled:

$$(|\alpha_{i-1o}|+|\alpha_{io}|)=\max[|\alpha_{i+1o}|+|\alpha_{i+1o}|], \text{ as well as } |\alpha_{io}|=\max[|\alpha_{i+1o}|]. \tag{5.41a}$$

The formalization of this procedure leads to a minimax representation of eigenvalues by the Kurant-Fisher variation theorem [62], which brings the condition of a sequential arrangement for the macromodel eiegenvalues' spectrum (Details are in sec.1.5.3).

The procedure leads to forming a monotonous sequence of the eigenvalues (5.39), which actually follows from the initial VP application.

Forming the IN.

There are two options in forming the IN:

(1)-identify the IN by collecting the current number of equal α_r^g for each cooperative group g_r -as an IN node, and then arranging these nodes into a whole IN;

(2)-building an optimal IN by collecting the triplet's $\alpha_r^{g=3}=\alpha_r^3$ and using the invariant relations (5.35),(5.36).

The sequence of the cooperating ordered eigenvalues $\alpha_r^3, r=3,5,7,..m$ moves to its minimal α_m^3 with the IN minimal dimension for a final node $n_{mo}\to 1$.

The cooperations and assigning to each triple its joint eigenvalue bring the eigenvalues' sum to its minimum: $\min \sum_{r=3}^{m} \alpha_r^3 = \alpha_m^3$.

The sequential cooperation of the ordered eigenvalues by triples, leads to repeating of the initial triplet's cooperative process for each following triplet with the preservation of two basic eigenvalues ratios:

$$\gamma_1^{\alpha}(\gamma) = \frac{\alpha_{1o}}{\alpha_{2o}} \to \frac{\alpha_{io}}{\alpha_{i+1o}}, \gamma_2^{\alpha}(\gamma) = \frac{\alpha_{2o}}{\alpha_{3o}} \to \frac{\alpha_{i+1,o}}{\alpha_{i+2,o}} ,$$

satisfying the equations

$$\gamma_1^{\alpha} = \frac{\exp(\mathbf{a}(\gamma)\gamma_1^{\alpha}\gamma_2^{\alpha}) - 0.5\exp(\mathbf{a}(\gamma))}{\exp(\mathbf{a}(\gamma)\gamma_2^{\alpha}) - 0.5\exp(\mathbf{a}(\gamma))}, \gamma_2^{\alpha} = 1 + \frac{\gamma_1^{\alpha} - 1}{\gamma_1^{\alpha} - 2\mathbf{a}(\gamma)(\gamma_1^{\alpha} - 1)}, \qquad (5.42)$$

where parameter γ is found from relation (5.35) at the known $\mathbf{a}_o$, and $\mathbf{a}(\gamma)$ from (5.36).

The preservation of $\gamma_i^{\alpha}, i = 1,2$ along the IN triplet's chain defines the IN's common parameter of γ that allows us to calculate the invariant quantity of information $\mathbf{a}(\gamma)$ transferred to each following triplet's eigenvalue (at every t_i).

Thus, the IN node collects information

$$\mathbf{a}(\gamma)(1 + (\gamma_1^{\alpha})^{-1} + (\gamma_2^{\alpha})^{-1}) = g(\gamma), \qquad (5.42a)$$

which is preserved at fixed γ.

The microlevel's information, generated at each $o(\tau_i)$-locality, evaluated by the invariants (3.176),(3.177):

$$S(\tau_i) = |\mathbf{a}_o(\gamma_i)| - |\mathbf{a}(\gamma_i)| \cong \mathbf{a}_o^2(\gamma_i) \qquad (5.42b)$$

is compensated by the optimal control's action at $\mathbf{a}_o(\gamma_i) \cong |\mathbf{a}(\gamma_i)| + \mathbf{a}_o^2(\gamma_i)$ (For details see ch.1.6).

Each triplet's discrete point is formed by a joint solution of the three local extremal equations that could be nonlinear. Such points are singular with a possibility of all kinds of chaotic dynamic phenomena. The controllable cooperative dynamics (that involve the chaotic resonances) automatically develop the IN hierarchical dynamic and geometrical organization through the enclosed sequence of the triplet's structures.

Each IN's node-triplet, cooperating in chaotic dynamics, in particular, memorizes a chaotic attractor.

Building the IN during a real-time *process follows the same procedure.*

In addition to that, fixing the real-time sequence of the time intervals allows selecting the corresponding sequence of the related IN nodes with their codes' sequence, which thereby establishes a current IN's genetic code (ch.1.6).

The real-time IN, possessing the structure of an initial IN, which is built on the *ordered* sequence of the same eigenvalues, would have the current code distinctive from the initial one.

Considering a multi-dimensional macroprocess, let us start the macromodel's cooperative examples with an initial pair of extremal segments.

At a first extremal segment, the equations (3.154),(3.161) have the solution

$$x_1(t) = x_1(t_o)(2 - \exp \lambda_1(t)t),$$

whose current eigenvalue $\lambda_1(t)$ (at a fixed

$$\lambda_1(t_o) = \lambda_1(t = t_o) = \lambda_{1t10})$$

is changing under the control's action according to

$$\lambda_1(t) = -\lambda_{1t10} \exp(\lambda_{1t10}t)(2 - \exp(\lambda_{1t10}t))^{-1}, \lambda_{1t10} = \lambda^{\alpha}{}_{1t10} + j\lambda^{\beta}{}_{1t10}, \qquad (5.43)$$

acquiring by the segment's end the value

$$\lambda_1(t = t_1) = \lambda_{1t1} = -\lambda_{1t0} \exp(\lambda_{1t10}t_1)(2 - \exp(\lambda_{1t10}t_1))^{-1}. \qquad (5.43a)$$

The second extremal segment starts with macrostates $(x_1(t_1 + o), x_1(t_1))$ and eigenvalue $\lambda_1(t_1 + o) = \lambda_{1t20}$ that is able to cooperate with λ_{1t1}, while $\lambda_1(t_1 + o) \cong \lambda_{1t20}$ by the end this segment reaches the value

$$\lambda_1(t = t_2) = \lambda_{1t2} = -\lambda_{1t20} \exp(\lambda_{1t20}t_2)(2 - \exp(\lambda_{1t20}t_2))^{-1}, \qquad (5.43b)$$

which might cooperate with the starting eigenevalue $\lambda_1(t_2 + o) \cong \lambda_{1t30}$ of the third segment, and so on for each *i,k:*

$$\lambda_i(t_k + o) \cong \lambda_{itk+1,0}. \qquad (5.44)$$

Each extremal segment should preserve invariants

$$\mathbf{a}_o = \lambda^{\alpha}{}_{1t10}t_1 = \lambda^{\alpha}{}_{1t20}t_2, ..., \text{ and } \mathbf{b}_o = \lambda^{\beta}{}_{1t10}t_1 = \lambda^{\beta}{}_{1t20}t_2, ..., \qquad (5.44a)$$

(where $\lambda^{\alpha}, \lambda^{\beta}$ indicate the real and imaginaty components of the model eigenvalues), even though the eigenvalues within the segment are changing by the control according to (5.43),(5.43a,b). (At $\lambda^{\beta}{}_{1t10} / \lambda^{\alpha}{}_{1t10} = \gamma_1$, the eigenvalue $\lambda_{10} = (\lambda^{\alpha}{}_{1t10}, \lambda^{\beta}{}_{1t10})$ is defined by the given $\lambda^{\alpha}{}_{1t10}$ and γ_1.)

From (5.44a) we find the first segment's time interval t_1 at fixed $\lambda^{\alpha}{}_{1t0}$, and the second time interval t_2 at the known $\lambda^{\alpha}{}_{1t1}$, and so on. It also determines such $\lambda_1(t_1 + o) \cong \lambda_{1t20}$ with whom λ_{1t1} is able to cooperate, and so on. This leads to building a chain of the sequentially cooperating extremal segments satisfying the VP, which start with $x_1(t_o)$, λ_{1t0} of the dimension *i*=1. The analogous chains can be built for other macromodel's starting eigenvalues with the cooperative segment's connections within each chain.

The segment's time interval is determined by sequentially applying relations (5.43a,b), and (5.44a). The macromodel's initial eigenvalues spectrum $\{\lambda_{1o}, \lambda_{2o}, \lambda_{3o}, ..., \lambda_{no}\}$ is determined for the chain of cooperating eigenvalues by the following the relations

$$\lambda_{1o} = \lambda_{1t0},\ \lambda_{2o} = \lambda_{1t20}, \lambda_{3o} = \lambda_{1t30}, \ldots,\ \lambda_{no} = \lambda_{1tn0}, \tag{5.44b}$$

where the temporal equations (5.43a) are fulfilled for each spatial dimensional sequence (5.44b). This means that such a spectrum can be built *directly* starting with λ_{1t0} by the use of the above equations. In this case, the invariant relations (5.44a) allocate the length of each segment belonging to different state's dimensions, which are cooperating sequentially each with other by forming a between dimensional cooperative chain.

Let us analyze the effect of control action on changing the model's eigenvalues, beginning with a single dimensional model and comparing both cases with the applied control and without it. Both equations:

$$\dot{x}_1(t) = \lambda_{1t10}(t)x_1(t) + v_1,\ v_1 = 2x_1(t_o),\ \dot{x}_1(t) = \lambda_1(t)x_1(t)$$

are equivalent at $\lambda_1(t)$ satisfying (5.43a).

This means that without control (at v_1 =0), we have $\lambda_1(t_1) = \lambda_{1t10}$.

The control v_1, applied at $t = t_o$ during the time $t = t_1$ brings the eigenvalue increment

$$\Delta\lambda_{1t1} = \lambda_{1t10} - \lambda_{1t1} = \lambda_{1t10}(1 + \exp(\lambda_{1t10}t_1)(2 - \exp(\lambda_{1t10}t_1))^{-1}, \tag{5.45}$$

which reduces the ratio of the initial eigenvalue:

$$\Delta_{v1} = \lambda_{1t1} / \lambda_{1t10} = -\exp(\lambda_{1t10}t_1)(2 - \exp(\lambda_{1t10}t_1))^{-1}, \tag{5.45a}$$

and increases the relative eigenvalue's contribution from the control:

$$\Delta^{\Delta}_{v1} = \Delta\lambda_{1t1} / \lambda_{1t10} = (1 + \exp(\lambda_{1t10}t_1)(2 - \exp(\lambda_{1t10}t_1))^{-1}. \tag{5.45b}$$

Considering separately the real and imaginary eigenvalue's parts and using the invariant relations, we get $\Delta_{v1} = inv$, as well as the corresponding *information* contribution $\Delta\lambda_{1t1}t_1 = inv$, which is a negative for a stable process with a negative eigenvalue λ_{1t10} <0, while λ_{1t1} (at the same sign of λ_{1t10}) brings a positive information contribution $\lambda_{1t1}t_1 = inv > 0$. So we have

$$\Delta^{\Delta}_{v1} = inv > 0, \Delta_{v1} = inv < 0. \tag{5.46}$$

These invariants determine a sequential decrease of both the initial eigenvalue along the *whole* cooperating chain and the eigenvalues invariant's ratio at each segment, accompanied by the invariant positive information contribution from the segment's controls.

The above regularities hold true for any cooperating chain satisfying (5.45)(5.45a).

Applying (5.46) to relations (5.45a), (5.45b) by considering only the real eigenvalue's ratios $|\lambda^{\alpha}_{io} / \lambda^{\alpha}_{i+1,o}| = \gamma^{\alpha}_{io}, i = 1, 2, \ldots n$, we get the invariant ratios

$$\gamma^{\alpha}_{io} = (\Delta^{\alpha}_{vi})^{-1} = inv, i = 1, \ldots, n, (\Delta^{\alpha}_{vi})^{-1} = |\, 2 / \exp \mathbf{a}_o\,(\gamma)\,|, \tag{5.47}$$

evaluating the information *distance* between the starting eigenvalues, which are acceptable for the following cooperations.

For example, at $\mathbf{a}_o \approx -\ln 2$ we get $\gamma_{io}^{\alpha} \approx 3$, which determines the sequential decrease the real eigenvalues forming the cooperative chain.

These invariant relations follow from the path functional's VP, which, being applied to the chain, forms the functional's "macropath" across the segments.

The needle control in this case:

$$v_1(\delta) = v_1(t_1) - v_2(t_{2t20}) \text{ at } v_2(t_{2t20}) \cong v_1(t_1 + \delta) \tag{5.48}$$

is formed by the controls from both connecting segments.

While the control v_1 delivers a positive information within a segment, the needle control spends the collected information (5.42b,c) on the segment's cooperation.

Using the control's information contributions and the related invariants, we express the information, collected from the previous segments' end and the following segment's start, in the invariant form:

$$\lambda_{1t}^{\alpha} t_1 = \mathbf{a}(\gamma_1),\ \lambda_{1t20}^{\alpha} t_{1t21} = -\mathbf{a}(\gamma_2), \tag{5.49}$$

where

$$\lambda_{1t}^{\beta} / \lambda_{1t}^{\alpha} = \gamma_1, \lambda_{1t20}^{\beta} / \lambda_{1t20}^{\alpha} = \gamma_2, sign\lambda_{1t}^{\alpha} = -sign\lambda_{1t20}^{\alpha}, \text{ or } \lambda_{1t20}^{\alpha} = -\lambda_{1t}^{\alpha}(t_1 + \delta). \tag{5.49a}$$

This brings the needle control's summary information spent on cooperation in the invariant form

$$v_1(\delta)_{\text{inf}} = \mathbf{a}(\gamma_1) + \mathbf{a}(\gamma_2). \tag{5.50}$$

In particular, at $\gamma_1 = \gamma_2 = 0.5$ we have $v_1(\delta)_{\text{inf}} \approx 0.5$, which coincides with (5.42c) at

$$\mathbf{a}_o \approx -\ln 2.$$

Thus, the model eigenvalue's spectrum, admissible for the cooperations, could not be an arbitrary.

The cooperating eigenvalues' ratio γ_{io}^{α} *is equal to the ratio of time intervals* $t_{i+1,o} / t_{io}$ *at a fixed invariant* **a**.

This means that at the given λ_{io}, t_{io}, the nearest eigenvalue $\lambda_{i+1,o}$ (or the related frequency) settles on as an equivalent of some "time delay" in delivering information between these starting segments.

In other words, the macromodel eigevalues are capable of assembling into a cooperative, if its information frequencies operate with the sequential delay, determined by the time intervals required for the following cooperations.

To satisfy this requirement the cooperative model arranges its starting eigenvalues (frequencies) according to an external communication providing the necessary time delays.

The above requirement is not a mandatory, but one of optional realization of the formal cooperative procedure.

The condition of binding the cooperative model's spectrum is imposed also on the formulas (5.42), which connect the IN's spectrum ratios with the model's invariants.

For *example*, suppose we know the triplet first segment's $\lambda_{io}^{\alpha} = \alpha_{io}, t_i$ and need to determine the time delays $\Delta t_{i,i+1}$ between the subsequent segments' t_i and t_{i+1} and $\Delta t_{i+1,i+2} = t_{i+1} - t_{i+2}$ accordingly, which depend upon these segments' $\lambda_{i+1}, t_{i+1}, \lambda_{i+2}, t_{i+2}$.

Using $\alpha_{io} t_i = \mathbf{a}_o(\gamma)$ and formula (5.36a) we find γ and then, applying (5.42), we obtain $\gamma_1^{\alpha}, \gamma_2^{\alpha}$. From that we get $\alpha_{i+1,o} = \alpha_{io} / \gamma_1^{\alpha}, \alpha_{i+2,o} = \alpha_{i+1,o} / \gamma_2^{\alpha}$, which at the known γ and the above invariant determine $\lambda_{i+1}, t_{i+1}, \lambda_{i+2}, t_{i+2}$ and then the time delays

$$\Delta t_{i,i+1}, \Delta t_{i+1,i+2} = t_{i+1} - t_{i+2}.$$

The vice versa operations are also true: starting from known t_i and given time delays, we determine t_{i+1}, t_{i+2} and then $\gamma_1^{\alpha}(\gamma) = t_i / t_{i+1}, \gamma_1^{\alpha}(\gamma) = t_{i+2} / t_{i+1},$ to find γ, the invariant and finally get $\alpha_{io}, \alpha_{i+1,o}, \alpha_{i+2,o}$.

Example. Let us evaluate the chain of the segment's real eigenvalues $\lambda^{\alpha}{}_{1ti}$ and the corresponding time intervals t_i using just the invariants

$$\mathbf{a}_o = \lambda^{\alpha}{}_{1t10} t_1 = \lambda^{\alpha}{}_{1t20} t_2, ..., \lambda^{\alpha}{}_{1ti0} t_i \approx |0.7|;\ \mathrm{a} = \lambda^{\alpha}{}_{1t1} t_1 = \lambda^{\alpha}{}_{1t2} t_2, ..., \lambda^{\alpha}{}_{1ti} t_i \approx |0.25|, \gamma = 0.5$$

at the sequential cooperation of the each segment's ending eigenvalues with each subsequent segment's starting eigenvalue $\lambda^{\alpha}{}_{1ti} = \lambda^{\alpha}{}_{1ti+10}$, i=1,2…5. At $\lambda^{\alpha}{}_{1t10}$=1 we get the following sequence of $\lambda^{\alpha}{}_{1ti}$: 0.357, 0.127, 0.0453, 0.0162, 0.00578 with the ratio $\gamma_{io}^{\alpha} \approx 2.83$ and the corresponding time intervals t_i: 0.7, 1.96, 5.52, 15.452, and 43.3 with the same ratio.

Therefore, by giving two numbers: γ and $\lambda^{\alpha}{}_{1t10}$ (and using the invariants depending on γ) we are able to uncover a whole path functional's trajectory with the set of cooperative states and the segment's time intervals.

The set of the states identifies a number of the ordered events covered by the information invariant and defined by the model's elementary quantity of information.

This number is limited by a total time T of the considered process for a given path functional, which restricts the related path of the IPF.

For a multidimensional process, this total time also restricts a maximal dimension of the cooperating local ordered processes with the related cooperative states and the segments time intervals (whose sum is limited by the T).

1.5.4. The IN Dynamic Structure

Let us first formulate and summarize the restrictions, imposed on the model's eigenvalue that cooperate into IN, taking into account a minimal path (MP) to each IN's node, which implements a local IPF and is characterized by a minimal time interval movement to the node along the sequence of extremal segments.

Based on the considerations in sec.1.5.2, the restrictions are:

1) the starting eigenvalues $\{\lambda_{io}\}, i = 1,...,n$, determined by the model's complex operator spectrum, are different;

2) the condition of joining of the eigenvalue's real parts requires vanishing of these eigenvalue's imaginary parts, taking place by the end of the starting time interval $t_i = \mathbf{a}_o / \alpha_{io}$ (for each cooperating segment).

The third restriction (3) we formulate by

Proposition 5.1.

To satisfy the MP and a stability of the cooperated segments, the components of the extremal spectrum should be assembled sequentially into the elementary units, composed by a pair of eigenvalues (doublets) and/or by a triple eigenvalues (triplets).

Let us show *primarily* that for each segment's cooperation, a minimal time interval of the cooperation is limited by the segment with a minimal eigenvalue (having a maximal segment's time interval).

Indeed. For a pair cooperation of *i*,*k*-extremal segments with the real eigenvalues α_{io} and α_{ko} accordingly, at $\alpha_{ko} > \alpha_{io}$, we get the segment's time intervals $t_k = |\mathbf{a}_o| / |\alpha_{ko}|$,

$t_i = |\mathbf{a}_o| / |\alpha_{io}|$ with $t_k < t_i$, where $t_i = t_{i\min}$ limits the cooperation, determined by a minimal real eigenvalue $\alpha_{io} = \alpha_{o\min}$.

This means that for the eigenvalue $|\alpha_k(t_k)| = |\mathbf{a}| / t_k > |\mathbf{a}| / t_{i\min} = |\alpha_i(t_{i\min})|$ to cooperate with $\alpha_i(t_{i\min})$, the minimal additional time $\Delta t_k = t_{i\min} - t_k$ is needed, while the total time necessary for the segments' cooperation cannot be less than $t_{i\min}$.

Joining $\alpha_k(t_k)$ with $\alpha_i(t_{i\min})$ requires applying the control to the *k*-extremal segment, which initiates the dynamic process

$$\alpha_k(t_{i\min}) = -\alpha_k(t_k)\exp(\alpha_k(t_{i\min} - t_k))[2 - \exp(\alpha_k(t_{i\min} - t_k))]^{-1}, \quad (5.51a)$$

leading to the coincidence of $\alpha_k(t_{i\min})$ and $\alpha_i(t_{i\min})$ by the end of $t_{i\min}$.

Adding a third real eigenvalue to this doublet, having a lesser eigenvalue $\alpha_{jo} < \alpha_{i\min} < \alpha_{ko}$, brings $t_j = |\mathbf{a}_o| / |\alpha_{jo}|$, at $t_j > t_{i\min} > t_k$, that only increases the doublet's time.

Joining these three segments according to process (5.51a) by applying the controls to $\alpha_k(t_{i\min}) = \alpha_i(t_{i\min})$ at the moment to $t_{i\min}$ or to any $\alpha_j(t_j)$ will add $\Delta t_j = t_j - t_{i\min}$.

In this case, the minimal time interval $t_{j\min}$ of the triple cooperation is limited by a minimal eigenvalue among all three segments: $\alpha_{jo} = \alpha_{j\min}$, while the MP for this triple's cooperative

process requires finishing the doublet's cooperation by the end of the $t_{j\min} = |\mathbf{a}_o| / |\alpha_{j\min}|$, Fig.5.4.

Adding a forth ordered eigenvalue *not only increases* a total minimal time cooperation, but will make the cooperation of this quadruple impossible, because the cooperation of more than three elements becomes *unstable* (with the unlimited trajectories [67] at such an interaction).

Secondary. At any ordered eigenvalue's manifold with m simultaneous and potentially stable cooperations, the MP would not exceed the time interval limited by a minimal final eigenvalue α_{mo} in this manifold. The cooperation, performed sequentially by triple or double, will keep the same MP preserving the stability at each of such elementary cooperation.

For example, at m =4, the two sequential double cooperations require the minimal time interval limited by α_{4o}, which is the same if the simultaneous cooperation of these four segments would be stable. For m =5, the two triple sequential cooperations occur (Fig. 5.4) during the time interval limited by minimal α_{5o}, which is the same if the simultaneous cooperation of these five segments would be stable.

Therefore, the MP is achieved for an elementary cooperative unit with a *maximum* of *three* eigenvalues (with $t_{j\min}$), where for a space distributed macromodel, a *minimal* number of cooperating segments is three, each one for every space dimension.

Besides that, the triple cooperation is able to deliver an elementary quantity of information contribution in 1 bit (sec.1.6). •

In addition to requirements (1-3) we request also

4)-the needle controls (secs.1.3.4, 1.3.5) have to be applied at the moments, when, with the fulfillment of (5.35) (in form of $|\alpha_k(t_{i\min})| = |\alpha_{io\min}|$), the conditions (5.40)-(5.41a) of the VP's maximum are satisfies.

The restrictions (1-4) lead to a sequential choosing such $\alpha_{i\min}$, which holds a maximum among all previously chosen eigenvalues from a spectrum, implementing thereby condition

$$\alpha_{i\min} = \min\max \alpha_i, \text{ or } \alpha_{i\min} = \max\min \alpha_i, \qquad (5.51b)$$

where each α_i is limited by the segment's invariant $\mathbf{a}_o$ and starting α_{io}.

Such a search (implementing the VP) will also order the spectrum's α_{io} sequence.

The above strategy we specify in the following statements.

Proposition 5.2.

The minimax condition for the macromodel eiegenvalues' spectrum leads to the spectrum sequential ordering that brings a monotonous sequence of the eigenvalues: $|\lambda_n| > ,\ldots > |\lambda_2| < \lambda_1|$ (with a minimal $|\lambda_n|$).

The result follows from the Kurant-Fisher's variation theorem [62] by a successive applying of the maximum condition to the minimal condition for the Relay's relation $q(x) = \dfrac{(x, Ax)}{(x, x)}$ (for the macromodel's matrix $A > 0$).

This leads to $|\lambda_i| \geq \frac{(x, A_t x)}{(x, x)} \geq |\lambda_{i+1}|, \ldots,$ or (in the indication of sec.1.5.2) to $|\alpha_{i-1}^g| > |\alpha_i^g| > |\alpha_{i+1}^g|$.

The geometrical meaning illustrates an ellipsoid, whose axes represent the model's eigenvalues. The method starts with the maximal eigenvalue $|\alpha_1(t_1)| = \max_x q(x)$, taken from a maximal axis of the ellipsoid's orthogonal cross section, that is rotating up to reaching a minimal axis of the ellipsoid's cross section, which should coincide with the following lesser eigenvalue $|\alpha_2(t_2)| < |\alpha_1(t_1)|$, and so on.

Because the model works with ranging the current $|\alpha_i(t_i)|$, the procedure brings also a monotonous sequence of the starting eigenvalues of the model's spectrum.

Each model's eigenvalue is an equivalent to a local average entropy Hamiltonian according to the VP.

The maximal part of the VP principle selects the maximal eigenvalue, corresponding to the maximal ellipsoid's cross section, while the minimal part selects an eigenvalue from a minimal axis of the ellipsoid's cross section in the current eigenvalue sequence.

This leads to alternating the sequence of min max $q(x)$ along the macrotrajectory model's segments, satisfying to the Kurant-Fisher's variation theorem.

Therefore, the VP application *automatically* orders the model's eigenvalues, executing the Kurant -Fisher's variation theorem. •

Proposition 5.2a.

For the macromodel with the complex ranged eigenvalues

$$\lambda_{i,o}^t = \alpha_{i,o}^t \pm j\beta_{i,o}^t, \lambda_{i+1,o}^t = \alpha_{i+1,o}^t \pm j\beta_{i+1,o}^t \text{ at } |\alpha_{i,o}^t| > |\alpha_{i+1,o}^t|:$$

(1)-the condition of the eigenvalues ordering by applying the equations for invariants $\mathbf{a}_o$, $\mathbf{a}$, $\mathbf{b}_o$ (5.34), brings the invariant relation for the eigenvalue's ratios as well:

$$\gamma_{i,m}^{\alpha^t} = \frac{\alpha_{i,m}^t}{\alpha_{i+1,m}^t} = \gamma_{i,m}^{\alpha} = Const(i, m, j); \tag{5.52}$$

(2)-the joint fulfillment of restrictions (1-6) with relation (5.52) allow cooperating of the nearest $(i, i+1)$ eigenvalues; that leads to a *minimal sum* of the discrete intervals (t_i, t_{i+1}) for each of the $(i, i+1)$ eigenvalues' couples, which can be reached if the discrete moment for joining of the first couple (for which $|\alpha_{i,o}^t| > |\alpha_{i+1,o}^t|$) *coincides* with the discrete moment for the second couple. Then an additional discrete interval is not required, satisfying the MP. (This result is applied for the joint solutions of the ranged IN eigenvalues spectrum at $|\alpha_{i,o}^t| > |\alpha_{i+1,o}^t|$, sec. 1.5.2);

(3)-the requirement of coinciding the eigenvalue $\alpha_i^t(t_i) = \alpha_i^t(\gamma, \alpha_{i,o}^t), \gamma = \dfrac{\beta_{io}^t}{\alpha_{io}^t}$ with $\alpha_{i+1}^t(t_{i+1}) = \alpha_{i+1}^t(\gamma', \alpha_{i+1,o}^t),\ \gamma' = \dfrac{\beta_{i+1,o}^t}{\alpha_{i+1,o}^t}$

at the condition

$$\gamma_i^{\alpha}(\gamma) = \frac{\alpha_{i,o}^t}{\alpha_{i+1,o}^t},$$

leads to

$$\gamma = \gamma' .\bullet \tag{5.52a}$$

Comments 5.1. At the cooperation of each of three sequential eigenvalues' spectrum (having the complex eigenvalues$\{\lambda_{i,o}^t\}$ at $|\alpha_{i,o}^t| > |\alpha_{i+1,o}^t| > |\alpha_{i+2,o}^t|$) and the fulfillment of (5.40)-(5.41a), the minimal sum of the discrete intervals (t_i, t_{i+1}, t_{i+2}) is achieved.

In this case, the moment (t_{i+2}) of the disappearance of the imaginary part of $\lambda_{i+2}^t(t_{i+2})$ coincides with the discretization moments, found from the solution for the i-and $(i+1)$-couple equations, when the real eigenvalues are equalized.

The total consolidation time of the considered triple does not exceed the interval of t_{i+2} defined by the third dynamic equation. This brings the following relations:

$$\alpha_{i+1}^t(t_{i+2}) = \alpha_{i+1}^t(\gamma', \alpha_{i+1,o}^t) = \alpha_{i+2}^t(\gamma'', \alpha_{i+2,o}^t) = \alpha_{i+2}^t(t_{i+2}), \tag{5.53a}$$

$$\alpha_i^t(t_{i+2}) = \alpha_i^t(\gamma', \alpha_{i,o}^t) = \alpha_{i+2}^t(\gamma'', \alpha_{i+2,o}^t) = \alpha_{i+2}^t(t_{i+2}), \tag{5.53b}$$

satisfying the requirements of the coincidence at

$$\gamma = \gamma' = \gamma''. \tag{5.54}$$

Figs. 5.4, 5.5a illustrate the above results.

Comments 5.2. The model's simulation [33] shows that the above condition for the eigenvalues' cooperation is sustained if the parameter γ is changed in the diapason $\gamma = (0.0 - 0.8)$. The system reaches an equilibrium at $\gamma = 0.5$, $\mathbf{a}_o(\gamma = 0.5) \cong \ln 2$, $\mathbf{a}(\gamma = 0.5) \cong 0.25$. Because the entropy functional is expressed in natural logarithmic measure, the above invariants values get the information measures (in bits): $\mathbf{a}_o(\gamma = 0.5) \cong 1$ bit and $\mathbf{a}(\gamma = 0.5) \cong 0.36$ bit. That is why all invariant's relations, applied (3.179a), hold above information measure.

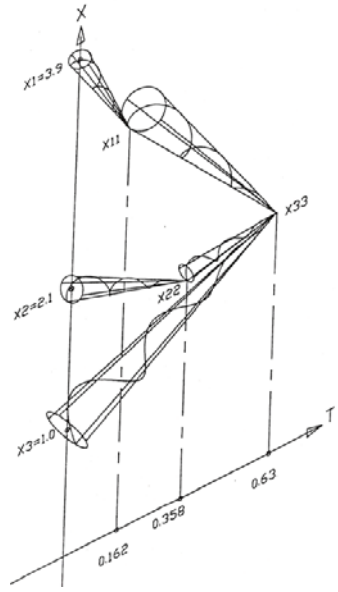

Figure 5.4. Forming a triplet's space structure.

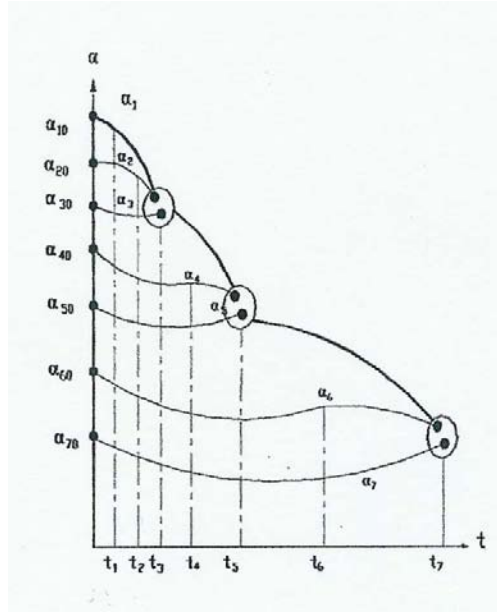

Figure 5.5. The equalization of the model's eigenvalues for the corresponding eigenvectors during the optimal movement with the triplet node's formation at the localities of the triple cones vertexes' intersections; $\{\alpha_{io}\}$ is a ranged string of the initial eigenvalues, which are cooperating (during the time dynamics) into the triplets, formed around the (t_1, t_2, t_3) locations.

Proposition 5.3.

The sequential cooperation of the ranged eigenvalues by threes (Fig.5.4) leads to forming the macromodel's information hierarchical cooperative network (IN) (Fig.5.5), where the IN's

nodes model the joint segments information contributions in the process of building a current optimal model with a successful equalization of the eigenvalues' spectrum and the consolidation of them into the final node's eigenvalue $\bullet$.

The result is supported by the following statements.

Proposition 5.4.

If the model's phase coordinates $\{z_i\}$ at the initial moment t_o are connected by relations

$$z_{i-1}(t_o)=\lambda^t_i(t_o)\ z_i(t_o), \tag{5.55}$$

then at the moment t_i the following equation for the phase coordinates:

$$z_{i-1}(t_i) = -2\dot z_i(t_i) \tag{5.56}$$

holds true with the relative accuracy, depending on γ:

$$\frac{\delta z_i}{z_i}(t_{i-1}) = \varepsilon(\gamma)=\exp[|\mathbf{a}_o|(\gamma^\alpha_{i,m}-1)]-2(1-|\frac{\mathbf{a}_o}{\mathbf{a}}|),\ \varepsilon \sim \Delta t^* = \frac{\delta t_i}{t_i}, \tag{5.57}$$

which at γ=0.5, $\mathbf{a}(\gamma)$, $\mathbf{a}_o(\gamma)$, $\gamma^\alpha_{i,m}(\gamma)$ does not exceed ε_i=0.0095-0.112.

The result follows from relations

$$\frac{x_i(t_i)}{x_{i-1}(t_{i-1})} = \frac{x_i(t_o)}{x_{i-1}(t_o)}, x_{i-1}(t_i) = x_{i-1}(t_{i-1})(2-\exp\lambda^t{}_{i-1}(t_i-t_{i-1})), \frac{\lambda^t_i}{\lambda^t_{i-1}} = \gamma^\alpha_{i,i-1}=inv,$$

$$\mathbf{a}(\gamma)=\alpha^t_{i-1}t_{i-1}=inv, \tag{5.58}$$

while the initial condition (5.56) can be fulfilled with the accuracy

$$\varepsilon_i=\frac{\delta x_i}{x_i}(t_{i-1}), \delta x_i(t_i) = 2\dot x_i(t_i)-x_{i-1}(t_i) = 2\ \lambda^t_i\ x_i(t_i)\ -\ 2x_{i-1}(t_{i-1})+$$

$$x_{i-1}(t_{i-1})\exp(-\lambda^t_{i-1}(t_i)t_{i-1})\exp(\lambda^t_{i-1}(t_i)t_i). \tag{5.59}$$

The last equation, after substituting (5.58), is reduced to the form (5.55), and it defines the limited accuracy's values in (5.57) at γ=0.5. $\bullet$

Comments 5.3. With growing γ, the error of performing the cooperation by the controls (ch.1.3) (that are coping the macrostates at τ_i), is enlarged: and at

$\gamma \to 1,\ |\frac{\mathbf{a}_o}{\mathbf{a}}| \to 1,\ \gamma^\alpha_{i,m} \to 1$ the error leads to $\varepsilon \to 1$.

This increases uncertainty of the cooperation, diminishes the binding of the eigenvalues' connections, and finally at γ=2.2 leads to the macromodel's decay.

From the connection of the first of two eigenvalues, we have $\gamma^\alpha_{1,2}$ =2.21 (at $\gamma = 0.5$) and $\varepsilon_1 \le 0.078$. By adjoining of the third of the ranged eigenvalue, we get $\gamma^\alpha_{2,3} = 1.75$ (at $\gamma = 0.5$), and $\varepsilon_2 \le 0.07$.

The errors are the same for joining each following eigenvalue of the *ranged* spectrum.

If the relative difference between their initial eigenvalues ratios $\gamma_{i,m}^{\alpha}$ is growing, the error is enlarging essentially.

The equality (5.55) is fulfilled with a zero error, if the initial conditions are bound by the relations

$$\dot{x}_i(t_o) = k_i x_{i-1}(t_o),\ k_i = \frac{\mathbf{a}_o}{2\mathbf{a}}(2 - \exp(\gamma_i^{\alpha}\mathbf{a}_o)) = inv\ , \tag{5.60}$$

or the discrete moments are defined from condition

$$t_i^{\,2} = t_{i-1}t_{i+1}\ . \tag{5.61}$$

The result is proved by a direct substitution of (5.60) into (5.57).

For the optimal model with t_i=2.21t_{i-1}, t_{i+1}=3.79t_{i-1}, γ=0.5, the admissible relative time's accuracy of the fulfillment (5.61) is equal to $\Delta t_i^* = \frac{\delta t_i}{t_i}$.

With this accuracy, or at the fulfillment (5.60), the self-forming of the optimal controls is possible:

$$u(t_i) = \lambda_i^t(t_i)v_i(t_i) = -2\lambda_i^t(t_i)z_i(t_i) = z_{i-1}(t_i)\,, \tag{5.62}$$

using the process $z_{i-1}(t_i)$ at the discrete moments t_i as a control $u(t_i)$, while the fixed values of the initial $z_i(t_o)$ perform the function of the applied starting control $v(t_o)$.

The obtained invariant equations simplify the problem solution for the self-control.

Comments 5.4. The system of equations (5.42), (5.51)-(5.54), and (5.62) allow the restoration of the macrodynamics by knowing the invariants, whose finding also use also use relations $\dot{r}_i\, r_i^{-1} = 2\alpha_i^t$, where $r_i\ (t) = E[\tilde{x}_i(t)^2]$, $i = 1,...,n$ are the covariation functions, identified by the observation of the microlevel's process $\tilde{x}_i(t)$.

Finding the α_i^t sequence determines γ_1^{α}, γ and the above invariants, which bring the values of the τ_i intervals.

This sets up processes $x_i(t) = F(x_{io}, t, \tau_i, \lambda_i^t(t))$, where $x_{io} = \tilde{x}_i(s)$, and $\lambda_i^t(t)$ is found from (5.32),(5.33), while the cooperative optimal control is found from (5.62).

The implementation of the above equations leads to a creation of the successively integrated information macrostructures, which accompany increasing of t_i and decreasing of

$$\alpha_i^t(t_i) = \frac{t_{i+1} + t_{i-1} - 2t_i}{(t_{i+1} - t_i)(t_i - t_{i-1})} \geq \frac{2\alpha_{i-1}^t(t_{i-1})\alpha_{i+1}^t(t_{i+1})}{\alpha_{i-1}^t(t_{i-1}) + \alpha_{i+1}^t(t_{i+1})} > 0, \tag{5.63}$$

where the condition of the eigenvalue's positivity is satisfied by switching the needle controls at the infinite nearest moments of time $(t_k,\ t_k + \delta, \delta \sim o(t_k))$: $\delta v(t_k) = v(t_k) - v(t_k + \delta)$.

The final eigenvalue of this integrated macrostructure (according to (5.40)) satisfies the relation

$$|\alpha_n^t(t_n)| = \min|\alpha_i^t(t_i)|, i=1,...,n,\ \alpha_n^t(t_n) = 1/2b_n(t_n)(\int_{t_n-o}^{t_n} b_n(\tau)d\tau)^{-1}. \quad (5.64)$$

According to (3.183)) the condition (5.64) acquires the form

$$\min|\alpha_i^t(t_i)| \to \min[1/2b_i(t_i)(\int_{t_i-o}^{t_i} b_i(\tau)d\tau)^{-1}] \to \min|h_i|, b_i(t_i) > 0, i=1,...,n, \quad (5.65)$$

where $h_i(t_i)$ is a local Hamiltonian at the time interval t_i.

The optimal strategy (that is chosen from the set of possible strategies) is a one that, transferring the dynamic process along the switching line ($\mathbf{a} = \alpha_i^t(t_i)t_i = Const$*), will bring the minimal* $|\alpha_i^t(t_i)|$ *at each discrete moment* t_i.

The *procedure* consists of joining of the two eigenvalues at first, and then adding a third eigenvalue to the doublet.

Optimal triplet structure includes the doublet as a *primary* IN's element.

By forming a triplet, each following doublet sequentially *orders and organizes* the eigenvalues' chain into the IN triplet's nodes.

For the ranged spectrum, defined from the joint solution (5.51)-(5.54),(5.63) (at $\gamma = 0.5$), the minimal sum of the discrete interval is a unique and a single.

This strategy is *implemented* for the spectrum of the initial eigenvalues, with a maximal value of parameter (5.51):

$$\gamma_n^{\alpha_0} = (2.21)(3.89)^{n/2},\ \gamma = 0.5, \quad (5.66)$$

which characterizes the *optimal-minimal filter*.

Thus, the eigenvalues $\{\alpha_{io}^t\}$ that are different from the optimal set:

$$\alpha_{i+1,o}^t = (0.2567)^i(0.4514)^{1-i}\alpha_{1o}^t,\ \alpha_{1o}^t = \max_i \alpha_{io}, i=1,2....n\ , \gamma = 0.5 \quad (5.67)$$

are filtered.

Using the dispersion of the synthesized process:

$$D = \prod_{i=0}^{i=n}(t_{i+1} - t_i), \quad (5.68)$$

we write it for the optimal set (5.66),(5.67) in the form

$$D^o = \frac{(2.042)^{n/2}(3.896)^{(n-2)/2}}{0.758 \bullet 2.215} t_1^n,\ t_1 = \frac{\mathbf{a}_o}{\alpha_{1o}^t}, \gamma = 0.5, \quad (5.68a)$$

The initial IN node's dimension n (allowing the cooperations) is found from the equation (5.67a) for the ranged eigenvalues:

$$[(\frac{\alpha_{1o}^t}{\alpha_{3o}^t})(\frac{\alpha_{1o}^t}{\alpha_{2o}^t})(\frac{\alpha_{2o}^t}{\alpha_{3o}^t}-1)]^{n/2-1}(\frac{\alpha_{3o}^t}{\alpha_{4o}^t}-1)^{n/2}=1 \qquad (5.69)$$

and from the relation (5.68a), written for given γ.

A vice versa, to find α_{1o}^t via n, is also true.

The IN dimension n can be expressed via a starting consolidating moment t_1 of the triplet's time sequence (t_1, t_2, t_3) according to formula

$$(\ln t_1 - 2.949876)n + 0.618485 = 0,\ \gamma = 0.5. \qquad (5.70)$$

Changing t_1, even with the fixed invariants $\mathbf{a}_o(\gamma)$ and the ratios of $\gamma_2^{\alpha} = t_2 / t_1(\gamma)$, $\gamma_3^{\alpha} = t_3 / t_1(\gamma)$, leads to a new model's dimension.

The spectrum of the initial eigenvalues, the consolidation procedure, and the optimal strategy are found for the model with a given (n, γ).

By the joint solution of the equations (5.76)-(5.80), the following model's parameters $\gamma_n^{\alpha}, \alpha_{1o}^t, \alpha_{no}^t, \gamma_{i,m}^{\alpha}; \{\alpha_i^t(t_i)\}_{i=1}^{n-1}$ are obtained successively.

This procedure allows us to build the macromodel even without using the microlevel (at given (n, γ)), implementing the above relations.

The given macromodel spectrum determines the total time of the IN's macrodynamic process $T_{IN} \cong t_n^m$, which we call the IN's life-time; here t_n^m is the α_{no}^t discrete interval at the last IN's triplet *m* that finalizes the IN cooperations.

The local MPs are integrated into the IN final node, whose information evaluates a total IPF.

Example. Let us determine the parameters of optimal IN for a given dimension $n = 10$ and $\gamma = 0.5$.

By solving the equations (5.64)−(5.70), (5.42)) we get $\mathbf{a}_o = 0.7606, \gamma_1^{\alpha} = 2.2155, \gamma_2^{\alpha} = 3.8955$, the staring segment's moment t_1, eigenvalue α_{1o}^t, and then we obtain the spectrum of the model's initial eigenvalues $\alpha_{io}^t, i = 1-10$ in the following ranged sequence:

8.43736, 3.80833,2.16592, 0.97762, 0.55600, 0.25096, 0.14273, 0.06442,0.03663, 0.01653,0.00940, along with the corresponding sequence of the discrete moments t_i: 0.08367, 0.18538, 0.32595, 0.72215,1.26976, 2.81316,4.94637,10.95870,19.26861,42.68961, 75.06089.

These numerical data allow us the complete restoration of the optimal macroprocess, optimal controls, and to build the corresponding IN (by analogy with Figs.5.4,5.5a,b), where each third t_i in the above sequence corresponds to forming the IN's triplet's node with a total number $m = n/2 + 1 = 6$.

1.5.5. Geometrical Structure of the Optimal Space Distributed Cooperative Macromodel (OPMC). The IN Geometrical Structure

The OPMC geometrical structure is a result of space transformations of the macrodynamic variables that satisfy the invariant condition for the path functional in the controlled process of the space-time motion.

The information structures, consolidating in the IN's nodes, accompany this movement.

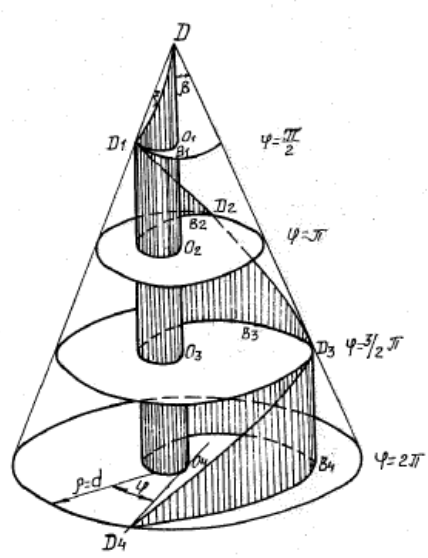

Figure 5.6. An OPMC spiral trajectory with radius $\rho = b\sin(\varphi \sin\beta)$ on the conic surface at the points D, D1, D2, D3, D4 with the spatial discrete interval DD1=μ, which corresponds to the angle $\varphi = \pi k / 2$, $k = 1,2,\ldots$ of the radius vector's $\rho(\varphi,\mu)$ projection of on the cone's base (O1, O2, O3, O4) with the vertex angle $\beta = \psi^o$ (in the text).

Theorem 5.1.

The trajectory of each of the n three-dimensional OPMC movements represents the parametrical equation of a helix curve on the cone (Figure 5.6) with the projection of radius-vector $\bar{r}$ (ρ, φ, ψ^o) on the *i*- cone's base:

$$\rho_i = b_i \sin(\varphi_i \sin \psi_i{}^o) . \tag{5.71}$$

The transfer from the one cone trajectory to another cone trajectory is executed at the points that satisfy the extreme condition of the equation (5.71) at the angles

$$\varphi_i(l_i) \sin \psi_i{}^o = \pi / 2 . \tag{5.72}$$

Proof. The trajectory of each of the n-three-dimensional movements with a nonorthogonal, in general, the matrix of rotation $\bar{A}$ (at the fixed angle with respect to one of the coordinate axis) has a simple representation in the cylindrical space coordinate system

$$l(\bar{r}) = l(\rho, \varphi, \psi^o):$$

$$z = r\cos\psi^o, z^2 = r^2 + \rho^2, z = \rho(tg\psi^o)^{-1}, dl^2 = d\rho^2 + \rho^2 d\varphi^2 + dz^2, \quad (5.73)$$

where φ is the angle's increment of the radius vector's coordinate ρ; ψ^o is the fixed angle's increment with respect to the axis z.

Then at the zero increment $d\psi^o$=0, corresponding to a fixed coordinate z, we get the equality

$$dl = [(\frac{d\rho}{d\varphi})^2 \sin^{-2}\psi^o + \rho^2]^{1/2} d\varphi. \quad (5.74)$$

The conditions (4.9),(4.10) are imposed on that equality, as a consequence of the VP fulfillment for $\bar{y} \equiv \frac{d\bar{l}}{dt}, \frac{dl_i}{dt} = \frac{dl}{dt} = c$.

From hence, the differential equations follow:

$$\bar{d}^2 = (\frac{d\rho}{d\varphi})^2 \sin^{-2}\psi^o + \rho^2; \bar{d} = \frac{dl}{d\varphi} = Const. \quad (5.75)$$

The solution of these equations leads to

$$\frac{\partial^2\rho}{\partial\varphi^2}\sin^{-2}\psi^o + \rho = 0, \frac{\partial\rho}{\partial\varphi} \neq 0, \quad (5.76)$$

$$\frac{\partial^2\rho}{\partial\varphi^2}\ \sin^{-2}\psi^o + \rho^2 \neq 0, \frac{\partial\rho}{\partial\varphi} = 0. \quad (5.77)$$

Relation (5.76) satisfies (5.71) automatically. Relation (5.77) that implements the extreme condition for $\rho = \rho(\varphi)$ is satisfied at the fulfillment (5.72), while (5.71) is a parametrical equation of the spiral curve on conic surface *S*:

$$\sum_{i,j=1}^{n} \frac{\partial^2 S}{(\partial l_j^i)^2}(\bar{l}_j^i - l_j^i)^2 = 0, \bar{l}_j^i = \sum \bar{e}^i{}_j l_j^i, \quad (5.78)$$

where $\bar{l}_j^i$ are the coordinates of the radius vector $\bar{r}$ at the fixed nonsingular point with orths $\bar{e}_j^i$, l_j^i are the current point's coordinates on the cone with the angle 2 ψ^o at the vertex with the $\bar{d}$, as the curve parameter, and with the projection ρ of the radius vector $\bar{r}$ on the cone base.

At the existence of a singular point, its location corresponds to the transformation from one cone's trajectory (5.71) to the analogous trajectory of another cone at a $\rho(\varphi)$ extreme.
The transformation $\bar{A}$ carries up the n spiral curves on the corresponding cones (Figs. 5.6, 5.4). •

Theorem 5.2.
Let us consider the directional cosines of the normal on the cones $(\bar{\alpha}_1^i, \bar{\alpha}_2^i, \bar{\alpha}_3^i)$, which are defined in the coordinate system, constrained by a particular cone at the discrete moments τ_i.

Then, in the cone vertex at the discrete space points $l_i(\tau_i) = \mu_{i+o}$, $i = 1,...,m$, taken on the helix curve, we get relation

$$\varphi(0)=0,\ \bar{\alpha}_1^i = \bar{\alpha}_1 = \cos\psi^o, \bar{\alpha}_2^i = \bar{\alpha}_2 = \sin\psi^o, \bar{\alpha}_3^i = \bar{\alpha}_3 = 0; \tag{5.79}$$

and in the cone base, defined at the discrete points: $l_k(\tau_k) = \mu_k + o$, $k = m-1$, ..., 7,5,3,1, we have

$$\varphi_k = 6\pi,\ \bar{\alpha}_2^k = 0, \bar{\alpha}_1^k = 0,\ \bar{\alpha}_3^k = 1. \tag{5.80}$$

At the intermediate points, the following relations hold true

$$\Delta\bar{\bar{l}}^j = (l(t) - \bar{l}^j(\tau_j))\,(\bar{\alpha}_1^j \bar{l}_1^j + \bar{\alpha}_2^j \bar{l}_2^j + \bar{\alpha}_3^j \bar{l}_3^j);\ j = (i,k). \tag{5.80a}$$

Proof. From the conditions of the orthogonality for the vectors $\bar{l}_i(t_i)$, $\bar{l}_{i+1}(t_{i+1})$ that are written via the directional cosines of the normal on the cone:

$$\bar{\alpha}_1^i = \frac{d\bar{l}_1^i}{d\mu_i} = \frac{d\rho_i}{d\varphi_i tg\psi_i^o} = \cos\psi_i^o \cos(\varphi_i \sin\psi_i^o);\ \varphi_i = \varphi(\tau_i),$$

$$\bar{\alpha}_2^i = \frac{d\bar{l}_2^i}{d\mu_i} = \sin\psi_i^o \cos\varphi_i \cos(\varphi_i \sin\psi_i^o) - \sin\varphi_i \sin(\varphi_i \sin\psi_i^o),$$

$$\bar{\alpha}_3^i = \frac{d\bar{l}_3^i}{d\mu_i} = \sin\psi_i^o \sin\varphi_i \cos(\varphi_i \sin\psi_i^o) + \cos\varphi_i \sin(\varphi_i \sin\psi_i^o) \tag{5.81}$$

the equation, connecting the above vectors, follows

$$\bar{l}^i(\tau_i)\,\bar{l}^i(\tau_{i+1}) = 0,\ \bar{\alpha}_1^i\bar{\alpha}_1 + \bar{\alpha}_2^i\bar{\alpha}_2 + \bar{\alpha}_3^i\bar{\alpha}_3 = 0, \tag{5.81a}$$

where

$$\bar{\alpha}_1 = \cos\psi_i^o \cos(\varphi(\tau_i)\sin\psi_i^o),\ \bar{\alpha}_2 = \sin\psi_i^o \cos\varphi_i(\tau_i)\cos(\varphi_i(\tau_i)\sin\psi_i^o) - \sin\varphi_i(\tau_i)\sin(\varphi_i(\tau_i)\sin\psi_i^o),$$

$$\bar{\alpha}_3 = \sin\psi_i^o \sin\varphi_i(\tau_i)\cos(\varphi_i(\tau_i)\sin\psi_i^o) + \cos\varphi_i(\tau_i)\sin(\varphi_i(\tau_i)\sin\psi_i^o). \tag{5.82}$$

Considering the equations (5.81a), (5.82) with the initial angle $\varphi(\tau_i)=\varphi(0)=0$, and substituting (5.81),(5.82) to (5.81a), we obtain

$$\cos^2\psi_i^o\cos(\varphi_i\sin\psi_i^o)+\sin^2\psi_i^o\cos(\varphi_i\sin\psi_i^o)$$

$$-\sin\psi_i^o\sin\varphi_i\cos(\varphi_i\sin\psi_i^o)=0. \qquad (5.83)$$

Solution of the last equation at $\overline{\varphi}=\varphi_i\sin\psi_i^o=\pm\pi/2$ leads to

$$\sin\varphi_i=0. \qquad (5.84)$$

The joint solution of (5.83),(5.84) defines the minimal real roots:

$$\varphi_i=\pm 6\pi,\ \psi_i^o=\pm 0.08343.$$

In a more general case, we get the relations

$$\varphi_i=k\pi,\ k=1,2,...\text{and at } k=1:\ \varphi_i=\pi.\ \bullet \qquad (5.85)$$

Corollary 5.1.

The parameters of spiral $b_i=b(\mu_i)=\mu_i(\varphi_i)^{-1}$ at the discrete points $(l_i\sim\mu_i)$, $l_i=l_i(\tau_i)$, $i=1,...,m$ for each cone are equal to

$$\rho_i=\rho_i(l_i(\tau_i))=\rho_i(\mu_i)=b_i\sin(\varphi_i\sin\psi_i^o)=b_i, \qquad (5.86)$$

and the angle at the cone vertex takes the values

$$\sin\psi_i^o=(2k)^{-1},\ k=1,2,..;\ \text{at } k=1,\ \psi_i^o=\pi/6; \qquad (5.86a)$$

with the angle

$$\varphi_i(l_i(\tau_i))=k\pi. \qquad (5.86b)$$

The results follow from (5.72) and (5.85). The obtained relations hold true for each cone, and because of that, all cones have the same angle at the vertex $\psi_i^o=\psi^o$ and the extreme value of the radius vector's projection $\rho_i=\rho_i(\mu_i)$ $(l_i\sim\mu_i)$ at this angle. •

Theorem 5.3.

At the moments of the matrix A^t eigenvectors' consolidation, when following equation holds true

$$\lambda^t{}_i(t_k)=\lambda^t{}_j(t_k)=\lambda^t{}_m(t_k+o(t_k)),\ k=1,3,5;\ i,j,m=1,2,\ldots,n \qquad (5.87)$$

the vectors $\bar{l}^i(t_k),\bar{l}^j(t_k),\bar{l}^m(t_k)$, which coincide by their direction with the corresponding eigenvectors of matrix A^t, get the *equal directions*.

This result follows from the definition of vectors $\bar{l}^i$ and the equalization of the eigenvalues. • .

Figure 5.6 illustrates the geometrical trajectory for each OMPC space coordinate. Using (5.80b),(5.87) we write the result of the Theorem 5.3 in the form

$$\frac{\Delta\bar{l}^{\,i}(t_k)}{|\Delta\bar{l}^{\,i}(t_k)|}=\frac{\Delta\bar{l}^{\,j}(t_k)}{|\Delta\bar{l}^{\,j}(t_k)|}=\frac{\Delta\bar{l}^{\,m}(t_k+o(t_k))}{|\Delta\bar{l}^{\,m}(t_k+o(t_k))|},\ i,j,m=1,2,..,n;\ k=1,3,5. \quad (5.88)$$

The condition of the equalization of the directions for the vectors $\bar{l}^{\,i}(t_k)$, $\bar{l}^{\,j}(t_k)$ acquires also the form

$$\frac{\bar{l}^{\,i-1}(t_{i-1})+\Delta\bar{l}^{\,i}(t_k)}{|\bar{l}^{\,i}(t_k)|}=\frac{\bar{l}^{\,j-1}(t_{i-1})+\Delta\bar{l}^{\,j}(t_k)}{|\bar{l}^{\,j}(t_k)|},\ \bar{l}^{\,i}=\bar{l}^{\,j-1}+b_i\varphi_i,\ i,j=1,...,n;\ k=1,3,5,\ldots \quad (5.89)$$

At the discrete moments t_k, considering the equal vector's modules in the form

$$|\bar{l}^{\,i}(t_k)|=|\bar{l}^{\,j}(t_k)|=|\bar{l}\ (t_k)|,\ \mu_i=l_i(t_i)-l_i(t_{i-1}), \quad (5.90)$$

we obtain, after differentiating (5.89) at $t\to t_k$, the relation

$$\frac{d\Delta\bar{l}^{\,i}(t_k)}{dt}=\frac{d\Delta\bar{l}^{\,j}(t_k)}{dt},\ c^{i}{}_{z}=\frac{d\Delta\bar{l}_z^{\,i}(t_k)}{dt}=\frac{d\Delta\bar{l}_z^{\,j}(t_k)}{dt}=c^{j}{}_{z}\ . \quad (5.91)$$

Taking into account the condition $\frac{d\Delta\bar{l}^{\,i}(t_k)}{dt}\neq\frac{d\Delta\bar{l}^{\,j}(t_k)}{dt}$, the equalities (5.91) for the vectors $\Delta\bar{l}^{\,m}(t_k+o(t_k))$ and $\Delta\bar{l}^{\,i}(t_k)$, or $\Delta\bar{l}^{\,j}(t_k)$ are not fulfilled generally.

The equality for the derivations (5.91) along the l_z-axis characterizes the equal speeds in a chosen direction of growing of the cooperative macrostructure, which is defined by the final discrete interval, where ($\frac{d\Delta\bar{l}_z^{\,i}(t_k)}{dt}\alpha_z^i$) is the projection of the linear vector speed; $\Delta\bar{l}_z^{\,i}\frac{d\alpha_z^i}{dt}$ is the projection of the angle speed for the vector $\Delta\bar{l}^{\,i}$.

The relations (5.88)-(5.91) hold true at the discrete intervals t_i, which correspond to the movement along the hyperbola defined by the model's VP dynamic invariant

$$\mathbf{a}(\gamma)=\alpha_i^t t_i=inv,$$

where $\gamma=\beta_{io}^t/\alpha_{io}^t$.

The local cone's helix trajectories are stuck along a geometric projection of the switching controls line on the hyperbola.

Let us consider the increment of the angle η for vector $\bar{l}^{\,i}=\bar{l}^{\,i}(\eta)$ for a sector of the entrance in the hyperbola, i.e., for $t_o\le t<t_i$.

Because $\bar{\bar{l}}^{\,i}(\eta)$ coincides by the direction with the corresponding eigenvector $\bar{z}^{\,i}(t)=z_\alpha{}^i(t\ )+jz_\beta{}^i(t\ )$ of the matrix $A(t)=(\alpha_i^t(t)+j\beta_i^t(t))_{i=1}^n$, this task can be resolved using the condition of the zero-equality for the *imaginary* component of the eigenvector at the moment of an entrance on the hyperbola $\alpha_i^t t_i=inv$, which is connected to another model's VP invariant $\mathbf{b}(\gamma)=\beta_i^t t_i=inv$.

We get the solution of the equation

$$z_{\beta}^{\ i}(t_0)(2\exp\alpha_i^t(t_0)\ t_i)\cos(\beta_i^t(t_0)\ t_i))= z_{\alpha}^{\ i}(t_0)\exp(\alpha_i^t(t_0)\ t_i)\sin(\beta_i^t(t_0)\ t_i), \quad (5.92)$$

$$tg\eta = \frac{z_{\beta}^{\ i}(t_0)}{z_{\alpha}^{\ i}(t_0)} = \frac{\exp(\alpha_i^t(t_0)t_i)\sin(\beta_i^t(t_0)t_i)}{2-\exp(\alpha_i^t(t_0)t_i)\cos(\beta_i^t(t_0)t_i)}. \quad (5.92a)$$

For the optimal model, this relation represents the invariant

$$c_x^0 = \frac{\exp(\mathbf{a}_o)\sin(\gamma\mathbf{a}_o)}{2-\exp(\mathbf{a}_o)\cos(\gamma\mathbf{a}_o)},$$

which depends on the invariant $\mathbf{a}_o(\gamma)=\alpha_{io}^t t_i$.

At γ=0.5, $\mathbf{a}_o=-0.706$, we get c_x^0=7.0866 that determines the angle η=1.430611($\approx 82^o$).

At the inverse directional movement, starting at the moment t_o, the condition of the zero-equality for the *real* component of the eigenvector determines the angle $\dot{\eta}$ of reaching the starting point on hyperbola $\mathbf{b}(\gamma)=\beta_i t_i$ at the moment $\tau_i^1 = t_i - t_o$**:**

$$tg\eta^1 = \frac{z_{\alpha}^{1i}(t_0)}{z_{\beta}^{1i}(t_0)} = \frac{2-\exp(\alpha_i^t(t_0)t_i-)\cos(\beta_i^t(t_0)t_i-)}{\exp(\alpha_i^t(t_0)t_i-)\sin(\beta_i^t(t_0)t_i-)} = c_x^{10}, c_x^{10} = \frac{2-\exp(\mathbf{b}_o)\cos(\gamma\mathbf{b}_o)}{\exp(\mathbf{b}_o)\sin(\gamma\mathbf{b}_o)}, \quad (5.92b)$$

which is defined by the invariant $\mathbf{b}_o(\gamma)=\alpha_i^t(t_0)t_i-$, where t_i- is the interval *preceding* to the initial moment t_o.

At γ=0.5, $\mathbf{b}_o=-0.517$, we obtain c_x^{10}=523.1 that defines $\eta^1 \approx \pi/2$.

Assume that the optimal motion takes place on the surfaces of the corresponding cones along the surfaces' normals, with the spiral's equation (5.71) at the discrete moments, which are defined by the invariants $\mathbf{b}_o$, $\mathbf{a}_o$ and $\mathbf{a}$.

Then the increment of the angle η of the spiral rotation φ_η is determined from condition

$$\bar{\alpha}_1^i(t_0)\bar{\alpha}_1^i(t_i) + \bar{\alpha}_2^i(t_0)\bar{\alpha}_2^i(t_i) + \bar{\alpha}_3^i(t_i)\bar{\alpha}_3^i(t_i) = \cos\eta,\ i=1 \quad (5.93)$$

and it holds the form

$$\varphi_\eta^i(t_i) = \varphi_\eta. \quad (5.94)$$

At γ=0.5, $\mathbf{a}_o=-0.706$, this angle equals $\varphi_\eta \approx 18\,rad$.

Relation (5.93) (with $\cos\eta^1$ on the right side) defines the space angle of a spiral rotation φ_η^1 to reach the cone vertex at the moment $t' = t_i-$.

At γ=0.5, $\mathbf{b}_o=-0.517$, $\eta^1 \approx \pi/2$, this angle takes the value $\varphi_\eta^1 \approx 18.9\,rad$.

The condition (5.91) for the vector $\bar{\bar{l}}_k^{\,i}(t_k + o)$ on the k-cone with respect to the vector $\bar{\bar{l}}_k^{\,m}(t_k + o)$, defined on this cone, at the μ_{i+1} discrete interval, and taken on the geometric projection of the switching controls line, acquires the form

$$\frac{d\Delta\bar{l}_z(t_k + o(t))}{dt} = \frac{d\Delta\bar{l}_z^{\,m}(t_k + o(t))}{dt},\ k = 1,3,5;\ m = 1,2,.. \tag{5.95}$$

The fulfillment of (5.88)−(5.94) assembles the above vectors mutual positions on the considered cones in the space and governs the alternations of the vectors $\bar{\bar{l}}^{\,i}(t)$, $(i = n, n-1,...,1)$ in the immobile coordinate system (l_x, l_y, l_z).

Let us form this coordinate system on the m-cone:

$$(\bar{l}_1^{\,m}, \bar{l}_2^{\,m}, \bar{l}_3^{\,m}) = (l_x, l_y, l_z), \tag{5.96}$$

at the discrete moment that finalizes the consolidation procedure.

As a result of the fulfillment of (5.88), we arrive at the following equations of the normals in the immobile coordinate system:

$$\Delta\bar{l}^{\,6} = (l - l^6)(\bar{\alpha}_1^6 l_x + \bar{\alpha}_2^6 l_y + \bar{\alpha}_3^6 l_z),\ \bar{\alpha}_1^6 = \alpha_x,\ \bar{\alpha}_2^6 = \alpha_y,\ \bar{\alpha}_3^6 = \alpha_z,$$

$$\Delta\bar{l}^{\,5} = (l - l^5)(-\bar{\alpha}_1^5 l_x + \bar{\alpha}_2^5 l_y + \bar{\alpha}_3^5 l_z),$$

$$\Delta\bar{l}^{\,4} = (l - l^4)[(\bar{\alpha}_1^4 \cos\psi\cos\psi^o + \bar{\alpha}_2^4 \sin\psi\cos\psi^o + \bar{\alpha}_3^4 \sin\psi^o) l_x +$$
$$(\bar{\alpha}_1^4 \cos\psi\sin\psi^o + \bar{\alpha}_2^4 \sin\psi\sin\psi^o - \bar{\alpha}_3^4 \cos\psi^o) l_y + (\bar{\alpha}_1^4 \sin\psi - \bar{\alpha}_2^4 \cos\psi) l_z],$$

$$\Delta\bar{l}^{\,3} = (l - l^3)[(\bar{\alpha}_1^3 \sin\psi\cos\psi^o + \bar{\alpha}_2^3 \cos\psi\cos\psi^o + \bar{\alpha}_3^3 \sin\psi^o) l_x +$$
$$(\bar{\alpha}_2^3 \cos\psi\sin\psi^o + \bar{\alpha}_1^3 \sin\psi\sin\psi^o - \bar{\alpha}_3^3 \cos\psi^o) l_y + (\bar{\alpha}_1^3 \cos\psi - \bar{\alpha}_2^3 \sin\psi) l_z]. \tag{5.97}$$

Considering the angles of rotation of the local coordinate systems (ψ, ψ_1), applied for the subsequent discrete intervals $\Delta\bar{l}^{\,2}$, $\Delta\bar{l}^{\,1}$, we introduce below the indications that will simplify the equations forms. For example at $n = 6$ we get

$$A_x^2 = (\bar{\alpha}_1^2 \cos\psi_1 + \bar{\alpha}_2^2 \sin\psi_1)\sin\psi^o - \bar{\alpha}_3^2 \cos\psi^o,$$
$$A_y^2 = (-\bar{\alpha}_1^2 \cos\psi_1 + \bar{\alpha}_2^2 \sin\psi_1)\cos\psi^o + \bar{\alpha}_3^2 \sin\psi^o,$$
$$A_z^2 = \bar{\alpha}_1^2 \sin\psi_1 - \bar{\alpha}_2^2 \cos\psi_1,\ A_x^1 = (\bar{\alpha}_1^1 \sin\psi_1 + \bar{\alpha}_2^1 \cos\psi_1)\sin\psi^o + \bar{\alpha}_3^1 \cos\psi^o,$$
$$A_y^1 = \bar{\alpha}_1^1(\cos\psi_1 - \sin\psi_1)\cos\psi^o - \bar{\alpha}_3^1 \sin\psi^o,\ A_z^1 = \bar{\alpha}_1^1 \cos\psi_1 - \bar{\alpha}_1^2 \sin\psi_1 \tag{5.98}$$

Generally, for the sequential set of discrete intervals we obtain the equation

$$\Delta\bar{l}^{\,j} = (l - l^j)[(A_y^j \cos\psi_1 \cos\psi^o + A_x^j \sin\psi_1 \sin\psi^o + A_z^j \sin\psi^o) l_x +$$

$$(A_y^j \cos\psi_1 \sin\psi^o + A_x^j \sin\psi^o - A_z^j \cos\psi^o) l_y + (A_x^j \cos\psi_1 - A_y^j \sin\psi_1) l_z]; \; j=2,1, \tag{5.99a}$$

where $(\bar{\alpha}_1^i, \bar{\alpha}_2^i, \bar{\alpha}_3^i)$ are the directional cosines of the normal in a local coordinate system of the i-cone; i=1,..., 6; j=2,1.

As it follows from (5.98),(5.99a), the condition (5.88) can be fulfilled with an accuracy of a some angle (ψ, ψ_1), approximating the rotation of a local coordinate system.

We choose the values of the angles $(\psi, \psi_1, ..., \psi_k)$ derived from the condition (5.91):

$$c_z^{3-5}(t_5) = (\frac{\partial \Delta l_z^{\,3-5}}{\partial\varphi}\frac{d\varphi}{dt})(t_5) = (\frac{\partial \Delta l_z^{\,4-5}}{\partial\varphi}\frac{d\varphi}{dt})(t_5) = c_z^{4-5}(t_5), \tag{5.99b}$$

$$l_z^{\,4-5} = l_z^{\,4} + b_5\varphi(\bar{\alpha}_1^4 \cos\psi + \bar{\alpha}_2^4 \sin\psi), \; l_z^{\,3-5} = l_z^{\,3} + b_3\varphi(\bar{\alpha}_2^3 \cos\psi \mp \bar{\alpha}_1^3 \sin\psi), \tag{5.99c}$$

where the signs in the last equation correspond to the different spiral directions on the considered cones (the “minus” for the right spiral direction and the “plus” for the left spiral direction). We arrive at the relations for derivatives:

$$\dot{l}_z^{\,4-5} = c_z^{4-5} = b_3(c/b_3)(\bar{\alpha}_1^4 \cos\psi + \bar{\alpha}_2^4 \sin\psi) + b_5\varphi(\dot{\bar{\alpha}}_1^4 \cos\psi + \dot{\bar{\alpha}}_2^4 \sin\psi),$$

$$\dot{l}_z^{\,3-5} = c_z^{3-5} = c(\bar{\alpha}_2^3 \cos\psi \mp \bar{a}_1^3 \sin\psi) + b_3\varphi(\dot{\bar{\alpha}}_2^3 \cos\psi \mp \dot{\bar{\alpha}}_1^3 \sin\psi), b = b_3,$$

$$\bar{\alpha}_1 = \cos\psi^o \cos(\varphi \sin\psi^o),$$

$$\bar{\alpha}_2 = \sin\psi^o \cos\varphi \cos(\varphi \sin\psi^o) - \sin\varphi \sin(\varphi \sin\psi^o),$$

$$\dot{\bar{\alpha}}_1 = -(c/b)\sin\psi^o \cos\psi^o \sin(\varphi \sin\psi^o),$$

$$\dot{\bar{\alpha}}_2 = -(c/b)[(1+\sin^2\psi^o)\cos(\varphi \sin\psi^o) + 2\sin\psi^o \sin\varphi \cos(\varphi \sin\psi^o)]. \tag{5.99d}$$

Because these relations hold true at the point of the triple consolidation, where $\varphi = 6\pi$ and $\varphi \sin\psi^o = \pi/2$, we obtain

$$\bar{\alpha}_1 = 0, \bar{\alpha}_2 = 0, \; \dot{\bar{\alpha}}_1 = -(c/b)\sin\psi^o \cos\psi, \; \dot{\bar{\alpha}}_2 = -(c/b)(1+\sin^2\psi^o);$$

$$c_z^{4-5} = -6\pi c\,[\sin\psi^o \cos\psi^o \cos\psi + (1+\sin^2\psi^o)\sin\psi],$$

$$c_z^{3-5} = -6\pi c[(1+\sin^2\psi^o)\cos\psi \mp \sin\psi^o \cos\psi^o \sin\psi], \tag{5.100}$$

or from $c_z^{3-5} = c_z^{4-5}$ we have

$$(1 - \sin\psi^o \cos\psi^o + \sin^2\psi^o)\cos\psi = (1 \pm \sin\psi^o \cos\psi^o + \sin^2\psi^o)\sin\psi,$$

$$tg\psi = \frac{(1-\sin\psi^o\cos\psi^o+\sin^2\psi^o)}{(1\pm\sin\psi^o\cos\psi^o+\sin^2\psi^o)}\ , \tag{5.101}$$

which at $\psi^o = 0.08343$, brings $\psi = 0.70311$; for the equal directed spirals at a small angle ψ^o, we get $\psi = \pi/4 - \psi^o$.

For the spirals of the opposite directions, it follows that $\psi' = \pi/4$.

For the intervals $t_1 - t_3$, $t_2 - t_3$, we come to the analogous procedure for the definition of the angles ψ_1, ψ_2, which take the same values.

We get the equalities

$$\psi = \psi_1 = \psi_2 = ... = \psi_k = \pi/4 - \psi^o;\ \psi_1' = \psi_2',\ \psi_2' ... = \psi_k' = \pi/4. \tag{5.102}$$

At the last interval of the optimal motion, for the example, with $(t_5 - t_7)$, $(t_6 - t_7)$, we obtain following equations

$$l_z^{5-7} = l_z^5 + b_5\varphi\alpha_z^5; l_z^{6-7} = l_z^6 + b_6\varphi\alpha_z^6; c_z^{5-7} = c\alpha_z^5 + b_5\varphi\dot{\alpha}_z^5; c_z^{6-7} = c\alpha_z^6 + b_6\varphi\dot{\alpha}_z^6\ , \tag{5.103}$$

where at $\varphi = 6\pi$, $\dot{\alpha}_z^j = 0, \alpha_z^j = 1$, $j = 5, 7$, and we get $c_z^{5-7} = c_z^{6-7} = c$.

This corresponds to the resulting macrosystem movement along the l_z axis with the constant speed c after finalizing the optimal process.

Because of that, the extremal condition for $\Delta\bar{l}_z(\psi)$ (5.103) (which is in agreement with the increments of this coordinate by other cones) defines its maximal increments.

By analogy, we determine the conditions of the cones' *coordination* (formed before the entrance on the hyperbola) *with* the cones that correspond to the movement along the hyperbola switching line.

The condition (5.99a) leads to the equation in the form (5.97).

From the condition (5.93), (5.94) we find the angle $\psi_0 = \varphi_\eta^i(\Delta\bar{l}_j)$ of the relative rotation of the above cones.

Considering, for example, the cones between the points $(l_6 - l_7), (l_0 - l_6)$ we come to the following equations:

$$c_z^6(t) = \dot{\varphi}(t)b_3\alpha_z^6(t) + b_3\varphi(t)\dot{\alpha}_z^j(t)\ ;\ \text{at}\ t = t_6 + o\ ,\ c_z^6(t_6+o) = 0;\ \varphi(t_6+o) = 0\ ;$$

$$c_z^6(t') = b_3[\dot{\varphi}(t')(\alpha_x(t')\cos\psi_0 - \alpha_y(t')\sin\psi_0) + \varphi(t')(\dot{\alpha}_x(t')\cos\psi_0 - \dot{\alpha}_y(t')\sin\psi_0)],$$

$$\dot{\alpha}_x = -(c/b)(\cos\psi^o\sin\psi^o), \dot{\alpha}_y = -(c/b)(1+\sin^2\psi^o)\ ;\ \text{at}\ t' = t_6 - o\ ;\ \varphi(t_6-o) = 6\pi,$$

$$\alpha_x(t') = \alpha_y(t') = 0, b = b_3, c_z^6(t') = -6\pi c[(1+\sin^2\psi^o)\sin\psi_0) - \sin\psi^o\cos\psi^o\cos\psi_0]. \tag{5.104}$$

By the equalization of the speeds $c_z^6(t) = c_z^6(t')$, we get equality for

$$tg\psi_0 = \frac{\sin\psi^o \cos\psi^o}{1+\sin^2\psi^o}; tg\psi_0(\gamma = 0.5) = 0.0075, \psi_0 \approx 0. \qquad (5.105)$$

The same relations follow from the consideration of the condition (5.94) for the cones between the points $(l_3 - l_5), (l_0 - l_3)$.

Finally, we come to the following recurrent equations for the projection of the normals (defined by the n-cones) with respect to the immobile coordinate system:

$$l_x^i = l_x^i(t_j) + b_i\varphi_i(\Delta t_j)\alpha_x(\Delta t_j);\ \Delta t_i \to \tau_i,\ j=0,1,...,n;\ i=1,...,n;$$

$$l_y^i = l_y^i(t_j) + b_i\varphi_i(\Delta t_j)\alpha_y(\Delta t_j);\ \varphi(0)=0,\ \varphi(\tau_j) = 6\pi,$$

$$\psi_0 = 0,\ l_z^i = l_z^i(t_j) + b_i\varphi_i(\Delta t_j)\alpha_z(\Delta t_j);\ \eta = \eta^o = 6\pi,\ \psi = \pi/4. \qquad (5.106)$$

Using the obtained results, we can express the relative angle of rotation of the local coordinate system as the function of the model's *basic parameters* (of dimension, uncertainty, and curvature) (n, γ, k):

-at the moment of the *entrance* of each cone's helix on the hyperbola (describing the mutual cone's locations):

$$\psi_0 = arctg[\sin\psi^o \cos\psi^o (1+(\sin^2\psi^o)^{-1}] = Const(n, j, k); \qquad (5.107)$$

-at the discrete intervals of the helix *movement* along the hyperbola, when the condition (5.88) holds true:

$$\psi(t_i) = arctg(1 - \sin\psi^o \cos\psi^o + \sin^2\psi^o) \times (1 \pm \sin\psi^o \cos\psi^o + \sin^2\psi^o)^{-1}, \qquad (5.108)$$

where the signs " +" and "– " are related to the spirals of the opposite directions.

The different values of γ bring the variations to the angles' values $\varphi, \psi^o, \eta, \psi, \psi_0$.

The fulfillment of (5.98)–(5.108) for the coordinates determines the mutual positions of the cones in the space and the changes of the geometrical macromodel coordinates in the immobile system. The obtained geometrical structure reflects the necessity of memorizing the macrodistributions in space, created by the undistinguished states in the consolidation process.

Let us find the increment of information volumes $\Delta V_{m,m+1}$ between the volumes of a two IN triplet's nodes m and (m+1): V_m, V_{m+1}, considering their relative difference $\Delta V^*_{m,m+1} = (V_{m+1}/V_m - 1)$.

Volume $V_m \sim V(t_5)$, for example, is formed at the moment t_5 of the m-th node's triplet's cooperation, and volume $V_{m+1} \sim V(t_7)$ is formed at the moment t_7 of the (m+1)-th node's triplet cooperation.

Forming the above triplet's volumes satisfies the relation following from the geometry of the IN node's hierarchy (Fig. 5.5):

$$V(t_5) = 2V_c[3(t_5 - t_3)^3 + (t_5 - t_4)], V(t_7) = 2V_c[3(t_7 - t_5)^3 + (t_7 - t_6)],$$
$$V_c = 2\pi c^3 / 3(k\pi)^2 tg\psi^o, \quad (5.109)$$

where t_j, $j = 3,4,5,6,7$ are the cooperative moments of the processes, related to the triplets shown on Fig.5.5, and V_c is the constant volume determined by the fixed space speed *c*, the cone geometrical parameter *k*, and the angle at each cone vertex ψ^o (Fig.5.6).
From that we get

$$\frac{V(t_7)}{V(t_5)} = \frac{t_6^3[3(t_5/t_6)^3((t_7/t_5)^3 - 1)) + ((t_7/t_6) - 1)]}{t_4^3[3(t_3/t_4)^3((t_5/t_3)^3 - 1)) + ((t_5/t_4) - 1)]}, \quad (5.109a)$$

while the following relations are satisfied at forming the IN: $t_5/t_3 = t_7/t_5 = \gamma_{3,5}^{\alpha}, t_6/t_5 = t_4/t_3 = \gamma_{3,4}^{\alpha}, t_5/t_4 = t_7/t_6 = \gamma_{4,5}^{\alpha},$
$t_6/t_4 = (t_6/t_5)(t_5/t_4) = \gamma_{3,4}^{\alpha}\gamma_{4,5}^{\alpha} = \gamma_{3,5}^{\alpha}$.

This brings the relative volume's ratio to

$$\Delta V^* = \frac{V(t_7)}{V(t_5)} - 1 = (\gamma_{3,5}^{\alpha})^3 - 1, \quad (5.109b)$$

where at $\gamma = 0.5$, $\gamma_{3,5}^{\alpha}$=3.89.

The relative increment of the volume $\Delta V_{m,m+1}^*$ in the considered node's (*m*+1) volume $V_{m+1} \sim V(t_7)$ regarding the previous node's volume $V_m \sim V(t_5)$ is increasing in $\Delta V^*(\gamma = 0.5) = 57.863$ times.

Comments 5.2. The triplet's asymmetry.
The *geometrical* meaning of consolidation at the triplet's discrete points consists of an *additivity* of the cone volumes V_k^i, and their increments $\Delta V_k^{i,i+2}$, which form the triplet as a *new* macrostructure:

$$V_k(t_k) = V_k^i + V_k^{i+1} + V_k^{i+2}, \ \Delta V_k^{i,i+2} = 2(3V_k^{i-2,i} + V_k^{i-1,i}), \ i=3,5,7. \quad (5.110)$$

The ratio of the volume V_k^i to the surface F_k of the forming structure characterizes its relative strength Π (as a specific force of the pressure, measured by a weight's ratio of the volume V_k^i to F_k^i):

$$\Pi = \frac{V_k^i}{F_k^i} = \frac{b}{3}\sin(\varphi \sin\psi^o)\cos\psi^o, F_k^i = \frac{\pi\rho_i^2}{\sin\psi^o} + \pi\rho_i^2. \quad (5.110a)$$

The condition of an extreme for the relative strength ($\frac{\partial \Pi}{\partial \varphi} = 0$) is satisfied by the execution of relation (5.73), i.e., with a maximum of $\rho(\varphi)$.

Each triplet is a structure with an odd symmetry order, determined by the equations

$$\Pi_c = \frac{2\pi}{\pi/2 \pm (\pi/4 - \arcsin(2k)^{-1})},\ \Pi'_c = \frac{2\pi}{\pi/4 - \arcsin(2k)^{-1}},\ \text{at } k=1:$$

$$\Pi_c(\psi) \cong 3, \Pi_c(-\psi) \cong 7, \Pi'_c(\psi) = 9, \tag{5.111}$$

where ψ is the angle of rotation of the local coordinate systems in (5.102).

Moreover, the positions of the triplet's local coordinate axes in the space are *not* repeated precisely (Fig. 5.7), and any symmetrical transformations cannot bring the above axes to an equivalent position.

The transformation of local coordinates is preserved at $\varphi = \pm\pi$ (k =1), and it possesses the symmetry of a *reflection*.

Angle ψ, as a third space coordinate, depends on parameter k.

With growing k, the number of three-dimensional coordinates is increasing, and the restricted relation, admitting an even symmetry order, takes place:

$$\lim_{k\to\infty} \Pi_c(+\psi) = 2.66,\ \lim_{k\to\infty} \Pi_c(-\psi) = 8,\ \lim_{k\to\infty} \Pi'_c(\psi) = 8, \tag{5.112}$$

even though, a "conditional " symmetry order is the odd at any limited k.

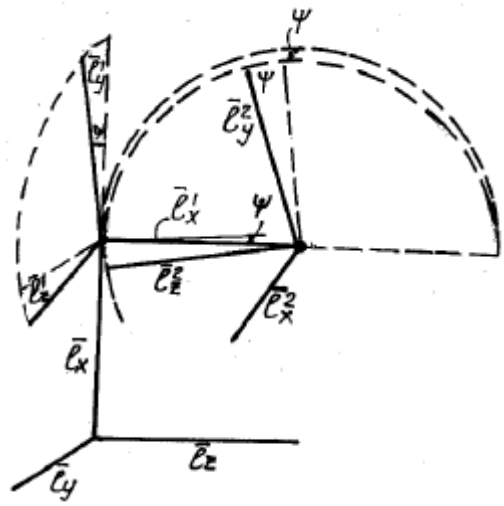

Figure 5.7. The OPMC space's asymmetry, illustrated by the *nonrepeating* positions of the triplet's local coordinate axes (in the space) $(\overline{l}_x^1, \overline{l}_y^1, \overline{l}_z^1)$ and $(l_x^2, \overline{l}_y^2, \overline{l}_z^2)$ at the coordinate system's *symmetrical* transformations.

The considered space movement is accompanied by a rotary movement along each spiral, and therefore it possesses a rotating moment $M(i)$, defined by the scalar multiplication of a system's impulse $imp(i)$ and the radius vector of the rotation $\bar{r}(i)$ for each local extremal piece (i):

$$M(i) = imp(i) \bullet \bar{r}(i), imp(i) = \frac{\partial L(x_i, \partial x_i / \partial t)}{\partial C_i},\ C_i = \frac{d\bar{l}_i}{dt}. \qquad (5.113)$$

The impulse is preserved for a Hamiltonian model, and at a vicinity of the discretization moment, when the macrostructure is forming, the radius of rotation acquires a maximal value $\bar{r}(i) = b(i)$, where $b(i)$ is the radius of the i-cone base.

This defines the maximal value of each local rotating moment (for considered the i-piece of extremal movement) around the corresponding coordinate axis:

$$M_{t=\tau}(i) = M_m(i) = imp(i) \bullet b(i). \qquad (5.114)$$

The $M_m(i)$ moment is able to rotate the initial matrix's eigenvalues for the eigenvector's space cooperation.

This rotary moment of the macromodel movement exists independently on electrical and magnetic fields as an intrinsic *information-geometric* characteristic during the formation of informational structure.

The obtained OPMC equations are important not only for the implementation of general relations (chs.1-4), but also in bringing this math approach to the solution of the actual applied problems [63].

Applying the above equations allowed us to develop the algorithmic procedure and the software packet for the restoration of the OPMC dynamic and geometrical structures for a given (γ,c, k, α_{io}) (part2)..

The OPMC is implemented in the IN models a complex object by a hierarchy of the sequentially reduced number of independent macrocoordinates.

Chapter 1.6

THE IMD MODEL'S PHENOMENA AND INFORMATION CODE

1.6.1. The Model Time Course and the Evaluation of the Information Contributions into IN. The Triplet Genetic Code

The model possesses two scales of time: a reversible time that equals to the sum of the time intervals *on* the extremals segments $T^r = \sum_{i=1}^{i=n} t_i$, and the irreversible time T_e^{ir} that is counted by the sum of the irreversible time intervals δt_i *between* the extremals segments, while the time arrow $\delta \vec{t}_i$ arises when the needle controls connect the segments.

Proposition 6.1.
The ratio of the above *elementary* time intervals is evaluated by the formula

$$\frac{\delta t_i}{t_i} = \frac{\Delta S_i^{\delta}}{\mathrm{a}_o^2} - 1, \tag{6.1}$$

where ΔS_i^{δ} is the information contribution delivered between the time intervals t_i and $(t_i + \delta t_i)$, $\mathbf{a}_o$ -invariant

Proof. Because the needle control connects the extremal segments by transferring information between the segment's window δt_i, i.e. from the *i*-segment's information $\Delta S_i^{\delta} / t_i$ to the $(i+o)$-segment's information $\Delta S_i^{\delta} /(t_i + \delta t_i)$, the information contribution from the needle control $\delta \alpha_i^o$, delivered during δt_i is

$$\Delta S_i^{\delta} / t_i - \Delta S_i^{\delta} /(t_i + \delta t_i) = \frac{\Delta S_i^{\delta} \delta t_i}{t_i^2 + t_i \delta t_i} = \delta \alpha_i^o . \tag{6.2}$$

From other consideration, $\delta\alpha_i^o$ is evaluated by an increment of information production at δt_i:

$$\delta\alpha_i^o = \delta\frac{\partial\Delta S_{io}^{\delta}}{\partial t} \approx \frac{\partial^2\Delta S_{io}^{\delta}}{\partial t^2}\delta t_i,$$

where following equations sec.1.3.5 we have at the segment's beginning:

$$\frac{\partial^2\Delta S_{io}^{\delta}}{\partial t^2} = \lim_{\delta t_i\to 0}\frac{\partial^2\Delta S_i}{\partial t^2}(\delta t_i) = -\frac{\partial H_{io}}{\partial t}, H_{io} = 1/2\alpha_i^t,$$

$$\dot\alpha_i^t = -2(\alpha_{i-1}^t)^2\exp(\alpha_{i-1}^t t_i)(2-\exp(\alpha_{i-1}^t t_i)^{-2}, \tag{6.3}$$

$$\lim_{t_i\to 0}\dot\alpha_i^t = -2(\alpha_{i-1}^t)^2,\ \dot H_{io} = -(\alpha_{i-1}^t)^2, \delta\alpha_i^o = (\alpha_{i-1}^t)^2\ \delta t_i\ . \tag{6.3a}$$

By substituting (6.3) into (6.2), at $\mathbf{a}_o\ (\gamma) = \alpha_{i-1}^t t_i$, we get $\delta t_i = (\frac{\Delta S_i^{\delta}}{\mathrm{a}_o^2} - 1)t_i$. •

Proposition 6.2

(1)-Each extremal segment's time interval t_i retains $\mathbf{a}_o(\gamma)$ units of the information entropy;

(2)-The regular control, applied at the interval t_i beginning, brings the negentropy $\mathbf{a}(\gamma) = \alpha_i^t t_i$, and this control is memorized at each of the *i*-segment's DP-locality.

Proofs of (1),(2) follow from the invariant relations sec.1.3.5 and an essence of the control actions. •

Corollary 6.1.

By evaluating the information contribution on the t_i-extremal by both the segment entropy's invariant $\mathbf{a}_o$ and the regular control's negentropy invariant $\mathbf{a}$, we come to

$$\Delta S_i^{\delta} = \mathbf{a}_o - \mathbf{a},$$

and

$$\frac{\delta t_i}{t_i} = |\frac{\mathbf{a}_o^2 - \mathbf{a}_o + \mathbf{a}}{\mathbf{a}_o^2}| = \delta^*(\gamma). \tag{6.3b}$$

Corollary 6.2.

The model life-time ratio $T_*^{ir} = T^{ir}/T^r$ is evaluated by the following invariant ratio $\delta^*(\gamma)$ at t_i-extremal:

$$T_*^{ir} = |\frac{\mathbf{a}_o^2 - \mathbf{a}_o + \mathbf{a}}{\mathbf{a}_o^2}| = \delta^*(\gamma). \tag{6.4}$$

Indeed, using $\delta t_i = |\frac{\mathbf{a}_o^2 - \mathbf{a}_o + \mathbf{a}}{\mathbf{a}_o^2}| t_i$, $T^{ir} = \sum_{i=1}^{n} \delta t_i$ and $T^r = \sum_{i=1}^{i=n} t_i$, we come to $T_*^{ir} = |\frac{\mathbf{a}_o^2 - \mathbf{a}_o + \mathbf{a}}{\mathbf{a}_o^2}|$.

Comments 6.1. Let us count T_e^{ir} by $\delta^*(\gamma)$ at $\gamma \in (0,1)$ and $T^r = \sum_{i=1}^{i=n} t_i \approx 2t_{n-1}$ (for IN's dimension *n*).

Then the $\delta^*(\gamma)$-function takes the values from 0.0908, at γ=0.1 to 0.848, at γ=1, with a minimal value 0.089179639, at γ=0.5.

And at *n*=22, t_{n-1}=2642, we have $T_e^{ir} = 471.225$, at γ=0.5, with a maximal $T_e^{ir} = 4480.832$.

Corollary 6.3.

A minimal $\frac{\delta t_i}{t_i} \to 0$ leads to equality

$$\mathbf{a}_o(\gamma) - \mathbf{a}(\gamma) - \mathbf{a}_o^2(\gamma) \approx 0, \tag{6.5}$$

which is approximated with an accuracy $\delta^* \mathbf{a}_o^2$=0.044465455, $(\gamma = 0.5)$. (6.5a)

Corollary 6.4.

Because each extremal segment's t_i interval retains $\mathbf{a}_o(\gamma)$ units of the information entropy, and the regular control brings the negentropy $\mathbf{a}(\gamma) = \alpha_i^t t_i$ for interval t_i, while a needle control is also applied on this interval, the fulfillment of equation (6.5) means that the information contribution, delivered by needle control for interval t_i ($\delta(v_i)_{\inf}$), is evaluated by invariant

$$\delta(v_i)_{\inf} = \mathbf{a}_o^2(\gamma). \tag{6.6}$$

This equality is preserved only if the needle control actually connects the segments.

At the needle control absence, the contribution $\delta(v_i)_{\inf} = 0$, even though the invariant $\mathbf{a}_o(\gamma) \neq 0$, which actually evaluates information of the segment internal process.

Therefore (6.5) expresses the balance of information at each t_i-extremal at condition $\frac{\delta t_i}{t_i} \to 0$, which at $\gamma = 0.5$ is approximated with the accuracy (6.5a).

The same result follows at $\Delta S_i^{\delta} = \mathrm{a}_o^2$ when the needle control compensates the between segment's contribution.

At $\Delta S_i^{\delta} > \mathrm{a}_o^2$ we have $\frac{\delta t_i}{t_i} > 0$, and at $\Delta S_i^{\delta} \to \infty$ (sec.1.3.5) we come to $\frac{\delta t_i}{t_i} \to \infty$.

Comments 6.2. Since the needle control joins the extremal segments by delivering information $\mathbf{a}_o^2(\gamma)$, we might assume that $\delta^*\mathbf{a}_o^2$ represents a defect of the $\mathbf{a}_o^2$ information, which is conserved after cooperation.

Taking this into account leads to a precise fulfillment of the balance equation in the form

$$\mathbf{a}_o(\gamma) - \mathbf{a}(\gamma) - \mathbf{a}_o^2(\gamma) - \delta^*\mathbf{a}_o^2(\gamma) = 0, \qquad (6.6a)$$

while $\delta^*\mathbf{a}_o^2$ is the information spent on the segments cooperation. This result also corresponds to relation (5.42c) evaluating (at the equal $\mathbf{a}(\gamma_1)$and $\mathbf{a}(\gamma_2)$) the information contribution from the needle control by $\sim 2\mathbf{a}(\gamma) \cong \mathbf{a}_o^2(\gamma)$.

Proposition 6.3. *The information structure of a triplet.*

A triplet, formed by the three-segments cooperative dynamics during a *minimal* time (secs.1.5.2,1.5.3), encloses information $4\mathbf{a}_o^2 + 3\mathbf{a} \cong 4$ bits at $\gamma = 0.5$, while each of the IN's triplet's *node* holds information $\mathbf{a}_o^2 + \mathbf{a} \cong 1$ bit.

Proof. The triplet's dynamics include two extremal segments, joining first into a doublet, which then cooperates with a third extremal segment (Fig.6.1).

Forming the triplet during a minimal time requires building the doublet during the time interval of a third extremal segment, while all three dynamic processes start simultaneously with the action of three starting controls. Each two extremals consist of two discrete intervals $(t_{i11}, t_{i12}, t_{i21}, t_{i22})$ where i is the triplet number; t_{i11}, t_{i12} are the first and second discrete intervals of the first dynamic process; t_{i21}, t_{i22} are the first and second discrete intervals of the second dynamic process; t_{i3} is a single discrete interval of the third dynamic process.

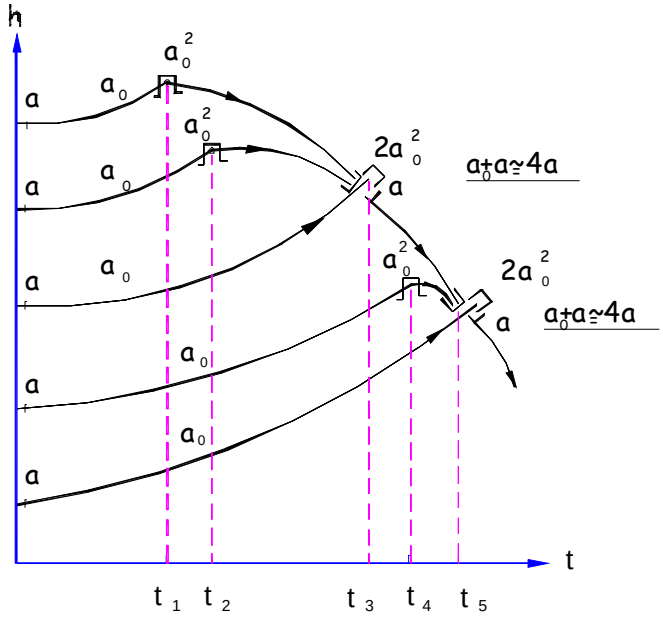

Figure 6.1.

The above requirement for a triplet with minimal process' time implements the following equations on the first discrete interval for the first and second dynamic process:

$$-\alpha_{i1o}t_{i11} + \alpha_{i11}t_{i11} + \mathrm{a}_o^2 = -\mathbf{a}_o + \mathbf{a} + \mathbf{a}_o^2 - \delta^* \mathrm{a}_o^2,$$
$$-\alpha_{i20}t_{i21} + \alpha_{i21}t_{i21} + \mathrm{a}_o^2 = -\mathbf{a}_o + \mathbf{a} + \mathbf{a}_o^2 - \delta^* \mathrm{a}_o^2, \qquad (6.7)$$

where $\alpha_{i1o}t_{i11} = \mathbf{a}_o$, $\alpha_{i20}t_{i21} = \mathbf{a}_o$, $\alpha_{i21}t_{i21} = \mathbf{a}$, $\alpha_{i12}t_{i12} = \mathbf{a}$.
This means that at each of these discrete intervals, the information balance is fulfilled with the accuracy $\delta^* \mathrm{a}_o^2$.
The first and second dynamics, at the second time interval, convey the total contribution $\alpha_{i13}t_{i13} + \alpha_{i23}t_{i23} + 2\delta^* \mathrm{a}_o^2$, followed by applying the needle control, which joins both dynamics into the doublet. This brings the balance condition in the form $\alpha_{i13}t_{i13} + \alpha_{i23}t_{i23} + 2\delta^* \mathrm{a}_o^2 = \mathrm{a}_o^2$. (We count here the information contribution from a defect $2\delta^* \mathrm{a}_o^2 = \delta^*(\gamma)$ at both intervals t_{i21}, t_{i22}).

Joining the third segment's discrete interval with the doublet at the IN node requires applying another needle control, acting at the end of third interval (Fig.6.1).

This leads to the balance equation for third discrete interval in the form

$$-\alpha_{i30}t_{i31} + \alpha_{i31}t_{i31} + \mathbf{a}_o^2 - \delta^* \mathbf{a}_o^2 \cong -\mathbf{a}_o + \mathbf{a} + \mathbf{a}_o^2 \cong 0, \qquad (6.8)$$

at $\gamma = 0.5$.

It can be seen that a total information, delivered to the triplet, is equal to $4\mathbf{a}_o^2 + 3\mathbf{a}$, which compensates for the information being spent on the triplet's cooperative dynamics:

$$3\mathbf{a}_o + \alpha_{i12}t_{i12} + \alpha_{i22}t_{i22} + \alpha_{i31}t_{i31} + \alpha_{i13}t_{i13} + \alpha_{i23}t_{i23} + 2\delta^* \mathrm{a}_o^2. \qquad (6.9)$$

Let us verify this result by direct computation of the contributions $\alpha_{i13}t_{i13}$ and $\alpha_{i23}t_{i23}$, using the following formulas for each of them:

$$\alpha_{i13}t_{i13} = \alpha_{i12}(t_{i3} - t_{i11})\exp(\alpha_{i12}(t_{i3} - t_{i11}))[2 - \exp\alpha_{i12}(t_{i3} - t_{i11})]^{-1},$$

$$\alpha_{i23}t_{i23} = \alpha_{i12}(t_{i3} - t_{i21})\exp(\alpha_{i12}(t_{i3} - t_{i21}))[2 - \exp(\alpha_{i12}(t_{i3} - t_{i21})]^{-1},$$

$$\alpha_{i12}(t_{i3} - t_{i11}) = \alpha_{i12}t_{i11}(t_{i3}/t_{i11} - 1) = \mathbf{a}(\gamma_{13} - 1),$$

$$\alpha_{i22}(t_{i3} - t_{i21}) = \alpha_{i22}t_{i21}(t_{i3}/t_{i21} - 1) = \mathbf{a}(\gamma_{23} - 1), \qquad (6.10)$$

where the triplet's parameters at γ=0.5 take the values $\gamma_{13} \cong 3.9$, $\gamma_{23} \cong 2.215$, $\mathbf{a} \cong 0.252$.

The computation shows that the regular controls, acting at t_{13} and t_{23}, deliver information $\mathbf{a}(\gamma_{13}-1)$=0.7708 and $\mathbf{a}(\gamma_{23}-1)$=0.306 accordingly, while the macrodynamic process at these intervals consumes $\alpha_{i13}t_{i13} \cong 0.232$ and $\alpha_{i23}t_{i23} \cong 0.1797$. Including the defect $\delta^*(\gamma)$, we get the information difference $\cong 0.50088 \cong \mathbf{a}_o^2$ (at $\gamma = 0.5$).

This means that both regular controls, acting on the second doublet's intervals, provide necessary information to produce the needle control, and therefore, the doublet satisfies the balance equation that does not need additional external information for the cooperation.

The doublet cooperation with the third extremal segment forms the triplet IN node, which encloses the information contribution from both the doublet's and the third segment's needle controls (6.8), providing the defect $\delta^* \mathrm{a}_o^2$ that satisfies the balance in (6.8).

The triplet information at $4\mathbf{a}_o^2 + 3\mathbf{a} \cong 2.75$ (at $\gamma = 0.5$) is measured in Nats (according to the basic formula for entropy (ch.1.1)), or by $3.96 \cong 4$ bits in terms of $\log_2$ measure.

Because the IN's triplet node consists of the doublet, which encloses information $\delta^* \mathrm{a}_o^2$, and the third segment that transfers information $\mathbf{a}_o^2 + \mathbf{a} - \delta^* \mathrm{a}_o^2 \cong 0.70535$ Nats to the node, the total node information is $\mathbf{a}_o^2(\gamma) + \mathbf{a}(\gamma) \cong 1.0157 \cong 1$bits.

This means that, in addition to the sec. 1.5.3 statement 1.5.1, the triplet represents also an *optimal* formation carrying both VP information invariants. •

Comments 6.2a. To form a triplet, each of its component's *initial* complex eigenvalues should have a nonzero imaginary part, because every first triplet segment's time interval is associated with disappearance of the imaginary eigenvalue's component, and a third segment's time interval coincides with the equalization of the real components of the two previous segments.

Therefore, one of the necessary conditions of forming an optimal triplet consists of having its

$$\gamma_{io} = \beta_{io} / \alpha_{io} \neq 0 \text{ at } \gamma_{io} = \beta_{io} / \alpha_{io} \geq 0. \tag{6.10a}$$

Taking into account the necessary condition $\gamma_{io} = \beta_{io} / \alpha_{io} \leq 1$, limiting a creation of the macrodynamics, we come to the admissible range of the basic model's parameter γ_{io}:

$$1 \geq \gamma_{io} \geq 0, \tag{6.10b}$$

which restricts both the triplet's and the cooperative dynamics' formations.

A number of joining triplet's nodes is limited by a total external information, delivered for their cooperation.

Below we evaluate this information and determine the amount of information necessary for cooperating of any number of the segment's chain.

Comments 6.3. The triplet's both regular and needle controls produce four switches (Fig.6.1), which carry information $\cong$ 4 bits. Since each switch can encode one bit symbol, or a single letter, it follows that a triplet is a carrier of a four letter's information code.

This is a triplet's *genetic* code, initiated at the triplet's *formation*.

Therefore, the creation of an *external* code with the same information switches, applied to the given initial eigenvalues spectrum, would be able to *restore* the triplet's information structure.

This means that such a code might reproduce the triplet's dynamics, which the genetic code had encoded.

Comments 6.4. The triplet's information structure could serve as an information model of a DNA's triplet, which is the carrier of the DNA four letter's code.

Comments 6.5. There are two options for building the triplet's information network.

One of them (IN1) consists of *starting* each new triplet with every three subsequent eigenvalues of the model initial spectrum.

The second (IN) includes a *continuing* the cooperation of an initial triplet with a next doublets and so on (Figs.5.4, 5.5).

In this procedure, the following next doublet is formed by the third dynamics of the previous triplet and the first dynamics of the current triplet.

Such dynamics, satisfying equations (6.9), (6.10), deliver information $\mathbf{a}_o^2$ necessary to form the doublet. The doublet's cooperation with the current triplet's second extremal segment also needs the needle control information $\mathbf{a}_o^2$. Thus, the second joint triplet includes the information from the third segment $\mathbf{a}_o^2+\mathbf{a}$, which brings also a total information contribution $4\mathbf{a}_o^2+3\mathbf{a} \approx 2.75$ Nats (at $\gamma = 0.5$), or ≈ 4 bits (at $\gamma = 0.5$).

Actually, the second triplet contains an original and independent information delivered by two regular controls and three needle controls (Fig.6.1), which is equal to $3\mathbf{a}_o^2+2\mathbf{a} \approx 2$ Nats, or ≈ 3 bits. This means that 4-3=1 bit is transferred from the previous triplet's node, being an equivalent of $\mathbf{a}_o^2(\gamma)+\mathbf{a}(\gamma) \approx 1.02$ bits. Information transferred to a first IN node is equal $\mathbf{a}_o^2(\gamma)+\mathbf{a}(\gamma)$, because the triplet's doublet conserves information - $\delta^* a_o^2$.

Therefore, each IN's triplet contains the same information 4 bits, while each IN's node encloses information $\mathbf{a}_o^2(\gamma)+\mathbf{a}(\gamma) \approx 1.02$ bits**,** which is transferred between the IN's nodes, as the node's *information code*. Each third code's digit integrates the triplet's code, and a fourth code's digit integrates all fourth code's digits.

Finally, each triplet is characterized by the four letters genetic code, whose first letter is created by the previous triplet and is transferred from the previous IN's node. Such a code letter is dependable. Thus, each IN's current node includes the fourth letter of the previous triplet, while the triplet's encoding needs just three letters of the independent genetic code.

Even *if* the IN node's code is the same (for an entire IN), its information value depends on the node location within the IN's structure, because each following IN's node gets information from the previous IN's node.

At this case, both the first and the second IN's triplets require 4+3=7 bits information code. The following third triplet contains the same information 3.0 bits, while all three triplets require information 10.0 bits.

Therefore, each three joint triplets contain 9 bits, except the first one having 10 bits. The *n*-dimensional macromodel, cooperating into the IN with *m=(n*+1)/2 triplets, is able to enclose information [$(m-1)3+4$]=(3*m*+1) bits.

Corollary 6.5.

A linear chain of the cooperated *n*-extremal segments contains a total information $n\mathbf{a}(\gamma)+(n-1)\mathbf{a}_o^2(\gamma)$, which is able to encode the chain.

Corollary 6.6.

The *n*-dimensional macromodel, cooperating into the IN1 with *m=(n*/3*)* triplet's nodes, accumulates 4*m* bits information code that encodes information of the entire IN1.

This means that the same *n*-dimensional macromodel, cooperating into the IN, can be encoded by the information code having the $(m-1)$ bits less than the IN1 code.

Comments 6.6. The information, transformed from each previous to a following triplet's node, has an increasing information *value*, because each following triplet's node encapsulates information from all previous triplet's nodes.

The information *valuelessness* (as a measure of *quality* of information) we define as a code's unchangeability with other code's having the same quantity of information.

Each triplet contains the same quantity of information $4\mathbf{a}_o^2+3\mathbf{a}\cong 4$bits, and transfers $\mathbf{a}_o^2+\mathbf{a}\cong 1$bits to the following node, but the above information quantities of the previous triplet do not evaluate the structure of the following triplet.

Since the information invarints $\mathbf{a}_o$ and $\mathbf{a}$ of the following triplet include the structural information from the previous triplet, which depends on its location within the IN, both triplets' equal quantities of information are *unexchangeable*.

The triplet's sequential connections by the successful contributions of the information to each following triplet binds the IN's nodes is such a way that its final node's invariants $\mathbf{a}_o^2+\mathbf{a}$ accumulate an entire IN network's entropy, having a *maximum* valuelessness.

Finally, a time-space sequence of the applied controls, generating the doublets and triplets, represents a discrete control *logic*, which creates the IN's *code* as a *virtual communication language* and an algorithm of *minimal* program.

The code is formed by a time-space sequence of the applied inner controls (both regular and needle), which automatically join the ranged discrete moments (DP) of the successive equalization of the model's phase speeds (eigenvalues) in the process of generation of the macromodel's spectrum.

The equalized speeds and the corresponding macrostate's variables are *memorized* at the DPs by the optimal control actions.

A selected sequence of the minimal nonrepeating DPs, produced by a *minimal ranged spectrum* of eigenvalues and the corresponding initial macrostates, generates the *optimal* IN's code, which initiates the ranged sequence of the IN's doublets, cooperating sequentially into triplets.

The optimal code consists of a sequence of the double states, memorized at the $\{t_i\}_{i=1}^{n-1}$ DP-localities, and each *n* dimensional model is encoded by **n** intervals of DP.

Each triplet joins and memorizes three such discrete intervals, generating the triplet's digital code.

The above results reveal the procedure of the *transformation of dynamitic information into a triplet's code*, which is governing by the VP.

The number of the spectrum's cooperating strings (frequencies) is limited by the amount of delivered external information, which is restricted by both the admissible γ_{io} *and a total information provided by the macrodynamics (3.152), (5.42c), (6.11a,b).*

Because these limitations depend on chosen model and the external information, the triplet's digital program is a finite, allowing to avoid the Kolmogorov principal incompatibility (the details are in ch.1.9).

1.6.2. The Model Information Geometry (IG), Its Specific, and the Structure

The solution of the optimal VP problem, which generates the geometrical space distributed structures, connects the macrodynamics and the *information geometry* (IG), initiated by macrodynamics.

The IG describes the time-space locations, configurations, and shapes of the information macrostructures, created by information contribution of the space distributed macrodynamics, with a feedback on macrodynamics.

The IG reflects the geometry of informational (virtual) connections, imposed by the information path functional's measure of uncertainty, as an additional to a regular way of representing and storing information.

The following specifics describe the IG essentials.

The information time-space macromovement along the extremal segments is modeled by a spatial trajectory, which is represented by a *spiral* shaped helix curve, located on the conic surface (Fig.5.6). Both the spiral and cones' equations satisfy the VP (chs.1.3,1.4).

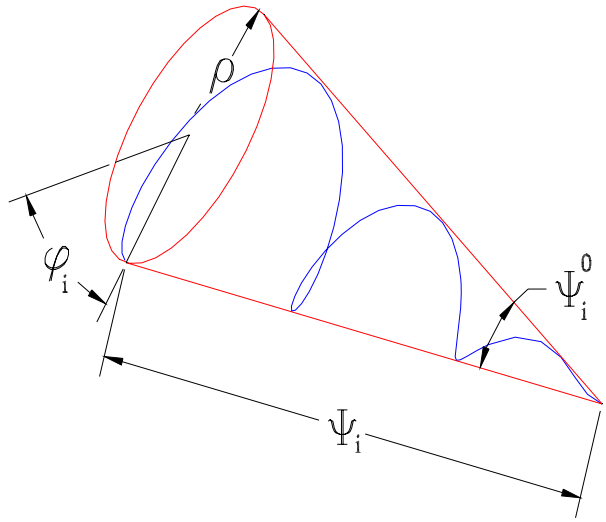

Figure 6.2. The cone's parameters (the indications are in the text).

A *three-dimensional* information geometry is formed at a locality of each joint three linear spiral segments at the cones' surfaces, forming a space-time *triplet* (Figs.5.4) with the cone's parameters (Fig.6.2).

Such a triplet is an *optimal elementary* macrostructure satisfying to the VP.

The consolidated macrodynamics originate a *space* movement, forming a *sequence* of the *triplet's geometrical macrostructures*, organized in the *information hierarchical structured network* (IN) (Figs.5.5, 6.3).

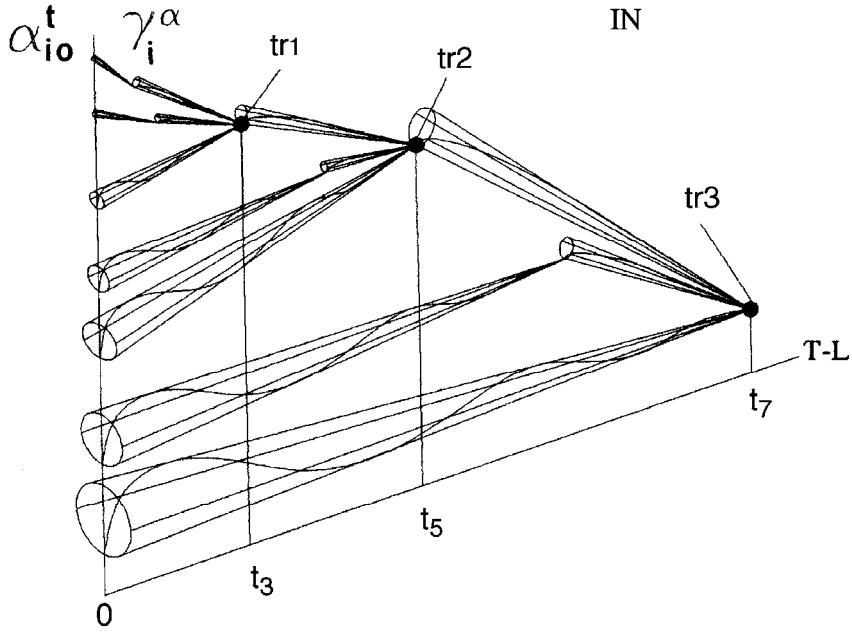

Figure 6.3**.** The IN geometrical structure represented by the hierarchy of the IN cones' spiral space-time dynamics with the triplet node's (tr1, tr2, tr3, ..), formed at the localities of the triple cones vertexes' intersections, where $\{\alpha^t_{io}\}$ is a ranged string of the initial eigenvalues, cooperating around the (t_1, t_2, t_3) locations; T-L is a time-space.

The *optimal IN nodes' cooperation*, fulfilling the VP minimum, consists of forming a *set* of triplet's substructures, which are sequentially *enclosed* according to the IN's hierarchy, in such a way that each following triplet includes the previous triplet, and the ending IN's node encapsulates the IN nodes' total information. This leads to an entire optimal IN's structure as a sequential connection of the triplets' hierarchy, where each second triplet joins one of the cone's spiral of a previous triplet and the doublet' structure, formed by the connection of the two following cone's spirals. The IN space macrostructure is built by a set of the time-space helix's form trajectories, located at an external shape of the local invariant cones.

The segment's macrodynamics at each current moment is determined by the local cone's space coordinates presented on Fig. 6.2: the spiral radius vector's projection on the cone's base ρ, depending on the radius' parameter b and the angle φ at the base; the fixed angle at the cone's vertex ψ^o; and a relative position of the cone's local coordinate systems ψ.

At the moment t_i of the segment's cooperation in the triplet's node, these coordinates (at γ=0.5) acquire the increments

$$\delta\varphi(t_i)\sin\psi^o = \pi/2\,,\ \delta\rho(t_i) = b(\mu(t_i))\,, \tag{6.11}$$

with a total cone's space angle $\psi(t_i) = \sum_{k=1}^{i}\delta\psi(t_k)$, counted from the process' beginning up to the end of each t_i interval.

The increment of the space angle $\delta\psi(t_i)$ between the IN nearest nodes $(m-1)$ and m determines an orientation of the m-th node's local coordinate system.

The information contribution from the segment's macrodynamics at a triplet's node m, formed at t_i, is defined by the invariants $\mathbf{a}(t_i)=\mathbf{a}_m(\gamma)$, $\mathbf{a}_o(t_i)=\mathbf{a}_o(\gamma)$, which in the geometric space depend on both the scalar invariant's value and the cone's relative position's space angle $\psi(t_i)=\psi_m$:

$$\mathbf{a}_m(\gamma,\gamma_m^{\alpha})=\mathbf{a}_m(\mathbf{a},\psi_m)\,,\ \mathbf{a}_o(\gamma,\gamma_m^{\alpha})=\mathbf{a}_{mo}(\mathbf{a}_o,\psi_m)\,, \tag{6.12}$$

where γ_m^{α} is the IN node's multiplier (equals to the eigenvalues' ratio for the nearest nodes), which determines the node position within the IN, connected to the space angle ψ_m. Relations (6.12) at a given IN with the fixed (γ,ψ_m) mean that each the IN node encloses the invariant information quantity defined by

$$\mathbf{a}_m^{in}=\mathbf{a}_m(\mathbf{a},\psi_m)+\mathbf{a}_{mo}(\mathbf{a}_o,\psi_m)\,, \tag{6.12a}$$

which is dependable on the node's location within the IN. At each time-space discrete interval (t_i, l_i) the information geometry produces information $\mathbf{a}_m(\gamma)=\mathbf{a}_m(\mathbf{a},\psi_m^o)$ that is revealed only by the end of this interval. Within this interval, the geometrical field conceals the information $\mathbf{a}_o(\gamma)=\mathbf{a}_{mo}(\mathbf{a}_o,\psi_m^o)$.

The *rotations and shifts* of the local triplet cone's geometry are performed according to the IN's enclosed control's logics, which determines an *algorithm* of the program and the triplet's *code* that organizes each IN level of the model.

A set of the control's switched points determine the model's switching control lines L_c (Fig.6.4), which is formed by the model's spatial intervals: $\Delta l_i = l_i(t_i) - l_{i+1}(t_{i+1})$.

Each of them has a *discrete* cellular geometric structure, formed by the joint triple cones (Fig.6.4a,b). The cell represents the IG elementary unit, which encloses invariant information (6.12a), measured in the bits of a c*ellular information code.*

The cell's space metric ds_m, defined by the cell diameter, acquires the invariant information measure

$$mes_{\inf}\,\|\,ds_m\,\|=\mathbf{a}_m^{\,i}\,, \tag{6.13}$$

which depends upon the cell *m* location within the IG.

This means that in the cellular IG, with different locations have distinctive information values depending on their position in the IN's node hierarchy, which, in turn, depends on the model's eigenvalues for this node. (In the Euclid space geometry, the metric invariant does no depend on the metric's location, while in the Riemann space it depends on a space curvature via the fundamental metrical tensor).

Thus, the IG cell's *geometry* is generated by the IN node's consolidated *space* movement, while each level of the IG hierarchical organization is evaluated by the *number* of the enclosed nodes into the considered IN node's level.

The related triplet node's information code posseses a hierarchical dependency on the node's position within the IN hierarchy.

The sequence of the cones' vertexes and bases' connections carries out the entire spatial optimal macrotrajectory, located at the surfaces of joint cones, which integrate the IN nodes' total contributions.

A certain IN's space trajectory enfolds a specific cell's code, which determines the particular IN's organization, enclosed into the *cooperative* cellular IG.
Such a time-space trajectory with the code cells is simulated on Figs.6.3,6.4.

Each IN node collects the contributions of the information path (MP, sec. 5.3) coming to this node (along the IN hierarchy of preceding nodes and accumulating a prehistory of this node formation.

The IN's node's information volume is increasing according to the ratio (5.109a,b).

Each space information invariant $\mathbf{a}_m^{in}$ represents a vector, measured by the quantity of information **a** and the angle $\vec{\psi}_m = \psi_m(\vec{l}^{\,m})$ of the IN node coordinate system's $\vec{l}^{\,m}$ location, which gets its increment on angle $\delta\vec{\psi}_m$ at the next IN's node.

This invariant can *only* be applied to the above specific node location.
Let us find a scalar distance $\| \vec{a}_{m+1} - \vec{a}_m \| = \| \Delta\vec{a}_m \|$ between two vectors:

$$\vec{a}_m = \mathbf{a}_m(\mathbf{a}, \vec{\psi}_m) \text{ and } \vec{a}_{m+1} = \mathbf{a}_{m+1}(\mathbf{a}, \vec{\psi}_{m+1}), \vec{\psi}_{m+1} = \vec{\psi}_m + \delta\vec{\psi}_m,$$

which at $\delta\vec{\psi}_m = \pi/4$ equals to $\| \Delta\vec{a}_m \| \cong 07653\,\mathbf{a}_m$ at the movement from triplet $(m-1)$ to *m*.

At $\delta\vec{\psi}_m = -\pi/4$, we get the negative increment $\| \Delta\vec{a}_m \| \cong -07653\,\mathbf{a}_m$,
which corresponds to the movement from triplet *m* to (*m*-1).

From that, the space invariant $\mathbf{a}_{m+1}$ for triplet (*m*+1) acquires the value $\mathbf{a}_m$(1+07653). The same increment gets the triplet's (*m*+1) space invariant $\mathbf{a}_{m+1,o}(\mathbf{a}_o, \vec{\psi}_{m+1})$ with regard to $\mathbf{a}_{mo}(\mathbf{a}_o, \vec{\psi}_m)$ at the shift of the triplet's coordinate system on angle $\vec{\psi}_{m+1} = \vec{\psi}_m + \delta\vec{\psi}_m$: $\mathbf{a}_{m+1,o} = \mathbf{a}_{mo}$(1+0.7653), as well as the triplet's eigenvectors scalars: $\lambda^l_{m+1,o} = \lambda^l_{m,o}$ 1.7653, $\alpha^l_{m+1,o} = \alpha^l_{m,o}$ 1.7653, and $\alpha^l_{m+1,1} = \alpha^l_{m1}$ 1.7653.

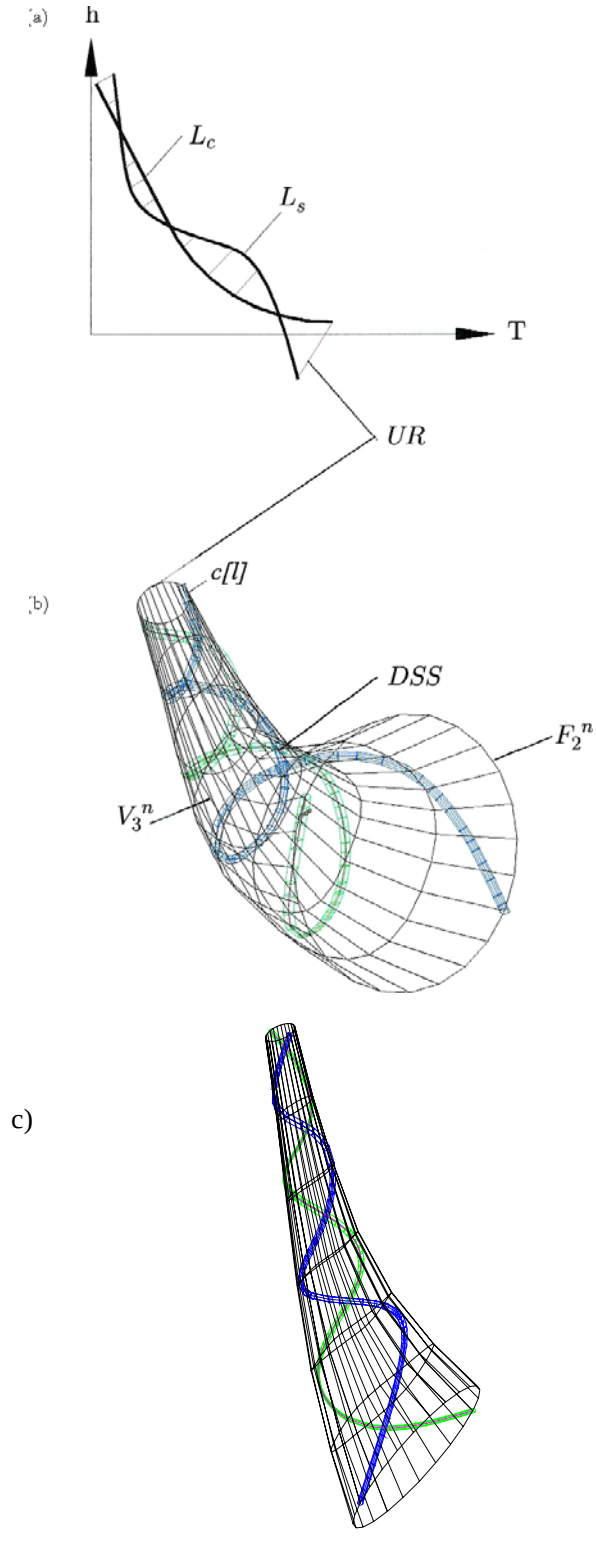

Figure 6.4. Continued on next page.

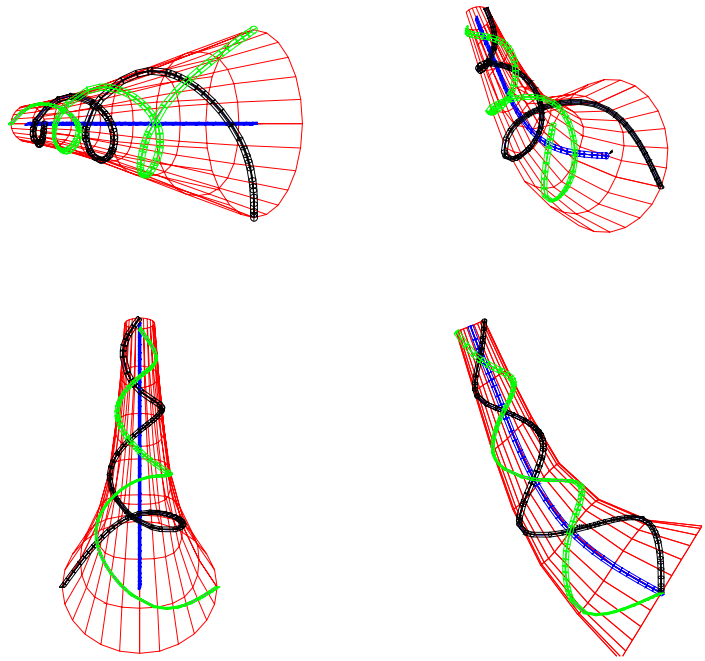

Figure 6.4. Simulation of the double spiral cone's structure (*DSS*) with the cell (c[l]), arising along the switching control line *Lc* (a); with a surface F_2^n of uncertainty zone (*UR*) (b), surrounding the *Lc*-hyperbola in the form of the *Ls*-line, which in the space geometry enfolds a volume V_3^n (b,c).

That is why the invariant's scalar relations: $\mathbf{a}_{m+1} = \alpha^l_{m+1,1} l_{m+1}$ and $\mathbf{a}_m = \alpha^l_{m1} l_{m1}$, as well as the the invariant $\mathbf{a}_{m+1,o} = \alpha^l_{m+1o,1} l_{m+1,o}$, $\mathbf{a}_{m,o} = \alpha^l_{mo} l_{mo}$ will be preserved, determining the same space distances $l_{m+1} = l_{m1}$ and $l_{m+1,o} = l_{mo}$, if only the above angle is changed. These changes in the information invariants bring the increasing contributions from both the extremal segment's information $\mathbf{a}_{m+1,o}$ and the information contribution from the regular control $\mathbf{a}_{m+1}$ in 1.7653 times, as well as the increase of the information contribution from the needle control $(\mathbf{a}_{m+1,o})^2$ in ~3.116 times. This means that both the needle control's and regular control's information *effectiveness grow* with the addition of each following (m+1) triplet, which increases a "strength" of the following triplet's connections.

These contributions also change the considered balance in equations (6.8).

Each IN node's space location, which encloses information $\mathbf{a}^{in}_m$, also virtually accumulates the space locations of the previous nodes, which had been distributed along in the IN's hierarchy and concurrently are consolidated in this node's geometrical spot.

The *cooperation* is accompanied by growing the node's information geometrical "density" $\mathbf{a}^{in}_m / \Delta l^k_m$, along the IN hierarchy, where Δl^k_m is a space location of a node *k*.

For example, if a node's 1 geometrical location Δl^1_m contains information $\mathbf{a}^{in}_m$, its density is $\mathbf{a}^{in}_m / \Delta l^1_m$; the node's 2 geometrical location Δl^2_m, containing the same information $\mathbf{a}^{in}_m$, has

the geometrical density $\mathbf{a}_m^{in}/\Delta l_m^2$. A node 3 that also contains information $\mathbf{a}_m^{in}$ in a geometrical location Δl_m^3, formed by the cooperation of the nodes 1 and 2, satisfies the relation $\mathbf{a}_m^{in}/\Delta l_m^3 = \mathbf{a}_m^{in}/\Delta l_m^1 + \mathbf{a}_m^{in}/\Delta l_m^2$.

At $\Delta l_m^1 = \Delta l_m^2$, the geometrical density of the third node increases twice ($\Delta l_m^3 = \Delta l_m^1/2$).

Each such a node acquires a raise in information geometrical intensity, or a "strength" through growing numbers of the enclosed triplet-nodes.

This leads to an increase of a local intensity of both regular and needle controls, determined by the values of the corresponding information invariants $\mathbf{a}$ and $\mathbf{a}_o^2$ accordingly.

The above hierarchical dependencies are carried out by the spatial-time chain of the model's cooperating segments, whose invariants, controls, discrete intervals, and related information code satisfy these phenomena.

Knowing the space-time distribution of the model's invariants allows us to predict both the intensity of each segment's needle control, and the following discrete interval which determines the related code.

Both the controls and the code are fixed and *memorized* in the IG space points and the nodes at the transformation from the temporal to the spatial macrodynamics.

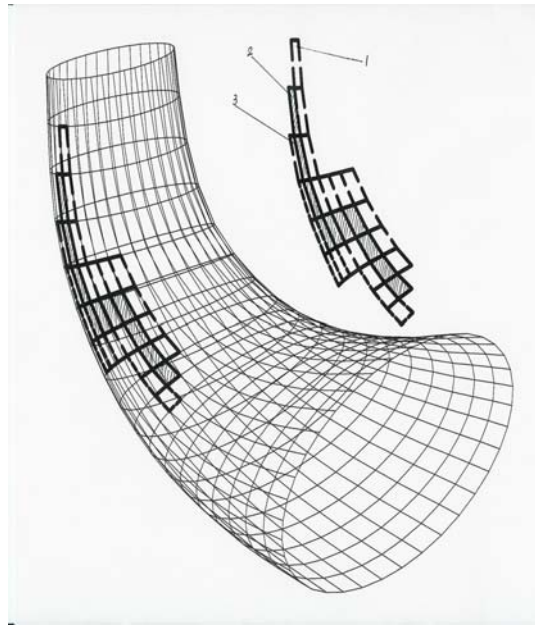

Figure 6.5. Structure of the cellular geometry, formed by the cells of the DSS triplet's code, with a portion of the surface cells (1-2-3), illustrating the space formation.

A set of the possible IN space trajectories (at the different model's eigenvalue's spectrum) defines an entire model's potential information geometrical structure, which encodes the creation of a wide *diversity* of the IN's macrostructures, generated by the genetic code. Along the switching controls line (Fig.6.4a), the IN set of the macromodel's cones allocates a geometrical volume V_3^n, whose external surface F_2^n identifies the IN geometrical

macrostructure's border (BMS) where the macromodel is open for the external interactions. The distribution of the local BMSs along IN's hierarchy allows each node to keep an external communication through its BMS and have an access to the entire model's IG structure.

The total F_2^n geometrical structure is composed by the quadrilateral curved *cellular* structure (Fig.6.5), which carry a set of *genetic* model's *codes* for a source-generation of all *potential* macrodynamic time-space *structures*, capable of developing in the future.

The geometrical IG phase space fabric (where the VP regularities act), consists of the code-cells (Fig.6.5), which form an expending hyperbolic cone's curved structure.

Any trajectory, located in this space, acquires a corresponding sequence of the code cells.

The model's *genetic* code is *enfolded* into this cellular structure in the triplet form a juncture of the local triplet's spirals. Such a code's *universal structure* can be applied to a variety of specific macromodels built for diverse processes. Since the optimal control is formed by a duplication of the model's macrostates at each t_i, both the switching control line and the macrostates are represented by a *double spiral curve*, carrying the cell's information code (Fig.6.4). The optimal IN code's *double spiral-helix* triplet structure (DSS) (Fig.6.4b-c), is shaped at the localities of the sequential connected cones' spirals, which form the time-space path-line of transferring the IN's information through the triplet's hierarchy.

The IN *optima*l *code* is both a *virtual communication language and* an algorithm of a minimal program for *building* the IN. The code's information language describes the object's uncertainty. Both the code and algorithm are the *attributes* of systemic connections, generated by the macromodel's primary information string $\{\alpha_{io}\} \sim \{\omega_{io}\}$ and the formed IN's structure. The code *embraces* both the micro-and macrolevel's regularities and their structuring into the optimal system's *organization*.

Both a system and its model could be *restored* by knowing their *genetic* code. Because the space cell encloses the code-message at any interactions, satisfying a law, this IG code conceals information regarding a law.

The information, transformed from each previous to a following triplet, has an increasing value, because each following triplet encapsulates and encloses information from all previous triplets. The *location* of the node within the IN's hierarchy determines the *value* of information, encapsulated into this node.

The information *valueness* as a measure of *quality* of information is defined by a code's *unexchangeability* with other codes having the same quantity of information.

The hierarchy of the triplet code's creates the IN code's *enclosed hierarchical dependency*, where the encoded macrostructures depends not just on the total number of corresponding code's symbols, but essentially on the coding structure, presented by the hierarchy of the previous code's symbols. This information systemic property allows a sequentially *integrating* of the encoded information into the code words of the *same length*.

A particular code's optimal program identifies the details of each space-time macrodynamics. The object's identification leads to the possibility of a *reconstruction* of optimal code for each class of the modeled processes allowing their universal information description. In general, this approach's information valueness defines it as a mutual unexchangeability of the information with other information having the same quantity. In the IN, this quality is measured in the cell's bits information code, *attached* to the specific node's eigenvalue and its dimension, which models the *particular* event's information.

The classical information theory does not make any difference between a quality of information obtained, for example, from a number of people and from the same number of cards or stocks, and so on.

Multiple attempts to introduce in information theory a constructive definition of a quality (value) of information [62, 63, others] were unsuccessful.

1.6.3. The Model Uncertainty Zone and Its Evaluation

Each *o*-window between the extremal segment is a source of uncertainty generated by the random microlevel. Such a local uncertainty also exits at each locality of the triplet's node where the doublet joins a third extremal segment.

The optimal controls, by closing the *o*-window, overcome the uncertainty, connect the segments, and initiate a sequential time-space movement from each previous to a following IN node. Along such a movement, a zone of uncertainty UR, surrounding all IN node, is formed (Fig.6.4).

Each local uncertainty is evaluated by the irreversible time interval

$$\delta t_i = \delta^*(\gamma) t_i(\gamma, \gamma_i^\alpha, m),\ t_i(\gamma, \gamma_i^\alpha) = \alpha_{io}^t(\gamma_i^\alpha, m) / \mathbf{a}_o(\gamma), \tag{6.14}$$

which depends on the ratio of the imaginary and real segment's eigenvalues: $\gamma = \dfrac{\beta_{io}^t}{\alpha_{io}^t} = \mathbf{b}_o(\gamma)/\mathbf{a}_o(\gamma)$, the ratio of real eigenvalues γ_i^α, and the IN's node number *m*. This allows evaluating the UR it terms of the above model's invariant, which determines also the information invariants and the IN parameters (sec.1.5.3).

As it follows from the numerical computations of (6.14), at any γ<0.5, or γ>0.5, the UR is enlarging, compared to γ=0.5. This allows selecting the UR of minimal geometrical shape. A minimal δt_i grows with rising *m*. This leads to the UR extension at increasing *m* (Fig.6.4).

The UR information *border* can be evaluated by the relative increment of a segment's entropy (concentrated in UR) through the model's invariants:

$$h_o = (\mathbf{a}_o(\gamma = 0) - \mathbf{a}_o(\gamma^*)) / \mathbf{a}_o(\gamma = 0) \tag{6.15}$$

at changing γ from 0 to $\gamma = \gamma^*$, where γ^* corresponds to the segment's location at the UR border. The above consideration extends the VP application, allowing us to *conclude* that

(1)-*the model's invariants not only characterize the region between the model's real and imaginary eigenvalues, but also limit the UR*;

(2)-*the model's irreversible time* (ch.6.1) *emerges as an information measure of uncertainty for both each local o-window and the whole UR;*

(3)- *the UR* (6.14) *can be measured in a discrete unit of uncertainty, expressed in the bits of the information code, as well as by the cell's number and sizes in the IG.*

The last result is naturally connected to the notion of information as a measure of uncertainty, allowing us to introduce its geometrical representation and a new meaning via the

irreversible time. Both the reversible time intervals $t_i = \mathbf{a}_o(\gamma)/\alpha_{io}^{t}$ and the equivalent of the scalar space intervals $l_i = \mathbf{a}_{mo}/\alpha_{io}^{l}$ are determines by the model's eigenvalue spectrum.

The entire UR's geometry identifies the IN's geometrical border where the macromodel is open for the external interactions, modeled by the microlevel.

The conic structures, generating the UR cells, form an information background for the IN. At the given cells' logic, the sequence of cells' blocks forming the IN nodes, can be built. For such an IN, both $\mathbf{a}_m(\gamma)$ and $\mathbf{a}_{mo}(\gamma)$ are the *m* space distributed node's invariants, preserving their scalar components ($\mathbf{a}$, $\mathbf{a}_o$) for a fixed γ, and depending geometrically on the relative shifts of angle $\psi_m(\gamma_m^{\alpha})$ and the angle starting position.

1.6.4. Creation of the IN geometry and a Genetic Code of the Information Cellular Geometry

Let us describe the IG *creation* from the microlevel stochastics, considering the generation in a random space the distributed n-dimensional state vector $x(\bar{l})$ at the initial moment t_o, where $\bar{l}$ is the three-dimensional vector of geometrical coordinates, composed by the set of local geometrical vectors $\bar{l}\{\bar{l}_i(t_o)\}$ for each of $i \in n$ components of state vector x. We assume that the ensemble of the random initial $x(\bar{l}(t_o))$ is able to generate the $(n \times 3)$-dimensional differential macrooperator $A(\bar{l}(t_o))$ with the set of three-dimensional space vectors $\bar{l}_i(t_o)$ for each of i-dimension, defined by the $A(\bar{l}(t_o))$ eigenvectors (ch.1.4). Each vector from the $\bar{l}_i(t_o)$ set has an equal length and different initial angle. Total possible sets of these $\bar{l}_i(t_o)$-vectors can be represented by the rotation of the initial one, having a constant length vector $\bar{l}_i^{o}(t_o)$, on the angle, equivalent to the discrete space interval $l_i(t_i)$.

Applying the VP, we presume the existence of a rotating moment $M(i)$ (sec 1.5.4) acting on these $\bar{l}_i^{o}(t_o)$ at each of the *i* extremal segments. Under the rotating moment, each three dynamic eigenvectors of the macrooperator is undergoing the transformation of rotation, which is capable of successive diagonalizing the dynamic operator.

Such a set of vectors generates a family of cones and helixes on the cones, rotating around the chosen vector $\bar{l}_i^{o}(t_o)$. The $(n \times 3)$ dimensional set of all possible $\bar{l}_i^{o}(t_o)$-vectors $i \in n$ generates the rotating hierarchy of the cones, forming the IN's triplet's dynamic structures (Figs.5.5,6.5).

The model's switching control lines L_c are formed by the model's spatial intervals: $\Delta l_i = l_i(t_i) - l_{i+1}(t_{i+1})$. Both the multiple "left" $l_i(t_i)$ and right $l_{i+1}(t_{i+1})$ points can be found from the corresponding relations $l_i = \mathbf{a}_m(\mathbf{a}, \psi_m)/\alpha_{im}^{l}(l_i)$ and

$l_{i+1} = \mathbf{a}_m(\mathbf{a}, \psi_m) / \alpha^l_{i+1m}(l_{i+1})$, depending on the ranged discrete spectrum of the model's real eigenvalues, which grow up with rising the node's number $m = m(\gamma^{\alpha}_m)$.

The rotating L_c sets form the hyperbolic strip $L_s \in L_c$, which includes all L_s, located within the UR (Figs.6.4a-c).

The UR is determined by the admissible error E at the region of the joint triplet's cones. Each local cone's region error $\varepsilon_i \in E$, which covers the *o*-window for the IN's node, forms a circle area that defines the E-strip's error E. The rotating L_c allocates the UR geometrical volume V^n_3 with an external surface F^n_2. Particular switching control lines L_c could be located inside *and* on the surface of the uncertain zone UR. An entire UR's external surface F^2_n identifies the IN's geometrical border where the macromodel is open for the external interactions. Total F^n_2 is composed by the set of cross-sections of L_s-strips with E-strips: $E(\varepsilon_i) \times L_s(l_i(t_i))$ at the region of assembling the triple's cones. Each cone's region $\varepsilon_i \in E$ is orthogonal to the cone's vertex having the line-projections $\{l_{ci}\}$ of the surface F^n_2.

The time-space region $L_s(l_i(t_i))$ between each of the following $E(\varepsilon_i)$ cone's regions limits the UR's boundaries, which are determined by the above cross-sections. This leads to the external shape of the *elementary* UR's boundaries ($\varepsilon_i \times l_i$), which form a cell $C_F[l_i \times l_{ci}]$, where the F^n_2 unifies the C_F cell's sets for the L_s-strips, limiting the UR boundaries, while the F^n_2 shape takes into account the cells' specific forms.

The initial three segments, consolidated on the switching control line L_c, form a three-dimensional IG. Their following merge to the next four eigenvalues' strings can generate 7=4+3 *extra dimensional* space-time IG (with a total of 11 extra dimensions if the next four strings are added), which are located at the ε_i-region, wrapped into the UR [34].

The mutual dimensions' information interactions at DP-localities (with potential chaotic dynamics) are compensated by the needle controls, which bind these dimensions.

The information geometry is represented by a *discrete* cellular structure of invariant cones with a set of time-space piece-vise trajectories, located on the cones' surfaces.

Since each IN's node contains the same number of the IG cells, the cells sizes, characterized by the cell's volume V^n_3 and external surface F^2_n, are growing with the rising of the node number *m*. The cells' number within the node conceals the node's code, while the cell's space size, depended on *m*, specifies the node's position within IN.

The particular model's F^{ε}_2 structure carries the cells code as the IG logical information universal characteristic, which encloses both the creation of model's dynamics and geometry and includes all their phenomena.

A set of possible space trajectories at F^2_n define the potential creation of a wide *diversity* of macrostructures, generated by the genetic code.

We apply to the hyperbolic structure (Fig.6.4) with volume V and an external surface F both the Gauss-Ostrogradsky theorem in the form

$$\int_V grad\,\Delta s_v dv = \int_F \Delta s_v df$$

and the model's dynamic equation

$$\frac{\partial x_v}{\partial t} = l_v grad\,\Delta s_v, \frac{\partial x_v}{\partial t} = i_v,$$

where i_v is a differential information flow, determined by a differential information potential $grad\,\Delta s_v$ and an elementary volume's kinetic matrix l_v, while Δs_v is an information produced within an elementary volume by the model's information dynamics.

At a $grad\,\Delta s_v$, directed to the external surface, where information interactions with the environment takes place, the generated information is concentrating on the structure's external surface F.

The external surface's square and its cellular geometry impose a limit on the generated external information, which also restrict the model's dynamic flow.

Finding the external surface's square of the hyperbolic structure, formed by the IN's cellular geometry

Let find the square *F* of the surface Fig.6.5, which is obtained by a rotation of the switching control line in the form of a hyperbola $y = a / x$ around some axis.

Generally, a surface, formed by rotating a curve with an elementary length $dl=(1+dy/dx^2)^{1/2}$ around an axis (0-*x*), has the equation

$$F = 2\pi \int_A^B y[1+(dy/dx)^2]^{1/2} dx. \qquad (6.16)$$

Integrating this equation by using $y = a / x$ leads to the following relations:

$$F = 2\pi \int_A^B y[1+a^2x^{-4}]^{1/2} dx, y = ax^{-1}, x = ay^{-1}, dx = -ay^{-2}dy, x^{-4} = a^{-4}y^4,$$

$$F = 2\pi \int_A^B y[1+a^2x^{-4}]^{1/2} dx = -2\pi \int_A^B y[1+a^{-2}y^4]^{1/2} ay^{-2}dy = -2\pi \int_A^B [1+a^{-2}y^4]^{1/2} ay^{-1}dy,$$

$$y^4 = z, 4y^3dy = dz, dy = 1/4y^{-3}, F = -1/2\pi a \int_A^B [1+a^{-2}z]^{1/2} y^{-1}y^{-3}dz = -1/2\pi a \int_A^B [1+a^{-2}z]^{1/2} z^{-1}dz,$$

$$F = -1/2\pi a\{2[1+a^{-2}z]^{1/2} + \ln[[(1+a^{-2}z)^{1/2}-1][(1+a^{-2}z)^{1/2}+1]^{-1}]\}|_A^B.$$

. (6.16a)

And finally we come to

$$F = -\pi a[1+a^{-2}y^4]^{1/2} - 1/2\pi a \ln[[(1+a^{-2}y^4)^{1/2}-1][(1+a^{-2}y^4)^{1/2}+1]^{-1}]\}|_A^B. \qquad (6.17)$$

Let us estimate both the surface's square *F* and the length *L* for IN's dimension *n*, using the approximate relations

$$F \cong \mp \pi y^2,$$

at

$$y_A = r = 0.083c, y_B = R = 75c, [c] = m/\sec. \quad (6.17a)$$

We get

$$F = 17,671.458c^2, L \cong F/\pi R = 75c. \quad (6.17b)$$

Each of the IN's *m* node spot occupies area $f_o = F/m$ square per node.

Assuming that each spot's f_o contains a known number of a code's cell per the spot m_c, we get the spot's area per a single cell $f_o^c = f_o/m_c$ at each node-spot, or $f_o^c = 1$.

At $F = F_o c^2$, this relation allows us to determine the mode's linear speed c for each given m_c, F_o, and *m*:

$$c = (m_c m)^{1/2}(F_o)^{-1/2}. \quad (6.18)$$

In particular, at $m_c = 4cells$, $m = n/2$, we get

$$c = (2n)^{1/2}(F_o)^{-1/2}. \quad (6.18a)$$

The code's cells are produced by the IN's information $s_m^o = S_m/m$ in bits per node.

Thus, each f_o^c brings a related s_o^m and vice versa: $f_o^c \Leftrightarrow s_o^m$, from which it follows

$$S_m \Leftrightarrow F/m_c. \quad (6.19)$$

At $m_c = 4cells$ or 4bits, we come to

$$S_m \Leftrightarrow 1/4F, \quad (6.20)$$

establishing the direct relation between the external surface's square and the information accessible through the cell's geometry, distributed on *F*.

This imposes the *restrictions* on the model's ability to accept external information and communicate with an environment.

Let us estimate c using (6.18) at the model example's parameters (6.17a) with

$n = 10, F_o = 17,671458$. We get $c \cong 0.0363m/\sec$.

At $n = 22, R = 4645.4314$, we have $c \cong 0.00947m/\sec$.

Therefore, both a given code-word m_c (per the IN node) and the IN's surface F (6.17) *limit* the mode's linear speed c, which determines the node's movement along the surface and the code-cells' distribution on this surface.

Actually, the hyperbolic structure's volume is filled up by a set of the INs having dimensions from n=2 up to $n=N$=96. Their information codes, embedded in F, are the sources of communication for the internal models with the environment.

Let us ***Summarize the results*** *of secs.* 1.6.1-1.6.4.

The genetic DSS code, reflecting a particular information network for each macrostructure geometry, carries the logic of the quadrilateral curved cell connections. Optimal space trajectory creates an *optimal* genetic code, while a manifold of non-optimal trajectories may produce a variety of the codes.

The code's geometry is an essential component of the IN's double spiral structure, where the particular code sequence is an indicator of the quality of information.

Thus, the model's information geometry is represented in the following structure:

-the cone spiral space trajectories of model's macrostates;

-the cone's spiral trajectories of model's controls, formed by the doubling of the macrostates at the DP;

-the double spiral trajectory, which encloses the space dynamics of both the macrostates and controls;

-the eigenvector's space movement;

-the triplet's node formed by both the joint cone's trajectories and the joint eigenvectors;

-the IN node's hierarchy created by the multiple triplet cone's cooperative movement;

-the triple code, resulting from a sequence of the discrete intervals and the switching controls, measured in the bits of information (as the control's attribute);

-the IN's uncertainty zone's cellular discrete geometry with the code's information measure, and

-the external UR surface, carrying the DSS genetic code, while a total F_2^n shape, integrating the UR of all possible IN's structure, encloses the DSS genetics of those multiple macrostructures.

Finally, the information geometry describes the bound hierarchical information structure as a *systemic* category, which includes a *smooth* field of dynamic trajectories *within* each cone, the *discrete* cone's geometry with their *functional connections* (in the IN's form), and *curved* cellular discrete *geometry* with its information measure by the triple's genetic code. Within the UR, a chaotic movement could arise [34,45], which generates the *fractal* geometry at the sticking cone's vicinities, characterized by a specific geometric invariant.

A source of the bound dynamic information $\mathbf{a}_m(\mathbf{a}, \psi_m)$, determined by the invariant $\mathbf{a}(\gamma)$ and the node's location $\psi_m(\gamma)$, is able to create the IG space structure, which memorizes both the IN's organization and DSS code.

The quantity of information, enclosed into the IN final node, represents the IPF information measure, which integrates total information from the sequence of the IN nodes. Thus, either the IPF value, or the IN triplet's code embodies the whole IN, and is able to recreate a specific IN.

1.6.5. The Minimal Admissible Uncertainty and Its Connection to Physics

Let us evaluate a minimal $h_o = (\mathbf{a}_o(\gamma = 0) - \mathbf{a}_o(\gamma^*)) / \mathbf{a}_o(\gamma = 0)$ in (6.15) by finding such $\mathbf{a}_o(\gamma^*)$, which serves a threshold of changing the macromodel dimension m_n.

This means that at such a feasible $\gamma = \gamma^* > 0$, a *minimal* increment of the model dimension would be

$$\Delta m(\gamma^*) = m(\gamma = 0) - m(\gamma^*) = 1, \tag{6.21}$$

and any $\gamma < \gamma^*$ will not affect the model dimension, being an admissible within a given dimension.

Because the macromodel's dimension depends on the number of cooperations, the condition (6.21) limits both the actual increment of dimension and the model's ability to cooperate.

We evaluate $\Delta m(\gamma^*)$ using the model invariants that determine (6.21).

Proposition 6.4.

The minimal h_o, satisfying (6.21), is

$$h_o = 0.00729927 = 1/137, \tag{6.22}$$

with $\mathbf{a}_o(\gamma^*) = 0.762443796$, $\mathbf{a}(\gamma^*) = 0.238566887$, and $\gamma^* = 0.007148$.

Proof. According to sec.1.6.1, both regular and needle control produce an elementary negentropy $(\mathbf{a}_o^2(\gamma) + \mathbf{a}(\gamma))$, available for each cooperation, while each extremal segment consumes an elementary quantity of information $\mathbf{a}_o(\gamma)$.

So, the difference $(\mathbf{a}_o^2(\gamma) + \mathbf{a}(\gamma) - \mathbf{a}_o(\gamma))$ determines a cooperative information surplus, which can be used for forming the additional information cooperations, and the ratio

$$m_k(\gamma) = (\mathbf{a}_o^2(\gamma) + \mathbf{a}(\gamma)) / (\mathbf{a}_o^2(\gamma) + \mathbf{a}(\gamma) - \mathbf{a}_o(\gamma)) \tag{6.22a}$$

characterizes the number m_k of such potential cooperations.

At $\gamma = 0$, we have

$$m_o = m(\gamma = 0) = (\mathbf{a}_o^2(\gamma = 0) + \mathbf{a}(\gamma = 0)) / (\mathbf{a}_o^2(\gamma = 0) + \mathbf{a}(\gamma = 0) - \mathbf{a}_o(\gamma = 0)),$$

and at $\gamma = \gamma^*$, we get

$$m(\gamma^*) = m_1 = (\mathbf{a}_o^2(\gamma^*) + \mathbf{a}(\gamma^*)) / (\mathbf{a}_o^2(\gamma^*) + \mathbf{a}(\gamma^*) - \mathbf{a}_o(\gamma^*)). \tag{6.22b}$$

Applying the common formula (sec.1.5.2) connecting the above invariants:

$$\mathbf{a}_o(\gamma) = \exp(-\mathbf{a}_o(\gamma))(1 - \gamma^2)^{1/2}$$

$$\times [4 - 4\exp(-\mathbf{a}_o(\gamma))\cos(\gamma \mathbf{a}_o(\gamma)) + \exp(-\mathbf{a}_o(\gamma))]^{-1/2}, \ \mathbf{a}_o(\gamma) > 0,$$

we have at $\gamma_o = 0$:

$$\mathbf{a}(\gamma_o) = \exp(-\mathbf{a}_o(\gamma_o))[4 - 4\exp(-\mathbf{a}_o(\gamma_o)) + \exp(-2\mathbf{a}_o(\gamma_o))]^{-1/2}.$$

By substituting $\mathbf{a}_o(\gamma = 0) = 0.76805$ we get the corresponding $\mathbf{a}(\gamma = 0) = 0.231960953$ and $m_o = 15.2729035$.

Using the same formulas that connect $\mathbf{a}_o(\gamma^*)$ and $\mathbf{a}(\gamma^*)$ and satisfy (6.22b), we come to equation

$$15.2729035 - 1 = m_1,\ m_1 = f(\mathbf{a}_o(\gamma^*)),$$

from which we get $\mathbf{a}_o(\gamma^*) = 0.762443796$ and $\mathbf{a}(\gamma^*) = 0.238566887$.

These relations bring $m_1 = 14.2729035$ and evaluate both parameter $h_o = 0.0729927 = 1/137$ and $\gamma^* = 0.007148$, allowing us to estimate a minimal elementary uncertainty $\mathbf{a}(\gamma^*)$ separating the model's dimensions. ●

Comments 6.7. The minimal $\Delta\gamma^* = \gamma^* - \gamma_o = \gamma^*, \gamma_o = 0$ that restricts reaching $\gamma = 0$ is able to provide both invariant quantities of information $\mathbf{a}_o(\gamma^*)$ and $\mathbf{a}(\gamma^*)$, which can generate an elementary negentropy $s_h = \mathbf{a}_o^2(\gamma^*) + \mathbf{a}(\gamma^*) = 0.819887424$ that conceals a minimal cooperative uncertainty.

In a three dimensional space, this γ^* provides information necessary to form a corresponding triplet $m_k^3 = 3$, which carries 4 bits of genetic DSS code.

The minimal number m_1 (at this threshold) (6.22b) brings the three dimensional $m_1^3 \cong 42.8187105 \cong 43$ with a potential genetic code carrying 172 bits of a non redundant information. A non removable uncertainty is inherent part of any interaction (cooperation).

Its minimal relative invariant h_o evaluates an elementary increment of the model's dimensions in (6.21), while the absolute quantity of the hidden invariant information s_h is able to produce an elementary triple code, enclosed into the hyperbolic structure Fig.6.5.

This means that *a non removable uncertainly enfolds a potential DSS information code.*

Comments 6.8. The found model's structural invariant $h^o \cong 1/137$ coincides with the Fine Structural constant in physics [66]:

$$\alpha^o = 2\pi \frac{e^2}{4\pi\varepsilon^o hc},$$

where e is the electron charge magnitude's constant, ε^o is the permittivity of free space constant, c is the speed of light, h is the Plank constant.

The equality $h^o \cong \alpha^o$ between the model's and physical constant allows us to show, for example, that the Plank constant can be found using the model's structural parameter and by knowing the other constants in the formula:

$$h = \frac{e^2}{2\varepsilon^o c h^o} = C_h \alpha_h, \alpha_h = (h^o)^{-1} = inv,\ \frac{e^2}{2\varepsilon^o c} = C_h, \qquad (6.23)$$

where C_h is the energy's constant (in[J.s]), which transforms the invariant α_h into h.

The above equalities lead to a connection of the IMD model to Quantum Mechanics QM, considered below and in ch.1.9.

The structural invariant α_h determines the relative negentropy increment necessary for the creation of a single information element. This is a minimal increment, potentially generated by the deviations of the model's eigenvalue $\Delta\lambda_i$, corresponding to the deviations of related conjugated model's variables $\Delta\hat{x}_i, \Delta\hat{X}_j$.

Using the model's Hamiltonian equations:

$$\int_t \hat{H}_i dt = \hat{x}_i \hat{X}_j > 0,\ \hat{H}_i = 1/2\lambda_i, \tag{6.24}$$

for a conjugate λ_i, let us evaluate the minimal increment of $\Delta\hat{x}_i, \Delta\hat{X}_j$ corresponding the minimal limit of $\Delta\lambda_i$. We have

$$1/2\int_t (\lambda_i + \Delta\lambda_i)dt = (\Delta\hat{x}_i + \Delta\hat{x}_i)(\hat{X}_j + \Delta\hat{X}_j) \geq 0 \tag{6.25}$$

that brings the information increment $\Delta\hat{S}_i$, generated by $\Delta\lambda_i$:

$$\Delta\hat{S}_i = 2(\hat{x}_i\Delta\hat{X}_j + \Delta\hat{x}_i\hat{X}_j + \Delta\hat{x}_i\Delta\hat{X}_j) \geq 0, \tag{6.26}$$

which follows from (6.25), (6.26).

The *minima*l value of the positive sum on the right side of (6.26) is

$$\Delta\hat{S}_i^o = \min \int_t \Delta\lambda_i dt = 2\Delta\hat{x}_i\Delta\hat{X}_j \geq 0\ , \tag{6.27}$$

(which reaches its limited minimal value equal to $2\Delta\hat{x}_i\Delta\hat{X}_j \geq 0$ if $\hat{x}_i\Delta\hat{X}_j + \Delta\hat{x}_i\hat{X}_j = 0$), while the model's structural invariant $\alpha_h = (h^o)^{-1}$ is found from the minimal relative information increment, generated by the minimal $\Delta\lambda_i$.

Therefore, α_h is the relative measure of information $\Delta\hat{S}_i^o$, and h is an energy's measure of the same information $\Delta\hat{S}_i^o$.

If the informational conjugate variables acquire such physical measures: $\Delta Q_i \sim \Delta\hat{x}_i, \Delta P_j \sim \Delta\hat{X}_j$, that their multiplication will bring the energy's measure of $\Delta\hat{S}_i^o$, then this multiplication corresponds to the minimal energy's measure of α_h, which is proportional to h. Because the α_h is a minimal increment, $\Delta\hat{S}_i^o$ is also a minimal information increment, corresponding to h, and therefore, any increments of physical

variables, being adequate to the minimal α_h, should not exceed h, restricted by the inequality

$$2\Delta Q_i \Delta P_j \geq h. \tag{6.29}$$

We got the famous Heisenber's uncertainty relation $\Delta Q_i \Delta P_j \geq 1/2h$ for the physical conjugate variable $\Delta Q_i, \Delta P_j$, representing the increments of a coordinate and momentum (or the increments of energy and time), whose multiplication satisfies to the above energy's measure h in QM.

Both these variables are identifiable separately, but cannot be measured simultaneously.

The invariant quantity of information $\mathbf{a}_o$ at the model's local equilibrium $\gamma = 0.5$ is an equivalent of an elementary information quanta $\ln 2$, which can be delivered through the needle control (in the form $\mathbf{a}_o^2$), accompanies each segment's interaction.

According to formulas (6.24), (6.25), a superimposition of the conjugate variables $\hat{x}_i$ and $\hat{X}_j = \hat{x}_i^*$ corresponds to their conjugate eigenvalues λ_i, λ_i^* being additive:

$$\int_t \hat{H}_i dt = \hat{x}_i \hat{X}_j \ , \ \hat{H}_i = 1/2(\lambda_i + \lambda_i^*). \tag{6.29a}$$

Following this formula, the model's conjugate eigenvalues $\lambda_{io} = \alpha_{io} + j\beta_{io}, \lambda_i^* = \alpha_{io} - j\beta_{io}$, *at the above superimposition, produce the elementary quantity of information*

$$\delta \hat{S}_{ij} = 1/2 \int_t (\lambda_i + \lambda_i^*) dt = 1/2 \int_{t_i} (2\alpha_{io}) dt = \alpha_{io} t_i = \mathbf{a}_o. \tag{6.30}$$

during an elementary time interval t_i.

The invariant quantity of information $\mathbf{a}_o$ at the model's local equilibrium $\gamma = 0.5$ has a distinctive information value depending on its location at the IN's hierarchy (sec.6.2).

If the cooperating segments are represented by the conjugate variables capable of superimposition, the quantum $\mathbf{a}_o$, as well as the needle control's negentropy can be delivered by the information, generated by the above superimposition.

The VP differential constraint's equation (secs. 1.3.3, 1.3.5), imposed by stochastics, originates the local dynamics within each extremal segment and connects the segments' macrodynamics. This constraint is responsible for arising the cooperative dynamics and the macrodynamic regularities.

From the VP solution with the differential constraint follows the equation (ch.1.3, 1.3.38) with the information *forces*, acting between the local dynamic processes. In the regular Hamiltonian mechanics these forces are not present.

1.6.6. Information Structure of the model Double Spiral's (DSS) Control Mechanism

Let us reveal the formation of the DSS along the model's line of switching controls L_c, which joins the macrojectory's segments into a consolidating process $x(t)$.

This consolidated trajectory, as a macrostate's carrier, has a single dimension for both cone's spiral lines and phase trajectories, described by the equations:

$$\dot{x}(t) = \alpha^t(t_i)(x(t) + v(t_i)),\ v(t_i) = v(\tau_i) = -2x(\tau_i),\ i = 1,....,m, \qquad (6.31)$$

(at given initial $x_o = x(t_{io}), \alpha^t(\tau_{i-o})$), where $v(t_i)$ are the optimal controls, applied at the DP's $\{t_i\}_{i=1}^{n-1}$ by doubling the macrostates $x(t_i)$ at each DP moment τ_i.

After applying this control at each at $t_i = \tau_i$ the equations (6.31) acquire a form

$$\dot{x}(\tau_i) = -\alpha^t(\tau_i)x(\tau_i), \qquad (6.31a)$$

and then, after transferring in a next segment with $t_{i+1} > \tau_i$, the process continues according to equation

$$\dot{x}(t_{i+1}) = -\alpha^t(\tau_i)(x(t_{i+1}) + v(t_{i+1})) \quad (6.32)$$

up to a moment $t_{i+1} = \tau_{i+1}$, when at

$$v(t_{i+1}) = v(\tau_{i+1}) = -2x(\tau_{i+1})$$

holds

$$\dot{x}(\tau_{i+1}) = \alpha^t(\tau_{i+1})x(\tau_{i+1}). \qquad (6.32a)$$

At the subsequent segment with $t_{i+2} > \tau_{i+1}$, the process follows the equation

$$\dot{x}(t_{i+2}) = \alpha^t(\tau_{i+1})(x(t_{i+2}) + v(t_{i+2})). \qquad (6.32b).$$

It is seen that at $t_i = \tau_i$, the control

$$u(\tau_i) = \alpha^t(\tau_i)(x(\tau_i) - 2x(\tau_i)) = -\alpha^t(\tau_i)x(\tau_i) = \dot{x}(\tau_i)$$

coincides with $\dot{x}(t_{i+1})$ from the next segment's equation (6.32) at $t_{i+1} = \tau_i, v(t_{i+1}) = 0$:

$$\dot{x}(t_{i+1}) = -\alpha^t(\tau_i)x(t_{i+1}).$$

The following control

$$u(\tau_{i+1}) = \alpha^t(\tau_{i+1})x(\tau_{i+1}) = \dot{x}(\tau_{i+1})$$

coincides with $\dot{x}(t_{i+2})$ from (6.32b) at $t_{i+2} = \tau_{i+1}, v(t_{i+2}) = 0$, and so on.

The optimal trajectory moves along the sequential segments with the alternating opposite signs of the local information speed $\alpha^t(t_i)$, and the optimal controls

$$u(\tau_i) = \dot{x}(\tau_{i+1}), i = 1,..., m$$

are formed by the phase speeds $\dot{x}(\tau_{i+1})$ at the beginning of each following segment with alternated $\alpha^t(t_{i+1})$.

In reality, these controls $u(\tau_i)$ (or the corresponding reduced controls $v(\tau_i) = -2x(\tau_i)$) should be formed and applied *before* the moment τ_i comes and/or when the following segment's process starts with $\dot{x}(\tau_{i+1}) = \dot{x}(t_{i+1})$.

This is possible at a simultaneous existence of two complementary mutually controlled processes during each t_i: one of them is (6.31) , another one has an opposite sign of $\alpha(t_i)$ according to (6.32). (Or considering the complementary processes (6.32) and (6.32b)).

Then, the control $u(\tau_i)$ can be formed at the beginning of the segment's process (6.32), while this process is running in opposite time direction starting at $-t_{io}$.

In that case, the macrostate $x(\tau_i - o)$ would be equivalent to $x(-t_{io} + o)$ at a concurrent time for both processes $(\tau_i - o) = -t_{io} + o$.

The states $x(\tau_i - o)$ and $x(-t_{io} + o)$, taken simultaneously from both spirals, bring $2\,x(\tau_i - o)$, which corresponds the required reduced controls in (6.31).

As a result, the controls is formed *automatically* by getting the phase speeds (or the reduced control's macrostates) at a moment $\tau_i - o$ (preceding τ_i) and applying them at τ_i sequentially from the alternating segment's spirals processes.

The related controls

$$u(-t_{io} + o) = \dot{x}(-t_{io} + o), v(-t_{io} + o) = -2x(-t_{io} + o)$$

are applied to the alternating segment's process.

Let us consider how this strategy might be implemented.

Instead of using the time coordinate, the analogous solutions of these equations can be presented along the opposite spiral's linear geometrical directions: $+l_i$ and $-l_i$ (with corresponding $\alpha^l(l_i^\tau)$ and $-\alpha^l(l_i^\tau)$ accordingly):

$$\dot{x}(l_i) = \alpha^l(l_{i-1}^\tau)(x(l_i) + v(l_i)),\ v(l_i) = -2x(l_i^\tau),$$
$$l_i = Ct_i, \alpha^l = \alpha^t C^{-1}, l_i^\tau = C\tau_i, i = 1,..., m, \qquad (6.33)$$

$$\dot{x}(-l_i) = -\alpha^l(l_i^\tau)(x(-l_i) + v(-l_i)), -l_i = C(-t_i), v(-l_i) = +2x(l_i^\tau), \qquad (6.33a)$$

where C is the process's linear speed, l_i is a current linear space interval corresponding t_i, l_i^τ corresponds τ_i. (Such opposite directional cone's spirals and their parameters are considered in sec. 1.5.4).

The macrostate's control $-2x(l_i^\tau)$, governing the equation (6.33), is prepared during the interval $-l_{i-1}$ and applied, starting at the point $(l_i - o)$.

The macrostate's control $+2x(l_i^{\tau})$, governing the equation (6.33a), is prepared during the interval l_{i-1} and applied, starting at the point $(-l_i + o)$, Fig. 6.6.

This means that both complimentary trajectory' segments mutually control each other, alternatively switching the process from equation (6.33) to equation (6.33a) and vice versa.

Using the positive and negative *time* directions, we would not be able to introduce the complimentary processes, because of the nonequivalence of these time directions in the IMD irreversible equations. By inverting the l_i-line directions of these linear chains we preserve the same positive time course for both complimentary processes.

Such self-forming controls and the initiated them the corresponding segments with the macrotrajectories are mutually complementary, producing the replicated macrostates.

The $\delta(l_i) = o_i$ and $\delta(-l_i) = -o_i$ localities of each l_i and $-l_i$ accordingly correspond to the UR existence around L_c. Because of that, the considered regular reduced controls: $v(l_i)$, $v(-l_i)$ generate automatically also the needle $\delta(v_i)$ controls, applied simultaneously to each equation (6.33), (6.33a) at each small interval $\delta(l_i) = o_i$ $\delta(-l_i) = -o_i$.

The needle controls secure a correct direction of the extremal segments and the corresponding cones' spirals, providing a structural process' stability along the cone sequence. The complimentary processes can also be generated by the two interacting operator's eigenvalues at the IN last interval $t_{n-1}(l_{n-1})$, providing a feedback to the IN's staring process.

The spatial trajectory segments, corresponding the considered complimentary processes, are represented by the cones' spirals of the opposite directions forming the information double spiral structure DSS (Fig. 6.6). The DSS's right directional (R) and the left directional (L) spirals could locate on the same L_c cone's sequence.

The DSS's linear structure is a carrier of both the doubling macrostates $x(l_i^{\tau})$ and corresponding controls, which produce the considered IN's code. The controls generate the code sequence simultaneously at each of $(l_i - o)$, $(-l_i + o)$ for both spiral's complimentary segments.

The specific code depends on the current locations l_i, l_i^{τ} along the L_c for each IN's triplets. At the locality of each cone's vertex, both spirals generate the macrostate's codeword $c[x(l_i)]$ (with $\mathbf{a}_o$ information measure), which carries both $v(l_i)$ and $\delta v(l_i)$ control codes in DSS.

The DSS code can also be *created* by the *superimposing* processes of different dimensions that are not necessary located on the L_c line. The controls, *generated* by a DSS primary code, can initiate this process. Actually, the DSS can be used for both the generation of the control code and encoding the renovated double spiral macrostates.

The DSS geometrical structure is determined by the cone radius vector ρ_i, the angle at the cone vertex ψ_i^o, the angle of spiral rotation φ_i, and the spatial angle ψ_i between the nearest cone's spirals (Figs. 5.6, 6.2). All these parameters are the functions of VP invariant, because the spatial cone trajectories are the geometrical representations of the VP extremals. For example at γ=0.5: $\varphi_i = \pi$, $\psi_i^o \approx \pi/6$, $\psi_i \approx 5.73\pi$, $\rho_i = l_i/\pi$.

The double chain and its coding language are similar to ones for DNA.

As an alternative to *experimental* DNA discovery, the double chain and its coding language implements the IMD mathematical model.

Moreover, we consider the double chain spiral structure as a *general genetic generator for any optimal macrostructures, following from the VP for the IPF*.

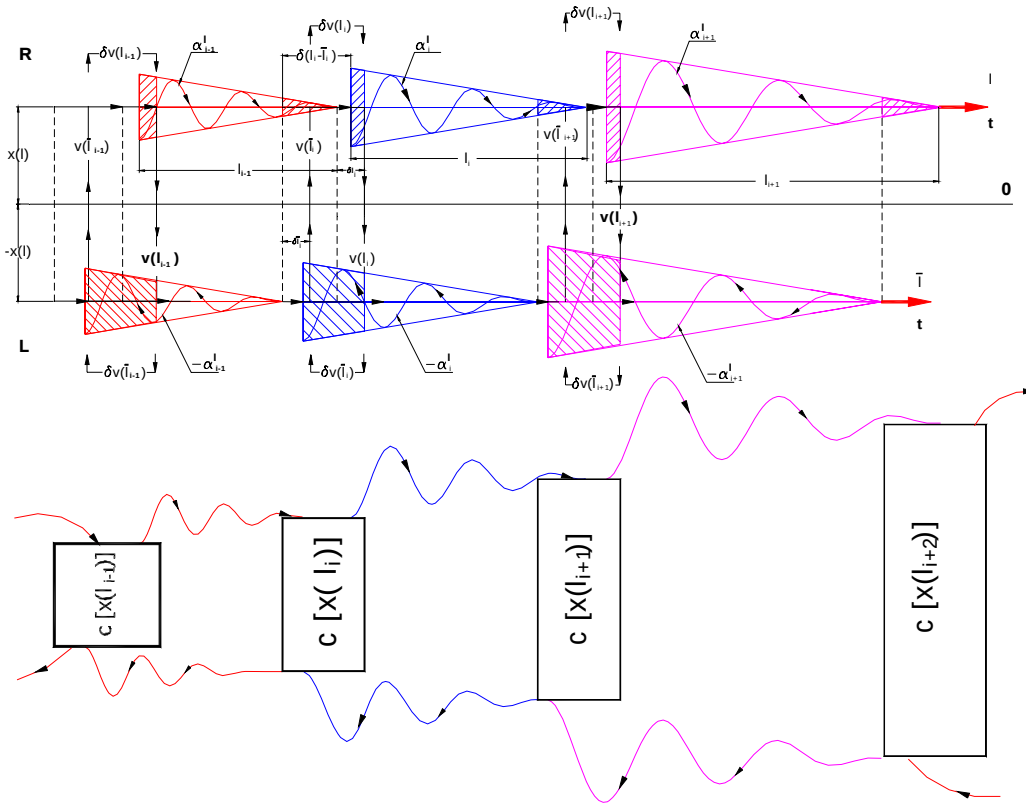

Figure 6.6. Information structure of the double spiral *DSS*: a).The self-controlling processes, located on opposite spiral's cones, which take into account the delay of applying controls: $\delta(l_i)$ and $\delta(\bar{l}_i)$, formed by a UR or the spiral's shift; b). *DSS*'s structure, obtained from a) with the code $c[x(l_i)]$ generated by each spiral pieces.

1.6.7. Examples of the DSS codes

Each triplet in the IN structure can be encoded, as a minimum, by the sequence of three IN string's symbols $\alpha_{1t}, \alpha_{2t}, \alpha_{3t}$, for example at $\gamma = 0.5$, they are α_{1t}=3.011, α_{2t}=1.35918, α_{3t}=0.77301, which are characterized by the following sequence of the triplet IN's ratios: $\gamma^{\alpha}_{12} = \alpha_{1t} / \alpha_{2t}$=2.2155, $\gamma^{\alpha}_{23} = \alpha_{2t} / \alpha_{3t}$=1.7583.

Let us encode the above sequence $\alpha_{1t}, \alpha_{2t}, \alpha_{3t}$ by a subsequence of the alphabet's letters: ABC, whose ratios code have the same values of the γ^{α}_{12}, γ^{α}_{23}.

The triplet, encoded by the subsequence BCA, has the different ratios: γ_{23}^{α}=1.7583, γ_{31}^{α}=0.2667, and the subsequences CAB, BAC have the ratios: γ_{31}^{α}=0.2667, γ_{12}^{α}=2.2155 and γ_{21}^{α}=0.451365, γ_{13}^{α}=3.8955 accordingly.

Actually to specify the very first letter of the code, the correct sequence of the ratios needs one more symbol, for example (3.011, 2.2155, 1.7583) for the code ABC.

A more *general* code's structure is $(N_{ik}, \gamma_{ij}^{\alpha}, \gamma_{jk}^{\alpha})$, where N_{ik} identifies the ordinal number of the first letter (among the chosen and ranged alphabet's letters) whose ratios $\gamma_{ij}^{\alpha}, \gamma_{jk}^{\alpha}$ are used for the i,k coding sequence.

For example, the above codes have the following general structure: ABC: $(1, \gamma_{12}^{\alpha}, \gamma_{23}^{\alpha})$, BCA: $(2, \gamma_{23}^{\alpha}, \gamma_{31}^{\alpha})$, CAB: (3, γ_{31}^{α}, γ_{12}^{α}), and so on.

Therefore, each sequence of the alphabet letters for a triplet's code can be represented by the corresponding sequence of the IN string's symbols or their ratios. The vice versa is also correct: any sequence of the IN's string can be encoded by the subsequence of the chosen alphabet's letters, and each triplet can be encoded by a minimum of three letters of this alphabet. The *m*=*n*/2 sequence of this code describes the sequential IN's model of the independent *m*-triplets. The sequence of the enclosed triplet's codes represents the IN's nested structure with a code *hierarchy*.

The initial string's symbols describe the object's specifics and can be found by the object's identification procedure, which could also classify the different objects in terms of their basic parameters (**n**, γ ,**k**) (representing the object's information functional). For a given IN with a fixed (**n**,γ ,**k**), each triplet has the same three letter's code, but the microlevel's influences and mutations, which affect the parameters (**n**,γ ,**k**), are able to modify the code of following triplets. This leads to diverse combinations of the code's letters, generating an *evolving* DSS chain, which encodes the variations of the object's characteristics.

The *fourth* letter-symbol of the IN's fixed triplet's code carries a repeating ratio of the triplet's letters, for example α_{4t}=0.34891, brings the same ratio $\gamma_{34}^{\alpha} = \alpha_{3t} / \alpha_{4t}$=2.2155 as γ_{12}^{α} has. This fourth letter compensates the possible code's errors, contributing the adaptation potential (ch.1.). Under the mutations, this letter brings new ratio to the code sequence, which together with the ratio γ_{12}^{α}, identifies the code ratios γ_{23}^{α}, γ_{34}^{α} for a new modified triplet.

This letters-symbol also serves as a bridge to each next triplet's code, carrying the code's hierarchical dependency. Each triplet's code is responsible for generation of the three *superimposing processes*, and created them the cross-phenomena.

The fixed IN's DSS encodes the **n** such processes (ranged by the α_{it} string values) which are able forming a real solid macrostructure with $m = \mathbf{n}/2$ cross-phenomena, sequentially enclosed each other.

The DSS can also encode the triplets' sequence belonging to the distinct fixed INs that brings a variety of the macrostructures with different basic parameters. Such a DSS code is not predetermined by the code of first triplet. The four letters of this extended DSS code would have more combinations in an expanded long sequence.

The above examples show a possibility to develop a *universal code's structure* for variety of systems models.

1.6.8. A System's Harmony, Regularities, and the VP

Harmony is a fundamental and universal phenomenon of any system in nature, which accompanies a mutual frequency locking in the cooperating resonances. It provides information about the tendencies of the nature to share rhythm, vibrating in harmony like music. This rule of music is also applied to a harmony of life.

The question is: Does it exist a law of harmony as a formal mathematical principle? What is a universal mechanism generating this phenomenon?

It was the Pythagoras discovery that the simultaneous sound of two strings becomes a harmonic if their frequencies are related *approximately* as 1/2 for an octave (seventh) and 2/3 for a quintet (fifth).

For the comparison, the IN triplets have a repeating frequency's ratios from f_{2o}/f_{1o}=0.407 and f_{3o}/f_{2o}=0.55 to f_{2o}/f_{1o}=0.507 and f_{3o}/f_{2o}=0.619 (by changing γ=(0−0.75)), which at γ=0.6 are f_{2o}/f_{1o}=0.469 and f_{3o}/f_{2o}=0.576 accordingly. These IN's nearest starting triplet's strings have the following sequence of the frequency's ratios: 2.13, 1.74; 2.13, 1.74, with the total frequency spectrum's ratios, counted from a maximal $f_o(n)$ (n-system's dimension): 2.13, 3.70, 7.89, 13.74, 29.26, where the octaves alternates with a quintets and each IN's triplet covers spectrum of 12 inner frequencies.

A particular system's spectrum depends upon both n and γ.

This means that the optimal IN satisfies the spectrum of most harmonic sounds, like a virtual music instrument, accepting only the incoming harmonics playable on this instrument.

The VP application leads to integrating and *cooperating* the extremal's segments into a system's chain, sequentially *synchronizing their frequencies*, automatically ranging and encoding the frequencies in the IN's nodes. The chain's segments become bound by the frequencies resonances, which, by transferring a maximal information along the chain, create the *harmony* for every one of the system's segments. Such a system, possessing the *internal* harmony, is sensible to perception of external *musical rhythms*.

In a cognitive reflection, the frequencies resonate with the individual's processes (intiated by a cognition's motivation) creating a satisfaction, while a fundamental inner harmony is a result of evolution, whose regularity follow from the VP.

The system's harmony is encoded by a sound of each frequency f_i, its time duration t_i, the "sound's strength" x_i, and the hierarchy of the node's geometrical locations.

Because the IN is an attribute of cooperative dynamics and both are the "product" of the variation principle, the VP is a basic *mathematical source of the harmony*, and the IN is *a universal information mechanism* generating the harmony.

We *assume* that the information field on the surface of the cooperative hyperbolic structure (Fig.6.5) unifies a multiple IN's harmonic coexistence.

Indeed. The volume V of this structure encloses a multiple collection of the IN's systems, which are able to interact by the IN node's information code (at the considered in sec.1.6.4 restrictions).

Information cooperation unifies different forms of interactions from various physical, economic exchanges, consumptions, up to diversity of *virtual* connections and communications.

The INs triplet's cooperating strings are generating the frequency's spectrum, producing the considered synchronized *harmonic* chain of the frequencies, bound by the frequencies resonances (which depend on the frequency's specific ratio γ, affecting the system's stability). These resonances, transferring along the IN chain, automatically range and encode the frequencies in the IN's nodes in the form of the code-cells (Fig. 6.5).

Therefore, a set the cooperating INs systems, occupying a common volume, are capable of a *harmonic coexistence* with a generation of their common code-cells on the volume surface *F*.

Thus, the information surface *F* and its code-cells can be *produced* by a harmonic IN's *collective*, occupying the volume.

An external system might interact with the cooperative through this surface by adding its specific cooperative connection to *F* for consuming a collective–generated *common* information, or producing it.

This collective information can also be used by the inner volume's systems for getting the needed information or generating it.

The codes-cells information distributed on the surface might be considered as an *information field*, bordered the volume, where collective productions of both the internal and external volume's systems are concentrated.

Because each IN, as well as the INs sets of different dimensions, models a *system*, the bordered information field might belong to variety of *collectives*, combining a diversity of their interactions and communications.

The *F*-information field, being not a directly visible, is formed around each functioning system-virtual and/or real. Such a collective information field is a *product* of a society and can be used by both particular system, society, and/or other interacting societies.

Finally, a society or their sets can coexist in a harmony and stability, determined by the system's nets synchronization frequencies, through the *cooperative exchanges and communications*, delivered the necessary information.

An external *intellect* (in any forms) could exist as an *information system*, producing negentropy, which might not obey physical laws, requiring a conservation of an energy.

But such a creative information system, to be a stable, should obtain an intellectual information from the environment, which might include a human society as a civilization.

Chapter 1.7

THE MACRODYNAMIC AND COOPERATIVE INFORMATION COMPLEXITIES

1.7.1. Introduction

Basic complexity measures have been developed in algorithmic computation theory [1-4], important indicators of complexity have been proposed in physics [5-8]; numerous other publications [9-20] are connected with these basics. (Please see the references to ch.1.7).

These complexity's measures focus on the evaluating complexity for an already formed complex system.

We intend to analyze an *origin* of complexity in an interactive dynamic *process* with its *elements' cooperation* into a joint system, accompanied by creation of new phenomena, which in turn, are the *potential* sources of a complexity.

The universality of information language allows a *generalization of* the description of various interactions in terms of the *information* interactions, considered independent of their specific forms and nature.

Focus on interactive *informational* dynamics leads to a study of a *dynamic* complexity resulting from the interactions of information flows, measured by the specific information speeds. That's why the dynamic information complexity should be connected with the information speeds rather than just with a quantity of information in the above publications.

An intuitive notion of a system's complexity, which distinguishes complexity from simplicity, is associated with the assembling of the system elements into a joint system during a cooperative process.

This means that the system complexity is naturally connected to its ability to cooperate, which depends on the phenomena and parameters of *cooperative* dynamics. It has been pointed out repeatedly that algorithmic complexity [1-4] does not fit the intuitive notion of complexity [5-6].

The system complexity, emerging from the cooperative dynamics of a *multiple* set of interacting processes (elements), having an adequate information measure, has not been studied yet [9-20].

The main questions are: What is a general mechanism of cooperation and the condition of its origin? Does there exist a general measure of a *dynamic* complexity independent of a particular physical-chemical nature of the cooperative dynamics with a variety of their phenomena and parameters? How can the dynamic *multi-dimensional* cooperative complexity be defined and measured?

The answers for these questions require a new approach leading us to a *unified notion of dynamic information complexity*, measured in terms of quantities and qualities of information by a corresponding information code.

The *objective* consists of the definition and formulation of the complexity's information measure, the analysis of the complexity's origin in cooperative dynamics, and both analytical and computational measure's connections to the informational dynamic parameters.

Compared to known publications, we analyze the complexity as an *attribute* of the process's *cooperative dynamics*, considering both the phenomenological concept and the formal measure of the complexity.

It is shown that a system complexity depends on both the between element's connections and the element's number.

Analysis of the *regularities* of collective dynamics, accompanied by a formation of cooperative structures, can be formalized using a variation principle (VP), applied to the informational path *functional* and equations of informational macrodynamics (IMD).

This chapter studies both the complexity of information macrodynamic process and the cooperative complexity, arising within a set of elements (dynamic objects) from the elements' involvement in the information transitions and communication with a focus on dynamics of this process. The cooperative complexity indicates and mesasures an *information difference between* the independent elements *and* their integration into a cooperating untit.

The complexity is evaluated by an increment of each element's internal concentration of information (an object's information capacity), measured by a change of an external relative information flow. It is shown that a common indicator of the origin of the cooperative complexity is the *specific* entropy's speed (related to a speed of the object's volume), rather than only the entropy, as it was accepted before. The results are extended on a wide class of complex systems with the physical and virtual interactions of different nature represented by transitions of information, including a communication process. We also consider the complexity's connections to other related complexity measures.

The chapter objective is implemented by introducing a formal notion of MC complexity and its information indicator (secs. 1.7.1, 1.7.2), and then considering the MC-emergence in an elementary cooperative dynamic process with the connection between diffusion and kinetics in cooperative dynamics (sec. 1.7.3).

Then, sec. 1.7.4 introduces the complexities' informational invariant measure; sec. 1.7.5 generalizes the MC complexity measure for multi-dimensional cooperative dynamics, integrated in the hierarchical IN with the MC *hierarchical invariant* information measure, which evaluates both quantity and quality by the IN's triplet's code, and provides the MC direct computability; sec.1.7.6 applies the information geometry's space equations to determine an intensity of information attraction and the complexity measures; sec.1. 7.7 connects the MC-complexity's to K complexity.

1.7.2. The Notion of Interactive and Cooperative Complexities and Their Information Measures

Definition 7.1. Let us have two sources of information S_i, S_k concentrated in geometrical volumes V_i, V_k accordingly, which are able to *interact* (affecting one another) by some portions of their information (entropy) $\Delta S_{ik} \neq 0$, accompanied by a change of a shared volume $\Delta V_{ik} \neq 0$, and characterized by the corresponding information speeds $-\frac{\partial \Delta S_{ik}}{\partial t} = H_{ik}$.

Then the *information measure of complexity* for the interacting sources: MC_{ik} is defined by an information speed, concentrated in shared volume ΔV_{ik}:

$$MC_{ik} = \frac{H_{ik}}{\Delta V_{ik}}, \tag{7.1}$$

or an instant entropy's concentration in this volume: $\frac{\partial \Delta S_{ik}}{\Delta V_{ik} \partial t}$ (the entropy production), which evaluates the specific information contribution, transferred during the source's interaction in *dynamics*.

Definition 7.2. Let us consider an increment of complexity $\delta(MC_{ik})$, generated by an increment of the specific information (entropy) speed:

$$\delta\left(\frac{\partial \Delta S_{ik}}{\Delta V_{ik} \partial t}\right) = \delta\left(\frac{\partial \Delta S_{ik}}{\partial t}\right) / \Delta V_{ik} - \frac{\partial \Delta S_{ik}}{\partial t} \frac{(\delta \Delta V_{ik})}{(\Delta V_{ik})^2} = \frac{(\delta \Delta V_{ik})}{(\Delta V_{ik})}\left[\delta\left(\frac{\partial \Delta S_{ik}}{\partial t}\right) / (\delta \Delta V_{ik}) - \frac{\partial \Delta S_{ik}}{\Delta V_{ik} \partial t}\right],$$

which at a small ΔV_{ik}^{δ} satisfies $\delta \Delta V_{ik} \cong \delta V_{ik}^{\delta} \neq 0$, and we get

$$MC_{ik}^{\delta} = \delta MC_{ik} + MC_{ik}, \tag{7.1a}$$

where $\delta MC_{ik} = -\delta\left(\frac{\partial \Delta S_{ik}}{\Delta V_{ik} \partial t}\right), \delta H_{ik} / \delta V_{ik}^{\delta} = MC_{ik}^{\delta}$ at $-\frac{\partial \Delta S_{ik}}{\partial t} = H_{ik}$.

Then MC_{ik}^{δ} is *the information measure of a differential interactive complexity, defined by* the increment of the information flow $-\frac{\partial \Delta S_{ik}}{\partial t}$ per a small volume increment δV_{ik}^{δ} (within

the shared volume ΔV_{ik}), or $MC_{ik}^{\delta} = \frac{\partial H_{ik}}{\partial t} / \frac{\partial \Delta V_{ik}}{\partial t}$ is defined by the ratio of the above speeds. The MC_{ik}^{δ} automatically includes both the MC_{ik} and its increment δMC_{ik}.

Comments 7.1. The information transition includes transferring both the information flow and the volume. At $\delta^* V_{ik}^{\delta} = \frac{\delta V_{ik}^{\delta}}{\delta \Delta V_{ik}} \to +1$, the shared volume ΔV_{ik} increases, and at $\delta^* V_{ik}^{\delta} = \frac{\delta V_{ik}^{\delta}}{\delta \Delta V_{ik}} \to -1$, the shared volume decreases, shrinks.

The shrinkage is associated with the sources' S_i, S_k volumes assembly (cooperation) while the volume enlargement is associated with the volume source's disassembly. According to the second thermodynamic law, the internal (per volume) entropy production can only increase: $\delta(\frac{\partial \Delta S_{ik}}{\partial t}) / \Delta V_{ik} > 0$ at a fixed ΔV_{ik} and any changes of the S_i, S_k external entropies. This leads to $\delta H_{ik} < 0$ at a fixed ΔV_{ik}.

Therefore at $\delta V_{ik}^{\delta} < 0$, $\delta H_{ik} < 0$, the corresponding differential information complexity $MC_{ik}^{\delta} > 0$, and at $\delta V_{ik}^{\delta} > 0$ we have $MC_{ik}^{\delta} < 0$. In both cases, an existence of the information flows ($H_{ik} \neq 0$ and its increment $\delta H_{ik} \neq 0$) determines the involvement of both S_i, S_k in the information transition, which are a *necessary* condition that connects S_i, S_k and can assemble them into a cooperative.

The corresponding $MC_{ik}^{\delta} > 0$ we call the *cooperative complexity measure*.

Comments 7.2. At $H_{ik} < 0$ and $\delta \Delta V_{ik} / V_{ik} = -1$, we get also $MC_{ik} > 0$ at the cooperation. A single source S_i, possessing the volume's concentration of entropy $s_i = \frac{S_i}{V_i}$, can generate outside a total potential increment $\delta s_i = \frac{\delta V_i}{V_i}(\frac{\delta S_i}{\delta V_i} - s_i)$.

Using the corresponding *information* measures:

$S_i = -H_i, s_i = -h_i$ at $|\frac{\delta V_i}{V_i}| = 1$

we get the increment in the form $MC_i^{\delta} = \delta h_i + h_i$, where $MC_i^{\delta} = \frac{\delta H_i}{\delta V_i}$ is the source's *potential* differential complexity, which can also be written through the ratio of the source's information speed to volume speed: $MC_i^{\delta} = \frac{\dot{H}_i}{\dot{V}_i}$.

In particular, at $\frac{\delta V_i}{V_i} = -1$ and $\delta H_i < 0$ the complexity $MC_i^{\delta} > 0$.

Comments 7.3. The examples of the interacting information sources by their differential entropies, in particular, represent each of the IN node (Figs.1.5.5a, b), and the nodes' space distributed volume is their shared volume.

The volume's specific formulas are in ch.1.5: (1.5.109, 1.5.109a, and b). Whereas each IN node is created by the interaction of three space segments (a triplet) of the path functional's extremals.

1.7.3. The Information Indicator of a Cooperative Complexity

Let us have *a set of elements (objects)* $\Delta v_j \in \Delta V$, $j = 1,...,k,..,n$ with the internal concentrations of information $\frac{\Delta h_j}{\Delta v_j} = h_j^v$ (as an object's information capacity) for each *j and* the element's possibility of a free movement within a volume ΔV. Suppose within the set exists an element k with an *internal* $\frac{\Delta h_k}{\Delta v_k} = h_k^v$, whose increment δh_k^v is a *source* of information transferred to other elements in a communication process. This means that an *external* increment of the element's k information δh_k^e (related to the element's surface) is transmitted to some other element j, satisfying relation $\delta h_k^e \geq \delta h_j^e$, where δh_j^e is the j-external increment (related to the element's surface) that affects its internal h_j^v according to the corresponding equality $\delta h_j^e \geq \delta h_j^v$.

Proposition 7.1.
The information indicator of the element's *k* involvement in the cooperative association with other elements of the above set serves the ratio

$$\delta_h(\Delta h_k) = \frac{dh_k}{dt} / \frac{\Delta v_k}{\Delta t_k} > 0, \tag{7.2}$$

where $-\frac{d(\Delta s_k)}{dt} = \frac{dh_k}{dt}$ is the element's information flow and $\frac{\Delta v_k}{\Delta t_k}$ is the volume's fixed speed (both of them assume to be controllable); *then* the above ratio (7.2) is applicable in both cases:

(i)-at a fixed volume Δv_k, relation (7.2) leads to

$$\delta_h(\Delta h_k) = \frac{d(\Delta h_k)}{dt} \tag{7.2a}$$

within the time interval Δt_k ; *and*

(ii)-if the transmission of the information flow is accompanied by transferring of the element's information volume, *then* the ratio (7.2) acquires the form

$$\delta_{h,v} h_k^v = \frac{\delta h_k}{\Delta v_k} - \frac{h_k \delta v_k}{(\Delta v_k)^2} = \frac{\delta v_k}{\Delta v_k}\left(\frac{\delta h_k}{\delta v_k} - h_k^v\right) = \frac{\delta v_k}{\Delta v_k}\left(\frac{\dot{h}_k}{\dot{v}_k} - h_k^v\right), \qquad (7.2b)$$

Indeed. Considering the increment δh_k^v at a fixed volume Δv_k, we come to relations

$$\delta_h\left(\frac{\Delta h_k}{\Delta v_k}\right) = \frac{\delta(\Delta h_k)}{\Delta v_k}, \delta(\Delta h_k) = \frac{d(\Delta h_k)}{dt}\Delta t_k,$$

where Δt_k is a time interval at a fixed $\frac{d(\Delta h_k)}{dt}$ and at a fixed Δv_k, which brings (7.2a).

When the transmission includes both the information flow and the element's information volume, then (7. 2) leads to (7. 2b). •

Corollary 7.1.

At a small Δv_k relation (7.2b) admits $\frac{\delta v_k}{\Delta v_k} \to 1$ and it leads to $\frac{\dot{h}_k}{\dot{v}_k} = \delta_{h,v} h_k^v + h_k^v$.

In this case, the information transition includes a relative increment of information volume. At $\frac{\delta v_k}{\Delta v_k} \to +1$, the element's initial volume increases, and at $\frac{\delta v_k}{\Delta v_k} \to -1$, the element's volume could be transferred to others during the transitions. In both (7.2a), (7.2b), function $\delta h_k^v > 0$ characterizes the element source's *informational potential* as its ability to involve other elements in the information connection. •

An ability of the element's for the information connections by adjoining other elements into a cooperative we *call* the element's *potential complexity* MC_{pk}^{δ}, which is measured in both formulas by the ratio of the information speed to the volume speed.

Specifically, at transferring the information flow:

$$MC_{pk}^{\delta} = \frac{dh_k}{dt} / \frac{\Delta v_k}{\Delta t_k} > 0,$$

this function coincides with the element information potential.

With changing both the element's information flow and the volume, the potential complexity also includes the element's volume internal concentration of information h_k^v:

$$MC_{phv}^{\delta} = \frac{\dot{h}_k}{\dot{v}_k} = (\delta_{h,v} h_k^v + h_k^v) > 0.$$

The positivity of both above functions can be reached at the same signs of $\frac{dh_k}{dt}$ and $\frac{\Delta v_k}{\Delta t_k}$ (or $\dot{v}_k$). More generally, at $\frac{\delta v_k}{\Delta v_k} \neq |1|$, both functions acquire the form

$$MC^{\delta v}_{phv} = \frac{\dot{h}_k}{\dot{v}_k} = \frac{\delta_{h,v} h^v_k}{\delta v_k} \Delta v_k + h^v_k . \tag{7.3}$$

Comments 7.4. At $MC^{\delta}_{pk} \to 0$, the information flow, transferred by the element, decreases, reaching zero in a limit, when the element is unable to make any connections to others and becomes independent, even though it could be located close to its neighbors. At $MC^{\delta}_{phv} \to 0$, we get the same result, even though $\delta_{p,v} h^v_k > 0$, because from MC^{δ}_{phv}=0 follows equality $\delta_{h,v} h^v_k = -h^v_k$, that means this increment is consumed by the element itself without transferring its information outside. Therefore, a positivity of above complexity measures is an *indicator* of the element's a *cooperative activity*, which characterizes the element's ability to transfer its information to other elements.

Proposition 7.2.

The information increment *transferred between* elements (k, j) follows from (7.3) in the form: $$h^v_{k,j} = \frac{dh_{k,j}/dt}{\Delta v_{k,j}/\Delta t_{k,j}} \geq 0,$$

with $$\delta_h h^v_{k,j} = \frac{dh_k/dt}{\Delta v_k/\Delta t_k} - \frac{dh_j/dt}{\Delta v_j/\Delta t_j} = MC^{\delta}_k - MC^{\delta}_j = \Delta MC^{\delta}_k \geq 0 , \tag{7.4}$$

where the elements are coordinated in such a way that one of them (k) is a source of information for others $(j, m, ..., n)$, which serve as the information consumers, and $MC^{\delta}_j = \frac{dh_j/dt}{\Delta v_j/\Delta t_j} > 0$ becomes an *indicator* of the *element* j *cooperative activity*, which for this element characterizes its ability to be involved in communication with a source (and possibly with other elements). (Generally, we assume that an object possesses an ability for a *self-controllable* cooperative activity *or* an opposite ability for inactivity). •

Corollary 7.2.

With changing both the element's information flow and the volume, the element's *j* cooperative activity acquires the form analogous to (7.3):

$$MC^{\delta v}_{jhv} = \frac{\dot{h}_j}{\dot{v}_j} = \frac{\delta_{h,v} h^v_j}{\delta v_j} \Delta v_j + h^v_j . \tag{7.4a}$$

Condition (7.4), applied to transferring both the information flows and volumes between the elements, leads to relations

$$\delta_{h,v} h^v_{k,j} = \delta_{h,v}(h^v_k - h^v_j) = \frac{\delta v_k}{\Delta v_k}\left(\frac{\dot{h}_k}{\dot{v}_k} - h^v_k\right) - \frac{\delta v_j}{\Delta v_j}\left(\frac{\dot{h}_j}{\dot{v}_j} - h^v_j\right) . \tag{7.5}$$

At $\frac{\delta v_k}{\Delta v_k} = -\frac{\delta v_j}{\Delta v_j}$ the element j's volume relative increment decreases by transferring it to the increasing element k's relative volume increment. In this case, $\dot{v}_k > 0$ and $\dot{v}_j$ <0, that it's why $\frac{\dot{h}_k}{\dot{v}_k} > 0$ and $\frac{\dot{h}_j}{\dot{v}_j} < 0$.

For $h_j^v = \frac{\Delta h_j}{\Delta v_j}$, we have

$\Delta v_j = -\Delta v_k \frac{\delta v_j}{\delta v_k}$ and h_j^v <0 at Δh_j >0.

At $\frac{\delta v_k}{\Delta v_k} = -\frac{\delta v_j}{\Delta v_j}$ =1,

a total volume of the element j passes to the element's volume k, merging with that element's volume. This merge corresponds to a binding of both element's information flows and volumes. At this case, relation (7.5) acquires form

$$\frac{\dot{h}_k}{\dot{v}_k} - \frac{\dot{h}_j}{\dot{v}_j} = (\delta_{h,v} h_k^v + h_k^v) - (\delta_{h,v} h_j^v + h_k^v), \tag{7.6}$$

where $\Delta v_j = -\delta v_j$. This means that each element's connection is measured by the same form of indicator

$$MC_j^\delta = \frac{\dot{h}_j}{\dot{v}_j} > 0, \tag{7.6a}$$

which is subtracting from the source's potential complexity measure

$$MC_{phv}^\delta = MC_k^\delta = \frac{\dot{h}_k}{\dot{v}_k}.$$

<u>*Comments*</u> <u>7.5.</u> *Decreasing of a current difference* ($MC_{phv}^\delta - MC_{jhv}^\delta$) means *arising* of the information connections between the elements' volumes $\Delta v_j \in \Delta V$, or growing of the involved information flows and the volumes that bind them. A total number of the elements, satisfying condition (7.4), or (7.6) (in the MC_{phv}^δ environment), measures a current cooperative complexity of the set. Each partial transition of the initial relative flow $\frac{\dot{h}_k}{\dot{v}_k}$ to the following elements' $\frac{\dot{h}_j}{\dot{v}_j}$, $j, m, ..., n$ decreases the potential complexity and increases the

number of elements merged into the cooperative. A current difference: $(\frac{\dot{h}_k}{\dot{v}_k} - \frac{\dot{h}_j}{\dot{v}_j}) = \Delta MC^{\delta}_{kj}$, represented by (7.5) or (7.6), measures an *increase* of cooperative complexity in the *dynamics*, accompany by a growing number of the involved elements.

We call ΔMC^{δ}_{kj} the *dynamic* cooperative complexity.

A total decline of a current potential complexity

$$\Delta MC^{\delta}_{phv} = MC^{\delta}_{phv} - \sum_{j}^{N} MC^{\delta}_{jhv} \tag{7.7}$$

is measured by the sum of cooperative activities of the elements *N* involved in the information transitions.

At ΔMC^{δ}_{piv}=0, the potential complexity covers the cooperative activities of all involved elements, even though their individual activities are not equal. If all involved elements are joined into a complex, then MC^{δ}_{pkv} measures the complexity of this cooperative.

If the individual measure of activity $MC^{\delta}_{j} = \frac{\dot{h}_j}{\dot{v}_j}$ is equal for all connected *n*-elements $j, m, ..., n$, and the potential complexity's measure covers the sum of the element's individual measures, satisfying relation $MC^{\delta}_{pkv} - nMC^{\delta}_{j}$=0, $n = MC^{\delta}_{pkv} / MC^{\delta}_{j}$, *N*=*n*, then all n elements, joined into the cooperative, get the equal divided source's information contributions, as the equal information consumers.

Even though the individual's MC^{δ}_{j} are not equal, their total number, deducted from MC^{δ}_{pkv}, measures the number of elements. Thus, both the potential complexity $MC^{\delta o}_{pkv}$ and the individual element's cooperative activity MC^{δ}_{j}, as well as the dynamic complexity, are measured by the *specific* information speed related to the speed of volume at the information transitions, which join the elements.

A maximal cooperative complexity

$$\max_{h,v} MC^{\delta}_{pkv} = \max_{h,v} \frac{\dot{h}_k}{\dot{v}_k} = \max_{h,v} (\delta h^{v}_{k} + h^{v}_{k}) > 0 \tag{7.8}$$

limits the complexity of the cooperative.

Assigning measure (7.6a) to each acting element (object) allows ordering and classification of the objects in terms of their cooperative activities. The object, having a higher MC^{δ}_{j} value, compared to some others (*m*, *m+1*,.. , *n*, *n+1)*, possesses the corresponding cooperative potential ΔMC^{δ}_{jm}, being able to attract these elements and form a local cooperative (in an addition to a "central" cooperative with the *k*-source).

A sequential forming of such local cooperatives leads to appearance of an information *cooperative net* with the spanning branches (Fig. 7.1).

In the process of cooperation, the potential in (7.7) decreases, bringing finally ΔMC_{kj}^{δ} to zero at joining the elements into a common unit. As a result, the elements, belonging to a cooperative, possess the MC_j^{δ} values depending on the local potential in (7.7) that initiates the elements' movement into a local cooperative. A higher potential allows to activate and cooperate more element's numbers, serving the complexity measure for each local cooperative, whose potential covers the complexities of the joint elements. Ranging the local cooperative's complexities by their values allows classifying the local cooperatives (by their *real* complexities) and creating a hierarchy of the cooperatives with different element's number. This leads to a hierarchical information network (IN), whose hierarchy is formed by the nodes, enfolded the local cooperatives and arranged by these complexities (Fig.7.1a).

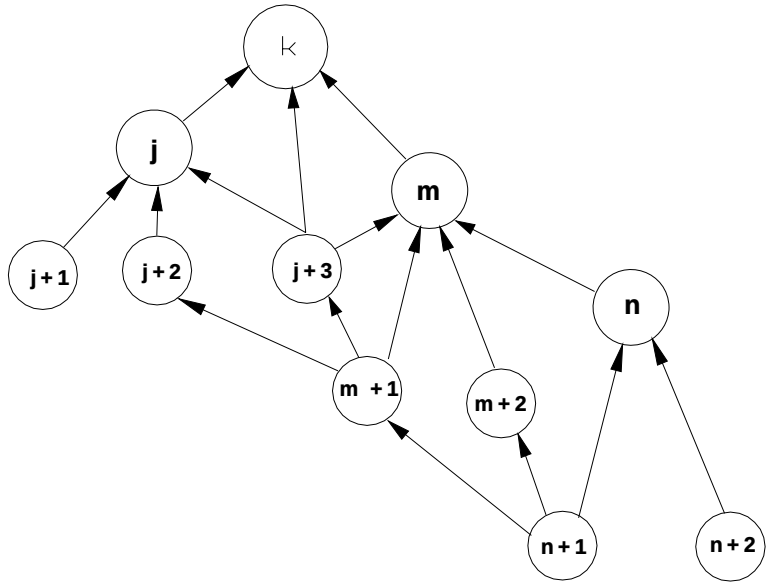

Figure 7.1. An information cooperative net with the arbitrary spanning branches.

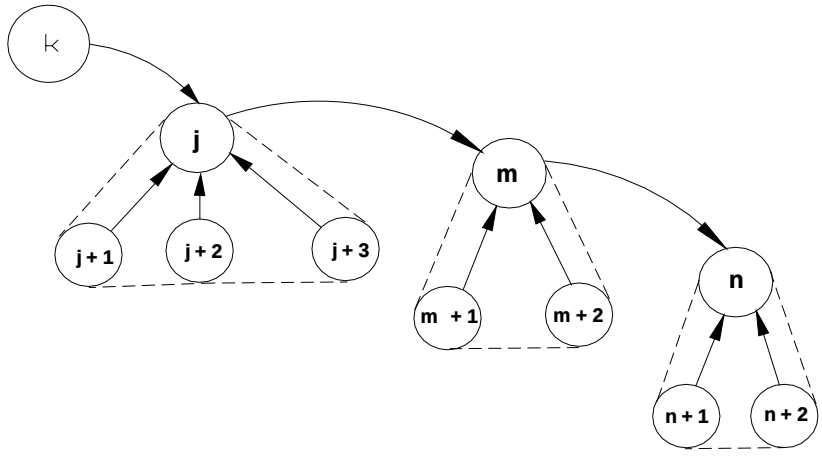

Figure 7.1a. An hierarchical information cooperative network (IN) with the ranged complexity's of the local cooperatives and their sequentially cooperations into a common unit.

In a such an IN, a sequence of nodes with the decreasing ranged *potential* activities, representing the potential complexities of the local cooperatives, could be a capable for a further cooperation, consecutively involving the IN's neighbor levels and reaching finally a complete cooperation at a lower IN's level.

This IN orders the cooperative activities of the elements' set in such a *system*, where a whole set of active elements, cooperating into a common unit, is evaluated by the complexity measures at each step of the local cooperation, as well as by the complexity of the joint unit.

Compared to an arbitrarily spanned branching net (Fig.7.1), the IN presents the ordered organized sequence of the merging local cooperatives, holding a cooperative *organization* of a given set of dynamic objects.

Indicator (7.7) is applicable to both cases (Figs.7.1,7.1a) allowing a discrimination of the connections and the restoration of a current information net, formed during the communications.

The time-space distributed *information network* (chs.1.4-1.5) allows the automatic allocation and measurement of the MC-local complexities for a multi-dimensional process, taking into account their *mutual dependencies*.

Collective macrodynamics in (chs.1.4-1.6) is a source of the IN with an *optimal* distribution of the information flows between the collective's elements according to the element's measure of cooperative activity. These processes' cooperative complexity is encoded in the IN communication code, allowing the ΔMC^{δ}_{kj} transmission through the common communication channels, including the specific ΔMC^{δ}_{kj} allocation at each IN *j*'s hierarchical level, with the following evaluation of the IN level by the ΔMC^{δ}_{kj} *-digital* measure. (The IN's code is an analog of the MC^{δ}_{pkv} code.)

1.7.4. Illustration of Arising of the Information Cooperative Complexity at Discrete Points of Applied Controls

Methods of Irreversible Thermodynamics (IT) [21,23,26,28] have been applied to describe superimposing phenomena, in terms of the generalized thermodynamic flows and forces of a cooperative process.

The universality of the information language and the informational nature of the IPF led us to Informational Macrodynamics (IMD) as an *information analogy* of the IT.

The macroprocess at the VP's extremals is described by the IT-IMD n-dimensional kinetic equation

$$\dot{x}_t = LX \ , \ \dot{x}_t = I, X = \frac{\partial S}{\partial x}, \tag{7.9}$$

where I is a vector of a generalized flow is, X is a vector of a generalized force, L is a kinetic matrix.

The microprocess, at a *locality* of the considering punched locality, is modeled by the n-dimensional stochastic differential equation (1.1.1).

The VP, applied to both $\tilde{x}_t, x_t$ (at a fixed $\tilde{x}_t^1$), imposes the constraint on vector X, binding these processes at the above $o(\varepsilon)$ locality:

$$X(\tau) = (2b_\tau)^{-1}\dot{x}_t(\tau),\ b_\tau = b(\tau), \tag{7.10}$$

where diffusion matrix σ of (1.1.1) is defined through a dispersion matrix b. Comparing the equations (7.9) and (7.10), we come to the equalities, connecting the kinetics and diffusion, the model's micro- and macrolevels

$$L_\tau = 2b_\tau, L_\tau = L(\tau),\ L_\tau = \sigma\sigma^T(\tau). \tag{7.11}$$

Several superimpositions increase the number of cooperative processes and their contributions. These and other essential properties of collective dynamics, accompanied by a formation of cooperative structures, can be formalized using the minimax variation principle (VP), applied to the IPF (ch.1.3). In the macrodynamic model, the controls implement the variation principle (VP) by selecting the macrotrajectory extremal's segments in a multi-dimensional process, divided by a "punched" microlevel's window, which is a source of new information. The additional "needle" controls, applied *between* the extremal's segments, *stick them into a cooperative structure* and successively join into a multi-dimensional cooperation.

The "needle" control action consists of *switching* the macroprocess from the extremal's section, satisfying the entropy minimum, to the extremal's piece, satisfying the entropy maximum, and back to the entropy minimal extremal's section for each model's dimension. The control's switch closes a jump of the corresponding entropy influx at each border's "punched" point (DP) between the section's discrete intervals. During the jump, a local chaotic motion might arise with a chaotic resonance, which is able to join the current equal probable macrostates (at the section's borders) into the cooperative structure (physically performing the control function). For the considered transformation, at the border of the $o(\varepsilon)$-window, we get the following *conditions*, defining the *indicators of a conjugation between the kinetics and diffusion*: at $L(\tau - o) \geq \sigma\sigma(\tau)$, -the kinetic flow transfers to diffusion; at $\sigma\sigma(\tau) \geq L(\tau + o)$ -the diffusion flow transfers to kinetics. Changing the signs of macrodynamic forces in the border (3.173a) links up to the arise, change , and decrease of these flows. The cooperation brings a physical analogy of the states' superposition into a *compound* state, accompanied by a nonsymmetry of the formed *ordered* macrostructures.

The jump is a source of the *irreversibility* at each DP and the nonlinear phenomena, accompanied by the creation of new properties. In particular, the above collective properties are also associated with forming dissipative structures [23].

Forming a dissipative structure possesses fundamental features of cooperative dynamics, such as *transformation* of macrodynamics to stochastic fluctuations, accompanied by instabilities, thermodynamic irreversibility and asymmetry, and the *transformation* of the stochastics $\tilde{x}(t)$ into the renovated and stable dynamics $x(t)$, leading to the creation of a complex system. At the dynamic-stochastic border, for the irreversible phase transformations [23, 24], the probability function in a limit acquires the form

$$P(x, \tilde{x}) = C_t\delta(x(t-o) - \tilde{x}(t)) + C_{t+o}\delta(\tilde{x}(t) - x(t+o)),$$

(in our indications, where a weight sum of the coefficients equals 1). A development of these phenomena, in particular, depends on the system's starting conditions [25] securing an

instability on the border and a subsequent stability of a formed composite structure. Many chemical kinetic processes, providing a cooperation of molecules [22], are accompanied by a push-pull impulse, which captures and joins the interacting molecules into a cooperative structure (Fig.7.2). Actually, the *multi-dimensional* superimposition also embraces a potential *inner* "needle" control action, associated with modeling of the process' jump-wise interactions (by an analogy with Fig. 7.2). This control (sec. 1.3.5), implementing condition (1.3.160), connects the process' segments of different dimensions that decreases the process' initial dimension and creates the cooperative dissipative structures.

In the case of a piece-wise *self-forming* control, such a decrease of the dimension is associated with forming the Prigogine dissipative structures [21].

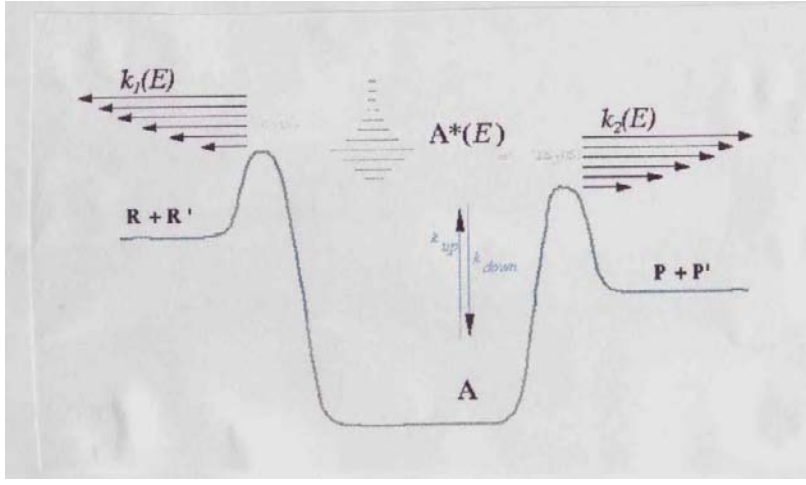

Figure 7.2. Simulated in [22], the chemical kinetic mechanism for the behavior of fluorocarbons in flames; $k_1(E), k_2(E)$ are the energy's (E) densities for a sum of states, generated by the cooperative reactions $R+R^I$, $P+P^I$ at the A –locality, under the k_{up}, k_{down} the push-pull's impulse.

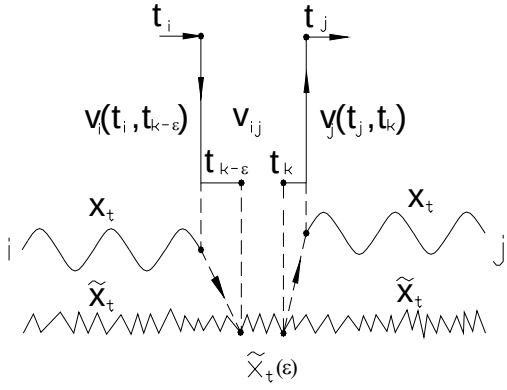

Figure 7.2.a. The illustration of transformations of the i - extremal's segment with a macroprocess x_t to a segment $\tilde{x}_t(\varepsilon)$ of the microprocess $\tilde{x}_t$ and back to the j -segment x_t under controls $v(t_i, t_{k-\varepsilon}), v(t_k, t_j)$ .It seen that this figure is analogous to Fig. 7.2.

Macrocomplexity MC_{ik} (sec.7.1-2) measures the specific quantity of information concentrated *within* and at a border of each extremal segment, while the complexity for a total macrotrajectory MC^{Σ} measures a summary of such information contributions from all of the macrotrajectory's sections. The differential complexity: MC^{δ}_{ik} characterizes the phenomena arising from the transformation from one extremal section to another by the *jump* through a microlevel's stochastic process during the $o(\varepsilon)$ -window between the *i*,*j* sections (Fig. 7.2a).

These phenomena consist of transferring first the macromovement from an ordered local stable (but a nonequilibrium) macroprocess (at one extremal segment) to a disordered microlevel's process with a local instability. The second transformation brings this microlevel process back to the macromovement at the subsequent stable extremal segment. MC^{δ}_{ik} evaluates the contributions to complexity, generated by the *appearance* of new features during the information interaction with the environment, while MC_{ik} evaluates an *accumulation* of this complexity at each interval of an *assimilation* of these features.

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Later on we find the concrete forms for both complexity's measures and constructively evaluate their values.

Let's start with MC^{δ}_{ik}, whereas the increment is generated by the model's needle control, performing the extremal segment's *cooperation* and bringing the above contributions. In this case, MC^{δ}_{ik} acquires a meaning of *cooperative complexity*.

Assume we have a couple of the extremal's sections i, j along a macroprocess with the needle control connecting them, and let's evaluate the *information* contribution by the needle control δv_{ij} actions, implementing both of above transformations at a small time interval between the segments (Fig.7.2a):

$$\delta v_{ij} = v_i(t_i, t_{i-\varepsilon}) + v_j(t_\varepsilon, t_j)\,. \tag{7.12}$$

We illustrate δv_{ij} using a *finite* approximation of an external needle control (Fig. 7.2a) that consists of two parts:

$$v_{ij} = v_i(t_i, t_{k-\varepsilon}) + v_j(t_k, t_j)\,, \tag{7.12a}$$

where ε is a small time interval between the end of control $v_i(t_i, t_{k-\varepsilon})$ and the start of control $v_j(t_k, t_j)$. (Theoretically, $\varepsilon = 0$, and a total needle control's impulse (during $(t_j - t_i)$ we divide here on two step-wise discrete functions $v_i(t_i, t_k)$ and $v_j(t_k, t_j)$).

Control $v_i(t_i, t_{k-\varepsilon})$ transfers the macroprocess x_t (at t_i), defined by a kinetic matrix $L_i = \sigma\sigma^T(t_i)$, to a diffusion process $\tilde{x}_t$ with duffusion matrix $\sigma\sigma^T(t_{k-\varepsilon})$; control $v_j(t_k, t_j)$ transfers this process' matrix $\sigma\sigma^T(t_{k-\varepsilon})$ to $\sigma\sigma^T(t_k)$ and then back to the macroprocess with a kinetic matrix $L_j = \sigma\sigma^T(t_j)$.

According to (ch.1.3.5), these transformations bring the changes of the corresponding eigenvalues, taken on a macrotrajectory, from $\lambda_i(t_i)$ to $\lambda_i(t_{k-\varepsilon}), \lambda_j(t_k)$ and then to $\lambda_i(t_j)$, where the moments $(t_i, t_{k-\varepsilon}, t_k, t_j)$ are defined by the nonrandom controls (7.12a) acting on the operator shift of (1.1.1), and therefore on both micro- and macrolevels.

A total eigenvalue's increment delivered by the control is

$$\Delta\lambda_{ij} = \lambda_j(t_j) - (\lambda_j(t_k) + \lambda_i(t_{k-\varepsilon})) - \lambda_i(t_i) \to (\lambda_j(\tau + o) - \lambda_{ij}(\tau) - \lambda_i(\tau - o)), (\lambda_j(t_k) + \lambda_i(t_{k-\varepsilon})) \to \lambda_{ij}(\tau) .. \tag{7.12b}$$

From this consideration and the difinitions (sec.7.1) we get

Proposition 7.3.

(i)-The MC_{ik}^{δ} complexities measures, generated by the needle control v_{ij} (7.12a), are

$$MC_{ik}^{\Delta} = \Delta H_{ik} / \Delta V_{ik}^{\Delta},\ \Delta H_{ik} = -\Delta(\frac{\partial S_{ik}}{\partial t}) = -\frac{\partial S}{\partial t}(t_i) + \frac{\partial S}{\partial t}(t_{k-\varepsilon}), \tag{7.13}$$

$$MC_{kj}^{\Delta} = \Delta H_{kj} / \Delta V_{kj}^{\Delta},\ \Delta H_{kj} = -\Delta(\frac{\partial S_{kj}}{\partial t}) = -\frac{\partial S}{\partial t}(t_k) + \frac{\partial S}{\partial t}(t_j), \tag{7.14}$$

with a total information increment

$$\Delta H_{ij} = \Delta H_{ik} + \Delta H_{kj} = -\Delta(\frac{\partial S_{ij}}{\partial t}) \text{ at } \frac{\partial S}{\partial t}(t_{k-\varepsilon}) \neq \frac{\partial S}{\partial t}(t_k), \tag{7.15}$$

where by applying the needle control δv_{ij} (7.12) at $\Delta \to \delta$ we get these complexities in the forms

$$MC_{ik}^{\delta} = \delta H_{ik} / \delta V_{ik}^{\delta}, MC_{kj}^{\delta} = \delta H_{kj} / \delta V_{kj}^{\delta}, MC_{ij}^{\delta} = \delta H_{ij} / \delta V_{ij}^{\delta}, \tag{7.16}$$

(*ii*)- The entropy increments (7.15), (7.16) are determined by the increments of related eigenvalues (7.12b) (delivered by controls $v_i(t_i, t_k)$ and $v_j(t_k, t_j)$), with a total information contribution

$$\delta H_{ij} = 4\delta\lambda_{ij}, \tag{7.16a}$$

(following from (3.154a)), which is a necessary for the cooperation and is supplied by control v_{ij}. •

(At the fulfillment of (7.16a), the complexities (7.1) – (7.8), (7.13) – (7.16a) can be expressed directly via the models Hamiltonian (3.145b) at the same indications for the above information contributions and the related Hamiltonian).

Example 7.1. We illustrate the application of (7.15) on the example from physical kinetics for a chain of connected chemical reactions.

Following [28], the kinetic equation between the number N_i of components for i-reaction and the number N_k of components for k-reaction is

$$\frac{\partial N_i}{\partial t} = (-\lambda_{ik} N_i + \lambda_{ki} N_k),\ \lambda_{ik} + \lambda_{ki} = 1, \tag{7.17}$$

where $-\lambda_{ik} N_i$ represents a decrease of N_i at ik-transition, while $\lambda_{ki} N_k$ represents an increase of N_k at ki-transition; $\lambda_{ik}, \lambda_{ki}$ here are the probabilities of transitions for these components, and are independent of time. The transition takes place during a time interval Δt_{ik} under a fixed increment of chemical potentials $\Delta\mu_{ik} = \mu_i - \mu_k$ for these reactions.

The specific entropy production for this transition is

$$\frac{\partial \Delta S_v}{\partial t} = \frac{\partial N_i}{\partial t}\Delta\mu_{ik} = (-\lambda_{ik} N_i + \lambda_{ki} N_k)\Delta\mu_{ik}, \tag{7.18}$$

where ΔS_v is an increment of specific entropy.

Applying (7.15),(7.16) at $\Delta t_{ik} \to \partial t$ and using the derivation, we come to the cooperative complexity

$$MC^d_{ik} = -\lambda_{ik}\frac{\partial N_i}{\partial t}\Delta\mu_{ik} + \lambda_{ki}\frac{\partial N_k}{\partial t}\Delta\mu_{ik} = -\frac{\partial \Delta S_{vi}}{\partial t} + \frac{\partial \Delta S_{ki}}{\partial t}, \tag{7.18a}$$

which is measured by the above increment of the entropy productions between the above reactions, generating new components.

These transitions join the above reactions in a *cooperative*.

For a chain of these reactions, we get a sum, taken at the corresponding time-space's locations.

An example of a cooperative behavior is also the Belousov-Jabotinsky (BJ) reaction [21,29] with a mutual transformation of its components during the oscillations.

The "three molecular" model of chemical reactions (brusselator) represents "an ideal system for studying cooperative processes in chemical kinetics" [23].

In the brusselator, a mutual interaction of two reactions gives rise to a third one, which has a feedback action on the first two, producing their cooperative dissipative structures. Both BJ's and brusselator's cooperative complexities are evaluated analogously to eq.(7.18a).

Comments 7.6. Applying relations (7.10) we can write (7.9) in the form:

$$\dot{x}_t(\tau) = A(\tau)x(\tau),\ A(\tau) = 1/2\dot{r}(\tau)r^{-1}(\tau),\ r(\tau) = M[\tilde{x}(\tau)\tilde{x}(\tau)^T],\ A = A_o(x+v), \tag{7.19}$$

where $r(\tau)$ is a covariation matrix, determined at the $o(\varepsilon)$-locality, with control v.

This allows us also to connect both the kinetic and covariance matrixes, opening an option for directly measuring the kinetics in (7.9) by (7.11).

The increment of forces ΔX_{ij} is initiated directly by the entropy production:

$$\Delta X_{ij} = \frac{\partial}{\partial \dot{x}}(\Delta \frac{\partial S_{ij}}{\partial t}), -\frac{\partial S_i}{\partial t} = H_i = 1/2\dot{x}^T(t_i)X(t_i), -\frac{\partial S_j}{\partial t} = H_j = 1/2\dot{x}^T(t_j)X(t_j), \tag{7.20}$$

where for a stable system (7.10) at $\tau = t_j$, $A(\tau = t_j) < 0$, the force $X(\tau = t_j) < 0$; and at a local instability around t_i with $A_i(t_i + o) > 0, X_i(t_i + o) > 0$, the increment

$$\Delta X_{ij} = X_i - X_j = -(2b_{i\tau})^{-1}A_i(t_i + o)x(t_i + o) + (2b_{j\tau})^{-1}A_j(t_j)x(t_j) \tag{7.20a}$$

acquires a sum of negative values. This indicates that the resulting force generates an *attracting* action, which joins the extremal's segments, where at $A_i(t_i + o) > 0, \frac{\partial S_i}{\partial t}(t_i + o) > 0$, and the entropy on the extremal reaches a local *maximum*: $\Delta S(x(t_i + o)) \to \max$, while at $A_j(t_j) < 0$, $extr\Delta S(x(t_j)) \to \min$.

The above *conditions* (related to (3. 1.64c), (3.173a)) are the main indicators of the origin of the cooperative complexity, which generates the processes of self-organization, complex biophysical and chemical transformations, accompanied by the formation of composite structures.

These results follow from the VP application, which involves the transformation from a kinetic instability to diffusion and back to stable kinetics during a time-space $o(\varepsilon)$ -window.

These cooperative phenomena are associated with an alteration between the VP intervals of the entropy's local minima and maxima, while the applied control are cable of modeling the process's interactive dynamics.

1.7.5. The Complexity Invariant Measure in a Cooperative Dynamic Process

As a rather a formal example, let us find the complexity of the model's *cooperative* dynamics, using (7.14) at

$$\lim_{\delta t \to 0} MC^{\delta} = \frac{\delta H / \delta t}{\delta V^{\delta} / \delta t} = \dot{H} / \dot{V}^{\delta} = MC_t^{\delta}, \tag{7.21}$$

The considered mechanism of cooperative dynamics reveals the complexity origin and its connection to the dynamic and physical phenomena and parameters.

The complexity invariant information measure emerges as an indicator of the cooperative phenomena, in particular, the contributions from different superimposing processes.

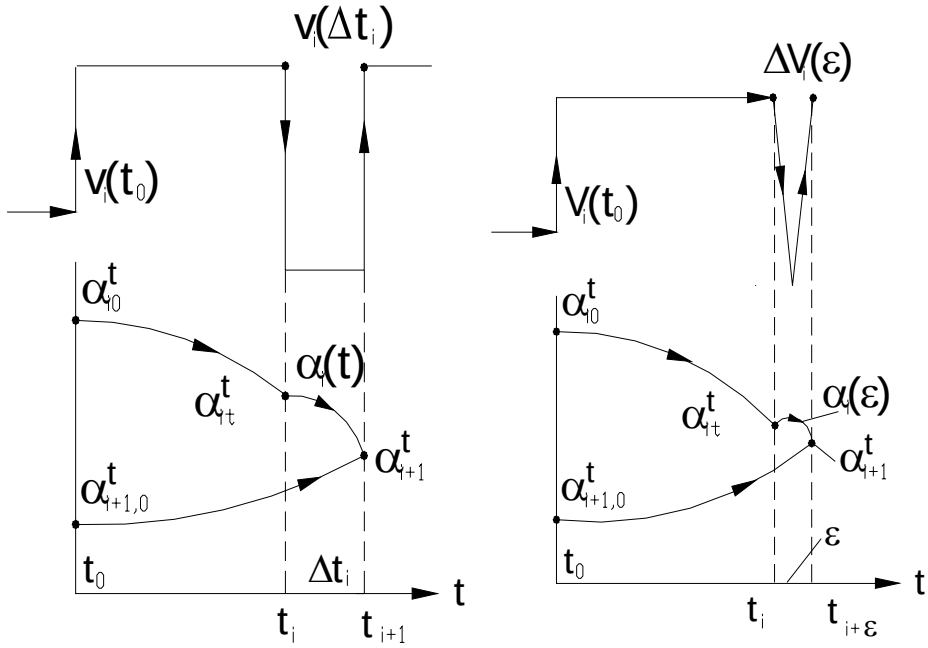
Proposition 7.4.

Let us consider the cooperative dynamics for the two nearest extremal segments of the macrotrajectory, accompanied by joining their corresponding eigenvalues $(\alpha_i^t, \alpha_{i+1}^t)$ of the model's operator spectrum $A = \{\alpha_i^t\}, i = 1,..,k,.,n$ by the needle control (7.12), and following the solutions of the variation problem for the model's operator eigenvalues (7.16a).

Then complexity (7.1), reduced to the beginning of a segment's time interval (t_i), acquires the form

$$MC_t^{\delta}(t_o) = \dot{H}_i(t_i) \,/\, v_i(t_i) = (\mathbf{a}_o)^2 \,/\, \pi c_i^3 t_i^4, \tag{7.22}$$

where $(\mathbf{a}_o)^2$ is the segment information needle control's contribution, measured by the information invariant $\mathbf{a}_o = \alpha_{io}^t t_i ; v_i(t_i) = \dot{V}_i / t_i = \pi \rho_i^3 t_i$ is an elementary segment's space volume increment at the time interval t_i, with a current $\dot{V}_i = \pi c_i \rho_i^2$, determined by the fixed model's space speed c_i and the volume's cross-section $F_i = \pi \rho_i^2$, $\rho_i = c_i t_i$, $\dot{V}_i = c_i F_i$.
(Here and below the information contribution, expressed via the eigenvalues (7.16b), should be multiplied by 4).



a) b)

Figure 7.3.a,b. The illustration of cooperation of two exremal's sections under the discrete control $v_i(\Delta t_i)$ (a) and the needle control $\delta v_i(t_i)$ (b) (other indications are in the text).

Proof. According to the IMD, the process starts with applying a regular controls $v_i(t_o)$ (Fig.7.3a) to the initial eigenvalues α_{io}^t and $\alpha_{i+1,o}^t$, initiating the dynamics of $\alpha_i^t(t)$ and $\alpha_{i+1}^t(t)$ at both segments within the time intervals t_i and t_{i+1}, accordingly.

The corresponding dynamic equations are determined by the model invariant relations, following from the solution of the variation problem:

$$\alpha_{it}^t = \alpha_{io}^t \mathbf{a}/\mathbf{a}_o\,,\ \mathbf{a}=\alpha_{i=1,t}^t t_i\,,\mathbf{a}_o(\gamma)=\alpha_{io}^t t_i\,,\alpha_{i+1,t}^t=\alpha_{i+1,o}^t\mathbf{a}/\mathbf{a}_o\,,\mathbf{a}(\gamma)=\alpha_{i+1,t}^t t_{i+1}\,,\mathbf{a}_o=\alpha_{i+1,o}^t t_{i+1}\,, \tag{7.22a}$$

where α_{io}^t, $\alpha_{i+1,o}^t$ are the eigenvalues at the beginning of time intervals t_i, t_{i+1} accordingly, α_{it}^t, $\alpha_{i+1,t}^t$ are the eigenvalues at the end of these time intervals.

The above information invariants, which depend on the model parameter γ, evaluate a closeness of the nearest initial eigenvalues (α_{io}^t and $\alpha_{i+1,o}^t$).

The needle control $\delta v_i(\Delta t_i)$ (Fig. 7.3b), applied at the moment t_i^v to the eigenvalue α_{it}^t and acting during a finite time interval $\Delta t_i = t_{i+1}^v - t_i^v$, provides the consolidation of the eigenvalue α_{it}^t with $\alpha_{i+1,t+1}^t$ at the moment t_{i+1}^v.The corresponding derivation of local model's Hamiltonian $\dot{H}_i(\alpha_i^t)=\partial\alpha_i^t/\partial t$ is determined by function

$$\alpha_i^t(t_{i+1}^v)=\alpha_{it}^t\exp(\alpha_{it}^t\Delta t_i)(2-\exp(\alpha_{it}^t\Delta t_i))^{-1},$$

where $\alpha_{it}^t=\alpha_i^t(t_i^v)$ is the eigenvalue at the beginning of Δt_i preceding the cooperation, $\alpha_i^t(t_{i+1}^v)$ is the eigenvalue at the moment t_{i+1}^v of cooperation.

Both of them depend on the model's dimension n and parameter γ.

Using the indication

$$p=\exp(\alpha_{it}^t\Delta t_i)(2-\exp(\alpha_{it}^t\Delta t_i))^{-1}, \tag{7.23}$$

we get the corresponding derivation

$$\dot{H}_i(\alpha_i^t)=(\alpha_{it}^t)^2(p+p^2), \tag{7.23a}$$

here p is the parameter of the dynamic cooperation of α_{it}^t with $\alpha_{i+1,t+1}^t$ at Δt_i:

$$\alpha_{it}^t\ p=\alpha_{i+1,t}^t\ . \tag{7.24}$$

By substituting relations (7.23), (7.23a), and (7.22a) into (7.24), we represent p in (7.23-7.24) by the ratio of the initial eigenvalues: $p=\alpha_{i+1,o}^t/\alpha_{io}^t$ that relates p to the model parameter of multiplication $\gamma_i^\alpha=\alpha_{io}^t/\alpha_{i+1,o}^t$ (which particularly, for the IN is $\gamma_i^\alpha=2.21$ at γ=0.5, where $\alpha_{it}^t=\alpha_{it}^t(n,\gamma)$).

Because generally $\Delta t_i \neq t_i$, this p differs from $(\gamma_i^\alpha)^{-1}$.

For example, at $p=(\gamma_i^\alpha)^{-1}=2.21^{-1}$, we get $\Delta t_i/t_i=0.47/\mathbf{a}(\gamma)$, which for γ=0.5 brings $\Delta t_i/t_i\cong 1.88^{\cdot}$.

This allows us to express the interval of cooperation: $\Delta t_i = p(\gamma_i^\alpha)$ and directly obtain the increment (7.23a) during the cooperative dynamics, using the model invariants and the initial α_{io}^t:

$$\dot{H}_i(\alpha_i^t)=\partial\alpha_i^t/\partial t(n,\gamma)=(\alpha_{io}^t)^2(\mathbf{a}/\mathbf{a}_o)^2[(\gamma_i^\alpha)^{-1}+(\gamma_i^\alpha)^{-2}], \tag{7.25}$$

where a maximal α_{io}^t depends on the model γ and dimension n : $\max_{\gamma\to 0}\alpha_{it}^t(n,\gamma)=\alpha_{it}^{to}(n)$.

Applying the needle control at a very small time interval δt_i:

$$\delta v_i(\delta t_i)=\lim_{\delta t_i\to 0}\delta v_i(\delta t_i)=\delta v_i,$$

we get the corresponding derivation of local Hamiltonian $\dot H_i(\alpha_i^t)$ in a limit:

$$\dot H_i(\delta v)=\lim_{\delta t\to 0}(\partial\alpha_i^t/\partial t),$$

which is determined by function $\alpha_i^t(\delta v)=\alpha_{it}^t\exp\alpha_{it}^t\delta t(2-\exp\alpha_{it}^t\delta t)^{-1}$.
From this we obtain the actual needle control's contribution

$$\dot H_i(\delta v)=\lim_{\delta t\to 0}(\partial\alpha_i^t/\partial t)=2(\alpha_{it}^t)^2, \tag{7.26}$$

and using the information invariants (7.22a), we come to

$$\dot H_i(\delta v)=\dot H_i(t_i)=2(\mathbf{a})^2/t_i^2=2(\alpha_{io}^t)^2(\mathbf{a}/\mathbf{a}_o)^2, \tag{7.27}$$

where

$$\dot H_i(t_o)=2(\alpha_{io}^t)^2=2(\mathbf{a}_o)^2/t_i^2, \tag{7.28}$$

and $\dot H_i(t_o)=\dot H_i(t_i)(\mathbf{a}_o/\mathbf{a})$ is the information contribution at the beginning of the segment, reduced to the segment's time interval t_i.

The value of $\dot H_i(t_o)/2$ (7.28) can be applied to each consecutive starting segment.

To evaluate the elementary cooperative complexity at each extremal segment's MC_t^δ, we use any of equations (7.27),(7.28) and the model's current $\dot V_i=\pi c_i\rho_i^2$, determined by the fixed model's space speed c_i and the volume's cross-section $F_i=\pi\rho_i^2$.

Because both invariants in (7.27),(7.28) depend on a segment's time *interval*, we reduce the volume's increment also to the considered time interval t_i. At the radius' increment $\Delta\rho_i=c_i t_i$ and $\Delta\dot V_i=\pi c_i^3 t_i^2$, using the contribution (7.28),we get (7.22). ●

Comments 7.7. Let us evaluate the numerical values of $MC_t^\delta(t_i)$ in (7.22) (at a fixed volume increment) for the model admissible variations of the parameter $\gamma=(0.0072-0.8)$ and a reasonable range of the dimensions $n=(2-20)$.

The MC_t^δ dependency on two variables (n,γ) splits the $MC_t^\delta(n,\gamma)$ into two functions: $MC_{t_i}^\delta(\gamma)$ and $MC_{t_i}^\delta(n)$. (In both cases, the complexity at the *o*-windows undergoes the *jump-vise change).*

Then $MC_{t_i}^\delta(\gamma)=(\mathbf{a}/\mathbf{a}_o)^2[(\gamma_i^\alpha)^{-1}+(\gamma_i^\alpha)^{-2}]$ takes the following computed values: $MC_{t_i}^\delta(\gamma=0.0072)\cong 0.052866353$, $MC_{t_i}^\delta(\gamma=0.8)\cong 0.5936826$ with the ratio's range $MC_{t_i}^\delta(\gamma=0.8)/MC_{t_i}^\delta(\gamma=0.0072)\cong 11.23$.

Complexity $MC^{\delta}_{t_i}(n)=(\alpha^{t}_{n(io)})^2$, at $n(io)=2-20$ (where $\alpha^{t}_{n(io)}$ is the initial eigenvalue of dimension $n=n(io)$) gets the following computed values: $MC^{\delta}_{t_i}(n=2)\cong 0.6058$ and $MC^{\delta}_{t_i}(n=20)\cong 58708.8$ with the ratio's span $MC^{\delta}_{t_i}(n=20)/MC^{\delta}_{t_i}(n=2)\cong 96911.2$.

This means that the MC^{δ}_{t} complexity's dependency on dimensions is in $\cong 100,000$ times stronger than the dependency on γ. At a given *n*, the complexity's growth is limited by a maximal $\gamma\to 1$ when the system decays.

In addition to the cooperative complexity, the model possesses the local $MC(t_i)$ and $MC(t_{i+1})$ complexities, determined by the macromovement *within and at the borders* of the time intervals t_i, t_{i+1} and the volumes $v^{-1}_{io}, v^{-1}_{i+1,o}$ – produced *during* these times:

$$MC(t_i)=(\alpha^{t}_{io}\ t_i)\,v^{-1}_{io}=\mathbf{a}_o\ (\gamma)\ v^{-1}_{io}\ ,\ MC(t_{i+1})=(\alpha^{t}_{i+1,o}\ t_{i+1})\,v^{-1}_{i+1,o}=\mathbf{a}_o\ (\gamma)\ v^{-1}_{i+1,o}\ , \quad (7.29)$$

$$MC_{\Sigma t_i}=MC(t_i)+MC(t_{i+1})=\mathbf{a}_o\ (\gamma)\,(v^{-1}_{io}+v^{-1}_{i+1,o}), \quad (7.30)$$

where the dynamics are initiated by the actions of regular controls $\nu_i(t_o)$.

The complete model's complexity, generated by the cooperative macrodynamics of joining α^{t}_{i} to α^{t}_{i+1}, is the sum:

$$MC_{\Sigma t_i}+MC^{\delta}_{t}(n,\gamma)=MC_{\Sigma}(n,\gamma), \quad (7.31)$$

where both the invariants and the elementary volumes in (7.17) – (7.18), (7.30), (7.31)(and in ch.1.5) are determined by the basic model's parameters of dynamics (n,γ) and geometry (*k*).

Computer simulation [30] of the model's *adaptive* self-organizing process shows that at small *n*, a number of neighboring subsystems with a similar complexity does exist.

With growing *n*, the number of close-complex neighboring subsystems decreases sharply.

Comments 7.8. The MC complexity, defined by the system's dynamics and geometry and expressed through the IN's code, allows *classifying* both the systems and corresponding macromodels in the terms of their MC value.

This classification also evaluates a difference between the systems (models) with the certain model parameters (n,γ, *k*) and some fixed (n_o,γ_o,k_o), measured by the corresponding MC (n,γ,k) and $MC_o\ (n_o,\gamma_o,k_o)$ accordingly.

$$MC=\frac{\sum_{i=1}^{n}\alpha^{t}_{io}(n,\gamma)}{cF(n,\gamma,k)}, MC_o=\frac{\sum_{i=1}^{n}\alpha^{t}_{io}(n_o,\gamma_o)}{c_oF_o(n_o,\gamma_o,k_o)}, cF(n,\gamma,k)=dV/dt, c_oF_o(n_o,\gamma_o,k_o)=dV_o/dt. \quad (7.32)$$

From that, the systems with the equal MC-complexity's measures but different model's parameters are connected:

$$\frac{\sum_{i=1}^{n}\alpha_{io}^{t}(n,\gamma)}{cF(n,\gamma,k)}=\frac{\sum_{i=1}^{n}\alpha_{io}^{t}(n_o,\gamma_o)}{c_oF_o(n_o,\gamma_o,k_o)}\ , \tag{7.33}$$

at

$$MC=H_{\Sigma}\Delta V^{-1}, H_{\Sigma}=Tr[A(0)](\Delta t)^{-1}, \Delta V=cF\Delta t, Tr[A(0)]=\sum_{i=1}^{n}\alpha_{io}^{t}\ . \tag{7.34}$$

Function (7.33) serves as a numerical indicator of the system's similarity, which also allows identifying the model's parameters (n, γ, k) by some known (basic) model's parameters (n_o, γ_o, k_o).

It leads to finding the specific dynamic and geometrical parameters and the code of the identifying system.

Comment 7.9. Since $-\delta(\frac{\partial\Delta S_{ik}}{\Delta V_{ik}\partial t})$ is the increment per volume of information produced in this volume, which defines the *cooperative complexity* MC_{ik}^{δ}, this negentropy increment cannot be more (by the absolute value) than the related internal increment of *physical* entropy $\delta(\frac{\partial\Delta S_{ik}^{\text{int}}}{\Delta V_{ik}\partial t})$ had been spent on the above negentropy production, according to the second thermodynamic law.

That is why the MC_{ik}^{δ} maximum is limited by the above maximal internal increment of the entropy, admitted by the *physical* properties of this volume:

$$\text{Max}|\,MC_{ik}^{\delta}\,|\le\max|\,\delta(\frac{\partial\Delta S_{ik}^{\text{int}}}{\Delta V_{ik}\partial t})\,|. \tag{7.35}$$

The analogous limitations are true also for the information complexity

$$MC_{ik}=-\frac{\partial\Delta S_{ik}}{\Delta V_{ik}\partial t}\ \textbf{:}\ \max|\,MC_{ik}\,|\le\max|\,\frac{\partial\Delta S_{ik}^{\text{int}}}{\Delta V_{ik}\partial t}\,|, \tag{7.36}$$

where $\frac{\partial\Delta S_{ik}^{\text{int}}}{\Delta V_{ik}\partial t}$ is the internal entropy production per the volume, limited by this volume (and by $\gamma\to 1$**)**. The maximal complexity within each time interval t_i (at the preservation of invariants **a, a** $_o$ and a system's maximal admissible entropy production within each t_i) can be reached, *if* this time interval acquires the *minimal* value, allowable by the VP during the macrodynamic process. A minimal complexity corresponds to a minimal admissible $\gamma\to 0$, which limits a minimal system's structural uncertainty (at γ=0.0072)).

The MC connection to physics involves also specific thermodynamics evaluations and limitations of complexity at its implementation through the computations.

The leading studies were performed by Ch. Bennett [40], R. Feynman [41], others. ●

The considered mechanism of cooperative dynamics reveals the MC origin and its connection to the dynamic and physical phenomena and parameters.

The MC's invariant information measure emerges as an indicator of the cooperative phenomena, in particular, from different superimposing processes.

This invariant complexity measure also evaluates the quantity of information required to join an object *j* with an object *k* (secs.7.1,7.2), imposing, in a particular, a *discrete* connection of these objects by the needle control, which can be expressed by the information code.

A sequential cooperation of all n eigenvalues for the model's operator spectrum generates the cooperative information hierarchical network (IN) (Fig.6.5), which we use for the information evaluation of a cooperative complexity in the multi-dimensional processes (sec.1.7.5).

1.7.6. The IN Cooperative Mechanism with the Complexity Measure

The following questions arise: What is the general *mechanism coordinating* the formation of cooperative information *structures* in the *multi*-dimensional cooperative dynamics?

What are an indicator and a measure of the *structural* cooperation?

Below we show that the *general mechanism is an information cooperative network* (IN), initiated by the VP application with its MC function as the indicator and measure of the embedded cooperative dynamics.

The IN incoming information string of the model eigenvalues is ranged automatically according to this mechanism(ch.1.5), which assigns the information measure following from the VP solution.

The MC_m^{δ} for the IN's current hierarchical level $l_i(m_i)$ depends on the triplet's number m_i enclosed into this triplet's level from the previous levels, while the differential MC_t^{δ} (7.1a) evaluates an instant production of information at every $(i, i+1)$ cooperation. Cooperation with each following node sequentially changes the IN's current MC(m_i, γ). Complexity of a total IN is determined by MC(m_n, γ), at $m_n = n/2-1$ of a final IN's node, according to formulas (7.29-7.31) and at preserving γ within the IN.

The information measure of the starting eigenvalues' string, together with the values of $\alpha_{1o}^t, \gamma_i^{\alpha}$, γ, and **a**(γ) allow us to calculate the IN's dimension (n), restore the complete cooperative dynamics with the IN's space-time hierarchical information structure (Fig. 5.5), including the sequence of cooperative resonance frequencies, the MC-function at each IN's hierarchical level, and for a whole IN.

Let us find the *limitations* on the IN's cooperative dynamics, its parameters, and the complexity.

Proposition 7.5 (P7.5).

The model eigenvalues' sequence ($\dots \alpha_{i-1,o}^t, \alpha_{io}^t, \alpha_{i+1,o}^t$), satisfying the triplet's formation, is *limited* by the boundaries for $\gamma \in (0 \to 1)$, which

(a) at $\gamma \to 0$ forms a geometrical progression with

$$(\alpha_{i-o}^t)^2 = (\alpha_{io}^t)^2 + \alpha_{i-1,o}^t \alpha_{io}^t, \tag{7.37}$$

representing the geometric "gold section" at $\alpha^t_{io} \cong 0.618\,\alpha^t_{i-1,o}$ and the ratio

$$G = \frac{\alpha^t_{i+1,o}}{\alpha^t_{io}} \cong 0.618\,; \tag{7.38}$$

and

(b) at $\gamma \to 1$ the sequence $\alpha^t_{io}, \alpha^t_{i+1,o}, \alpha^t_{i+2,o}, ...$ forms the Fibonacci series, where the ratio $\alpha^t_{i+1,o} / \alpha^t_{i+2,o} = \gamma^{\alpha}_2$ determines the "divine proportion" $PHI \cong 1.618$, satisfying

$$PHI \cong G + 1; \tag{7.39}$$

and the eigenvalues' sequence loses its ability to cooperate.

Proof. At $\gamma \to 0$, the solution of (5.42)(ch.1.5) at $\mathbf{a}(\gamma \to 0)$=0.231 brings $\gamma^{\alpha}_1 \cong 2.46$ and $\gamma^{\alpha}_2 \cong 4.47$, $\gamma^{\alpha}_2 / \gamma^{\alpha}_1 = \gamma^{\alpha}_{23} \cong 1.82$.

The ratio $(\gamma^{\alpha}_1)^{-1} = \alpha^t_{io} / \alpha^t_{i-1,o} \cong (2.46)^{-1} \cong 0.618$ for the above eigenvalues forms a "golden section" satisfying for $G = (\gamma^{\alpha}_1)^{-1} = \alpha^t_{io} / \alpha^t_{i-1,o} \cong (2.46)^{-1} \cong 0.618$ to the equation (7.37); *and* the "divine proportion" $PHI \cong 1.618$. The above relations hold true for each primary pair of the triplets' eigenvalues sequence, while for the third eigenvalue we get the ratio $\alpha^t_{i+1,o} / \alpha^t_{io} = (\gamma^{\alpha}_{23})^{-1} \cong 0.549$. These prove P7.5a.

The solution of (1.5.42) at $\mathbf{a}(\gamma \to 1)$=0 brings $\gamma^{\alpha}_1 = \gamma^{\alpha}_2 = 1$, at which we have $\alpha^t_{i-1,o} = \alpha^t_{io} = \alpha^t_{i+1,o} = \alpha^t_{i+2,o} = , ...$ and the eigenvalues' sequence loses its ability to cooperate, disintegrating into the equivalent and not connected elements, which proves P7.5b ●

Comments 7.10. In a chain of connected information events, the appearance of an event carrying $\gamma \to 1$ leads to a chaos and decoupling of the chain; the moment of this event's occurrence a real world is dangerous.

The above relations allow one to *predict* it by measuring a current event's information $\mathbf{a}_o$ and using (5.36a) to compute γ.

At approaching $\gamma \to 1$, we get $\mathbf{a}_o \to -0.58767$, which is an equivalent of 0.8462448 bit, and the corresponding time's ratio of the following and preceding intervals is $\tau_{i+1} / \tau_i = 1.8254$ with the eigenvalues' ratio $\alpha_{io} / \alpha_{it} \cong -1.9956964$.

At $\gamma = 1$ this equation gives $\mathbf{a}_o\,(\gamma = 1) = 0$ with both information contributions for the regular control $\mathbf{a}\,(\gamma = 1) = 0$ and the needle control $\mathbf{a}^2_o(\gamma = 1) = 0$.

This means that at a locality of $\gamma = 1$, both the event's information $\mathbf{a}_o$ *and* the time undergo the jump, which could be the indicators of approaching $\gamma = 1$.

Comments 7.11. With a decay of the cooperative model (at $\gamma \to 1$), its current dimension n_i is changed. A minimal IN potential's growth (from the n_i dimension) corresponds to a possibility of adding the two cooperating eigenvalues (doublet) to the last triplet's eigenvalue of the n_i dimensional system, forming a new system of n_{i+1} dimension, and so on.

Thus, we get the sequence of the feasible systems' dimensions n_i, n_{i+1}, n_{i+2}, which satisfy a simple relation $n_{i+1} = n_i + 2$, forming the Fibonacci sequence.

The proportion PHI (at $\gamma \to 1$) expresses the completion of the cooperative dynamics in one subsystem and a potential start of the cooperation in a subsequent subsystem, exhibiting a boundary between the evolved subsystems.

Corollary 7.3.

The complexities of the IN's node's dimensions n_{i-1}, n_i at $\gamma \to 0$ are connected by relations

$$MC^{\delta}(n_i) = MC^{\delta}(n_{i+1}) + \sqrt{|\, MC^{\delta}(n_i) MC^{\delta}(n_{i+1})\,|}\,, \tag{7.40}$$

following from (7.37), where according to (7.28):

$$MC^{\delta}(n_{i-1}) = \alpha^{t}_{i-1,o}(\mathbf{a}/\mathbf{a}_o)\nu^{-1}_{i-1,o};\; MC^{\delta}(n_i) = \alpha^{t}_{io}(\mathbf{a}/\mathbf{a}_o)\nu^{-1`}_{io}; \tag{7.41}$$

and subsequently

$$MC^{\delta}(n_{i+2}) = (\alpha^{t}_{i+1,o})^2\ (\mathbf{a}/\mathbf{a}_o)\nu^{-1}_{i+1,o}\,.$$

ecause the triplet's eigenvalues sequence forms a triplet's *code* (ch.1.6) the ability of generating this code is also *limited* by the conditions of P7.4a,b. ●

Let us study the elementary cooperation of two processes (i,k), represented by the corresponding IN_i, IN_k with *different* γ_i, γ_k and determine the cooperative complexity.

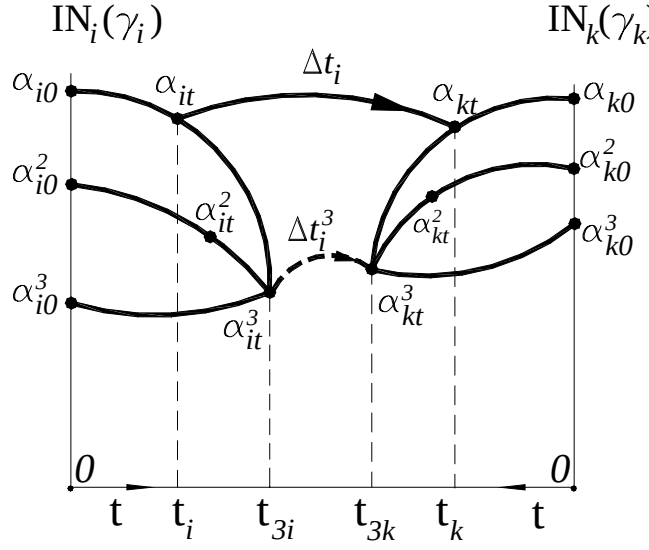


Figure 7.4. The cooperative schema for joining of two subsystems (i,k), represented by corresponding IN_i, IN_k with different γ_i, γ_k.

Proposition 7.6

Suppose the first eigenvalue α^1_{it} of the IN_i joins the first eigenvalue α^1_{kt} of the IN_k during a time interval Δt_i (Fig.7.4) according to the equation (7.22), (7.25), and assume that both $IN_i(\gamma_i)$, $IN_k(\gamma_k)$ are given, their starting dimensions n_i, n_k and $\alpha^1_{it}(\gamma_i, n_i)$, $\alpha^1_{kt}(\gamma_k, n_k)$ are known, as well as all other networks' eigenvalues.

Then the complexity increment, reduced to the segment's time t_i and related to the corresponding volume's contribution $v_i(t_i)$, acquires the form

$$MC^{\delta}_{ikt} = \mathbf{a}_o(\gamma_i)^2\ \rho^1_{\mathrm{ik}}\ \mathbf{a}(\gamma_i)/\mathbf{a}_{\mathrm{o}}(\gamma_i)\mathbf{a}(\gamma_k)/\mathbf{a}_{\mathrm{o}}(\gamma_k) \times\{1+\ \rho^1_{\mathrm{ik}}\ \mathbf{a}(\gamma_k)/\mathbf{a}_{\mathrm{o}}(\gamma_k)[\mathbf{a}_{\mathrm{o}}(\gamma_i)/\mathbf{a}(\gamma_i)/\ ]^{-1}\}/v_i(t_i), \tag{7.42}$$

which is determined by the invariants

$$\mathbf{a}(\gamma_i)=\alpha^1_{it}\,t_i\,,\ \mathbf{a}(\gamma_k)=\alpha^1_{kt}t_k\ \ ,\ \mathbf{a}_{\mathrm{o}}(\gamma_i)=\alpha^1_{io}t_i\,,\ \mathbf{a}_{\mathrm{o}}(\gamma_k)=\alpha^1_{ko}t_k\ , \tag{7.43}$$

and the relations

$$\alpha^1_{it}p_i=\alpha^1_{kt},\ p_i=\alpha^1_{it}\exp(\alpha^1_{it}\Delta t_i)(2-\exp\alpha^1_{it}\Delta t_i)^{-1},\ \alpha^1_{ko}=\alpha^1_{io}\rho^1_{\mathrm{ik}}\ . \tag{7.43a}$$

Proof. From (7.43,7.43a) follow

$$\alpha^1_{io}\ \ p_i\,\mathbf{a}(\gamma_i)/\mathbf{a}_{\mathrm{o}}(\gamma_i)=\alpha^1_{ko}\,\mathbf{a}(\gamma_k)/\mathbf{a}_{\mathrm{o}}(\gamma_k),\ p_i\,\mathbf{a}(\gamma_i)/\mathbf{a}_{\mathrm{o}}(\gamma_i)=\rho^1_{\mathrm{ik}}\,\mathbf{a}(\gamma_k)/\mathbf{a}_{\mathrm{o}}(\gamma_k), \tag{7.44}$$

from which we have

$$p_i=\rho^1_{\mathrm{ik}}\,\mathbf{a}(\gamma_k)/\mathbf{a}_{\mathrm{o}}(\gamma_k)[\mathbf{a}(\gamma_i)/\mathbf{a}_{\mathrm{o}}(\gamma_i)\ ]^{-1}\ . \tag{7.44a}$$

Applying (7.44a) to equality

$$\dot\alpha^1_{it}=(\alpha^1_{io})^2\,(\mathbf{a}(\gamma_i)/\mathbf{a}_{\mathrm{o}}(\gamma_i)\ )^2\ (p_i\ +p_i^2), \tag{7.44b}$$

and using (7.44), we get the information contribution in the forms:

$$\dot H^1_{ik}=(\alpha^1_{io})^2\ \rho^1_{\mathrm{ik}}\,\mathbf{a}(\gamma_i)/\mathbf{a}_{\mathrm{o}}(\gamma_i)\mathbf{a}(\gamma_k)/\mathbf{a}_{\mathrm{o}}(\gamma_k) \times\{1+\ \rho^1_{\mathrm{ik}}\,\mathbf{a}(\gamma_k)/\mathbf{a}_{\mathrm{o}}(\gamma_k)[\mathbf{a}(\gamma_i)/\mathbf{a}_{\mathrm{o}}(\gamma_i)\ ]^{-1}\}, \tag{7.45}$$

$$\dot H^1_{ik}=\mathbf{a}_o(\gamma_i)^2\ /t_i^2\ \rho^1_{\mathrm{ik}}\,\mathbf{a}(\gamma_i)/\mathbf{a}_{\mathrm{o}}(\gamma_i)\mathbf{a}(\gamma_k)/\mathbf{a}_{\mathrm{o}}(\gamma_k) \times\{1+\ \rho^1_{\mathrm{ik}}\,\mathbf{a}(\gamma_k)/\mathbf{a}_{\mathrm{o}}(\gamma_k)[\mathbf{a}_{\mathrm{o}}(\gamma_i)/\mathbf{a}(\gamma_i)\ ]^{-1}\} \tag{7.46}$$

and finally we come to (7.42) at the corresponding volume's contribution $v_i(t_i)$. ●

Comments 7.12. The obtained relations allow us to evaluate both the condition of cooperation (7.44a) and the cooperative complexity (7.42) by the INs' information invariants

and the ratio ρ^1_{ik} of the initial cooperated eigenvalues. If the (i,k) cooperation is performed by joining of the first IN_i - IN_k *triplets* during the time interval Δt_{3i} when the IN_i node's eigenvalue α^3_{it} cooperates with the IN_k node's eigenvalue α^3_{kt}, then the formula (7.44a) acquires the view

$$p_{3i}\,\mathbf{a}(\gamma_i)/\,\mathbf{a}_o(\gamma_i) = \rho^3_{ik}\,\mathbf{a}(\gamma_k)/\mathbf{a}_o(\gamma_k). \tag{7.47}$$

Using the ratio of starting triplets' eigenvalues and the p_{3i}, expressed through Δt_{3i}, we get

$$\rho^3_{ik} = \alpha_{3ko}/\alpha_{3io},\ p_{3i} = \alpha^3_{it}\exp(\alpha^3_{it}\Delta t_{3i})(2-\exp\alpha^3_{it}\Delta t_{3i})^{-1}. \tag{7.47a}$$

For the triplets, by exchanging ρ^3_{ik} with ρ^1_{ik}, we come to the formula for the MC complexity, analogous to (7.42).

The related formulas follow from joining any other corresponding IN_i - IN_k nodes.

From equations (7.47),(7.44) we receive

$$p_{3i}/p_i = \rho^3_{ik}/\rho^1_{ik}, \tag{7.48}$$

which shows the proportionality between the time's parameters and the ratio's of cooperating eigenvalues. Within each IN, the ordered eigenvalues' ratios $\gamma^\alpha_{13}(\gamma_i) = \alpha^1_{i0}/\alpha^3_{i0}, \gamma^\alpha_{13}(\gamma_k) = \alpha^1_{k0}/\alpha^3_{k0}$, depending on the fixed (γ_k, γ_i), are preserved. Because of that, the ratio

$$\rho^1_{ik}/\rho^3_{ik} = \gamma^\alpha_2(\gamma_k) = \gamma^\alpha_2(\gamma_i) = \alpha^i_{2t}/\alpha^i_{3t} = \alpha^{i=k}_{4t}/\alpha^{i=k}_{5t}(\gamma_i)$$

(on the right side of (7.48)) is also fixed for both the INs' ranged cooperating eigenvalues.

This leads to the fixed ratio (on the left side of (7.48)), or to the possibility of an *automatic* cooperation of all interacting IN_i - IN_k eigenvalues *if* the cooperation of any two eigenvalues of these networks has occurred.

Actually, the automatic cooperation requires the sequential supply of the produced information from each previous cooperating eigenvalue to each following cooperation (see also ch.1.6).

Such a supply is possible for any ordered cooperating sequence satisfying the VP, which leads to the automatic creation of each IN.

If the ratio on the right of (7.48) is fixed for the IN_i - IN_k cooperation, then the MC complexity also gets fixed and stays the same for all cooperating eigenvalues.

This means that the MC is changed only with the cooperation of eigenvalue having a distinctive γ.

Proposition 7.7.

Let us consider the cooperative dynamics for a set of *triplets* with different γ, assuming that this set forms an IN where the cooperation of a triplet's i eigenvalue α^i_{3t} with triplet's k eigenvalue α^k_{3t} satisfies the relation (7.44), (7.44a) and

$$\alpha^i_{3t}\, p^{ik}_{\Delta t} = \alpha^k_{3t},\ p^{ik}_{\Delta t} = \exp(\alpha^i_{3t}\Delta t_{ik})(2-\exp(\alpha^i_{3t}\Delta t_{ik})^{-1}. \tag{7.49}$$

Then the entropy's consumption for the triplet's node k, corresponding (5.42a), gets the invariant form:

$$\begin{aligned} \mathrm{g_k}(\gamma_\mathrm{k},\gamma_\mathrm{i}) &= 2\mathbf{a}(\gamma_i)/\gamma_1^{\alpha}(\gamma_k) + \mathbf{a}(\gamma_k)/\gamma_2^{\alpha}(\gamma_k) + \mathbf{a}(\gamma_k) \\ &= \mathbf{a}(\gamma_k)\,[2\mathbf{a}(\gamma_i)/\mathbf{a}(\gamma_k)\ (\gamma_1^{\alpha}(\gamma_k))^{-1} + (\gamma_2^{\alpha}(\gamma_k))^{-1} + 1], \end{aligned} \tag{7.49a}$$

where the $\mathrm{g_k}(\gamma_\mathrm{k},\gamma_\mathrm{i})$ establishes the information connection between any cooperating i, k triplets, modeling the IN's cooperative phenomena.

Proof. For such an IN, the α^k_{3t} is obtained by the cooperating a pair $(\alpha^k_{4t},\alpha^k_{5t})$ that forms the doublet

$$\begin{aligned} \alpha^k_{3t} \Rightarrow \alpha^k_{5t} = \alpha^k_{4o}\,\mathbf{a}(\gamma_k)/\mathbf{a}_\mathrm{o}(\gamma_k)\ \ p^k_2 = \alpha^k_{5o}\,\mathbf{a}(\gamma_k)/\mathbf{a}_\mathrm{o}(\gamma_k), \\ p^k_2 = \alpha^k_{5o}/\alpha^k_{4o} = \alpha^k_{5t}/\alpha^k_{4t} = (\gamma_2^{\alpha}(\gamma_k))^{-1}, \end{aligned} \tag{7.49b}$$

while the sequence of the cooperating triplet's eigenvalues $\alpha^i_{3t}(\alpha^i_{1t},\alpha^i_{2t}), \alpha^k_{3t}(\alpha^k_{4t},\alpha^k_{5t})$ is characterized by the ratios

$$\alpha^k_{3t}/\alpha^i_{3t} = \rho^{ki}_3 = (\gamma_1^{\alpha}(\gamma_k))^{-1},\ \gamma_1^{\alpha}(\gamma_i) = \alpha^i_{1t}/\alpha^i_{3t}, \gamma_1^{\alpha}(\gamma_k) = \alpha^i_{3t}/\alpha^k_{3t}\ . \tag{7.50}$$

In such a sequence, the ratios $p^{ik}_{\Delta t} = p^k_2$ (and therefore the interval Δt_{ik}) are connected to γ_k through the parameter of multiplication $\gamma_1^{\alpha}(\gamma_k)$ of the adjoint triplet.

This connects (5.42a) to the related equation (7.49a) at

$$\mathbf{a}(\gamma_i)/\mathbf{a}(\gamma_k) = 1,\ \mathrm{g_k}(\gamma_\mathrm{k},\gamma_\mathrm{i}) = g\ (\gamma).\ \bullet$$

<u>*Comments*</u> <u>7.13.</u> The $\mathrm{g_k}(\gamma_\mathrm{k},\gamma_\mathrm{i})$ component $2\mathbf{a}(\gamma_i)/\gamma_1^{\alpha}(\gamma_k)$ (which depends on the triplet's invariant and the nearest ratio $\gamma_1^{\alpha}(\gamma_k)$) measures the information *contribution* of a merging *phenomenon* (defined by the ratio), created by the cooperation.

A *complex system represents a composition of these phenomena, which indicate an emergence of complexity.*

The IN structure and the invariants allow the numerical evaluation of these phenomena by the amount of generated *quantity and quality* of information.

Because each *k*-triplet's node encloses information $\mathrm{g_k}(\gamma_\mathrm{k},\gamma_\mathrm{i}) = 2\mathbf{a}(\gamma_k)$ following from (1.5.42a), equation (7.49a) acquires the form

$$\mathbf{a}(\gamma_k)\,[2\mathbf{a}(\gamma_i)/\mathbf{a}(\gamma_k)\ (\gamma_1^{\alpha}(\gamma_k))^{-1} + (\gamma_2^{\alpha}(\gamma_k))^{-1} + 1] = 2\mathbf{a}(\gamma_k),$$

and we come to

$$2\mathbf{a}(\gamma_i)/\mathbf{a}(\gamma_k)\ (\gamma_1^{\alpha}(\gamma_k))^{-1} + (\gamma_2^{\alpha}(\gamma_k))^{-1} = 1,$$

or

$$(1-(\gamma_2^{\alpha}(\gamma_k))^{-1})\,\gamma_1^{\alpha}(\gamma_k) = 2\mathbf{a}(\gamma_i)/\mathbf{a}(\gamma_k), \tag{7.50}$$

which determines the invariant condition, connecting γ_i, γ_k of the nearest triplets.

Application of this condition allows us to get a *chain* of cooperating triplets, forming a *harmonic ensemble* of the sequentially coordinating frequencies, defined by the eigenvalues ratios (ch.1.6).

This leads to a synchronization of cooperating triplets-nodes, stabilizing the chain.

The *computational example* (for (7.50)) brings the following sequence of the IN's cooperating triplets' parameters:

$$(\gamma_1 \to \gamma_2 \to \gamma_3 \to \gamma_4) \Rightarrow (0.72 \to 0.42 \to 0.62 \to 0.5),$$

which corresponds to the ratio's sequence of information frequencies related to $\gamma_i^{\alpha}(\gamma_k)$, where the first one starts from the last frequency of the triplet with γ_1=0.72.

The computation results [35,42] show that for a range of the feasible $\gamma \to (0.0072 - 0.8)$, only a limited collection of $\gamma_k \to (0.4 - 0.7)$ exists with the frequency's ratios between 4.08 and 2.049, which satisfies (7.50).

In spite of these limitations, the absolute range of actual cooperating frequencies is essentially wider, depending on the specific frequencies' band for such a *cooperating subsystem* according to its particular dimension and the locations of ending frequency, as well as this frequency band.

For example, even for

$\gamma_k = \gamma = 0.5$, $\gamma_1^{\alpha} = \gamma_{12}^{\alpha}$=2.215, $\gamma_2^{\alpha} = \gamma_{13}^{\alpha}$= 3.895, and *n*=22,

the frequency band is between 0.00015 and 500, i.e. 1:3300000.

At the subsystems' cooperation, this band extends more radically, leading to the synchronization and generation of the stable subsystems' chain (within the limited γ_k).

Comments 7.14. Each triplet encapsulates three symbols representing a *code* of the cooperating eigenvalues $(\alpha_1, \alpha_2, \alpha_3)$, which carry the information transferred by these eigenvalues and evaluated by quantity of information 3**a**(γ)(ch.1.6).

The model's control adds one more symbol (v_1) to this code: $(\alpha_1, \alpha_2, \alpha_3, v_1)$, evaluated by a total quantity of information 4**a**(γ), which possesses 1.44 bits at $\gamma = 0.5$.

In particular, this is precisely the quantity of information necessary to encode each DNA codon or each 3 nucleotides of 20 aminoacides.

If the IN's starting eigenvalues' string is aranged by the invariant ratios of the nearest eigenvalues, evaluated by quantity of information 3**a**(γ) with the eigenvalues ratio

$\gamma_{12}^{\alpha}(\gamma) = \alpha_1 / \alpha_2 = \alpha_3 / \alpha_4, ..., \gamma_{13}^{\alpha}(\gamma) = \alpha_1 / \alpha_3 = \alpha_3 / \alpha_5, ...$,

then this four letter's code's word preserves also its sequence: $(\alpha_1, \alpha_2, \alpha_3, v_1) \Rightarrow (\alpha_3, \alpha_4, \alpha_4, v_2) \Rightarrow (\alpha_{i-1}, \alpha_i, \alpha_{i+1}, \dots)$,

while the last symbol of the previous code transfers the node's information to the first symbol of subsequent code, and so on.

This code possesses a *hierarchical complexity* for both the code's dynamic and geometrical structures [36].

Such a code's geometry consists of a set of the curved quasi-quadrate cells, whose sequence forms a hyperbolic cellular structure located along the IN nodes in the information geometric space**,** and the IN geometrical boundary is encoded by this code (ch. 1.6.3).

The above structures possess the cooperative complexity satisfying relations (7.37), (7.38), and (7.50).

The quantity of information $\mathbf{a}_o$, accumulated by each following IN's triplet includes the structural information from the previous triplet (by its third extremal segment), which also depends on the node's geometric location.

The corresponding *hierarchical* information measure for each node has an increasing *quality* of information.

Even though the triplet's code preserves information of both the code word's and the code symbols' sequence, each IN's node's code consists of a *new* sequence of distinctive *frequencies* $f_{i-1} \sim \alpha_{i-1}$, $f_i \sim \alpha_i$, $f_{i+1} \sim \alpha_{i+1}$, which are capable of encoding all initial string symbols. A single triplet can generate various combinations of the four code symbols at different γ bringing an analogy to the DNA triplet's code.

A system's genetic information is encoded by a sequence of multiple combinations of these four symbols.

Comments 7.15. The MC specific consists of providing a precise complexity measure for a *dynamic irreversible process* and evaluating the above complexities by the *hierarchical structure* of information contributions in bits of information.

The VP's defined intervals of observations, when the contributions are measured, allow the formalization of both an *observer* and the *localities* of the contributions, their hierarchical *structuring* according to *quantity and quality* of information, and a successive computation of the total complexity of a distributed IN.

The IN structure, as an information model of a complex system, with its MC depends on both the number of its elements-segments and mostly on the links-nodes, connected them in a cooperative.

A cooperative system can be controlled via an element (link) binding the cooperative together.

Information form of such a link is a common code, which not only can sustain a ranged subsystem's connection but also assemble not physically bound subsystems through their feed-back communication.

Such a code is also an indicator and a measure of the system's complexity.

Finally the initial complexity measures (secs.7.1, 7.2) are expressed via the information *invariants* of a cooperative process and the related information code, allowing both *constructive measurement and computation* of complexities for a real observed system.

These complexity measure's essentials connect the formal *mathematical and thermodynamic* meaning of the complexity with theory information and communication theory.

1.7.7. The Equations of the Spatial Information Cooperative Dynamics. Information Attraction and Complexity

Here we examine the role and contribution of space information dynamics to cooperation and complexity.

The information cooperative dynamics include the equalization of the model eigenvalues and the consolidation of the equal eigenvectors into a common information unit.

The regular and needle controls, applied as a function of time, provide the equalization of the eigenvalues, while the consolidation of eigenvectors requires a space movement.

Let us analyze this process, considering the cooperation of two local, dynamic, space models (triples) transformed into diagonal form:

$$dx_i / dt = \lambda_i x_i ,\ dx_k / dt = \lambda_k x_k , \tag{7.51}$$

where x_i , x_k are the model's macrocoordinates, λ_i , λ_k are the eigenvalues corresponding to the matrices

$$A_i^l = \| \lambda_i \|_{i=1}^3 , A_k^l = \| \lambda_k \|_{k=1}^3 . \tag{7.51a}$$

If the equalization of the model's eigenvalues had been reached at the moment $t_i + o$:

$$\lambda_i (t_i + o) = \lambda_k (t_i + o) \tag{7.52}$$

during the cooperative dynamics, then the cooperation is accomplished after the coincidence of the model's eigenvectors $\vec{x}_i$, $\vec{x}_k$, generally defined in a Riemann's space R^3 with the invariant metric

$$ds^2 = g^3 d\vec{x}_i^T d\vec{x}_i , g^3 = \| g_{ij}^3 \|_{i,j=1}^3 , \vec{x}_i = (x_1^i \bar{e}_1^i , x_2^i \bar{e}_2^i , x_3^i \bar{e}_1^i),\ g^3 \neq 0 , \tag{7.53}$$

where the vectors $\vec{e}_i = (\bar{e}_1^i , \bar{e}_2^i , \bar{e}_3^i), \vec{e}_k = (\bar{e}_1^k , \bar{e}_2^k , \bar{e}_3^k)$, represent the local orths (fundamental orthogonal triad's vectors), defined in the considered Riemann's space R^3, whereas each of $\vec{e}_i , \vec{e}_k$ is a tangent vector to a curve–trajectory of the eigenvector space movement in R^3.

(We assume that the phase speed's vector and the corresponding state's vector are defined at each R^3 point).

Definition 7.3. Let us move vector $\vec{x}_i$ toward equalization with vector $\vec{x}_k$, considering an increment $d\bar{x}_i$ on the path interval $ds_i = ds$ between $\vec{x}_i$, $\vec{x}_k$.

Then, at the VP fulfillment, metric (7.53) in the cooperative motion is described by the equation of a geodesic line in R^3, written in the traditional coordinate form [38]:

$$\frac{d^2x_j^i}{ds^2} = -K_{mn}^j \frac{dx_m^i}{ds}\frac{dx_n^i}{ds},\ j,m,n = 1,2,3, \tag{7.54}$$

where K_{mn}^j is a Gaussian curvature at a $\delta\vec{x}_{\boldsymbol{i}}(\delta s_{ik})$-locality, which along the geodesic is defined by the vector equation

$$d\vec{x}_i / ds = -K_i\vec{x}_i; \tag{7.55}$$

K_i is the operator's representation of the K_{mn}^j components.

At a positive cooperative speed $d\vec{x}_i / ds > 0$ and $\vec{x}_i > 0$, the space gets a negative curvature $K_i < 0$ corresponding an attraction; and at $K_i > 0$, we get repulsion.

That is why the vector speed in (7.55) defines an *intensity of information attraction (or a repulsion)*, determined by the space curvature K_i. (The vector's derivations here and below mean that the derivations for each vector's components are similar to (7.54).

Along the geodesic line (7.54), the vector undergoes a parallel displacement, where the vector increment according to (7.55) depends linearly and homogeneously upon the increments of the related coordinates, and only the first vector of curvature is significant).

Comments 7.16. The space curvature, defining the cooperative speed, is a fundamental source of both the complexity and the cooperative dynamics.

At the VP filfullment, the K_i satisfies the following expressions via the fundamental tensor g^3:

$$K_i = 1/2(g^3)^{-1}dg^3 / ds\ . \tag{7.56}$$

The formula for a scalar curvature K^i is expressed through the determinant of the fundamental tensor in this eigenvector's space**:**

$$K^i = (\sqrt{g}\ )^{-1}\partial\sqrt{g}\ / \partial s\ , g\ = \det \| g_{ij}^3 \|, (\sqrt{g}\ )^{-1} = | x_1^i x_2^i x_3^i |. \tag{7.57}$$

Proposition 7.8.

Assume that the eigenvalues of matrix $A_i^l = \| \lambda_i \|_{i,j=1}^3$ (7.51a) are simple with the distinctive roots, and the related eigenvectors $(x_1^i\overline{e}_1^i, x_2^i\overline{e}_2^i, x_3^i\overline{e}_1^i)$ are mutually orthogonal (the matrix satisfies the conditions for a Vandermonde matrix [39]).

Then the curvature in the considered geometrical phase space (7.57) has the form

$$K^i = -3\frac{\alpha_o^i d\alpha_o^i}{ds}, \tag{7.58}$$

where $\alpha_o^i = Re\lambda_o^i, \lambda_1^i = \lambda_2^i = \lambda_3^i = \lambda_o^i$.

Proof. The above matrix's conditions allow expressing the space components (x_1, x_2, x_3) for each eigenvector in (7.53) through the eigenvalues:

$$x_1 = \{1, \lambda_1^i, (\lambda_1^i)^2\}, x_2 = \{1, \lambda_2^i, (\lambda_2^i)^2\}, x_3 = \{1, \lambda_3^i, (\lambda_3^i)^2\}, \ \lambda_{j=1,2,3}^i \neq 0. \quad (7.59)$$

According to the cooperative dynamics, the eigenvectors $(\vec{x}_1, \vec{x}_2, \vec{x}_3)$ underwent the consolidation into a single eigenvector $(\vec{x}_1, \vec{x}_2, \vec{x}_3) \to \vec{x}_o, \vec{x}_o = (1, \alpha_o, \alpha_o^2)\vec{e}_o$ with the real eigenvalues $\alpha_o = Re\lambda_o$.

The eigenvector's space coordinates are formed in the process of the equalization and transformation of the corresponding eigenvalues

$$\{(1, \lambda_1^i, (\lambda_1^i)^2), (1, \lambda_2^i, (\lambda_2^i)^2), (1, \lambda_3^i, (\lambda_3^i)^2)\} \to (1, \alpha_o^i, (\alpha_o^i)^2),$$

which specifically take the form:

$$\{(1, \lambda_1^i, (\lambda_1^i)^2) = (1, \lambda_2^i, (\lambda_2^i)^2) = (1, \lambda_3^i, (\lambda_3^i)^2)\} \to (1, \alpha_o^i, (\alpha_o^i)^2).$$

Applying (7.57) we get

$$(\sqrt{g}\,)^{-1} = |1\alpha_o^i(\alpha_o^i)^2|, \partial\sqrt{g}\,/\partial s = \partial(\alpha_o^i)^{-3}/\partial s = -3(\alpha_o^i)^{-2}\partial(\alpha_o^i)/\partial s. \quad (7.59a)$$

Assuming that vector $\vec{x}_o^i = (1, \alpha_o^i, (\alpha_o^i)^2)\vec{e}_o^i$ is cooperating with the analogous eigenvector $\vec{x}_o^k = (1, \alpha_o^k, (\alpha_o^k)^2)\vec{e}_o^k$, at $|3\alpha_o^i| = |\alpha_o^k|$, the curvature (7.57) gets the form (7.58). ●

Comments 7.17. At the transformation of an initial fundamental tensor (in (7.57), with three eigenvectors (7.59)) *to* its form (in (7.59a), with the cooperated eigenvectors (7.59)), the curvatures changed twice: first, at the eigenvectors' equalization, and second, after forming a triplet with $|3\alpha_o^i| = |\alpha_o^k|$.

As a result, we come to *three different topological structures* at the cooperation of the curved subspaces with the above eigenvectors (including the initial one with the three different eigenvectors (7.59)).

Because these transformations arise under the jump-wise control actions; the conversions to these structures have the forms of *discrete* (jump-wise) transitions.

This means that the discrete geometrical boundaries between these structures are *topological indicators* of the merged cooperative phenomena.

The cooperative complexity's connection to the curvature implies that the discrete boundaries are *also* the topological indicators of complexity for a *structure* in information geometry. (A shared volume of the cooperating structures could be formed by "stitching" of the merged boundaries).

The DSS code-cells, distributed in a fabric of information space geometry (Fig. 6.5, ch.1.6), present an elementary form of these discrete structures.

Specific code sequence determines the geometrical structure to be built in this IG space location.

Comments 7.18. Unification of the cooperating eigenvectors produces an *information cooperative mass* M_i, which is defined by the information value of an elementary phase volume υ forming at the cooperation:

$$M_i = \upsilon 3 divS_i, \tag{7.60}$$

where $3divS_i$ is the instant quantity of information generated at joining of the above three eigenvectors. Using the relations

$$cdivS_i = -\partial S_i / \partial t, H^i = -\partial S_i / \partial t, \dot{H}^i / \upsilon = MC_t^\delta$$

at $\dot{H} / \dot{V}^\delta = MC_t^\delta, \dot{V}^\delta = \upsilon$, the space speed $c = ds / dt$, and (7.60), we write curvature (7.58) in the form

$$K^i = -MC_t^\delta M_i, \tag{7.61}$$

where

$$M_i = 3H^i c^{-1} \upsilon, M_k = H^k c^{-1} \upsilon, H^k = \alpha_o^k = 3\alpha_o^i = 3H^i, \tag{7.61a}$$

and α_o^k, H^k are the model's eigenvalue and Hamiltonian accordingly after the cooperation, at a joint vector $\overline{x}_k$.

The information, localized in the space (due to $H^k = \alpha_o^k = -\partial S_k / \partial t$, $d\alpha_o^i / ds = dH^i / ds$), generates an increment of fundamental metric's tensor (in the forms (7.56, or 7.57)), analogously to the "metric's relative density", which changes the curvature. It is clearly seen that the curvature K^i turns to zero *if* any of

$$\partial S_i / \partial t = -\alpha_o^i, \partial^2 S_i / (\partial t)^2 = -\dot{\alpha}_o^i$$

equals to zero, or $c \to \infty$ (which contradicts to $\alpha_o^i \neq 0$ at $a^u \neq 0$ in 1.1.1, sec. 1.1.1).

Because $\partial S_i / \partial t$ and $\partial^2 S_i / (\partial t)^2$ are the entropy production and its derivative (or an amount of the entropy's acceleration), memorized at the cooperation, the curvature conceals the corresponding *information of cooperation*, possessing the cooperative system's complexity and information mass.

Moreover, the curvature is a result of both the cooperation and the memorized information mass, or a cooperated information mass, which generates complexity.

The cooperation decreases uncertainty and increases information mass.

According to (7.61a), the information mass $M_k = 3\alpha_o^i c^{-1} \upsilon$ can be encoded (in terms of $3\alpha_o^i$ (ch.1.6)) by the IN triplet code for each phase volume υ related to speed c.

Therefore, the curvature in (7.61) also can be ecoded using both the complexity's MC_t^{δ} and mass' M_i codes.

By substituting $\partial S_i / \partial t = -\alpha_o^i$ and $c = \partial s / \partial t$, we get the information mass' expression by the entropy derivation along ds for a phase volume υ:

$$M_k = -3\upsilon \partial S_i / \partial s \text{ , or } M_k = -3\upsilon dS_i / ds \,. \tag{7.61b}$$

Considering an entropy divergence $-\partial S_i / c_i \partial t = divS_i$, let us find a maximal admissible space speed c_{io}, carrying $divS_i$.

Using the IN invariant's relation $\partial S_i / \partial t = |\mathbf{a}_o| / t_i$, at the minimum admissible $t_{io} \approx 1.33 \times 10^{-15}$ (defined by the minimal time-interval of the light wavelenght $l_{io} = 4 \times 10^{-7} m$), and a maximum of the normalized $div^* S_i = divS_i / |\mathbf{a}_o| \approx 1/137$ (sec.1.6.4), we get the maximal information speed

$$c_{io} = (t_i div^* S_i)^{-1}, \tag{7.62}$$

evaluated by $c_{io} = c_{mo} \approx 1.03 \times 10^{17}$ Nat/s.

This maximum restricts the cooperative speed and a minimal information curvature at other equal conditions.

From (7.62) it also follows that a bound into space information ($div^* S_i$) limits the maximal speed of incoming information, imposing an *information* connection on the time and space.

The ratio $c_k^* = c_{mo} / c_{ko} \cong 0.343 \times 10^9$ *Nat/m*$= 0.343$ *gigaNat/m* (in a light wavelength meter) limits a maximal information space speed. In this case, each light wavelength carries $\cong 137$ Nats during 1.33×10^{-15} sec, which are delivered with speed of light.

The physical mass-energy that satisfies the law of preservation energy (following the Einstein equation [38]), is distinguished from the information mass (7.60), which does not obey this law.

According to [38] the multiplication of a mass on $\sqrt{g}$ defines the mass *density* M_i^*, which following (7.58) and (7.60), acquires the form

$$M_i^* = (\alpha_o^i)^{-3} \upsilon 3 \alpha_o^i c^{-1} = 3\upsilon c^{-1} (\alpha_o^i)^{-2} \,. \tag{7.63}$$

In the simulated IN hierarchy [30, 35] (see also Fig.5.5), the values of cooperating eigenvalues α_o^{kl} decrease with a growing number of the hierarchical level $l = 1, 2, .., k, .., m$, which leads to an increasing of $M_i^*(l)$.

Comments 7.19. According to equation (7.51), function $\alpha_o^i = (x_i)^{-1}\partial x_i / \partial t$ corresponds to a relative information flow.

Let us consider an information macrostate x_i to be an elementary information charge q_i, whose flow $\partial q_i / \partial t = -c_i divq_i$ characterizes the charge's divergence from a volume $\upsilon_i = \upsilon$, we get relation $-q_i(divq_i)^{-1} = q_i^*$ describing a relative concentration of the charge within the volume.

With growing the divergence, the relative information charge q_i^* decreases, and vice versa, the charge concentration increases with decreasing the divergence from the volume.

Writing the mass density M_k^* (7.63) in the form $M_k^*(c_k)^3 = (q_k^*)^2\upsilon_k$ and taking into account $\upsilon_k = \dot{V}_k = c_k f_k$, where f_k is an elementary cross section of volume υ_k, we get

$$M_k^*(c_k)^2 = (q_k^*)^2 f_k \ , \tag{7.64}$$

which is associated with an internal information charge's *power*, carried by an elementary space square.

With growing the divergence, the concentration of the charge power in a volume diminishes, indicating a decline of a *quality* of the information power.

The information power at $c_k = c_{mo}$:

$$M_k^* c_{mo}^2 \cong M^*_{k c_k=1} \times 10^{34}, \ M^*_{k c_k=1} = [(q_k^*)^2 f_k]_{c_k=1} \tag{7.64a}$$

reaches a maximum at a fixed $M^*_{k c_k=1}$,whose mass's density characterizes the information power's concentration at a fixed $c_k = 1$, or it describes a related power's "quality" (connected to a decrease of a uncertainty density).

The power, corresponding to the information equivalent of speed of light $c_{ko} \cong 3\times 10^8$ m/s, is

$$M^*_{ko} c_{ko}^2 \approx 9\times 10^{16} M^*_{ko c_k=1} \ . \tag{7.65}$$

The information power for a *relative speed* $c_k^* = c_{mo} / c_{ko}$:

$$M_k^*(c_k^*)^2 = [(q_k^*)^2 f_k]_{c_k^*=1} 1.176\times 10^{18}, \tag{7.65a}$$

at $c_k^* = 1$ defines the related information mass $M_{kr}^* = M_k^*(c_k^* = 1)^2$, which is equal to $M^*_{k_{c_k^*=1}} = [(q_k^*)^2 f_k]_{c_k^*=1}$.

From (7.65a) we get the ratio

$$p_{c_k}^{kn} = [(q_k^*)^2 f_k]_{c=c_k} / [(q_k^*)^2 f_k]_{c_k=1} = (c_k / c_{mo})^2 \times 10^{-34}, \quad (7.65b)$$

or from (7.64a), (7.65a) we have

$$p_{c_k^*}^{kn} = [(q_k^*)^2 f_k]_{c=c_k} / [(q_k^*)^2 f_k]_{c_k^*=1} = (c_k)^2 , \quad (7.65c)$$

which presents in each case the *information power*, related to the power at $c_k = 1$ and at $c_k^* = 1$ accordingly; while, in general, both power's components are unequal:

$$[(q_k^*)^2]_{c=c_k} \neq [(q_k^*)^2]_{c_k=1}, [f_k]_{c=c_k} \neq [f_k]_{c_k=1}. \quad (7.65d)$$

Applying the IMD approach to equation (7.51), we come to its information analogy: $\dot{q}_i = \rho_{qi} X_{qi}$, where $X_{qi} = grad_{qi} S_i$ is an information force acting on a charge q_i within a flow $\dot{q}_i$, and ρ_{qi} is a medium's conductivity to transfer the flow.

The IMD equation for the VP's constraint (ch.1.3) in the form $\partial X_i / \partial x_j = -2 X_i X_j$, at $X_i = \sigma_i \alpha_i, X_j = \sigma_j \alpha_j$, with the corresponding coefficients of a resistance: $\sigma_i = (\rho_{qi})^{-1}, \sigma_j = (\rho_{qj})^{-1}$ *and* the information flows accordingly (connected to the information mass (7.60)):

$$\alpha_i = M_{ki} c_i / \upsilon_i, \alpha_j = M_{kj} c_j / \upsilon_j ,$$

lead to the equations

$$F_{ij} = G_m M_{ki} M_{kj} , \; F_{ij} = -1/2 \partial X_i / \partial x_j , \quad (7.66)$$

where the parameter of the resistance's ratio

$$G_m = \sigma_i^f / \sigma_i^f, \sigma_i^f = \sigma_i / f_i, \sigma_j^f = \sigma_j / f_j, \text{ at } f_i = \upsilon_i / c_i, f_j = \upsilon_j / c_j \quad (7.66a)$$

characterizes an ability of a medium to resist for a mutual exchange of the information flows at the interaction of the information mass M_{ki}, M_{kj} under the action of a force F_{ij}; and σ_i^f, σ_i^f are the medium's relative coefficients of resistance (related to the elementary squares f_i, f_j accordingly).

Equation (7.66) determines the force acting *between* the masses, which are generated by the corresponding space curvatures (7.60) in the interactive cooperative dynamics.

The mass is formed by an influx of information charges $\{q_i\}$ and by the force F_{qij}, controlling the flows *interaction*, where the force F_{qij} is a derivative of the elementary information force X_{qi}:

$$F_{qij} = -1/2\partial X_{qi} / \partial q_j . \tag{7.66b}$$

If q_i represents an elementary "information fermion" (particularly a "graviton"), *then* X_{qi} corresponds to an elementary "information boson", acting on q_i, which has a physical analogy of an elementary gravitation particle that carries a gravitation force.

Physical matter's particle-a boson affects a fermion like a "messenger", which transfers information, potentially, in some information code. The hypothetical Higgs boson supposes to bind the elementary particles into a matter (similar to the needle control's binding the information).

We may assume that the message code is based on the four symbols' optimal information code, which the Nature has already invented by the DNA code.

Thus, the information boson-fermion pairs' communication code pretends to be a *united information language, which describes all known four elementary interactions.*

For example, such a code could be memorized by gluons, holding a silent in the early Universe (as possible seeds of an external intelligence).

Gluons, as the elementary color exchange particle, underlying the particle interactions, are represented by eight pairs of bi-colored charges-octets (carrying three kinds of color and anti-color units). The generated color's charges specify the binding exchange forces, acting between the interacting quarks and gluons (as an analogy of needle control).

This means that a specific color force can be *encoded by any pair of such a three bits' information code* (whose binary digit handles eight messages), which is memorized in anticipation of interactions. The triplet's code can encode these *three* color-anticolor forces, which binds the quarks' color *triplet*.

Such a potential *general information systems theory with a unified information language could serve as an information approach to a theory of everything.*

Example 7.2. Let us find the curvature for an elementary triplet, formed by joining of three of the cones' vertices with the fundamental tensor's coordinates (sec.1.5.4):

$$\begin{aligned} g_1 &= \cos\psi \cos(\varphi \sin\psi) \\ g_2 &= \sin\psi \cos\varphi \cos(\varphi \sin\psi) - \sin\varphi \sin(\varphi \sin\psi) \\ g_3 &= \sin\psi \sin\varphi \cos(\varphi \sin\psi) + \cos\varphi \sin(\varphi \sin\psi) \end{aligned} \tag{7.67}$$

where φ is the projection of the space angle for the spiral's radius vector ρ at the cone's base (Figs. 5.6, 6.2), ψ is the angle at the cone's vertex. The invariant metric of information space has the following expression via the model's information invariant, defined at the moments of cooperation t_i:

$$mes \,||\, ds \,||= mes \,||\, \partial\rho_i(t_i)\partial\varphi_i(t_i) \,||= mes \,||\, \mu(t_o) \,|| = mes \,||\, inv_{\inf} \,|| = \mathbf{a}, \tag{7.68}$$

where mes is the information measure of the metric. Indeed. For each the i-cone, the radius vector ρ_i is equal $\rho_i = b_i \sin(\varphi_i \sin\psi_i)$, where b_i is the parameter for each cone's radius vector $b_i = \mu_i / \varphi_i$, μ_i is a piece of the dynamic trajectory at the cone's surface.
At the moment of cooperation, the above vector and the angles acquire the increments

$$\delta\rho(t_i) = b(\mu(t_i)),\ \delta\ \varphi(t_i) = \pi,\ \delta\ \psi(t_i) = \pi/6,\ \delta\varphi(t_i)\sin\delta\psi(t_i) = \pi/2, \quad (7.68a)$$

and $mes \,\|\, \mu(t_o) \,\|$ obtains the invariant informational measure

$$mes \,\|\, inv_{\inf} \,\| = \mathbf{a}. \quad (7.68b)$$

Each of the IN cell-code's (ch.1.6) space metric ds_m, defined by the cell *m* diameter, acquires the invariant information measure $mes_{\inf} \,\|\, ds_m \,\| = \mathbf{a}_m(\mathbf{a}, \psi_m)$, which depends upon both the invariant $\mathbf{a}(\gamma)$ and the cell space angle $\psi_m\ (\gamma)$ of its location within the IN.
Writing the formula (7.57) for the metric's components (7.67) at $\psi_i = \psi = const$, we have

$$K_{123} = 1/2 \,|\, g_j \,|^{-1} \sum_{j=1}^{3} \partial g_j / \partial\varphi\ . \quad (7.69)$$

By taking the derivative $\partial g_j / \partial\varphi$ of $g_j = (g_1, g_2, g_3)$ and considering $\partial g_j / \partial\varphi$ at the condition (7.67), after the substitution to (7.69), we get $K_{123} \approx -0.4$.

We come to the information attraction of the triplet's eigenvectors.

Finding the information mass and complexity requires to know the triplet's cone phase volume:

$$\dot{V}_i(t) = (c_i)^3 t^2 \pi^{-2} (tg\psi_i)^{-1}, t = \Delta t_i,$$

where the triplet's space speed c_i is defined by applying a natural constrain $\Delta S(\mu(t_i)) \to 0$ to the triplet's space geometry. This determines $c_i = c_i(\Delta t_i)$.

For instance, at $i = 3, \Delta t_3 = t_3 - t_1 = 1.2$, we get $(c_3)^{-1} \approx 0.0125$ and $\dot{V}_3(\Delta t_3) \approx 0.1957 \times 10^{-3}$.

Using formula (7.63) in the form

$$M_i^*(\Delta t_3) \approx 3\dot{V}_3(\Delta t_3)\ (\mathbf{a}/\Delta t_3)^2\ (c_3)^{-1}, \text{ we get } M_i^*(\Delta t_3) \approx 0.169 \times 10^{-3}.$$

The calculated complexity in (7.61) leads to $MC_{t_3}^{\delta} = -0.4435 \times 10^{-3}$ and the corresponding curvature is $K^i = -0.433$.

Therefore both formulas (7.57) and (7.61) for the curvature illustrate the comparative results with a relative (to their average) error $\pm 4\%$, even at a rough approximation.

Example 7.3. Let us find a *maximal* information curvature, formed at a locality of a single triplet's node when all three eigenvectors are joint into a single one dimensional vector. According to formula (7.58) the curvature is

$$K = -3\alpha_{3t}\dot{\alpha}_{3t}, \text{ where } \dot{\alpha}_{3t} = 2(\alpha_{3t}^{t})^2 \text{ and } K_3 = -6t_o^{-3}\mathbf{a}^3 \text{ at } \alpha_{3o} = \mathbf{a}_o t_o^{-1}. \quad (7.70)$$

We can evaluate this result using the minimal time interval $t_o = t_{io} \approx 1.33\times 10^{-15}$, corresponding the light's wavelength $l_o = l_{io} = 4\times 10^{-7} m$, and the invariant $\mathbf{a}(\gamma \to 0) \cong 0.25$, we get $K_3 \cong -2.5\times 10^{45}$.

We also evaluate the cooperative complexity at this location:

$$MC_t^{\delta} = 6(\alpha_{3t})^2 v_{3t}^{-1} = 6(\pi c_m)^{-3} t_o^{-7}\mathbf{a}^2, \quad (7.70a)$$

at $c_m = c_{mo} \cong 1.03\times 10^{17}$, getting a maximal $MC_t^{\delta} \cong 1.64\times 10^{51}$, which corresponds to a maximal IN's compression to a single one-dimensional node at $\gamma \to 0$.

The above c_m is obtained in Nat/s at $\mathbf{a}_o \cong \ln 2 \cong 1.44$ bit.

This maximal information speed, evaluated in gigabit per sec, is $c_m \cong 1.483\times 10^8$ gigabit/s.

We assume that this is a limit, imposed on information transmission not by the channel's characteristics, but a source's ability to produce a maximal information during a minimal admissible time interval of the light wave.

At this condition, each elementary light wavelength's time interval carries $10^{34}\times 1.33\times 10^{-15} = 1.33\times 10^{19}$ Nat.

Because each light wavelength's space interval carries the same information, each light wave's meter holds $1.33/4\times 10^{19}\times 10^7 = 0.3325\times 10^{26}$ Nat, or 0.3325×10^{17} gigaNat/m, which corresponds to $0.4788\times 10^{17} \cong 50\times 10^{15}$ gigabit/m, delivering each second $\cong 0.5\times 10^9$ gigabit with speed of light. This is a limit of a maximal information compression up to a single bit in the final IN's triplet's node. ●

The equations (7.58), (7.65) show how the curvature of the geometrical space R^3, determined by the space fundamental tensor, enfolds an inner uncertainty-entropy, becoming an attribute of cooperative dynamics.

The cells' structure of the curved information space conceals information of the cooperative dynamics.

This space curvature evaluates an intensity of information cooperative attraction, complexity and information mass, generated at cooperation.

The bound information along with the curvature imposes the restrictions on a linear information speed, which jointly with the information mass determine an information cooperative power.

In the above information space, the geometrical metric is determined by information invariants, whereas in the physical space, the geometrical metric is determined by physical invariants.

Dynamic processes in both spaces are governed by related variation principles: for the information and physical path functionals accordingly.

The physical and informational attractions are determined by the same curvature of the geometrical space, which are both a source of attraction and a carrier of information.

Since a physical space's curvature generates gravitational attraction, the gravitational and informational attractions are connected by an inner uncertainty, enclosed by both physical and informational fields and expressed by *information invariants.*

Such an invariant information measure (as an equivalent) of physical attraction links physical and information spaces, information and physical geometries, as well as the related dynamics, serving for their mutual transformation.

Because the cooperation requires a curvature, it causes both uncertainty and curvature. This means that the curve space is a *result* of the cooperative dynamics, whereas both uncertainty and curvature are inseparable and mutually dependable space's attributes.

The curved space embraces the cooperated information structures and the IN's information codes, which the geometrical curvature enfolds.

Therefore, the IN's code is a primary information structure of a curved space.

Actual formal originator of cooperative dynamics is the variation principle, which also virtually causes the formation of the curved space and the code.

1.7.8. Connection to Kolmogorov's Complexity

Algorithmic Kolmogorov's (K) complexity [1] is measured by the relative entropy of one object (*k*) with respect to other object (*j*), which is represented by a shortest program in bits.

The ΔMC_{kj}^{δ} complexity measures the quantity of information (transmitted by the relative information flow), required to join the object *j* with the object *k*, which can be expressed by the algorithm of a minimal program, encoded in the MC_{m}^{δ} (IN) communication code.

This program also measures a "difficulty" of obtaining information by *j* from *k* in the transition *dynamics*.

The ΔMC_{kj}^{δ} represents the information measure between *order and disorder* in stochastic dynamics and it can detect determinism amongst the randomness and singularities.

Because the IPF has a limited time length and the IPF strings are *finite*, being an upper bound, the considered cooperative complexity is *computable* in opposition to the *incomputability* of Kolmogorov's complexity.

The MC-complexity is able to implement the introduced notion and measure of information independent on the probability measure by applying the IN information code for the object's processes.

In the IPF-IMD, an object is represented by random processes, while their observations are measured by dynamic processes; and this approach's aim is to reveal the object's information in a form of its *genetic code.*

This approach differs from both the Shannon information of an object's *random events' observation* and the Kolmogorov encoding of *an individual* object's description (in a binary string) by a shortest algorithm; the algorithmic complexity is not required the description's probability function.

Chapter 1.8

THE EVOLUTIONARY DYNAMICS AND THE INFORMATION LAW OF EVOLUTION

The bi-level macromodel, following from the solution of variation problem for information path functional, embraces the following regularities of the evolutionary dynamics: creation of an order from stochastics, evolutionary hierarchy, stability, adaptive self-controls and a self-organization with copying information and a genetic code.

The equations' regularities allow us to formulate the mathematical law of evolution, based on a minimax variation principle (VP) for the informational path functional.

The chapter shows that the VP single form of information law, which defines the above regularities, is capable of a prognosis the evolutionary dynamics and its specific components: the evolution potentials, diversity, speed, and genetic code.

1.8.1. Introduction

In spite of numerous publications on evolution theory [1-10, other], the central questions concerning the existence of a general evolutionary law for Life, remains unanswered. (Please see the references to ch.1.8).

E. Schrödinger [11], analyzing a physical aspect of Life, concludes that an organism supports itself through orders that it receives from its environment via the *maximization of a negative entrop*y, as a general principle, connecting its dynamics and stochastics.

Darwinian law's modern formula [12], stating that "evolution is a result of genetic variances through ordering by the elimination and selection", focuses on a competitive "*struggle* for Life" amongst organisms.

G. Dover [13] emphasizes on the genes' *cooperative* phenomena: "the central feature of evolution is one of tolerance and cooperation between interacting genes and between organisms and their environment… Genes are born to cooperate".

An informational aspect of evolution in [14] symbolizes "the cooperation and ordering with a creation of information code that transfers the novelties during an evolutionary cycle." Are there any connections between all these formulas, covering them under a general law?

Most publications in mathematical biology [15-16, others] do not allow for a quantitative prognosis of the law phenomena, and only support the law by experimental and/or simulation data. For example, [16] describes the result of *simulation*, depending on priory *chosen* parameters, without a detailed insight of the *dynamic* mechanism of micro-macroevolution.

In such approaches, many essential evolutionary phenomena are missing.

We show that instead of a "punctuated equilibrium" [17,5], evolution depends on a punctuated *nonequilibrium* where the microlevel's stochastic fluctuations contribute to the macrolevel's dynamics, generating a dynamic potential of evolution.

A principal aspect of evolutionary law is an ability to *predict* the process development and phenomena based on the law mathematical forms.

The existing mathematical formalism does not satisfy these requirements.

Some mathematical publications [6,7,17-20, others] present the evolution equations, which are not bound by a general principle related to a unique law.

The existing situation in evolution theory is dominated by diverse assumptions, concepts, methods, and hypotheses. The unique formalism of evolutionary law, as well as general systemic regularities of evolution, expressed in information form are still unknown.

This chapter goal is to find the mathematical forms of *general regularities* of evolutionary process, based on a single variation principle as a mathematical law. Approaching to this problem with a broad and rather formal point of view, we apply and finalize our results (ch.1.1-1.6), and introduce the *informational* evolutionary mechanisms of evolution, not dependable on specific material substances [22].

An observed *evolutionary* process is affected by *stochastic* perturbations, which activate the process dynamics, involving a wide diversity of superimposing process' of distinct nature. Such a complex random object can be modeled by *information* interactions, making the information description a *universal* language for disclosing the object's information regularities. The solved variation problem (ch.1.3) leads to a dynamic model of *open system* with an *irreversible* macroprocess, originated by a random microprocess.

The known *variation* problems produce the equation of a *close* system with a *reversible* processes. Applying the variation principle to the path functional's information measure leads to revealing a *systemic* connection of the functional process' events and finally to building an *information network* not only for the process events but also for a complex object multi-dimensional process, which models their systemic *regularities*.

Following the information path functional's and variation principle's results (ch.1.1-1.7), we establish a mathematical form of evolutionary law and study the law regularities.

1.8.2. The Equations Regularities and the Evolutionary Law

We finalize the above results focusing on the equations regularities following from the VP as an evolutionary law.

Proposition 8.1(P1). At the fulfillment of relations

$$\dot{x} = a^u, \dot{x} = L_k X,$$
$$\dot{x}_t(\tau) = -A(\tau)x(\tau), A(\tau) = 1/2r^{-1}(\tau)\dot{r}(\tau), r(\tau) = E[\tilde{x}(\tau)\tilde{x}(\tau)^T] \quad (8.1)$$

the dynamics and stochastics are connected by equations:

$$L_k(x,\tau+o)=2b(x,\tau+o),\ a^u(\tau-o)=\sigma\sigma^T(\tau-o)X(\tau-o), \tag{8.1a}$$

$$A(\tau)=b(\tau)(2\int_{\tau-o}^{\tau}b(t)dt)^{-1}=1/2\frac{d}{dt}|\ln(\int_{\tau-o}^{\tau}b(t)dt)|_{t=\tau},$$

$$2\int_{\tau-o}^{\tau}b(t)dt=E[\tilde{x}(\tau)\tilde{x}(\tau)^T], \tag{8.2}$$

where the dynamic operator depends on the *gradient of dynamic potential*

$$gradX(\tau)=\frac{\partial X(\tau)}{\partial x(\tau)}=-(2\int_{\tau-o}^{\tau}\sigma\sigma^{\mathrm{T}}(t)dt)^{-1},\ X(\tau)=-1/2(\int_{\tau-o}^{\tau}\sigma\sigma^{\mathrm{T}}(t)dt)^{-1}x(\tau), \tag{8.3}$$

defined by the process diffusion at the DP localities.

Proof. The joint consideration of equations (8.1), following from ch.1.3.T3.2 (1.3.47, 1.3.48), (sec.1.3.5), connects the shift vector and the diffusion component (of the initial stochastic equation) at the DP's border in the forms (8.1a),(8.2) and brings the conjugate vector and its gradient to the forms (8.3) accordingly. •

<u>*Comment*</u> <u>8.1.</u> The potential, corresponding to the conjugate vector, which satisfies (8.3) at the DP, loses its *deterministic* dependency on the shift vector, becoming the function of *diffusion* and a state vector at the DP vicinity, and when $\sigma\sigma^{\mathrm{T}}\to 0$, it acquires a form of the δ-function. Out of the DP, the gradient does not exist as well as the potential function in form (8.3) and the potential relations (8.1).

At the dynamic gradient's growth, the diffusion part in (8.3) tends to decrease, which *diminishes* an effect of randomness on the dynamics, and vice versa.

<u>*Comment*</u> <u>8.2.</u>The DPs divide the macrotrajectory on a *sequence* of the extremal segments limited by the "punched" DP's localities.

The microlevel randomness provides the various information contributions at each DP locality, determined by function (3.163), which (taking into account (8.1a)) depends on $b=b(\tilde{x}_t(\tau)$ being limited by a specific microlevel environment.

Each DP locality is defined by a relative irreversible time interval **a**

$$\delta t_k^*=\frac{\delta t_k}{t_k}=(\frac{\Delta S_k^{\delta}}{\mathbf{a}_o^2}-1),\ \delta t_k\sim(\tau_{k+1}^o-\tau_k^1), \tag{8.4}$$

which is reduced with decreasing γ and growing $\mathbf{a}_o(\gamma)$, and vice versa.

At small γ, relation (1.5.42c) is evaluated by

$$\Delta S^{\delta} = \Delta S(\delta t_k) \cong \mathbf{a}_o^2(\gamma), \tag{8.4a}$$

which for (8.4) leads to $\delta t_k^* \to 0$.
This means that decreasing γ *diminishes* the influence of randomness on the macrolevel dynamics. But actual ΔS^{δ} is random that cannot satisfy exactly (8.4a).

Proposition 8.2(P2).
The evolution dynamic equation within each k-segment:

$$|A(\tau_k^1)| = |A(\tau_k^o)| \exp[-(|A(\tau_k^o)|\tau_k^1)]\{2E - \exp[-(|A(\tau_k^o)|\tau_k^1)]\}^{-1} \tag{8.5}$$

depends on the operator $A(\tau_k^o)$ at the beginning τ_k^o of the segment time interval

$t_k = \tau_k^1 - \tau_k^o$, identified at a moment $\tau_{k+1} = \tau_k^1 + o$, while the segment's ending operator

$A(\tau_k^1)$ is renovated at a moment τ_{k+1}^o (which does not belong to this segment).

These operators, identified at the punched localities $\tau = (..., \tau_k, \tau_{k+1}, ...)$ according to (8.1), (8.2), satisfy the relations

$$|A(\tau_k)| = |A(\tau_k^o)|, |A(\tau_{k+1})| = |A(\tau_{k+1}^o)| \neq |A(\tau_k^1)|. \tag{8.6}$$

Existence of the joint solutions (8.4), (8.5) and (8.2) corresponds to the model *ability* for renovation under the microlevel randomness and controls.

Proof follows from P1 and (ch.1.3). •

Comments 8.3. The results of chs.1.3 and ch.1.4 allows finding the length of the extremal segment and the related time intervals for equations (8.1-2),(8.5) applying the invariant relations (sec.1.3.5). The operator renovation is accompanied by changing the operator sign.

In particular, at $v_i(\tau_k^o) = -2x_i(\tau_k^o)$ we get $A(\tau_{k+1} - o) = -A(\tau_k^1)$, and at

$$\delta v_i(\tau_k^1, \tau_{k+1}, \tau_{k+1}^o) = -2x_i(\tau_k^1) + 2x_i(\tau_{k+1}^o)(\tau_k, \tau_{k+1}), i = 1,..n, k = 1,..., m, \tag{8.7}$$

we have

$$A(\tau_k) = -A(\tau_k^o) = -(\lambda_i(\tau_k^o)), A(\tau_{k+1}) = (\lambda_j(\tau_{k+1})) = -A(\tau_{k+1}^o) = -(\lambda_j(\tau_{k+1}^o)), \tag{8.8}$$

where $A(\tau_k), A(\tau_{k+1})$ satisfies (8.1) and $A(\tau_k^o)$, $A(\tau_{k+1}^o)$ satisfies (8.5).

Comments 8.4. According to (8.5) the evolutionary *dynamics* (within a segment) depend on the segment operators' $A(t), t \le t_k$ function of time, *determined by the model's inner (self) controls* and followed both dynamics' *and controls' renovations* at each DP.

Switching the self-control at the moments τ_k^1, τ_{k+1}^o coordinates the dynamics' consumed information *and* the external information, supplied at each DP (τ_k, τ_{k+1}).

This coordination is *driven* by the macrodynamics, which provides the self-control switch in a hunt for external information surplus and for the above coordination, following from the VP.

Therefore, the actual hunt is directed on getting a *maximum* information, corresponding to the VP fulfillment, and *such* a model is *motivated* by this drive, which *seeks the evolutionary changes*. The models, which do not possess the motivated drive, lose the ability for a continuation of the coordinated dynamics and stochastics, and might stop their evolutionary process because of either a lack of necessary information, or an impossibility of assimilation (accumulation) of its surplus. These models do not satisfy the VP law.

Comments 8.5. The *segment's local* information *speed* (along a chain of extremal segments):

$$|\frac{dx_i}{dt}(\tau_k^1)x_i^{-1}(\tau_k^1)| = |\frac{dx_j}{dt}(\tau_{k+1}^o)x_j^{-1}(\tau_{k+1}^o)| \tag{8.9}$$

connects an extremal segment's eigenvalue ((at moment $\tau_k^1 = \tau_{k+1} - o$):

$|\lambda_i(\tau_k^1) = |\frac{dx_i}{dt}(\tau_k^1)x_i^{-1}(\tau_k^1)|$, the current eigenvalue $\lambda_j(\tau_{k+1}) = |\frac{dx_j}{dt}(\tau_{k+1})x_j^{-1}(\tau_{k+1})|$

identified at each τ_{k+1}, and the segment's eigenvalue (at the following moment τ_{k+1}^o):

$|\lambda_j(\tau_{k+1}^o)| = |\frac{dx_j}{dt}(\tau_{k+1}^o)x_j^{-1}(\tau_{k+1}^o)|$.

This connection expresses a requirement for the extremal segments to *fit* each other in the *evolutionary cooperative* dynamics(with building a related IN hierarchy, ch.1.5).

A decrease of the contribution from diffusion with an increase of the gradient (while both comply with the VP) intensifies a *growing impact of the each following dynamics* on the evolutionary development. These link up the *gradient of dynamic potential, local evolutionary speeds, and the evolutionary conditions of a fitness.*

Proposition 8.3 (P3).
An average information speed along the trajectory of evolutionary dynamics:

$$E[H(\tau + o)] = 1/4Tr[A(\tau + o)] = 1/4\sum_{i=1}^{n}\lambda_i(\tau_k^o), k = 1,..., m$$

$$= -1/8\sum_{i=1}^{n} d/dt[((gradX_i(t))^{-1}]_{t=\tau_k+o}\, gradX_i(\tau_k + o), k = 1,...m; \tag{8.10}$$

is generated by the trace-sum of the local speeds–the negentropy productions, or by the sum of the above potential's gradient at the DP's localities.

Proof follows directly from (8.4) and sec. 1.3.4, Comments 4.2. •

Comments 8.6. The average speed depends on the *entire* spectrum's eigenvalues, or a total dynamic potential, representing a whole system with space distributed subsystems (sec.1.7.5), called a "population", which accumulates an emergence of new random features for this population.

The speed grows with adding each following eigenvalue, even though this eigenvalue decreases.

A local speed, measured by a *decreased eigenvalue* in the above trace, *declines;* that weakens the evolutionary dynamics' average speed.

A minimal eigenvalue, corresponding to a local maximum of the dynamic gradient, limits the evolutionary speed.

The evolution speed grows with enlarging the population's dimension.

Corollary 8.1.

Minimization of the entropy $\Delta S(t_k)$ at each segment's time interval (according to the VP) leads to a maximization of a gradient of dynamic potential at the segment end, or to a minimization of the corresponding diffusion contribution (on the right part of (8.3)).

Indeed, after integrating (8.10):

$$-E\int_{t_k}\frac{\partial S}{\partial t}(t)dt = -1/8\int_{t_k}\sum_{j=1}^{n} d\,/\,dt[((gradX_j(t))^{-1}]gradX_j(t)dt \tag{8.11}$$

during the segment time interval $t_k = \tau_k^1 - \tau_k^o$, we get

$$E(\Delta S_{(j)}(t_k) = 1/8[-\ln|gradX_j(\tau_k^1)| + \ln|gradX_j(\tau_k^o)|]$$
$$= 1/8\ln[|gradX_j(\tau_k^o)|/|gradX_j(\tau_k^1)|].$$

The condition

$$\min E(\Delta S_{(j)}(t_k) = \min[|gradX_j(\tau_k^o)|/|gradX_j(\tau_k^1)|] \tag{8. 11a}$$

for each fixed $gradX_j(\tau_k^o)$ at beginning of the interval leads to

$$\max|gradX_j(\tau_k^1)||$$

at the interval end, or to

$$\min[\int_{\tau_k^1}^{\tau_{k+1}-o}\sigma_j^2(t)dt],$$

according to (8.3).

Thus we get

$$\min E[\Delta S_{(j)}(t_k)] \to \max|gradX_j(\tau_k^1)| \to \min[\int_{\tau_k^1}^{\tau_{k+1}-o}\sigma_j^2(t)dt]. \tag{8.12}$$

Comments 8.7. For a stable process at each segment and the fulfillment

$$\min E \mid H_i(\tau + o) \mid = \min Tr \mid A(\tau + o) \mid > \min \sum_{i=1}^{n} \lambda_i(\tau_k^o), \lambda_i(\tau_k^o) < 0 \ , \tag{8.13}$$

(sec.1.3.5), the eigenvalue at the beginning of the extremal segment:

$$\mid \lambda_i(\tau_k^o) \mid = \mid \lambda_i(t_k) \mid (\tau_{k+1} - \tau_k) \sim t_k \tag{8.13a}$$

decreases, which leads to decreasing also $\mid \lambda_i(\tau_k) \mid$ at each DP-locality.

Comments 8.8. While by the end of each extremal' segment, the gradient reaches a local maximum, these maximums are decreasing along the extremal segments' chain with the *diminishing impacts of the dynamic contribution* from a stochastic drive at DPs, as well as lessening of the local eigenvalues at the following segments.

Decrease of each $\lambda_i(\tau_k^o)$ leads to related increase of the intervals between the punched localities $(\tau_{k+1} - \tau_k) \sim t_k$ (following the invariant relations sec.1.3.5)).

Proposition 8.4.

(*i*). Let us have a sum of the relative derivations of the model eigenvalues spectrum, preserving the spectrum *stability* at the current derivations:

$$D = \sum_{i=1}^{n} \frac{\Delta \lambda_i}{\lambda_i}, \tag{8.14}$$

while each spectrum's *variation* is produced by the increment of diffusion according to (8.1-8.3), or the related dynamic potential. We call (8.14) a model's *diversity*.

Then the relative increment of the averaged speed in (8.10):

$$H_\Delta = \frac{\Delta \hat{H}}{\hat{H}} \tag{8.15}$$

is determined by the model diversity $H_\Delta = D$. (8.15a)

Because a sum of the identified eigenvalues for each extremal segment is equal to an averaged local Hamiltonian according to (8.10), the *proof* follows straight.

(ii). Let us have $\max D = D_o$ **,w**hich defines the spectrum's maximal variation, limited by a model's ability to preserve the spectrum *dimension n*.

Then $H_\Delta^o = D_o$ measures a maximal increment of the average evolutionary speed (H_Δ^o), which we call a *potential of evolution* $P_e = H_\Delta^o$ (for a given macrosystem's dimension, related to the population); P_e evaluates a maximum among the admissible model's variations, accumulated during the model operator's renovations under the stochastic drives (for given subsystems' population).

From these follow that *extending the population by increasing dimension n and growing the sum in (8.10), raises the potential of evolution.* •

Proposition 8.5 (P5).

The model limitations on the potential of evolution $P_e = \max \sum_{i=1}^{i=n} a_{io}^*(\gamma)$, which counterbalances to the admissible eigenvalue's deviations (ch.1.7, Prop.7.1), are determined by the fulfillment of the following equations preserving each triplet's dimension:

$$\alpha_{io}^* = 1 - \frac{\alpha_{i+1,o}^t \alpha_{i-1,o}^t}{(\alpha_{io}^t)^2} = 1 - \frac{\gamma_{i-1}^\alpha}{\gamma_i^\alpha} = \frac{\Delta\gamma_i^\alpha}{\gamma_i^\alpha} = \varepsilon, \ \mathbf{a}(\gamma)\gamma_i^\alpha(\gamma) = inv,$$

$$\gamma_i^\alpha(\gamma) = \frac{\alpha_{io}^t}{\alpha_{i+1,o}^t} = \frac{t_{i+1}}{t_i} = inv, \quad \frac{t_{i-1}}{t_i} = \frac{t_i}{t_{i+1}} = inv, \tag{8.16}$$

where $\gamma_{i-1}^\alpha(\gamma), \gamma_i^\alpha(\gamma)$ are the triplet eigenvalue's ratios. •

Comments 8.9. For a triplet, this ability will be compromised, if either α_{io}^t approaches α_{i-1}^t, or α_{i+1}^t approaches $\alpha_{i+1,o}^t$, *because* if both $|\alpha_{io}^t - \alpha_{i-1,o}^t| \to 0, |\alpha_{io}^t - \alpha_{i+1,o}^t| \to 0$, the triplet disappears, and the macromodel dimension decreases according to $m = \frac{n-1}{2}$. Actually, the above minimal eigenvalue's distance is limited by the admissible minimal relative $\frac{\Delta\alpha_{io}^t}{\alpha_{io}^t} \cong 0.0072$ at perturbations $\alpha_{it}^*.\ \alpha_{it}^*$.

The maximal distance between both $\alpha_{i-1,o}^t$ and α_{io}^t, α_{io}^t and $\alpha_{i+1,o}^t$ at the fixed $\alpha_{i-1,o}^t$, $\alpha_{i+1,o}^t$ satisfies to the known condition of dividing a segment with ($\alpha_{i-1,o}^t, \alpha_{io}^t, \alpha_{i+1,o}^t$) in the mean and extreme ratio: $\frac{\alpha_{i-1,o}^t}{\alpha_{io}^t} = \frac{\alpha_{io}^t}{\alpha_{i+1,o}^t}$, which coincides with the above formulas for the triplet's invariant (8.16).

The triplet's *information capacity* to counterbalance the maximal admissible deviations defines the triplet's potential P_e^m:

$$P_e^m = \max \frac{\Delta\gamma_i^\alpha}{\gamma_i^\alpha} = \max |\varepsilon(\gamma)|. \tag{8.17}$$

For an entire macromodel with m such triplets, the total potential is $P_e^n = m\ P_e^m$.

Proposition 8.6.

The macromodel potential P_e^n is limited by the maximum *acceptable* increment of dimension that sustains the macrostates' cooperation:

$$P_e^n \cong 1/3\,m,\ m = \frac{n-1}{2}. \tag{8.18}$$

A *maximal* $\varepsilon(\gamma)$ at the permissible deviations of $\gamma_i^{\alpha}(\gamma)$, which preserves the triple macrostates' *cooperation* and the triplet formation, corresponds to the minimal admissible $\gamma \cong 0.00718$, which for $\gamma_1^{\alpha} = 2.46$, $\gamma_2^{\alpha} = 1.82$ brings $\max \varepsilon \cong 0.35$ and $P_e^n = m\ P_e^m \cong 1/3\,m$. •

Comments 8.10. Potential P_e^n differs from P_e, which generally does not support the evolution process's hierarchy. The model's *acceptable* potential P_e^n that can adapt variations not compromising the IN hierarchy, we call *adaptive potential* $P_a = P_e^n$.

Relation $P_e^n \le P_e$ limits the variations, acceptable by the model that restrict the maximal increment of dimension and sustain the model's cooperative functions.

At the variations within the P_a capabilities, the generated mutations enable creating a new dimension with a trend to minimize both γ and the system uncertainty (ch.1.6). Extending the P_a capability can bring instability, accompanied by growing γ and a possibility to jeopardize the system cooperation.

The adaptive model's *function* is implemented by the adaptive *feedback-control*, acting within the P_a capabilities.

The *triplet's robustness*, which preserves the triplet's invariants under admissible maximal error (at current and fixed $\gamma = \gamma^*$), is satisfied if the related adaptive potential holds the relation analogous to (8.17):

$$P_a = \max \varepsilon(\gamma^*). \tag{8.19}$$

In a chain of connected information events, the appearance of an event carrying $\gamma \to 1$, leads to $\mathbf{a}_o\,(\gamma = 1) = 0$ when both information contributions from the regular control $\mathbf{a}\ (\gamma = 1) = 0$ and the needle control $\mathbf{a}_o^2(\gamma = 1) = 0$ turn to zero.

At a locality of $\gamma = 1$, both the event's information $\mathbf{a}_o$ and the related time undergo a jump, which could be the indicators of approaching $\gamma = 1$.

This means, the appearance of an event carrying $\gamma \to 1$ leads to a chaos and decoupling of the chain, whereas the moment of this event's occurrence could be predicted (ch. 1.7).

Comments 8.11. Let us show that at the equal deviations of the model parameter γ_i^{α}: $\pm\Delta\gamma_1^{\alpha}, \pm\Delta\gamma_2^{\alpha}$, the model threshold $|\varepsilon(\Delta\gamma)|$ is asymmetric.

Indeed. The macromodel with $\gamma = 0.5$ has $\gamma_1^{\alpha} = 2.21$, $\gamma_2^{\alpha} = 1.76$ and get $\varepsilon_o = 0.255$. Admissible deviations $\Delta\gamma_1^{\alpha} = 0.25, \Delta\gamma_2^{\alpha} = 0.08$ correspond to the

macromodel with $\gamma \cong 0.01$, $\gamma_1^{\alpha} = 0.246$, $\gamma_2^{\alpha} = 1.82$, which determines $\varepsilon_1 \cong 0.35$ and $\Delta\varepsilon(\Delta\gamma_1^{\alpha}, \Delta\gamma_2^{\alpha}) = \varepsilon_1 - \varepsilon_o = 0.095$.

At $\Delta\gamma_1^{\alpha} = -0.25$, $\Delta\gamma_2^{\alpha} = -0.08$ we have the macromodel with $\gamma = 0.8$, $\gamma_1^{\alpha} = 1.96$, $\gamma_2^{\alpha} = 1.68$, which determines $\varepsilon_2 \cong 0.167$ and $\Delta\varepsilon(-\Delta\gamma_1^{\alpha}, -\Delta\gamma_2^{\alpha}) = \varepsilon_2 - \varepsilon_o = -0.088$.
It is seen that at the equal deviations, the model potential, defined by

$$\max \varepsilon = \max \varepsilon_1 \cong 0.35, \tag{8.20}$$

tends to *increase at a decreasing* of γ, and vice versa.
An essential *asymmetry* of $|\varepsilon(\Delta\gamma)|$ and therefore the dissimilar P_m are the result of the macromodel fundamental quality of *irreversibility*. •

Therefore, evolution dynamics, which follow from VP, is accompanied by the following main *features*:

-A tendency of decreasing γ that *diminishes* the influence of randomness on the macrolevel dynamics;

- A decrease of the contribution from diffusion with the increase of the dynamic gradient that intensifies a *growing impact of the each following dynamics* on the evolutionary development;

-An average evolutionary information speed (for a population of subsystems) is declined, weakening during the time of evolution;

-A nonsymmetrical adaptive potential leads to both rising the system's adaptive capability (at decreasing γ) with expanding the potential ability to correct a current error and increasing of the impact of a dynamic prehistory on current changes.

1.8.3. A Mechanism of an Enhancement of the Acceptable Mutations

According to P.8.4, the sum of the relative increments of the eigenvalues spectrum measures the system's potential diversity

$$D = \sum_{i=1}^{n} \frac{\Delta\lambda_i}{\lambda_i},\ \Delta\lambda_i(\tau_k) = 1/2\Delta[\sigma_i\sigma_i^T(\tau_k)(\int_{\tau_{k-o}}^{\tau_k} \sigma_i\sigma_i^T(t)dt)^{-1})], \tag{8.21}$$

which determines the system's *admissible variations*, preserving the spectrum *stability* at *current* deviations.

A $\max D = D_o$ defines the spectrum's maximal variation, limited by a system's ability to preserve the spectrum *dimension n*; $D_o = P_e$ measures a maximal potential of *evolution* P_e and brings a maximal increment of the average *evolutionary speed*

$$H_{\Delta} = \frac{\Delta \hat{H}}{\hat{H}} = D, D_o = \max H_{\Delta} = H_{\Delta}^{o} = P_e \qquad (8.22)$$

for a given macrosystem's dimension, related to the subsystem's population.

The required spectrum's *stability* at current derivations brings a direct P_e dependency on the maximum *admissible deviations*, preserving the fixed triplet's *invariants*, or a fixed γ and the triplet code.

If the fluctuations generate the admissible deviations, the model enables generating the *feedback-controls* that support a *structural robustness* for each IN's node.

In this case, $P_e(\gamma) = P_r$ represents the model *potential of robustness*, which expresses the model ability to counterbalance the admissible eigenvalue's deviations (acting within the P_r capacity).

The P_r restriction (by the triplet's invariants and the code) limits its capacity for the model's evolution. A potential code, which has been generated by the *potential of robustness*, is a DSS's evolutionary *predecessor*.

Creating new macromodel characteristics is possible by changing the invariant and the code.

Let us *analyze* when it is possible.

During the o-window of the applied control, when the model is open for the environmental interactions, the external perturbations could *cohere* with the microlevel randomness (Fig.8.1). The perturbation of a high frequency represents a noise, which is the subject of the model filtration. Thus, the model frequency' coherence with the environmental frequency and/or with other interacting macromodels might occur at a lower frequency, theoretically, at any o-windows within the segment's chain of a given dimension, and also at the chain end, corresponding to the o-window of the IN final node.

The *mutations*, generated by the coherent frequencies, are more powerful, compared with others *competing* within the window. Such a coherence creates a kind of *cooperation* between the external and internal model's processes, in particular, during the stochastic resonance.

A current eigenvalue $\alpha_k(\tau_k) \sim \omega_k(\tau_k)$ could be modified by a resonance frequency $\omega_k^1(\tau_k)$, which can bring an increment of the eigenvalue in (8.21): $\delta\alpha_k(\tau_k) \sim \delta\omega_k(\tau_k)$ according to relations $\alpha_k(\tau_k) + \delta\alpha_k(\tau_k) = \alpha_k^1(\tau_k + \delta\tau_k)$.

The new $\alpha_k^1(\tau_{k\delta}^o) = -\alpha_k^1(\tau_k + \delta\tau_k)$, fixed at the beginning of following segment $\tau_{k\delta}^o$, changes the primary invariant $\mathbf{a}(\tau_k) = \alpha_k(\tau_k^o)\tau_k^o$ that acquires a new value $\mathbf{a}^1(\tau_{k\delta}^o) = \alpha_k^1(\tau_{k\delta}^o)\tau_{k\delta}^o$.

An essence of stochastic resonance consists of a coincidence of a deterministic frequency, defined by the eigenvalue $\alpha_k(\tau_k)$, with a resonating frequency of a random environment $\omega_k^1(\tau_k)$.

This leads to the modified time-space dynamics at the following segments and changing the quantity of information, carried by corresponding section of DSS code.

In the stochastic resonance, the primary frequency $\omega_k(\tau_k)$ attracts some frequencies of external fluctuations, *selecting* the nearest of them, while forming the resonance frequency $\omega_k^1(\tau_k)$.

According to the VP, the newborn $\alpha_k^1(\tau_k + \delta\tau_k)$ should bring a local maximum to the information speed, which allows getting a maximum of the information (negentropy) production from the environment.

In a common environment, such this model's eigenvalue $\alpha_k^1(\tau_k + \delta\tau_k)$ possesses an advantage in the survival, comparing with other competing models, having the smaller eigenvalues and the information productions.

This allows the model detecting and accepting the mutations *beneficial* for the model's processes and providing a *directed* evolution [9, 14], accompanied by the code's renovation.

The information environment should be able to supply this maximum via a cooperative interaction of an external information source with the internal $\alpha_k^1(\tau_k + \delta\tau_k)$, therefore establishing the information exchanges-communications between them.

Without the cooperation with a coherent frequency $\omega_k^1(\tau_k)$, a new $\delta\alpha_k(\tau_k)$ could not be obtained (at a first place) from the random microlevel's environment.

The external frequency resonance $\omega_k^1(\tau_k)$ during interval $\delta\tau_k$ provides **an** external entropy $\partial S_e(\tau_k)$ taken from the microlevel random process for the cooperation between $\omega_k^1(\tau_k)$ and $\alpha_k^1(\tau_k + \delta\tau_k)$ (while $\partial S_e(\tau_k)$ is limited by the parameters of stochastic equation, ch.1.1). This elementary entropy defines the renovated invariant $\mathbf{a}_o^1(\tau_{k\delta}^o)$, which determines the following interval of discretization $t_{k\delta} = \tau_{k\delta}^1 - \tau_{k\delta}^o$ with

$\tau_{k\delta}^1 = \mathrm{a}_o^1(\tau_{k\delta}^o) \,/\, \alpha_k^1(\tau_{k\delta}^o)$ at known $\alpha_k^1(\tau_{k\delta}^o) = -\alpha_k^1(\tau_k + \delta\tau_k)$.

Each macromodel, built according to the VP with the limitation ch.1.7), which enters to the environment, should have a *limited* maximum of invariant $\mathbf{a}_o^1(\tau_{k\delta}^o)$ to be acceptable by the existing environmental information systems, providing $\partial S_e(\tau_k)$.

At this condition, the macrolevel's $\alpha_k^1(\tau_k + \delta\tau_k)$ would be able to *cooperate* with an external frequency (eigenvalue), having the coinciding information speeds, to produce information $\mathbf{a}_o^1(\tau_{k\delta}^o)$, available in a current information environment.

Such ability depends on both the model's *individual* characteristics (invariants, eigenvalues, and a capacity of generating an attraction frequency) and the *local environment*, delivering the model's resonance frequency.

On Fig.8.1 we analyze the possible cooperative schemas in the local macrosystems' environment. Some of the $\alpha_k^1(\tau_k + \delta\tau_k)$ could be attracted by the eigenvalues of the same spectrum, forming a doublet or triplet of the IN current spectrum (the IN node's attractor).

Other, with a higher $\alpha_k^1(\tau_k + \delta\tau_k)$, can not be included in the nearest systems' INs, but are capable to attract another local eigenvalue with a lesser value to start a new IN and its DSS code. Such a local macromodel opens a possibility of creating a new generation of the systems.

Each of the IN's local hierarchical level could be a dead end of a specific evolutionary tree.

Indeed. The modified code's section (with a changed parameter of uncertainty γ_k) might encode a *distinctive phenomenon's component*, compared to a preceding (primary) code's section. The modified eigenvalue and a new γ_k change the primary spectrum of the model eigenvalues, and a perhaps its dimension, taking into account also a possibility of attaching the new eigenvalue to the final IN node.

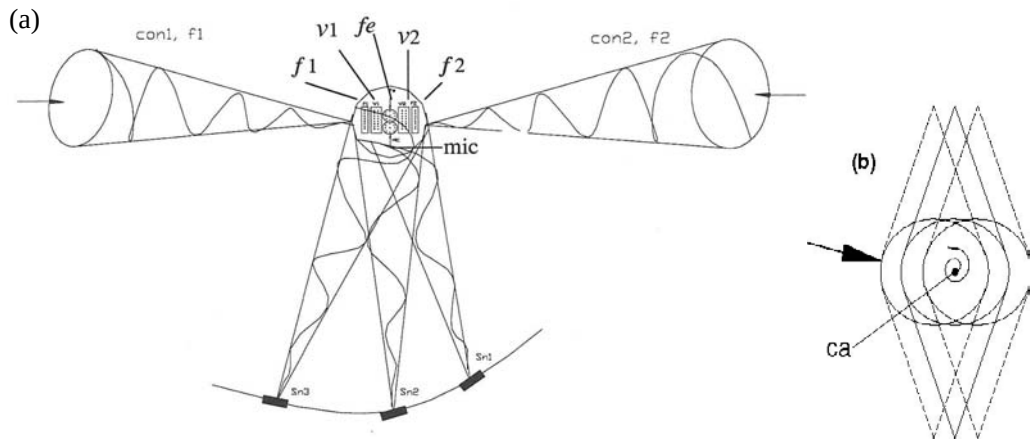


Figure 8.1. An illustration of the competition and selection for the state's variations at the DP locality, brought by: the external frequencies $f1, f2$, the controls $v1, v2$, the microlevel mic, and the external influences fe, with forming the multiple node's spots $sn1, sn2, sn3$, generated by a chaotic attractors; the node, selected from the spot, is memorized. b) Chaotic dynamics, generating an attractor (ca) at joining the cone's processes, which forms the following cone's basin.

Therefore, the renovations of both the model's eigenvalues and the invariant modify the code and initiate the above mutations, which enhance and memorize the appearance of new features (including new subsystems) and a future model's development.

The random mutations, generating a rising macromodel's dimension, contribute to growing P_e.The average amount of the triplet's potential:

$$P_a \approx 1/3m\,(\gamma = 0.5)\,, \tag{8.23}$$

generates an information capacity equals to $\approx 33\%$ of the quantity of the non-redundant information in $\mathbf{a}_o$ bits, accumulated by each triplet.

This information is encoded only by three letters of the double spiral optimal code (DSS), generated by the controls (ch.1.6).

The maximal acceptable variation of information, corresponding to the 33% capacity of triplet's potential, might change just one letter to the non-redundant three letters of the minimal DSS code.

According to the model's mechanism, the applied control can provide the forth letter to the initial minimal three triplet's letters.

The control works like a switch changing the letter's sequence.

The switch represents a memorized control signal $v(\tau) = -2x(\tau)$, (memorized at each τ_k^o) which acts upon the subsequent dynamics.

The code's switch provides an additional capability for adapted evolution.

The existence of the adaptive IN capacity creates a model's *potential redundancy (PR)*, which is automatically *added* to the initial non-redundant IN. The *PR* leads to an ability of correcting the errors under the admissible perturbations, preserving the invariants, which provides the model's *error correction mechanism* to the IN and its DSS code.

The adaptive PR *control minimizes the error of accepted external information, preserving and protecting the internal IN's cooperative structure.*

Following the VP, the most probable evolution of the modified eigenvalues goes in the direction of their lessening, leading to a decrease of the model invariant $\mathbf{a}(\gamma_k)$ (reducing a contribution of an external control) and a decrease of the model's uncertainty parameter γ that is beneficial to improvement of a system's functioning.

These effects associate with the above nonsymmetry of potential P_e, which provides an increase in an internal negentropy production and shows a *self-organization* trend and automatic improvement under mutations.

The model stability assumes a balance between the IN's geometrical border BMS (ch.1.6), being a source of model negentropy, and the external entropy.

Such a coordination is performed along all BMS surface, with some space distributions of the dynamic operator's $\lambda(e)$-eigenvalues spectrum.

A diversity of the external environment creates an essential variety of the BMS forms and their cooperative key-lock connections (Fig.8.2) with different $\lambda(e)$-space distributions, which are supposed to compensate for the environmental influence.

A lack of such variety or a lack of some noncompensated values of this spectrum brings the model's instability.

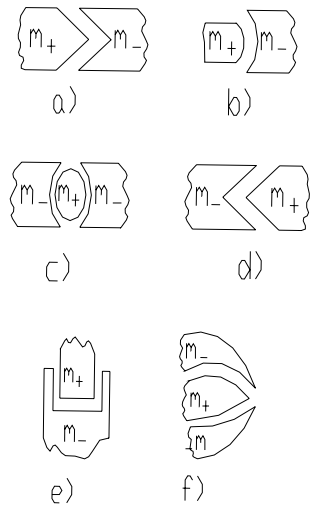


Figure 8.2. The BSM geometrical structures: a),b),c),d),e), f)- formed by the control's information cooperative key-lock connections; m_-, m_+ are the interacting information masses (sec.1.7.4) (virtual or a real).

The discrete eigenvalues spectrum determines the allocation of the BMS's $\lambda\ (e)$ with some discrete spatial interval δl and the space curvature, defined by the model dynamic invariants.

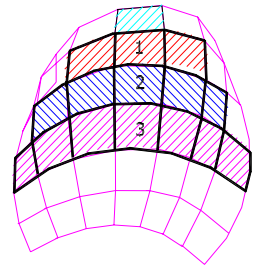


Figure 8.2.a. A BSM surface made by a sequence of the triplets (1,2,3,…).

The BMS' entropy is proportional to the entropy density and the value of the BMS square F_2^n (Fig.6.4b). Assuming a fixed entropy density for the *maximal* $\delta l\,(\gamma \to 0)$=0.372=const, we may determine the maximal elementary BMS square (δF) for each triplet: $\delta \mathrm{F} = \pi 3/4(\delta l)^2$. A BMS surface, made by the triplet's sequence (1,2,3), is shown on Fig.8.2a. The total square for all BMSs with (**n**-1)/2 triplets approaches

F($\gamma \to 0$) $\cong$ 0.32(**n**-1)/2. This F is close to the value of the adaptation potential P_m (8.23), providing a maximal contact between the external mutations and the adaptive model's resistance throughout a "genotypic fitness surface" [2].

The information geometry (Fig.6.5), enfolding the varieties of genetic codes, is an actual space of interactions, variances, and mutations.

An intrinsic macrodynamic geometry reveals its new role as a source of the memorized defect of dynamic information**,** at the formation of the macrostructure**,** and as an *attribute of adaptation*.

1.8.4. The Conditions of the Model's Self-control, Adaptation, and Self-organization

Let us find such a control, which, acting automatically, is able to accomplish model performance within capacities of both the model's adaptive and evolution potentials.

We consider the model's optimal controls function

$$u_i(t_i) = \alpha_i^t v_i(t_i)\,,\; v_i(t_i) = -2x_i(t_i)\,, \tag{8.24}$$

applied at discrete moment t_i, while $x_i(t)$ satisfies to the controllable local differential equation for a segment (in a diagonal form):

$$\dot{x}_i(t) = -\alpha_i^t\, x_i(t). \tag{8.25}$$

Then, the optimal control

$$u_k(t_i) = -2\alpha_k^t v_k(t_i) = -2\dot{x}_k(t_i)\;, \tag{8.26}$$

can also be applied using a derivative $\dot{x}_k(t_i)$.

Considering a sequence of macrostates, satisfying the relations for the *phase* coordinates [14]: $\dot{x}_i(t_i) = x_k(t_i)$, $x_l(t_{i+1})$, $x_m(t_{i+2})$ for the subsequent discrete moments (t_{i+1}, t_{i+2}), we come to the macrostates' connections:

$$x_k(t_i) = \dot{x}_i(t_i)\,,\; x_l(t_{i+1}) = \dot{x}_k(t_{i+1})\,,\; x_m(t_{i+2}) = \dot{x}_l(t_{i+2})\,. \tag{8.27}$$

If the macrostates' sequence is numbered according to the time's sequence, we come to the relations

$$x_f(t_{i-1}) = x_i(t_{i-1}), x_k(t_i) = \dot{x}_i(t_i), x_m(t_{i+1}) = \ddot{x}_i(t_{i+1}), x_l(t_{i+2}) = \dot{x}_m(t_{i+2}) = \dddot{x}_i(t_{i+2}), \ldots \tag{8.28}$$

which satisfy to $x_{k-1}(t_i) = \dot{x}_k(t_i)$, or $x_{k-1}(\tau) = \dot{x}_k(\tau)$ for any following $k-1, k, k+1$ macrostates at fixed $t_i = \tau$.

This shows that each $x_{i-1}(t_i)$ is able to perform the optimal control function, applied to the following $x_i(t_i)$, *and* the generation the model optimal processes can be possible by

originating only an initial optimal macrostate $x_i(t_o)$, applied as a control $x_i(t_o) = u_{i+1}(t_o)$ to the equation

$$\dot{x}_{i+1}(t\ ) = \alpha^t_{i+1,o} x_{i+1}(t\ ) + u_{i+1}(t_o)\,. \tag{8.29}$$

At $u_{i+1}(t_o) = \alpha^t_{i+1,o} x_{i+1}(t_o)$, we get the initial $\dot{x}_{i+1}(t_o + o) = 2\alpha^t_{i+1,o} x_{i+1}(t_o + o)$,

which can be applied as a control

$$u_{i+1}(\tau_o) = -2\dot{x}_{i+1}(\tau_o) = -2\alpha^t_{i+1,o} x_{i+1}(\tau_o)$$

at the moment τ_o, following the initial t_o, where $t_o - \tau_o = \delta t_o$.

The sequence of the regular control's differences:

$$\delta u_{i+1}(\tau_o) = u_{i+1}(t_o) - u_{i+1}(\tau_o) = x_i(t_o) - 2\dot{x}_{i+1}(\tau_o)\,, \tag{8.30}$$

applied at a small δt_o, forms the starting needle control.

The above relations provide the fulfillment of

$$\alpha^t_{i+1,o} x_{i+1}(t_o) = \dot{x}_{i+1}(t_o) = x_i(t_o) = u_{i+1}(t_o)\,. \tag{8.31}$$

Starting from the moment τ_o, the optimal control $u_{i+1}(\tau_o) = -2\dot{x}_{i+1}(\tau_o)$ will be applied to the equation (8.31), generating the model optimal processes.

Such processes that mutually control each other are the *superimposing* processes [14].

Let us show that *if* the optimal model's phase coordinates are connected by relation

$$x_{i-1}(t_o) = \alpha^t_{io} x_i(t_o) \tag{8.32}$$

at the initial moment t_o,

then at the DP moment t_i, relation $x_{i-1}(t_i) = -2\dot{x}_i(t_i)$ is correct with the relative error

$$\varepsilon_i = |\,\delta x_i / x_i\,| = \exp(-\mathbf{a})\exp(\gamma_i^{\alpha}\mathbf{a}) - 2(1 - \mathbf{a}/\mathbf{a}_o) = inv\,, \tag{8.33}$$

where the assumption (8.32) corresponds to the considered equality (8.31), and the equation (8.33) fulfills Prop.1.5.4.

Indeed, let us find the difference $|\,\delta x_i\,| = |\,2\dot{x}_i(t_i) - x_{i-1}(t_i)\,|$ between the actual $2\dot{x}_i(t_i)$ and $x_{i-1}(t_i)$, determined by the optimal solution of $\dot{x}_{i-1}(t) = \alpha^t_{i-1} x_{i-1}(t\ ) + v_{i-1}(t_i)$ at fixed $x_{i-1}(t_{i-1}) : x_{i-1}(t_i) = x_{i-1}(t_{i-1})(2 - \exp\alpha^t_{i-1}(t_i - t_{i-1}))$,

where $\alpha^t_{i-1} t_{i-1} = \mathbf{a}(\gamma)$, $\alpha^t_{i-1} t_i = \gamma_i^{\alpha}(\gamma)\mathbf{a}(\gamma)$ and, therefore $(2 - \exp\alpha^t_{i-1} t_i - t_{i-1})) = inv$.

This leads to

$$\frac{x_{i-1}(t_i)}{x_{i-1}(t_{i-1})} = inv\,(\gamma).$$

Using for each *fixed* $x_i(t_i), x_{i-1}(t_{i-1})$ the solutions:

$$x_i(t_i) = x_i(t_o)(2 - \exp\alpha_{io}^t(t_i - t_o)), x_{i-1}(t_{i-1}) = x_{i-1}(t_o)(2 - \exp\alpha_{i-1,o}^t(t_{i-1} - t_o))$$

at

$$(2 - \exp\alpha_{io}^t(t_i - t_o)) = (2 - \exp\alpha_{i-1,o}^t(t_{i-1} - t_o)) = inv,$$

we get

$$\frac{x_i(t_i)}{x_i(t_{i-1})} = \frac{x_i(t_o)}{x_{i-1}(t_o)}.$$

From that, taking into account (8.32), we get

$$\frac{x_i(t_o)}{x_{i-1}(t_o)} = (\alpha_{io}^t)^{-1} = \frac{x_i(t_i)}{x_i(t_{i-1})}.$$

By the substitution it into

$$|\delta x_i| = |2\alpha_i^t x_i(t_i) - 2x_{i-1}(t_i) + x_{i-1}(t_i)\exp\alpha_{i-1,}^t(t_i - t_{i-1}))$$

we obtain

$$\varepsilon_i = |\frac{\delta x_i}{x_{i-1}}(t_{i-1})| = 2\alpha_i^t \frac{x_i(t_i)}{x_i(t_{i-1})} - 2 + \exp\alpha_{i-1}^t(t_i - t_{i-1}))$$

$$= |2\frac{\alpha_i^t}{\alpha_{io}^t} - 2 + \exp(\gamma_i^{\alpha}\mathbf{a})\exp(-\mathbf{a})|,$$

where $\frac{\alpha_i^t}{\alpha_{io}^t} = \frac{\alpha_i^t t_i}{\alpha_{io}^t t_i} = \mathbf{a}/\mathbf{a}_o$.

Finally we come to (8.33). •

Let us find the condition at which an analog of relation (8.32) can be true with $\varepsilon_i = 0$. Assuming

$$\dot{x}_{i+1}(t_o) = k_i x_{i-1}(t_o), \tag{8.34}$$

we will determine such k_i, which brings $\varepsilon_i = 0$, proving that $\varepsilon_i = 0$ is achieved at

$$k_i = \mathbf{a}_o / 2\mathbf{a}(2 - \exp(\gamma_i^{\alpha}\mathbf{a})\exp(-\mathbf{a})). \tag{8.35}$$

Indeed, substituting (8.34) in the form $x_{i-1}(t_o) = \frac{\dot{x}_{i+1}(t_o)}{k_i}$ into equation

$$\varepsilon_i = |2\alpha_i^t \frac{x_i(t_o)}{x_{i-1}(t_o)} - 2 + \exp(\gamma_i^{\alpha}\mathbf{a})\exp(-\mathbf{a})|,$$

we get

$$\frac{x_i(t_o)}{\dot{x}_{i+1}(t_o)} k_i = \frac{x_i(t_o)k_i}{x_i(t_o)\alpha_{io}^t}$$

and have

$$\varepsilon_i = |\frac{2\alpha_i^t k_i}{\alpha_{io}^t} - 2 + \exp(\gamma_i^{\alpha}\mathbf{a})\exp(-\mathbf{a})| = |2\mathbf{a}/\mathbf{a}_o\, k_i - 2 + \exp(\gamma_i^{\alpha}\mathbf{a})\exp(-\mathbf{a})| \,. \quad (8.35a).$$

By requiring $\varepsilon_i = 0$, we come to the k_i satisfying to (8.35).

Thus, the arrangement of the model's initial conditions as the phase coordinates, satisfying to the relations (8.27) and (8.32), guaranties the model's self-controllability, which is an equivalent of applying the optimal control $v = -2x(t_i)$.

We also need to prove that the model's self-control $u_i(t_i) = -2\alpha_i^t x_i(t_i)$, applied at each t_i, is able to achieve the adaptivity within the admissible gate threshold ε.

Each of these optimal controls, applied at t_i within τ_i, will be determined by the corresponding $(\alpha_{it} \pm \Delta\alpha_{it})$:

$$\Delta u_i^*(t_i) = \frac{\Delta u_i(t_i)}{u_i(t_i)} = \mp \frac{\Delta\alpha_i^t}{\alpha_i^t} = \alpha_{it}^* \,. \quad (8.36)$$

If α_{it}^* belongs to the *admissible* $\alpha_{i\tau}^*$, which preserves the invariant $\mathbf{a}(\gamma)$, then this control also preserves both model's invariants $\mathbf{a}_o(\gamma)$. Those keep γ constant and, therefore, also preserves $\gamma_i^{\alpha}(\gamma)$ and thus provides $\mathbf{a}(\gamma)\gamma_i^{\alpha}(\gamma) = inv$ at the above admissible $\alpha_{i\tau}^*(\gamma)$.

Such a self-control accomplishes the model's *robustness*.

Some of a self-control, acting *within* $\max\varepsilon$, is able to *minimize* the maximal admissible triplet's error

$$1 - \frac{t_i^2}{t_{i+1}t_{i-1}} = \Delta t_i^*(\gamma)$$

under the perturbations of $\alpha_{it}^*(\Delta\gamma)$ for *any allowable* $\Delta\gamma \in \Delta\gamma_m$.

Such a control compensates the *acceptable* $\alpha_{it}^*(\Delta\gamma)$, unrestricted by the *robustness* potential P_r, but *limited* by adaptive potential P_a.

The implementation of $\max\varepsilon = P_a$ initiates the *adaptive* control, which provides the preservation of each triplet within the n dimensional IN and *adapts* the acceptable variations.

Thus, the macromodel's self-controls are able to support both the *robustness'* and adaptive potentials that guarantees the stability, preservation of the model triplets, and the total model's dimension n.

The adaptive potential's asymmetry contributes decreasing γ and growing the potentials of both dynamics and adaptation.

The evolutionary dynamics, created by a *multidimensional* eigenvalues' spectrum, forms a chain of interacting extremal segments, which are assembled in an *ordered organization* structure of the information hierarchical network (IN).

A cooperative coupling stabilizes the forming structure, while each triple generates an order attracting other couples and triples, and so on.

The space distributed IN's structural robustness is preserved by the feedback actions of the inner controls, which provide *a local stability at the admissible* variations.

This control supports a limited $P_e(\gamma) = P_r$ that determines the *potential of robustness.*

The requirements of preserving the evolutionary hierarchy (Eqs (5.5.1)-(5.53)), impose the restrictions on the maximal potential of evolution P_e and limit the variations, *acceptable* by the model.

The model's adaptive potential $P_a \le P_e$, which adapts the variations, not compromising the IN hierarchy, restricts the maximal increment of dimension, contributed to the adaptation. The punched evolution's nonequilibrium accumulates the changes by an hierarchy of the selections and adaptations, with following a local equilibrium at each hierarchical level.

The adaptive model's *function* is implemented by the adaptive *feedback-control*, acting within the P_a capabilities.

The self-control function is determined by the conditions (8.26), (8.27), and (8.36) of a proper *coordination* for a chain of superimposing processes, where each preceding processes adopts the acceptable variations of each following process.

The above optimal controls are synthesized, as an *inner feedback*, by the *duplication* of and *copying* the current macrostates at the beginning of each segment, which are *memorized* and *applied* to the segment.

The adaptive potential's *asymmetry* contributes the model's evolutionary improvement.

A sequence of the sequentially enclosed IN's *nodes* represents a *discrete control logic* to *design* the IN.

The applied control, which adds the forth letter to the initial minimal three triplet's code letters, provides the model's *error correction mechanism* to the *IN's DSS code*, and also provides a discrete filtering of the randomness, acting at the *o*-window.

The IN geometrical border forms the external surface where the macromodel is open for the outside interactions. At this surface, the interacting states *compete* for delivering a maximum of dynamic potential's gradient. Selected states are copying and memorized by the model control, contributed to the code.

The control provides a *directional* evolution with the extraction of a maximum information from the environment of competing systems, while the acquired DSS code can be passed to a successor [22].

1.8.5. The Evolution of the Model Invariants and a Potential Macroprocess' Cyclicity

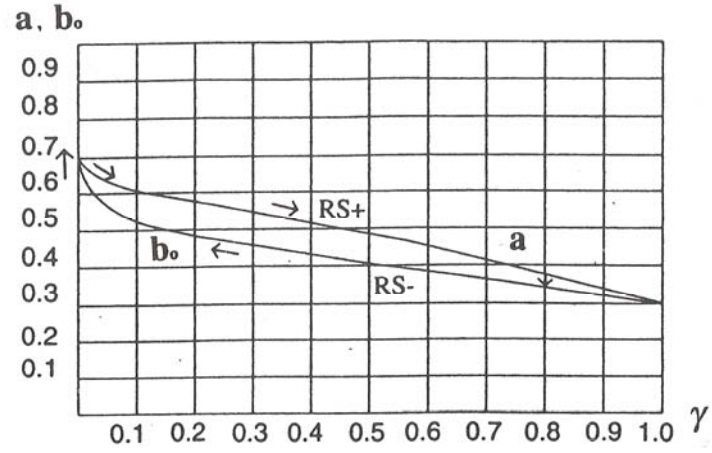
The model invariants' $\mathbf{a}_o$, $\mathbf{b}_o$ dependency on γ follows from equations (sec.1.3.5, 1.5.4):

$$2(\sin(\gamma\mathbf{a}_o)+\gamma\cos(\gamma\mathbf{a}_o))-\gamma\exp(\mathbf{a}_o)=0\ ; \tag{8.37}$$

$$2\cos(\gamma\mathbf{b}_o)-\gamma\sin\gamma(\mathbf{b}_o)-\exp(\mathbf{b}_o)=0\,. \tag{8.38}$$

The macromodel evolves toward a minimal $\gamma\to 0$ during the process of adaptation, while the evolution increases the number **n** of consolidated subsystems. With increasing $\gamma\to 1$, **n** is decreasing. At $\gamma\to 0$, the movement approaches the uncertainty zone UR.

a).



b).

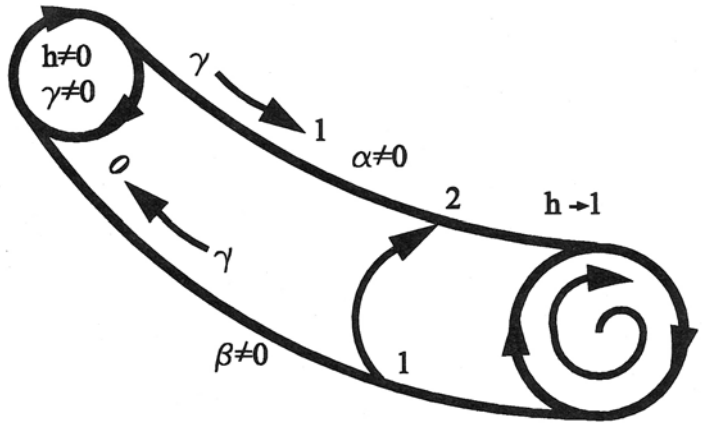


Figure 8.3. a). The evolution of the model invariants with a loop between information subspaces RS + and RS − ; b). A schematic of the cyclic process.

At the UR locality, both equations (8.37),(8.38) have the nearest solutions: $\mathbf{a}_o(\gamma\to 0)\approx 0.768,\quad \mathbf{b}_o(\gamma\to 0)\approx 0.7.$ Other nearest solutions correspond to $\mathbf{a}_o(\gamma\to 1)\approx 0.3, \mathbf{b}_o(\gamma\to 1)\approx 0.3.$

This allows representing the invariant's evolution at $\gamma \to (0 \rightleftarrows 1)$ by Fig. 8.3a.

Let us analyze this evolution process.

The consolidation process, preserving invariant $\mathbf{a}_o$, develops along the line of switching controls, corresponding $\mathbf{a}(\gamma) = \alpha_i^t t_i = inv$, at a region of the real entropy's (RS+) geometrical locality. The macrosystem can evolve along this line at a constant total entropy $S=S_e+S_i$ if the internal entropy ($S_i>0$) is compensated by an external negentropy ($S_e<0$), delivered from the environment. Until the entropy production is positive, the time direction is also positive, and the entropy is a real S^a. The irreversible states create the systemic order. Another character of the macrodynamics takes place along the switching line $\mathbf{b}_o(\gamma) = \alpha_i^t(t_0) t_i^- = inv$ (where t_i^- is the time interval *preceding* to the starting moment t_o) at a region of the geometrical locality (RS−), where the macromovement with an imaginary time of $jt = t_i^-$, holds the imaginary entropy S^b. The movement along this line preserves the entropy S^b but the second law as well as any real physical law is not fulfilled.

The evolution at $\gamma \to (0 \rightleftarrows 1)$ keeps the invariant ratio

$$S^b / S^a \sim \mathbf{b}_o(\gamma) / \mathbf{a}_o(\gamma). \tag{8.39}$$

The macrosystem, which satisfies the VP information law, is adaptive and self-controlled. The controlled macroprocess, after a complete cooperation, is transformed into an one dimensional consolidated process

$$x_n(t) = x_n(t_{n-1})(2 - \exp(\alpha_{n-1}^t t), \tag{8.40}$$

which at $t_{n-1} = \dfrac{\ln 2}{\alpha_{n-1}^t}$ is approaching the final state $x_n(t_n) = x_n(T) = 0$ with an infinite phase speed

$$\frac{\dot{x}_n}{x_n}(t_n) = \alpha_n^t = -\alpha_{n-1}^t \exp(\alpha_{n-1}^t t_n)(2 - \exp(\alpha_{n-1}^t t_n)^{-1} \to \infty. \tag{8.41}$$

The model cannot reach the zero final state $x_n(t_n) = 0$ with $\dot{x}_n(t_n) = 0$.

A periodical process arises as a result of alternating the movements with the opposite values of each *two* relative phase speeds $\dfrac{\dot{x}(t_{n+k})}{x_n(t_{n+k})}$, $k = 1,2$ and under the control switches; the process is limited by the admissible error $\varepsilon^* = \dfrac{\Delta x_n}{x_n}$ and/or the related time deviations of switching control $\varepsilon_m^* \le \Delta t_i / t_i = \alpha_{io}^*$. • (sec.1.8.3). Such a nonlinear fluctuation can be represented [23] by the superposition of linear fluctuations with the frequency spectrum $(\omega_1^*, ..., \omega_m^*)$ proportional to the imaginary components of the eigenvalues $(\beta_1^*, ..., \beta_m^*)$, where ω_1^* and ω_m^* are the minimal and maximal frequencies of the spectrum accordingly.

Proposition 8.7.

A nonlinear fluctuation is able to generate a new model with the parameter

$$\gamma_{lo} = \frac{\beta_{lo}(t_o)}{\alpha_{lo}(t_o)} = \frac{\beta_n^i(t_{n+k})}{\alpha_n^i(t_{n+k})} = \frac{2\cos(\beta_{n+k-1}^i t) - 1}{2\sin(\beta_{n+k-1}^i t)}, \tag{8.42}$$

where $\alpha_{lo}(t_o) = \alpha_i^*(t_{n+k})$, $\beta_{lo}(t_o) = \beta_i^*(t_{n+k})$ are the new model's starting real and imaginary eigenvalues.

Proof. During the oscillations, initiated by the control action, a component β_i^* is selected from the model's imaginary eigenvalues $\mathrm{Im}\lambda_n^i(t)=\mathrm{Im}[-\lambda_{n-1}^i(2-\exp\lambda_n^i t)^{-1}]$, at each $t = (t_{n+k-1}, t_{n+k})$.

We come to relation

$$\mathrm{Im}\,\lambda_n^i(t_{n+k}) = j\beta_n^i(t_{n+k}) = -j\beta_{n+k-1}^i \frac{\cos(\beta_{n+k-1}^i t) - j\sin(\beta_{n+k-1}^i t)}{2 - \cos(\beta_{n+k-1}^i t) + j\sin(\beta_{n+k-1}^i t)}, \tag{8.43}$$

at $\beta_i^* = \beta_{n+k}^i$, $\beta_i^* \neq 0 \pm \pi k$. It seen that $\beta_n^i(t_{n+k})$ includes a real component

$$\alpha_n^i(t_{n+k}) = -\beta_{n+k-1}^i \frac{2\sin(\beta_{n+k-1}^i t)}{(2-\cos(\beta_{n+k-1}^i t))^2 + \sin^2(\beta_{n+k-1}^i t)}, \tag{8.44}$$

at $\alpha_i^* = \alpha_i^*(t_{n+k}) \neq 0$, with the corresponding parameter

$$\gamma_i^* = \frac{\beta_n^i(t_{n+k})}{\alpha_n^i(t_{n+k})} = \frac{2\cos(\beta_{n+k-1}^i t) - 1}{2\sin(\beta_{n+k-1}^i t)}. \tag{8.45}$$

These eigenvalues $\lambda_i^*(t_{n+k}) = \alpha_i^*(t_{n+k}) \pm j\beta_i^*(t_{n+k})$, at some moment $t_0 > t_n$, could give a start to a new forming macromodel with $\lambda_{lo} = \alpha_{lo}(t_o) \pm j\beta_{lo}(t_o)$, with the initial real $\alpha_{lo}(t_o) = \alpha_i^*(t_{n+k})$, the imaginary $\beta_{lo}(t_o) = \beta_i^*(t_{n+k})$, and the parameter $\gamma_{lo} = \dfrac{\beta_{lo}(t_o)}{\alpha_{lo}(t_o)}$ equals to (8.42). •

Comments 8.12. This new born macromodel might continue the consolidation process of its eigenvalues. Therefore, returning to some primary model's eigenvalues and repeating the cooperative process is a quite possible after ending the preceding consolidations and arising the periodical movements. This leads to the cyclic macromodel functioning when the state integration alternates with the state disintegration and the system decay. (The time of the macrosystem decay increases with an increase in the accuracy $\varepsilon^* = \dfrac{\Delta x_n}{x_n}$ of reaching the given final state). Because the system instability corresponds to $\gamma \geq 1$, while $\gamma \to 0$ corresponds to the start of the cooperative process, we might associate the cycle *start* (at the end the invariant loop (Fig.8.3b)) with the model's oscillations at $\gamma \geq 1$ (generating the spectrum $\lambda_i^*(t_{n+k})$), and the cycle *end* with the start of the new born macromodel at $\gamma \to 0$.

In this case, $\gamma \to 0$ can be achieved at $2\cos(\beta_{n+k-1}^i t) \to 1$, or at

$$(\beta_{n+k-1}^i t) \to (\pi/3 \pm \pi k), k = 1, 2, \ldots,$$

with

$$\alpha_n^i(t_{n+k}) = \alpha_{lo}^m(t_o) = \lambda_{lo}^m \cong -0.577\beta_{n+k-1}^i, \beta_{lo}(t_o) \cong 0,$$

whereas $\gamma = 1$ corresponds to $(\beta_{n+k-1}^i t) \approx 0.423 rad(24.267^o)$,

with

$$\beta_i^*(t_{n+k}) \cong -0.6\beta_{n+k-1}^i.$$

Here $\beta_i^*(t_{n+k}) \cong \alpha_{lo}^m(t_o)$ determines the maximal frequency ω_m^* of the fluctuation by the end of the optimal movement. The new macromovement starts with this initial frequency.

Proposition 8.8.

The maximal *frequency's ratio*, generated by the initial (*n*-1) dimensional spectrum with an imaginary eigenvalue $\beta_{n-1,o}(t_{n-1,o})$ (at the end of the cooperative movement at (γ=1)):

$$\frac{\beta_i^*(t_{n+k})}{\beta_{n-1,o}(t_{n-1,o})} = l_{n-1}^m \tag{8.46}$$

is *estimated by the invariant*

$$\frac{0.577\pi/3}{\mathrm{a}_o(\gamma)/\mathrm{a}(\gamma)\ln 2} = l_{n-1}^m(\gamma = 1). \tag{8.47}$$

Proof. Let us estimate (8.46) using the following relations.

Because $\beta_{n-1,o}(t_{n-1,o}) = \alpha_{n-1,o}(t_{n-1,o})$ at $\gamma = 1$ and $\beta_i^*(t_{n+k}) \cong \alpha_{lo}^m(t_o)$, we come to

$$l_{n-1}^m = \frac{\alpha_{lo}^m(t_o)}{\alpha_{n-1,o}(t_{n-1,o})}.$$

Applying relations $\alpha_{n-1,o}(t_{n-1,o}) = \alpha_{n-1,t}(t_{n-1})\,\mathbf{a}_o(\gamma)/\mathbf{a}(\gamma)$ and using $\alpha_{lo}^m(t_o) = -0.577\pi/3$, we have

$$\frac{\alpha_{lo}^m(t_o)t_o}{\alpha_{n-1,o}(t_{n-1})t_o} = \frac{0.577\pi/3}{\mathrm{a}_o(\gamma)/\mathrm{a}(\gamma)\alpha_{n-1,t}(t_{n-1})t_o} = l_{n-1}^m(\gamma = 1). \tag{8.48}$$

Assuming that a minimal t_o starts at $t_o = t_{n-1} + o(t) \cong t_{n-1}$, and using (8.42): $\alpha_{n-1,t}(t_{n-1})t_o = \ln 2$, at $\alpha_{n-1,t}(t_{n-1}) = \alpha_{n-1}$, we get the invariant relation (8.47). •

Comments 8.13. The ratio (8.47) at $\mathbf{a}_o(\gamma$=1)=0.58767, $\mathbf{a}(\gamma$=1)=0.29 brings $l_{n-1}^m(\gamma = 1) \approx 0.42976$. Because $\alpha_{n-1,o}(t_{n-1,o})$ is the initial eigenvalue, generating a starting eigenvalue $\alpha_{lo}^m(t_o)$ of a new model, their ratio $\frac{\alpha_{n-1,o}(t_{n-1,o})}{\alpha_{lo}^m(t_o)} = \gamma_l^{\alpha m}$ determines the parameter of the eigenvalue multiplication for the new formed model, equal to $\gamma_l^{\alpha m} = (l_{n-1}^m)^{-1}$, or at $\gamma = 1$, we get $\gamma_l^{\alpha m} \approx$2.327. For the new model, the corresponding parameter $\gamma_l \to 0$, at which the cooperative process and the following evolutionary development might start, holds

true. For a new formed potential triplet, satisfying relations (8.16), we get the eigenvalues and their parameters of multiplication in the forms:

$$\alpha^{m}_{l-1,o}(t_o) = \alpha_{n-1,o}(t_{n-1,o}) \approx -1.406,\ \alpha^{m}_{lo}(t_o) \approx -0.60423,\ \alpha^{m}_{l+1,o}(t_o) \approx -0.26,$$

with

$$\gamma^{\alpha m}_{l-1}(\gamma) = \frac{\alpha^{m}_{l,o}(t_o)}{\alpha_{l+1,o}(t_o)} \approx 2.3296 \text{ and } \gamma^{\alpha m}_{l-1,l+1}(\gamma) = \frac{\alpha^{m}_{l-1,o}(t_o)}{\alpha_{l+1,o}(t_o)} \approx 5.423. \qquad (8.49)$$

If the optimal cooperation in the new formed model is continued, both parameters γ_l and $\gamma^{\alpha m}_{l}$ will be preserved, which determines both model's invariants and the information genetic code. The transferred invariants are the carriers of the evolutionary hierarchical organization, self-control, and adaptation (while the adaptation process could change both γ_l and $\gamma^{\alpha m}_{l}$).•

The above analysis shows that even after ending the consolidations and initiating the periodical movements with a potential system's decay, it is quite possible the automatic generation of a new macromodel by the end of functioning of a current model, and with transferring of the emerged model's basic parameters, invariants, and code to the new model. This leads to the model's *cyclic functioning*, initiated by the two mutual controllable processes, which should not consolidate by the moment of the cycle renovation.

The initial interactive process may belong to different macromodels (as "parents"), generating a new macrosystem (as a "daughter"), which "inherits" the "parents" code's genetics, accumulated in the transferred basic parameters and invariants. Thus, in the cyclic macromodel functioning, after the model disintegration, the process can repeat itself with the state integration and the transformation of the imaginary into the real information during dissipative fluctuations at the end of the "parents" process and the beginning of the "daughter's generation. An individual IN's macromodel *life-time* (ch.1.5) is *limited* by its number *n* of the ranged eigenvalues (the local entropy's densities) and a maximal value of this density. At growing these information densities, the macromodel's *life-time is shortening*.

The structural stability, imposed by the VP, affects a restoration of the system structure in the cyclic process through a reproduction. A new born system sequentially evolves into anew IN that embraces its dynamics, geometry, and the DSS code. The optimal control, which *preempts* and determines the process dynamics at each extremal segment, is memorized at the segment starting time moment. This means that such a memorized control *sequence* (a code) can remember both the previous and the prognosis segment's dynamics.

Therefore, IN's DSS code, represented by the digits of these control sequence, is able to remember a preceding behavior of a complex system and pass it to a successor.

As a result, the successor might do both repeats some behavior of its predecessor and anticipates its own future behavior.

These information system's features, following from the VP (that determines the optimal control and optimal dynamics), probably explain the biosystems behavior, which can anticipate some of their future behavior.

Among the behavioral set, remembered by the code (and initiated by the VP), is also the system's *reproduction* with transferring its code to the successor. That is why a successor, possesing this code (which memorizes the previous behavior and can encode the future

dynamics), is able to anticipate and preempt its behavior including the reproductive dynamics. From that also follows that the path functional's punctuated information dynamics is also responsible for a prediction of the reproductive dynamics

Comments 8.14. At the fixed ε_m^*, the environment generates the *random* initial eigenvalues at the beginning of the cycle through the mutation.

Therefore, the model cannot precisely repeat both the cycle time and its amplitude.

Since the law of increasing entropy is not violated, each of the following cycles has a tendency of fading and chaotization of the periodical processes.

The model of the cyclic functioning includes the generator of random parameters that renovates the macromodel characteristics and peculiarities. The model's adaptive potential increases under the actions of the random generator even if the randomness carries a positive entropy. Such an adaptive, repeating, self-organizing process is the *evolutionary cycle*.

The above conditions for the existence of the adaptation potential and the model's cyclic renewing impose a *limitation* on the minimal admissible macromodel's diversity. With similar reproductive dynamics for all population, the reproduction, applied to a set of the macromodels, having a maximum diversity, brings a maximum of adaptive potential, and therefore, would be more adaptive and beneficial for all set, potentially spreading through the population. Such an evolution trend is confirmed in a latest publication [25].

Generally, the nonsymetrical P_m creates a possibility of the automatic macromodel's evolution and improvement. Each macrosystem dimension **n** is characterized by an appropriate optimal code and the universal mechanism of the code formation. This coding life program encapsulates all essential information about a given macrosystem, including its hierarchical structure's organization and a possibility of the restoration of the initial system macrostructure. The existence of such a universal mechanism of the code transformation allows opening the communications between interacting macrosystems. The macrosystem that is able to continue its life process by renewing the cycle, has to transfer its coding life program into the new generated macrosystems and provide their secured mutual functioning.

A specific invariant information unit of this program $h_{oo} = \mathbf{a}_o(\gamma) = C_p$ defines channel capacity C_p, which incorporates the initial model's parameters and positive time course, carrying the second law.

An imaginary information's equivalent is a potential source of these substances.

At the $\mathrm{RS}- \rightarrow \mathrm{RS}+$ transformation, a positive time can create a start of systemic cooperation. The genetic code can reproduce the encoded macrosystem by both decoding the final IN's node with a reproduction of a particular IN, and decoding the specific position of each node within the IN structure.

An *association* of INs with different $(\mathbf{n}, \gamma)$ can mutually interact as a *system*, creating the negentropy-entropy exchanges between them analogous to a crossover. Their ranged frequencies could be wrapped up in each other changing the subsystem invariants and inner codes. The corresponding subsets of minimal programs, encoded the system information, can communicate (by $C_p(\gamma^o)$) using a common code language, generated by the unified parameters (γ^o, n^o) according to relations:

$$S^o = \sum_i S_i(\mathbf{a}_i(\gamma_i)n_i,\ S^o = \mathbf{a}_o(\gamma^o)n,\ S_i = \mathbf{a}_i(\gamma_i)n_i\ , \tag{8.50}$$

where S_i, $\mathbf{a}_i, \gamma_i, n_i$ are a subsystem's entropy and invariants, S^o, $\mathbf{a}_o, \gamma^o, n^o$ are the entropy and invariants of the unified system. The external subsystems, with distinct invariants and multiple interactions can provide more diversity and adaptivity to the communicated system.

The ability of a macrosystem to counterbalance the perturbations, preserving both stability and adaptivity, characterizes the macrosystemic robustness.

The macrosystem with a growing dimension has longer discrete intervals, and therefore possesses more robustness. The increase of adaptive potential with a growing dimension, following from (8.18), has a *limit*, defined by the condition of non coincidence of the nearest eigenvalues α_{io} with $\alpha_{i+1,o}$, which leads to changing the macrosystem's dimension.

Growing the diversity has two options:

(1)-uncoupling the macrostate connections between triplets and/or doublets, which is limited by admissible time deviations of switching controls $\varepsilon_m^* \le (\Delta t_i / t_i)_m$. The limitation is $\varepsilon_m \le a_{io}^*$. Within this limitation, the adaptive potential provides a robustness to the control code, managing these connections;

(2)-changing the macromodel's dimension by a decrease of a distance between the nearest eigenvalues α_{io}, $\alpha_{i+1,o}$, or by augmenting the additional independent eigenvalues $\alpha_{n+1,o}$.

The ratio of the considered thresholds $m^* = \varepsilon_m / \varepsilon_m^*$ depends on γ, taking the values $m^*(\gamma = 0.1) = 3$, $m^*(\gamma = 0.5) = 4$, $m^*(\gamma \to 1) = 7$.

This means that the creation of a new dimension requires from 3 to 7 times more deviation from the initial eigenvalues than it's necessary to change the robustness of the adaptive code. Each of these thresholds decreases with growing γ, but ε_m^* declines at faster rate. The adaptive macromodel, whose adaptive potential is able to compensate the admissible ε_m^*, can preserve both the robustness and the dimension within a limited diversity according to the relation $\varepsilon_m \le m_m^* \varepsilon_m^*$, $m_m^* = m^*(\gamma \le 0.8)$.

At $\gamma \to 0$ the movement approaches the uncertainty zone UR.

At the allocation of the total macromodel's spectrum within the UR, each DP's moment of uncoupling the process correlations and the intervals between them, including the last one, turn to zero with a maximal $\gamma \to 1$.

1.8.6. Requirements for the Model Self–organization. The Results of Computer Simulations

Let us find a minimal dimension n_o, which is able to generate the both controls for a single triplet in an equilibrium ($\gamma = 0.5$). Because, according to sec.1.6.1, each triplet requires $\cong 4$ bits information, while producing $\cong 1$bit, it is needed a minimum 3 starting triplets to generate 3 bits, and the additional $a_o^2 + \mathbf{a} \cong 1$ bit can be obtained by connecting the

third node to the 8-th extremal segment of the spectrum's dimension. Thus, n_o=8 is the macromodel's minimal dimension that is able to produce a single triplet dimension 3.

We come to a chain of the *self-productive* subsystems dimensions 8 and 3. A sequential adjoining a new generated triplet to 8-th dimensional subsystem not only successively increases the triplet's chain, but also creates the progressive growth of the cooperated dimensions, increasing gradually from the set of the 8-dimensional subsystems by attracting the generated them triplets from the different space-time subsystem's location.
These *self-generated system's chains can* spread and expand gradually like a spider's web.

The *maximal* admissible *dimension* of the macromodel capable for the cooperative attraction is found from $n^m = \gamma^m / \gamma^o$, where parameter γ^m evaluates a *maximal* threshold that *limits* the macromodel's decay, and parameter γ^o evaluates a minimal interval between the starting eigenvalues, where producing a single cooperation is possible.

It is shown that each $\gamma^* = \gamma^o \approx 0.007148$ can bring a minimal increment of the cooperating dimension Δn^o=1. The value of γ^o also determines a macromodel's sensitiveness to perturbations at possible deviations of γ at time interval t_i (within the *o*-window). At $\gamma^m \approx 2.2$, the computation brings the interval's ratio $t_i / t_{i+1} = 1$, which leads to repeating the interval's sequence and the impossibility of the model for further cooperation. Using the above values we get $n^m \cong 308$.

For a macrosystem, operating in an equilibrium (γ=0.5), the minimal admissible relative distance between the nearest eigenvalues, belonging to different dimensions, is $\frac{\Delta \alpha_{io}^t}{\alpha_{io}^t} \approx 0.025$ (for $\varepsilon \approx 0.975$), which corresponds to $\gamma_o^e \approx 0.0244$ and determines the maximal *realistic* dimension $n_o^e \approx 90$, potentially crated in the equilibrium, with the triplet's number $m_o^e = 46$.

The adaptive mechanism, operating within the cycle with its nonsymmetrical adaptive potential, can deliver a positive negentropy of mutations, selected from external fluctuations. This leads toward the decreasing of $\gamma \to 0$ and a possibility of a growing dimension of $n = n(\gamma)$, accompanied by increasing the MC-complexity.

So, each cycle may deliver a positive feedback, carrying the system's improvement and acceleration of the reproductive process. This creates the macromodel's *self-organization* as a tendency of its components, producing the negentropy surplus, toward an automatic formation of the joint macrostructures.

The mutations and control's actions, changing the model's structural information, are the sources of the negentropy surplus for the self-organization.

Let us analyze the increment of the model the structural information, generated at cooperation.

A total entropy increment (S_{dv}), generated by both the macrodynamics (S_d) and geometry (S_v), affects changing the information volume in process of spatial movements.

According to sec.1.7.6-1.7.7, S_v should be deducted from $S_d = m_i \alpha_{io} t_i$:

$$S_{dv} = S_d - S_v = m_i \alpha_{io} t_i - \ln V^*,\ V^* = V(m_i)/V_o,\ \alpha_{io} t_i = \mathbf{a}_o \qquad , (8.51)$$

where m_i is the number of the initial phase flows $\{\alpha_{io}\}$ joined together at DP; $V(m_i)$ is the volume of the consolidated phase flows, considered comparatively to the volume of a single phase flow V_o at that moment. If V_o is equal to the initial triplet's volume (V_{o3}), then this triplet's entropy is $S_d = \mathbf{a}_o$, and the increment total entropy of the dimension **n** has view:

$$S_{dv} = \mathbf{a}_o(\gamma)\mathbf{m} - \ln V_3^*(\mathbf{m}, \gamma, \mathbf{k});\ V_3^* = V_{n+1}/V_{o3};\ \mathbf{m} = (\mathbf{n}-1)/2, \tag{8.52}$$

where V_3^* is the relative information volume, equals to the ratio of the total volume (V_n), (formed during the **n** DP's intervals of the optimal movement) to the information volume (V_{o3}), (sec.1.5.4), considered as the initial one in the adaptive self-organization.

Gradient of the S_{dv} increment, produced at changing the system dimensions from a current (n) to some nearest (n'):

$$grad S_{dv}(n - n') = S_{dv}(n) - S_{dv}(n') \tag{8.53}$$

measures the degree of functional ordering in the considered evolutionary self-organizing cooperative process.

The model asymmetry and self-organization are a motivating power of evolution, whose moving evolutionary force is defined by the equality

$$grad_\gamma S = -\partial \Delta S_{dv} / \partial \gamma > 0, \tag{8.54}$$

where ΔS_{dv} is an increment of S_{dv}<0 during each considered evolutionary cycle, associated with changing γ, which can bring also a new dimension.

The computer simulation demonstrates the following peculiarities and limitations of the model's evolution and self-organization.

There exists a *crucial limit* of complexity that gives a start to the jump-wise changing of the systemic characteristics creating the negentropy production necessary for the self-organization.

Renewing of the macromodel lifetime depends on its ability for the self-organization.

The initiation of the macromodel's self-organization is possible by starting its at a minimal dimension in the phase space with **n**>8.

A total space entropy, generated by m triplets, according to (8.51), decreases while the triplets' volume $V(m)$, related to a single phase volume V_o, increases.

Simulation reveals an increase of S_{dv} with the rise of dimensions **n** and then decrease it, at changing the S_{dv} sign by approaching **n**<18, γ<0.55; the S_{dv} acquires the most negative values at the higher dimensions (**n**>50).

In these cases, the number of information states, defining the informational volume $V(n)$, increases faster than the rise of the entropy in the macrodynamic process. So, the negentropy, generated at **n**=50, γ=0.522, is enough to compensate for the total entropy, which is necessary to generate a new single triplet with S_{3d}=0.684 (γ=0.522)$\bullet$3=2.052.

At forming each new triplet, a maximal relative volume $V^* = V(n)/V_o(n_o)$ (related to the initial macrounit volume $V_o(n_o)$) is growing ~ 10 times.

The volume ratio $V_o^* = V(n=7)/V_o(n_o=1) \cong 10^3$ is a necessary condition for starting the process of self-organization. A maximal volume, corresponding to the most "active" cooperation, is characterized by the negentropy maximum (at $\gamma \to 10^{-4}$).

Therefore, the simulated self-organization is possible at dimensions $\mathbf{n} \geq 50$.

The mutations, even if they are able to change the INcode, can create only a *discrete* set of the model's macrostructures, capable for the consolidations.

This means that not all of the potential n-dimensional macromodels with an *arbitrary* initial operator's spectrum (which are possible in the cooperative self-organization process) could even *exist*.

The IN consolidating macrostructure permits the existence of only a selected, *limited* set of the macromodel's, defined by the admissible values of the parameter of multiplication of the operator's eigenvalues $\gamma_i^{\alpha}(\gamma)$, for $\gamma \in (0, 0.8]$.

Results of simulation [21] shows that at $\gamma \to 0.8$ the model decays in a chaotic disintegration. After that, there exists a possibility of a model reproduction in a cyclic evolutionary process by transferring the DSS code to the new model [22].

The admissible $\gamma \in (0.0072 - 0.8)$ and $n_{\max}$, along with the time irreversibility, lead to the following limited ratio of the models' maximal and minimal volumes [24]: $V_{\max} / V_{\min} = 287.5 \times 10^{11}$.

The simulation of the model adaptive self-organizing process shows that at small **n**, a number of neighboring subsystems with a similar MC complexity do exist. With growing **n**, the number of close-complex neighboring subsystems decreases sharply.

The MC (ch.1.7) defines the degree of the macrosystem organization and the resulting entropy increment that is able to *overcome* a crucial limit of complexity, creating self-organization. The value of the minimal attracting forces (sec.1.7.6) can be evaluated by the local value of the function macrosystemic complexity MC_{n-1} at the final interval of consolidation t_{n-1}. The limit on maximal dimension n^m imposes the restriction on minimal value of attracting force enables maintain the cooperation.

This limit is computed by knowing $t_{n-1} = t_{n^m-1}$ and $\mathbf{a}_o(\gamma)$.

As the MC-complexity increased, the gap between the nearest subsystems' dynamic and geometrical properties, increases radically. The informational "individuality" of the subsystem is continually supported, as "further" away (in terms of the MC complexity's values) this subsystem is located from the neighbor subsystems.

Such distance has a limit, defined by a minimal stable parameter γ, and a maximal **n**, corresponding to a dynamic equilibrium between the system and its environment.

The condition of self-organization is fulfilled at the optimal ratio of the above systems dimension $\mathbf{n}/n_1 = 1.14$. In particular, at $\mathbf{n}=60$, the dimension $n_1 = 52$ corresponds to the generation of the subsystem of minimal dimension $\Delta n = n_o = 8$, which is capable for starting the self-organization.

The considered minimal volume ratio V_o^* determines a *threshold* or starting the model's process of self-organization.

The conditions of cooperation, self-organization (at a macrosystem's formation) and a macrosystem's stability (to resist the environmental perturbations and struggle for a survival) impose an essential *limit* on the diversity of all possible macrosystems.

A macrosystem that possesses a negentropy maximum (at $\gamma = 0$), is most "active" for self-organization. The model's $\gamma = 0$ can only be approached, but not reached, because of

impossibility of the complete elimination of the uncertainty of any macrosystem.

Moreover, because each macrostate is wrapped in the uncertainly zone, none of the macrostates can be measured with zero error. Both the IN code and its communication language contain the intrinsic uncertain *errors* that constrain the formation of information macrostructure and impose the limitations on the coding language correctness.

A time-space sequence of the considered cycles defines the evolution's trend.

Finally, the macromodel attributes are: complexity; structural stability; adaptivity; robustness, evolution, and self-organization; the joint mechanism of selection, competition, cooperation; the limited lifetime of the ordered processes' existence; the generation of new peculiarities in a renewed periodical process (by the end of the cooperation and ordering); the possibility of generating macromodels and systemic structures by transferring the model code during the evolutionary cycle *that creates systemic novelties.*

Because the natural macrostructures are distributed in space, ordering and self-organization are possible for a natural system in a real-time process without violating the second law. Condition (8.54), as an indicator of extensive negentropy's self-reproduction in the evolutionary cycle, with the above attributes and the MC complexity of cooperative structure, composes an *informational definition of Life.*

1.8.7. Evaluation of Some Prognostic Parameters of the Evolutionary Dynamics. Example

At the adaptation, the model intends to approach a minimal $\gamma \to 0$ (independently on the dimension). In the theoretical limit, the invariant acquires the value $\lim_{\gamma\to 0} \mathbf{a}_o(\gamma) = \mathbf{a}_o^o(\gamma) = -0.768$, which is unable to generate any new dimension.

At the nearest $\gamma^* = 0.0072$, $\mathbf{a}_o(\gamma^*) = -0.762443796$, the model is capable of producing just a single dimension. The corresponding gradient of dynamic potential at $\Delta S(t_i^*) = \mathbf{a}_o(\gamma^*)$, acquires the value $gradX(t_i^*) = \exp[\,\mathbf{a}_o(\gamma^*)\,] = 2.143508122$, whose information can generate a single dimension.

At the above theoretical limit $\lim_{\gamma\to 0} \mathbf{a}_o(\gamma) = \mathbf{a}_o^o(\gamma) = -0.768$, $gradX(t_i^o) = \exp[\,\mathbf{a}_o^o\,] = 2.155451038$, the generation of new dimensions is blocked.

During the evolution, the model adapts a decreasing diffusion contribution, limited by

$$r(\tau^o) = \min \int_{\tau}^{\tau+\delta\tau} \sigma\sigma^T dt = 0.46394 \quad \text{at } \mathbf{a}_o^o(\gamma), \text{ and } r(\tau^*) = 0.466525 \text{ at } \mathbf{a}_o(\gamma^*).$$

At any $\gamma \to 0.8, \gamma \in (0.0072 - 0.8), \gamma < \gamma^*$, the related potentials and variations cannot change in the model dimension. This limits the increment spectrum, the diversity, evolution potential, and the average speed of evolution. At a minimal admissible $t_i^* = 2.66 \times 10^{-16}$ (defined by the light wave time interval), we get the maximal eigenvalue $\alpha_{io}^* = \mathbf{a}_o(\gamma^*)/t_i^* = 0.286633 \times 10^{16}$, which also limits the related maximal entropy production within t_i^*. At the preservation of invariant $\mathbf{a}_o$ and a system maximal admissible

entropy production within each t_i, the model time intervals acquire the minimal values, allowable by the VP during the evolutionary process. The maximal model's dimension, determined by a sequence of *all* possible INs is limited by $n_{\max} = 1.59 \times 10^{178}$.

Both the limited γ^* and $n_{\max}$ restrict a maximal model's complexity (ch.1.7).

The above relations constrain the evolutionary development and model's improvement. The evolution equations and limitations allow the computer simulation and a comprehensive prognosis of the evolutionary dynamics.

Results of simulation [21] shows that at $\gamma \to 0.8$ the model decays in a chaotic disintegration. After that exists a possibility of a model reproduction in a cyclic evolutionary process by transferring the DSS code to a new model.

The admissible $\gamma \in (0.0072 - 0.8)$ and $n_{\max}$, along with the time irreversibility (ch.1.6), lead to the following limited ratio of the models' maximal and minimal volumes: $V_{\max} / V_{\min} = 287.5 \times 10^{11}$ [24].

1.8.8. Information Mechanism of Assembling the Node's Frequencies and Automatic Selection

The IN models a mechanism of assembling the initial information frequencies into the doublets and triples, encoding their combinations by the IN's code. The primary cooperative structure is a doublet. Adding a single component to the doublet creates a triplet.

The assembling mechanism forms a zone of key-lock connections by applying the needle controls at DPs (Fig. 8.4) and generates the chaotic attractors by interfering the frequency components of the doublet or triplet within this zone. The basin of cooperative chaotic attractor is able to "capture" the augmented nodes. A minimal optimal number of these attractors is equal to 2, 3, or 7 (as an extra triplet).

The dynamic movement of each frequency-code toward cooperation is modeled by the spiral on the IN's cone (Figs.8.4) in the spatial dynamics.

Applying the needle control at the DP changes impulsively the control sign needed for the doublet's cooperation. In the IN geometry, this is associated with the formation of the opposite directional orthogonal cones within the key-lock zone (for the doublet) (Fig.8.4). The superimposing frequencies interfere in a common base of the opposite cones, initiating the chaotic dynamics, which synchronize both frequencies.

The chaotic attractor models the IN cooperative node.

The consolidation dynamics are governed by the optimal control, which copies, duplicates, and memorizes the macrostates at the DP. This means, each previous state controls each following state, involving in the consolidations and building a self-controlling process.

According to IMD formalism, an ensemble of the model's equal probable *macrostates*, generated by random initial conditions, is able of transferring to a random ensemble of the model's *microstates* at the DP locality. By applying the optimal control at DP, the IMD model selects the most informative and the equal probable *microstates* as new macrostates, working as a discrete filter at each DP locality within a given model's dimension.

By duplicating and copying, both the regular and needle controls deliver the additional macrostates toward the cooperation. They should compete for the subsequent selection and

cooperation with the set of the random states, brought by the microlevel.

The cooperated macrostates, formed by the ensemble of the probable states, carry the assembling frequencies, which represent a new model's dimension.

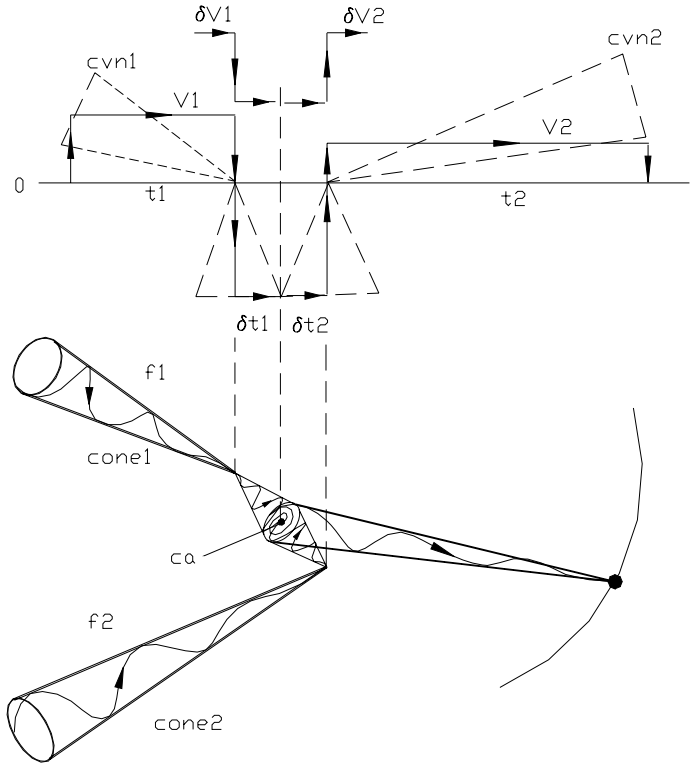


Figure 8.4. An elementary schema of assembling of a pair of frequencies into the encoded spot-node by applying both regular (v1,v2) and impulse controls (δ v1, δ v2) with a schematic location of the corresponding cone's 1, 2 spirals having the initial frequencies f1,f2; cd-chaotic dynamics, created by the superimposing f1,f2 with the chaotic attractor ca.

The macromodel automatically works as an error correction mechanism at each DP's locality, selecting among all interfering frequencies the most probable ones and memorizing their resulting chaotic attractor as the node. The selected synchronized frequency maximizes a contribution from the competing neighboring frequencies, affecting the cooperation.

After the cooperation, the model enables the error correction at a new dimension.

The resulting dynamic node has an appropriate cone geometry (Fig.8.4), which is characterized the cone sizes, defining its length, and the direction.

Within the IN's geometrical structure, the vertex of this cone forms a specific limited spotted area that encodes the cone's *location.*

The cone's vertex actually "sharpens" the selected common synchronized movement into the node spot. The process of filtering and selecting of the synchronized frequencies is accompanied not only by the influence of the random microlevel's ensemble.

The resulting dynamic node has an appropriate cone geometry (Fig.8.4), which is characterized the cone sizes, defining its length, and the direction. Within the IN geometrical

structure, the vertex of this cone forms a specific limited spotted area that encodes the cone *location*. The cone vertex actually "sharpens" the selected common synchronized movement into the node spot.

The model process of filtering and selecting of the synchronized frequencies is accompanied not only by the influence of the random microlevel's ensemble. At each DP, the environmental influence brings the external random perturbations to the macrolevel states.

Applying both the regular and needle controls contributes to multiplying the competing copies. Each DP opens a gate where the random microlevel and external perturbations may affect the macrolevel, bringing together the six ensembles: two from each macrostates, delivered for cooperation, two copies of each these ensembles, carried by the control's duplication, the ensemble of microstates, and an ensemble of the external perturbations.

This essentially multiplies the number of possible frequencies for subsequent cooperation, creating a geometrical ensemble of the opposite cones and the spirals on them (Fig.8.4), which open a possibility of forming different combinations of assembling frequencies and multiplying the number of the node spots.

But not all possibilities could be accomplished, because of existing the limited differences on these frequencies, defined as the model admissible frequency's *cooperative gate*.

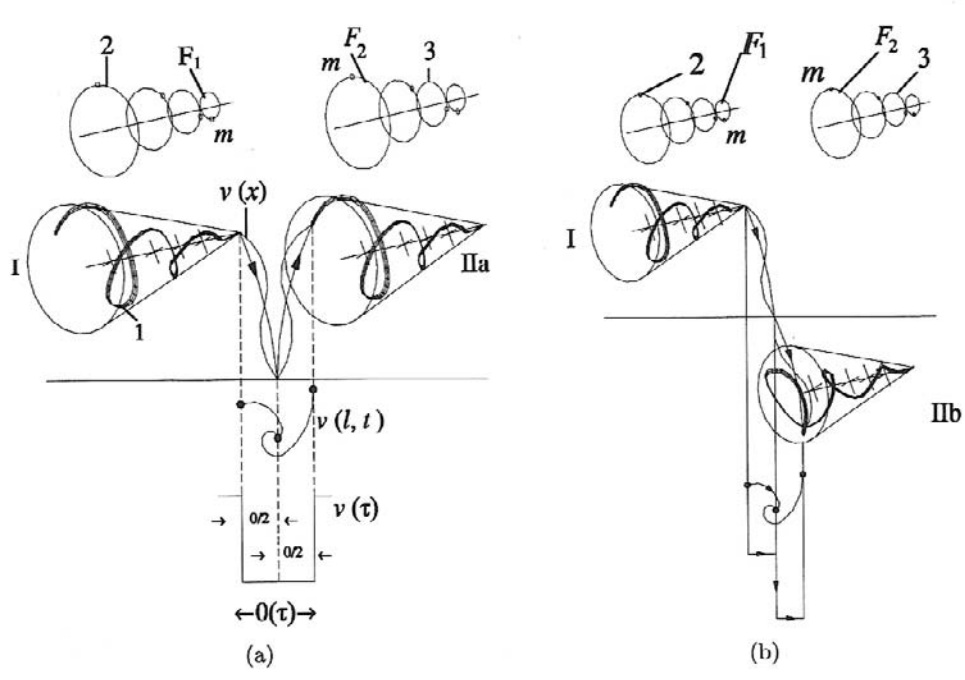


Figure 8.5. a,b).The schematics of the communication process between the assembling nodes: 1-the cone' spirals, 2-the sequence of the communicating cone's cross sections $F_1(t_i)$ $F_2(t_i)$ with the pieces of cone's spirals 3 on them.

The selected competing frequencies should be able to overcome an admissible *threshold*, as the gate border. The frequencies that unable overcoming the gate make the potential

competing random *variations*. If the cooperation is not accompanied by the presence of the microlevel ensemble (when only four of the above macro-ensembles are interfering), the assembling frequencies would be renovated only under the controls actions.

The influence of the microlevels' states would add new properties to this a primary cooperative ensemble, increasing the number of different states for subsequent cooperation. The error correction mechanism will select a certain renovated maximal probable ensemble among the five of considered ensembles. The survived frequencies, which have passed through the threshold gate, or theirs interfering combinations can bring *new encoded* combinations, multiplying the number of assembling frequencies.

The influence of external perturbations on the cooperation can bring, first, the additional changes in the joint assembling macrostates, and second, increases the number of the multiplying nodes, satisfying both the admissible threshold and the error filtering.

This can generate the additional node spots, which are not *predictable* by the initial input.

Applying both the regular and needle controls contributes to the reproduction of the competing copies that increase the effectiveness of the selective process.

This finally creates a local *evolution* process at each DP's locality, which brings the multiple variations and renovations to the states, competing for cooperation, and selects such of them-successful that survive under both the limited threshold gate and the error filtering.

The evolution generates a *creativity* in a reflection the external information, which is automatically memorized in the multiple renovated node' spot.

Copying in digital code, which is the base of the IN coding language, significantly contributes to the selection's effectiveness, comparing with possible analogous processes.

The IN triplet's digital code is carried by the macromodel's double spiral chain (Fig.6.4).

The sequentially interaction of the IN information frequencies binds the initial code's symbols into the enclosed coding string in such a way that each following node includes the inner code of each previous node. The final IN's node binds all initial encoded symbols, accumulating a total network's nonreduntant information.

This means, knowing the final node opens a possibility for disclosing a total chain of initial symbols.

The impulse δ-needle control (Fig.8.5a) connects the transmitter's and receptor's nodes and initiates the signal propagation, transmitted between them. This is associated with adding a macrostate (carried by the control) to transmitting node at the moment of transmission, and removing it after the transmission occurs. On the receptor's node, the propagation's and control's action initiates adding a macrostate to the current macrostate, which indicates the occurrence of the transmission, and releasing the macrostate after the propagation occurs.

A piece of specific spiral on the cone surface represents a signal carrier at each fixed moment. The δ-control serves only as a *messenger*, which does not carry these pieces, but rather *induces* the formation of the corresponding macrostate-piece by the receptor.

During the time of communication, a spiral form of a signal, initiated by the transmitter, is reproduced by the receptor. This signal carries the IN inner code of the primary message, and each particular spiral's piece on the cone surface's cross section also carries the current code's symbol. Each code word corresponds to some IN's frequency.

The spiral curve (as the macroprocess' space representation), transfers this transmitter's frequency to the receptor's frequency. The δ-control *process'* space representation, which is modeled by the considered conjugated nonlinear oscillations (Fig.8.5a), connects these

frequencies, but does not carry the spiral pieces. Specifically, the fist part of the δ-control impulse adds the $-2x(\tau)$ to a current macrostate $+x(\tau)$ of transmitting node forming a virtual macrostate $-x(\tau+o/2)$; the second part brings $+2x(\tau+o)$ to the $-x(\tau+o/2)$ that transfers new macrostate $x(\tau+o)$ to the receptor's node.

This means applying a virtual regular the control $-x(\tau)-x(\tau+o)=-2x(\tau)$ between the nodes. If both macrostates of the communicated nodes have opposite signs ($\pm x(\tau)$, $\mp x(\tau+o)$) at the moment of cooperation $(\tau,\tau+o)$, then the actual regular control $-2x(\tau)\cong -x(\tau)-x(\tau+o)$ performs the same function. On Figs.8.5a,b are shown both cases. For the nodes with the macrostates of the equal signs (Fig.8.5a), the controls are both $-2x(\tau)$ at $(\tau,\tau+o/2)$ and $2x(\tau+o)$ at $(\tau+o/2,\tau+o)$, or the corresponding $-v(x(\tau))+v(-x(\tau))$ and $v(-x(\tau+o))-v(x(\tau+o))$, applied to the cones I-IIa. For the nodes with the macrostates of the opposite signs (Fig.8.5b), the control $-2x(\tau)$ is applied at the moment $(\tau,\tau+o/2)$ to the corresponding cones I and IIb.

The considered nonlinear oscillations $v(l,t)$ of opposite directions, connecting the nodes at the singular point (chaotic attractor ca), are generated by a an actual nonlinear form of the control function $v(x)$ (which approximates the discrete controls) Figs.8.5a,b.

The control also manages a propagation channel, connecting the nodes geometry, which memorizes the transmitting macrostate by the key-lock connection after the cooperation.

Synchronization of each of cooperating nodes stimulates a sequential synchronous resonance's *excitation* of a total network, accompanied by a wave, propagating with some velocity. A possibility of self-exciting synchronization exists by overcoming a threshold of mutual re-excitation, by analogy with an ensemble of nonlinear oscillators.

A sequence of the *self-assembling* INs can be involved in this self-organizing process.

This means, that a whole assembling synchronized process, accompanied by the IN node's cooperation, growing their dimensions, generating the controls, and the different IN's connections, can be governed by the selective *adaptive mechanism* of considered evolutionary cycle.

The environment may affect any of these key-lock connections at every of the *o*-moments of time. The ensembles can grow under both the direct competition and a better surviving.

An appearance of a memorized state depends upon the threshold of the resonance excitation for the adjacent nodes. In the IN each node is evaluated by both particular quantity of information and the path to this node by the information functional measure. This brings an information *measure* of a selective novelty for each node and their assemblies.

The corresponding attractors, capturing the informational distinctive patterns, are nonredundant.

A selective informational real-time search, competition, cognition, and recognition may be very effective.

An automatic initiation of common information measure (during the evolution) occurs by fixing the intervals t_i between the resonances and the coherent frequencies ω_i, which define the information invariant $\mathbf{a}(\gamma)\sim t_i\,\omega_i$ as an elementary quantity information. Selecting two or three intervals with equal $\mathbf{a}(\gamma)$ and decreasing ω_{i+1} will not only preserve their common information measure, but also (with increasing t_{i+1} and a decreases of the contribution from

the diffusion) bring to the corresponding macrostate, having a higher adaptive potential and a capacity for cooperation.

A sequential memorizing them as a single node automatically (at each fixed t_i and synchronized frequencies ω_i) produces the IN code of evolutionary development.

The selective process is governed by the controls, self-generated in the evolution.

The asymmetry of the adaptive potential is accompanied by increasing the macrosystemic stability and growing the macrocomplexity.

The automatic selection works among the resonance's (assembling) nodes *competing* for a limited area space. The preference would be automatically given to such ones, which are able to occupy this area. All others will be excluded.

A primary DSS four letter's code defines γ that can predict future $\mathbf{a}(\gamma)$ with τ_j, ω_j. But obtaining the very primary DSS code and starting the evolution process require applying a *non random initial* control (chs.1.1-1.3).

The evolutionary mechanism of competition and selection *automatically* carries the common information measure.

The common information scale leads to a *common* evaluation of the different node's (logical) combinations for their comparison and choosing of a less redundant.

This brings a growth of a novelty for the selected nodes.

1.8.9. The Functional Schema of the Evolutionary Informational Mechanisms. A Summary

The considered laws and regularities determine the unified functional informational mechanisms of evolution presented on Fig. 8.6, which include:

-the system macrodynamics MS, defined by the model operator $A(t, x(\tau), v)$ that is governed by the inherited double spiral structure DSS^o;

-the control replication mechanism RE1 that transforms the DSS^o code into the initial controls v_o and delivers v_o as the MS input programmable controls;

-the IN, formed in the process of the macrostates' cooperation and the macromodel renovation, generating a renovated $DSS1$;

-the mechanism of mutations MT, delivering external perturbations, which act on the total system;

-the adaptative and the self-organizing mechanisms AS, stimulated by the MT, which generate (G) the fluctuations ξ;

-the replication control mechanism RE2, which selects the macrostates $x(\tau)$ at DP $t' = \tau$ and forms the current control $v(\tau) = -2\,x(\tau)$ by the duplication of $x(\tau)$,

-the coupling of the two macrostates CP that carry both parents' $DSS1$ and $DSS2$ invariants;

-the generation of stochastic dissipative fluctuations SDF after coupling, while forming new macrosystemic invariants (γ^o, n^o) that define a new DSS_o^o, initiating the new MS and

IN, which are renovating under the MT and the AS, in the process of functioning (a previous inherited $DSS2$ minimizes a possible SDF set, generating a new DSS_o^o);

-repeating the whole cycle after coupling and transferring the inherited invariants to a new generated macrosystem.

The IMD software packet (part 2) has simulated the main mechanisms of the evolutionary cycle.

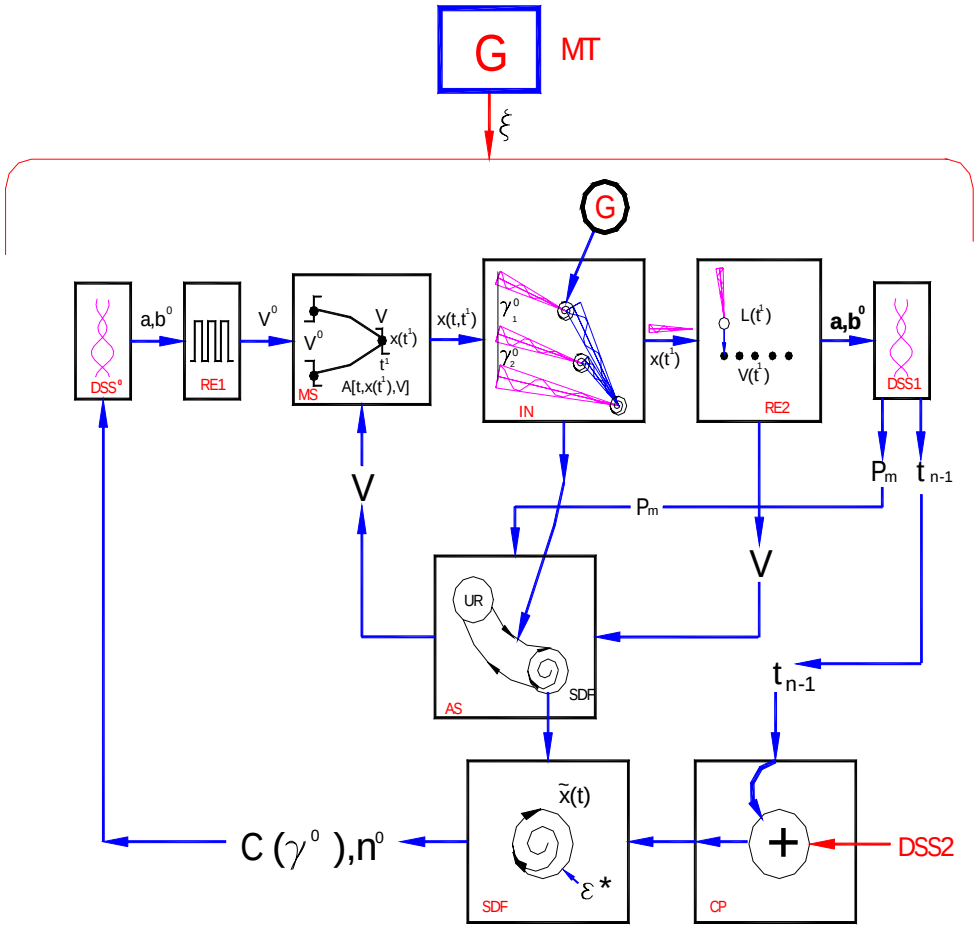


Figure 8.6. The functional schema of the evolutionary informational mechanisms.

A Summary

The evolutionary dynamics, created by the multi-dimensional eigenvalues' spectrum, form a chain of interacting extremal segments, which are assembled in an *ordered organization* structure of the information hierarchical network (IN).

The space distributed IN's structural robustness is preserved by the feedback actions of the inner controls, which provide *a local stability at the admissible* variations.

This control supports a limited $P_e(\gamma) = P_r$ that determines the *potential of robustness.*

The requirements of preserving the evolutionary hierarchy impose the restrictions on the maximal potential of evolution P_e and limit the variations, *acceptable* by the model.

The model's adaptive potential $P_a \le P_e$, which adapts the variations, not compromising the IN hierarchy, restricts the maximal increment of dimension, contributed to the adaptation.

The punched evolution's nonequilibrium accumulates the changes affecting the hierarchy of the selections and adaptations, with following a local equilibrium at each hierarchical level.

The adaptive model's *function* is implemented by the adaptive *feedback-control*, acting within the P_a capabilities.

The self-control function is determined by the conditions of a proper *coordination* for a chain of superimposing processes, where each preceding process adopts the acceptable variations for each following process and controls it.

The optimal controls are synthesized, as an *inner feedback*, by the *duplication* of and *copying* the current macrostates at the beginning of each segment, which are *memorized* and *applied* to the segment.

The adaptive potential's *asymmetry* contributes the model's evolutionary improvement.

A sequence of the sequentially enclosed IN's *nodes*, represented by a *discrete control logic*, creates the IN *code* as a *virtual communication language and* an algorithm of minimal program to *design* the IN.

The optimal IN' code has a *double spiral* triplet structure (DSS), shaped at the localities of the sequential connected cones' spirals, which form the time-space path-line of transferring the IN's information through the triplet's hierarchy.

The applied control adds a forth letter to the initial minimal three triplet's code letters, which provides the model's *error correction mechanism* to the IN and its DSS code.

It also provides discrete filtering of the randomness, acting at the DP-window.

The control's *potential of robustness* might generate a DSS's evolutionary predecessor.

The IN's geometrical border forms the external surface where the macromodel is open for the outside interactions. At this surface, the interacting states *compete* for delivering a maximum of the dynamic potential's gradient.

The selected states are copied and memorized by the model control, contributed to the code.

Information attraction (sec.1.7.6) plays a crucial role in biological mutations and mating, which become the *selective* evolutionary processes (instead of a poor random) by a discriminative matching to some complementary information curvatures (characterized by a similar quality and quantity of information).

Any naturally made curvature conceals genetic information; which is a source of the curvature formation.

The informational DNA code; produced by Nature during a long term evolution; is a general and a native systems language.

The code, as a source of the information curvature (sec.1.7.6), can be considered as a *predecessor* of information geometry and evolution dynamics.

The evolution dynamics are accompanied by the following main *features*: a tendency of decreasing γ that *diminishes* the influence of randomness on the macrolevel dynamics; a decrease of the contribution from diffusion with the increase of the dynamic gradient that intensifies a *growing impact of the each following dynamics* on the evolutionary

development; an average evolutionary information speed (for a population) is declined, weakened during the time of evolution; a nonsymmetrical adaptive potential (at decreasing γ) leads to both rising the system adaptive capability and increasing an impact of a dynamic prehistory on current changes.

In the evolution dynamics, adaptation, self-organization, ordering, and copying arise naturally if the IPF VP is fulfilled.

The evolutionary model possesses two scales of time: a reversible life-time equals to a summary of the time intervals on the *extremals* segments

$$T_e^r = \sum_{i=1}^{i=n} t_i \,,$$

and the irreversible life-time counted by a summary of the irreversible time elementary intervals *between* the segments equal to

$$\delta t_i = (\frac{\Delta S_i^{\delta}}{\mathrm{a}_o^2} - 1) t_i \,,$$

where ΔS_i^{δ} is an elementary information contribution between the segments generated by a random process.

If each information contribution from a random process between the segments ΔS_i^{δ} is compensated by the needle control action $\Delta S_i^{\delta} = \mathbf{a}_o^2$, then the irreversible time does not exist, means that the macroprocess's dynamics cover the random contributions.

At any $\Delta S_i^{\delta} > \mathbf{a}_o^2$ the stochastics affect dynamics bringing the macroprocess' irreversibility.

The above results describe *the regularities of evolutionary hierarchy, stability, potential adaptation, adaptive self-controls and a self-organization; coping, genetic code, and the error correction, which follow from the VP as a single form of mathematical law that defines the above regularities and is able to prognosis the evolutionary dynamics and its specific components: potential, diversity, speed, complexity.*

The *particular* biological regularities, generalized by the information law, are confirmed in numerous experimental data, described in many specific publications [1-13, 15-17, others], supporting Darwinian evolutionary theory.

Chapter 1.9

THE PHYSICAL ANALOGIES RELATED TO THE INFORMATION PATH FUNCTIONAL

1.9.1. The Connection between the Information Path Functional (IPF) and the Kolmogorov's (K) Entropy of a Dynamic System, and the Relations to Physics

The K-entropy is an entropy per unit of time, or the entropy production rate, measured by a sum of the Lyapunov characteristic exponent (LCE) [1-5]. (Please see the references to ch.1.9).

LCE describes a separation between the process trajectories, created by the process dynamic peculiarities.

In the IMD model, the separated extremal segments are resulted from the piece-wise dynamic approximation of a random process' entropy functional.

The partition (and the following merge) is initiated by the model controls actions (fulfilling the VP), which also carry out the transitions between the process dimensions, physically associated with the phase transformations, chaotic movement and related physical phenomena [6-9].

Let us find the LCE for the IMD model.

At the DP, each of these controls *switches* the process extremal segment *from* a local movement $x_{it} = x_{io} \exp(-\lambda_i t)$, satisfying a local process' stability, *to* a local movement

$$x_{i\tau} = x_{i\tau o} \exp(\lambda_{i\tau} t), t \in (\tau - o, \tau), \tag{9.1}$$

corresponding to a local process' instability, which brings a separation between these two process' movements. Here x_{io} is an initial condition at a beginning of the *i*-segment; with the macroprocess' eigenvalue λ_i, $x_{i\tau o}$ is a starting state at the moment $t = \tau - o$ (near the segment end), $\lambda_{i\tau}$ is the eigenvalue at $\tau - o$ approaching τ (which depends on

$gradX(\tau - o)$ (ch.1.8.1) that potentially initiates these dynamics, approximating the between segment's stochastics at $t \to \tau$).

The LCE is measured by a mean rate of exponential divergence (or convergence) of two neighboring trajectories: one of them describes an initial nondisturbed movement x_{it} , another one is the disturbed movement $x_{i\tau}$ (for this model at DP). A local LCE:

$$\sigma_i = \lim_{t\to\tau} \frac{1}{t} \ln(\frac{x_{i\tau}}{x_{i\tau o}}) = \lambda_{i\tau} \tag{9.1a}$$

expresses the exponential divergence $x_{i\tau}$ from the movement x_{it} along the extremal segment ($x_{i\tau}$ starts at the moment $t=\tau - o$ by the end of the movement $x_{it}\,|_{t=\tau-o} \to x_{i\tau o}$, which precedes the beginning of the disturbed movement $x_{i\tau}$).

At $\lambda_{i\tau}>0$, the process is instable and chaotic: the nearby points, no matter how they close, will diverge to any arbitrary separation. These points are instable.

At $\lambda_{i\tau}<0$, the process exhibits asymptotic stability in a dissipative or a non-conservative system. The LCE zero at $\lambda_{i\tau}=0$ indicates that the system is in a steady state.

A physical system with this exponent is a conservative. Such a system exhibits Lyapunov stability. Although this system is deterministic, there is no process' order [8,9] in this case.

Exponent (9.1) approximates the dynamic divergence of the extremal segments at a window between the segments; and the LCE (9.1a) characterizes the *information dynamic* peculiarities arising at the DP localities; some of them have been studied in chs.1.4-1.8. (See also the LCE example in ch.2.2.1, (2.2.23a)).

In particular, under the optimal control, applied to $\lambda_{i\tau}$ at the nearest moment $\delta\tau$ following τ , the eigenvalue changes according to equations

$$\lambda_{i\tau}(v_{i\tau}) = -\lambda_{i\tau} \exp(\lambda_{i\tau}\delta\tau)[2 - \exp(\lambda_{i\tau}\delta\tau)]^{-1},$$

which at $\delta\tau \to 0$ reaches a limit: $\lim_{\delta\tau\to 0} \lambda_{i\tau} = -\lambda_{i\tau}$.

Such a discrete (jump-wise) LCE renovation, is a phenomenon of a controllable process, specifically at the process' coupling, and could serve as a LCE indicator of this phenomenon.

The K entropy is the nonlinear dynamics counterpart of physical the Boltzmann-Gibbs entropy [10], which is directly connected to the Shannon information entropy.

In the IMD model, the DPs are the crucial points of changes in a dynamical evolution with the fixed entropy path functional's production rates (PFR), given by the sum of positive LCE.

According to relation (secs.1.3.5,1.8.1), the PFR, being equal to the sum of the operator positive eigenvalues:

$$E[-\frac{\partial S^i}{\partial t}(\tau)] = E[H(\tau] = Tr[A(\tau)] = \sum_{i=1}^{n} \lambda_i(\tau_i) > 0\,, \tag{9.1b}$$

coincides with the K entropy at these crucial points.

This additivity of the discrete linear rate (at DPs) for both the K entropy and PFR corresponds to a thermodynamic extensivity of the Boltzmann-Gibbs entropy [11], which is important in a connection between statistical mechanics and chaotic dynamics.

The extensivity of entropy is an essential requirement, with which thermodynamics can be constructed [11-13]. This may be the case even if a system energy is nonextensive [12].

A sufficient important is the linear growth of the K entropy and the thermodynamic extensivity of the Boltzmann-Gibbs entropy only in the *long-time* limit and in the thermodynamic limit, respectively.

As it's known [11], a physical quantity to be a temporally extensive should satisfy its linear grow in time. Thus, for example, the K entropy possesses the temporal extensivity for chaotic dynamical systems.

The IMD model holds the open system's qualities such as a nonlinearity and irreversibility (at the DP), and the stationarity and reversibility within each extremal segment, corresponding to a system's conservativity.

These phenomena allow applying the IMD model for a wide class of real systems, which can exhibit the above alternative behaviors at different stages of dynamic evolution [14, 15].

Most publications on this subject are based on the models of the linear phenomenological irreversible thermodynamics, using fluctuations from a stationary state, or a quasi equilibrium process [16-19].

Foundation of nonlinear irreversible thermodynamics in [20] is based on the *n*-dimensional correlators and their connections to the measured physical macrovariables.

We consider *information* approach with the macroprocess' irreversibility arising from a random movement at the entropy functional's punched localities; while the relations for preservation energy might not be fulfilled.

The main problem consists of math difficulties of applying a *macro* evolution approach to a random process and random entropy.

Some publications use an informational approach to a process of self-organization, applying a control's *parameter* for the evaluation of irreversibility in the state's transition [21].

The equations for a *controllable* irreversible information macroprocess, applying the VP for the information path functional (defined on the solution of a controllable stochastic equation), brings the *irreversible kinetic macroequation* with its connection to *diffusion*.

The irreversibility is a consequence of the jump-wise changing of the equation operator with transferring kinetics into diffusion and vice versa along with the renovating operator on a new extremal.

Applying the Shannon entropy measure to *n*-dimensional random *process* with the statistical dependent events leads to the unsolved problem of the long terms *n*-dimensional correlations, while these events are a *naturally connected* by the entropy path functional.

The lack of additivity—even for the statistically independent events—leads to the problem related to the lack of thermodynamic extensivity [13].

Considered in [22,23] a degree-α and α-norm entropy measures satisfy a "pseudo-additive" relation, associated with a *nonextensive* thermodynamics, rather than the *additive* relation, provided by the Shannon and Renyi [24] entropies.

The evolutionary path functional entropy is defined by a simple sum of the local entropies at each DP, according to (1.3.150), (1.3.156a), sec.1.3.5, and (9.1b), which is applied to an extensive dynamic system.

But the extensivity is locally violated at the random window between the extremal segments.

The IN evolutionary PFR forms a ranged sum, satisfying the VP.

The maximal and minimal PFR values characterize the maximal and minimal speeds of evolutionary process according to ch.1.8. A current PFR is defined by a sequential enclosure each of a previous model's eigenvalue to the following one, which are connected by the IN structure. This allows getting the cooperative complexity for all process (ch.1.7), as well as the PFR measure at each stage of evolution.

The IN final node's eigenvalue characterizes both the system's terminal evolutionary speed and the system cooperative complexity.

Applying the IPF leads to revealing complex regularities and uncertainties of a random process by building a system of the INs with the information invariants and encoding a chain of the events, covered by the random process.

IPF measures the process uncertainty by the entropy functional and allows minimizing uncertainty by the optimal control actions.

The IPF Hamiltonian, which determines both the instant entropy production and the macromodel operator, also defines the LCE, representing Lyapunov's function of the process stability. That connects the stability to the process uncertainty.

The process optimization by the controls actions changes the LCE sign at the DP allowing the stability of cooperative process concurrently with the minimization of its uncertainty.

The above results connect the model's Uncertainty, Regularity, and Stability.

1.9.2. An IPF Analogy with the Feynman Path Functional in Quantum Mechanics and Informational Form of Schrödinger's Equation

Feynman's path functional introduces the integral of a wave probability function ($\overline{\varphi}$) in Quantum Mechanics (QM):

$$\overline{P}(x(s),x(T)) = \int_{x(s)}^{x(T)} \overline{\varphi} Dx(t),\ \overline{\varphi} = \exp(j\overline{S}/\hbar)\,, \tag{9.2}$$

where $\overline{S}$ is a function of action, and $Dx = \prod_{i=1}^{n\to\infty} dx_i$ is is a differential along a given "particle" trajectory $x_t = x(t)$, which measures a total probability $\overline{P}(x(s),x(T))$ for the trajectory to pass through a sequence of the gates, defined by discrete values of $\Delta x_i = \int_{\Delta t_i} dx_i(t_i)$; $\hbar$ is Plank's constant.

Formula (1.1.14), ch.1.2 is an analog of (9.2) at $Dx(t) \sim P(d\omega), \overline{\varphi} \sim \exp\{-\varphi_s^t\}$.

These relations become apparent when the path functional (9.2) is written through the mathematical expectation taken along the trajectory (according to [27]) with some measure $\overline{P}_{s,x}(dx)$:

$$\overline{P}(x(s),x(T)) = E_x[\overline{\varphi}] = \int_{x(s)}^{x(T)} \overline{\varphi}\overline{P}_{s,x}(dx), \tag{9.3}$$

even though the trajectories, along which the Feynman path functional was defined, are nonrandom [26, 27]. Actually, such expressions for both path functional and the corresponding Wiener integral lead to Schrödinger's equation [27] in QM.

Let us consider a transitional probability $\tilde{P} = \tilde{P}(s,\tilde{x}_t,t,B)$ (ch.1.1) of a diffusion process $\tilde{x}_t = \tilde{x}_t(t,x,\xi_t)$ and find the probability function at a small interval Δt following t:

$$\tilde{u}(s,x) = P_{\Delta t}(s,x,t,B), \tag{9.4}$$

which is connected to the process' additive functional $\varphi_s^t(\omega)$ by relation

$$E_{s,x}[\int_{\tilde{x}(t)\in B} \exp\{-\varphi_s^t(\omega)\}] = \tilde{u}(s,x). \tag{9.5}$$

Then the above function $\tilde{u} = \tilde{u}(s,x)$ satisfies the Kolmogorov differential equation [28]

$$-\frac{\partial \tilde{u}}{\partial s} = a(s,x)\frac{\partial \tilde{u}}{\partial x} + b(s,x)\frac{\partial^2 \tilde{u}}{\partial x^2} - V(s,x)\tilde{u}, \tag{9.6}$$

where function $V(s,x)$ we specify below.

Using the representation of a function $\tilde{u}(s,x,\lambda^o)$ (with a paramerter $-\infty < \lambda^o < \infty$) via a characteristic function of a random real functional $\tilde{\Psi}$, we have :

$$\tilde{u}(s,x,\lambda^o) = E_{s,x}\exp[j\lambda^o\tilde{\Psi}], \tilde{\Psi} = \int_s^\tau \tilde{\psi}(t,\tilde{x}(t))dt, \tag{9.6a}$$

where τ is a moment of exit from some interval Δt, or it might be a constant $\tau = T$ [28].

In this case, $\tilde{\Psi}$ corresponds to the additive functional φ_s^T (ch.1.1):

$$\varphi_s^T = 1/2\int_s^T a^u(t,\tilde{x}_t)^T(2b(t,\tilde{x}_t))^{-1}a^u(t,\tilde{x}_t)dt + \int_s^T(\sigma(t,\tilde{x}_t))^{-1}a^u(t,\tilde{x}_t)d\xi(t). \tag{9.7}$$

Such a function $\tilde{u}(s,x,\lambda^o)$ at a fixed λ^o, for which holds true (9.6a), satisfies differential equation (9.6) with a real or complex non random function

$$V(s,x) = -j\lambda^o \tilde{\psi}(s,x). \tag{9.7a}$$

The equation for the function $\tilde{u}(s,x,\hat{h}) = \hat{u}(s,x)$ with (9.7a) and $\lambda^o = \hat{h}^{-1}$ (where $\hat{h}$ can be the uncertain constant h_o in ch.1.6, or be the Plank constant $\hbar$) acquires the form

$$-\frac{\partial \hat{u}}{\partial s} = a^u(s,x)\frac{\partial \hat{u}}{\partial x} + b(s,x)\frac{\partial^2 \hat{u}}{\partial x^2} + j\hat{h}^{-1}\tilde{\psi}(s,x)\hat{u}, \tag{9.8}$$

where function

$$\tilde{\psi}(s,x) = 1/2a^u(s,x)^T(2b(s,x))^{-1}a^u(s,x) \tag{9.8a}$$

is connected to the entropy functional (ch.1.1):

$$\tilde{S} = 1/2E_{s,x}[\int_s^T a^u(s,\tilde{x})^T(2b(s,\tilde{x}))^{-1}a^u(s,\tilde{x})dt]. \tag{9.9}$$

Using the information path functional $S = S[x_t]$, defined on its extremals $x_t = x(t)$, and applying the sec.1.3.5 results, we get the information Hamiltonian

$$H = 1/2(\dot{x})^T(2b)^{-1}\dot{x}, \dot{x} = a^u, \tag{9.10}$$

which satisfies (9.8a) in the form

$$\tilde{\psi}(s,x) = H(s,x). \tag{9.10a}$$

For this Hamiltonian the fulfillment of the variation principle: $-\frac{\partial S}{\partial t} = H$ leads to equation

$$-\frac{\partial S}{\partial t} = 1/2(a^u)^T(2b)^{-1}a^u. \tag{9.10b}$$

Following ch.1.3, we consider a distribution of functional (9.9) on $(t,x) \in Q$ as a function of current variables $\tilde{S} = \tilde{S}(t,x)$, which satisfies the Kolmogorov (K) equation, applied to the math expectation of functional (9.9) in the form

$$-\frac{\partial \tilde{S}}{\partial t} = (a^u)^T\frac{\partial \tilde{S}}{\partial x} + b\frac{\partial^2 \tilde{S}}{\partial x^2} + 1/2(a^u)^T(2b)^{-1}a^u. \tag{9.11}$$

Equations (9.10a), (9.11) can be satisfied at each point $(t,x) \in Q$ of the functional field, except a certain set $Q^o \subset Q$, $Q^o = R^n \times \Delta^o, \Delta^o = [0,\tau], \tau = \{\tau_k\}, k = 1,...,m$
where hold true the relations

$$(a^u)^T \frac{\partial \tilde{S}}{\partial x} + b\frac{\partial^2 \tilde{S}}{\partial x^2} = 0, a^u = a^u(t,x), b = b(t,x). \tag{9.12}$$

The differential constraint (9.12) corresponds to operator's equation

$$\tilde{L}S[x_\tau] = 0, \tilde{L} = a^u \frac{\partial}{\partial x} + b\frac{\partial^2}{\partial x^2},\ x_\tau = x(\tau)\ , (x,\tau) \in Q^o, \tag{9.13}$$

which can be applied to (9.8) in the operator form

$$\tilde{L}u[x_\tau] = 0, \tilde{L} = a^u \frac{\partial}{\partial x} + b\frac{\partial^2}{\partial x^2}. \tag{9.13a}$$

This constraint, imposed on equation (9.8), brings the *dynamic* solutions for function $\hat{u}(t,x)$, while (9.13) allows fulfilling the variation conditions for the IPF and selecting the extremal trajectories $x_t = x(t)$.

The constraint (9.13), establishing a connection between the microlevel's diffusion and macrolevel's dynamics, is relevant only at some "punched" *points* of the *space* $Q^o = R^n \times \Delta^o$, while macroequations (9.10), (9.10a) acts along each extremal, except the related discrete *points* (DP) (1.3.153) of states $x_\tau = x(\tau) = \{x(\tau_k)\}$, for which the information path functional (IPF) coincides with the entropy functional.

The solutions of (9.13) allow selecting the discrete states $x(\tau) = \{x(\tau_k)\}, k = 1,...,m$, considered to be the *boundary* points [28] of a diffusion process: $\lim_{t\to\tau} \tilde{x}(t) = x(\tau)$, which (at the limitation in sec.1.3.5), *bring a quantum character of generation for both the macrostates and the macrodynamic's information at the VP fulfillment.*

Actually, the macrostate $x(\tau)$, approaching a random $\tilde{x}(\tau)$, is not a dynamic state, and the moments τ can be determined with some probability [28].

At these conditions, applying a current time course $s \to t$ to the equations (9.8), (9.13a), we come to their dynamic form as an *information analogy* of Schrödinger equation

$$-\frac{\partial \hat{u}}{\partial t} = j\hat{h}^{-1} H\hat{u}, \tag{9.14}$$

where the IPF's $S = S(t,x)$, defined on the extremals, performs a role of a function of action along the trajectory $x_t(t,x)$ of $\hat{u} = \hat{u}(x_t)$.

The solutions of (9.14) $\hat{u} = \hat{u}(t,x)$ at some *given* Hamiltonian $H = H(\dot{x},x,t)$ are not distinguished from the solutions of a *traditional* Schrödinger equation [26,27,29].

Each particular solution of this equation determines the complex amplitude of a wave function; the corresponding probability is equal to the square of absolute value of the wave function.

At the condition

$$|\hat{u}|^2 = |\hat{u}(t,x)\hat{u}(t,x)^*|, \tag{9.14a}$$

the wave functions interfere, became entangled.

Imposing the constraint (9.13) on classical dynamics binds the equations for conjugate variable in n-dimensional system (ch.1.3,3.3.56c).

Constraint (9.13a) for function $\widehat{u} = \widehat{u}(x_t)$ on the extremals, at the punched localities $x_t = x_t(\tau)$, acquires the form

$$\frac{\partial \widehat{u}}{\partial x_t(\tau)}(\frac{\partial \widehat{u}}{\partial x_t(\tau)})^T = |\frac{\partial \widehat{u}}{\partial x_t(\tau)}|^2, \tag{9.14b}$$

which for the conjugated

$$\frac{\partial \widehat{u}}{\partial x_t(\tau)} \text{ and } (\frac{\partial \widehat{u}}{\partial x_t(\tau)})^*$$

leads to

$$|\frac{\partial \widehat{u}}{\partial x_t(\tau)}|^2 = \frac{\partial \widehat{u}}{\partial x_t(\tau)}(\frac{\partial \widehat{u}}{\partial x_t(\tau)})^*. \tag{9.14c}$$

Conditions (9.14b,c), following from the IPF dynamic constraint, are written via the probability *densities at the* $x_t(\tau)$ *localities* for the process trajectories, which specify (9.14a) at these localities.

Each complex solution of (9.14), satisfying (9.14b,c), allows selecting a specific locality with particular bound state, where the dynamic solutions for $\widehat{u}(t,x)$ interfere, becoming entangled.

In addition to this, for Hamiltonian (9.10), the solutions of (9.13) allow selecting the *quantum information states* (x_τ), which provides the instants of the solution's $\widehat{u} = \widehat{u}(x_\tau)$ interference. According to IMD, the related extremal segments join together at these instants forming the cooperative (bound) states. Example 9.1 illustrates it.

For the considered multi-dimensional process, Hamiltonian H is defined for each extremal segment of this process, while for a whole process' trajectory should be applied an averaged Hamiltonian $\hat{H} = E[H]$.

The corresponding function $\hat{u} = \hat{u}(t,x)$ satisfies the equation

$$-\frac{\partial \widehat{u}}{\partial t} = j\widehat{h}^{-1}\hat{H}\widehat{u}, \tag{9.15}$$

where this Hamiltonian can be expressed via the model's eigenfunctions according to Prop.1.3.5.6, (1.3.162),(1.3.162a), while the set of quantum macrostates is determined by the solutions of (9.13), considered for each multi-dimensional extremal.

For such an ensemble of extremal trajectories, conditions (9.14b,c) determine a set of the punched localities with corresponding ensemble of bound states.

Applying the equations of extremals (1.3.154) with matrix (1.3.171) we get $\hat{H} = Tr[A^v]$, where at each $\tau_k, k = 1...,m$ the matrix' eigenvalues $A^v = (\lambda^v)|_{i=1}^n$ take the real values and its imaginary components turn to zeros.

The result, as it is shown in ch.1.3, follows from the constrain equation (9.13).

This means that for the complex conjugated $(\lambda_i^v, \lambda_{i+1}^v)$ with the solutions of (9.15):

$$\hat{u}_i(t) = \hat{u}(t_{io}) \exp[-j\hat{h}^{-1}(\int \lambda_i^v dt)],\ \hat{u}_{i+1}(t) = \hat{u}(t_{i,o}) \exp[-j\hat{h}^{-1}(\int \lambda_{i+1}^v(t) dt)] \quad (9.15a)$$

at the moment τ_k hold true the relations

$$\lambda_i^v(\tau_k) = \lambda_{i+1}^v(\tau_k),\ \mathrm{Im}\,\lambda_i^v(t)_{t=\tau_k} = 0. \quad (9.15b)$$

Specifically for $\lambda_i^v = \alpha_i^v + j\beta_i^v, \lambda_{i+1}^v = \alpha_i^v - j\beta_i^v$, according to (9.15a), we have

$$\hat{u}_i(t) = \hat{u}(t_{io}) \exp[-\hat{h}^{-1} \int (j\alpha_i^v(t) - \beta_i^v(t)) dt)],$$

$$\hat{u}_{i+1}(t) = \hat{u}(t_{i,o}) \exp[-\hat{h}^{-1} \int (j\alpha_i^v(t) + \beta_i^v(t) dt)],$$

and following (9.14a) we get these solution's amplitude

$$|\hat{u}_i|^2 = |\hat{u}_i(t)\hat{u}_{i+1}(t)^*| = \hat{u}(t_{io})^2 \exp[-\hat{h}^{-1} \int (j\alpha_i^v(t) - \beta_i^v(t) + (-j\alpha_i^v(t) + \beta_i^v(t)) dt] = \hat{u}(t_{io})^2, \quad (9.15c)$$

where the moment of the solution's interference is found from (9.17a) in the form $\beta_i^v(\tau_k) = 0$. These solutions also satisfy (9.14c), which is confirmed by the substitutions.

Example 9.1.

Let us have $\lambda_{io}^v = j\beta_{io}, \lambda_{i+1,o}^v = -j\beta_{io}$ at a fixed $\beta_{io} \neq 0$.

Then according to Example 1.3.1 we get

$$\lambda_i^v = \alpha_i^v + j\beta_i^v, \lambda_{i+1}^v = \alpha_i^v - j\beta_i^v \text{ at}$$
$$\alpha_i^v = -2\beta_{io} \sin(\beta_{io} t)[5 - 4\cos(\beta_{io} t)]^{-1}, \beta_i^v = [2\cos(\beta_{io} t) - 1][5 - 4\cos(\beta_{io} t)]^{-1}.$$

At $\beta_i^v(\tau_k) = 0$, $2\cos(\beta_{io}\tau_k) - 1 = 0$, we have $\beta_{io}\tau_k = \pi/6$, $\tau_k = \pi/6\beta_{io}$, and $\alpha_i^v(\tau_k) = -(3)^{-1/2}\beta_{io}$.

At this moment, the solutions:

$$\hat{u}_i(\tau_k)=\hat{u}(t_{i,o})\exp[-\hat{h}^{-1}\int_{t_o}^{\tau_k}(j\lambda_i^v(t)dt)],\ \hat{u}_{i+1}(\tau_k)=\hat{u}(t_{i,o})\exp[-\hat{h}^{-1}\int_{t_o}^{\tau_k}(j\lambda_{i+1}^v(t)dt)],$$

with $\lambda_i^v(\tau_k)=\lambda_{i+1}^v(\tau_k)=\alpha_i^v(\tau_k), \operatorname{Im}\lambda_i^v(\tau_k)=\operatorname{Im}\lambda_{i+1}^v(\tau_k)=0$ coincide, and the solution's interference satisfying (9.14a), (9.14c). •

The considered *information* Hamiltonian (9.10) is different from the energy Hamiltonian in QM; and $\hat{u}(t,x)$ or $\hat{u}(t,x)$ represents an *information* wave function, while the QM's related function is amplitude of probability for a *physical* particle.

A more general information description can be applied to any information events including the physical ones.

Both (9.13) and (9.14) determine the information wave functions as the *dynamic* approximation of the diffusion process by the corresponding amplitudes of the probabilities.

IMD with the random microlevel, the discrete intervals of imaginary information, whose *interaction* (collapse) generates real information, and the code, can be considered as an *information* analogy of QM.

It's possible that the QM dynamics cover stochastic dynamics laying underneath, which carry the information code (to be transformed into a matter, sec.1.9.3)).

A specific of these stochastics consists of the fulfillment of equation in partial derivations (9.12), which can be used for finding the discrete interval between quantum interactions.

The dynamic macroprocess and its wave function characterize a prognostic movement among the probable random trajectories in information dynamics.

This approach connects the process' probability functional (defining the path functional) with the process entropy functional and expresses both of them via the integrand (9.7) of related stochastic equation.

Considering the random process as a starting source for a macroprocess and using the macrolevel's complex wave functions equations, we found that the interference (a "collapse") of the information wave functions corresponds to transferring the macroprocess with a complex operator to a microprocess with a real operator at each discrete point (DP).

DPs open an access to random process for the control and identification of the renovated macrooperator at each extremal segment's end via the measurement of correlation functions (ch.1.3, other).

From this follows a *discrete* character of *measurements* in the IMD bi-levels model at the DPs, where the eigenfunctions of the macrooperator undergo a jump-wise change. Prior to the DP occurrence, both the macro- and microvariables in this model are unobserved.

Thus, the joint solutions of (9.13), (9.14) allow to determine a *formal "observer*",which provides a quantum *measurement* of a random object that extracts its maximal information.

Such an observer also self-transforms an initial information measure (EF) into its dynamic form (IPF) and then to the information algorithm and code.

These IMD specifics connect it to a measurement and control in quantum dynamic systems [30].

The corresponding DSS quantum code (ch.1.6), generated at DP localities (according to 1.3.153), potentially can be applied for a quantum computation [31].

The quantum states define the quants of a *discrete time*, which, using the information invariants (ch.1.3), determines the quants of related *information*.

This means that a discrete time carries information.

1.9.3. About the Invariant Transformation of the Model Imaginary Eigenvalues and Information

The macromodel initial pure imaginary eigenvalues $\lambda_{1,2}(\pm jt_*)$ can be transformed into the real eigenvalues using transformation (sec.1.3.5):

$$T_o = -\frac{\exp(\pm jt)}{2-\exp(\pm jt)}$$

carried by the imaginary *control's* feedback $v = -2x_j(t_o)$ and applied at some initial moment t_o. This transformation can be considered as a special conformal reflection of the line $\pm j$ into a shape $w(z) = \frac{az+b}{cz+d}$ at $z = \exp(\pm jt)$, a=1, b=0, c=– 1, d=2 with the invariant points $z_1 = 0, z_2 = 1$, $w(z=0) = 0, w(z=1) = 1$.

Such a reflection transforms the $\pm j$ lines of an Euclid geometry into the curved lines on the Riemann shape $w(z)$.

The angle between a pair of curves at each point z is transformed into an equal angle (by its value and the marked course direction) between the curves in the shape $w(z)$.

This leads to the transformation of the initial parallel pair of lines $\pm j$ into the intersecting curves on the Riemann's shape $w(z)$.

Following the connection to Quantum Mechanic, each of these eigenvalues $\pm j$ corresponds to the pair of wave functions $\psi_1 = \exp(jt)$, $\psi_2 = \exp(-jt)$.

The property of being the harmonic wave functions is an invariant under the conformal mapping.

Because of that, transformation T_o creates a pair of transformed wave functions

$$\varphi_1 = -\frac{\psi_1}{2-\psi_1},\ \varphi_2 = -\frac{\psi_2}{2-\psi_2},$$

which can actually intersect on the Riemann shape $w(z)$.

A set of imaginary events, described by the wave functions, have a unobserved probabilistic tendency $P = |\varphi_1 + \varphi_2|^2$ that is transformed into a real observed event at a point of their intersection.

A curved shape, corresponding to Riemann geometry, is formed by the local IN's space dimensions at the DP vicinities.

A virtual transformation above, that is able to generate a positive entropy production (chs.1.3,1.7), becomes *irreversible*.

This means, that the transformation implementation requires consuming an energy that *converts* its virtuality into a reality.

A similar transformation undergoes each third extremal segment of the IN node (starting with the imaginary eigenvalues).

The transformation can also be expressed via the model imaginary invariants (secs.1.3.5, 1.5.4).

By knowing an elementary imaginary information $\mathbf{b}_o = \beta_{io}\tau$ and using the above transformation in the form $\mathrm{Re}\{(j\mathbf{b}_o)\exp(j\mathbf{b}_o)[2-\exp(j\mathbf{b}_o)]^{-1}\} = \mathbf{a}$, can be found the elementary real information $\mathbf{a}$ generated by $\mathbf{b}_o$.

According to Example 1.3.2, a pure imaginary initial $\lambda_{io} = \pm j\beta_{io}$ corresponds to the segment's invariant $\mathbf{b}_o = \pi/6$, which at $\mathbf{a} = 2\mathbf{b}_o \sin(\mathbf{b}_o)[(5-4\cos(\mathbf{b}_o)]^{-1}$ is able to produce, by the segment end, the real elementary information $\mathbf{a} \cong 0.303$ Nat, or $\cong 0.44$ bit. Using relation $\mathbf{a}_o^2 \cong 2\mathbf{a}$, we have $\mathbf{a}_o^2 \cong 0.606$ Nat and $\mathbf{a}_o \cong -0.778$ Nat.

Applying formula ch.1.6 for the relative irreversible time:

$$\delta t_i^* = \frac{\mathbf{a}_o^2 - \mathbf{a}_o + \mathbf{a}}{\mathbf{a}_o^2},$$

we get $\delta t_i^* \cong 0.215$.

Thus, the above real information allows starting a needle control and irreversible real time.

Real information starts with a positive entropy production, evaluated by an elementary α_{io}, which is able to initiate a real time course with starting elementary time interval $\tau_{io} = \mathbf{a}_o/\alpha_{io}$, following the generation of $\mathbf{a}$, $\mathbf{a}_o^2$, $\mathbf{a}_o$ and α_{io}.

At $\lambda_{io}^o = \pm j$, $\beta_{io} = 1$, the time interval, corresponding the imaginary information, holds an imaginary time $\tau_{io}^o = \pi/6$.

In the dynamic systems theory [1,35, other], the transformations exist with invariant measures, which preserve an entropy as their metric or topological invariant (depending on the symbolic or topological dynamics). A numerical value of the entropy invariant serves as a measure for classification of these transformations.

The transformations with invariant measure are purely mathematical and virtual even though they are able to produce information.

For example, the transformation of a symbol's x doubling $T(x) = 2x$ has topological entropy $h(T) = \log 2$; for the rotation on a circle, the entropy $h(T) = 0$, and for the transformation of the full shift on *k* symbols, the entropy is $h(T) = \log k$.

Thus, there exists a wide class of transformations of symbolic information, which does not require consuming energy.

The IPF variation principle *virtually transforms* the entropy functional's uncertainty into both imaginary and real informational dynamics.

The imaginary dynamics satisfy Hamiltonian dynamics for a close system, while the real dynamics take place in an open system.

IMD operates with two kinds of transformations: reversible within each segment's interval and irreversible out of these intervals. Each reversible transformation preserves the invariant amount of information (for example, expressed by the imaginary invariant), which satisfies the conservation law for information as it *virtual logical substance*.

For the irreversible transformation, an open system gets additional information from the environment (including a control), and both direct and inverse transformations become

nonequivalent. The irreversible transformation, even being a virtual, might produce real information and require energy.

Memorizing by copying does not consume energy in a virtual computation. But each renovation of the copied information erases the previous one and actually consumes energy. Both irreversible logic and memory, implemented in a software for a dynamic virtual computation, can transform the encoding information into material (energy) substance.

Supposedly, an external intelligence might transform information (a mind) into a matter.

1.9.4. The Superimposing Processes, Control and Asymmetry. The IMD Relation to Nonequilibrium Thermodynamics (NT)

IMD represents the *informational form* of NT equations considered under the action of control functions. The entropy functional has an analogy to the Onsager-Machlup functional in the NT [19], (ch.1.3).

The IMD minimax principle, requiring a maximum entropy's generation at the DP and its minimum at the extremals (where the information spending is minimal), is connected to the Prigogine minimal principle [15, 16].

Its implementation initiates irreversibility, instability, cooperation, and appearance of the new ordered asymmetrical formations by an analogy with the Prigogine dissipative structures.

The IMD irreversible macrodynamics and the general information mechanisms describe the regularities of the optimal synthesis of information macrostructures, which are associated not only with physics.

There are various IMD connections with the NT equations.

In a particular, the Onsager condition in irreversible linear thermodynamics [17] is a consequence of the detail equilibrium principle and the condition of symmetry of the linear operator $\hat{l}$ in the equation $\dot{x} = \hat{l}X$.

The related n-dimensional IMD kinetic equation (ch.3) is

$$\dot{x} = \bar{L}X\,,\ \dot{x}_t = I, X = \frac{\partial S}{\partial x}\,, \tag{9.16}$$

where I is a vector of a information flow, X is a vector of a information force, $\bar{L} = 2b$ is a kinetic matrix, which generally could be asymmetrical and a nonlinear.

The considering "punched" DP-localities provide a *conjugation between the kinetics and diffusion*: at $\bar{L}(\tau - o) \geq \sigma\sigma^T$ the kinetic flow transfers to diffusion; at $\sigma\sigma^T \geq \bar{L}(\tau + o)$ the diffusion flow transfers to kinetics.

The cooperation brings a physical analogy of the states' superposition into a *compound* state, accompanied by a nonsymmetry of the formed *ordered* macrostructures, the *irreversibility* at each DP, and the emerged nonlinear phenomena, accompanied by the creation of new properties.

For a physical model, each its dimension represents a particular physical process, such as chemical, electrical, diffusion, mechanical, etc.; and their intersection creates new cross

phenomena like thermoelectrical, electrodiffusion, electrochemical, and electromechanical phenomena.

The *multi-dimensional* superimposition embraces a potential *inner* "needle" control action, associated with modeling of the process' jump-wise interactions.

Indeed. In a chain of the superimposing processes $x_i(t,\tau), i = 1,2,.....n$, each current process (i) controls one or more of the following chain's processes $(i+1=j)$ with a possibility of changing the operator in the NT macroequation

$$\frac{dx_i}{dt} = \sum_{i,j=1}^{n} L_{ij} X_j, i \neq j = 1,...,n \ . \tag{9.16a}$$

The mutual cross phenomena, modeled by the applied control functions, connect the chain by relations

$$\frac{dx_i}{dt} = L_i X_i + u_i, \ u_i = \sum_{j=1}^{n} L_{ij} X_j, i \neq j \ , \tag{9.16b}$$

where the controls at the current *i*-segment:

$$u_i = u_i(\delta v(t',l')) \tag{9.17}$$

include the corresponding needle controls, applied at the DP-*localities* (t',l'):

$$\delta v(t',l) = -2x(t',l) + 2x(t'+o,l) \ , \ \delta v(l',t) = -2x(l',t) + 2x(l'+o,t) \ , \tag{9.17a}$$

which arrange the cross connections and change the operator components with a potential operator's renovation.

Example 9.2.

Let us consider a single dimensional segment interacting with a following segment by applying controls in the IMD model:

$$\dot{x}_1 = a_{11}(x_1 + v(\tau_1)) + a_{12}(x_2 + v(\tau_2)) \ ,$$

which corresponds to the NT form:

$$\dot{x}_1 = L_{11} X_1 + L_{12} X_2 \ ;$$

with

$$\dot{x}_{11} = I_{11} = L_{11} X_1 = a_{11}(x_1 + v(\tau_1)), \ \dot{x}_{12} = I_{12} = L_{12} X_2 = a_{12}(x_2 + v(\tau_2)) \ ;$$

where τ_1, τ_2 are the moments of applying these controls at the segment beginning and end accordingly:

$$u_1(\tau_1) = a_{11} v(\tau_1), u_1(\tau_2) = a_{12}(\tau_1) v(\tau_2) \ .$$

Under the action of the needle control $\delta v(\tau_2) = v(\tau_2)$ at segment's end, when $x_2(\tau_2) = 0$, we have at this moment

$$L_{12}(\tau_2)X_2(\tau_2) = a_{12}(\tau_2)\delta v(\tau_2) = u_1(\tau_2).$$

It seen that the segment's control $u_1(\tau_2)$, applied at the segments' end, generates the between segment's cross-information flow

$$I_{12}(\tau_2) = L_{12}(\tau_2)X_2(\tau_2),$$

while an increment $a_{12}(\tau_2)$ is a result of $\delta v(\tau_2)$ action and the eigenvalue renovation. •

This means that, in modeling complex dynamics, the considered controls performs *function of a cross-interacting superimposing processes,* connecting the segments (in addition to the control functions in *Comments* 3.14).

The vice versa is also true: in complex dynamics, the superimposing processes are modeled by the above control's functions (see also ch.1.7 and [38]).

The path functional extremals, as the solutions of the irreversible NT–IMD equations, also provide an analogy of the Onsager conditions at the end of each extremal segment.

The kinetic operator

$$\bar{L}(x(t',l')) = 2b(x(t',l')) = E_{x(t,l')}[x(t,l)\frac{\partial x}{\partial t}(t,l)^*] = E_{x(t',l)}[\frac{\partial x}{\partial t}(t,l)x(t,l)^*] \quad (9.17b)$$

is changing by a jump at each time-space point (t',l') of applying the discrete control where the relations

$$\frac{I_i}{X_i}(t_j',l'_j) = g_i(t_j',l'_j) = g_k(t_j',l'_j) = \frac{I_k}{X_k}(t_j',l'_j), \quad (9.18)$$

take pace; here g_i ,g_k are the subsequently equalized components of the generalized transient conductivity (admittance) (see also sec.1.7.6).

The needle controls select them based on the variation condition (ch.1.3).

The sequence of the chain dependable n-controllable components of these conductivities can be reduced to a single currently controllable conductivity, for example, to electrical conductivity (which is proportional to the diffusion conductivity) and whose measuring is the simplest.

Following this and the connection between the diffusion and electroconductivity, an average entropy production of the controlled process in the IMD equations:

$$\frac{\partial \Delta \hat{S}}{\partial t} = 1/2\dot{\sigma}_e\sigma_e^{-1}, \int_t^{t+\tau} b(t)dt = \sigma_e(\tau), \quad (9.19)$$

can be expressed via the nonlinear electroconductivity $\sigma_e = \sigma_e(\tau)$, measured at the discrete points τ ; where (9.19) serves as an indirect *indicator* of the IPF averaged Hamiltonian.

The nonlinearity of matrix $\bar{L}$ (in 9.17) is a result of interactions, new effects and phenomena at the superimposition, which are modeled by the consolidation process of forming the cooperative macrostructure.

In the space consolidated model, these processes involve diagonalizing of the dynamic operator under the periodical rotations of a symmetrical transformation (chs.1.4,1.5).

Such a procedure also decreases the number of components of the path functional's Lagrangian, minimizing the entropy production.

This means that the space movement, directed toward diagonalizing of the dynamic operator (as an attribute of the space consolidation process, (secs.1.4.2, 1.4.3)), is a source of the generation of an *additional* negentropy for the cooperation.

During this process, the ranged sequence of the triplet eigenvalues preserves the pair-wise ratios:

$$\lambda_1 / \lambda_2 = G1, \lambda_2 / \lambda_3 = G2, \lambda_3 / \lambda_4 = G1, \lambda_4 / \lambda_5 = G2, \ldots, .. \qquad (9.20)$$

defined by the IN multiplicators $G1 \sim \gamma_1^{\alpha}$, $G2 \sim \gamma_2^{\alpha}$, which are determined by the ratios of local entropies.

The rotation spreads these ratios along each coordinate axis: $l_x \rightarrow$G1, $l_y \rightarrow$G2, $l_z \rightarrow$G1, and the ranged eigenvalues are distributed along each geometrical axis with their pair-wise multiplicators.

For example, along the axes l_x, l_y, l_z can be selected the extremal segments that hold the following related multiplicators:

$$\begin{aligned} &G(x)=\lambda_1 / \lambda_4 = G1 \bullet G2 \bullet G1; G(y)=\lambda_2 / \lambda_5 = G2 \bullet G1 \bullet G2; \\ &G(z)=\lambda_3 / \lambda_6 = G1 \bullet G2 \bullet G1 = G(x). \end{aligned} \qquad (9.20a)$$

These segments define the formed macrostructure cell's volumes (by analogy with Fig.6.4), which preserve the ratios of geometrical sizes proportional to invariant $\mathbf{a}_o$ as an elementary quantity information for each macrocell.

The rotating matrix, applied to the ranged triplet eigenvalues, retains a symmetry of the group transformation with the preservation of the above multiplicators.

The transformation of symmetry acts only within each discrete interval when VP is satisfied.

The macrocells can be mutually transformed by a translation, analogous to crystal structures.

Such a well-known transformation consists of the macrocell rotation on angle $\varphi = 2\pi / n$ with changing the linear sizes in *p*-times, where *n* and *p* are connected by relation $n = 2\pi(\arccos\frac{1-p}{2})^{-1}$.

The indexes *n* and *p* of an order in the rotating symmetry take the values: *n*=(2,3,4,6), *p*=(3,2,1). The considered macromodel can keep these values for $G1 \bullet G2 \geq 2$, $G2 \geq 1$ (at $\gamma \cong (0.0 - 0.8)$), which bring $p \geq 1$.

For a real macrosystem, having $G1 \bullet G2 \geq 2$, G2>1.5, the above relations define *p*=3, *n*=2. In this case, a macrocell (*x*) with $G_x = G1 \bullet G \bullet G1 = 3$ has the linear sizes in three times longer than the related macromodel's initial cell, that had been rotated on angle φ.

Therefore, the above transformation of rotation allows building the consolidated macrosystem from the macrocell's size formed at any starting model's DPs.

The needle controls bind the macrocells according to the joint model's multiplicators.

The consolidation in form of triplet structures and the distribution of the eigenvectors by three with preservation of their multiplicators are a consequence of the three-dimensionality of the geometrical space, where existence of the geometrical symmetrical transformations follows from VP. This brings the *existence* a common symmetry of transformation for all macrocells, representing a phenomenon that connects dynamics and geometry.

The macrostructures with considered multiplicators enable the preservation of the symmetry only at very narrow values of the macrocell parameters.

Generally, macrocells with the arbitrary multiplicators cannot be transformed into each other by symmetrical transformations (ch.1.5). By the consolidation moment, three of such macrocells have almost equal sizes, and they are able to form the joint macrocell with the triple linear sizes, having a single dimension. By the end of discrete interval, the matrix elements complete diagonalzing, allowing the joint macrocells' rotation toward a subsequent triplet's cooperation.

The consolidated cells are asymmetrical; they cannot coincide using the symmetrical transformations.

The needle controls consolidate the asymmetrical local instable irreversible macrostates into stable cooperative structures at each new discrete interval.

The triple cell's formations are analogous to the three-critical phase transformations of the second order with a specific connection the kinetics, diffusion and the symmetry order in crystals.

The space distributed model's process has a spiral space structure (chs.1.5, 1.6) on a cone with a possible left (L) or right (R) direction of the spirals, which are typical for a real process' superposition. In a macrosystem with superimposing processes, each *i*-asymmetrical (L or R) process can control the subsequent (*i*+1) (R or L) asymmetrical process.

The superimposing processes of an opposite asymmetry's direction are able to create a force (analogous to Lorentz's force), which can be a physical source of rotation of the geometrical coordinate system in the space consolidated model.

This means that in the superimposing processes, the asymmetry is a capable to control a system's geometry and change the macrodynamics by rotating the dynamic operator (Fig 9.1). Both the rotating symmetry and its inner rotating mechanism are specific characteristics of macromolecule polymers [31].

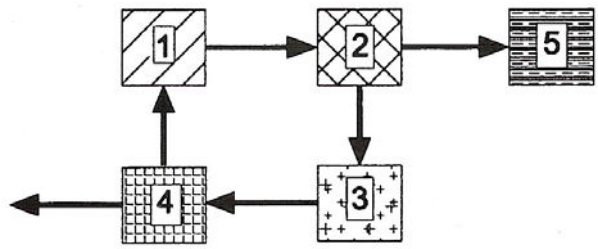


Figure 9.1. Operations with superimposing processes: 1-Asymmetry, 2-Rotation, 3-Diagonalization of dynamic operator, 4-Dynamics, 5-Geometry.

A set of the macromodel triplets create a specific shape of corresponding key-lock connections according of their LRL-RLR-… asymmetry sequence, which could be used for the macromodel recognition and creation of a communicating information language (initiated by such a sequence) (part 2).

IMD introduces a common information description for both *superimposition and controls* by revealing the *information mechanism of the cross phenomena*.

In particular, the IN cooperative nodes model the integral phenomena of such superpositions as thermodiffusion, electrokinetic, electrochemical cross interactions, others.

The process' superimposition is a source of the IN code.

The controls, *generated* by this code, in turn, might initiate these processes, which can lead to a self-organization system.

*Nature produces a conjunction of natural interactive processes, while the IMD models this by the functions of the inner control and the INs***.**

Physically, the discrete intervals are the distances between the diffusion barriers, generated by microlevel at the DP δ-localities.

The points of compensation diffusion and kinetics (sec.1.7.3) hold the chain connection. The interacting processes form the spiral space chain of macrostructures based on their subsequent dynamic consolidation.

The coefficients of kinetics $\bar{l}_i$, diffusion b_i, and correlations r are connected according to the equations of chs.1.3,1.7 and can be expressed through the model invariant $\mathbf{a}(\gamma)$.

The information analogies of thermodynamic forces X and mass are considered in sec. 1.7.6.

Let us analyze the connection between the object's observed physical states and the modeled information states and operators.

Assume we observe the physical states $\hat{x}_j, j = 0,1,...,n$, determined by the object's thermodynamic processes. These states are connected with the information states $x_i, i = 1,...,n$ using relations [20]:

$$\hat{x}_o = -\theta^{-1}, \hat{x}_i = x_i\theta^{-1}, \text{‘} \tag{9.21}$$

where θ is a thermodynamic temperature.

The kinetic equation for the physical states acquires the form

$$\dot{\hat{x}}_i = \hat{A}_i\hat{x}_i, \hat{A}_i = (\hat{\lambda}_i)_{i=1}^n , \tag{9.22}$$

where each eigenvalue $\hat{\lambda}_i$ can be identified applying the considered correlation relations:

$$\hat{\lambda}_i = 1/2\dot{\hat{r}}_i\hat{r}_i^{-1}, \hat{r}_i = E[\tilde{\hat{x}}_i^2], \tag{9.23}$$

which is determined by the controlled random $\tilde{\hat{x}}_i$.

Let us find the connection between the eigenvalues (9.23) and ones in the information form of (9.22):

$$\dot{x}_i = A_i x_i, A_i = (\lambda_i)_{i=1}^n \tag{9.24}$$

using for λ_i the analogous relations $\lambda_i = 1/2\dot{r}_i r_i^{-1}, r_i = E[\tilde{\hat{x}}_i]$.

Substituting (9.21) into (9.23) and using (9.24) we get

$$\hat{\lambda}_i = \lambda_i - \theta^{-1} d\theta / dt \,. \tag{9.25}$$

Integrating both sides of (9.25) at the corresponding segment's time interval t_i we receive

$$\ln \theta(t_i) + C_i = \lambda_i t_i - \hat{\lambda}_i t_i \,, \tag{9.26}$$

at $\lambda_i = \alpha_i + j\beta_i$, $\hat{\lambda}_i = \hat{\alpha}_i + j\hat{\beta}_i$ and $\hat{\beta}_i = \beta_i$.

For a real temperature, relation (9.26) acquires view

$$\ln \theta(t_i) + C_i = a_i t_i - \hat{\alpha}_i t_i \,, \text{ where } \alpha_i t_i = \mathbf{a}(\gamma) \,.$$

It is seen that temperature at each t_i becomes a function of the identified $\hat{\alpha}_i$ and parameter γ.

Indeed. Using the invariant relation $\hat{\beta}_i t_i = \beta_i t_i = \mathbf{b}(\gamma)$ at the known $\hat{\beta}_i = \beta_i$ and t_i, we find $\mathbf{b}$ and then, applying the function $\mathbf{b}(\gamma)$ (ch.1.5), we get the corresponding γ that allow finding $\mathbf{a}(\gamma)$.

After that the temperature can be found by formula

$$\ln \theta(t_i) + C = \mathbf{a}(\gamma) - \hat{\alpha}_i t_i \,, \tag{9.27}$$

at the known $\mathbf{a}(\gamma)$ and $\hat{\alpha}_i, t_i$.

In the case of unknown t_i, it can be found from a joint solution of equations (9.27) and

$$t_i = \mathbf{a}(\gamma) \; (\hat{\alpha}_i + \theta^{-1} d\theta / dt)^{-1} \,, \tag{9.27a}$$

following (9.25) at $t_i = \mathbf{a}(\gamma) / \alpha_i$, which leads, in particular, to the solution of

$$\int y dt = \mathbf{a}(\gamma) \; y(\hat{\alpha}_i + y)^{-1} \,, y = \theta^{-1} d\theta / dt \,. \tag{9.27b}$$

Knowing θ and the controlled $\hat{x}_i$ allow finding the information states x_i and apply all IMD results.

For the controllable physical states and the identified object's operator, some thermodynamic relations and functions can be available.

The inner energy U can be found using the Hamiltonian, determined by the observed physical states: $\hat{\mathrm{H}} = Tr[\hat{A}(\hat{x}, t)]$:

$$U = \int \hat{\mathrm{H}} dt \,.$$

These relations allow us to identify a thermodynamic potential

$$\Gamma = U\theta^{-1} - S\,,$$

or a free energy

$$\Phi = U - \theta S\,,$$

which leads to finding the main thermodynamic functions including a pressure

$$\mathrm{P} = -\partial\Phi / \partial V$$

at a given volume V or its model's relations (chs.1.5, 1.7).

It is important to point out that a notion of a local equilibrium temperature θ_{ik} in our model can be useful only to the cooperated states (i,k), satisfying condition

$$\partial U_i / \partial S_i = \partial U_k / \partial S_k = \theta_{ik}\,. \tag{9.28}$$

where the cooperation occurs at $t_{ik} = o(\tau)$.

Applying the above relations to the model:

$$\partial U_i = \hat{\lambda}_i t_i,\ \partial S_i = \mathbf{a}_o(\gamma_i) - \mathbf{a}(\gamma_i), \partial U_k = \hat{\lambda}_k t_k\,, \partial S_k = \mathbf{a}_o(\gamma_k) - \mathbf{a}(\gamma_k),$$

we get accordingly

$$\theta_i = (\mathbf{a}(\gamma_i) - t_i\theta_i^{-1}d\theta_i)((\mathbf{a}_o(\gamma_i) - \mathbf{a}(\gamma_i))^{-1}; \tag{9.29a}$$

$$\theta_k = (\mathbf{a}(\gamma_k) - t_k\theta_k^{-1}d\theta_k / dt)((\mathbf{a}_o(\gamma_k) - \mathbf{a}(\gamma_k))^{-1}. \tag{9.29b}$$

At the moment of cooperation we have

$$\theta_i(t_{ik}) = \theta_k(t_{ik}) = \theta_{ik}(t_{ik})\,, \tag{9.30a}$$

and the condition of a local equilibrium leads to

$$d\theta_i / dt(t_{ik}) = d\theta_k / dt(t_{ik}) = 0\ . \tag{9.30b}$$

Therefore, the condition (9.28) requires just $\gamma_i = \gamma_k$, which is fulfilled for a given IN.

This means that all nodes of a given physical IN (where (9.28), (9.30a,b) hold true) keep the same thermodynamic temperature.

The IN models the superimposition of macroprocesses with revealing of the created phenomena and singularities.

1.9.5. About the Notion of a System

Let us define a system as a *set of interactions exchanging* their *common substance*, e.g., energy, material, information, etc.

The term "interaction" includes an *acceptance and generation* of this substance for the sets.

The term *set* is defined in a mathematical meaning as a *collection* of the interactive elements.

The entropy functional (sec.1.1.1) integrates the sets' information measures into the common *information* measure, bringing a *unity* of the *information* interactions, which constitutes the notion of a *system*.

Thus, this formalism defines a *system* by a set of interactions exchanging *information* as the *common* substance.

This substance carries *uncertainty* of different forms of multiple physical and/or virtual interactions having universal information measure.

Uncovering a system's regularities by minimizing its uncertainty is formalized by a variation principle which, by imposing the constraint (ch.1.3), provides the systems' dynamic model.

Actually, a formal preservation of a process' invariant measure leads to a variation principle [33, 34], which for an open dynamic system requires information exchanges–interactions and brings a capability for the information integrations (ch.1.5).

Universality of the probability measure and its information form (sec.1.1.1) makes *information* the *basic* substance for a diverse nature of interactions.

The information, unifying the interaction's description, *integrates* them into some *primary* information units-subsystems and then binds them into a system.

The essence of information interactions is concretized in and in ch.1.3, where the exchanges between the system's external and internal entropies occur at a set of the macrostates' discrete points (DP)(sec.1.3.5), which *determine* the set of interactions, where also the state's cooperation takes place. In these exchanges-interactions, the internal entropy, being a source of the information dynamics, is also spent on building the *structure* of a cooperative hierarchical IN, which contains the process' *genetic* information (ch.1.6).

The external information, delivered at the DP, compensates the total internal information.

The IN represents a formal connection of its element into a system, assembled by the mathematical equations, following from a single variation principle, as a distinction of existing systemic approaches [37], where such connections are usually formed artificially by different combinations of postulates, and/or, intuitive, arbitrary, and even rational concepts.

A system *structure* we specify by the IN, with an *admissible range* of γ; while each IN with a *fixed* and different γ describes the structure of a subsystem.

Then the considered IN's invariants (depending on γ) determine the parameters of any particular subsystem's triplet and its complexity.

The subsystems' set depends on the collection of γ (bound by Prop.7.4, ch.1.7)), whose admissible values $\gamma \to (0.0 - 0.8)$ follow from the model's simulation [35] and the condition of a subsystems' *stability*, consistent with the preservation of the total model's entropy *S*.

A collection of such subsystems, which keeps the total *S* constant, forms a *system*, where *S* consists of two parts: imaginary *I* and real information *R*.

For example, a source of information (TV, radio, others) generates information waves; an observer accepts only its part, which is a *real information for* the observer, resulting from the observer's interaction and entanglement with the wave.

Other part of information, which actually exists, but is unavailable for the observer, constitutes *an imaginary information for the observer*.

In the considered macromodel, the relation between the imaginary and real information is determined by the model's parameter γ evaluating a ratio of the imaginary and real parts of eigenvectors (eigenvalues) in the form $\gamma = I/R$, and we get $S=(1+\gamma)R$, or $\gamma = S/R - 1$.

At $\gamma = 0.8$, $S/R = 1.8$, and at $\gamma = 0$, we come to $S/R = 1$.

Thus, the R/S range is between 1 and 0.555, and the *I/S* range is between 0 and 0.445.

At the minimal feasible $\gamma = 0.0072$, the practical rage is narrower: $R/S = 0.9928$ to 0.555, and $I/S = 0.0072$ to 0.445. (In sec. 1.9.3 we consider one of the invariant transformations of the model's imaginary eigenvalues to the real eigenvalue).

The number of diverse subsystems within a single system (distinguished by the γ of a starting eigenvalue's sequence) is determined by the ratio $(0.8 - 0.0072)/0.0072 = 110.111\ldots$, while each subsystem, having the same fixed γ, might consist of a maximal element's number $n = 90$.

This means that each system may include any of 110^{90} different combinations of those 90 subsystems representing the maximal number of diverse systems, having the distinct information characteristics and are capable of producing specific codes.

The complexity arises by joining of the above sets and subsystems into a system, becoming an integral system's characteristic.

An individual macrosystem's complexity, measured by its minimal algorithm' program, encoded in the IN code, allows the systems' classification by the level of complexity.

The model's controls, as a carrier of the subsystems' integration and the code, are also a source and a part of the system's complex dynamics.

The IPF-IMD notions of a system, system structure, complexity, and the code establish a foundation of *information systems science* with the related computer-based methodology, software, and applications (part 2).

PART 2.
THE INFORMATION PATH FUNCTIONAL'S AND IMD'S APPLICATIONS

Chapter 2.1

SOLUTION OF THE CONTROL PROBLEMS FOR A COMPLEX OBJECT

2.1.1. The Control Problems for a Complex Object

The complex objects with a wide diversity of interactions and the superimposing processes of distinct nature are extensively studied in theory of dynamic and stochastic systems, control and systems theories, theory of information, and physics.

Multiple interactions create uncertainty and randomness, associated with the object's probabilistic description, and/or with the *information* necessary for the object measurement and identification.

Modeling of the complex interactions still remains an actual problem, which involves identification the random processes by a corresponding dynamic model.

The object optimization problem includes an optimal control of a stochastic system [1-5, other] with a selection (filtering) of a suitable (needed) dynamic signal by a control, minimizing a random noise. (Please see the references to ch.2.1).

For the complex objects, a practical interest represents the model restoration during a *joint* solution of both the identification and the optimal control problems (in a real-time), allowing to obtain a *current* optimal model of the observed process, which could have been changed during observation.

Since the remarkable publications [6-10], many years passed but *significant new* results in this area have not been achieved.

A *complete solution of this problem is unknown*.

The conventional methods for the model's identification by an object's experimental data [11-13, other] do not use the *concurrent* identification, combined with optimal control.

The IMD with the information path functional (IPF), applied for a *dynamic modeling* of a variety of *information* interactions, integrates their multiple *information* interactions.

The IMD, instead of modeling each particular random object (by the known methods), builds a *class* of information *dynamic models* with a *class of synthesized control* functions (applied to a wide class of random objects), and then specifies both of them to each particular

random object. This allows avoiding some crucial obstacles, complicating the solution of the dual control [9] and other related problems.

Building of such a common model requires, first, using a broad-spectrum class of models for the *initial controllable* random object, secondary, developing a mathematical formalism and methodology, being applicable to this class of models, to produce a wide-ranging dynamic model, reflecting the *regularities* of this class of objects.

The initial control problem (ch.1.1,1.3) of reaching a maximal closeness of a controllable random process with a given programming process is performed via the approximation of the random process by a macroprocess with a minimum uncertainty, using the IPF minimum as a *performance criterion*.

Applying the IPF mathematical formalism (part1), we focus this chapter on the IPF *applied methodology and the procedure* for the solution of the above control problems illustrated by *detailed examples.*

The methodology and procedure have been implemented in the developed package of the computer programs.

The *specifics* of the obtained *solutions* are the following:

- the initial random process is approximated with a *minimal* functional uncertainty (maximal probability on the *trajectories*) by the IPF extremal segments;
- between the extremal segments exists a "*window*", where the random information affects the dynamic model's operator, creating its piece-wise *dependency* upon the observed data;
- the synthesized optimal controls start at the beginning of each segment, act along the segment, and connect the segments in the macrodynamic optimal process;
- the discrete interval of the applied controls, associated with the segment's length (which is defined by the VP invariants), are found in the process of identification of the segment's operator.

These specifics allow *proceeding the identification* of the dynamic operator at each extremal segment in *real-time under* the optimal control action and during the object's current motion. The optimal controls, connecting a set of the extremal segments for a *multi-dimensional object*, are able also to solve the problem of the object's *state consolidation and aggregation* with building a *cooperative hierarchical information network* (IN) of a complex object. Moreover, this procedure can proceed concurrently during the object optimal motion, combined with optimal control and the operator identification.

A complex object is characterized by a diverse nature of different superimposing processes, creating the cross-phenomena (thermo-diffusion, electro-kinetics, and many others). Modeling of such objects presents a very complicated problem, studied in irreversible thermodynamics, where it can be solved only for simple examples.

In the above dynamic model, these cross-phenomena arise between the extremal segments, when the model dynamic operator is renovated at the beginning of each following segment. This allows us to identify those phenomena concurrently during the procedure of the operator restoration.

The IPF connection to information theory brings both a *universal information* structure of the dynamic macromodel and a *common information language* (including the model's generated *code*) for description of the complex object's regularities in distinct

interdisciplinary objects with a *diverse nature* of superimposing processes in a variety of complex objects.

The above outline shows that the IPF solutions of the considered control problems differ essentially from ones used in the control and identification of the stochastic and deterministic systems (including all cited references). That is why our references to the related contributions are limited.

Actually, the IPF presents an *information analogy* of Feynman's path functional, applied in quantum mechanics, with the references [ch.1.1-1.9], which are not related to control theory. Below we analyze more specifically the comparison of the IPF approach with the known contributions to the considered control *problems*.

The IPF dynamic model, identification equations, and optimal control follow directly from the *analytical* solution of the *single* variation problem for the *considered class* of the random objects and are *not introduced separately* as the explicit equations, or specific problems (as it is widely accepted).

The *applied* optimal control and the *correctly restored* (by using the identification equations) dynamic operator for *each segment* are fit precisely to their analytical solutions and do *not* require the *sequential step-wise approximations*. The procedure of the operator restoration does not involve any *iteration* if the needed correlations (for the identification equations) are measured and computed correctly (by well known methods).

The known methods of joint identification and optimization (generalized, particularly, in [14], others) include first identification and then finding the optimal control, which does not allow obtaining a *current* optimal model of an observed object.

Most of these methods apply a *special* control for the object *identification* and *then* form *another* optimal control *after* the solution of identification problem, using the statistical *estimations* and sequential *iterations*. The IPF approach is not connected to Kalman's filtration [10] that deals with the parameters' *estimation* in *adaptive* control.

Compared to the Feldbaum [9] dual control, the IPF approach does not contain a *sequential improvement* of both observation and controls. (Both the applied optimal control and the correctly restored dynamic operator for each segment are identical to their analytical solutions).

The known entropy and probability methods (including the related extremal problems), applied in control systems [15], traditionally evaluate an object *process* by the probability measure for the random *states* and corresponding Shannon's entropy measure as the uncertainty *function* of the states.

The *process*' IPF integral information measure evaluates a process' *trajectory* by an *integral* probability, defined on the random *trajectories*. That is why the IPF approach and methodology differ from the Kalman adaptive control, the Feldbaum dual control, the adaptive design [25,26], and other related contributions, even thought a final target for some of them appears the same. Because the IPF is defined by started functions of shift and diffusion, the IPF optimum predicts each extremal's segments movement not only in terms of a total functional path goal, but also by setting at each following segment the renovated values of these functions, identified during the optimal movement, which currently correct this goal. The concurrently synthesized optimal control provides each action with a maximal Markov's probability at each segment. This optimal dual strategy at each current movement cannot be improved (even theoretically) because it defined by an extremal of a *total path* to a terminal state, which is updated at each optimal control's action.

The IPF, as a functional measure of a priory given performance criterion, can be applied to any specific performance criterion [28, others].

The chapter organizes as the flowing.

In secs.2.1.2.1, 2.1.2.2 we introduce the methodology and the examples solving the identification problem for both concentrated and distributed objects. In sec.2.1.3 we solve the above control problems, focusing on the joint solution of the optimal control and identification problems (secs.2.1.3.1, 2.1.3.2), *and* the solution's methodology *with* the procedure including the consolidation problem, which is finalized by building the object's cooperative information network (in ch.2.1.3.3). Both methodology and procedure are supported by the examples, which also demonstrate the concurrent object's identification under the optimal control actions. The IPF applied equations and the theoretical results with the detailed proofs can be found in part1. The examples, implementing the procedure, formalize the practical applications and illustrate both IPF approach and the methodology.

2.1.2. Solving the Identification Problem

2.1.2.1. The Identification of the Concentrated Object's Models

Let us consider the problem of restoration of operator of the random process that is defined by the differential equation of a homogenous system:

$$\dot{x} = A\ x + \tilde{g},\ A = A\,(t',x), \tag{1.1}$$

where $\tilde{g}$ is a known function of time, depending on initial conditions of the state vector $x = x(0)$, with its probability density function $p = p(\tilde{x}(0))$.

If equation (1.1) is obtained, for example, by averaging a stochastic differential equation, then $\tilde{g}$ is defined by the moments of random process $\tilde{x}(t)$.

We will show that matrix A can be expressed through the moments of vector x.

Writing the solution (1.1) in the form:

$$x = x(0) + \int_0^t \exp(-A\tau)\tilde{g}(\tau)d\tau,$$

we get the covariation matrix r of vector x, defined on this solution:

$$r = E(xx^T)\ r = E(xx^T) = E\{\exp At[x(0) + \int_0^t \exp(-A\tau)\tilde{g}(\tau)d\tau]$$

$$\times[x^T(0) + \int_0^t \tilde{g}^T(\tau)\exp(-A^T\tau)d\tau]\exp(-A^T t)\}\ .$$

and its derivative

$$\dot{r} = Ar + rA^T + k + k^T,\ k = E[x(t)\tilde{g}^T(t)],\ k^T = E[x^T(t)\tilde{g}(t)]. \tag{1.1a}$$

At the fulfillment of the symmetry condition

$$Ar = rA^T \tag{1.2}$$

we get the relation

$$A = 1/2(\dot{r} - k - k^T)r^{-1}.$$

In particular, for a conservative model with $\tilde{g}(t) = 0$, we have

$$A = 1/2\dot{r}r^{-1} = R. \tag{1.3}$$

Because matrix A is a nonrandom, it can be taken out of the mathematical expectation's operation according to the equality

$$R = E[\dot{x}x^T] = E[x\dot{x}^T] = AE[xx^T]\{E[xx^T]\}^{-1} = A.$$

The identification of stationary and nonstationary objects is performed by analogy.

Considering $x = x^1, \dot{x} = x^2$, $x^1 = (x_1, \ldots x_n)$, $x^2 = (\dot{x}_1, \ldots \dot{x}_n)$ as an object's input variable, we come to the identification equations

$$x^2 = Rx^1, R(t,\tau) = r_{12}(t,\tau)r_{11}^{-1}(t,\tau),\ r_{12}(t,\tau) = E_{x^1(t,\tau)}[x^2(t)x^1(t)^T],$$

$$r_{11}(t,\tau) = E_{x^1(t,\tau)}[x^1(t)x^1(t)^T].$$

Form $r_{12}(t,\tau) = R(t)r_{11}(t,\tau)$ coincides with the equation for an optimal linear operator in the problem of optimal linear filtration.

If the random functions $x^1(t), x^2(t)$ are connected stationary, then this equation acquires the form

$$r_{12}(t,\tau) = R(t)r_{11}(t-\tau).$$

If $x^1(t)$ and $x^2(t)$ are connected by a differential equation, with a corresponding impulse transitive function $g\ (\tau) = g, g(-\tau) = 0$, then the following equations are satisfied:

$$x^2(t) = \int_0^\infty g^T(\tau)x^1(t-\tau)d\tau, r_{21}(\tau) = \int_0^\infty x^2(s-\tau)x^1(s)^T ds;$$

$$x^2(s-\tau) = \int_0^\infty g^T(v)x^1(s-t-\tau)dv,$$

$$r_{21}(\tau) = \int_0^\infty \int_0^\infty g^T(v)x^1(s-t-\tau)x^{1T}(s)dsdv, r_{21}(\tau) = \int_0^\infty g^T(v)r_{11}(t-v)dv.$$

The last one corresponds to well-known Wiener-Hoph's equation.

By substitution it into the initial identification equation we get

$$\int_0^\infty R(t,\tau) = \int_0^\infty g^T(v)r_{11}(t-v)dv \bullet [r_{11}^{-1}(t,\tau)],$$

where the weight-function satisfies to the Wiener-Hoph equation, as a condition of *optimal* filtration, based on minimization of the average mean square error.

Because of that, R is also the optimal operator for this problem.

This analysis shows the connections of the path fuctional's identification equations ch.1.3 with the above problems.

The condition of the operator's symmetry is fulfilled automatically for path functional extremal's differential operator satisfying the VP.

Example. The object at microlevel is described by equation

$$\ddot{\tilde{x}} = -(\alpha + \sigma\xi)\dot{\tilde{x}} + \sigma_1^2\xi_1^2\tilde{x},\ \alpha < 0.$$

A simple macroequation for mathematical expectations is

$$\ddot{x} = -\alpha\dot{x} - \sigma E[\xi\dot{\tilde{x}}] + \sigma_1^2 E[\xi_1^2\tilde{x}].$$

Using the indications $x_1 = x$, $\dot{x}_1 = ax_1 + x_2$, we write this equation in the form

$$\ddot{x} = a\dot{x}_1 + \dot{x}_2 = -\alpha(ax_1 + x_2) - \sigma E[\xi\dot{\tilde{x}}] + \sigma_1^2 E[\xi_1^2\tilde{x}],$$
$$\dot{x}_2 = -\alpha(ax_1 + x_2) - \sigma E[\xi\dot{\tilde{x}}] + \sigma_1^2 E[\xi_1^2\tilde{x}] - a(ax_1 + x_2).$$

We obtain the system

$$\dot{x}_1 = ax_1 + x_2,\ \dot{x}_2 = -a(\alpha + a)x_1 + (\alpha + a)x_2 - \sigma E[\xi\dot{\tilde{x}}] + \sigma_1^2 E[\xi_1^2\tilde{x}],$$

which corresponds to the initial equation (1.1) at

$$\tilde{g}_1 = 0,\ \tilde{g}_2 = -\sigma E[\xi\dot{\tilde{x}}] + \sigma_1^2 E[\xi_1^2\tilde{x}].$$

We can chose parameter a to get the symmetrical matrix A in the form

$$A = \begin{Vmatrix} -a, 1 \\ 1, a \end{Vmatrix},\ \text{or } a(\alpha + a) = 1,\ a = -\alpha/2 \pm (\alpha^2/4 - 1)^{1/2}.$$

In this case, the condition (1.2) is satisfied and we can use it for the identification of equation (1.3).

If equation (1.1) is a nonlinear, for example because of $\tilde{g} = \tilde{g}(x)$, then the correlation matrix k in (1.1a) depends on the parameters of the function $\tilde{g}(x)$, and for the solution we first, express (1.1) through the equation for mathematical expectations $m = E[x]$:

$$\dot{m} = Am + E[\tilde{g}(x,t)]. \tag{1.4}$$

Secondly, we substitute (1.1) into (1.4):

$$\dot{m} = 1/2\dot{r} + E[\tilde{g}(x,t)] - E[x\tilde{g}^T(x,t)] - E[\tilde{g}(x,t)x^T], \tag{1.5}$$

and then solve the algebraic equation with respect to the above parameters of function $\tilde{g}(x)$. Using a serial representation of this function, we may specify the coefficient of decomposition via the moments of x-vector.

Example. Let $\tilde{g}(x) = xx^T B$, then

$$E[\tilde{g}(x)] = rB, k = M[xB^T xx^T],$$

and by substitution of above equations into (1.5) we obtain the linear equation with respect to the components of vector B.

For a second order object's equation we have

$$\tilde{g}(x)=\left\|\begin{matrix} b_1x_1^2+b_2x_1x_2 \\ b_1x_1x_2+b_2x_2^2 \end{matrix}\right\|, E[\tilde{g}(x)]=\left\|\begin{matrix} b_1r_{11}+b_2r_{12} \\ b_1r_{12}+b_2r_{22} \end{matrix}\right\|,$$

$$k=\left\|\begin{matrix} b_1E[x_1^3]+b_2E[x_1^2x_2], b_1E[x_1^2x_2]+b_2E[x_1x_2^2] \\ b_1E[x_1^2x_2]+b_2E[x_1x_2^2], b_1E[x_1x_2^2]+b_2E[x_2^3] \end{matrix}\right\|. \quad (1.6)$$

Example. The identification of the nonlinear object's model:

$$\dot{x}_i=a_i(x_j)(x_i+v_i),\ i,j=1,..,n,$$

where v_i is a control, and the function $a_i(x_j)$ can be an essential nonlinear; for this example, in the form of a jump-vise changing operator

$$a_i(x_j)=\left\{\begin{matrix} \varphi_i(x_j) \\ a_isign[x_j(t)-x_j(\tau)] \\ \phi_i(x_j) \end{matrix}\right., \left\{\begin{matrix} \alpha_2<[x_j(t)-x_j(\tau)]<\alpha_1 \\ \alpha_1<[x_j(t)-x_j(\tau)]<\alpha_o \\ \alpha_o<[x_j(t)-x_j(\tau)]<\alpha_3 \end{matrix}\right\}. \quad (1.7)$$

In this case, the identification of unknown operator is possible at the moments of decoupling of the time correlations at DP$\{\tau_i\}$ when the control executes the condition

$$\dot{r}(\tau_i)=2E_{v_i}[a_i(x_j(\tau_i))(x_j(\tau_i)+v_i)^2]=2a_i(x_j(\tau_i))E_{v_i}[(x_j(\tau_i)+v_i)^2] \quad (1.8)$$

and the extraction of unknown operator from operation of conditional mathematical expectation takes place. Indeed. According to (1.2,1.8) we get the sought operator:

$$R(\tau_i)=1/2\dot{r}(\tau_i)r^{-1}(\tau_i)=a_i(x_j(\tau_i))E_{v_i}[(x_j(\tau_i)+v_i)^2]$$
$$\times\{E_{v_i}[(x_j(\tau_i)+v_i)^2]\}^{-1}=a_i(x_j(\tau_i)). \quad (1.9)$$

Example. Let the object model at microlevel is described by the equation

$$\dddot{\tilde{x}}+\alpha\ddot{\tilde{x}}+\alpha\beta\tilde{x}\dot{\tilde{x}}+c\tilde{x}+\tilde{u}=0,$$

with the random parameters α,β,c, where α,c have the continuous uniform distributions with the means α_o ,c_o accordingly.

Writing the model in a system form, we get

$$\dot{\tilde{x}}_1=\tilde{x}_2,\ \dot{\tilde{x}}_2-c\tilde{x}_1-\alpha\tilde{x}_2-\alpha\beta\tilde{x}_1(\tilde{x}_2+\tilde{v}),\ \alpha\beta\tilde{x}_1\tilde{v}=\tilde{u}. \quad (1.10)$$

The control $\tilde{v}$ is chosen from the condition of decoupling the correlation:
$E[\alpha\beta\tilde{x}_1(\tilde{x}_2+\tilde{v})^2]=\beta E[\alpha]M[\tilde{x}_1]M[(\tilde{x}_2+\tilde{v})^2]=\alpha_o\beta E[\tilde{x}_1]M[(\tilde{x}_2+\tilde{v})^2]$, (1.11)

which can be fulfilled at the control jump at DP.

Then the matrix $R(\tau_i)$ identifies the above operator.

The macromodel of the microlevel equations:

$$\dot{x}_1 = x_2 \frac{x_1}{x_1}, x_1 = E[\tilde{x}_1], x_2 = E[\tilde{x}_2],\ \dot{x}_2 = -(c_o \frac{x_1}{x_2} + \alpha_o \beta\ x_1 + \alpha_o),$$

$$u = -(c_o \frac{x_1}{x_2}\ v + \alpha_o \beta\ x_1\ v + \alpha_o\ v), \tag{1.12}$$

is identified by the equations

$$\left\| \begin{matrix} \dot{x}_1 \\ \dot{x}_2 \end{matrix} \right\| = \left\| \begin{matrix} R_{11}, 0 \\ 0, R_{22} \end{matrix} \right\|,\ R_{11} = \frac{\dot{x}_1}{x_1} = \frac{x_2}{x_1}, R_{22} = \frac{\dot{x}_2(\tau)}{x_2(\tau) + v} = -(c_o \frac{x_1}{x_2} + \alpha_o \beta\ x_1(\tau) + \alpha_o). \tag{1.13}$$

2.1.2.2. The Identification of the Space Distributed Object's Models

Let us consider the object with a matrix differential equation

$$\frac{\partial x}{\partial t} = A \frac{\partial x}{\partial l}; A = A(x,t,l), \tag{1.14}$$

where x is vector of the state coordinates, l is a vector of the space coordinates.

This form (ch.1.4) is applied to some equations of mathematical physics, such as the diffusion, heat transfer, wave functions, Laplace's and Helmgoltz's equations.

The identification problem consists of the restoration of the operator by observation of the equation's solutions (processes) in the form

$$x(t,l) = T_1 T_2 x_0;\ T_1 = T_1(t,l_0), T_2 = T_2(t_0,l) \tag{1.15}$$

with the random initial conditions $x_o = x(t_o, l_o)$, and a given probability density $p(x_o)=p_o$. The boundary conditions $x = x(t_o, l) = x(l)$ for the observed process, we assume, are included naturally into equation (1.15).

At first, we determine the operator of the ordinary differential equation

$$\frac{\partial x}{\partial t} = A_1\ x\ ,\ A_1 = A_1(x,t,l), \tag{1.16}$$

which according to equation (1.15), takes the form

$$\frac{\partial x}{\partial t} = T_1' T_1^{-1}\ T_1 T_2\ x_0\ ,\ T_1' T_1^{-1} = A_1\ ,\ T_1' = \frac{\partial}{\partial t}\ T_1. \tag{1.17}$$

Using for the identification of equation (1.16) the relations

$$R_1 = E[x\, x^*]^{-1} E[x(\frac{\partial x}{\partial t})^*],$$

$$E[x\, x^*] = E[T_1 T_2\, x_0\, x_0^*\, (T_1 T_2)^*] = T_1 T_2 E[x_0\, x_0^*](T_1 T_2)^* \tag{1.18}$$

with the symmetry conditions

$$E[x(\frac{\partial x}{\partial t})^*] = E[(\frac{\partial x}{\partial t})x^*],\ E[x(\frac{\partial x}{\partial l})^*] = E[(\frac{\partial x}{\partial l})x^*]$$

and the solutions (1.15), we come to equations

$$E[x(\frac{\partial x}{\partial t})^*] = E[(T_1 T_2\, x_0\, x_0^*\, T_2^*\, T_1^*\, T_1^*\, (T_1^*)^{-1}\, T_1^*'].$$

The nonrandom functions can be taken out of the operation of mathematical expectation.

We get the equality $R_1 = A_1$ from relation

$$E[x(\frac{\partial x}{\partial t})^*] = T_1 T_2 E[x_0\, x_0^*](T_1 T_2)^* (T_1' T_1^{-1})^* = T_1' T_1^{-1} E[x\, x^*]. \tag{1.19}$$

Let us determine the operator of the equation

$$\frac{\partial x}{\partial l} = A_2\, x;\ A_2 = A_2(l, t, x) \tag{1.20}$$

that can be written in the form

$$\frac{\partial x}{\partial l} = T_2 T_2^{-1}\, T_2 T_1^*\, x_0^*,\ T_2 T_2^{-1} = A_2,\ T_2' = \frac{\partial}{\partial t} T_2\ . \tag{1.21}$$

By the substitution the solution of equation (1.15) into relation

$$R_2 = E[x\, x^*]^{-1} E[x(\frac{\partial x}{\partial l})^*], \tag{1.22}$$

we obtain

$$E[x\, x^*] = T_1 T_2 E[x_0\, x_0^*](T_1 T_2)^*,$$

and

$$E[x(\frac{\partial x}{\partial l})^*] = T_2' T_2^{-1} E[x\, x^*],\ R_2 = T_2' T_2^{-1} = A_2. \tag{1.23}$$

After substituting (1.16), (1.20) to the initial equation, we come to equality $A_1 x = A A_2 x$, which has to be satisfied for all nonzero x in (1.15),(1.17),(1.19),(1.21),(1.23).

Writing this equality in the form $A_1 x\, x^* = A A_2 x\, x^*$ and applying the math expectations, we determine the unknown operator by the formula $A = A_1 A_2^{-1}$.

Operator A_2^{-1} can be identified directly using relation

$$R_2^{-1}=E[x\,x^*]\{E[x(\frac{\partial x}{\partial l})^*]\}^{-1}.$$

If the operators depend on the state coordinates, then the extraction of the considered nonlinear functions follows from the condition of decoupling of the time-space correlations, for example, by applying the controls $u(t,l)=u(t,l_o)u(t_o,l)$ to the object.

For this purpose we write the equation (1.16) in the form

$$\frac{\partial x}{\partial t}=A_1(x+v_1),$$

where $A_1v_1=u(t,l_o)$ is the time dependable control, applied at the space point l_o of the space distribution; $v_1=v_1(t,l_o)$ is the control, reduced to the state coordinates $x(t,l_o)$.

By analogy, we write equation

$$\frac{\partial x}{\partial l}=A_2(x+v_2),$$

where $A_2v_2=u(t_o,l)$ is the control, applied at the moment t_o to the space distribution; $v_2=v_2(t_o,l)$ is the control, reduced to the state coordinates $x(t_o,l)$.

Operators (1.19), (1.23) are identified during the process of optimal movement, at first, by applying the optimal control $u(t,l_o)$ at the fixed point (l_o), and then by the distribution of the controls as the function of l.

Such an identification and the optimal control are combined in the time and space.
If the increments of the *space* coordinates become zeros, then we get a concentrated model.
If the increments of the *time* state coordinates become zeros, then we get a static model.
On the contrary, we will continue the identification procedure until we obtain the distributed model in the form (1.20).

Some of the mass', heat transfer's and chemical kinetic's processes are described by the integral-differential equations:

$$\frac{\partial x(l)}{\partial t}=\int_{l'}A(l,l')x(l')dl'. \tag{1.24}$$

Such a form can also be reduced to the equations in partial derivations:

$$\frac{\partial x}{\partial t}=divL\nabla X \tag{1.25}$$

with $x=x(t,l)$, $X=X(t,l)$ as the conjugate vectors and $L(x,l,t)$ as a matrix function of the kinetic coefficients.
At $X=h\,x$ (with *h* as an inverse covariation function), the equation (1.25) acquires the form

$$\frac{\partial x}{\partial t}=\nabla L\nabla hx, \tag{1.26}$$

where ∇ is a Hamilton operator in the equation for a gradient and divergence:

$$\nabla X=\text{grad}\,X,\ \nabla L=div\,L.$$

In a particular, the equation (1.25) at constant $L(l)$, $h(l)$ leads to a diffusion equation (ch.1.4) with the operator

$$R=\frac{dr}{dt}r^{-1},\ r=E[x\ x^{*}].$$

The transformation of the equation (1.26) to form (1.24) is given in [16].

Following this transformation, we write operator (1.25) in the form

$$\int_{l'}\nabla L\nabla hx\delta(l-l')dl'=\int_{l'}Adl'=\nabla L\nabla h,$$

where $\delta(l\text{-}l')$ is a three-dimensional δ-function.

We will identify the integral operator assuming that the solutions (1.24) are observed:

$$x=T_1T_2\,x_0^{*};\ T_1=T_1(t,l_0),\ T_2=T_2(t_0,l),\ x_0=x(t_0,l_0). \tag{1.27}$$

Using (1.27) we write the equation

$$\frac{\partial x}{\partial t}=A_0\,T_1\,T_2\,x_0^{*},\ A_0=T_1'T_1^{-1}, \tag{1.28}$$

whose operator is identified by applying (1.18),(1.19). We obtain the equalities

$$R_0=A_0,\ R_0=E[x(l)x(l)^{*}]^{-1}E[\frac{\partial x}{\partial t}(l)x(l)^{*}].$$

Using (1.24) and (1.28), and integrating both equality's sides by l, we get

$$\int_{l}A_0xdl=\int_{l}[\int_{l'}Adl']x\,d\,l.$$

Integral equation $A_0=\int_{l'}Adl'$ defines the unknown operator.

For the space variable l' we get

$$A=-\,div\,A_0.$$

The symmetry condition for (1.24) leads to equation

$$\frac{\partial R_0(l,l',t)}{\partial l}=\frac{\partial R_0(l',l,t)}{\partial l'}.$$

Example. Let us consider the distributed one-dimensional object, described by the heat-transfer equation

$$\frac{\partial x}{\partial t}=A\frac{\partial^2 x}{\partial l^2},\ x=x_1,\ l=l_1,\ A=a^2=\text{Const} \tag{1.29}$$

at a given initial probability density and a known initial distribution $p(x_o) = p(x_o, l_o)$ along the line $x(t_o, l) = x(l)$, with the solution (1.29):

$$x(t,l) = x_o \exp(ikl - k^2 a^2 t), \tag{1.30}$$

where *k* is a constant determined by a boarder conditions.

The problem consists of the restoration of operator A.

We find the solution by two steps.

First, we identify the operator of the ordinary differential equation

$$\frac{\partial x}{\partial t} = A_1 x, A_1 = A_1(x,t,l)$$

by relation

$$R_1 = E[x(t,l)^2]^{-1} E[\frac{\partial x}{\partial t}(t,l)x(t,l)]. \tag{1.31}$$

By substituting the solution (1.30) into (1.31) we obtain

$$R_1 = -k^2 a^2 = A_1. \tag{1.32}$$

Then we identify the operator of equation $\frac{\partial^2 x}{\partial t^2} = A_2 x$ in its system form:

$$\frac{\partial x_1}{\partial t} = r_{12} x_2, \frac{\partial x_2}{\partial l} = r_{21} x_1, x = x_1, \frac{\partial x_1}{\partial t} r_{12}^{-1} = x_2. \tag{1.33}$$

At $r_{12} = r_{21}$, the symmetry condition for this operator:

$$R_2 = \left\| \begin{matrix} 0, r_{12} \\ r_{21}, 0 \end{matrix} \right\|, \frac{\partial^2 x}{\partial t^2} = r_{12} r_{21} x_1, A_2 = r_{12}^2,$$

is fulfilled.

From that we find the unknown operator of the initial equation

$$A = A_1 A_2^{-1} = a^2. \tag{1.34}$$

Writing the system (1.33) in the form

$$\frac{\partial x_1}{\partial t} = x_2, \frac{\partial x_2}{\partial l} = A_1 x_1,$$

we satisfy the symmetry condition at $A_2 = -k^2 = 1$, which leads us directly to (1.34).

Example. The distributed object is described by one-dimensional wave equation

$$\frac{\partial^2 x}{\partial t^2} = A \frac{\partial^2 x}{\partial l^2}, A = c^2 = const, \tag{1.35}$$

at the given initial conditions $x(t_o, l) = x(l)$, and the known initial probability density $p(x_o) = p(x(t_o, l_o))$.

The solution of the equation has the form

$$x(t,l) = x(t_o, l_o)\exp(\pm ikl \pm ickt)\,. \tag{1.36}$$

Let us determine operator R_1 of the system

$$r_1(l_2,l_1,t) = d_1, d_2 = \frac{\partial x_1}{\partial l} = a_{12}x_2, \frac{\partial x_2}{\partial l} = a_{21}x_1, R_1 = \begin{Vmatrix} 0, a_{12} \\ a_{21}, 0 \end{Vmatrix}, a_{12} = a_{21},$$

which we represent by the analogy with (1.33) in the form

$$\frac{\partial x_1}{\partial t} = r_{12}x_2\,,\ \frac{\partial x_2}{\partial l} = r_{21}x_1\,,\ R_2 = \begin{Vmatrix} 0, r_{12} \\ r_{21}, 0 \end{Vmatrix}\,,\ r_{12} = r_{21}.$$

Using the relations (1.20), (1.31) and the solutions (1.36), we get the solutions**:**

$$R_1 = \begin{Vmatrix} 0, \pm ick \\ \pm ick, 0 \end{Vmatrix},\ R_2 = \begin{Vmatrix} 0, \pm ik \\ \pm ik, 0 \end{Vmatrix}.$$

Returning to the initial form of both equations, we come to equations

$$\frac{\partial^2 x}{\partial t^2} = A_1 x\,, A_1 = a_{12}^2\ , \frac{\partial^2 x}{\partial l^2} = A_2 x\,, A_2 = r_{12}^2\ , A = A_1 A_2^{-1} = a_{12}^2 r_{12}^{-2} = c^2\,.$$

Example. The identification of an object, described by an integral-differential equation in the form

$$\frac{\partial x}{\partial t}(l_1,t) = \int_{l_2} A(l_1,l_2,t)x(l_2,t)dl_2;\ \frac{\partial x}{\partial t}(l_2,t) = \int_{l_1} A(l_1,l_2,t)x(l_1,t)dl_1 \tag{1.37}$$

at given initial probability distribution $p[x(l_1^o, l_2^o, 0)] = p_o$.

First, we identify the operators

$$1/2\int_{l_1}\int_{l_2} \dot{r}_1(l_1,t)r_1^{-1}(l_1,l_2,t)dl_2dl_1 = \int_{l_1} R_1 dl_1\,, 1/2\int_{l_2}\int_{l_1} \dot{r}_2(l_2,t)r_2^{-1}(l_1,l_2,t)dl_1dl_2 = \int_{l_2} R_2 dl_2\,,$$

using relations

$$r_1(l_2,l_1,t) = E[x(l_2,t)x^*(l_1,t)]\,,\ r_2(l_1,l_2,t) = E[x(l_1,t)x^*(l_2,t)]\,,$$

$$r_1\,(l_1,t) = E[x(l_1,t)x^*(l_1,t)]\,,\ r_2(l_2,t) = E[x(l_2,t)x^*(l_2,t)]$$

and the solution of system (1.37) in the form $x(l_1,t) = x_1\,,\ x(l_2,t) = x_2$.

At computation of the functions

$$\frac{\partial r_1}{\partial t}(l_1,t) = 2E[\frac{\partial x}{\partial t}(l_1,t)x^*(l_1,t)]\,,\ \frac{\partial r_2}{\partial t}(l_2,t) = 2E[\frac{\partial x}{\partial t}(l_2,t)x^*(l_2,t)]\,,$$

we use the initial object's equation and their solutions.

We have equation

$$\frac{\partial r_1}{\partial t}(l_1,t) = E\{[\int_{l_2} A(l_1,l_2,t)x(l_2,t)dl_2]x^*(l_1,t)\}. \quad (1.38)$$

Because $x(l_1,t)$ does not depend on l_2, we may input it under the integral sign. Then (1.38) acquires the form

$$\frac{\partial r_1}{\partial t}(l_1,t) = E\{[\int_{l_2} A(l_1,l_2,t)x(l_2,t)x^*(l_1,t)dl_2]\}. \quad (1.39)$$

Because $A(l_1,l_2,t)$ is a nonrandom function(as well as dl_2), we may write

$$\frac{\partial r_1}{\partial t}(l_1,t) = \int_{l_2} A(l_1,l_2,t)E[x(l_2,t)x^*(l_1,t)]dl_2, \quad (1.39a)$$

$$\int_{l_1} R_1 dl_1 = \int_{l_1}\int_{l_2} A_1(l_1,l_2,t)E[x(l_2,t)x^*(l_1,t)]E[x(l_2,t)x^*(l_1,t)]^{-1}dl_2dl_1$$
$$= \int_{l_1}\int_{l_2} A_1(l_1,l_2,t)dl_2dl_1,$$
$$\int_{l_2} R_2 dl_2 = \int_{l_2}\int_{l_1} A_2(l_1,l_2,t)E[x(l_1,t)x^*(l_2,t)]E[x(l_1,t)x^*(l_2,t)]^{-1}dl_1dl_2$$
$$= \int_{l_2}\int_{l_1} A_2(l_1,l_2,t)dl_1dl_2,$$

and we can get the sought operators from the relations

$$\frac{\partial R_1}{\partial l_2} = A_1(l_2,l_1,t), \frac{\partial R_2}{\partial l_1} = A_2(l_1,l_2,t).. \quad (1.40)$$

The symmetry conditions in the form

$$A_1(l_1,l_2,t) = A_2(l_1,l_2,t)$$

and the equations for

R_1, R_2

lead to the equalities

$$\frac{\partial R_1}{\partial l_2} = \frac{\partial R_2}{\partial l_1}, \frac{\partial[\int_{l_1}\dot{r}_2(l_2,t)r_2^{-1}(l_1,l_2,t)dl_1]}{\partial l_1} = \dot{r}_2(l_2,t)r_2^{-1}(l_1,l_2,t) = d_1$$
$$= d_2 = \dot{r}_1(l_1,t)r_1^{-1}(l_1,l_2,t) = \frac{\partial[\int_{l_2}\dot{r}_1(l_1,t)r_1^{-1}(l_1,l_2,t)dl_2]}{\partial l_2},$$

which determines the equalization of local operators d_1, d_2 at the DP, preceding their subsequent cooperation.

2.1.3. Solving the Optimal Control Problem

2.1.3.1. A Joint Solution of the Optimal Control and Identification Problems. The Basic Results

Solving this problem is based on results of ch.1.3. Specifically from theorems T.3.1 and T.4.1 follow that the DPs divide the macrotrajectory on a sequence of the extremals' segments, defined by the solutions of the model equations, and the regular (*step-wise*) controls are applied at each segments. These extremals provide a piece-wise approximation of the initial entropy functional with the aid of the controls.

Both regular and needle δ -controls, solving the VP, we call the *optimal* controls, which start at the beginning of each segment, act along the segment, and connect the segments in the macrodynamic optimal process.

The δ -control, acting between the moments (τ_{k-o}, τ_k) also performs a decoupling (a"decorrelation") of the pair correlations at *these* moments.

The reduced control presents a projection of control u_t on each of the state macrocoordinates, which is consistent with the object's controllability and identifiability [17, other]. This control *specifies* the structure of the controllable drift-vector $a^u = A(x+v)$ and the model

$$dx/dt = A(x+v) \tag{2.1}$$

dynamic operator, which is identifiable using the *identification equations* for the correlations functions, or the equation, connecting directly the operator

$$A(\tau) = 1/2b(\tau)(\int_{\tau-o}^{\tau} b(t)dt)^{-1}. \tag{2.1a}$$

with the dispersion matrix *b(t)*.

The control provides also the fulfillment of equality

$$|\frac{dx_i}{x_i dt}(\tau_k)| = |\frac{dx_j}{x_j dt}(\tau_k + o)|, x_i(\tau_k) \neq 0, x_j(\tau_k + o) \neq 0,\ i, j = 1,...,n\ . \tag{2.1b}$$

which identifies each following DP according to equations sec.1.3.5.

The reduced controls, built by the *memorized* macrostates, are an important part of the macrosystem's structure, providing a mechanism of a self-control synthesis.

These controls are also applied for a direct programming and the process' prognosis.

The solution of the *optimal control problem*, *combined with the identification*, we consider for the object, observed discretely at the moments $\tau \in \{\tau_k\}$, $k = 1,\ldots,m$, and transformed by the applied control to the terminal state x_T=0.

Let us apply a transformation *G* to the model (2.1), transforming it to a diagonal form. We get the equations

$$dz/dt = \bar{A}(z+v), \bar{A} = G^{-1}AG, G = (G_{ij}) \in L(R^n), detG \neq 0, \forall\, t \in \Delta^o,$$
$$z = Gx,\ \bar{v} = Gv, \tag{2.2}$$

$$x_T = (o_{ij})_{i,j=1}^n = O \Leftrightarrow z_T = (o_{ij})_{i,j=1}^n = O, \bar{v} = -2z(\tau),$$

$$\bar{A}^v = \bar{A}(I + (\frac{\bar{v}_j(\tau,\bullet)}{z_i(t,\bullet)}\delta_{ij})_{i,j=1}^n = (\lambda_i(t))_{i=1}^n, \tag{2.2a}$$

where the piece-wise matrices $A, \bar{A}$ are fixed within the intervals of the control discretization $t_k, k = 1,...,m-1$, and are identifiable at each of these intervals, while the matrices eigenvalues (2.2a) are connected according to relations (2.1b); I is identity matrix.

Theorem 2.1 (T2.1).

Transferring the system (2.2),(2.2a) to an origin of its coordinate system by the optimal controls, applied at the time intervals $t_k, k = 1,...,m$, requires the existence of a minimum of *two* matrix's $\bar{A}^v = (\lambda_i^k)_{i=1}^n$ eigenvalues, which at each of these moments satisfy the condition of connecting these intervals (1.3.64a,b), sec.1.3.5 in the form:

$$|\lambda_i^k| = |\lambda_j^k|, i, j = 1,...,n, k = 1,...,m-1 \tag{2.3}$$

with the *number of the control discrete intervals equal to* n.

Proof. By applying (2.2), and (2.3) using the matrix function (2.2a) under the control $\bar{v} = -2z(\tau_{k-1})$, we come to the recurrent relations connecting the nearest λ_i^k, λ_i^{k-1}:

$$\lambda_i^k = -\lambda_i^{k-1}\exp(\lambda_i^{k-1}t_k)(2 - \exp(\lambda_i^{k-1}t_k))^{-1}. \tag{2.4}$$

Then solutions of (2.2) acquire the form

$$z_i^k(t_k) = (2 - \exp(\lambda_i^{k-1}t_k)z_i^{k-1}(t_{k-1}). \tag{2.5}$$

By writing the solution on the last control's discrete interval $t_m = T$:

$$z_i(T,\bullet) = (2 - \exp(\lambda_i^{m-1}T)z_i(t_{m-1}) = 0, z_i(t_{m-1}) \neq 0, i == 1,...,n, \tag{2.6}$$

we get the relation, defining *T* through a preceding eigenvalue, which satisfies to all previous equalizations:

$$T = t_{m-1} + \ln 2/|\lambda_i^{m-1}|, \lambda_1^{m-1} > 0,\ \lambda_1^{m-1} = \lambda_2^{m-1} .... = \lambda_n^{m-1} > 0. \tag{2.7}$$

The positivity of the above eigenvalues can be reached at applying the needle controls in addition to the above step-wise controls.

If these controls are not added, more general conditions below are used.

The equalizations of the eigenvalues at other discrete intervals, leads to the chain of the equalities for $n \geq m$:

$$|\lambda_1^{m-1}|=|\lambda_2^{m-1}|=....=|\lambda_n^{m-1}| \tag{2.8}$$

$$|\lambda_1^{m-2}|=|\lambda_2^{m-2}|=.....=|\lambda_{n-1}^{m-2}|,\ldots,$$

$$|\lambda_1^{m-i-1}|=|\lambda_2^{m-i-1}|=....=|\lambda_{n-i}^{m-i-1}|,\ldots,$$

$$|\lambda_1^{1}|=|\lambda_2^{1}|=.....=|\lambda_{n-m+2}^{1}|, \tag{2.8a}$$

and for $m \geq n$ leads to the following chain of the equalities:

$$|\lambda_1^{m-1}|=|\lambda_2^{m-1}|....=|\lambda_n^{m-1}|, \tag{2.9}$$

$$|\lambda_1^{m-2}|=|\lambda_2^{m-2}|=.....=|\lambda_{n-1}^{m-2}|,\ldots,$$

$$|\lambda_1^{m-i-1}|=|\lambda_2^{m-i-1}|=....=|\lambda_{n-i}^{m-i-1}|,\ldots,$$

$$|\lambda_1^{m-n+1}|=|\lambda_2^{m-n+1}|. \tag{2.9a}$$

The system of equations (2.8), (2.9) defines the sought (*m*-1) moments of the controls discretization.

In a particular, from equation (2.8) the relation (2.8a) follows, which is inconsistent with the condition of a pair-wise equalization of the eigenvalues (2.3) at *n*>*m*.

The system (2.9) is a well defined, it agrees with (2.1),(2.2) and coincides with (2.8) *if* the number of its equations equals to the number of the equation state's variables.

Thus, equations (2.7), (2.8), (2.9) have a sense only when *n*=*m*.

The *n*-dimensional process requires *n* discrete controls applied at (*n-1)* intervals, defined by (2.8), (2.3) at the given starting conditions for equations (2.2). •

Remark. In the case of the matrix' renovation, each following solution (2.5) begins with a renovated eigenvalue, forming the chain (2.8), (2.9).

Theorem 2.2 (T2.2).

The fulfillment of conditions (2.3) leads to an indistinctness in time of the corresponding *transformed* state's variables:

$$\hat{z}_i = \hat{z}_j, \begin{pmatrix} z_i \\ z_j \end{pmatrix} = \hat{G}_{ij} \begin{pmatrix} \hat{z}_i \\ \hat{z}_j \end{pmatrix}; \hat{G}_{ij} = \begin{pmatrix} \cos\varphi_{ij}, -\sin\varphi_{ij} \\ \sin\varphi_{ij}, \cos\varphi_{ij} \end{pmatrix},$$

$$\varphi_{ij} = arctg\left(\frac{z_j(\tau_k) - z_i(\tau_k)}{z_j(\tau_k) + z_i(\tau_k)}\right) \pm N\pi, N = 0,1,2... \tag{2.10}$$

in some coordinate system, built on the states $(0z_1...z_n)$ and rotated on angle φ_{ij} in (2.10).

To *prove* we consider the geometrical meaning of the condition of equalizing of the eigenvalues as a result of the solutions of the equations (2.1), (2.2).

Applying relations (2.3) to the solutions of (2.8) for a nearest i, j, $i \neq j$, we get

$$\frac{dz_i}{z_i dt} = \frac{dz_j}{z_j dt} \; ; \; z_j(t,\bullet) = \frac{z_j(\tau_k.)}{z_i(\tau_k.)} z_i(t,\bullet), \; i,j = 1,...,n, \; k = 1,...,(n-1), \quad (2.10a)$$

where the last equality defines a hyper plane, being in a parallel to the axis $z_i = 0, z_j = 0$

in coordinate system $(0z_1...z_n)$.

By rotating this coordinate system with respect to that axis, it is found a coordinate system where the equations (2.10a) are transformed into the equalities for the state variables $\hat{z}_i$ in form (2.10). The corresponding angle of rotation of coordinate plane $(0z_i z_j)$ is determined by relation (2.10). Due to the arbitrariness of $k = 1,...,(n-1)$, $i,j = 1,...,n$ the foregoing holds true also for any two components of the state vector and for each interval of discretization. By carrying out the sequence of such $(n-1)$ rotations, we come to the system $(0\hat{z}_1...\hat{z}_n)$, where all the state variables are indistinguishable in time. •

Comments 2.1. If a set of the discrete moments ($\tau_k^1, \tau_k^i, \tau_k^{N_k}$) exists (for each optimal control v_k) then a unique solution of the optimization problem is reached by choosing a minimal interval τ_k^i for each v_k, which accomplishes the transformation of the above system to the origin of coordinate system during a minimal time.

The macrovariables are derived as a result of memorizing of the states $z_i(\tau_k)$, $i,k = 1,...,n$, being an attribute of the applied control in (2.2), which are fixed along the extremal segments.

The transformation $(G \times \hat{G}_{ij})$ transfers $\{x_i\}$ to new macrovariables $\{\hat{z}_i\}$, whose pair-wise indistinctness at the successive moments $\{\tau_k\}$ agrees with the reduction of numbers of independent macrocoordinates. This reduction has been referred as the *states' consolidation.* The successive equalization of the relative phase speed in (2.10a), accompanied by *memorization* of $z_i(\tau_k)$, determines an essence of the mechanism of the states' ordering.

Therefore, the problem of forming a sequentially *consolidated* macromodel is solved in a real–time process of the optimal motion, combined with identification of the renovated operator. Whereas both equalization and cooperation follow from the solution of the optimal problem for the path functional.

The macromodel is reversible within the discrete intervals and is irreversible out of them. Thus, a general structure of the initial object (1.1.1.1)(used also in physics), allows modeling a wide class of complex objects with *superimposing* processes, described by the *equations of irreversible thermodynamics* (ch.1.9).

According to the extremal properties of the information entropy, the segments of the extremals approximate the stochastic process with a maximal probability, i.e., without losing information about it. This also allows us to get the *optimal and nonlinear filtration* of the stochastic process within the discrete intervals [18].

2.1.3.2. The Procedure of the Joint Identification, Optimal Control, and Consolidation

Let an object (as the initial stochastic equation, ch.1.1.1) is characterized by unknown drift vector $a^u = a^u(t,x)$ (which includes the function of applied control) and a diffusion matrix $\sigma = \sigma(t,x)$, measured via dispersion matrix $b = 1/2\sigma\sigma^T$.

We assume that the object initial conditions $\tilde{x}_s$ as well as initial $r_s = E[\tilde{x}_s \tilde{x}_s^T]$ and σ_s are known.

The object dynamic model is described by the macrolevel equations

$$\dot{x} = Ax + u, u = Av, \dot{x} = A(x + v),$$

whose matrix $A = A(\tau, t_k)$ is a subject of the object identification under control $v = -2x(\tau),$ formed as the function of the object state vector's macrocoordinates at the moments τ (of the matrix identification) and fixed during the discrete intervals t_k between the moments τ of the object observation.

The object drift vector and the model are connected by relation

$$a^u(\tau, t_k, x(\tau,t)) = A(\tau, t_k)(x(\tau,t) + v(\tau, t_k)) \tag{2.11}$$

with the same applied control.

Solving *jointly the problems of optimal control and identification* includes the *following sequence*.

The procedure starts with applying initial control $v(\tau_o^o) = -2E_{\tau_o}[\tilde{x}_t(\tau_o)]$ at the moment $\tau_o = s + o$ where a related non random macrostate can be defined also via $x_i(\tau_o) \cong | r_i^{1/2}(\tau_o) |$.

Under this control action the initial matrix is identifies by relations

$$A(\tau_o) = b(\tau_o) r^{-1}(\tau_o), A(\tau_o) = (\lambda_i(\tau_o), i = 1, ..., n. \tag{2.11a}$$

The starting external control $u(\tau_o) = b(\tau_o) r^{-1}(\tau_o) v(\tau_o^o)$ also follows from these equalities.

Matrix (2.11a) is changing under the above controls' action: $A_o = A(\tau_o, t)$, and by the end of a first interval $t = t_{k=1}$, some of the matrix eigenvalues will satisfy the condition (sec.1.3.5):

$$\mathrm{Im}\, \lambda_i(t_{k=1}) = 0, \text{ or } | \lambda_i(t_{k=1}) | = | \lambda_j(t_{k=1}) |,$$

which determine the interval duration.

At this interval's end τ_1, the macrostate $x(\tau_1)$ is defined and a second control $v_1 = -2x(\tau_1)$ is applied.

Under this control action, the model matrix is identified by relations

$$A(\tau_1) = b(\tau_1)r^{-1}(\tau_1), r(\tau_1) = E_{\tau_1}[x(\tau_1)x^T(\tau_1)]. \quad (2.11b)$$

After that, this matrix continues to change: $A_1 = A(\tau_1, t)$ within a next interval $t = t_{k=2}$ of the applied control $v_1 = -2x(\tau_1, t)$, when by its end τ_2 the next control $v_2 = -2x(\tau_2)$ is applied, and the identification of the related matrix takes place:

$$A(\tau_2) = b(\tau_2)r^{-1}(\tau_2), r(\tau_2) = E_{\tau_2}[x(\tau_2)x^T(\tau_2)],$$

and so on.

Thus, the problem solution consists of measuring the object covariation matrix r_v and its derivative $\dot{r}_v = 2b(\tau)$ (or using just $b(\tau)$ in (2.1a)) to identify $A(\tau)$, and then, under action of the currently applied control, computing the discrete interval for applying the following control, when by interval's end, the identification of new macromodel's operator proceeds.

Finding the discrete intervals using condition (1.364a,b), (2.3) for the macromodels of the second and third orders we illustrated in Examples 1.5.1, 1.5.2a, ch.1.5.

For the model (2.2), number of the discrete intervals is equal to the number of the independent state variables (T2.1), and (n-1) moments of the switching control are determined by the considered system of the equalities at given initial conditions.

This control is able to transfer the object to a given final state along an optimal trajectory.

If there exists a set of the moments $(\tau_k^1, \ldots, \tau_k^j, \ldots, \tau_k^{N_1})$, then the unique solution of the optimization problem (in the terms of selection of a single τ_k for each found control v_t) is achieved by choosing the minimal $\tau_k^j = \tau_k$ for each k.

A chosen τ_k ensures a minimal time for the above transformation.

For example, considering $C_n^2 = \dfrac{n!}{(n-2)!\,2!}$ of the possible equalities (2.3):

$$\lambda_i^1 = \frac{\lambda_i^o \exp(-\lambda_i^o \tau_1)}{2 - \exp(-\lambda_i^o \tau_1)} = \frac{\lambda_j^o \exp(-\lambda_j^o \tau_1)}{2 - \exp(-\lambda_j^o \tau_1)} = \lambda_j^1,\ i, j = 1, \ldots, n, i \neq j, \quad (2.12)$$

we find all roots for each of the equalities, and select a such one that corresponds to the minimal $\tau_1 = \min_{j=1,\ldots,N_1}(\tau_1^1, \ldots, \tau_1^j, \ldots, \tau_1^{N_1})$, which defines the first moment of switching control.

Using the indications

$$\alpha = \frac{\gamma_{ij}^o - 1}{\gamma_{ij}^o} = \alpha(i, j),\ \gamma_{ij}^o = \frac{\lambda_i^o}{\lambda_j^o},\ \eta = \exp(-\lambda_i^o \tau_1) > 0,$$

the equality (2.12) for $(\lambda_i^1, \lambda_j^1)$) is reduced to a simple form

$$\eta^{\alpha} - \frac{\alpha}{2}\eta + \alpha - 1 = 0, \tag{2.12a}$$

which follows directly from the equalization of the eigenvalues at τ_1 (following (2.3)).
All discrete moments $\{\tau_k\}$ are found by the analogy:

$$\tau_k = \tau_{k-1} + \min_{j=1,\ldots,N_1} \{\tau_k^j\}_{j=1}^{N_k},\ \tau_k > \tau_{k-1},$$

where $\{\tau_k^j\}$ are the roots of equations (2.12).
The last moment of discretization τ_n=T (when the control is turned off), is found from equation (2.7), or by solving the corresponding Cauchy problem [19-21,other].

Let us consider the procedure of the macrostate consolidation.

The condition of the eigenvalues equalization satisfying (2.3) for the matrix:

$$a = \begin{pmatrix} a_{11}, a_{12} \\ a_{21}, a_{22} \end{pmatrix} \overset{def}{=} \begin{pmatrix} A_{ii}^{v}(\tau_k), A_{ij}^{v}(\tau_k) \\ A_{ji}^{v}(\tau_k), A_{jj}^{v}(\tau_k) \end{pmatrix},\ a = a^T, \tag{2.12b}$$

$$\lambda_{1,2} = \frac{Tr(a)}{2} \pm [(\frac{Tr(a)}{2})^2 - \det a]^{1/2},\ (\frac{Tr(a)}{2})^2 - \det a = (a_{11}a_{22})^2 + 4(a_{12})^2 = 0,$$

leads to a_{11}=a_{22}, a_{12}=0, e.g., to the matrix diagonalization.
The model is reduced to the diagonal form $\dot{z} = \overline{A}(z + \overline{v})$, $t \in (t_{k-1}, t_k)$, and then is transferred to a new rotating coordinate system toward achievement of the equalization of phase coordinates in (2.10).
Angle φ_{ij} of the coordinate plane's $(0 z_i z_j)$ rotation is found from (2.10):

$$\varphi_{ij} = arctg \frac{z_j(\tau_k,.) - z_i(\tau_k,.)}{z_j(\tau_k,.) + z_i(\tau_k,.)} \pm N\pi, \tag{2.12c}$$

for N=0,1,2,....Using the above relations for any two components of the state vector and for each discrete intervals (t_{k-1}, t_k), and providing (n-1) of such rotations, we arrive at the coordinate system $(0\hat{z}_1 \ldots \hat{z}_n)$, where all state variables are undistinguished in time.
The transformation of the initial state coordinates into this coordinate system leads to an origin of new macrostate variables.

Thus, during the model optimal motion, the problem of the successive states' cooperation gets the simultaneous solution with the identification problem.

Because, the IPF extremal segments approximate the random process with a maximal information, the optimal controls, selecting these segments, also provide an optimal discrete filtration of random process (within each discrete interval).

The controlled discrete filter passes a signal through only at the moments of the object operator renovation, when the information entropy reaches a maximum.

Such a device, implementing the decoupling of time-correlations, selects a stochastic equivalent of the extremal states by applying the discrete controls, which cut the random process at the moment of reaching the above maximum.

The discrete filter that provides both control and selection of the above points, also forms, computes, and applies the optimal controls.

The nonsearch control device [18] implements the above control and filtration.

Let us apply the dynamic model to a class of *nonlinear* objects, whose model can be presented by equation

$$\dot{x}_i = a_i(x_j),\ a_i(x_j) = \begin{pmatrix} \phi_i(x_j), \delta_2 < |\delta| < \delta_1 \\ a_i \operatorname{sgn}\delta, \delta_1 < |\delta| < \delta_0 \\ \psi_i(x_j), \delta_0 < |\delta| < \delta_3, \end{pmatrix}, \delta = (x_j(t_o) - x_j(\tau)),$$
$$i \neq j, i, j = 1, ..., n, \qquad (2.13)$$

where $\delta_0, \delta_1, \delta_2, \delta_3$ are fixed at the intervals of observations (τ, t_o); $\phi_i(x_j), \psi_i(x_j)$ are the nonlinear functions.

For such objects, the identification of unknown operator $a_i(x_j)$ is possible, using matrix $R_v(\tau) = 1/2\dot{r}r^{-1}$ at the Markovian moments τ_i of decoupling the time correlations, when the control $v_i = v_i(x_i)$ fulfills the condition

$$\dot{r}_i(\tau_i) = 2E_{v_i}[a_i(x_j(\tau_i))(x_i(\tau_i) + v_i)^2] = 2a_i(x_j(\tau_i))E_{v_i}[(x_i + v_i)^2], \qquad (2.13a)$$

which leads to selecting the nonlinear operator out of the sign of the conditional mathematical expectation. By substituting both $\dot{r}_i(\tau_i)$ and $r_i(\tau_i) = E_{v_i}[(x_i + v_i)^2]$ into $R_v(\tau)$ we come to

$$R_v(\tau_i) = a_i(x_j(\tau_i)).$$

Fixing $R_v(\tau_i)$ simultaneously with $x_i(\tau_i)$ allows restoring this, previous unknown nonlinear operator.

The condition of consolidation (2.3) for equations

$$\dot{x}_i = a_i(x_j(\tau_k))(x_i + v_i),\ \dot{x}_j = a_j(x_i(\tau_k))(x_j + v_j),$$

with controls

$$v_j = -2x_j(\tau_k),\ \ v_i = -2x_i(\tau_k)$$

is satisfied at

$$|a_i(x_j(\tau_k))| = |a_j(x_i(\tau_k))|,$$

which defines the operator of cooperative model.

Example. Let us start with the second order object's stochastic equation as the microlevel model at $t \in [s, T]$:

$$d\tilde{x}_1(t,\bullet) = a_1(t,\tilde{x}_t,u)dt + \sigma_{11}(t)d\xi_1(t,\bullet) + \sigma_{12}(t)d\xi_2(t,\bullet),\ \tilde{x}_1(s,\bullet) = \tilde{x}_{1s},$$
$$d\tilde{x}_2(t,\bullet) = a_2(t,\tilde{x}_t,u)dt + \sigma_{21}(t)d\xi_1(t,\bullet) + \sigma_{22}(t)d\xi_2(t,\bullet), \tilde{x}_2(s,\bullet) = \tilde{x}_{2s}. \quad (2.14)$$

Suppose the task at the macrolevel is given by a constant vector $\overline{\overline{x}}_t^1 = \overline{\overline{x}}_o^1 \forall t \in \Delta,\ \overline{\overline{x}}_o^1 \in R^2$, which is chosen to be a beginning of coordinate system ($0\,\overline{\overline{x}}_1\ \overline{\overline{x}}_2$).

Then at $x_t = \overline{\overline{x}}_t - \overline{\overline{x}}_t^1$ (ch.1.1), $x_t = \overline{\overline{x}}_t$.

The macrolevel model $\dot{x}_t = A_t(x_t + v_t)$ requires the identification of matrix A_t using

$$R_v^1(\tau) = 1/2\dot{r}_1(\tau)r_v(\tau)^{-1}, \dot{r}_1(\tau) = 2b(\tau), \qquad \tau = (\tau_o, \tau_1),\ \tau_o = s + o,$$

(2.14a)

which we specify by the following equations:

$$\dot{x}_1(t,\bullet) = A_{11}(t)(x_1(t,\bullet) + v_1(t,\bullet)) + A_{12}(t)(x_2(t,\bullet) + v_2(t,\bullet)),\ x_1(s,\bullet) = x_{1s},$$
$$\dot{x}_2(t,\bullet) = A_{21}(t)(x_1(t,\bullet) + v_1(t,\bullet)) + A_{22}(t)(x_2(t,\bullet) + v_2(t,\bullet)), x_2(s,\bullet) = x_{2s},$$

$$x_t = \begin{pmatrix} x_1(t,\bullet) \\ x_2(t,\bullet) \end{pmatrix},\ v_t = \begin{pmatrix} v_1(t,\bullet) \\ v_2(t,\bullet) \end{pmatrix}, A(t) = \begin{pmatrix} A_{11}, A_{12} \\ A_{21}, A_{22} \end{pmatrix},$$

(2.14b)

$$b = \begin{pmatrix} b_{11}, b_{12} \\ b_{21}, b_{22} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} (\sigma^2{}_{11} + \sigma^2{}_{12}), (\sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22}) \\ (\sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22}), (\sigma^2{}_{21} + \sigma^2{}_{22}) \end{pmatrix},$$

$$\dot{r}_1(\tau) = \begin{pmatrix} (E_\tau[\dot{x}_1(x_1(t,\bullet) + v_1(t,\bullet))]), (E_\tau[\dot{x}_1(x_2(t,\bullet) + v_2(t,\bullet))]) \\ (E_\tau[\dot{x}_2(x_1(t,\bullet) + v_1(t,\bullet))]), (E_\tau[\dot{x}_2(x_2(t,\bullet) + v_2(t,\bullet))]) \end{pmatrix} = 2\begin{pmatrix} b_{11}(\tau), b_{12}(\tau) \\ b_{21}(\tau), b_{22}(\tau) \end{pmatrix},$$

$$r_v(\tau) = \begin{pmatrix} (E_\tau[(x_1(t,\bullet) + v_1(t,\bullet))]^2), (E_\tau[(x_1(t,\bullet) + v_1(t,\bullet))(x_2(t,\bullet) + v_2(t,\bullet))]) \\ (E_\tau[(x_2(t,\bullet) + v_2(t,\bullet))(x_1(t,\bullet) + v_1(t,\bullet))]), (E_\tau[(x_2(t,\bullet) + v_2(t,\bullet))^2]) \end{pmatrix},$$

$$E[\bullet] = \left\{ \begin{array}{l} E_o = \int\limits_{R^2} [\bullet] P_o(\tau_o, x)dx, x = x_{0,}, t \in [\tau_o, \tau_1), \\ E_{\tau=\tau_1} = \int\limits_{R^2} [\bullet] P_{\tau_1}(\tau_1, x)dx, x = x_{\tau_1,}, t \in [\tau_1, T) \end{array} \right\}. \quad (2.15)$$

The object observation is a discrete time process with the elements of A_t as the piece-wise functions of time, which are fixed within each of two discrete intervals $[\tau_o, \tau_1), (\tau_1, T]$, while the controls $v_t(\tau \pm o) = -2x_t(\tau \pm o)$ are applied at the localities of these moments:

$$\tau \pm o = (\tau_o^o = (\tau_o + o), \tau_o^1 = (\tau_1 - o), \tau_1^1 = (\tau_1 + o), \tau_1^1 = (T - o)). \quad (2.15a)$$

This *identification problem* consists of the restoration $A_t(\tau)$ by known $b(\tau)$, $r_1(\tau)$, using here the model solutions $x_t(t,\bullet)$ (with the applied controls $v_t(t,\bullet)$), which are considered to be the equivalents of the object under observation.

Using (2.14a,b),(2.15), we write (2.14a) via these solutions in matrix forms:

$$r_v(t) = E[(x_t(t,\bullet) + v_t(t,\bullet))(x_t(t,\bullet) + v_t(t,\bullet))^T]$$

and

$$\dot r_1(t) = E[\dot x_t(t,\bullet)(x_t(t,\bullet) + v_t(t,\bullet))^T + (x_t(t,\bullet) + v_t(t,\bullet))\dot x_t(t,\bullet)^T]$$

$$= A_t E[(x_t(t,\bullet) + v_t(t,\bullet))(x_t(t,\bullet) + v_t(t,\bullet))^T] + E[(x_t(t,\bullet) + v_t(t,\bullet))(x_t(t,\bullet) + v_t(t,\bullet))^T]A_t,$$

which at $t = \tau$, $v_t(\tau) = 0$, acquire the view

$$r_v(\tau) = E[x_t(\tau)(x_t(\tau)^T] = r(\tau)\ \dot r_1(\tau) = A_t(\tau)r(\tau) + r(\tau)A_t(\tau).$$

After substitution these relations in (2.14a), at symmetrical $r(\tau)$, $A_t(\tau)$, we come to

$$R_v^1(\tau) = A_t(\tau) \text{ or } R_v^1(\tau) = b(\tau)r^{-1}(\tau) = A_t(\tau), \quad (2.15b)$$

which validate the correctness of the identification relation (2.14b).

It's seen that $R_v^1(\tau)$ does not depend on the probability distributions of an *initial* state vector, which is important for many applications.

Let us have the matrix, identified at the first moment τ_o:

$$A(\tau_o) = \begin{pmatrix} 2,3 \\ 3,10 \end{pmatrix}$$

with eigenvalues $\lambda_1^0 = 11,\ \lambda_2^0 = 1$.

At $x_t + v_t = y_t$, the fundamental system and general solutions of (2.14a) within interval $t \in [\tau_o, \tau_1)$ under control $v_t(\tau_o^o, \tau_o^1)$ have the forms::

$$Y_t = \begin{pmatrix} y_{11}(t), y_{12}(t) \\ y_{21}(t), y_{22}(t) \end{pmatrix} = \begin{pmatrix} \exp(11t), 3\exp(11t) \\ -3\exp(11t), \exp(t) \end{pmatrix},\ x_0 = \begin{pmatrix} x_{10} \\ x_{20} \end{pmatrix},\ y_0 = -x_0,$$

$$y_1(t,\bullet) = C_1 \exp(11t) - 3C_2 \exp(t),\ y_2(t,\bullet) = 3C_1 \exp(11t) + C_2 \exp(t).$$

Using the following initial conditions and constants C_1, C_2:

$$C_1 - 3C_2 = -x_{10},\ 3C_1 + C_2 = -x_{20},\ C_1 = -0.1(x_{10} + 3x_{20}),\ C_2 = 0.1(3x_{10} - x_{20}),$$

we get the solution of Caushy problem in the form

$$y_1(t,\bullet) = -0.1[x_{10}(\exp(11t) + 9\exp(t)) + 3x_{20}(\exp(11t) - \exp(t))],$$
$$y_2(t,\bullet) = -0.1[3x_{10}(\exp(11t)) - \exp(t)) + x_{20}(9\exp(11t) + \exp(t))].$$

Then we find the moment τ_1 of switching the optimal control using condition (2.3) in the form

$$\frac{\dot{x}_1(t_1)}{x_1(t_1)} = \frac{11C_1\exp(11t_1) - 3C_2\exp(t_1)}{-v_1(\tau_o,\bullet) + C_1\exp(11t_1) - 3C_2\exp(t_1)}$$
$$= \frac{\dot{x}_2(t_1)}{x_2(t_1)} = \frac{33C_1\exp(11t_1) + C_2\exp(t_1)}{-v_2(\tau_o,\bullet) + 3C_1\exp(11t_1) + C_2\exp(t_1)},$$
$$v_1(\tau_o^o) = -2x_1(\tau_o), v_2(\tau_o) = -2x_2(\tau_o). \quad (2.16)$$

We get equation

$$5\exp(11t_1) - 11\exp(10t_1) + 1 = 0, t_1 > 0 \quad (2.16a)$$

having the unique root $\tau_1 \cong 0.7884$.

Application of formula (2.12a) also leads to (2.16), using parameters

$$\gamma_{12}^o = \gamma_{12}^o = \frac{\lambda_1^o}{\lambda_2^o} = 11,\ \alpha = 10/11,\ \eta = \exp(11t).$$

These relations illustrate the independency of the discrete moment on a chosen coordinate system.

The model solution within interval $t \in (\tau_1, T]$ have the forms

$$x_1(t) = (2 - \exp(0.7t)x_1(\tau_1); x_2(t) = (2 - \exp(0.7)x_2(\tau_1). \quad (2.16b)$$

We also obtain the eigenvalues at moments τ_1 and the final *T*:

$$\lambda_1^1 = \lambda_2^1 \cong 11, T = \tau_1 + \frac{\ln 2}{\lambda_1^1} \cong 0.851,\ \overline{A}(\tau_1) \cong \begin{pmatrix} 11, 0 \\ 0, 11 \end{pmatrix}. \quad (2.17)$$

If the identified matrix is negative:

$$A(\tau_o) = \begin{pmatrix} -2, -3 \\ -3, -10 \end{pmatrix},$$

then the moment τ_1 is found by analogy:

$$\frac{11C_1\exp(-11\tau_1)}{-v_1(\tau_o) + C_1\exp(-11\tau_1) - 3C_2\exp(-\tau_1)} = \frac{33C_1\exp(-\tau_1) + C_2\exp(-\tau_1)}{-v_2(\tau_o) + 3C_1\exp(-11\tau_1) + C_2\exp(-\tau_1)} \quad (2.17a)$$

This equality leads to equation

$5\exp(-11t) - 11\exp(-10t) + 1 = 0, t_1 > 0$ having root $\tau_1 \cong 0.193$.

The negative eigenvalues at this moment τ_1:

$$\lambda^1_1 = \lambda^1_2 \cong -\ 0.7 \qquad (2.17b)$$

are changed by applying the needle control, which brings

$$\lambda^1_1 = \lambda^1_2 \cong 0.7,$$

with T=0.193+$\dfrac{\ln 2}{0.7} \cong 1.187$.

Applying both identification's and model's equations, we can find the model matrix at the moment τ_1 whose elements are determined by relations

$$A_{11}(\tau_1) = \frac{2\exp(12\,\tau_1) - 2.2\exp(11\tau_1) - 1.8\exp(\tau_1)}{\exp(12\,\tau_1) - 2\exp(11\tau_1) - 2\exp(\tau_1) + 4},$$
$$A_{12}(\tau_1) = A_{21}(\tau_1) = \frac{3(\exp(12\,\tau_1) - 2.2\exp(11\,\tau_1) + 0.2\exp(\tau_1))}{\exp(12\,\tau_1) - 2\exp(11\,\tau_1) - 2\exp(\tau_1) + 4},$$
$$A_{22}(\tau_1) = \frac{10\exp(12\,\tau_1) - 19.8\exp(11\,\tau_1) - 0.2\exp(\tau_1)}{\exp(12\tau_1) - 2\exp(11\tau_1) - 2\exp(\tau_1) + 4}. \qquad (2.17c)$$

The numerical solutions for this matrix are:

$$A\ (\tau_1) \cong \begin{pmatrix} 11.006, -0.00077 \\ -0.00077, 11.004 \end{pmatrix} \cong \begin{pmatrix} 11, 0 \\ 0, 11 \end{pmatrix} \qquad (2.17d)$$

Comparing both results for $A\ (\tau_1)$ (2.17) and (2.17d) we come to the conclusion that we have identified $A_t \forall t \in (\tau_1, T)$ with a high precision (defined by a computation accuracy), which does not depend on a chosen coordinate system.

The step-wise control provides changing the matrix $A(\tau)$ sign at any of τ-localities.

The optimal processes within the discrete interval $t \in (\tau_1, T)$ with the matrix' eigenvalues (2.17d):

$$x_1(t) = (2 - \exp(11t)x_1(\tau_1); x_2(t) = (2 - \exp(11t)x_2(\tau_1),$$

are distinctive only by the starting states ($x_1(\tau_1)$, $x_2(\tau_1)$).

Analogous form has optimal processes with the matrix negative eigenvalues in (2.17b):

$$x_1(t) = (2 - \exp(0.7t)x_1(\tau_1); x_2(t) = (2 - \exp(0.7t)x_2(\tau_1).$$

Therefore, the matrix' identification proceeds *during the optimal control action* at each extremal segment.

Let us determine the phase trajectories of the dynamic model at both discrete intervals.

At first, we will find these trajectories for the diagonalized system at $t \in [\tau_o, \tau_1)$:

$$\frac{dz_1}{dz_2} = -\ \frac{-\lambda^o{}_1}{-\lambda^0{}_2}\frac{z_1}{z_2},\ z_2 = \pm|\imath|\, z_1^{\lambda^o_2/\lambda^o_1},\ \imath \in R^1,\ \pm|\imath| = z_2(\tau_o)/z_1(\tau_o)^{\lambda^o_2/\lambda^o_1}.$$

The phase trajectories of this system (Fig.1.1a,b) present the ι -parametrical family of the curves with a singular tangle-point in (0,0) (a "knot").

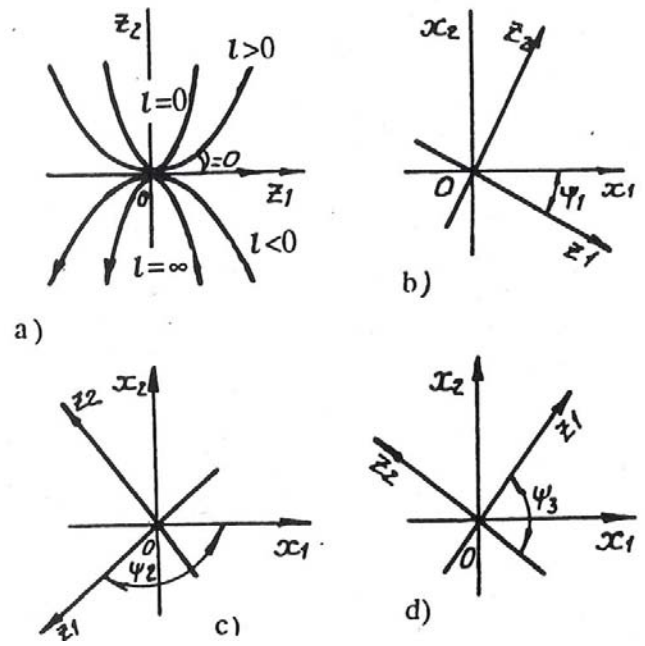


Figure 1.1.(a-c). The phase pictures of dynamic model in the initial coordinate system at the first discrete interval.

The phase picture (Fig.1.1a) is turned over on the angle ψ , defined by transformations

$$G_t = G = \begin{pmatrix} \cos\psi, \sin\psi \\ -\sin\psi, \cos\psi \end{pmatrix},\ t \in [\tau_o, \tau_1),\ G^{-1} = G^T,\ \det G = 1,$$

and we come to the following equations

$$z_t = G\ x_t,\ \dot{z}_t = G\ \dot{x}_t = GA(\tau_o)\,(x_t + v_t) = GA(\tau_o)G^T\,(z_t + \bar{v}_t),$$

$$GA(\tau_o)G^T = \begin{pmatrix} -\lambda_1^0, 0 \\ 0, -\lambda_2^0 \end{pmatrix}.$$

At the initial eigenvalues $\lambda_1^0 = -11, \lambda_2^0 = -1$, let us find angle ψ .
We get relations

$$GA(\tau_o)G^T = \begin{pmatrix} ((GA)_{11}\cos\psi + (GA)_{12}\sin\psi), (-(GA)_{11}\sin\psi + (GA)_{12}\cos\psi) \\ ((GA)_{21}\cos\psi + (GA)_{22}\sin\psi), (-(GA)_{21}\sin\psi + (GA)_{22}\cos\psi) \end{pmatrix}$$

$$= \begin{pmatrix} 11,0 \\ 0,1 \end{pmatrix},$$

$$GA(\tau_o) = \begin{pmatrix} (a_{11}\cos\psi + a_{21}\sin\psi), (a_{22}\sin\psi + a_{12}\cos\psi) \\ (-a_{11}\sin\psi + a_{21}\cos\psi), (-a_{12}\sin\psi + a_{22}\cos\psi) \end{pmatrix},$$

$$a_{11}\cos^2\psi + a_{12}\sin 2\psi + a_{22}\sin^2\psi = 11,$$

and the equations for the angles ψ are:

$$\frac{1}{2}(a_{22} - a_{11})\sin 2\psi + a_{12}\cos 2\psi = 0,\ a_{11}\sin^2\psi - a_{12}\sin 2\psi + a_{22}\cos^2\psi = 1;$$

$$tg2\psi = 2\frac{a_{12}}{a_{22} - a_{11}} = -0.75,\ \psi = \frac{1}{2}arctg2\frac{a_{12}}{a_{22} - a_{11}} + \frac{k\pi}{2},$$

$$k = 0, \pm 1, \pm 2, \ldots,\ \psi_1 = \psi|_{k=0} \cong -\frac{\pi}{2},\ \psi_2 = \psi|_{k=-1} \cong -0.6\pi,\ \psi_3 = \psi|_{k=1} \cong 0.4\pi.$$

From the same equations we get the equality

$$\cos 2\psi = \frac{(11-1)(a_{22} - a_{11})}{(2a_{12})^2 + (a_{22} - a_{11})^2} = -0.8,$$

which leads to the equivalent expressions for ψ at both intervals, because of the fulfillment of

$$\cos^2 2\psi \equiv (1 + tg^2 2\psi)^{-1}.$$

The model phase picture in the initial coordinate system $(0\, x_1\, x_2)$ is given on Figs.1.1b-d, which we got after turning the coordinate system on Figs.1.1a (with the phase picture) on the angles (ψ_1, ψ_2, ψ_3), determined by the matrix's identified components (2.17c).

We come to

$$a_{11}(x_1 + v_1)x_2 + a_{12}(x_2 + v_2)x_2 = a_{21}(x_1 + v_1)x_1 + a_{22}(x_2 + v_2)x_1;\ a_{12} = a_{21}; \qquad (2.18)$$
$$a'_{11}x_1^2 + 2a'_{12}x_1x_2 + a'_{22}x_2^2 + 2a'_{13}x_1 + 2a'_{23}x_2 + a'_{23} = 0,$$

where

$$a'_{11} = a_{21};\ a'_{12} = 1/2(a_{22} - a_{11});\ a'_{22} = -a_{12};\ a'_{13} = 1/2(a_{21}v_1 - a_{22}v_2);$$
$$a'_{23} = -1/2(a_{11}v_1 + a_{12}v_2),\ a'_{33} = 0.$$

The equation of a second order for a line (2.18) we will transform to a canonic form after transferring the beginning of the line's coordinate system into the line's center, using equation

$$a'_{11}(x'_1)^2 + 2a'_{12}x'_1x'_2 + a'_{22}(x'_2)^2 + \frac{I_3}{I_2} = 0, \qquad (2.18a)$$

where $x=x_1^{'}+x^o$ and $x^o=(x_1^o, x_2^o)$ are the coordinates of the beginning of system coordinates $(0\,x_1^{'}\,x_2^{'})$ being transformed into the system coordinates $(0\,x_1\;\;x_2)$, which satisfy the equations

$$a_{11}^{'}x_1^o + a_{12}^{'}x_2^o + a_{13}^{'} = 0, a_{12}^{'}x_1^o + a_{22}^{'}x_2^o + a_{23}^{'} = 0,$$

with the parameters of transformation

$$I_2=det\begin{pmatrix} a_{11}^{'}, a_{12}^{'} \\ a_{12}^{'}, a_{22}^{'} \end{pmatrix}=det\begin{pmatrix} a_{12}, 1/2(a_{22}-a_{11}) \\ 1/2(a_{22}-a_{11}), -a_{12} \end{pmatrix}=inv,\; I_2=-25,$$

$$I_3 = det\begin{pmatrix} a_{11}^{'}, a_{12}^{'}, a_{13}^{'} \\ a_{12}^{'}, a_{22}^{'}, a_{23}^{'} \\ a_{13}^{'}, a_{23}^{'}, a_{33}^{'} \end{pmatrix}=det\begin{pmatrix} 3, 4, 1/2(3v_1+10v_2) \\ 4, -3, -1/2(3v_1+2v_2) \\ 1/2(3v_1+10v_2), -1/2(3v_1+2v_2), 0 \end{pmatrix},$$

$$I_3 = -1/4\,(33\,v^2{}_1+327v^2{}_2+196v_1v_2)=inv,$$
$$\frac{I_3}{I_2}=-\,0.01(33\,v^2{}_1+327v^2{}_2+196v_1v_2). \qquad (2.18b)$$

After a simplification we obtain the equations

$$a^{''}{}_{11}(x_1^{''})^2+a^{''}{}_{22}\,(x_2^{''})^2+\frac{I_3}{I_2}=0,\;\begin{pmatrix} x_1^{''} \\ x_2^{''} \end{pmatrix}=\begin{pmatrix} \cos\vartheta, \sin\vartheta \\ -\sin\vartheta, \cos\vartheta \end{pmatrix}\begin{pmatrix} x_1^{'} \\ x_2^{'} \end{pmatrix},$$

$$ctg\,2\,\vartheta=\frac{a^{'}{}_{11}-a^{'}{}_{22}}{2a^{'}{}_{12}}=0.75;$$

$$I_2=inv<0;\; I_2=det\begin{pmatrix} a^{''}{}_{11}, a^{''}{}_{12} \\ a^{''}{}_{12}, a^{''}{}_{22} \end{pmatrix}=det\begin{pmatrix} a^{''}{}_{11}, 0 \\ 0, a^{''}{}_{22} \end{pmatrix}=a^{''}{}_{11}a^{''}{}_{22}<0\,,$$

$$a^{''}{}_{11}=a^{'}{}_{12}\sin 2\,\vartheta+\frac{a^{'}{}_{11}-a^{'}{}_{22}}{2}\cos 2\,\vartheta\,,\; a^{''}{}_{11}\overset{def}{>}0,$$

$$a^{''}{}_{22}=-a^{'}{}_{12}\sin 2\,\vartheta-\frac{a^{'}{}_{11}-a^{'}{}_{22}}{2}\cos 2\,\vartheta\,,\; a^{''}{}_{22}\overset{def}{<}0.$$

At $I_3<0$ we get the canonical equation of a hyperbola with respect to the real axis $(0^{'}x_1^{''})$ and the imaginary axis– $(0^{'}x_2^{''})$ (Fig. 1.2a):

$$\frac{(x_1^{''})^2}{[(\frac{I_3}{I_2a^{''}{}_{11}})^{\frac{1}{2}}]^2}-\frac{(x_2^{''})^2}{[(\frac{I_3}{-a^{''}{}_{22}I_2})^{\frac{1}{2}}]^2}=1. \qquad (2.19)$$

At $I_3>0$ we come to other hyperbola on Fig.1.2a, with respect to the real axis $(0^{'}x_2^{''})$ and the imaginary axis– $(0^{'}x_1^{''})$:

$$\frac{(x_1^{"})^2}{[(-\frac{I_3}{I_2 a^{"}_{11}})^{\frac{1}{2}}]^2} - \frac{(x_2^{"})^2}{[(\frac{-I_3}{-a^{"}_{22} I_2})^{\frac{1}{2}}]^2} = - 1. \qquad (2.20)$$

At I_3=0 we get a couple of the equations that satisfy to the coordinates of the points, located on the straight lines, represented the asymptotes of the hyperbolas (2.19),(2.20) (Figs. 1.2b,c):

$$\frac{x_1^{"}}{(\frac{1}{a^{"}_{11}})^{\frac{1}{2}}} + \frac{x_2^{"}}{(\frac{1}{-a^{"}_{22}})^{\frac{1}{2}}} = 0, \quad \frac{x_1^{"}}{(\frac{1}{a^{"}_{11}})^{\frac{1}{2}}} - \frac{x_2^{"}}{(\frac{1}{-a^{"}_{22}})^{\frac{1}{2}}} = 0. \qquad (2.21)$$

On the coordinate plane ($0' x_1^{"} x_2^{"}$), the phase picture of relation (2.16) represents a couple of the conjugated hyperbolas with the asymptotes, defined by equation (2.21) and a saddle singular point (0,0) (Fig. 1.2d).

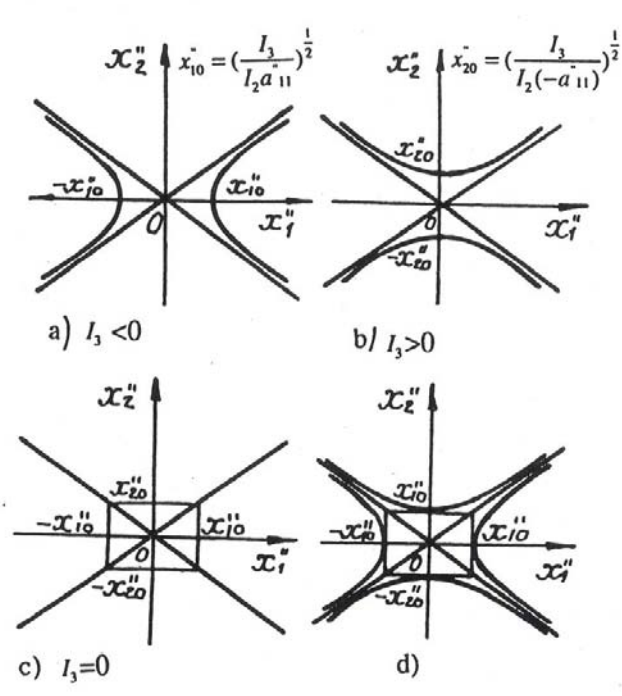


Figure 1.2. The phase pictures of the dynamic model (a-c) and the relation (2.16) (d) at the second discrete interval after the transforming to the initial coordinate system.

The phase trajectories of the dynamic system at the second discrete interval, after switching the control, are

$$\dot{y}_1(t,\bullet) = A_{11}(\tau_1 + 0) y_1(t,\bullet) = \lambda_1^1 y_1(t,\bullet),\ t \in (\tau_1, T),$$

$$\dot{y}_2(t,\bullet) = A_{22}(\tau_1 + 0)\, y_2(t,\bullet) = \lambda_2^1\, y_2(t,\bullet), \quad \lambda_1^1 = \lambda_2^1; \quad \frac{dy_1}{dy_2} = \frac{y_1}{y_2}, y_1 = \pm|C|\, y_2, \quad C \in R^1,$$

$$\pm|C| = \frac{y_1(\tau_1,\bullet)}{y_2(\tau_1,\bullet)} = \frac{x_1(\tau_1,\bullet)}{x_2(\tau_1,\bullet)}; x_1 + v_1 = \frac{x_1(\tau_1,\bullet)}{x_2(\tau_1,\bullet)}(x_2 + v_2).$$

The phase picture at $t \in (\tau_1, T)$ presents a family of the straight lines

$$x_2 = \frac{\dot{x}_1(\tau_1,\bullet)}{x_1(\tau_1,\bullet)} x_1 \tag{2.22}$$

with parameter $\dfrac{x_1(\tau_1,\bullet)}{x_2(\tau_1,\bullet)}$.

The phase picture of equality

$$\frac{\dot{x}_1(\tau_1,\bullet)}{x_1(\tau_1,\bullet)} = \frac{\dot{x}_2(\tau_1,\bullet)}{x_2(\tau_1,\bullet)}, \; t \in (\tau_1, T)$$

has the form

$$A_{22}(\tau_1 + 0)\, v_2(\tau_1 + 0)\, x_1(t,\bullet) = A_{11}(\tau_1 + 0)\, v_1(\tau_1 + 0)\, x_2(t,\bullet), \; x_1 = \frac{x_1(\tau_1,\bullet)}{x_2(\tau_1,\bullet)} x_2. \tag{2.23}$$

At the second discrete interval, the phase pictures of the dynamic model (2.23) and relation (2.22) coincide.

Because of that, their relative phase speeds are equal at $t \in (\tau_1, T)$, when the dynamic system's differential constraint (2.17) is imposed on the extremals by the microlevel's stochastics (via Kolmogorov's equation (ch.1.3)).

The comparison of the Figs.1.1,1.2 for (2.22),(2.23) illustrates the geometrical interpretation of the constraint action.

At the moment of applying the control, the phase pictures of the dynamic model and relation (2.15) are changed by the jumps. This leads to the renovation of matrix $A(\tau_1 + 0)$ with respect to matrix $A(\tau_1 - 0)$, and it creates the new model's peculiarities.

Let us find the jump of the phase speed at τ_1:

$$\begin{aligned}
&\delta\dot{x}(\tau_1,\bullet) = (\tau_1,\bullet) = \dot{x}(\tau_1 + 0,\bullet) - \dot{x}(\tau_1 - 0,\bullet) \\
&= -A(\tau_1 + 0)\, x(\tau_1,\bullet) - A(\tau_1 - 0)(x(\tau_1,\bullet) + v_0) \\
&= -(A(\tau_1 + 0) + A(\tau_1 - 0))\, x(\tau_1,\bullet) + 2A(\tau_1 - 0)\, x_0 \\
&= [-A(\tau_1 + 0) + A(\tau_1 - 0)\, \overline{Y}(\tau_1) + 2A(\tau_1 - 0))]\, x_0.
\end{aligned}$$

Remark.

The values

$$\varpi_i = \lim_{t \to \infty} \frac{1}{t} \ln \frac{\delta x_i(x_i, t)}{\delta x_i(x_i, 0)} \tag{2.23a}$$

define the Lyapunov indicators (ch.1.9) (as an averaged speed of the exponential divergence of the nearest trajectories), whose positive sum is connected to Kolmogorov's differential entropy h_ϖ: $h_\varpi = \sum_i \varpi_i$.

For the classical integrated Hamiltonian systems, all indicators equal to zero and $h_\varpi \equiv 0$ •.

From the phase speed's expressions and the previous relations we have

$$A(\tau_1+0)+A(\tau_1-0)\cong\begin{pmatrix}13,3\\3,21\end{pmatrix},\ 2A(\tau_1-0)=\begin{pmatrix}4,6\\6,20\end{pmatrix};$$

$$-\begin{pmatrix}13,3\\3,21\end{pmatrix}\bar{Y}(\tau_1)+\begin{pmatrix}4,6\\6,20\end{pmatrix}=K=\begin{pmatrix}K_{11},K_{12}\\K_{21},K_{22}\end{pmatrix},\ \delta\dot{x}(\tau_1,\bullet)=Kx_0,$$

$$K_{11}=2.2\exp(11\tau_1)+10.8\exp(\tau_1),\ K_{12}=K_{21}=6.6\exp(11\tau_1)-3.6\exp(\tau_1),$$
$$K_{22}=19.8\exp(11\tau_1)+1.2\exp(\tau_1)-22.$$

At $\tau_1=0.7884$, we obtain the numerical results:

$$K=\begin{pmatrix}12848.65,38532.75\\38532.75,115602.66\end{pmatrix},$$

which determine the values of both jumps:

$$\delta\dot{x}_1(\tau_1,\bullet)=51381.4,\ \delta\dot{x}_2(\tau_1,\bullet)=154135.41 \text{ at } x_0=\begin{pmatrix}1\\1\end{pmatrix}.$$

Therefore the changes of model's original nonlinear operator are also identified at the DP (τ_i) *by the jump-wise sequence of* $\Delta A(\tau_i)$.

For the solution of the *consolidation* problem, we rotate the initial coordinate system on angle φ to find such a coordinate system $(0\,z_1'\,z_2')$, where the optimal processes are undistinguished.

Using the relations for consolidation, we get

$$\varphi_{12}=\varphi=\text{arctg}\left(\frac{x_2(\tau_1,\bullet)-x_1(\tau_1,\bullet)}{x_2(\tau_1,\bullet)+x_1(\tau_1,\bullet)}\right)+k\pi,\ k=0,\pm1,\pm2,...,$$

$$x(\tau_1,\bullet)=\begin{pmatrix}x_1(\tau_1,\bullet)\\x_2(\tau_1,\bullet)\end{pmatrix}=\begin{pmatrix}5839.294,17511.88\\17511.88,52537.66\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}23351.17\\70049.54\end{pmatrix},\ \text{at } x_0=\begin{pmatrix}1\\1\end{pmatrix}$$

and the angle

$$\varphi=\text{arctg}\frac{46698.37}{93400.71}+k\pi\cong\text{arctg}0.5+k\pi,\ \varphi|_{k=0}\cong0.1472\pi.$$

Applying to the equal model eigenvalues (2.17d) a transformation, leading to the equalization of corresponding state coordinates $x_1(\tau_1)=x_2(\tau_1)$, allows us to characterize each pair of the state vectors by a *single joint* vector.

The consequent realization of this process for a multi-dimensional system leads to *building* the object's cooperative information network (IN), considered in the next section.

The identification method depends on the accuracies of computing the correlations functions $\tilde{r}_{ij}(\tau_k)$ and on the feedback effect in a close-loop control.

The identification of the *actual* object's operator in the close-loop control has been implemented on the *electro-technological process* using a direct measuring of the diffusion conductivity according to (sec.1.9.4).

It expedites the close-loop dynamics and minimizes the related error, compared to statistical method of computing the correlation function.

This error was not exceeded 10%.

The considered results explain the procedure and the numerical solutions of the joint optimal control, identification, and consolidation problems, which brings the state's *cooperation during the optimal control and identification.*

Let us show that the synthesized optimal controls *provide both* the *identification* of *A* (and/or $\overline{A}$) and *solving* the *Boundary-value* problem.

We prove it by reducing the object's equation (2.1) to a diagonal form via transformations

$$z = \overline{T}x, \overline{A} = \overline{T}^{-1}A\overline{T}\,.$$

The problem consists of transferring the equation's solution

$$z(t) = \exp(\lambda(t-\tau))z(\tau) + (\exp(\lambda(t-\tau))-1)u \tag{2.24}$$

from its initial condition $z(t_o = s) = z(s)$ to a given final state $z(T)$=0 by applying the optimal control $u = u(s,\tau)$.

This control is represented by the piece-wise function of the state vector $z(s,\tau) = z(\tau_i)$ at the control's discrete points $\tau = \{\tau_i\}$ and with some coefficients of control's amplification

$$\mu = \{\mu_i\} : u = \mu z(s,\tau).$$

Matrix $\overline{A}$ is a piece-wise function, whose eigenvalues $\{\lambda_i\}$ are fixed at $\tau = \{\tau_i\}$ and are identified by the corresponding correlation functions $r(t)$ using

$$R(t) = 1/2\dot{r}(t)r^{-1}(t), R = \|R_i\|, r = \|r_i\|,\ r(t) = E[z(s,t)z^T(s,t)] \tag{2.25}$$

with $z(s,t)$ (2.24) at the corresponding time interval.

The amplification's coefficients satisfy the condition

$$z(\mu_1,...,\mu_{n-1},T) = (0_i)_{i=1}^n \tag{2.26}$$

at a sought final moment *T*.

We analyze this problem solution considering a three-dimensional matrix (3×3) in (2.25), with a maximum of three discrete intervals $(i = 1,2,3)$, applying the optimal controls at these intervals:

$$u_1 = -2z(s),\ u_2 = \mu_2 z(s+\tau_1) + (\mu_2+1)u_1,\ u_3 = \mu_3 z(s+\tau_2)\,. \tag{2.27}$$

The eigenvalue within the first discrete interval is identified by the relation

$$R_1(t-s)=-\lambda_1\exp(\lambda_1(t-s))[(2-\exp\lambda_1(t-s)]^{-1},$$
$$R_1(s-0)=\lambda_1, R_1(s+0)=-\lambda_1, \tag{2.28}$$

and within the second discrete interval we get

$$R_2(t-(s+\tau_1))=\lambda_2((t-(s+\tau_1))=\lambda_1[1-2(2-\exp(\lambda_1\tau_1)^{-1}]\exp[\lambda_2(t=(s+\tau_1)]$$
$$\times\{\exp[\lambda_2(t-(s+\tau_1)](1+\mu_2)[1-2(2-\exp(\lambda_1\tau_1)^{-1}]+$$
$$[2(1+\mu_2)(2-\exp(\lambda_1\tau_1)^{-1}-\mu_2]\}^{-1}. \tag{2.28a}$$

From that, at the moment $t=(s+\tau_1)$ we get

$$R_2(s+\tau_1)=\lambda_2=\lambda_1[1-\mu_2]^{-1}.$$

The last equation identifies the second eigenvalue as the function of the control's coefficient. The eigenvalue at third discrete interval $t=s+\tau_1+\tau_2$ is identified by relation

$$R_3(s+\tau_1+\tau_2+0)=\lambda_3=\lambda_1[1-2(2-\exp(\lambda_1\tau_1)^{-1}]\exp[\lambda_2(\tau_2)]$$
$$\times\{(1+\mu_3)[\exp[(\lambda_2\tau_2)](1+\mu_2)[1-2(2-\exp(\lambda_1\tau_1)^{-1}]$$
$$+[2(1+\mu_2)(2-\exp(\lambda_1\tau_1)^{-1}-\mu_2]\}^{-1}. \tag{2.28b}$$

Let have

$$\mu_2[2(2-\exp(\lambda_1\tau_1)^{-1}-1]+2(2-\exp(\lambda_1\tau_1)^{-1}=\beta[1-2(2-\exp(\lambda_1\tau_1)^{-1}]$$
at $\beta=-[\mu_2+2\exp(-\lambda_1\tau_1)^{-1}]$.

Then formula for λ_3 acquires the form:

$$\lambda_3=\frac{\lambda_1\exp[\lambda_2(\tau_2)]}{(1+\mu_3)\{[\exp[(\lambda_2\tau_2)](1+\mu_2)-[\mu_2+(2\exp(-\lambda_1\tau_1)^{-1}]\}} \tag{2.29}$$

and the solution of the equation at the last discrete interval is

$$z(t-(s+\tau_1+\tau_2))=[\exp(\lambda_3(t-(s+\tau_1+\tau_2))(1+\mu_3)-\mu_3]z(s+\tau_1+\tau_2). \tag{2.30}$$

This solution reaches the final state $z(T=s+\tau_1+\tau_2+\tau_3)=0$

if $\exp[\lambda_3(T-s+\tau_1+\tau_2)]=\mu_3[1+\mu_3]^{-1}$.

The equation (2.29) has a solution only at the condition

$$[\exp[(\lambda_2\tau_2)](1+\mu_2)-[\mu_2+(2\exp(-\lambda_1\tau_1)^{-1}]>0. \tag{2.30a}$$

Indeed. At

$$\mu_3 > 0,$$

we have

$$\mu_3[1+\mu_3]^{-1} < 1,$$

and therefore

$$\exp[\lambda_3(T - s + \tau_1 + \tau_2)] < 1,$$

from which

$$\lambda_3 > 0;$$

and at the condition (2.31) is fulfilled because

$$\lambda_1 < 0,\ 1 + \mu_3 < 0.$$

At

$$\mu_3 < -1 < 0, \mu_3[1+\mu_3]^{-1} > 1$$

we get

$$\exp[\lambda_3(T - s + \tau_1 + \tau_2)] > 1.$$

From that follows $\lambda_3 > 0$ and the fulfillment of (2.31), because

$$\lambda_1 < 0, 1 + \mu_3 > 0.$$

Using **c**ondition (2.30a) we come to relations

$$\mu_2(\exp(\lambda_2\tau_2) - 1)\exp(\lambda_2\tau_2) - 2\exp(-\lambda_1\tau_1) > 0,$$
$$\mu_2(\exp(\lambda_2\tau_2) - 1) > \exp(\lambda_2\tau_2) - 2\exp(-\lambda_1\tau_1). \qquad (2.30b)$$

Let

$$\mu_2 < -1,\ \lambda_2 < 0,\ \exp(\lambda_2\tau_2) < 1,$$

then the relation

$$\mu_2(\exp(\lambda_2\tau_2) - 1$$

is a negative at $\mu_2 > 0$, or it is a positive at $-1 < \mu_2 < 0$ and **ca**nnot exceed 1.
Relation

$$2\exp(-\lambda_1\tau_1) - \exp(\lambda_2\tau_2) > 1,$$

and therefore inequality (2.30b) cannot be true at $\mu_2 < -1$.
Thus we come to

$$\mu_2 < -1,\ \lambda_2 > 0 \text{ and } \exp(\lambda_2\tau_2) > 1,$$
$$0 > \mu_2 > [2\exp(-\lambda_1\tau_1) - \exp(\lambda_2\tau_2)][\exp(\lambda_2\tau_2) - 1]^{-1}.$$

This means that we have relations:

$$[2\exp(-\lambda_1\tau_1) - \exp(\lambda_2\tau_2)] < 0,$$
$$\exp(\lambda_2\tau_2) > 2\exp(-\lambda_1\tau_1),$$
$$\exp(\lambda_2\tau_2) > \exp(\ln 2)\exp(-\lambda_1\tau_1),$$
$$\lambda_2\tau_2 > \ln 2 - \lambda_1\tau_1,\ \lambda_1[1+\mu_2]^{-1}\tau_2 > \ln 2 - \lambda_1\tau_1,$$

$$\mu_2 > \frac{2\exp(-\lambda_1\tau_1) - \exp(\lambda_2\tau_2)}{\exp(\lambda_2\tau_2) - 1},$$

$$\mu_2 > \frac{2\exp(-\lambda_1\tau_1) - \exp(\lambda_2\tau_2) - 1 + 1}{\exp(\lambda_2\tau_2) - 1} = \frac{2\exp(-\lambda_1\tau_1) - 1}{\exp(\lambda_2\tau_2) - 1} - 1, \mu_2 < -1;$$

$$2\exp(-\lambda_1\tau_1) - 1 > 1,\ \lambda_2 > 0,\ \exp(\lambda_2\tau_2) - 1 > 0.$$

The following condition:

$$\mu_2 > -1 + \frac{2\exp(-\lambda_1\tau_1) - 1}{\exp(\lambda_2\tau_2) - 1} > 0$$

contradicts to $\mu_2 < -1$.

This means that μ_2 can be chosen equal -2.

Therefore, we finally get

$$\lambda_3 \cong - \lambda_1[1 + \mu_3]^{-1}$$

at the initial $\lambda_1(s = t_o) > 0$, and $\mu_3 = - 2$.

The optimal process is analogous to shown on Fig.1.5.3 (having just two discrete time intervals).

Thus, the optimal control in the form $v = -2x(\tau)$ *satisfies the considered Boundary-value problem.*

The IMD approach leads to the solution of both the optimal problem for information path functional and, in particular cases, to the Cauchy (Boundary-value) problem for a many-dimensional dynamic object.

Comparing to Bellman's method of dynamic programming [24], the IMD achieves the solution of a "bottleneck"-dimensional problem by the successive *consolidation* and reducing the model's dimensions and finally reaching a *single dimension* in the IN's upper level nodes (ch. 1.5).

The IMD model is able to reveal a nonlinear structure of an object, whose dynamics and structure are changed in the process of functioning, with a possibility of the sequential adaptive control and identification at each model's time-space interval.

The IMD also brings a constructive *integral measure* of current information, which evaluates both the concurrent observation and modeling, and leads to a direct computer implementation of the considered method and procedure of the joint optimal control, identification, consolidation, and filtration.

2.1.3.3. Building the Object Cooperative Information Network

The object random process, approximated by a sequence of the extremal segments; is identified by the object diffusion matrix $b_j(\tau_k)$, $j = 1,..n, k = 1,..,m$ (according to (2.1a)) at the segment's punched locality, preceding each current segment.

Thus, a total model's macrotrajectory is determined by the sequence of both the identified punched localities (window) and above diffusions at the window.

A cooperative macroprocess is formed by the sequential selecting of the extremal segments and sticking the nearest segments in a chain by applying both the regular and "needle" controls.

The object macrotrajectory with *n* extremal segments requires $(n-1)$ of such controls. Each of them needs a minimal quantity of information $(\mathbf{a} + \mathbf{a}_o^2) \cong \mathbf{a}_o$ bit to perform both control tasks.

Hence, the macrotrajectory can be encoded by a sequence of the segment diffusions (identified at the windows) and both the regular and the needle controls, producing the segments' chain.

If a quantity information needed to encode the diffusions is H_b, then a total chain can be encoded by quantity of information H_c equals

$$H_c \cong H_b + \mathbf{a}_o(n-1), \tag{2.31}$$

where a minimal codeword length is found using the Shannon formula for optimal coding.

A sequential process' consolidation leads to a cooperative information network (IN), built on a multi-dimensional spectrum of the object operator's eigenvalues, *identified* during the optimal motion.

The eigenvalues' spectrum forms an interacting chain, which is assembled in an *ordered organization* structure of the IN hierarchy.

Each IN node is encoded by the triplet code, and a total IN is encoded by a last node triplet's code.

The IN synthesis for a real object implements the formal results, considered in chs.1.5-1.6,1.8.

The space distributed IN's structural robustness is preserved by the feed-back actions of the inner controls $(v(\tau), \delta v(\tau))$, which provide *a local stability at the admissible* variations.

This control, acting within the P_a capabilities, supports a limited $P_e(\gamma) = P_r$ that determines the *potential of robustness* (ch.1.8) .

The potential's code, generated by the *potential of robustness*, is a DSS's evolutionary *predecessor*.

The requirements of preserving the evolutionary hierarchy (1.8.1)-(1.8.3) impose the restrictions on the maximal potential of evolution P_e and limit the variations, *acceptable* by the model.

The model adaptive potential $P_a \le P_e$, which adapts the variations, not compromising the IN hierarchy, restricts the maximal increment of dimension.

The punched evolution's nonequilibrium accumulates the changes by the hierarchy of selections and adaptations, with the following local equilibrium at each hierarchical level.

The self-control function includes the conditions (secs. 1.5.3, 1.8.3) of a proper *coordination* for a chain of superimposing processes, where each preceding processes adopts the acceptable variations of each following process.

The above optimal controls are synthesized, as an *inner feedback*, by the *duplication* of and *copying* of the current macrostates at the beginning of each segment, which are *memorized*.

The adaptive potential's *asymmetry* contributes the model's evolutionary improvement.

A sequence of the sequentially enclosed IN's *nodes*, represented by the *code logic*, is a *virtual communication language and* an algorithm of minimal program to *design* the IN.

A *double spiral* triplet structure, shaped at the localities of the sequential connected cones' spirals, forms the time-space path-line of transferring the IN information through the triplets' hierarchy.

The applied control adds the forth letter to the initial minimal three triplet's code letters, which provides the model *error correction mechanism* to the IN and its DSS code.

It also provides discrete *filtering* of the randomness, acting at the DP-window.

The IN's geometrical border (ch.1.6) forms the external surface where the macromodel is open for the outside interactions.

At this surface, the interacting states *compete* for delivering a maximum of dynamic potential's gradient.

The selected states, copied and memorized by the model control, contribute to the code. The control provides a *directional evolution* with the extraction of a maximum information from the environment of the competing systems, while the acquired DSS code can be passed to a successor.

A new born system sequentially evolves into anew IN that embraces its dynamics, geometry, and the DSS code.

The synthesized INs for the specific objects are shown on Figs. 1.5.5,1.6.3, 1.6.4.

Chapter 2.2

THE INFORMATION MODELING OF THE ENCODING-DECODING PROCESSES AT TRANSFORMATION OF BIOLOGICAL AND COGNITIVE INFORMATION

2.2.1. The Objective and Methodology

Using the Informational Macrodynamics (IMD), the chapter focuses on information modeling of the encoding-decoding mechanisms for the transformation and synthesis of some biological and cognitive functions that these mechanisms enable to generate.

Through the synthesis and encoding of information in the cooperative dynamics of cognitive information processing, we are trying to understand the information nature of some neurodynamic processes and intelligence.

As an alternative to *experimental* DNA discovery, the considered double spiral information structure (DSS) and its coding language implement the IMD analytical model. The pieces of the DSS information code are synthesized into meaningful units, integrated by an information network.

Moreover, we consider the double chain spiral structure (DSS) as a *common genetic generator for any optimal macrostructures.*

Like the fundamental DNA bio-mechanisms, many biological processes are well-known, while others are still subjects of extended bio-research and finding the systemic regularities.

The following important questions still are not resolved:

How is *valuable* information *encoded* into the DSS? How is the encoded DSS information *translated* along with its *decoding?* Are there any mechanisms that *count* the number of such translations? What is an effective *procedure* for the integration, synthesis, and compression of information into the DSS? Can these procedures be automatically created? What are the limitations on these processes?

By searching for the answers on these questions, we focus on the *information systemic essence* of these mechanisms, which are crucial for the understanding the regularities of the corresponding *biological mechanisms*.

The analysis of the cognitive information processing shows that its mechanism of information synthesis includes a preliminary encoding-decoding procedure, whose code's duration embodies the total time required for the translation of information, which characterizes also its life-time duration.

The applied methodology is based, first, on analyzing and modeling a *general* information structure of the information network and its DSS code for encoding and transferring information; secondary, on developing a mathematical model dealing with the interactive information flows for this general IN's structure; third, on revealing the regularities, mechanisms, and effective procedures for encoding, translating, decoding, concentrating, and synthesizing of information along with the limitations on these processes; and fourth, on utilizing these results for a wide scope of biological and cognitive applications.

The well-known compression methods [1,others](see the references to ch.2.2) are based on eliminating redundant information that increases the entropy, for example, the repeating symbols and images. These traditional methods compress only redundant information.

In addition, there are many specific technical restrictions on the compression ratio for each particular method.

We believe, a capability of *compression of a nonredundant* information represents a *uniqueness* of biosystems and a specific of human intellectual processing.

Conventional compression methods do not use the *dynamic* information network and its *dynamic* logic, which are created *naturally* by the observed processes.

Our goal is a maximal compression of *nonredundant* information, that means performing the compression *after* the elimination of redundancies, for example, by applying traditional methods and its "cognitive versions" considered below.

2.2.2. An Inner Information Structure of the IN with the Ranged and the Nonranged Sequences of the Starting Eigenvalues. The DSS Code

The developed information dynamic network (IN) (ch.1.5) represents a sequentially enclosed, nested, dynamic structure that arranges the input information into the IN starting string and then orders it by creating the hierarchical organized information nodes.

The IN structure describes the hierarchy of the informational connected spatial dynamic macromodels, generated by the corresponding stochastic microlevel's information processes, which are also used for the model identification.

The IN nodes carry information that enfolds the parameters of the model operator.

The nodes' interactive dynamics model both the micro-and macrolevel's interactions.

The series of IN nodes' information interactions convey a *dynamic logic* of the entire micro-macro model.

The IN is able to compress the total information into the ending node that accumulates the IN complete information, which can be used for the restoration of the enclosed spatial macrodynamics.

The IN functions include modeling the micro-macrodynamics, dynamic logic, concentration, ordering, and synthesis of information. The IN consists of the hierarchy of

interacting information doublets, forming the triplets, sequentially enclosed each other (Fig.1.5.5). Each triple node is a product of the *inner* interactive processes.

Here we intend to analyze the detailed (within a node) dynamics, which were substituted before by a "solid" node (Figs.1.5.5, 1.6.3).

The specifics of the IN consist of the possibility of binding the nodes' sequence by the *defects* of information, produced by the information flows that interact at each node.

A comprehensive analysis of the IN dynamics below contributes to the constructive understanding of the concentration and synthesis of information by the IN double spiral structure (DSS) of the code (ch.1.6).

Let us consider an arbitrary sequence of incoming information symbols $(s_1, s_2, \ldots, s_i, \ldots, s_m)$, which are encoded into a corresponding sequence of the IN symbols $(\gamma^1, \gamma^2, \ldots, \gamma^i, \ldots, \gamma^n) = \{\gamma^n\}$ (Fig.2.1).

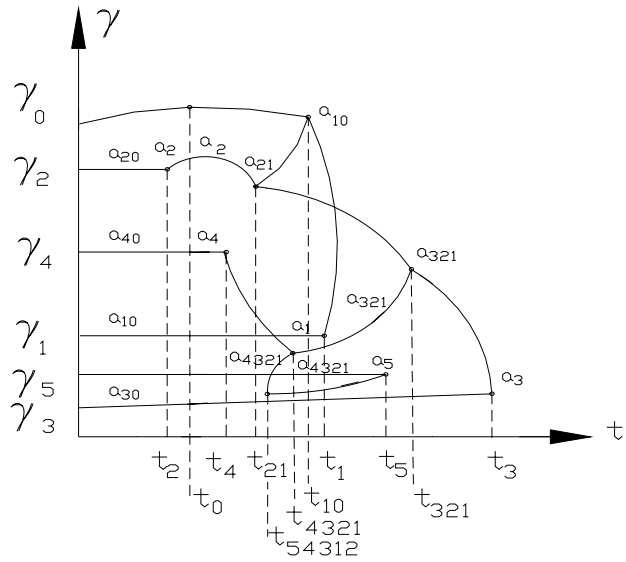


Figure 2.1. The sequence of the initial data symbols $\gamma_i(\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5)$, which are encoded into the nodes: $a_{i1}(a_1(t_1), a_{21}(t_{21}), a_{321}(t_{321}), a_{4321}(t_{4321}), a_{54321}(t_{54321}))$, $a_{i2}(a_1(t_2), a_{21}(t_{21}), a_{321}(t_{321}), a_{4321}(t_{4321}), a_{54321}(t_{54321}))$, $a_{i3}(a_3(t_3), a_{321}(t_{321}), a_{4321}(t_{4321}), a_{54321}(t_{54321}))$, $a_{i4}(a_4(t_4), a_{4321}(t_{4321}), a_{54321}(t_{54321}))$, $a_{i5}(a_5(t_5), a_{54321}(t_{54321}))$, $a_{i5}(a_5(t_5), a_{54321}(t_{54321}))$, and then are bound into the IN's final nodes $t_{ijkl,\ldots,1}(t_{54321})$; $\alpha_i(\alpha_1, \alpha_2, \ldots)$ are the network's arcs.

We assume that at the transformation of the current $\{s_i\}$ symbols to the $\{\gamma^i\}$ symbols, the repeating redundant symbols from the $\{s_i\}$ sequence are removed.

For this reason, the total m_s numbers of the $\{s_i\}$ sequence do not coincide in general with the total n_γ numbers of the $\{\gamma^i\}$ sequence. For example, a set of $\{\gamma^i\}$ symbols is able to encode a fixed alphabet, whose letters are ranged according to their information content, and

a current $\{s_i\}$ set carries a sequence of the nonranged word letters (w_i) from this alphabet. In this case, the corresponding collection of the IN starting symbols $\{\gamma^w\}$ will not be ranged. This is an example of an *arbitrary* current sequence $\{\gamma^i\}$.

If a *self-ranged* sequence $\{\gamma^i\}$ is generated for each corresponding incoming arbitrary sequence $\{s_i\}$, which is encoded by $\{\gamma^i\}$ in accord with some dictionary, the result is the *ranged* starting sequence, which was considered in ch.1.5.

The IN introduces a measure of information for each of these symbols: $(\alpha_{10}, \alpha_{20}, ..., \alpha_{i0}, ..., \alpha_{n0})$ that binds the sequence of $\{\gamma^i\}$ to the $\{\alpha_{i0}\}$ sequence, and the initial $\{s_i\}$ symbols acquire the IN information measure in terms of $\{\alpha_{i0}\}$.

The IN dynamic structure transforms $\{\alpha_{i0}\}$ to the corresponding $\{\alpha_i\}$ information symbols by a sequence of time-steps $\{t_i\}$ in such a way that the invariant relations $\alpha_{10} t_i = \mathbf{a}_o$ are accomplished at each step, where $\mathbf{a}_o$ is measured by a Shannon quantity of information that is preserved during each $\{t_i\}$ step's operations for a given IN.

The IN structural information invariant $\mathbf{a}_o(\gamma) = \mathbf{a}_{mo}(\mathbf{a}_o, \psi_m^o)$ (ch.1.6) provides the connection to a *total* Shannon quantity of the incoming information $\Delta S = n\mathbf{a}_o(\gamma)$ to be encoded into the IN, where n is the number of the initial $\{\gamma^i\}$ symbols or their information IN's equivalents $\{\alpha_{i0}\}$ (in particular at $n = n_\gamma$).

For the transformed sequence $\{\alpha_i\}$, the IN operations preserve another invariant $\mathbf{a}(\gamma)$ by the relations $\alpha_1 t_i = \mathbf{a}$, where both IN invariants are connected by the IN structural parameter γ, which depends on the sequences' ratio $\alpha_{i-1,0} / \alpha_{i0} = \gamma_i^{\alpha}(\gamma)$.

The IN cooperative dynamics are directed on a sequential assembling of the triplets' structures in such a way that each current triplet sends its information to the following doublet for joining with forming a new triplet.

The triplet's assembled information conveys it for generating the next triplet's structure.

This leads to the sequential encapsulation of the conveyed information into a final IN triplet, which accumulates exclusively the information $\mathbf{a}_o(\gamma)$ that has not been transferred to other IN's nodes.

The encapsulated information has increasing *value* (quality) (ch.1.6) compared to the incoming information. The information value of the current $\mathbf{a}_o(\gamma)$ is evaluated by the number of previous assembled triplets m through their information $\mathbf{a}_o(m)$, where m can also be identified via the corresponding resonance frequen**cy** $\omega_m(\alpha_m)$.(ch.1.6).

Thus, with a growing number of enclosed triplets, the IN dynamics sequentially reduce the flow of "free" information, transferred to each following nodes, with concentrating the minimal entropy $\mathbf{a}_o(\gamma)$ into the final node. This entropy defines the optimal code-word's length $l = \mathbf{a}_o(\gamma) / \ln n$ to encode-decode each of the n starting IN strings, which equals to $l \cong 1bit$ per n starting symbols at $\gamma = 0.5$.

For example, the IN can encapsulate the five subsequent nonredundant letters of the word: "robust" into the letter "t".

This word has a minimal Kolmogorov (K) complexity [2], measured by the word entropy (as a characteristic of the word randomness) and is incompressible by traditional methods.

By encoding this word into the IN starting symbols and applying the IN cooperation, the K complexity is minimized to its limit-the complexity of a single letter. Conventional methods minimize K and the corresponding "free" entropy by eliminating redundancies.

The considered compression minimizes the entropy *bound* into the IN nodes and provides the ability for a successively accumulation of the sequentially transmitted information.

Each triplet in the IN structure can be encoded, as a minimum, by the sequence of three IN string's symbols $\alpha_{1t}, \alpha_{2t}, \alpha_{3t}$. The examples and details are considered in sec.1.6.7, where its shown that each letter sequence of the triplet code can be represented by a corresponding sequence of the IN string's symbols or their ratios. The vice versa is also true: any sequence of the IN string can be encoded by the subsequence of the chosen alphabet's letters, and each triplet is encoded by a minimum of three letters from this alphabet.

The model *adaptive potential* (ch.1.8) generates additional informational capacity equal to ~1/3, bringing *one more letter* to the *nonredundant* three letters of the minimal DSS code. The adaptive capacity creates the model *potential redundancy*, which provides *the error correction mechanism* to the DSS minimal code with the total letter's permutations $4^3 = 64$.

The various (even redundant) combinations of these letters essentially extend the encoding string. This code's $m = n/2$ sequence describes the sequential IN model of the independent m-triplets. The sequence of the *enclosed* triplet's codes represents the IN's nested structure.

The quantity of information carried by the final triplet for a *four* letter's code equals

$$\mathbf{a}_o^2(\gamma) + \mathbf{a}(\gamma).$$

The initial string of symbols describe the object specifics and can be found by the object identification**;** the string can also classify the different objects in terms of their basic parameters (**n**, γ ,**k**) (representing the object's information path functional). The final IN's triplet carries the code of these parameters.

For a given IN with a fixed (**n=** n_*, $\gamma = \gamma_*$, **k=**k_*), each triplet has the same three letter code, but the microlevel influences and the mutations, which affect the parameters (**n**, γ ,**k**), are able to modify the code of the following triplets. This leads to diverse combinations of the code letters, generating an evolving DSS chain, which encodes the variations of the object characteristics. The fourth letter-symbol of the IN *fixed triplet code* carries the repeating ratio of the triplet's letters.

For example, α_{4t}=0.34891 brings the same ratio $\gamma_{34}^{\alpha} = \alpha_{3t} / \alpha_{4t}$=2.2155 as γ_{12}^{α} has (sec.1.6.7). This fourth letter compensates for the code possible errors, contributing to the adaptation potential. Under the mutations, this letter can bring a new ratio to the code sequence, which together with the ratio γ_{12}^{α}, identifies the code ratios γ_{23}^{α}, γ_{34}^{α} for a new modified triplet. This letter-symbol also serves as a *bridge* between each next triplet's code. Each triplet's code is responsible for the generation of three *superimposing processes*, and the *cross-phenomena* that they create.

The fixed IN's DSS encodes **n** such processes, arranged by the string's α_{it} values, which form a "solid" macrostructure with $m = \mathbf{n}/2$ cross-phenomena, sequentially enclosed in each other.

The DSS can also encode the triplets' sequences belonging to a *distinct*, fixed IN that brings a variety of macrostructures with different basic parameters.

Any such DSS code is not predetermined by the first triplet's code. The four letters of this extended DSS code would have more combinations in an expanded sequence.

The specific object's code depends upon particular triplet's sequence, which can be fixed, translated, or developed through the considered model of evolution under competition and superposition of the inheritance-mutation mechanisms (ch.1.8).

The code, being translated into appropriate environment, is able to build the correspondent object's spatial dynamic model, determined by the basic parameters (**n,** γ, **k**) that specify the model's operator eigenvalues $\{\alpha_{io}\}$ and invariants.

Example of the Bio-process' Code

The modeling of a simple auto regulatory process [3] by the IMD equations and the identified basic parameters (n=6, γ =0.2,k=6) reveals the following *minimal* DSS code for a particular IN (with the object's superimposing processes): (1, 2.423693, 1.821067).

The DSS code for other related model of the regenerative processes with basic parameters (**n**=6, γ =0.3,**k**=6) is (1, 2.366832, 1.798496).

The similarities between these codes can be seen. However, their differences vanish with essential growth the number of superimposing processes.

The examples demonstrate the possibility of developing a *universal* code structure for a variety of system models.

The information evolutionary cyclic model (ch.1.8) includes the mechanism of generating the genetic information through the inherited DSSo, mutations, adaptation, and self-organization, encoding, concentration and synthesis by the IN and the renovated DSS1 code, which can be transferred and inherited by other model's generation.

We consider here the process of encoding, concentration and synthesis of *new* information by the IN and the generation of the DSS1 code. The translation and decoding of this information belong to the generation of a newborn model.

2.2.3. Mathematical Model of the IN with an Arbitrary Sequence of the Starting Eigenvalues

Considering the *arbitrary* starting sequence $\{\gamma^i\}$, encoded into the IN corresponding nonranged $\{\alpha_{io}\}$ sequence, we develop the IN mathematical model for the Fig.2.1 schematics to reveal the IN *extensive regularities*.

The IN dynamics transform the $\{\alpha_i\}$ sequence into the $\{\alpha_i^*\}$ information sequence during a time interval T_i according to the formula

$$a_i^* = \frac{a_i T_i}{t_i}\{2\exp[a_i(T_i / t_i - 1)] - 1\}^{-1} \qquad (2.1)$$

where $a_i^* = \alpha_i^* T_i$ and $a_i = \alpha_i t_i = \mathbf{a}(\gamma_i)$, with the possibility of changing the structural parameter $\gamma \to \gamma_i$.

The same relation is true for any previous transformed symbols α_{i-1}^*:

$$a_{i-1}^* = \frac{a_i T_{i-1}}{t_{i-1}}\{2\exp[a_i(T_{i-1} / t_{i-1} - 1)] - 1\}^{-1},\ a_{i-1}^* = \alpha_{i-1}^* T_{i-1}\ . \qquad (2.2).$$

At the moment T_i, the information $a_{i-1}^*(T_{i-1})$ interacts with the information $a_i^*(T_i)$ generating information $A_i(T_i)$.

The reverse is also true: $a_i^*(T_i)$ interacts with $a_{i-1}^*(T_{i-1})$ generating the information $A_{i-1}(T_{i-1})$; A_{i-1}, A_i characterize information, *produced* at the node interactions.

The interactive information forms the elementary cyclic processes between the $i-1$ and i arcs and between the correspondent $i-1$, i nodes (Fig.2.2), and vice versa.

The interactions involve the sources of external entropies a_{i-1}^*, a_i^* at each t_{i-1} and t_i intervals accordingly, and the internal entropies A_{i-1}, A_i as the products of interactions at each previous T_{i-1}, next T_i, and following T_{i+1} intervals accordingly.

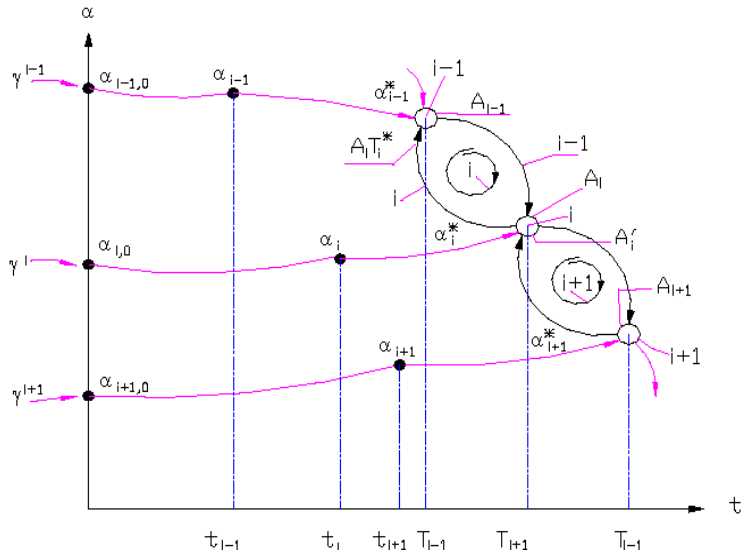


Figure 2. 2. An elementary cyclic information process between the IN's nodes $i-1, i$, and $i+1$.

The $A_{i-1}(T_{i-1})$ can also consume information from a previous $(i-2)$ cycle, and information $A_i(T_i)$ can contribute to a following $(i+1)$ cycle.

The information delivered to the i-node in a right directional cycle (Fig.2.2) is

$$A_{i-1} - a_{i-1}^* + a_i^*;$$

the information delivered to the $i-1$-node in an inverse cycle direction, is

$$A_i - a_i^* + a_{i-1}^*.$$

Both internal information (entropies) bind the nodes $(i-1,i)$ and satisfy the *balance* equation:

$$A_i - a_i^* + a_{i-1}^* = A_{i-1} - a_{i-1}^* + a_i^*. \tag{2.3}$$

The internal dynamic entropy for each cycle's arc, described by the equalities

$$A_{i-1}(\vec{T}_{i-1}^* - 1) \text{ and } \pm A_i(\overleftarrow{T}_i^* - 1) \tag{2.4}$$

of the corresponding entropy flows, should also satisfy the *balance* equation

$$A_{i-1}(\vec{T}_{i-1}^* - 1) = \pm A_i(\overleftarrow{T}_i^* - 1), \tag{2.5}$$

which takes into account the entropy flows in both directions.

The above equations use the indications

$$\vec{T}_{i-1}^* = \frac{T_i}{T_{i-1}}\{2\exp[A_{i-1}(T_i / T_{i-1} - 1)] - 1\}^{-1}, \tag{2.6}$$

$$\overleftarrow{T}_i^* = \frac{T_{i-1}}{T_i}\{2\exp[A_i(T_{i-1} / T_i - 1)] - 1\}^{-1}, \tag{2.7}$$

$$\vec{T}_i^* = \frac{T_{i+1}}{T_i}\{2\exp[A_i(T_{i+1} / T_i - 1)] - 1\}^{-1}. \tag{2.7a}$$

By coordinating the cyclical entropy flows with the external entropy sources for each node, we come to the following defects of information that contribute to the nodes' internal connections and virtually accumulate the bound information.

For the node with A_i, taking into account the signs $\mp A_i$ in (2.4), (2.5), we have the defect:

$$A_{i-1}\vec{T}_{i-1}^* \mp A_i - a_{i-1}^*.$$

For the node with A_{i-1} (at the signs $\mp A_i \overleftarrow{T}_i^*$) we get the analogous equality

$$\pm A_i \overleftarrow{T}_i^* + A_{i-1} - a_i^*.$$

With regard to the cycle's flows in the opposite direction, we obtain the balance equation

$$A_{i-1}\vec{T}_{i-1}^* \mp A_i - a_{i-1}^* = -(\pm A_i \overleftarrow{T}_i^* + A_{i-1} - a_i^*) \tag{2.8}$$

from which we come to

$$A_{i-1}\vec{T}_{i-1}^* + A_{i-1} - a_{i-1}^* = \mp A_i \overleftarrow{T}_i^* \pm A_i + a_i^*, \tag{2.8a}$$

or to

$$A_{i-1}(\vec{T}^{*}_{i-1}+1)-a^{*}_{i-1}=\mp A_i(\overleftarrow{T}^{*}_{i}-1)+a^{*}_{i}. \tag{2.8b}$$

By getting to the next $(i+1)$, or $(i-2)$ step, the signs + will be changed on $-$, and the equalities (2.6),(2.7),2.7a) keep $\overleftarrow{T}^{*}_{i} \neq \vec{T}^{*}_{i}$, because both of them depend on diverse functions. The set of equations (2.1-2.8), at the known independent and nonranged variables t_{i-1}, t_i, T_{i-1}, T_i, define a^{*}_{i-1}, a^{*}_{i}, and then A_{i-1}, A_i.

The arrows in the above equations reflects the *irreversibility* of the information transformations for both *within* each cyclic process and *between* the cycles.

This means that all direct and inverse processes are nonequivalent, creating an entropy production according to Irreversible Thermodynamics and Information Macrodynamics.

The interacting information flows (Fig.2.2) characterize the consolidating sequence of each pair of doublets, representing a cooperative triplet's structure in the IN.

The transfer of information from the i doublet to the $i+1$ doublet should satisfy the following balance equations, preserving both the entropy and its entropy production in the form

$$A_i + a^{*}_{i} = A^{'}_{i}, \tag{2.9}$$

$$(A_i + a^{*}_{i})(T_i)^{-1} = A^{'}_{i}(T^{'}_{i})^{-1}, \tag{2.10}$$

where the $A^{'}_{i}$ defect of information is transformed into the $i+1$ node at the moment $T^{'}_{i}$. Generally, $A_i \neq A^{'}_{i}$ and $T_i \neq T^{'}_{i}$.

However, the fulfillment of both equations (2.9) and (2.10) leads to $T_i = T^{'}_{i}$ at $A_i \neq A^{'}_{i}$. This is acceptable taking into account the continuity of the times T_i and $T^{'}_{i}$ at the same i node.

Finally, an arbitrary elementary triplet (Fig.2.2.) is described by the system of basic nonlinear equations: (2.9)-(2.10) and also

$$A_i - a^{*}_{i} + a^{*}_{i-1} = A_{i-1} - a^{*}_{i-1} + a^{*}_{i}, \tag{2.11}$$

$$A_{i-1}(\vec{T}^{*}_{i-1}+1) - a^{*}_{i-1} = \mp A_i(\overleftarrow{T}^{*}_{i}-1)+a^{*}_{i}, \tag{2.12}$$

$$A_{i-1}(\vec{T}^{*}_{i-1}-1) = \pm A_i(\overleftarrow{T}^{*}_{i}-1). \tag{2.13}$$

Looking at the functions in (2.1),(2.2),(2.6),(2.7),(2.7a), it's seen that these Eqs. depends of current t_i, T_i, A_i at given $t_{i-1}, T_{i-1}, A_{i-1}$.

By summing (2.11-2.12) and sequentially considering both of their opposite signs we get the equivalent equation

$$2\vec{T}^{*}_{i-1}A_{i-1} = a^{*}_{i} + a^{*}_{i-1}, \tag{2.14}$$

where the system (2.11, 2.13) and (2.14) models an initial doublet. The source of external entropy for this doublet is the initial string $\{\alpha_{io}^t\}$ according to the relations

$$\alpha_{io}^t t_i = \mathbf{a}_o(\gamma_i),\ \mathbf{a}(\gamma_i) = \alpha_i^t\, t_i = a_i,\ \alpha_i^t / \alpha_{io}^t = \mathbf{a}(\gamma_i)/\mathbf{a}_o(\gamma_i),\ i = 1,...,n, \qquad (2.15)$$

where $\mathbf{a}_o(\gamma_i)$, $\mathbf{a}(\gamma_i)$ are the invariants, which measure the elementary quantity of information for a chosen IN.

A_{i-1} stands for a current quantity of information, accumulated into a local $i-1$-node during each previous cycle; A_i stands for a current quantity of information, accumulated into a local i-node during each of the following cycles.

Each previous $i-1$-cycle is connected to the following i cycle by a common i node having, in general, the distinct parameters T_i, A_i and $T_i^{'}$, $A_i^{'}$ for its corresponding previous and following cycles.

This means that for each following i cycle, $T_i^{'}$ represents an analog of an initial moment T_{i-1} in the previous $i-1$-cycle.

This assumption allows us to sequentially apply the same equations (2.11)-(2.14) for both the previous and following cycles, with $A_i^{'}$, $T_i^{'}$, T_{i+1}, in the i cycle and with $A_{i+1}^{'}$, $T_{i+1}^{'}$, T_{i+2} in the $i+1$-cycle, where each cycle models the IN's doublet elementary structure.

Thus, applying the initial equations (2.11)-(2.14) for both the $i-1$-cycle and the following i cycle with $A_i^{'}$, $T_i^{'}$, T_{i+1}, we model an initial first triplet. Then considering the triple ($A_i^{'}$, $T_i^{'}$, T_{i+1}), ($A_{i+1}^{'}$, $T_{i+1}^{'}$, T_{i+2}), we model the second triplet, and so on.

The triplet is formed as a result of a sequential adjoining of information generated in each $i, i+1$ cycle, each of which models the interactions of the inner processes into a joint node (Fig. 2.2). The above equations describe the inner node's cyclic processes, which previously were substituted by a "solid" node.

The model (Fig. 2.2) coincides with basic IN's model (Fig.1.5.5) at the condition

$$T_1 = T_2 = T_3 = t_3 \qquad (2.15a)$$

for each triplet. In the above equations, each $i+1$-node of the IN system's structure is based only on incoming information from the i-node, and vice versa. Each $(i, i+1)$ element of the IN system's model enhances all necessary information about the IN total structure.

This result has a more general meaning: the identification of any system's element can provide a *comprehensive information* regarding the *whole system.*

The equations open the *possibility* of encoding a *nonranged* sequence of the IN's i-nodes into the final n-node, starting with the node i=1.

At given the initial string $\{\alpha_{io}^t\}$, the values of t_{i-1}, t_i,..., determined by the ~~above~~ invariant equations (2.15), can encode, for example, the known alphabet's letters.

Then, T_{i-1}, T_i and corresponding $T_i^{'}, T_{i+1}^{'}$ can be found. At given t_i, T_i and α_{io} (or any γ_i, A_i), a *potential* decoding procedure will be able to compute sequentially T_{i-1}, t_{i-1} and α_{i-o}.

A direct application of these nonlinear equations requires a complex mathematical analysis and an extensive *numerical* procedure that may not lead to a *precise, simple, and explicit* decoding procedure.

The computer program for the numerical solution of this system has been developed by Dr. A. Treyger.

An *analytical* solution would be more useful for analyzing the *regularities* in the transformation of information by the IN.

After the *linearization* of the equations (2.11)-(2.14) we come to the corresponding simpler system (2.16)-(2.18), which we will write at first for the initial elementary doublet and then apply it sequentially, according to the above assumption, to the first and following triplets:

$$\frac{A_1}{a} = \frac{A_2}{a} - 2\frac{T_2}{t_2} + 2\frac{T_1}{t_1},\ a = a_i = const, \tag{2.16}$$

$$\frac{A_1}{a}(\frac{T_2}{T_1} - 1) = \pm\frac{A_2}{a}(\frac{T_1}{T_2} - 1), \tag{2.17}$$

$$2\frac{A_1}{a}(\frac{T_2}{T_1}) = \frac{T_2}{t_2} + \frac{T_1}{t_1}. \tag{2.18}$$

Using only equation (2.18) we get

$$\frac{A_1}{a} = \frac{1}{2}\frac{T_1}{t_2}(1 + \frac{T_1}{T_2}\frac{t_2}{t_1}), \tag{2.19}$$

which by substitution into (2.16) leads to:

$$\frac{1}{2}\frac{T_1}{t_2}(1 + \frac{T_1}{T_2}\frac{t_2}{t_1}) = \frac{A_2}{a} - 2\frac{T_2}{t_2} + 2\frac{T_1}{t_1}, \tag{2.19a}$$

$$\frac{T_1}{t_1}(\frac{T_1}{T_2} - 4) = 2\frac{A_2}{a} - 4\frac{T_2}{t_2} - \frac{T_1}{t_2}. \tag{2.19b}$$

We come to equality

$$t_1 = \frac{T_1(\frac{T_1}{T_2} - 4)}{2\frac{A_2}{a} - \frac{T_2}{t_2}(4 + \frac{T_1}{T_2})}. \tag{2.20}$$

After the substitution t_1 from (2.20) and $\frac{A_1}{a}$ from (2.19) into (2.17), considering only the + sign, we get the quadratic equation regarding T_1 (or corresponding T_2):

$$T_1^2 - 4[\frac{T_2}{t_2}(\frac{a}{A_2}) + 1]T_2T_1 + [4\frac{T_2}{t_2}(\frac{a}{A_2}) + 3]T_2^2 = 0. \tag{2.21}$$

This equation allows the analytical solution by the formulas

$$\frac{T_1}{T_2} = 2[\frac{T_2}{t_2}(\frac{a}{A_2}) + 1] \pm [2\frac{T_2}{t_2}(\frac{a}{A_2}) + 1]. \tag{2.21a}$$

From which at the sign + we get

$$\frac{T_1}{T_2} = 4\frac{T_2}{t_2}(\frac{a}{A_2}) + 3, \tag{2.22}$$

or

$$\frac{A_2}{a} = \frac{4\frac{T_2}{t_2}}{\frac{T_1}{T_2} - 3} \tag{2.23}$$

that limits $\frac{T_1}{T_2} \neq 3$.

The solution of (2.21) at

$$\frac{T_2}{t_2}(\frac{a}{A_2}) = -\frac{1}{2}, \tag{2.24}$$

leads to $T_1 = T_2$, which corresponds to repeating the encoding sequence. The same result takes place at the sign $-$ in (2.21a). Only the condition

$$\frac{T_2}{t_2}(\frac{a}{A_2}) > -\frac{1}{2} \tag{2.24a}$$

determines not the trivial and the real solutions of (2.21) at the above limitation.

Considering an inverse process, we take the sign $-$ in (2.18), and for the system (2.16)– (2.18) we get the following quadratic equation

$$(T_1^-)^2 - [6 - \frac{4T_2^-}{t_2^-}(\frac{a}{A_2})^-]T_2^-T_1^- + [5 - \frac{4T_2^-}{t_2^-}(\frac{a}{A_2})^-](T_2^-)^2 = 0 \tag{2.25}$$

with the solution

$$T_1^- = \frac{T_2^-}{2}\{[6 - \frac{4T_2^-}{t_2^-}(\frac{a}{A_2})^-] \pm [(6 - \frac{4T_2^-}{t_2^-}(\frac{a}{A_2})^-)^2 - 20 + \frac{16T_2^-}{t_2^-}(\frac{a}{A_2})^-]^{1/2}\}. \tag{2.25a}$$

Taking the sign + we get

$$\frac{T_1^-}{T_2^-} = 5 - \frac{4T_2^-}{t_2^-}(\frac{a}{A_2})^- \ . \tag{2.26}$$

The sign $-$ leads to $\frac{T_1^-}{T_2^-} = 1$, which is useless at arbitrary t_2^- as well as the conditions

$$\frac{4T_2^-}{t_2^-}(\frac{a}{A_2})^- = 6,\ \frac{4T_2^-}{t_2^-}(\frac{a}{A_2})^- = 5, \text{ and } \frac{T_2^-}{t_2^-}(\frac{a}{A_2})^- = 1. \tag{2.26a}$$

These and above limitations are essential while using the solutions (2.22), (2.26) for both encoding and decoding.

Using the indications $\frac{T_1}{T_2} = x_1$, $\frac{T_1^-}{T_2^-} = x_1^-$, we write the solution (2.22) in the form

$$\frac{A_2}{a} = \frac{4T_2}{t_2(x_1 - 3)} \tag{2.27}$$

for any $x_1 \neq 3$ **,** and the solution (2.26) in the form

$$(\frac{A_2}{a})^- = -\frac{4T_2^-}{t_2^-(x_1^- - 5)} \tag{2.27a}$$

for any $x_1^- \neq 5$. The equation (2.18) acquires a simple form

$$A_1 = -x_1 A_2, A_1 = A_1^{'} \tag{2.28a}$$

for the first doublet and

$$A_i^{'} = -x_i A_{i+1} \tag{2.28b}$$

for any i-th doublet, where according to (2.23):

$$\frac{A_{i+1}}{a} = \frac{4T_{i+1}}{t_{i+1}(x_i - 3)} \ . \tag{2.28}$$

Considering the next doublet, we add to the system (2.16)-(2.19) the linearized equations (2.9)-(2.10) in the form

$$\frac{A_2^{'}}{a} = \frac{A_2}{a} + \frac{T_2}{t_2} \ . \tag{2.29}$$

By substituting ~~the~~ solution (2.27) into (2.29) we get the equation:

$$\frac{A_2^{'}}{a} = \frac{T_2}{t_2}(\frac{x_1 + 1}{x_1 - 3}) = \frac{A_2}{a}\frac{x_1 + 1}{4},\ \frac{A_2^{'}}{A_2} = \frac{x_1 + 1}{4} \tag{2.30}$$

that connects $A_2^{'}$ and A_2, at any $x_1 \neq 3, x_1 \neq -1$.

Using (2.28a), we come to the connection between the entropies for previous A_2 and the following doublets:

$$\frac{A_3}{A_2} = -\frac{x_1 + 1}{4x_2}, \tag{2.31}$$

which can be applied for any i, $i+1$ doublets or a corresponding triplet:

$$\frac{A_{i+1}}{A_i} = -\frac{x_i + 1}{4x_{i+1}}, \ \frac{A'_{i+1}}{A_{i+1}} = \frac{x_i + 1}{4}. \tag{2.31a}$$

Applying the solution (2.27) to the next doublet we have

$$\frac{A_3}{a} = \frac{4T_3}{t_3(x_2 - 3)}, \tag{2.32}$$

and considering the ratio of relations (2.27) and (2.28) we get

$$\frac{x_1 - 3}{x_2 - 3} = \frac{A_3 t_3}{A_2 t_2} x_2. \tag{2.33}$$

Substituting (2.31) into (2.33) we come to

$$\frac{x_1 - 3}{x_2 - 3} = -\frac{x_1 + 1}{4}\frac{t_3}{t_2}, \text{ or } b_2 = -\frac{(x_1 + 1)(x_2 - 3)}{4(x_1 - 3)} = \frac{t_2}{t_3}. \tag{2.34}$$

The equation (2.34) connects the ratio of encoding symbols t_2, t_3 to their IN's reflections x_1, x_2.

This formula can be extended to connect any $i, i+1$ nodes:

$$b_i = -\frac{(x_{i-1} + 1)(x_i - 3)}{4(x_{i-1} - 3)} = \frac{t_i}{t_{i+1}}, \tag{2.34a}$$

where for any $i-1, i$, $(x_{i-1}, x_i) \neq 3, x_{i-1} \neq -1$.

Using both the inverse solution (2.27) and the direct solution (2.26a), we can find their dependencies.

By substituting (2.27) into (2.26a) we get

$$x_1^- = 8 - x_1, \ x_1^- \neq -x_1, \ (x_1^-, x_1) \neq 8 \tag{2.35}$$

if holds true the equality

$$\frac{T_2^-}{t_2^-}\left(\frac{a}{A_2}\right)^- = \frac{T_2}{t_2}\left(\frac{a}{A_2}\right). \tag{2.35a}$$

Below we will show that equality (2.35a) is satisfied identically if (2.35) is true and vice versa.

These equations acquire a more general form for any $i-1, i$ doublets:

$$x_{i-1}^{-} + x_{i-1} = 8,\ \ x_{i-1}^{-} \neq -x_{i-1},\ (x_{i-1}^{-}, x_{i-1}) \neq 8, \tag{2.35b}$$

$$\frac{T_i^{-}}{t_i^{-}}(\frac{a}{A_i})^{-} = \frac{T_i}{t_i}(\frac{a}{A_i}). \tag{2.35c}$$

Using these equations, we can write the corresponding analogies of (2.30-2.31) for the inverse process:

$$(\frac{A_2^{'}}{A_2})^{-} = -\frac{x_1^{-} - 9}{4},\ \ A_2^{'-} = -x_2^{-}A_3^{-},\ (\frac{A_3}{A_2})^{-} = \frac{x_1^{-} - 9}{4x_2^{-}},\ x_1^{-} \neq 9. \tag{2.36}$$

By analogy to equation (2.33) we get

$$(\frac{A_3}{A_2})^{-} = \frac{x_1^{-} - 5}{x_2^{-}(x_2^{-} - 5)}\frac{t_2^{-}}{t_3^{-}}. \tag{2.36a}$$

Using both equations (2.36) and (2.36a), we get for the inverse process:

$$b_2^{-} = \frac{(x_1^{-} - 9)(x_2^{-} - 5)}{4(x_1^{-} - 5)} = \frac{t_2^{-}}{t_3^{-}} \tag{2.37}$$

and applying (2.35) we come to the connection of the discrete intervals for both processes

$$b_i^{-} = \frac{(x_{i-1}^{-} - 9)(x_i^{-} - 5)}{4(x_{i-1}^{-} - 5)} = -\frac{(x_{i-1} + 1)(x_i - 3)}{4(x_{i-1} - 3)} = b_i = \frac{t_i^{-}}{t_{i+1}^{-}}. \tag{2.37a}$$

Writing the ratio of the equations (2.35a) for $i-1, i$ doublets, we have

$$\frac{T_{i-1}^{-}t_i^{-}}{T_i^{-}t_{i-1}^{-}}(\frac{A_i}{A_{i-1}})^{-} = \frac{T_{i-1}t_i}{T_i t_{i-1}}(\frac{A_i}{A_{i-1}}),\ \text{or}\ \frac{x_i^{-}b_i^{-}(x_{i-1}^{-} - 5)}{x_i^{-}b_i^{-}(x_i^{-} - 5)} = \frac{x_i b_i (x_1 - 3)}{x_i b_i (x_i - 5)} \tag{2.37b}$$

after the substitution into (2.37b) the corresponding relations (2.36a,b).

The last relation is fulfilled identically by applying (2.35).

This proves that equation (2.35) is the necessary and the sufficient condition connecting the direct and inverse processes.

The generated inverse process interacts with the direct information process in such a way that both of them create a local dynamic stability within each examined cycle according to the balance equations.

The specifics of the system consist of enclosing the complete IN information's prehistory in the information, stored by the final node's $A_n^{'}$ according to the chain of relations:

$$A_n^{'} = A_n + a_n^{*},\ A_n = -\frac{A_{n-1}^{'}}{x_{n-1}},\ A_{n-1}^{'} = A_{n-1} + a_{n-1}^{*}, A_n^{'} = a_n^{*} - \frac{1}{x_{n-1}}(A_{n-1} + a_{n-1}^{*}),$$

$$A_{n-1} = -\frac{A_{n-2}^{'}}{x_{n-2}},\ A_{n-2}^{'} = A_{n-2} + a_{n-2}^{*}, A_n^{'} = a_n^{*} - \frac{1}{x_{n-1}}(-\frac{1}{x_{n-2}}(A_{n-2} + a_{n-2}^{*}) + a_{n-1}^{*}),$$

$$A_n^{'} = a_n^* - \frac{1}{x_{n-1}}(-\frac{1}{x_{n-2}}(-\frac{1}{x_{n-3}}(A_{n-3} + a_{n-3}^*) + a_{n-2}^*) + a_{n-1}^*),.... \quad . \tag{2.38}$$

It can be seen that the final node's information is not equal to the simple sum of the input information $a_{\Sigma}^* = \sum_{i=n}^{i=1} a_i^*$, because it contains the information $\{A_i\}$, generated by the interactions. In addition, the correspondent chain of nonlinear relations (by (2.8)– (2.10)) includes the inner connections between A_i and a_i^*.

All chain is actually *embedded* in the final node's information.

The IN *macrosystemic complexity* measures a *cooperative* complexity (ch.1.7) and arises not as a simple sum of the IN's interacting components but as their *superimposing* contributions accompanied by the creation of new information.

From equations (2.38) it follows that the final node does not simply accumulate the IN's compressed input information**,** but rather *synthesizes* it. In the IN's hierarchy, each previous node synthesizes the information of all preceding nodes.

The synthesis depends on the particular information contributions of the current pairs a_i^*, a_{i+1}^* to the acting IN's interactive dynamics and therefore cannot be predicted a priori.

The synthesized information incorporates the encoded symbols.

2.2.4. The Procedure of Encoding, Compression, Synthesis, and Decoding of the IN Information

Encoding begins with assigning the corresponding information measures $\{\alpha_{io}\}$ to the initial nonredundant $\{\gamma^i\}$ symbols. After that, the IN dynamics execute the transformations $\{\alpha_{io}\} \to \{\alpha_i\} \to \{t_i\} \to \{T_i\} \to \{A_i\}$, including the interactive dynamics between each of the $i-1, i, i+1$ cycles and within them. This leads to the encoding of each pair t_i, t_{i+1} into corresponding pairs T_i, T_{i+1}, or b_i into x_i according to formula (2.34a).

The initial conditions for the considered system T_1, A_1 are supposed to be given along with the known t_1, t_2. Instead of that, the very first x_1 can be given. By continuing sequentially the encoding procedure according to (2.34a), we come to the final node with T_n, A_n, t_n, or

$$x_{n-1} = \frac{4T_n a}{t_n A_n} + 3, \tag{2.39}$$

which accumulates the total encoded information.

This procedure can contain an *unlimited* number of encoding symbols. It does not require the memorization of the encoded $\{\gamma^i\}, \{\alpha_{io}\}, \{t_i\}, \{T_i\}$ sequences if the connection between $\{\alpha_{io}\}$ and $\{\gamma^i\}$ is given by the initial encoding relations. It is also unnecessary to range the initial symbols $\{\gamma^i\}$ as well as $\{\alpha_{io}\}$. The decoding procedure can start with the identification of x_{n-1} at the given T_n, A_n, t_n using (2.39).

However, to find the previous $T_{n-1}, A_{n-1}, t_{n-1}$, or x_{n-2}, and for a successive implementation of the formula (2.34a), some of these symbols have to be given, such as A_{n-1}, which according to formula (2.31a) defines x_{n-2} at given x_{n-1}, and so on. Assigning of the sequence of A_{n-1}, A_{n-2} ,...is the equivalent of a given sequence x_{n-2}, x_{n-3},....

The above decoding equations imply that the sequence of the initial symbols $\{\gamma^i\}$ should be fixed and assigned a priory and therefore the sequence of the t_{n-1}, t_{n-2} ,… should also be assigned. An initial arbitrary sequence of the $\{\gamma^i\}$ symbols cannot be decoded by the above procedure. Moreover, any equations (nonlinear or linear) directly connecting the sequence x_{n-2}, x_{n-3},… (in an addition to the basic encoding equation (2.34a)) during or after the encoding procedure, *bind* the initial $\{t_i\}$ symbols.

An assumption regarding the possible nonlinear connections t_{n-1}, t_{n-2} **, e**mbedded into a sequence of the parameters $b_{n-1}(t_{n-1}, t_n)$ and $b_{n-2}(t_{n-2}, t_{n-1})$, also brings the (t_{n-2}, t_{n-1}, t_n) dependency that makes decoding of the arbitrary symbol's sequence problematical.

The procedure should also take into account the restrictions, imposed by the relations (2.24a) and inequalities following from (2.34a).

The decoding of previous events' occurrence requires the reconstruction of the events' *prehistory*. To initiate the reconstruction and decoding, any known process $x_{n-1} = f(x_{n-2}, x_{n-3}, \ldots)$ can be used.

A simple one should be connected with the above systems' equations.

It would be most natural to apply the inverse process for this purpose, as an intrinsic part of the system.

We have the following connections between the inverse and direct processes:

$$\frac{A_2^{'-}}{A_2^-} = -\frac{x_1^- - 9}{4} = \frac{x_1 + 1}{4} = \frac{A_2^{'}}{A_2}, \frac{A_2^-}{A_2} = \frac{A_2^{'-}}{A_2^{'}}, \frac{T_2 a}{t_2 A_2} = \frac{x_1 - 3}{4}, \tag{2.40}$$

$$\frac{T_2^- a}{t_2^- A_2^-} = -\frac{x_1^- - 5}{4} = \frac{x_1 - 3}{4} = \frac{T_2 a}{t_2 A_2}, \frac{T_2^-}{t_2^-} = \frac{T_2 A_2^-}{t_2 A_2}, \tag{2.40a}$$

where relations (2.40a) directly prove the correctness of (2.35a).

Using (2.30),(2.36), we come to the equalities

$$\frac{T_2}{t_2} = \frac{A_2^{'}}{a}(1 - \frac{A_2}{A_2^{'}}), \frac{T_2^-}{t_2^-} = \frac{A_2^{'-}}{a}(1 - \frac{A_2^-}{A_2^{'-}}), \tag{2.41}$$

whose ratio at $\frac{A_2^{'}}{A_2} = \frac{A_2^{'-}}{A_2^-}, \frac{A_2^-}{A_2} = \frac{A_2^{'-}}{A_2^{'}}$ leads to $\frac{T_2^-}{t_2^-} = \frac{T_2}{t_2}$.

From that, by applying (2.40a), we get $\frac{A_2^-}{a} = \frac{A_2}{a}$.

At these conditions, equation (2.35a) leads to

$$\frac{T_2 t_3}{T_3 t_2} = \frac{T_2^- t_3^-}{T_3^- t_2^-},$$

which at the fulfillment of $b_2 = b_2^-$, brings the equality

$$x_2 = \frac{T_2}{T_3} = \frac{T_2^-}{T_3^-} = x_2^-. \tag{2.42}$$

Equations (2.40)-(2.42) sequentially bind the known x_{n-1} not only to x_{n-2} and the following $x_{n-3}, x_{n-4}, ...$ but also connect the sequence of $t_n, t_{n-1}, t_{n-2}, \ldots$

This does not allow us to encode and decode arbitrary symbols $\{\gamma^i\}$, even though a lossless decoding is possible according to (2.37a). Such decoding orders an arbitrary $\{\gamma^i\}$.

The fulfillment of (2.42) leads to the fixed $x_{n-1} = \frac{T_{n-1}}{T_n} = 4$ and to the sequence of

$$\frac{t_{n-1}}{t_n} = -\frac{x_{n-2}+1}{4(x_{n-2}-3)}, \ \frac{t_{n-2}}{t_{n-1}} = -\frac{x_{n-3}+1}{4(x_{n-3}-3)}, ..., \tag{2.43}$$

leading to repeating the time ratios:

$$\frac{t_2}{t_3} = -\frac{5}{4} = \frac{t_i}{t_{i+1}} \ \text{ and the following } \frac{t_2}{t_4} = \frac{25}{16}, \ \frac{t_2}{t_5} = -\frac{125}{64}. \tag{2.43a}$$

The process starts with $\frac{t_2}{t_3}$, because according to eq.(2.20): $t_1 = 0$ at $x_1 = 4$.

A secondary encoding, applied after that, brings periodically repeating b_i and x_i.

The inverse process, applied for decoding, therefore, is able to *regularize* the initial arbitrary and generally irregular (random) sequence of the $\{\gamma^i\}$ symbols leading to their *ordering*. In general, the law of regularity complies with the equations (2.35),(2.39),(2.40) for any inverse process that determines the sequence of

$$x_{i-2} = f(T_{i-1}, A_{i-1}, t_{i-1}),\ T_{i-1} = T_i x_{i-1},\ A_{i-1} = 4\frac{x_{i-1}}{x_{i-2}-3},\ t_{i-1} = t_{i-2}\,/\,b_i,$$

or $x_{i-2} = f(x_{i-1}, t_{i-1})$.

For instance, the last relation can be given in a simple form $x_{i-2} = kx_{i-1}$, which according to (2.34a) leads to

$$b_{n-1} = -\frac{(kx_{n-1}+1)(x_{n-1}-3)}{kx_{n-1}-3}. \tag{2.43b}$$

For example, at $k = 2.5, x_{n-1} = 2$, we get the sequence of

$$b_{n-1} = 3,\ b_{n-2} = -2.84,\ b_{n-3} = -12.13, ...$$

This means that given k, x_{n-1}, n, or k, x_1, n determine the encoding-decoding sequence and the IN structure along with the IN code.

These three parameters can be encoded in the final IN's node analogous to the basic IN.

The inverse process' symbols $\bar{x}_{i-1}, \bar{x}_{i-2}, ..$ carry the direct connection to the encoding process' symbols $x_{i-1}, x_{i-2}, ..$ if they execute the equality (2.42) for any $i-2, i-3, \ldots$ by moving in the inverse time direction and working with the assumption $\bar{t}_i = t_i, \bar{T}_i = T_i$.

However, even for such conditions, the encoded connection between each T_{i-1} and t_{i-1} is erased by the dependency on the inverse and direct processes. At the interaction of direct and indirect information flows, a chaotic process with a "mix up" might arise within each cycle (Fig. 2.2). This leads to an erasure of the prehistory and the impossibility of precise restoration of the encoded symbols. In an attempt to decode this bound process, a fixed sequence of the ranged symbols $t_i^*, t_{i-1}^*, t_{i-2}^*, \ldots$ is identified, which is dependent upon the *equation of the inverse* information process being used for decoding.

The IN's decoding procedure can exist only for a given fixed sequence of initial symbols, which can be ranged by the IN. The ranged sequence of $\{t_i^*\}$ *defines a code of a secondary created IN*, which is analogous to the above considered IN code.*

For the symbols, ranged in the IN (Fig.1.5.5), the connection between each fixed and previous node is given by the IN's structural parameter γ, and the decoding procedure can easily be performed by the recurrent relations $\alpha_{n-1,o} = \alpha_{no}\gamma_i^{\alpha}(\gamma)$.

Compared the model (Fig. 2.1) to the basic IN (Fig.1.5.5), satisfying the condition (2.15a), which corresponds to $x_1 = x_2 = 1, \frac{t_3}{T_2} = 1$, we get, according to eq.(2.34a),

$$\frac{t_2}{t_3} = -\frac{1}{2}, \frac{T_2}{t_2} = -2.$$

By substituting those into (2.39) we have $\frac{A_2}{a} = -4$, and then using the equation (2.20) we get $\frac{t_1}{T_1} = -1.5$ and $\frac{t_1}{t_2} = 3$.

These ratios are closely related to the basic IN parameters $\gamma_i^{\alpha}(\gamma)$, which acquire the values:

$$\frac{t_1}{t_2} = 2.5 - 2.1, \frac{t_2}{t_3} = 0.55 - 0.58, \gamma = (0.01 - 0.75). \qquad (2.44)$$

The basic IN, being not restricted by the considered limitations, has a more freedom in the variation of ranged $\{\gamma^i\}$ and corresponding codes.

The IN basic parameters n, γ are connected by the formulas $(\gamma_i^{\alpha}(\gamma))^{n-1} = \frac{\alpha_{1,o}}{\alpha_{no}}$, where $\alpha_{1,o} = \max \alpha_{io}$, $n = F_n(\alpha_{1,o})$, and hence $\alpha_{no} = F_{\alpha}(\alpha_{1,o})$.

The ranged symbols also define the IN invariants $\mathbf{a}_o(\gamma)$, $\mathbf{a}(\gamma)$ and therefore, the IN parameter of symbols' multiplication $\gamma_i^{\alpha}(\gamma) = \frac{\alpha_{io}}{\alpha_{i+1,o}}$.

Decompression or decoding assume applying an auxiliary (or inverse) process for a "reading" and obtaining the encoded symbols that uncover the prehistory.

In the considered encoding-decoding procedure (E-D) (Figs.2.3, 2.3a), the input arbitrary symbols generate an ordered sequenced logic as a "blueprint" for the secondary encoding. The ranging and ordering of the initial random symbols by the E-D in the process of compression-decompression is analogous to the creation of an incoming string $\{\alpha^*_{io}\} \sim \{t^*_i\}$ for a new secondary formed information network (IN*), where $\{t^*_i\}$ is the result of transformations of the initial sequence $\{t_i\}$ after the E-D.

The secondary IN* is able to *identically* compress and decompress the transformed ranged string of the initial information. Moreover, applying the E-D automatically creates this IN* based on the arbitrary input data. The secondary compression *operations* require a knowledge regarding the connection between $\{\alpha^*_{io}\}$ and $\{\alpha_{io}\} \sim \{\gamma_i\}$, or about a direct encoding $\{\gamma_i\}$ into $\{\alpha^*_{io}\}$. After the compression of initial symbols $\{\gamma_i\}$ into the primary IN and the decompression of the ranged symbols $\{t^*_i\}$, their connections with $\{\gamma_i\}$ are verified and corrected for a further compression into the IN* final node. During a secondary compression, formula (2.34a) and above relations are applied to the IN* (even though only the primary encoding works with arbitrary $\{\gamma_i\}$ up to their unlimited compression).

The primary E-D serves as a *preparatory* operation necessary for the automatic organization of incoming information. Theoretically, any initial information can be transformed and organized by IN*, and its potentially unlimited sequence will be compressed into the IN* final node. The decompression of information is possible only after its ranging, because the fixed time intervals are the same, determined exactly for both direct and backward movements, being automatically generated by the primary E-D.

In addition, the E-D encoded collection of symbols has been bound to this time-sequence by the secondary encoding. The compressed symbols, satisfied to equations (2.43),(2.43a), keep a fixed regularity of the considered time intervals $\{t^*_i\}$ like a clock.

This clock-time becomes embedded into the IN* final node after compression. For each random and finite $\{\gamma^i\} \sim \{b_i\}$ sequence, the total clock-time can be considered as a measure of the number of such symbols within the encoded sequence. Actually, this clock counts the number of encoding-decoding events corresponding to current $\{\gamma^i\}$ number.

The final node memorizes and stores the time-clock that had collected these numbers. The time-clock does not depend upon the specifics of each particular event γ^i within the $\{\gamma^i\}$ sequence. Any random sequence with the equivalent γ^i numbers is able to generate the same time-course. This means that the E-D leads to *averaging* of the random sequences into nonrandom $\{t^*_i\}$ intervals, or to a statistical correlation between the random γ^i.

The above $\{t^*_i\}$ sequence can be considered as a result of a large number of such a random E-D, not especially connected with any of them.

For a Gaussian random sequence, the entropy $H(\gamma^i, \gamma^j)$ is defined via the mutual correlation function R_{ij} [5]:

$$H(\gamma^i, \gamma^j) = 1/2 \ln(\det R_{ij}),\ R_{ij} = E[\gamma^i, \gamma^j]. \tag{2.44}$$

Because the IN total entropy $\Delta S = n\mathbf{a}_o(\gamma)$ is determined by the equality $H(\gamma_i, \gamma_j) = \Delta S$, the IN parameters n, $\gamma_i^{\alpha}(\gamma)$, and $\gamma_i^{\alpha}(\gamma)$ become connected by the correlation function of the initial sequence, which expresses its statistical relation $\gamma_i^{\alpha}(\gamma)$ to the clock course: $R_{ij} \sim R_{ij}^{\alpha} = E[\gamma_i^{\alpha}, \gamma_j^{\alpha}]$.

The ordered decompression can serve as a mechanism, which filters randomness and eliminates redundancies. The repeating code-word (carrying the quantity of information $\mathbf{a}_o$ that provides the sequential decompression) is able to generate the decoding process' periodicity as an *indicator* of the process' ordering.

Let us evaluate the difference in" reading" the symbols t_k, t_{k+1} by the IN direct: $t_k \to t_{k+1}$ and the inverse: $t_{k+1} \to t_k$ orders, which have the following relations to the inverse times: $t_{k+1} = t_k^-$, $t_k = t_{k+1}^-$.

Thus, even though their ratios are equal at $b_i = b_i^-$, the difference

$$\Delta b_k = \frac{t_k}{t_{k+1}} - \frac{t_{k+1}^-}{t_k^-} = \frac{[(x_{k-1}+1)(x_k-3)]^2 - 16(x_{k-1}-3)^2}{4(x_{k-1}-3)} \tag{2.45}$$

arises at each fixed t_k, t_{k+1} moments. At $x_k = x_{k-1} = 4$, we get $\Delta b_k = 0.45$.

The backward movement is accompanied by a nonequality of their entropy ratios, leading to the difference

$$\delta(\frac{A_k^{'}}{A_k}) = \frac{A_k^{'}}{A_k} - \frac{A_k^-}{A_k^{'-}} = \frac{(x_{k-1}+1)}{4} + \frac{4}{(x_{k-1}^- - 9)} = \frac{(x_{k-1}+1)^2 - 16}{4(x_{k-1}+1)}, \tag{2.46}$$

which at the same $x_k = x_{k-1} = 4$ brings $\delta(\frac{A_k^{'}}{A_k}) = 0.45$; and $\delta(\frac{A_k^{'}}{A_k}) = 0$ if $x_k = x_{k-1} = -5$, or $x_k = x_{k-1} = 3$.

The first equality's condition determines $\Delta b_k = 0$ at $\frac{t_k}{t_{k+1}} = \frac{t_{k+1}^-}{t_k^-} = 1$, which is useless, the second one leads to a chaotic destruction of all IN's information (at $\frac{A_k}{a} \to \infty, \frac{T_k a}{t_k A_k} \to 0$), meaning that the fulfillment of both $\delta(\frac{A_k^{'}}{A_k}) = 0$ and $\Delta b_k = 0$ is not possible.

The difference between each direct and indirect time interval is initiated by generation of the entropy's surplus at crossing the doublet border, which is a result of the model irreversibility. This difference is exposed through the IN information equivalents $\{\alpha_{ko}, \alpha_{k+1,o}\}$.

Each symbol's location α_{ko} among the ranged set $\{\alpha_{io}\}$ is characterized by the specific entropy productions $\alpha_{ko} = \mathbf{a}_o / t_k$, $\alpha_{k=1o} = \mathbf{a}_o / t_{k+1}$.

Their direct difference

$$\alpha_{ko} - \alpha_{k=1o} = \mathbf{a}_o ((t_k)^{-1} - (t_{k+1})^{-1}) = \delta s_{k,k+1}$$

generates the entropy surplus $\delta s_{k,k+1}$.

The inverse difference $\alpha_{k=1o} - \alpha_{ko} = -\delta s_{k,k+1}$ requires **a** consumption of the equivalent defect of entropy.

This determines the *nonequivalence* of moving from $t_k \to t_{k+1}$ and from $t_{k+1} \to t_k$.

The above relations evaluate an *inner irreversibility* of the symbols' order, encoded into the IN, and take into account both the information *value* of each symbol's pair, according to its location within the network, and the information *irreversibility* of the IN symbols' *processing*.

This reflects the irreversibility of the human language's symbols in both the words and a brain information processing.

Compared to the basic network, the built IN* represents a sequence of the joint encoded doublets' structures with a decoding logic

$$x_i^- \to (T_i, T_{i+1})^- \to (t_i, t_{i+1})^- \to (\gamma^i, \gamma^{i+1})^-,$$

performing a sequence of the direct $A \to B \to C \to D$ and the analogous indirect operations up to the initial (γ^i, γ^{i+1}).

The IN* represents a *limited* version of the basic IN, where the triplet's structures are formed by the sequential adjoining information generated by the doublets.

2.2.5. Summarized Results

The following *correspondence* between each pair of the symbols takes place:

$$(\gamma^i, \gamma^{i+1}) \to (t_i, t_{i+1}) \to (T_i, T_{i+1}) \to x_i$$

in the direct order operations of encoding.

In the opposite order operations of decoding, we assume that the inverse *correspondence* is correct between each corresponding pairs:

$$x_i^- \to (T_i, T_{i+1})^- \to (t_i, t_{i+1})^- \to (\gamma^i, \gamma^{i+1})^-.$$

When an equivalence is reached between x_i and x_i^-: $x_i = x_i^-$ (by applying the inverse operations) the inverse correspondence becomes incorrect.

The accordance between the encoded (t_i, t_{i+1}) and decoded $(t_i, t_{i+1})^-$ is violated, even though $(T_i, T_{i+1}) = (T_i, T_{i+1})^-$ is still true (by using (2.40) – (2.42)).

The inverse operation is able to erase the encoded symbols (T_i, T_{i+1}) using the reverse information flow, but cannot delete the inner logic defining the directed sequence of encoded symbols. Moreover, the logic is arranged by the erasure and is preserved without above symbols. The inverse information flow that provides $x_i = x_i^-$ carries the same entropy as the direct logic processes, which *cannot be erased* by the inverse operation.

A direct dynamic logic $A \to B \to C \to D$, determined by the interactive complex dynamics, requires a time for its performance. The restoration of an inverse logic $D \to C \to B \to A$ assumes the reconstruction of the logic prehistory associated with an inverse movement backward in time.

It was shown that such a process cannot precisely restore this logic because the IN's dynamic time is irreversible.

This analysis yields the following results:

1. The authentic, direct logical sequence $A \to B \to C \to D$ (or any other related direct series: $A \to C \to B \to D$, applied to auxiliary nonredundant events), which is performed in real time, become *false* in the inverse direction.

2. Such a deterministic logic incorporates entropy; therefore it is not a certain logic. An attempt to restore the direct logic releases uncertainty-entropy between both direct and inverse logics, determined according to (2.46). Each new combination of initial symbols, generated by an *extension* of the preceding results, introduces more *diversity* and provides more entropy-information into the direct logic. It is similar to the symbols of an alphabet and the rules of the proper grammar in a formal syntax. The introduction of a new extended language's alphabet or new formal axiomatic, as well as a new programming language, changes the initial entropy.

3. The impossibility of complete elimination of this uncertainty in traditional logic requires the development of an *unconventional logic* with an inner uncertainty, where the error of accuracy could be within the inherent uncertainty of applying a logic's code. For example, the operation of adjoining doublets into a triplet structure is executed not precisely but with an inevitable error (ch.1.5).(On Fig.1.5.5, this error is enlarged for a better illustration.) As a result, the correct mathematical operations describing the triplet's formation should carry an internal uncertainty that would automatically cover these and a possibly other inherent inaccuracies.

A human mind's inner uncertainly, we believe, is a consequence of these results. For this reason, we proposed [6] to operate with the IN information code (DSS), carrying the IN logic and its intrinsic uncertainty rather than using traditional mathematical logic in the Artificial Intellect (AI).

The DSS implies an *optimal three digit* code, which, compared to traditional two digits (yes-no) code, has extending characteristics and opportunities for future applications.
The DSS is related to *DNA code, unveiled by Nature*.

4. Because the inverse logic cannot precisely repeat uncertain direct logic, a truth of direct logic is unverifiable by a proof based on the opposite assertion. This result is in the agreement with G$\ddot{o}$del's theorem of incompleteness and Turing's uncomputability [7].

Even the ranged IN starting numbers such as 4, 2,1,0.5,0.25,...., taken by absolute values (for example $\alpha_{1o} = |4| \cong 3.5777 \pm 1.7888 j_1$, $|\alpha_{2o}| = |2| \cong 1.7888 \pm 8944 j_2$,

$$\alpha_{3o} = |1| \cong 0.8944 \pm 0.4472 j_3, .., \ \alpha_{io}, \ j_i = (-1)^{1/2}, ...)$$

bind a primary "free" entropy under the IN cooperations (chs.1.5-1.7).
In this example, the IN has the parameters: $\gamma = 0.5$, $\mathbf{a}_o \cong 0.72$, $\mathbf{a} \cong 0.245$ and it performs the following operations: $j_1 = 0$ at the moment $t_1 \cong 0.03494$, forming $\alpha_1 = 2 \bullet 3.5777 = 7.1554$; $j_2 = 0$ at $t_2 \cong 0.06988$, forming

$\alpha_2 = 2 \bullet 1.7888 = 3.5776; j_3 = 0$ at $t_3 \cong 0.13975$, forming $\alpha_3 = 2 \bullet 0.8944 = 1.7888$, and so on.

Approximately at the same moment $t_3^{'} = t_3 + o$ the equalization of $\alpha_1(t_3) \cong \alpha_2(t_3) \cong \alpha_3 = 1.7888$ takes place with their tripling $\alpha_3(t_3^{'}) = 3\alpha_3(t_3) = 5.3664$, while each IN algebraic operation (at the moments t_1, t_2, t_3) generates the equal entropy $\mathbf{a} \cong 0.245$, whose sum (in $\delta s = 3\,\mathbf{a}$) binds it into a single triplet's node $\mathbf{a}_o \cong 0.75$.

Thus, a simple algebraic operation with the initially independent (random) numbers, which connects the numbers, is capable of releasing an entropy-uncertainty.

In the considered example, the first of such operations is ranging the numbers.

The encoding of initial arbitrary symbols into the IN's ordered starting symbols performs the regularization that decreases the initial randomness. The ordered symbols' sequence has a minimal uncertainty compared to any nonordered ones and is characterized by a minimal Kolmogorov's complexity. The ordered sequence is incompressible.

5. The E-D has a more general meaning: creating an order through a backward movement. In other words, the inverse motion orders the events that occurred during the direct motion. The *same way that encoding regularizes the direct process, the equivalent decoding orders the inverse process.*

The inverse motion can also be performed *virtually* by an imaginable (virtual) analysis of already executed actions.

The scheme on Fig.2.3 establishes the connection of both the primary and the secondary E-D with the elements of Information Cognitive Processing (ICP) (Fig.2.3a).

The incoming information is primarily encoded by an individual's IN code and is compared to existing information, stored in the dictionary of knowledge (DK). Only new, nonredundant information is filtered by this cognitive filter and is fixed using the DK labels. In particular, the considered information interactions model a consonance in a harmony of sounds, associated with the correct acceptance of the natural language words and/or sentences' semantics.

A proper grammar is verified by the correct sounding word-sentence combinations; assuming that a *natural language itself automatically carries* a natural *harmony*, which enables *both regularize and connect* the language' grammar and semantics serving for their verification.

This approach is also relevant for the translation of visual information into the interacting frequencies. Through these interactions, the produced information not only binds the authentic symbols by the node frequencies ω_m but also synthesizes nodes into *meaningful integrated* units (words, sentences, thoughts);and the sequentially *enclosed* node's connection orderly integrates these units.

This means the final IN's node assembles not just an arbitrary collection of symbols but provides a *meaningful synthesis.*

The synthesized unit can easily be recognized and memorized as a *single* word-concept's symbol in the DK's label code, which also carries the substantial symbols usable for decoding.

Actually, fixing and labeling this information corresponds to memorizing it at the *secondary encoding* for *synthesis* and allows concentrating it for storage into long-term memory.

The memorization of composite symbols (that had been assembled into the final node for possible IN* decoding) takes place in the ICP's Data Base [6].

The primary E-D generates a *matrix logic structure* for the memorization.

Therefore, both the primary and secondary E-D mechanisms present an intrinsic part of the ICP in the process of acceptance of incoming information. The E-D is not executable for an **i**nitial nonredundant but an arbitrary symbol's sequence.

The transformation of an arbitrary symbols' sequence into its ordered sequence releases an entropy which can be used to encode and decode the arbitrary sequence.

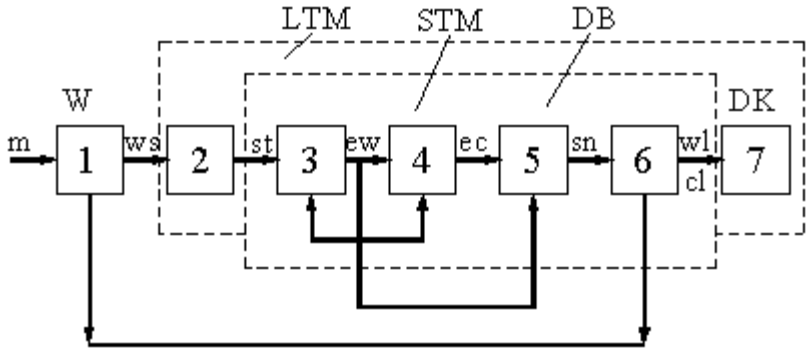


Figure 2.3. Scheme of the E-D operations within the ICP: W-Webster's model, DB-Data base, DK-dictionary of knowledge, STM-short-term memory, LTM-long-term memory, m-message, ws-word's symbol, sf-symbol's frequency, ew-encoded word, ec-encoded concept, sn-spot's node, wl-word's label, cl-concept's label.

The E-D and ICP mechanisms not only model a comprehensive process of transformation information but rather are analogous to human perception, cognition, and accumulation of information [4,6]. During the perception, the initial random events are primarily encoded-decoded by an individual**'s** code, a possibly stored in short-term memory; the compressed information is sent into long-term memory. The information retrieval depends upon its necessity in applications. If the stored information is not required, it can be forgotten.

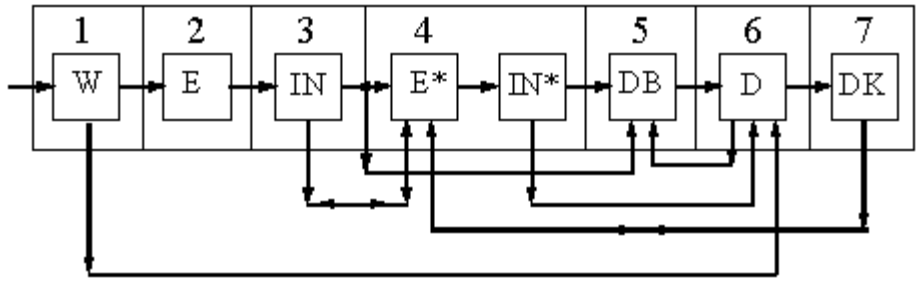


Figure 2.3.a. E is a primary encoding into IN, E* is a secondary encoding into IN*, D is a decoding; other indications are the same as on the Figs 2.3; the numbers 1-7 indicate a sequence of the above operations.

The final IN* node can be programmed to decompose and reveal the compressed information after reaching a fixed or limited number of the $\{\gamma^i\}$ event-symbols.

Another option is the initiation of a signal for the destruction or erosion of total information based of a given final clock's time.

This mechanism allows for both the effective acceptance of incoming information and its storage, memorization, and the automatic erasure of the unused information.

The considered model also opens a constructive method of automatically counting arbitrary information and effectively storing it in a very compressed form, enabling feed-back control for different information proceedings. The synthesis of information in the form of thoughts and concepts is based on its proper collection, concentration, ordering, and revealing the inherent logical connections. These mechanisms have important AI applications. In a cognitive observer, discrete intervals of imaginary information ended with *interaction* (collapse) of information waves (ch.1.9), which generates real information, and a code.

The cognitive dynamics include: selecting of a most informative events, ordering, memorizing, cooperating, encoding and integrating this information (into into an IN hierarchy), creating a base of knowledge, allowing its renovation, control, synthesis, creativity, and decision making.

Cognitive system, implementing the *evolutionary cooperative dynamics* (ch.1.8), *feeds itself* with valuable information. All these and other related components are the IMD parts (which are theoretically founded in chs. 1.1-1.9 and applied in part.2).

An observer's cognition transforms unobserved imaginary information into observed real information (after an interactive "collapse" of the wave functions, ch.1.9).

A notion of *intelligence* belongs to an observer as its ability to accept, use, and create the information. This ability we evaluate by an increasing *quantity and quality* of accepted information during the use and creativity. Such an increase becomes possible if an observer's inner (accumulated) information is utilized in the cooperative dynamics of processing the accepted information. In these cognitive processes, different forms of encoding information are instrumental for its reception, recognition, distinction, and creation.

An intelligence level is connected to an observer's ability of creating a highest level of its IN hierarchy, measured by the highest quality (ch. 1.6) and *cooperative complexity* (1.7) of accepted information being transformed during the cooperative dynamics.

A *maximum complexity of the most valuable information* could become a *scale* for the intelligence comparison.

Each E-D operation depends on specific $(t_n,\ x_{n-1})$ or (t_1, x_1) sequences that determines the corresponding time intervals, associated with the transformation of distinctive information.

The examined sequences of random events, which are embedded and compressed into a time clock, have a function analogous to that of a cell's telomerase at the DNA end [8].

Six DNA bases are translated into RNA and then, using RNA as a template, are encoded into each DNA end by a repeating sequence of these bases.

This is like *a secondary E-D*, which compensates the telomerase erosion after each cell reproduction and protects the DNA information from a potential loosing.

A limited number of the secondary encoded base's tandem and the sequential erosion of these bases at each cell division *restrict* the telomerase life-time.

Because this process depends on individual's telomerase length and DNA specifics, the time intervals that identify the time's course and the life-times are strongly individual.

The clock counts the time that is contributed to the entropy production (according to [8]) at each discrete interval of the transformation of information into a cell, which accumulates the entropy production.

The telomerase measures and controls the processes of transforming DNA information into RNA, which regulates the biological clock of cell's reproduction, controlling the life spare of the cell, and possibly determining the life clock of the total organism.

Until an individual consumes his negentropy, which compensates for the entropy production, he lives.

However, the ability to compensate and therefore to accept the negentropy is declined when the entropy production (associated with a process of destruction) exceeds the available limit. This leads to a total collapse. Before that occurs, the accumulated entropy production decreases a "free entropy reserve", which is capable of compensating for errors in a self-regulating system. This is a primary symptom of aging.

The above IN* functions extend the fundamental functional connection between the IMD and biological processes [3], including functional mechanisms of evolution (ch.1.8) and brain information processing [4].

A set of the IN final codes (that embed total information processing from a complex, multi-dimensional system) serves as an analogy of DNA code with the telomerase time-clock at its end.

The code's retrieval allows for the restoration of the initial complex process.

We believe that the collection, encoding, and encryption of information into the IN* are the functions of the *distinctive control mechanisms* (as the brain's analogies) in the diverse natural systems, developed by the evolutionary dynamics.

This means that the above biological mechanisms are created according to general *information systemic* principles implemented by biological processes and entities.

Moreover, each mechanism‘s macrostate is described by a local uncertainty that requires an adequate language, having an inherent uncertainty, to operate with the macrodynamic states and complex processes.

2.2.6. About Other Related Applications

The IN optimal average codeword's length l_n follows from Shannon's formula: $l_n = H / \ln n$, where H is entropy of a message, N is the number of message's alphabet symbols.

For the IN: with $N = n$, $H = \Delta S = \mathbf{a}_o(\gamma) n$ we get $l_n = \mathbf{a}_o(\gamma) n / \ln n$.

A code is decodable if its codeword lengths $l_i, i = 1, \ldots$ satisfy to Kraft's inequality [9]:

$$\sum_i N^{-l_i} \le 1 .$$

For the IN fixed codeword $l_n = \mathbf{a}_o(\gamma) n / \ln n$, this formula leads to $n n^{-l_n} \le 1$ that brings the following limitation: $\mathbf{a}_o(\gamma) \ln n / n$, or $\ln n / n \le \mathbf{a}_o(\gamma)$.

For $\mathbf{a}_o(\gamma) \le 0.765$ (Nats), the above inequalities are fulfilled at any $n > 1$.

Thus, the IN code is decodable by Kraft.

C. Shannon calculated the entropy of English as $H \cong 12$ bits per word [1]. Encoding the word into the IN* minimizes this entropy down to the entropy of final node $\mathbf{a}_o \cong 1bit$.

The difference $\Delta H \cong 11$ bits can be spent to classify, range, and encode (label) the word into a corresponding IN* according to their novelty.

The average optimal codeword's length to encode-decode each of m words is equal $l_m = \Delta H / \ln m$.

Using English alphabet's symbols for labeling these words brings m=26 and $l_m \approx 3$ (which is enough to encode $3^3 = 27$ symbols).

This means, a sequence of three symbols from a chosen new alphabet (for instance, a binary code) can encode each of m words and is able to label each of these words by one letter of English alphabet. A label of a latest word (with entropy $\mathbf{a}_o \cong 1bit$), being squeezed into the final IN_m node, identifies all compressed chain.

The ratio $H/\mathbf{a}_o$ determines the compression ratio $r \cong 12$ for each word. By applying $\mathbf{n}$ such IN*s we get $r \cong 12\,\mathbf{n}$, which even for a simple text with $\mathbf{n}$=10 words brings the ratio to 120.

For example, a complete book can be compressed and stored as a single symbol. Conventional methods cannot reach such compression.

The ordered decoding decreases the starting entropy by approximately the same ratio.

This means that the ordered decoding releases a source's random entropy in the same way that compression reduces the potential redundancies.

Therefore, the ordered decompression can serve as a mechanism of *filtering* and eliminating redundancies.

Further compression should include *nonredundant* information that requires non traditional methods.

The repeating code-word (carrying a quantity of information $\mathbf{a}_o$ that provides the sequential decompression) is able to generate the decoding process' periodicity as an *indicator* of the process' ordering.

The encoded compressed file is *encrypted* by both the secondary code and the decoding algorithm. Only a possessor of both of these has access to the compressed file.

This means that the encrypted information is well secure and uniquely protected, and can be sent for long term storage and memorization. Because the final element of the IN system compresses the total information, its capacity enhances and memorizes the total IN capacity. Cognitive processing [6], implementing a maximal acceptance of information with its minimal spending, orders and prioritizes incoming information in terms of a maximum informativeness, creating the ranged IN automatically.

The considered systemic information model has feasible connections with the publications [10-19] in neurodynamics and many other scientific results.

R. E. John [10], in a study of brain information processing through a purely biological point of view, provides the detailed and convincing results regarding the substances of the brain systemic biological phenomena. Such as a coherent chaotic activity, binding actions, self-synchronization; integration, hierarchical complexity, discretion of time, an occurrence of

the discrete stimulus' complex, and conscience; memory formation; neuron's communications; maximum entropy and generation of negentropy; quantum dynamic phenomena with a wave's collapse, and many others.

All of these, are consequences of systemic information regularities, mathematically expressed by the minimax variation principle.

It is believed that there are also other specific bio-mechanisms which follow from the information systemic model but have not yet been practically revealed.

This is due to the fact that a systemic approach and Informational Macrodynamics are a new and mostly unknown and unexplored field, requiring bio-scientists of different backgrounds and working in specific areas.

The importance of a systemic approach is supported by W.J. Freeman's research [11], which focuses on following mechanisms and features of the brain dynamics (in terms of information processing):

- -bifurcations of the individual's space distributed chaotic dynamics (at a basin of the local attractors) initiate the coherent motion of selected frequencies;
- -the local coherent dynamics are integrated sequentially and hierarchically;
- -these cooperative dynamics carry a code which controls the current assembling mechanism of this dynamically created hierarchy;
- -the hierarchy of these ordered aggregations forms an information network whose code is originated by the individual's genetics.

Trying to explain these mechanisms in a unified approach, W.J. Freeman wrote: "It might be some of the principles in accordance with which the whole brain takes short steps and creates its own path into the future". We believe that such a systemic principle exists in the form of the IMD minimax variation, from which the above mechanisms follow (chs.1.5-1.8).

R. Penrose, a well-known physicist and brain scientists [12], indicates the necessity of developing a new "*missing theory at the fundamental level*".

We think that the missing theory is not physics but Informational Macrodynamics, which lay beneath physics and classic dynamic equations and also deals with the "virtual information and the codes of natural phenomena" (chs.1.6,1.7), leading to the macrodynamics of uncertainties.

H. Stapp [13] emphasizes that the dynamic equations of classical physics, including the classical mechanics, cannot describe human consciousness.

K. Pribrams' reference [14] to brain information processing, maximum and minimum uncertainty, and quantum mechanics are very interesting and directly connected with [4].

Roy S. and Kafatos M. [15] provide interesting ideas regarding probabilistic space-time representation of neuronal dynamics.

The specifics of the considered (in chs.1.5**,**1.6) information geometry consist not just in expressing the initial probabilistic distributions in terms of information, but also in the ability to *generate* the distributed quantums of information (cells) according to its geometrical *topology* along with the initiation of a corresponding space-time *metric by* this information. The generated information enables the creation of a variety of functional geometrical configurations and diversity of hierarchical geometrical structures.

For instance, a brain's accumulated information generates a cerebral chain of cooperating geometrical structures, characterized by shapes of curved subspaces, whose merged discrete boundaries "stitch" sequentially (ch.1.7). The discrete's number and the curvatures of these topological structures, determined by the code of the accumulated information, are measured in the terms of *cooperative complexity and quality of bound information*.

These specifics could not be found in known publications.

The paper [16] provides the experimental results, which translate the entropy production into the measured signal, representing a carrier of different interactions. Such a verification supports the theoretical IMD equations connecting the different superimposing processes through interacting information flows, which pool into collective networks.

The phenomena of the IPF formalism, with asymmetry in a future and past time, are essential for dealing with **c**onscience's regularities [4, 6].

The formalism also provides a mechanism for the transformation of imaginary into real information (ch.1.9) having a connection to evolution.

Moreover, the mechanism reveals the information forces that arise at the border of the imaginary and real zones and estimates their ratios, evaluating a resistance, necessary for overcoming information barriers between these zones [4].

According to [17]: "...Information about intensity of a stimulus is usually transmitted to the brain by frequency coding, and information about a quality by label-line coding...", which has a direct connection to the above results.

The existence of DSS' triple code confirms the recent publication [18], uncovered that a neuron communicates by a *trinary code*, utilizing not only zeros and ones of the binary code but also minus ones.

A human way of organizing and assembling the valuable information is described in [20]. We believe that a notion of cognition is related mostly to a reflection of external information and its conversion by a human neurodynamics.

We focus on the *details* of the IMD connections to the neurodynamics experimental results in the following section.

2.2.7. The Connections between Some Physical Neuronal Functions and Mechanisms and Their IMD Information Analogies

We outline the common features and mechanisms of both neurodynamics and information macrodynamics (IMD), based on the following assumptions:

- The IMD time-space dynamics is described by a sequence of the path functional's extremal segments with the spiral trajectories located on a conic surface, while each segments represents a three dimensional extremal;
- The physical dynamics of propagation through axon is modeled by the IMD information dynamics at a specific extremal segment, while a neuron can be modeled by a three-dimensional dynamic system with an applied control;
- The spike generation is modeled by the impulse needle control during a window between the segments, while the needle control joins the corresponding segments;

- The IMD embraces both the individual and collective regularities of the information model and its elements, forming a multiple connection and a variety of information cooperative networks.
- In the information cooperative networks' nodes, the collected valuable information is conserved in an invariant form and sequentially enclosed with growing both information's concentration and its volume; the nodes' interaction is able to produce a new information.

A comprehensive review of the physical dynamic principles in neuroscience [19], allows us to provide the following comparison of both the general principles of information macrodynamics and neuroscience and the specific neurophysical and informational features[18,21,25,26, others]

- A spike is generated upon overcoming a threshold. The needle control arises at the end of the extremal segment, after the information, carried up by the propagating dynamics, compensates the segment's inner information (determined by the segment macroequation). Quantity of this information is evaluated by the segment information invariant $\mathbf{a}_o$, while the needle control's information is measured by $\mathbf{a}_o^2$.
  This establishes, first, the direct connection between the information analogies of both spike and the threshold, secondary, brings the spike information measure for each its generation.
- A spike, reaching the axon's terminal, triggers the emission of transmitter, which binds the receptor of the following neuron. At the consideration of information transmission between the segment-transmitted and a segment-receiver (receptor), the needle control connects them and initiates the signal propagation between them (sec.1.8.7). The propagation is associated with adding a macrostate (carried by a control) to transmitter, and removing it after transmission occurs. At the receptor side, the propagation's and the control's actions initiate the addition of a macrostate to the current receptor's macrostate, indicating the occurrence of the transmission, and release the macrostate after the propagation occurs. The needle control serves only as a messenger, which does not carry the macrostate, but rather induces the macrostate's formation by the receptor (Figs.1.8.4,1.8.5).
  The details of this mechanism are in [4].
- The interspike intervals carry the encoded information, the same way as the intervals of discretization between the applied needle controls do. An axon conductivity depends on the between neuron's electrical conductance, which is determined in the IMD model by the diffusion conductivity of the cooperative connection, computed via the derivation of the correlation function (ch.1.5). An external signal through this conductivity (correlations) might modify a topology of a single, as well as a multiple connection, changing its macrodynamic functions (and a possibly leading to the distinct networks) under different input.
- The IMD optimal elementary form of the multiple cooperative connections is a *triplet*, formed by the cooperation of two ordered segments into doublet, which is

coupling with a next ordered segment. The axon's branching geometrical structure [20] is an example of the triplet cone's connections (Fig.1.5.4), where at each triple point, the two cone's vertexes are connected with the base of a third cone like two inputs and one output. The neuron communicates by the *triplet* code of cooperative dynamics (ch.1.6), whose existence has been experimentally confirmed in the recent publications [18], others. Cooperation of interacting neurons leads to a stimulus dependent ordering in the neuron groups, multiple encoding of an input signal as a result of coordinated activity.

- Both neuronal dynamics and macrodynamics are strongly dissipative, based on stochastic dynamics of the controlled diffusion processes, whose macrobehavior includes chaotic dynamics with a possibility of different types of local and global bifurcations, associated with changing the structure, instabilities, and singularities. A three dimensional triplet's dynamics generally generate such bifurcations as the limited orbits, saddles and attractors. A structural stable macrosystem encloses a set of the stable attractors, forming the triplet's nodes of the information network (IN). As a neural system, the IN is organized as a hierarchical dynamic structure (in both the phase time-state and three dimensional spatial state) that utilizes such neural phenomena as the state's coordination, synchronization, and cooperation, which are probabilistically dependent on the input signal, transformed into the specific output. Each behavioral event forms a starting condition for the next widow of time, which connects the events also in a space; the same information states' sequence is generated by the "quants" (sec.1.3.5) at the widows between the information model's segments. Coexistence of the multiple nodes-attractors at a given initial conditions characterizes a multistability of both neurodynamic and macrodynamic systems. In the triplet's network, each previous triplet node enables sequential attraction of the following triplet's node, with a possibility of their synchronization, generation of adaptive rhythms, and forming a transient behavior. A proposed "polychronization" of a neuronal network with the time delay of axonal activities and a group's self-organization [21], are consistent with the IN cooperative dynamics. In particular, it was shown (sec.1.5.2) that the macromodel eigevalues are capable of assembling into a cooperative, if its information frequencies operate with the sequential delay of incoming information, determined by the time intervals required for the following cooperations. The IN triplet's connections can be changed depending on the current segment's sequence, its information quantity, and their number, which leads to an alteration of network structure, reacting on incoming inputs. Changing the network's connectivity is also associated with learning [19, 6].
- The IMD mechanism of the IN building includes an *automatic ordering* the model segments evaluated by the information quantity of the segment's eigenvalues (and the eigenvectors) in the process of this information acceptance for a specific sequence. Moreover, proceeding of these information quantities involves the automatic generation of the triplet's structures with the corresponding space movement of the *local space coordinate systems and forming a global IN's coordinate system*, which are determined by the eigenvectors' inputs (ch.1.4). This automatic procedure not only transforms the spatial-temporal input to its spatial form

and finally to spatial-temporal output, but also establishes an ordered geometrical mapping relationship between them, allowing the exact localization of the transformed inputs. We assume that this transformation is carried by the spiral wave modes, represented by the model's spirals on the cones.

- The IMD model is characterized by a sequential growing of the information *effectiveness* of the needle (and regular) controls actions along the IN spatial-temporal hierarchy. This is connected with changing the *quality* of the IN node's information depending on the node's *location* within IN geometry and corresponding spatial space (ch.1.6). The changes increase the *intensity* of the cooperative coupling, its competitive abilities, which make the segment's synchronization more stable and robust against noise. It also affects the length of the discretization intervals, and an ability to adjoin more cooperating elements; increases the model's hierarchical complexity (ch.1.7). In the IMD neuron model, this leads to the spike strengthening along the formed networks, affects the interspike intervals, increases the neuron's chain ability for connectivity and self-organization. Growing the network effectiveness, quality, and intensity allows conserving and concentrating more valuable invariant information with increasing information volume.
- The macrodynamics is reversible within a limited time-space intervals (contained by the segments) and irreversible out of these intervals (at the windows), representing an *open* system, for which the preservation laws are not satisfied at the moments of interaction when the external environment is able to change the model structure. The reversible and symmetrical macrodynamics, satisfying classical dynamics, are followed by the irreversibility and asymmetry, satisfying *nonequilibrium thermodynamics*. These uncertain macrodynamics, which characterize both neurodynamics and cognitive dynamics, connect randomness and regularities, stochastic and determinism, reversibility and irreversibility, stability and instability, symmetry and asymmetry.
- As a neuronal subsystem, the information cooperative subsystem is ensembled from the interconnected units at each IN's hierarchical level, all of which are similarly organized, preserving a local autonomy and a self-regulation at a diversity of the key-lock cooperative connections (Fig.1.8.2). Such a cooperative dynamics are characterized by its specific cooperative complexity, including the hierarchical dependency (ch.1.7).
- The multiple triplets and the IN nodes, created by various sensory inputs, do not assume establishing between them the *specific* encoder-decoder's relationships. It could be many simultaneous senders and different receivers with distinct messages and transmissions processes, but with a universal coding language, determined by the triplet's genetics. The universal time scale (and corresponding space locations) are established for a given system by its local clock course [22] (secs.2.2.3,2.2.4).
- The macrodynamics, as the brain dynamics, are regular and robust in the presence of both intrinsic and external noises, and are the sensitive to incoming inputs, preserving the self-control and adaptivity at the limited inputs, and leading to the self-organization at overcoming of some thresholds, especially in the evolutionary

dynamics (secs.1.8.1-1.8.5). The specifics of these features include the error correction mechanism, a non symmetry of the adaptive potential, the acceptable mutations at a system's diversity, and a renovation with growing the potential of evolution under these mechanisms. The IN evolution involves the triplet's *genetics*, which is able to encode the different inputs features into distinct output patterns. The evolutionary dynamics includes compatibility and selectiveness with adaptive self-controls, self- organization; genetic code, coping, and reproductivity.

- The cognition includes extraction, selection of and ranging the facts depending on its informativeness (both the quantity and quality of the fact's information content), building the fact's *hierarchical connections*-as a key for understanding. These cooperative connections cohere and organize this information, aiming at crafting its meaning and remembrance. Motivated by that, "the large goups of neurons in the brain synchronize oscillatory activity in order to achieve coherence". It seen that all these features possess both the neuron system and the IN information network. Actually, the short-term memory (STM) (or working memory) is modeled by the current control (both regular and needle), with fixing the double states at the segments' cooperation. The long-term memory (LTM) performs a sequence of the cooperating triplet's nodes-attractors, which guarantee the memory robustness. The LTM is generated after the STM forming, which has a limited capacity. Placing the LTM into a spatial IN occurs after removing the current multiple STM, which are not composing the IN nodes. STM is a dynamic process, while LTM is based on forming the *structural* connections. The working memory's limited capacity by four to seven items is following from the triplet's elementary dynamics at their minimal number and is in agreement with Miller's magical number [23]. Therefore STM works until structural connections have not been made, and until the STM limited capacity has not been exceeded. After that the LTM starts, and it depends on the specific triplet's element, which keeps the latest STM. "Each of the saddle point represents an event in sequence to be remembered."
- Learning needs a precise spike synchronization and the sequential memory encoding that requires a sequential generation of the temporal asymmetry, following from the properties of the network's connections. The nonstationary activities between the attractors initiate the transient responses and their *competing* connections through the information communications, which bring a cooperative behavior. Therefore, each IN's node-attractor, with a memory storage, enables transmitting information and attracting a following doublet or a triplet by overcoming some threshold (ch.1.8). Each triplet, attracting others, is capable of bringing an ordered cooperative connections, counteracting the between connections' instabilities and singularities [24]. An asymmetrical coupling goes through a stochastic synchronization, which after cooperation keeps robustness of the renovated structure. The cooperative connections also arise between the different networks (INs), as well as between these nets' individual elements: within each net and the elements-nodes, located at the end one of them and at the beginning of the next net [25].

- According to [19], "experimental evidence shows the existence of population code that collectively express a complex stimulus better than individual neurons" by a spatiotemporal code. And then… "the presence of network coding", i.e. "spatiotemporal dynamic representation of incoming message, has been confirmed in several experiments." Thus, the IN network's cooperative code, portrayed through the triplet's genetics, is a valid element of informational neurodynamics. Following the IMD, each component of this code has a dual representation: by the time moment (or the space location) of firing the needle control, or by the time (space) intervals between these impulses. The code's universal time (or space location) is set up by a given system of the model equations, which determine the eigenvalues and eigenvectors, as well as through the variation principle that defines the invariant relations for these eigenvalues, eigenvectors, and the above time-space intervals.
- The IMD model admits a creation of inner information, without a specific external input. It is possible by two ways: when a needle control connects the extremal segments by closing access of external information. In this case, a previously memorized information produces the needle control, which cooperates the segments, maintaining the new information structures that were not directly initiated by external inputs. Also the instable chaotic activity between the segments is a potential generator of new information at any of the considered dynamic moves. Another way uses the information triplet's surpluses, generated by the triplet's interactions. Physical processes associated with these mechanisms are connected to the phenomena of superimposing processes (ch.1.9). More powerful, three-dimensional visual information, compared with scalar auditory information, might trigger the production of this information. Actually the IN is activated by a string (eigenvalues) with a very low initial information, which is sequentially amplified through the contributions of cooperating triplets. Such a network is described by a system of the ordinary differential equations, which are joint by the cooperative double and triple connections (ch.1.5). The connection's activities are synchronized, generating the network's periodic rhythms. The IN possesses the inner harmony and anticipative functions (sec.1.6.9).

The actual question is: What can IMD bring to neuroscience?

We *summarize* the answer by concluding of what the IMD does:

- Integrates the neurosystem information's features, based on the path functional variation principle and its specific applications; these bring a complex information systemic approach;
- Explains the information specifics of some neuronal functions, allowing a prediction of their behavior; this includes the origination of neuron stochastic dynamics and its local and global bifurcations; the memory formation and its accumulation at the attractors, or at the saddles (depending on the bifurcation's specifics); details of the transmission information at the neuron's communication, dependency the spike strength on a prehistory; a flexibility of the spike's threshold, forming multiple

spatial locations, and many others, including the time delay's connection with the neuron's internal and cooperative dynamics;

- Provides the explicit explanation of the code origination, with both dynamic and information points of view, and identifies the universal double spiral code with its information dynamic and geometrical structure;
- Presents the formal information mechanisms of creation of both the triplet information structure and the hierarchical information network (IN), based on the ordered cooperative triplet's information dynamics and forming the IN node's attractors;
- Explains the IN node's sequential attraction, synchronization, competitions, anticipation of the future local operations, creativity, and forming cognitive structures;
- Joins the concepts of robustness, adaptive control, self organization, evolution, and reproductivity.
- Introduces the information mechanism of the error (noise) correction and the adaptive potential.

It sounds amazing the fact that the IMD reveals the *information analogies* for most of the physical features and the specific neuronal mechanisms, which the neuroscience obtains during many years of experimental and theoretical research, even though the IMD actually is not using this research.

This means, these features and mechanisms are governed by some general principle, or a law-the same as it's imposed on the IMD.

For the IMD, this law is formulated by the variation principle for a path functional, which models a formal acceptance of information regularities by a system (an observer).

The cognitive applications of this formal model bring the *cognitive dynamics* (both reversible and irreversible) with the above results.

Chapter 2.3

INFORMATION MODELING AND CONTROL OF SOME INDUSTRIAL TECHNOLOGY PROCESSES WITH COMPLEX SUPERIMPOSING PHENOMENA

This chapter's goal is to apply both the Nonequilibrium's (NT) and IMD's equations for modeling of the considered complex objects and using the path functional's (IPF) optimal Hamiltonian (as an object's performance criteria) for the solution of the objects' optimization problems.

In a technology of the solidification (crystallization) process, we illustrate the physical-technological connections between NT and IMD by identifying both physical meaning of the information macrocoordinates and the process' physical relations to the object's performance criterion.

For the considered electrotechnology, in addition to that, we use a direct measurement of the diffusion conductivity (according to chs.1.3, 1.5, 1.7 – 1.9) within the chain of superimpositions by the related electroconductivities. This allows us to apply a simple procedure for the identification of the model operator (avoiding the computation of the correlation functions), and to express the IPF performance's criterion directly through the measured electroconductivity.

Both of these applications expedite the close-loop dynamics and allow building the automatic optimization system, based on the *indirect* control of the multiple superimpositions.

These important results, which illustrate the IPF applications, have been implemented in practice.

2.3.1. Process Solidification and its Application in Casting Technology

Solidification (crystallization) plays an important role in many metallurgical and casting technologies.

Basic solidification models were studied in [2 – 4] with the important contribution by A.N. Kolmogorov [2].

Crystallization is created by the superimposing processes shown in Table.3.1, where phase coordinates (x^t_{n-k}, x^l_{n-k}) are presented as the derivatives and integrals of corresponding $(n-k+1)$ coordinate: temperature θ, rate of crystallization v_c, concentration Δ C, rate of mass transfer v_m, thermomechanical stresses σ, strains ε, density ρ, and pressure p.

Actual number of the total n variables for a specific object is a priory unknown.

We use both the IMD and Nonequilibrium (NT) equations [1] to connect the traditional physical approach with the IMD modeling formalism.

Let us illustrate first the NT application as a *method* of choosing thermodynamic coordinates and corresponding forces in the IMD equations.

We start with thermoconductivity (T) as the basic phenomenon generating TH, K, D (in Table 3.1).

Table 3.1. The main interrelated physical phenomena

Physical phenomena and their notations	**Phase-space coordinates** x^l		**Phase-time coordinates** x^t	
1. Thermomechanical stresses (TH)	x^l_{n+1}	$\int \theta dl \sim \sigma(l)$	x^t_{n+1}	$\sigma(t)$
2. Heat conductivity (T) (initial phenomenon)	x^l_n	$\theta(l) \sim \frac{\partial \sigma}{\partial l}$	x^t_n	$\theta(t) \sim \frac{\partial \sigma}{\partial l}$
3. Crystallization (K) under temperature gradient	x^l_{n-1}	$\frac{\partial \theta}{\partial l} \sim v_c$	x^t_{n-1}	$\frac{\partial \theta}{\partial t} \sim \Delta C$
4. Effective diffusion (D) under heat flow	x^l_{n-2}	$\frac{\partial^2 \theta}{\partial l^2} \sim \frac{\partial v_c}{\partial l}$	x^t_{n-2}	$\frac{\partial^2 \theta}{\partial t^2} \sim \frac{\partial \Delta C}{\partial t}$
5. Mass transfer (M)	x^l_{n-3}	$\frac{\partial^2 \Delta C}{\partial l^2} \sim v_m$	x^t_{n-3}	$\frac{\partial^2 \Delta C}{\partial t^2}$
		$\frac{\partial \sigma}{\partial l} \sim \frac{\partial \rho}{\partial t} \sim \frac{\partial \varepsilon}{\partial t}$		
6. Phase conversions (PC) causing stresses and pressure	x^l_{n-4}	$\frac{\partial v_m}{\partial l} \sim \frac{\partial^3 \theta}{\partial t^3}$	x^t_{n-4}	$\frac{\partial v_m}{\partial t} \sim \int p dl \sim \frac{\partial^2 \rho}{\partial t^2}$
7. Hydrodynamic transformations in liquid-solid (HD)	x^l_{n-5}	$\frac{\partial^2 v_m}{\partial l^2} \sim p(l)$	x^t_{n-5}	$p(t)$
8. Hydrodynamic mechanism (H) developing a pressure	x^l_{n-6}	$\frac{\partial p}{\partial l}$	x^t_{n-6}	$\frac{\partial p}{\partial t}$

In the process of transfer the heat and diffusion, the thermoconductivity is described by equation

$$a\frac{\partial^2\theta}{\partial l^2} = b\frac{\partial^2\theta}{\partial t^2} + \frac{\partial\theta}{\partial t}, \tag{3.1}$$

following from the Newton law for the heat flow:

$$q = -\lambda\nabla\theta - \frac{\partial\theta}{\partial t},\ b = a / v_q, \tag{3.2}$$

where v_q is the speed of the heat distribution.

In particular, at $v_q \to \infty$ we come to Fourier's equation

$$a\frac{\partial^2\theta}{\partial l^2} = \frac{\partial\theta}{\partial t} \tag{3.3}$$

with a as the coefficient of thermoconductivity.

The hyperbolic equation in the form (3.1) can be obtained from the preservation condition for superimposing processes by drawing Monge's surfaces [3], for example, using the surfaces of the phase transitions, or the chemical transformations.

An analogy represents the surface for a nonequilibrium entropy in the NT variation problem.

The T is connected with TH by equation

$$\sigma = \beta\int_{\beta_o} \alpha E\theta dl,\ \sigma \sim \int \theta dl\ , \tag{3.4}$$

which relates the integral of temperature to the thermomechanical tensions σ, where α is the coefficient of linear heat expansion for a non fixed metal ingot, E is a module of elasticity, $\theta(t,l)$ are the temperature distributions; β, β_o are the geometrical coefficients of the forming ingot's volume.

According to Stefan's equation [4], the linear speed of crystallization v_c is proportional to the temperature gradient:

$$\frac{\partial\theta}{\partial l} \sim v_c\ . \tag{3.5}$$

In the theory of growing crystals [3,4,other], the formed spiral dislocations are functions of linear crystallization speed.

Diffusion process is characterized by a difference of concentrations $\Delta\bar{c}$ between liquid and solid phases.

The material density ρ is connected to a specific molar concentrations of the $\bar{c}_k = \frac{\partial c_k}{\partial l}$ components by relation

$$\rho = \sum_{k=1}^{m} M_k \frac{\partial c_k}{\partial l},\ \rho \sim \frac{\partial\Delta\bar{c}}{\partial l}\ , \tag{3.6}$$

where M_k is a component of the molar concentrations.

The density's changes at phase transactions are characterized by the following equation of mass transfer:

$$\frac{\partial \rho}{\partial t} = -div(\rho w)\ ,\ w = v_d \Delta \bar{c}\ , \tag{3.7}$$

where v_d is a linear speed of movement for the particles of a given concentration.

Function $\frac{\partial \rho}{\partial t}$, according to relation $div(\nabla(\Delta \bar{c})) \sim \frac{\partial^2 \Delta \bar{c}}{\partial l^2}$ and the second Fick's law, satisfies to the following proportional dependencies

$$\frac{\partial \rho}{\partial t} \sim (\frac{\partial \Delta \bar{c}}{\partial l}, v_d \frac{\partial^2 \Delta \bar{c}}{\partial l^2})\ . \tag{3.8}$$

From other side, the relation

$$\frac{\partial m}{\partial t} \sim \frac{d}{dt}(\nabla(\Delta \bar{c})) \sim \frac{\partial \rho}{\partial t} \tag{3.9}$$

characterizes the increment of the carbon (or other components) mass, created by its concentration's differences in the liquid and solid phases.

When the solid phase is formed, at $\frac{\partial \bar{c}}{\partial l} = 0$, the relation $\Delta \bar{c}_1 \sim \frac{\partial m_1}{\partial t}$ characterizes the mass speed for a graphitization process in a casting technology.

Thermoflow $q_\theta \sim \frac{\partial \theta}{\partial t}$ and the difference of concentrations $\Delta \bar{c}$ in an ingot create a thermodiffusion.

Substance's flow I_c**,** arising under a temperature gradient**,** represents Sorre's effect, which is described by equation

$$I_c = cD_{c\theta} \frac{\partial \Delta \bar{\theta}}{\theta \partial l}\ , \tag{3.10}$$

where $D_{c\theta}$ is a coefficient of thermodiffusion, and c is an average molar concentration.

The corresponding flow is

$$I_c \sim \Delta \bar{c} \frac{\partial l}{\partial t}\ . \tag{3.11}$$

From that, taking into account (3.10), we get

$$\frac{\Delta c}{c} = \frac{\partial \Delta \bar{\theta}}{v^c \theta \partial l} D_{c\theta},\ v^c = \frac{\partial l}{\partial t}\ , \tag{3.11a}$$

where v^c is a linear speed of the carbon transfer, characterizing a linear graphitization speed.

The diffusion's overcooling is defined by the equation

$$\frac{\partial \Delta \overline{\theta}}{v_c \theta \partial l} = \frac{\theta_l - \theta_s}{\overline{D}} \ , \tag{3.12}$$

where θ_l, θ_s are the liquid and solid temperatures accordingly, v_c is the crystallization speed, and $\overline{D}$ is the coefficient of diffusion in the equation

$$\frac{\partial \Delta \overline{c}}{\partial t} = \overline{D} \frac{\partial^2 \Delta \overline{c}}{\partial l^2} \ . \tag{3.13}$$

Joint consideration of equations (3.11) and (3.12) leads to relation

$$\frac{\Delta \overline{c}}{c} = \frac{\overline{v}_c (\theta_l - \theta_s) D_{c\theta}}{v^o \theta \overline{D}}, \ \frac{\Delta c}{\Delta \theta} = \frac{v_c}{D} \ , \ v_c = \frac{\overline{v}_c}{v^o} \ , \ D = \frac{\overline{D}}{D_{c\theta}} , \tag{3.14}$$

which connects the relative increment of concentrations $\Delta c = \frac{\Delta \overline{c}}{c}$ and the relative increment of temperature $\Delta \theta = \frac{(\theta_l - \theta_s)}{\theta}$ (defined by the state diagram $Fe - C$), with the corresponding speeds of crystallization, graphitization, and the coefficients of diffusion.

Changes in the density characterizes the diffusion processes with phase transformations, which determines the deformations ε, creating the concentration's tensions σ_o:

$$\frac{\Delta \sigma_o}{\overline{\sigma}_o} = \sigma_o \sim \varepsilon \sim \frac{\partial \Delta \overline{c}}{\partial l} , \ \frac{\partial \varepsilon}{\partial t} \sim \frac{\partial \sigma_o}{\partial t} \sim \frac{\partial \rho}{\partial t} \ . \tag{3.15}$$

The hydrodynamic process we describe by the equation connecting the liquid mass' center speed v_m with a hydrodynamic force ∇p:

$$\frac{dv_m}{dt} = L_p \nabla p \ , \ v_m \sim \frac{dm}{dt} \sim \frac{d\rho}{dt} , \tag{3.16}$$

which in general case has the form of Nevier-Stock's equation:

$$\frac{dv_m}{dt} = -\frac{\nabla p + \nabla \eta \nabla v_m}{\rho} + \Delta (k + 1/3\eta) \nabla v_m + \rho^{-1} \sum_{i=1}^{n} \rho_i F_i \ , \tag{3.17}$$

where k is a coefficient of the volume's viscosity, η is the coefficient of the shift's viscosity, $\rho_i F_i$ is the force vector acting upon *i*-mass unit, and ρ is an average density, determined, for example, from equations (3.7), (3.8).

Equation (3.17) for the considered problem is reduced to form (3.16) by dividing the hydrodynamic force ∇p on two parts: $\nabla p = \nabla p_1 + \nabla p_2$, where ∇p_1 is the force of hydrodynamic pressure, ∇p_2 is the gravity force depending on the liquid height z and a gravity acceleration g.

The kinetic coefficient $L_p(\eta, f, \Sigma) = L_p$ depends on density ρ (in particular $L_p = \rho^{-1}$), where ρ is determined by η and depends on the phase state f and a technological weight's specific Σ **(**of the phase distributions along some square S), using the relations:

$$\rho \sim \eta / m, \quad m = m(f, \Sigma), \nabla p \sim \frac{\partial p}{\partial l}.$$

Equation (3.16) for a two phase zone, in particular, when the liquid speeds are small and $\frac{dv_m}{dt} \approx 0$, acquires the form

$$v_m = -\nabla(p + \rho g z) m(f, S) / \eta, \tag{3.18}$$

which corresponds to Darci's equation in the theory of filtration.

Finally the considered equations take into account a joint impact of thermal, thermodynamic, thermokinetic, thermodiffusion, strengthen, and hydrodynamic phenomena, which accompany the solidification process.

In addition, the above equations select the phase variables and establish their connections to the dynamics, generated by the cross phenomenon effects.

The phase variables, connected by relations

$$\frac{\partial x_{i+1}}{\partial t} = x_i^t, \ \frac{\partial x_{i+1}}{\partial l} = x_i^l, \ i = 1, \ldots, n, \tag{3.19}$$

with corresponding thermodynamic forces X_i^t, X_i^l and kinetic coefficients L_{ii}, L_{ij}, L_{ji} $i, j = 1, \ldots, n$, have a distinct tensor dimension in the NT equations.

For the considered thermodynamic phenomena, the selected phase variables (in relative units on Table 3.1) connect only the cross phenomena reflecting the systemic interconnections of the solidification processes. For the pair of thermokinetic (K) and thermodiffusion (D), the general thermodynamic flow and forces have the form

$$x_{n-1}^t = \frac{\partial \theta}{\partial t} = I_\theta^t, \ x_{n-1}^l = \frac{\partial \theta}{\partial l} = I_\theta^l, \ X_{n-1}^t = grad\Delta\ \theta, \ X_{n-1}^l = grad\Delta c, \tag{3.20}$$

and we come to the following NT equations for the selected variables:

$$\frac{\partial \theta}{\partial t} = L_{n-1,n-1}^t grad\Delta\ \theta, \ \frac{\partial \theta}{\partial l} = L_{n-1,n-2}^l grad\Delta c, \tag{3.21}$$

which are interconnected according to the time and space phase relations.

Introducing for general flow I_i the corresponding derivations $\frac{dx_{i+1}}{dt}$, in particular $I_\theta = \frac{d\theta}{dt}$, $I_c = \frac{d\Delta c}{dt}$, we come to the NT equations, which connect directly these phenomena:

$$\frac{d\Delta\theta}{dt} = L_{n-1,n-1} grad\Delta\theta + L_{n-1,n-2}^l grad\Delta c, \ \frac{d\Delta c}{dt} = L_{n-2,n-1} grad\Delta\theta + L_{n-2,n-2} grad\Delta c. \tag{3.22}$$

Applying the above equations for a quasi equilibrium's two phase zone, at

$$\frac{d\Delta\theta}{dt} = \frac{d\Delta c}{dt} = 0,$$

we come to well known relation, connecting the kinetic coefficients for the cross phenomena [1]:

$$\Delta\theta = K_L \Delta c,\ K_L = -L_{n-1,n-2} / L_{n-1,n-1} = -L_{n-2,n-2} / L_{n-2,n-1}. \tag{3.23}$$

Analogous equations take into account other related phenomena.

The variable for the basic phenomenon (n+1) generally depends on both time and space coordinates $(x^l{}_{n+1}, x^t{}_{n+1}) = x_{n+1}(t,l)$.

At the fulfillment of the variation principle (ch.1.3), we come to the equations for the state variables $x = \{x_i\} = \{x_i(t,l)\}, i = 1,\ldots,n$:

$$\frac{\partial x^t}{\partial t} = L^t X^t,\ \frac{\partial x^l}{\partial t} = L^l X^l, \tag{3.24}$$

where the kinetic matrices are determined by the process' statistical characteristics at the microlevel:

$$L^t = 1/2\dot{r}^t,\ L^l = 1/2\dot{r}^l. \tag{3.25}$$

Modeling of the solidification phenomena follow from the direct connections of the matrix kinetic coefficients, thermodynamic flows, and thermodynamic forces.

The scalar form of these equations:

$$\frac{\partial x_i}{\partial t} = L^t_{ii} X^t_i + L^t_{ij} X^t_j,\ \frac{\partial x_i}{\partial l} = L^l_{ii} X^l_i + L^l_{ij} X^l_j \tag{3.26}$$

shows the influence of the cross phenomena's forces and flows:

$$\dot{x}^t_{ij} = L^t_{ij} X^t_j,\ \dot{x}^l_{ij} = L^l_{ij} X^l_j, \tag{3.27}$$

during the model space-time movement, with a dependency of each space coordinate on the space speed, at C =const, $l = Ct$, according to relations

$$x^l{}_n = \frac{\partial x_{n-1}(t,l)}{\partial l} = \frac{\partial x_{n-1}(t,l)}{\partial t} C^{-1} = x^t{}_n C^{-1},\ l = \{l_x, l_y, l_z\}. \tag{3.28}$$

We get the following connections of the coordinates and kinetic coefficients:

$$1/2\dot{r}^t_{ij} (r_{jj})^{-1} x_j = \dot{x}^t_{ij} = L^t_{ij} X^t_j, 1/2\dot{r}^l_{ii} (r_{ii})^{-1} x_i = \dot{x}^l_{ii} = L^l_{ii} X^l_i. \tag{3.29}$$

The above relations take into account Curie's principle for interacting components of the same tensor dimension.

This means that each space-time connection holds true only for the same number of both phase coordinates (x^t_i, x^l_i).

System (3.26) – (3.29) is reduced to the Cauchy form

$$\dot{x}_i^l = \lambda_i^t\, x_i^t + \lambda_j^t x_j^t,\ \lambda_i^t = 1/2\,\dot{r}_{ii}^t\,(r_{ii}^t)^{-1},\ \lambda_j^t = 1/2\dot{r}_{ij}^t (r_{jj})^{-1},$$
$$\lambda_i^l = 1/2\,\dot{r}_{ii}^t\,(r_{ii}^l)^{-1} = C^{-1}\lambda_i^t,\ \lambda_j^l = 1/2\dot{r}_{ij}^t (r_{jj})^{-1} = C^{-1}\lambda_j^t, \qquad (3.30)$$

which takes into account both parts of the trajectory for each extremal segment.

One of them, at $\dot{x}_{ii}^t (t < t^{'})$, $\dot{x}_{ii}^l (l < l^{'})$, corresponds to the process trajectory's within the extremal's segments **(**between the DP points) here: $t^{'} = (t_1, t_2, \ldots, t_m), l^{'} = (l_1, l_2, \ldots, l_m)$, which reflects the peculiarities of the each independent processes.

Another one, at $\dot{x}_{ij}^l (t \ge t^{'})$, $\dot{x}_{ij}^l (l \ge l^{'})$, considered after passing the DP points of process interactions, reflects the processes' cross phenomena, arising at transferring kinetic into diffusion and back into kinetics (ch.1.7).

At these points, the superimposing processes interact sequentially during the model's space-time movement and forming a cooperative hierarchical structure (chs.1.5, 1.6).

In this hierarchy, each following extremal segment (at $(t \ge t^{'})$, $(l \ge l^{'})$) corresponds to a sequential equalization of the superimposing two or three process' components, accompanied by the local nonsymmetry and singularities.

Specifically, the IN node's hierarchical sequence for these segments describes the cross phenomena arising at the superimposition of the thermoconductivity, diffusion, and other processes in Table 3.1, whose interacting components are the main sources of solidification.

Character of optimal dynamics and controls for the processes at the Table 3.1 are shown on Fig.3.1

Considering the controls as the inner variables in the IMD-NT equations (chs.1.7,1.9), the synthesized optimal control functions, applied to these equations, are able to prognosis the most probable process evolution.

Let us find the *optimal hydrodynamic conditions*, defining the process ingot's maximal ordered and homogenous macrostructure without defects.

For the solution, we apply the minimal conditions for the path functional's space distributed Hamiltonian $h_v(t,l)$, connected to the optimal function of the specific entropy production (chs.1.4,1.7, 1.9) $\sigma_o = \sigma_o(t,l)$ (in our indication here).

The problem consists of finding the optimal functions for hydrodynamic variables: pressure p, velocity υ of the liquid phase movement, which are defined by the given function of the specific entropy production for a single space coordinate *l*=1: $\sigma_o = \sigma_o(\tau,1)$.

We consider the movement of a noncompressed Newton's liquid within a cylindrical channel (z, τ), described by equations

$$\frac{\partial \upsilon}{\partial \tau} = \Delta\upsilon - \frac{\partial p}{\partial z},\ z = \frac{z_o}{r_o}, \tau = \frac{t\nu}{r^2{}_o} \qquad (3.31)$$

with the border conditions:

$$\upsilon(0,R) = \psi(R),\ R = \frac{r}{r_o},\ \upsilon(\tau,1) = \varphi(\tau) = 0; \frac{\partial \upsilon}{\partial R} = -2g(\tau). \qquad (3.32)$$

The tangent tension on the border, separating the liquid and solid phases, is given by relation

$$f = a_1 \frac{\partial \upsilon}{\partial R},\ a_1 \sim \mu,\ \mu = \rho\nu. \tag{3.33}$$

Equations (3.31) – (3.33) are written for the relative variables: time τ, radius R, and one of the space coordinates $l = z$, with the initial absolute variable: time t, the kinematics' ν and dynamic' μ viscosities, the current r and the fixed r_o channel radiuses, the axis coordinate z_o and density ρ.

The entropy production is connected with the tangent tension f by relation

$$\sigma_f(R,\tau) = a^2 f^2,\ a^2 \sim \mu. \tag{3.34}$$

It is assumed that parameters a, a_1, ρ, ν are given and fixed.

The entropy production for the *optimal* IMD model is defined by equation

$$\sigma_o(\tau,1) = b^2 \alpha_o^t(\tau,1)\ , \tag{3.35}$$

where $\alpha_o^t(\tau,1) = \alpha^t$ is the model eigenvalue's given function and b here is the constant coefficient. Function g(τ) (3.32) – (3.34) we express via α^t using (3.35) in the form

$$\sigma_o(\tau,1) = \sigma_f(R,\tau),\ b^2\alpha^t = a^2 f^2 = a^2 a_1^2 \left(\frac{\partial \upsilon}{\partial R}\right)^2,\ g = \pm \frac{b}{2a_1 a}(\alpha^t)^{1/2}. \tag{3.36}$$

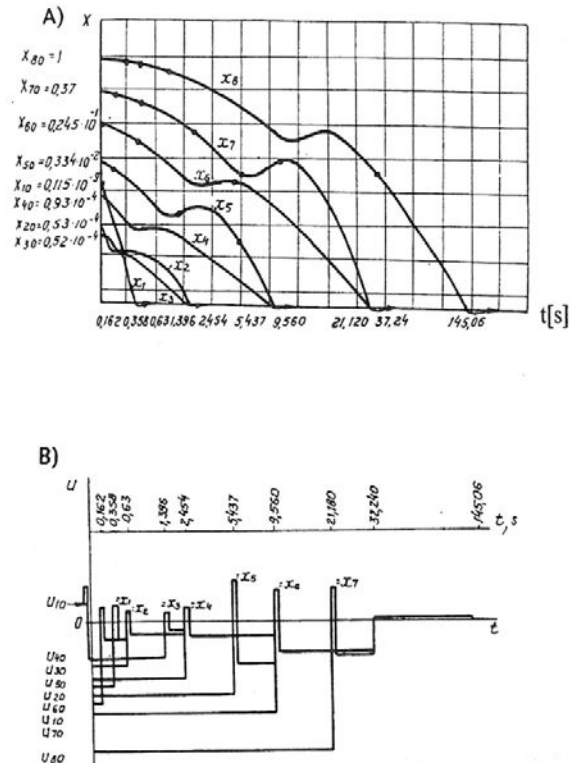


Figure 3.1. Optimal dynamic processes (A) and controls (B) for processes on the Table 3.1.

By solving equations (3.31) – (3.36) at an arbitrary function $\frac{\partial p}{\partial z} = f$ and at the given α^t, we obtain the following integral equation

$$\sum_{k=1}^{\infty} \int_0^{\tau} f(\varepsilon) \exp(1 - \lambda_k^2(\tau - \varepsilon)) d\varepsilon = C(\alpha^t)^{1/2}, \; C = \pm \frac{b}{a}, \tag{3.37}$$

where λ_k^2 are the eigenfunctions in equation (3.31). The solution gets representation

$$f = \sum_k f^k, \; \int_0^{\tau} f^k(\varepsilon) \exp(-\lambda_k^2(\tau - \varepsilon)) d\varepsilon = \frac{C}{2^k}(\alpha^t)^{1/2}. \tag{3.38}$$

After substituting (3.38) into (3.1 – 3.36), the field of velocities $\upsilon = \upsilon(R, \tau)$ can be found.

Let us consider the concrete results for function σ_o, given by equations

$$\sigma_o = \overline{\alpha}_o \overline{\sigma}, \; \overline{\sigma} = \overline{\sigma}(\alpha_o, \overline{\beta}_o, z), \; \overline{\alpha}_o = \alpha_o \left(\frac{\nu}{r^2_o}\right)^{-1}, \; \overline{\beta}_o = \beta_o \left(\frac{\nu}{r^2_o}\right)^{-1}, \; \alpha_o = \alpha_o(\gamma), \tag{3.39}$$

where $\overline{\alpha}_o$, β_o, α_o are determined by parameter γ of the optimal model at given r_o and ν.

The sought function $f' = \varphi$ acquires the form

$$\varphi = \pm 25.1 \left(\frac{|\alpha_o| r_o^2}{4 \nu 25.1} \frac{\dot{\sigma}_o(t)}{[\sigma_o(t)]^{1/2}} \frac{\dot{\sigma}_o(t)}{[\sigma_o(t)]^{1/2}} + [\sigma_o(t)]^{1/2} - \sigma_o(0)]^{1/2} \right). \tag{3.40}$$

From that, at the initial conditions $\sigma_o(0) = 0, \sigma_o(t) = \overline{\sigma}(t)$, we get the solutions:

$$\sigma_o(0) = 0, \sigma_o(t) = \overline{\sigma}(t), \alpha_o = -2.29, \beta_o = -1.1456, \gamma = 0.5, \; \varphi_o = \pm 25.1 \left(\frac{3.3\dot{\sigma}_o(t) + \sigma_o(t)}{[\sigma_o(t)]^{1/2}} \right). \tag{3.41}$$

The applied program computes the functions for the pressure $\varphi_o(t)$, its gradient $\Delta\varphi_o(t)$, and the velocity υ (t) at the channel axis z (Fig 3.2).

These functions define the optimal laws for the pressure and velocity to reach the maximal ordered macrostructure.

As a result of the optimization problem's solution, we have found the regularity of changes in the kinetic operator, characterized by a successive reduction in time of the optimal model's intrinsic eigenvalues, which are equalized at the specified points (DP).

The optimal dynamics at the boundary of the ingot cross section hold the boundary functional conditions for the distribution of the physical processes within the solidified ingot's cross section.

The computation procedure consists of identifying the real time-space parameters of the synthesized optimal model for a circular ingot's cross section.

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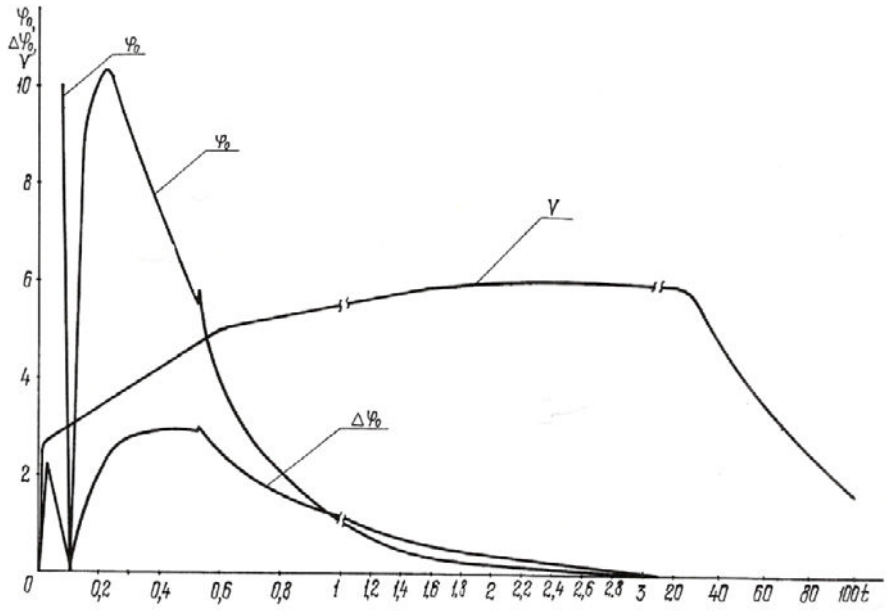


Figure 3.2. The optimal hydrodynamic processes: the function of time for the pressure φ_o(t), its gradient $\Delta\varphi_o$(t), and the velocity υ (t) at the channel axis z .

The specific Hamiltonian h_v, applied to calculate the renovated process parameters, is the main model's indicator of the process dynamics and quality

The synthesized optimal processes, found from the extremum of the path functional (ch.1.3), useful here for ordering of a metallic matrix's structure, in terms of an uniform distribution of graphite and chemical elements along this matrix's cross section (without a segregation).

The results have been applied in the technology of the horizontal casting of ductile iron (HCC), and compared with experimental data and numerical simulation methods. A good agreement was confirmed between the predicted and practical data, as well as between the new and traditional methods [5,6]. The regularities of the pressure changing (Figs. 3.2) were used for the forming of the process optimal feeding law (Fig.3.3), when the corresponding velocity of liquid metal at the channel axis fulfills the above relations.

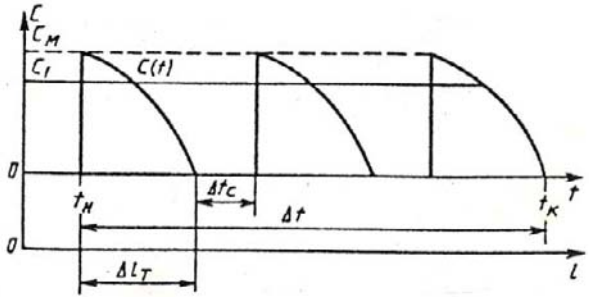


Figure 3.3. The optimal feeding law of the impulse withdrawing $C = C(\Delta l_T, \Delta t_c)$ (the interval of drawing Δl_T, the time of stopping Δt_c).

It is confirmed that for the implementation of the directional solidification, the external pressure**,** as a control function, should satisfy the relation for φ_o (t).

Other details of the practical results are in Appendix 2.3.1.

Example of the Technological Object's Code

The solidification modeling of casting technology processes, along with NT-IMD equations and the identified basic parameters (**n**=8, γ =0.3,**k**=6), reveals the following minimal DSS code for one of the particular IN for the superimposing processes: (1, 2.366832, 1.798496).

The *DSS* code for other related solidification model with basic parameters (**n**=8, γ =0.1,**k**=6) is (1, 2.460668, 1.82039755).

2.3.2. Some Electrotechnological Processes

In the considered electro metallurgical technologies, the produced process' outputs: ferrous, nonferrous metals, alloys', and concentrates, are formed by the physical-chemical superpositions analogous to Table 3.1.

In addition to that, important are electrical conductivity, electrokinetic and electromagnetic phenomena, hydrodynamic filtration, accompanied by chemical reactions, heat and mass diffusion transfer.

The mutual interaction of these process' components creates the object's IN, where a resulting superimposing process, which integrates the components interaction, is able to coordinate the local interactive processes.

The process' quality depends on the speed of physical and chemical transformations, a starting composition of the materials, which are able to change the technological interactions, the expense of energy, the metal loses, and the productivity.

These features characterize the technology as a multiple criterial process, for which a priori formulation of the performance criteria and applying the regular control methods are not possible.

A high temperature and the aggressive chemical reactions in the control furnace zone make a direct measurement of such key's technological parameters as temperature, speed of physical transformations, characteristics of chemical components, and the gas pressure very difficult.

The essential physical and cross-interacting processes are not separated without losing technological information. For such superimposing processes, the path functional, IMD methods of macromodeling, optimal control, and an indirect measurement of the integrated macroparameters are the most effective.

In particular, it is simple enough to use electrical conductivity for controlling the chain of superimposing technological transformations, as an intrinsic function of the integrated interactive processes.

Hamiltonian h_v of the controlled process, according to the IMD equations, can be expressed via the electroconductivity $\sigma_e = \sigma_e(t^{'}, l^{'})$ (ch.1.9) at the discrete points

$(\tau, l(\tau)) \sim (t^{'}, l^{'})$ (ch.1.4, 1.5):

$$h_v(t^{'}, l^{'}) = \dot{\sigma}_e \sigma_e^{-1} \sim k_e\ (t^{'}, l^{'}). \tag{3.42}$$

This Hamiltonian defines the optimization criterion in the form

$$minmax\ k_e = k_e^{\ o},\ k_e = \frac{\partial G_e}{\partial \mu_e} G_e^{\ -1},\ G_e \sim \sigma_e,\ \mu_e = \frac{h_e}{h_o}, \tag{3.43}$$

where the furnace's electrical conductivity G_e is measured in the process of the electrode's movement h_e, with h_o as a level of a melting metal in a furnace; μ_e is a relative electrode's movement (Fig.A.3.2.2).

The controlled function $k_e\ (t^{'}, l^{'})$ of this indirect macroparameter indicates *a path* through a chain of the superimposing processes. Its optimal value $k_e^{\ o}\ (t^{'}, l^{'})$ is changing along the IN's line of switching control, by analogy with Fig.1.6.4, specifying the process' sequence of the consolidating macrostates [19].

The connection of the measured k_e with the main technological parameters has been established as a result of the practical work on many industrial objects [8-18].

All through these projects, the methodology has been developed and implemented with the procedure of the minimization of the k_e -function, which actually provides the optimal technological process with a minimum energy loss.

The results of practical implementations the optimization procedure are in this chapter's Appendix A.3.1., A3.2.

Appendix. Additional Practical Results

A.3.1. Control System in the Casing Technology

The liquid metal feeding and the solidification processes in the systemic model are mutually interconnected by such a way that the speed of the liquid metal movement is coordinated with changing the density of the forming solidification bar.

The time of solid skin formation, required for the start of draw, is governed by the conditions needed to control the elastic deformations and for compensation of a shrinkage.

The optimal feeding law (Fig.3.3) $C(l_n)=C[l_n(t_n, \Delta l_T, \Delta t_c, \Delta t)]$ satisfies these conditions, where t_n is the starting moment of changing linear feeding velocity C(*t*); t_k is the fixed time interval, Δt is the current time interval, Δl_T is the space interval of the feeding impulse duration (as an optimal control); Δt_c is the time interval of stopping the feeding impulses; C_l is the average solidification speed.

An increment of the ingot's linear size satisfies the function

$$l_n = (C - C_l) t_{n-1},$$

where

$$C = \frac{\Delta l_T}{\Delta t}\left(1 - \frac{\Delta t_c}{t_k}\right),\ \Delta t = t_k - t_n$$

characterizes the average speed of feeding.

This increment is necessary to compensate the inglot's volume reduction during the time interval t_{n-1}.

The optimal informational model has been implemented as a CAD-CAM methodology for the design of the casting processes.

A special feature of the developed IMD model consists of finding the intervals of withdrawal Δl_T and pauses Δl_c (for each process) on the basis of its identified model, rather than being set externally according to the conventional approaches [5].

The method and algorithm allow the computation of the optimal value of casting speed, the drawing and pause intervals, the cast iron temperature in the receiver, the cast ingot's temperature, and the variation in the flow rate and water temperature in the die (Fig.A.3.1.1).

The model is also used to determine the parameters of a die design and the cooling conditions.

The IMD model calculates an amount of the chemical elements, for example Sn, Cu, Mo, which are capable to control the zone of effective diffusion under the real solidification conditions. The model calculates the phase transformations, in the presence of k points of overlapping phenomena, particularly, by the nodule count (NC) at the points where the element's numbers ($\mathbf{n}-2$+k) are formed.

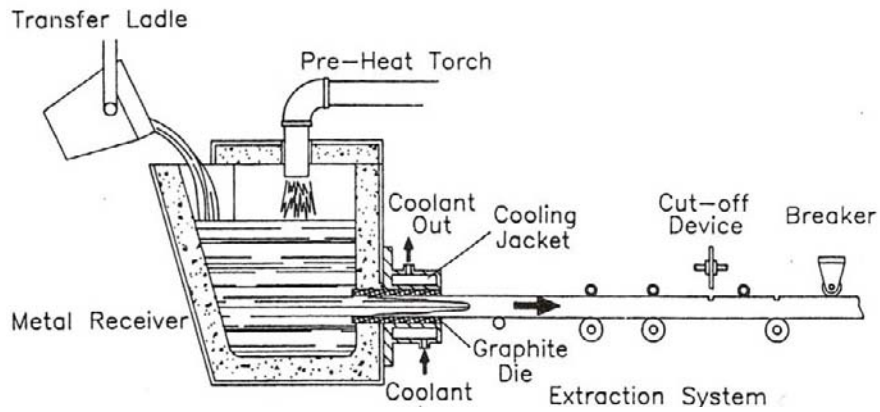


Figure A.3.1.1. Schematic of the casting process.

The model diffusion zone's length is characterized by the segment l(n+1) of the helix on the cone (at the fixed angle at its vertex), where the number of elements $L(\mathbf{n}+1)/L(\mathbf{n}+k-1) \bullet 2(\mathbf{n}-2-k)(l+\gamma^2)^2$ is located;

$L(\mathbf{n}+1)=L(\mathbf{n},\gamma)$ is the model space length in a solidification cone;

$L\mathbf{n}+k-1=L(\mathbf{n},\gamma)$ is the model length of the effective diffusion zone.

The nodule count per unit of a cross-sectional area $S=\pi R^2$, precipitated at the k-th point, is

$NC=4\pi/[S(\mathbf{n}-2+k)^2 (L\mathbf{n}+1/L\mathbf{n}+k-1)^2 (l+\gamma^2)^2]$,

where **n** and γ are the IMD model's parameters.

In the optimal model, graphitization occurs during spatial interval L(**n**-4) with the maximum NC (L(**n**-4)) nodule's count.

The computerized method calculates the NC with a sufficient accuracy [7] at any point along the entire cross section of the bar.

The Automatic Control System Design and Development

The developed information model has been used as an integrated computerized self-adaptive module to control the real HCC parameters (Fig. A.3.1.2).

The HCC machine, installed in jobbing foundry, was equipped with series of thermocouples to monitor and record iron temperature in the receiver, the cast bar temperature, and the water temperature in the die.

These data was automatically input into the computerized control system with the developed software, which was able to adjust and concurrently control automatically the drawing parameters (speed and intervals) related to the actual process variables.

Series of sequential experiments have confirmed that the practical value of the considered solidification model consists of not only in the process simulation, but also in the operative control of the casting process.

The developed microprocessor for the self-controlled HCC system has functioned practically [8].

The IMD approach has been successfully used for modeling of the ductile iron permanent mold casting, for engineering design of the feeding and cooling systems, in particular, for the synthesis and design of sectional die-coolant and drawing devices.

Dr. Y.S. Lerner was a co-principal investigator in this project [7].

A.3.2. Control System in the Electrotechnology

The optimization procedure has been automatized by designing the computer-based optimal control system (Fig. A.3.2.1).

The automatic system includes the central computer (optimizer) that optimizes k_e by changing the control strategy for the local regulators.

The optimizer's nonsearch procedure is based on a combination of the optimal control synthesis

$v = v\{k_e(t')\}$, $v(t')=(v_1, v_2, v_3)$ and the identification methodology.

The local controls consist of the following devices:

- the regulator of the furnace electrode's conductivity $G_e = G_e(\mu_e)$, $v_1 = G_e^o$;
- the regulator of the furnace voltage V, $v_2 = V^o$;
- the regulator of the furnace loading (q) by changing the raw materials, as the function of indirect parameter $q=q(\frac{\partial G_e}{\partial \mu_e})$, $v_3 = q^o$;

- the electrode device controlling h_o, which uses an additional electrode for measuring a total level of the melting metal in the furnace and the local contact's conductivities: $(G_e)_{mc}$, $(\frac{\partial G_e}{\partial \mu_e})_{mc}$;
- the above electrode devices also measures a level of the melting metal's *components* (h_{mc}), as a function of the electroconductivities $h_{mc} = h_{mc}(\frac{\partial G_e}{\partial \mu_e})_{mc}$ of the components;
- the controlling devices for automatic output of melting components from the furnace.

The optimal control system has been implemented in industry for some acting ferrous and non-ferrous electrical technological objects [17].

It was shown [15] that for these objects, the k_e- criterion is also an integrated indicator of a similarity of the technological features, which is useful for the comparison of different electrical furnaces in terms of their power, geometry and dynamics.

Such a comparative analysis is also important for a direct transfer of the experimental results into practice and the process' improvement, applied to industrial objects and technologies.

The direct transfer is also a part of the industrial applications of control systems, which had been developed on some experimental or similar objects.

The prognosis for the optimal technology is based on a methodology of the process' computer simulation with the optimal control.

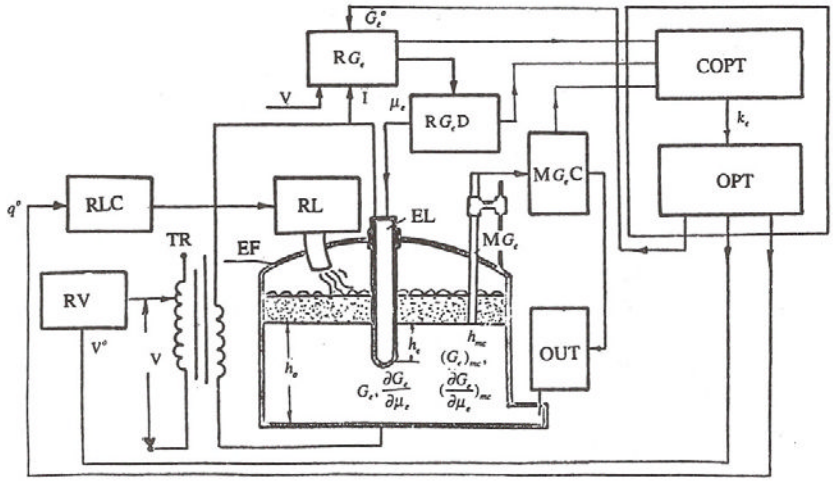


Figure A. 3.2.2. Optimal control system of the electrotechnological process.

Analogous results have been achieved for the metallurgical electrolysis processes with a chain of electrical, chemical, and thermodynamic cross phenomena.

Both technological optimization and control systems, using the indirect measurement of the electrical conductivity, have been implemented on industrial objects.

The electrolytic technology for manufacturing of the composite ligatures is an example of such an object [11].

The process of a microwire casting in a microwave furnace has wide applications.

Its high-speed technology is based on the superimposition of electromagnetic, heat and chemical interactions.

The macromodel of the optimal process' technology has been identified following the IMD identification methodology and the indirect measurement of the main furnace parameters.

The results of computer simulation have predicted the optimal furnace parameters and the control systems structure [14].

The designed optimal system has been implemented in acting microwave furnaces.

The methods of macromodeling, identification and optimization have also been applied for some chemical technologies [10].

The detailed original results of practical implementation are given in References [9 – 19] and also in [R] .

The above results demonstrate the IPF-IMD approach viability and the effectiveness for complex objects with superimposing phenomena.

Chapter 2.4

AN ELEMENTARY INFORMATION MACRODYNAMIC MODEL OF A MARKET ECONOMIC SYSTEM

Information represents a common and universal substance, active participating in a diversity of physical and/or virtual interactions, including various forms of economic interactions.

Applying the IMD, we study the information *regularities* of economical dynamics and the mathematical *evaluation* of the economical system's processes, focusing on an elementary *production-organization*, the production's *interaction* and *management*, and the different *market dynamics*, by building their *information systemic models*. An *organization* is modeled by the *hierarchical structure* of information cooperative dynamic space distributed *network*. The found formal information *mechanisms* govern the marke**t** cooperative dynamics and impose the information *restrictions* on these processes. The considered systemic mechanisms of self-control, adaptation, and evolution represent a general *attribute* of an economical system.

2.4.1. About Information Systems Modeling of a Modern Economy. The Objectives

In the modern economy, the main exchanges occur through the transferring of information (in different forms of signals, physical signs, coded communications, etc.) between customers, producers, banks, investors, firms, and across the market.

Even a business with a diversity of goods can be represented by an exchange of corresponding information structures and values, expressed, in particular, by their code substitutes. All participants in such a real and/or virtual business' information produce, transmit, exchange, or consume information.

Information appears as a *universal* equivalent of money, directly exchangeable with different commodities, including a human labor. Because exchanges exist in a social system, the quantitative and qualitative values of information can also evaluate *social* relations.

Since modern economics become an *information system*, it is imperative to apply information systems theory for *understanding the systemic regularities* of the information

economy.

A joint consideration of the economic system's components intends to expose not only the *inter components' relations, necessary for the coordinated dynamic cooperation, but also to detect the inner components' regularities,* required to carry out the systemic relations for a whole system.

The first problem in this direction is to *develop* the information models for each particular economic object (as a system's component) such as a local business production and organization, a market, a bank, and others, and then modeling the information dynamic relations between them.

The second is to model the *specific cooperative dynamics and the phenomena*, essential for the economic object**s**, and also to *unify* the models into the information system, using a *common* information language and the modeling methodology, applied to a variety of the object's interactions and communications.

The goal is to reveal the main *information regularities* of a market economic system, based on the mathematical formalism of *cooperative dynamics*, the modeling of the system's information *structure*, and an analysis the information exchange flows and communications throughout the whole system.

Since the *production and transformations* of information are the basics of any information object, we focus on dynamic regularities of their main economic entities: an elementary production system and a market, at the conditions of free competition and open, unrestricted supply-demand processes.

The developed methodology intends not only to *identify a current* object's model with all specific phenomena, but also to *improve* the object's functioning by optimal control and management, following from the system's modeling.

We believe that the information approach and the results of information *system theory* are able to bring a new understanding of economics, business relations, and communications, creating a breakthrough in the solution of important economic problems. The references**'** analysis (to this chapter) shows that a system's *information* model of economics has not been developed yet.

Evidently, the first information models of economics were considered by H. Theil [1]. There were the static models and the aggregation analysis, applied to the local economic problems with the entropy measure of employment, markets, incomes, industrial concentrations in the USA, and occupational diversity in cities, all of which can be linked to the growth and decline of social systems [2, 3].

J. Marschak [4, 5] applied Shannon's information theory to economics with the "teams" approach and decentralized organizations.

M. Aoki [6] developed a stochastic approach to macroecomic modeling based on stochastic dynamics and stochastic random combinatorial analysis. The approach uses Markov processes with probability distributions, determined by the Chapman-Kolmogorov equation. The author's goal is to unveil stochastic regularities of the multi-agent interactive models, including power-laws in share or stock markets. Even not using information theory, M. Aoki emphasizes [7] on the key role of uncertainty in standard economic analysis and economic policy.

J. Hirshleifer considers [8] uncertainty, arising in a market irreversible process of the "liquid" commodities, which can unfold the events over time. Actually, none of the known models directly minimize the uncertainty, providing the control actions and outcomes, in

terms of quantifiable and marketable production factors and products, or their prices.

J. Hirshleifer [9] reviews a potential broad use of information theory for a wide description of multiple micro- and macro economic processes, including market and trading relations, technological productions, organizations, banking, money flows, price, multi-person's behavior, others.

Unfortunately, none of the existing model covers such a broad approach.

The development of the *information dynamic* model of macroeconomics, taking into account the interaction of the material, social-labor, and biological processes on the basis of the united information mathematical formalism represents an actual, never before uninvestigated problem.

Considering a joint information description of different interacting elements, composing an economic system, we intend to find *common information mechanisms* and their mathematical expressions, governing the information economic processes and their regularities.

Starting with the primary system's components such as a cooperative behavioral production system and different forms of the market, we however will not take into account the government limitations, public and military policies, monopolies, and other specific restrictions and requirements, imposed on a free market economy. Even with such limitations and simplifications, our approach sounds productive and promising in revealing unknown information regularities of an economic system.

An aim of the Information Systems Modeling [10-11] is to build a *bridge* between the mathematical modeling and systemic formalism *and* the world of information and *information technologies* to reveal the common information regularities of a variety of modeling objects with the final goal of exposing a specific information *code* for each object. For the problems solution we apply the path functional (IPF) mathematical formalism of revealing information regularities, generated by *multiple random interactions*, which are considered *independent of the physical or virtual specifics*, while all interactions generate information.

The information model's *macroprocess* with a set of macrotrajectories is determined by the considered class of the random *microprocess* (ch.1.1), generated by a variety of interactions. The macroprocess' information *dynamic model* (ch.1.3) describes the information *structure and regularities* of each economic entity. These include finding the model's punched localities where a dynamic prediction is possible.

The IMD approach leads to a general *systemic* macromodel, which incorporates the cooperative structure of the time-space hierarchical information network, the functional mechanisms of adaptation, self-organization, and evolution.

Part 1 provides the fundamental basis for building the systemic information model of an economic system. The IPF specification for each object identifies its eigenfunctional, whose information form synthesizes a variety of economic, social, and intellectual processes of a modern economy. The IN, created by the particular functional, allows cooperating and structuring these processes in the forms of different organization, whose systemic integration is modeled by the IMD global cooperative structure.

The designed information models and developed computer algorithms and programs are successfully applied to a diversity of physical and non-physical systems, including the economics.

2.4.2. An elementary Local Production System (LP)

The LP represents an elementary information model of the interaction of an individual (labor process) and a production process in an organization, based on cooperative dynamics.

We model the information production process by a *synthesis* of information, carried by the interaction of different information sources and operating by an individual, which cooperates with an organization.

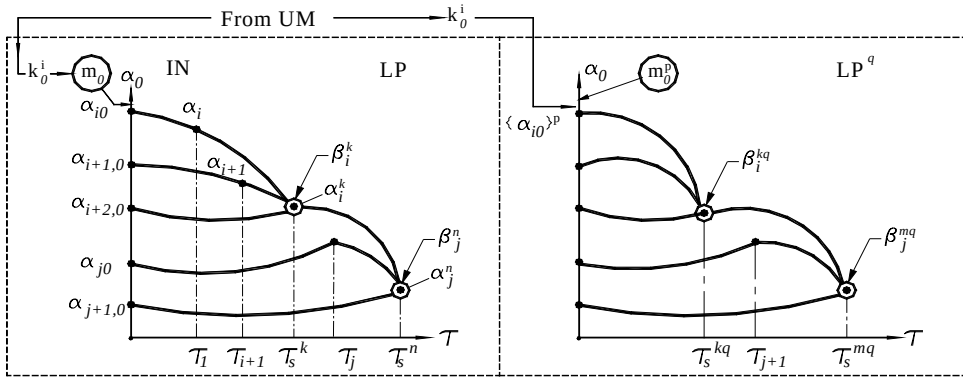


Figure 4.1. a) An information structure of an elementary LP; b) A schematic of the LP^q set, being a components of LP^p; k_o^i is a coefficient of the current information unit m_o, adjustable by the unit exchange market UM, Fig. 4.3.

An individual participates in modern production by utilizing his brainpower, provided by knowledge, intellectual capabilities, and skills, becoming a key ingredient of economic activities and operational information sources, which generate the production process. The synthesis performs the IN optimal information structure (Fig.4.1a,b), where $\{\alpha_{io}\}$ is the information, delivered by the source of production, including different materials and machines (and/or initial human labor), $\{g_{io}\}$ is synthesized information (in the form of goods, or an intellectual product), and $\{h_{io}\}$ is information required to control the synthesis, delivered by a human being. The IN hierarchical structure (Fig. 4.1a,b) presumes that the above information components are located at corresponding hierarchical levels starting with number 01 at the lowest level ($\{\alpha_{1o}\}$, $\{g_{1o}\}$, $\{h_{1o}\}$), where the process is initiating, and ending at the highest hierarchical level ($\{\alpha_{no}\},\{g_{mo}\}$, $\{h_{mo}\}$), where a final product is synthesized. Because the IN optimal structure consists of the triplet's sequence $(1,3,5,...,7,...,m)$, $\{g_{ko}\}$ and $\{h_{ko}\}$ belong to corresponding triplets whose information contributions also take into account the information exchanges between hierarchical levels.

Both the information productions g_{ko} and human participation h_{ko} depend upon their hierarchical location that determines the specific information values of information $\mathbf{a}_m^k$, located at level k with the total m levels of the IN's hierarchy.

The hierarchical level is determined by a valueness of its eigenvalue $\alpha_{i\tau}^{k}$, whose multiplication on time interval τ_i^k, preceding the synthesis, defines the invariant $\mathbf{a}_m^k = \alpha_{i\tau}^k \tau_i^k$. The amount $\mathbf{a}_m^k$ measures the corresponding *quantity* of information, while $\alpha_{i\tau}^k$ characterizes its quality at k level.

Therefore $\mathbf{a}_m^k$ accumulates both quantitative and qualitative measures of a *bound information*, where k characterizes the number of triplets being enclosed from the previous levels of hierarchy.

The VP carries out ranging of the initial information sources $\{\alpha_{io}\}$, productions $\{g_{io}\}$, and controls $\{h_{io}\}$ according to the information quantities and qualities, measuring the information economic valueness.

The VP implementation also creates a triplet as an *optimal* information structure, synthesizing a local maximum of an output information. In the IN, formed by optimal triplet's connections, each triplet produces the same quantity of information, generating the constant maximal output surplus $d_s(\gamma)=\mathbf{a}(\gamma)+\mathbf{a}_o^2(\gamma)$, reached at the moment τ_s of the triplet synthesis. A human plays a role of "synthesizer", which contributes the needle control's information $\mathbf{a}_o^2(\gamma)$ assembling every triplet: $\mathbf{a}_o^2(\gamma)=h_{ko}$. Each h_{ko}, spending the information $\mathbf{a}_o^2(\gamma)$, generates information surplus $d_s(\gamma)$, which produces the information form of goods-commodity $g_{ko}=d_s(\gamma)$. This determines a human relative information contribution $r_h(\gamma)=h_{ko}/g_{ko}=\mathbf{a}_o^2(\gamma)/(\mathbf{a}_o^2(\gamma)+\mathrm{a}(\gamma))$ at each level of the IN's hierarchy. The actual range of r_h follows from their numerical computations: $r_h(\gamma=0.1)\cong 0.715$, $r_h(\gamma=0.5)\cong 0.66$, $r_h(\gamma=0.8)\cong 0.595$, where $\gamma=0.5$, corresponds to a local equilibrium. Thus, decreasing γ corresponds to increasing a total information enclosure into a production (for example, by its modernization), leading as a result to an increase of the human contribution $r_h(\gamma)$. After producing the final information during the time interval τ_s^n, the production process repeats itself being a *periodical* at each level.

The outcome of information production is expressed in generating the maximal IN's information surplus equals $\Delta S = m(\mathbf{a}(\gamma)+\mathbf{a}_o^2(\gamma))$ of information *quantity* and $\Delta S_m=\mathbf{a}_m^m(\gamma)$ of a total information *quality*. Such an optimal ranged IN consists of repeating the ratios $\gamma_{1,2}^{\alpha}(\gamma)=\alpha_{1o}/\alpha_{2o}$ and $\gamma_{2,3}^{\alpha}(\gamma)=\alpha_{2o}/\alpha_{3o}$ for each subsequent triplet.

Let us take for a unit of information production g_j^k, the amount d_s^k for a level k, related to the time of synthesis information at this level τ_s^k: $d_s^k(\gamma)/\tau_s^k=\alpha_j^k$, where α_j^k has the same dimension as α_{io} does, which is measured by the units of information frequency $m_o s^{-1}$ with a the measure's multiplier m_o that assumes a periodical adjustment. The corresponding information production for $h_j^k=\mathbf{a}_o^2(\gamma)$, related to the time of synthesis information at this level τ_s^k, is $\mathbf{a}_o^2(\gamma)/\tau_s^k=\beta_j^k$.

The spectrum of $\{\alpha_{io}\}$ describes the input IN parameters, while the $\{\alpha_j\}$ represents the *spectrum* of output parameters with $\{\beta_j\}$, modeling an information unit of a human participation.

The set $\{\alpha_{io}^k, \alpha_j^k, \beta_j^k\}$, for each hierarchy level k, depending on γ, characterizes the level's k *structural* information, which measures the corresponding qualities of information according to their valueness. This can be used as an equivalent unit in the information-money exchanges on an information market.

The same information IN's structure has each of the other local information production's models LP^p: $p = 1,...,q$ (Fig.4.1b) with information inputs $\{\alpha_{io}\}^p = [\{\alpha_{io}\}^1,...,\{\alpha_{io}\}^q]$ and information outputs $\{\alpha_j^p\} = [\{\alpha_j^m\}^1,.\{\alpha_j^m\}^k.,\alpha_j^m\}^q]$, $\{\beta_j^p\} = [\{\beta_j^m\}^1,..\{\beta_j^m\}^k.,\{\beta_j^m\}^q]$**. Any** of these components $A_m^k = \{\alpha_{io}^k, \alpha_j^k, \beta_j^k\}$ correspond to the information frequencies $\Omega_m^l = \{\omega_{io}^k, \omega_j^k, \omega_j^k\}$, measured in a common scale (of $m_o s^{-1}$) for all information producers' INs. This leads to the existence of an adequate comparison's scale and equivalent mathematical expressions of different input and output belonging to diverse INs.

The A_m^k components measure the local entropy productions $\frac{\partial S_m^k}{\partial t} = \Omega_m^k$, while their multiplication on a time interval τ_s^k measures the corresponding quantity of information $Q_m^k = \Omega_m^k \tau_s^k$, generated at this interval.

In some cases, each local output component may represent an intermediate product of information synthesis and can be exchanged on the information market; the same applies to each component of information input being exchanged on the information market. The mechanism of the LP, LP^p internal and their mutual competition is analogous to that considered in cooperative dynamics.

Different electronic communications (LAN, INTRANET, INTERNET, others) implement the codes' connections and virtual interactions within each LP and between the LP^p.

2.4.3. An Information Model of a Local Market

We assume a local information market (LM) is a region of the whole market where the information exchanges of the local information producers (LP^p) take place.

This means that at the market, some information producers supply information to their information consumers, or the supplied information finds and satisfies the demand for information at the condition of mutual competition.

A set of mutual producers-suppliers interacts in such a market, exchanging a commodity.

Let says the produced+A_m^k value meets the demand of – A_m^l value, with their information quality difference (price) A_m^{k-l}, which can be reached during a dynamic communication process between k-supplier and l-customer.

The questions are: How can the dynamics of this process be modeled and in what total time? Can a particular supplier and consumer possibly reach a common price? Does this common price exist? How can potential prices be compared? What are the limitations on this process? What are the information and mathematical mechanisms governing a local market?

The answers can be found dealing with an IN set, modeled by the LP^p, which are described by the time-spatial cooperative macrodynamics.

The solution is based on the equilibrium of the information qualities, using the formula (analogous to (1.3.171):

$$\frac{\mathrm{A}_m^k \exp(\mathrm{A}_m^k \tau_m^{k-l}}{2-\exp(\mathrm{A}_m^k \tau_m^{k-l})} = \frac{(-\mathrm{A}_m^l)\exp(-\mathrm{A}_m^l \tau_m^{k-l})}{2-\exp(-\mathrm{A}_m^l \tau_m^{k-l})} = \pm \mathrm{A}_m^{k-l}, \tag{4.1}$$

where τ_m^{k-l} is the time interval between the start and end of the communication process. If the solution of this equation exists, the component's common price A_m^{k-l} can be reached.

The concrete sign of $\pm \mathrm{A}_m^{k-l}$ determines it as a new supply (with sign $-$) or as a new demand (with sign $+$). A buyer, based on his motivation, can change the final sign by applying the needle control. This transcendent equation expresses a partial solution of the main macrodynamic equation (chs.1.4 and 2.1) under the optimal control's action.

This action means that any equilibrium solution, described by the equation (4.1), is most motivated by both the supplier and customer-buyer.

After reaching the equilibrium price, both of them get the same quantity and quality of information, which expresses an *equivalent* exchange. The acquisition of this information is impossible if for the specific commodities the solution does not exist.

Moreover, the comparison of different existing solutions can determine both the final price and continuation of this process, which could be chosen based on the best satisfaction during the competition at the market (between the LP^p and their components).

The equation (4.1) describes the dynamics of potential exchanges between any possible combinations of the components $[\{\alpha_{io}\}^p, \{\alpha_j\}^p, \{\beta_j\}^p]$.

The competitions and exchanges can be performed also by the IN universal codes. In the case of spatial distribution of these components, the IMD equations transform the time intervals τ_m^{k-l} into corresponding *spatial* intervals of a considered space. This models a possible relocation of both LP^p and LM.

The equation, applied to any components of the corresponding frequencies (Ω_m^l, Ω_m^k):

$$\frac{\Omega_m^k \exp(\Omega_m^k \tau_m^{k-l})}{2-\exp(\Omega_m^k \tau_m^{k-l})} = \frac{-\Omega_m^l \exp(-\Omega_m^l \tau_m^{k-l})}{2-\exp(-\Omega_m^l \tau_m^{k-l})} = \pm \Omega_m^{k-l}, \tag{4.2}$$

defines a common *resonance* frequency Ω_m^{k-l}, reached in a process of the equalization of the initial unequal frequencies (Ω_m^k, Ω_m^l). Generally, the process acquires a form of stochastic resonance under the control's actions, analogous to that in cooperative dynamics.

The details of such cooperative resonance attraction mechanism are considered in chs.1.6,1.7.

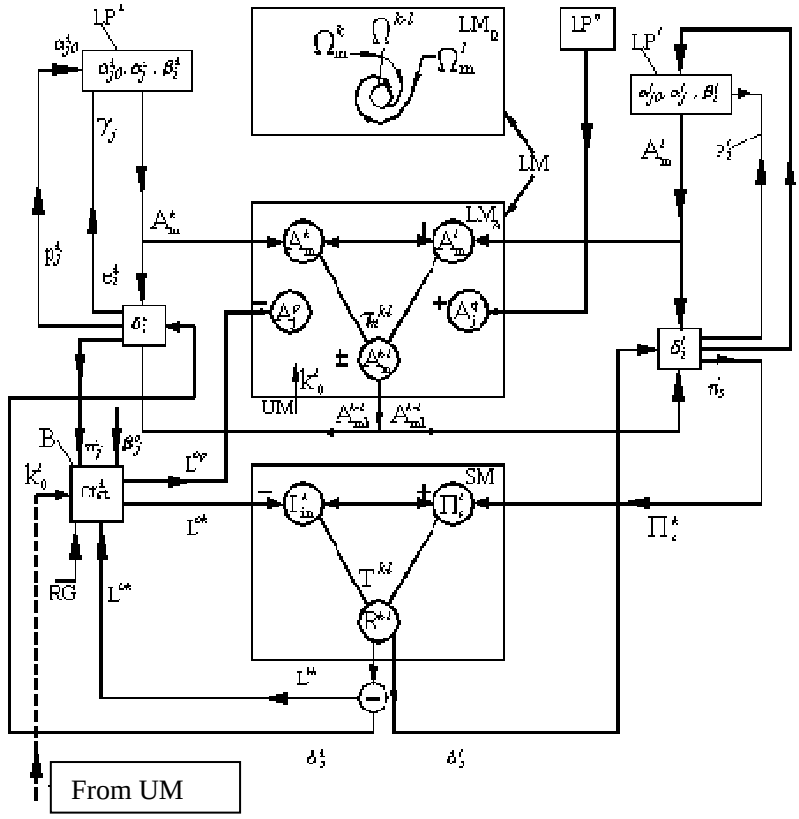


Figure 4.2. A scheme of interacting LP^l, LP^k on an information market LM and a stock market SM with borrowing loans L^{ol} from a bank B and offering stocks Π_s^k; cr is a function of the bank credit, established by a governing regulator GR.

Fig.4.2. shows the LM s' dynamics of both LM_A and LM_Ω, where the LM_A structure applies (4.1) , and the LM_Ω performs a cooperative resonance according to (4.2).

The considered market can also bring new inputs $\{\alpha_{io}^*\}$ to particular IN-LP production as a result of corresponding exchanges between different information commodities, including the initial $\{\alpha_{io}\}^p$, $\{\alpha_j\}^p$, and possibly $\{\beta_j\}^p$, whose every pair is described by the above equations. For instance, in the form

$$\frac{\alpha_{io}^k \exp(\alpha_{io}^k \tau_m^{k-l})}{2-\exp(\alpha_{io}^k \tau_m^{k-l})} = \frac{(-\alpha_j^l)\exp(-\alpha_j^l \tau_m^{k-l})}{2-\exp(-\alpha_j^l \tau_m^{k-l})} = \pm\alpha_{io}^{(l-k)*}, \qquad (4.1a)$$

where $\alpha_{io}^{(l-k)*}$ is a component of a new $\{\alpha_{io}^*\}$.

The renovated spectrum$\{\alpha_{io}^*\}$, according to the market reevaluation, brings new ratios $(\gamma_1^{\alpha j}(\gamma_j), \gamma_2^{\alpha j}(\gamma_j))$ in the input, changing the initial γ_j.

Fig. 4.2 illustrates a schematic of changes on the market for the LP^k, LP^l outputs-inputs, whereas LP^q could not find a buyer.

These changes should preserve the *optimal* values of the above ratios, affecting only γ_j, which maintains a *maximal* and *constant* output for each triplet within a given IN. The amount of this maximum depends on a particular γ with the preservation of the above optimal ratios.

For example, the maximal surplus d_s for different γ satisfies to the following table: $\gamma = 0.01, d_s = 0.998$; $\gamma = 0.5, d_s = 0.958$, $\gamma = 0.8, d_s = 0.912$, where decreasing γ corresponds to the modernization of production. The input reevaluation by the market with keeping an optimal relationship between the input components corresponds to the well-known "optimal input decision," minimizing "diminishing marginal returns" [12-15].

Equations (4.1)-(4.2) can also be applied to the informational exchanges between humans (for which the solutions exist). At the equal quantities and qualities $Q_m^k = (\Omega_m^k \tau_m^{k-l}) = (\Omega_m^l \tau_m^{k-l}) = Q_m^l$, equation (4.2) has a trivial solution $\Omega_m^l = \Omega_m^k = \Omega_m^{k-l}$, which means that the initial equivalent information values do not produce new information, being an analogy to a copying process.

The equations (4.1a), (4.2) also describe the transfer of information within each IN's level and between the levels. In this case, the equation (4.2) acquires the simple forms

$$\frac{-\alpha_{io}^k \exp(-\alpha_{io}^k \tau_s^k)}{2 - \exp(-\alpha_{io}^k \tau_s^k)} = \alpha_\tau^k \qquad (4.2a),$$

$$\frac{(-\alpha_{i+1,o}^{k+1}) \exp(-\alpha_{i+1,o}^{k+1} \tau_o^{k+1})}{2 - \exp(-\alpha_j^l \tau_m^{k-l})} = \frac{\alpha_\tau^k \exp(\alpha_\tau^k \tau_s^k)}{2 - \exp(\alpha_\tau^k \tau_s^k)} = \pm \alpha_\tau^{(k+1)}, \qquad (4.2b)$$

where $Q_m^k = \alpha_{io}^k \tau_s^k = \mathbf{a}_o(\gamma^*)$, $Q_m^l = \alpha_\tau^k \tau_{so}^k$, $\alpha_\tau^k \tau_s^k = \mathbf{a}(\gamma^*)$, $\tau_{so}^k = \tau_o^k + \tau_s^k - \tau_o^{k+1}$, and equation (4.2a) acts within level k, while (4.2b) acts between the level's k and $k+1$.

These equations model the possible information communications between the components $(\mathrm{A}_m^k, \mathrm{A}_m^l)$ within a given production LP^q.

The resulting price A_m^{k-l}, achieved during communication time τ_m^{k-l}, is distributed between the seller and buyer $(\mathrm{A}_{ml}^{k-l}, \mathrm{A}_{mk}^{k-l})$ generating their profits δ^l, δ^k.

The function of a whole market consists of minimizing the difference

$$\Delta_{n_l}^{n_k} = S_{m_k}^{n_k} - D_{m_l}^{n_l}, \; S_{m_k}^{n_k} = \sum_{k=1, m=m_{ko}}^{n_k, m_k} |\mathrm{A}_m^k|, \; D_{m_l}^{n_l} = \sum_{l=1, m=m_{lo}}^{n_l, m_l} |\mathrm{A}_m^l| \qquad (4.3)$$

between the total supply $S_{m_k}^{n_k}$, and total demand $D_{m_l}^{n_l}$, while maximizing each of them, where n_k is the number of suppliers, n_l is the number of customers-buyers, m_{ko} and m_k, m_{lo} and m_l are the minimal and maximal levels of hierarchy for all suppliers and all customers-buyers accordingly.

The above function implements the initial minimax principle using the considered information mechanisms that execute the equations (4.1-4.2).

Actually the condition of

$$\min_{n_k,n_l}\Delta_{n_l}^{n_k}=\max_{n_k,m_k}S_{m_k}^{n_k}-\max_{n_l,m_l}D_{m_l}^{n_l} \tag{4.4}$$

is accomplished by mutual compensations of the local equilibriums: $\sum_{k,l,m}\pm A_m^{k-l}$ in the form

$$\min_{n_k,n_l}\Delta_{n_l}^{n_k}=\min_{n_k,n_l}\sum_{k,l=1,m_{ko},m_{lo}}^{n_k,n_l,m_k,m_l}\pm A_m^{k-l}\ , \tag{4.4a}$$

where equations (4.1)-(4.2) fulfill the minimax principle at *each local equilibrium*, which satisfies to a best motivation of both suppliers and buyers.

The fulfillment of (4.4a) minimizes a shortage of both total supply and demand and also eliminates of both overflows and the queues.

The implementation of (4.4a) can also be done by Nash's equilibrium strategies [13, 14].

Because each component of (A_m^k, A_m^l) satisfies to the relations $A_m^k \sim \frac{\partial S_m^k}{\partial t}$, $A_m^l \sim \frac{\partial S_m^l}{\partial t}$, their nonzero difference:

$$\left|\frac{\partial S_m^k}{\partial t}-\frac{\partial S_m^l}{\partial t}\right|=\left|\frac{\partial \Delta S_m^{k,l}}{\partial \gamma}\right| \tag{4.5}$$

which initiates $\pm A_m^{k-l}$ (not equals to this difference), is measured by the components *unequal* values, determined by the relative information quantities at each level $(k-m_k)$ and $(l-m_l)$ accordingly. These quantities (at the other equal conditions, including the triplets' number) are defined by the IN's corresponding γ_k, γ_l. Thus, the existing difference (4.5) is characterized by the $grad(\gamma)$, which is measured by an information force

$$X(\gamma)=-\frac{\partial \Delta S_m^{k,l}}{\partial \gamma}=grad(\gamma)>0\ , \tag{4.6}$$

where $grad(\gamma)$ corresponds to a local distinction between γ_k, γ_l (at $\gamma_k \neq \gamma_l$), while each local maximum of $\frac{\partial S_m^k}{\partial t}$ and $\frac{\partial S_m^l}{\partial t}$ can be reached at a minimum of each IN's (γ_k, γ_l), satisfying to the VP. The tendency to

$$\min(\gamma_k)\to 0,\ \min\gamma_l\to 0, \tag{4.7}$$

at the existence of the force (4.6) (that guaranties the implementation of

$$\min(\gamma_k)\neq \min\gamma_l, n_k\neq n_l \tag{4.8}$$

for each IN on the market), leads to maximizing the surpluses $d_s(\gamma)$ of both supply and demand by modernizing an effectiveness productivity (for example, in terms of Pareto-efficiency [13-14]).

The maximization of force (4.6): $\max X(\gamma)$ intensifies this tendency, which serves as a moving force in economics. Conditions (4.6)-(4.8), represent a moving force in the considered model's evolution, accompanied by decreasing γ, increasing system's dimension, complexity, and the capability for self-organization (ch.1.8). The uncertainty functional for a total market $\Delta S(x_t(u)) = \Delta S$, defined on a market processes $x_t(u)$ with the matrix

$$A_n = (+\mathrm{A}^k_m, -\mathrm{A}^l_m), k, l = 1, \ldots, p, m = n/2 - 1, n = \sum_{k,l=1}^{p} (n_k + n_l) \tag{4.9}$$

and a potential control $u(cr^k_{in}, k^i_o)$ should satisfy to the VP.

These results answer the very practical questions posed above.

Different modern electronic communications (E-Commerce, E-trade, E-Bank, others) connect LP^p and LM and implement the inner LM exchange's operations.

2.4.4. Managing the LP. A Bank and a Stock Market

Assume that each of the LP^p (LP^k, LP^l, LP^q) manages its profit $\delta^k_i = \alpha^{k-l}_i - \alpha^*_{io} - \beta^k_i$ (Fig. 4.2) to achieve a maximum growth of the sale α^{k-l}_i and a minimum of the spending $(\alpha^*_{io} + \beta^k_i)$.

Let us analyze the potential distribution of this income to outline an *optimal* profit's management.

A part of income p^k_i should go toward internal consumption, including the part α^*_{io}. Another part e^k_i should be spent on modernization and a possible LP^k extension, or the income's accumulation. The difference $\delta^k_i - (p^k_i + e^k_i) = \pi^k_i$ can be used to get additional profit; an option is to buy and sell stocks.

As it was shown in [16], the optimal distribution between the components of informational income satisfies to the following relations: $p^k_i / \delta^k_i = 0.287$, $e^k_i / \delta^k_i = \pi^k_i / \delta^k_i = 0.3565$.

Therefore, each of the LP^p can go on a stock market SM (Figs. 4.2, 4.3) with the corresponding portions of income π^k_i, π^l_j, having different information values.

An individual, getting an information earning β^k_i, expects to distribute it in the related proportions, with the components $\beta^k_i = p^{k\beta}_i + e^{k\beta}_i + \pi^{k\beta}_i$, where portion $p^{k\beta}_i / \beta^k_i$ is spent on internal consumption, the portion $e^{k\beta}_i / \beta^k_i$ can be used for improvement, development, learning, other professional remedies, and part $\pi^{k\beta}_i / \beta^k_i$ can be used for getting additional profit.

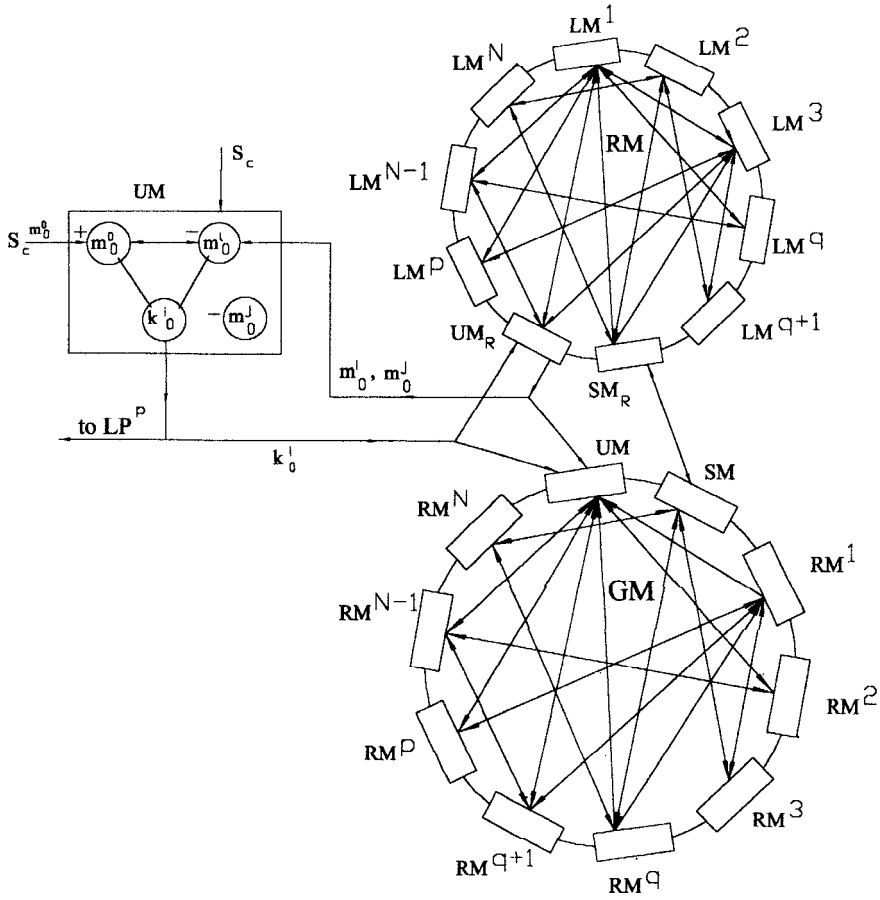


Figure 4.3. Regional market RM, including a stock market SM_R and the unit market UM_R; a schematic of UM; and Global Market GM with the acting regional markets RM^p, $p = 1,\dots,N$ and the interacting (SM, SM_R) and (UM, UM_R).

A potential LP^k investment Π^k_{in}=[π^k_i, $\pi^{k\beta}_i$,..., π^k_j, $\pi^{k\beta}_j$,…] (for example, to buy stock) varies upon each L^p portion of income π^k_i, π^l_j and depends on the bank's B credits-loans $L^k_{in} = [l^k_i, l^{k\beta}_i, \dots, l^k_j, l^{k\beta}_j]$. For the total investment $I^k_{in} = [l^k_i \pi^k_i, l^{k\beta}_i \pi^{k\beta}_i, \dots, l^k_j \pi^k_j, l^{k\beta}_j \pi^{k\beta}_j]$ the credits-loan is $I^k_{in} = L^k_{in} \Pi^k_{in}$, where the returned loans $L^{kb}_{in} = (1 + cr^k_{in})$ assume a fixed credit coefficient cr^k_{in}, which can be controlled by some regulating organization (RG).

At the SM, the supplied stocks $+\Pi^l_{st}$ and the investors' demands $-I^k_{in}$, interact (Fig. 4.2) in a competitive environment, according to the market master's equations (4.1)-(4.2).

The resulting cost R^{k-l}, achieved during the communication time T^{k-l}, is distributed between an investor and stock-seller, generating their profits δ^k_s, δ^l_s, where $\delta^l_s = R^{k-l}_l - L^{kb}_{in}$

goes back to the corresponding component of LP^l , and δ_s^k goes to the LP^k . An individual, earning β_j^p can also go to the local market LM , as a buyer A_j^p of a potential supply A_j^q, after getting a loan L^{op} from the bank (Fig. 4.2). Some individuals can also go directly to the stock market SM , with the loan's components included in L^{ol}, as an element of a complex investor I_{in}^l, Fig. 4.2. An individual-producer can get all necessary services and products without money, paying only with valuable information, which an organization provides to him through a bank. From the above relations it follows that a decrease of each sale α_i^{k-l} declines the profit and decreases each investment π_i^l .

A multiple decrease of sales on the market leads to a multiple decline of investments I_{in}^k, or vice versa that corresponds to the Keynesian acceleration principle [12].

The considered information economic functional relations are the model's *attributes* rather than the economic examples.

Multiple forms of electronic communications accomplish the interactions between LP^k , Π_{st}^l , I_{in}^l , B , SM , with a code transmission of the above information.

The multiple interactions also create an evolutionary process (ch.1.8) capable of adaptation and self-organization of the surviving subsystems.

The notion and results related to both macrosystemic and cooperative complexities (ch.1.7) are applied for the considered economic model. At a small system's dimension n , the number of neighboring subsystems having a similar complexity exists. But with growing n , the number of close-complex neighboring subsystems decreases sharply.

As the MC-complexity increased, the *gap* between the dynamic and geometric properties of the nearest subsystems *increased radically*.

The informational "individuality" of the subsystem is continually supported, as this *subsystem is located further away* (in terms of the classification complexity) from the neighboring subsystems. Such a distance has a limit, defined by a minimal stable parameter γ , and maximal n , which correspond to the dynamic equilibrium between the systems.

At a minimal stable $\gamma \to 0$, when the human component in the synthesis of information increases, such distance approaches a maximum. Its optimal value is established in the process of mutual exchanges in the market, as an optimal feedback, which also brings the optimal ratios γ_i^{α} . These ratios determine a related human contribution in each LP^p synthesis-production by $r_h(\gamma)$, averaging 60%. The above condition satisfies the *stability in an optimal* distributed dynamic system with the market's feed-back to LP .

The fluctuation of an average γ within the considered threshold $|\varepsilon(\Delta\gamma)|$ (ch.1.5,1.8) finally moves the model toward an equilibrium. But each of these γ changes the model's invariants and the above τ_s^n , which bring new information equivalents of the exchanges on the market. Decreasing γ increases the invariant and corresponds to a *deflation*, while increasing γ decreases the invariant, corresponding to inflation.

Both of these processes, accompanied by the return to final stability, have a *cyclic* character.

The $r_h(\gamma)$ value, exchanged on a market, directly connects the economical and social systems and in a limit leads to a social equalizing of all human beings (producers-exchangers) [17]. As a starting component of the information synthesis (α_{io}), a human being can also be involved in this process. Moreover, the synthesis may occur between just two people, while the parameter $r_h(\gamma)$ expresses a mutual relation between two human beings with equal contributions from $\{h_{mo}\}$. This means, a human contribution from $\{h_{mo}\}$ is equal to human contribution from $\{\alpha_{no}\}$. That is why the optimal $r_h(\gamma = 0.5) \cong 0.66$. Even a single human being can synthesize useful (exchangeable) information. In this case, the brain is a producer of information. Neurological research shows that brain cells produce approximately 50% more information than is fed into the processing cell [18]. This is consistent with the above evaluation of $r_h(\gamma)$, characterizing a *specific part of the labor's surplus, being averaged by all* LP^p**.**

Therefore, *an elementary equitable human's contribution should* be compensated an *average minimum* 50% back from a fair economy and a social system. The $r_h(\gamma)$ value also means that an elementary equitable human can control $\cong$ 66% of the average production at each LP hierarchical level. Actually, the concrete contribution of each particular participant (in a random microelement) is not equal to $r_h(\gamma)$, and each human-producer will not get back exactly 50% from every LP**.** Moreover, even for an averaged participant, $r_h(\gamma)$ is changed in the process of approaching a local equilibrium at $\gamma \to 0.5$. The value β_i^k, which depends on a human's location at the k-level of the LP hierarchy, determines the human's a total contribution and the potential income $\Delta S_{ik} = k\,\mathbf{a}_{oo}^{hk}$, $\mathbf{a}_{oo}^{hk} = \mathbf{a}_{ok}^{2}(\gamma)$.

With increasing $k \to m$, as the location is transferred on higher hierarchical level, this *contribution-income* grows, becoming *unequal* for the distinctive participants.

A higher location at the LP hierarchy requires more a brainpower in the LP, embodied into production. For example, at k=2 the average $\Delta S_{ik} \cong$ 100% guarantees a compensation to a worker for spending his labor in the synthesis. A worker, located at level k', produces additional surplus $\delta\Delta S_{ik}' = (k' - k)\,\mathbf{a}_{oo}^{hk}$. If the worker has not gotten a payment for this surplus, the ratio $r_h^{\delta} = (k' - k)/k$ becomes analogous to the Marx's rate of surplus-value s/v, which characterizes a degree of exploitation [17]. But in [17], the s/v surplus was not embedded into the LP exchange-final product, while the r_h^{δ} measure incorporates an entire relative difference of the information contributions to the synthesis, including a worker's portion. The k' growth is limited by an increase of the periodical time of production τ_s^n (Fig.4.1a), which is restricted for each worker by the admissible maximal working hours at each hierarchical level. Another possibility for an organization consists in paying a complete earning to the worker, which could collect it and becomes a source of additional income by the stock market exchange, or creating an LP.

The cooperation leads to both the labor's and social IN's hierarchical organization.

The above analysis shows that a precise human's role in the LP-production determines not only his specific *income* but also his *position* in a hierarchical *social* system.

For a human being, a minimax principle consists of getting *a maximum information* and *its minimum spending.*

2.4.5. Other Information Markets

Some local markets are specific, like energy, transportation, construction, agricultural, real trade, insurance, healthcare, difference services, etc. Each of them can be represented by the considered information model of a local market where the information values of specific supply and demand are provided by corresponding sources of the INs (Fig. 4.1). These local markets (LM 1, LM 2, ..., LM N) mutually interact (Fig. 4.3) by ranging their commodities to provide a shared worth, for example, in terms of aggregate supply and aggregate demand [19, 20, 21]. A set of local markets organizes a Regional Market RM, which includes a regional stock market SM_R that satisfies the equations (4.1)-(4.9), which minimize a difference between the regional supply S_{RG} and the total demands D_{RG}: Δ_{RG}.

A set of interacting Regional Markets (RM^p, $p = 1,\dots,N$) forms a Global Market (GM), where *an information unit's exchange market* (UN) plays a distinctive role. The information scale units m_o^i, used at different RM_i, $i = 1,\dots,n$ (countries), could not be the same and would need a periodical verification and adjustments. This can be done at the UN, where these units are exchangeable on an information unit, accepted as a *standard* information measure m_o^o (analogous to the gold standard, or a dollar equivalent), supplied by some m_o^o-source Sc (Fig. 4.3). The exchanges acquire a form of buying and selling the different m_o^i to determine how many k_o^i units of m_o^o each m_o^i contains: $m_o^i = k_o^i m_o^o$.

The exchange process satisfies the equations (4.1)-(4.2), while the supply consists of providing a flow of m_o^o, and the demands include the diversities of m_o^i. Some of the m_o^j could also serve as a supply for other m_o^k, motivating to buy them. The found resulting k_o^i changes the information scale for each LP^p, affecting every one of LM^p, B, SM_R, RM^p, and GM. The supplied flows of m_o^i (monetarism) and m_o^o can also regulate an economy. Interacting regional markets RM^p may include (along with SM_R) a local unit market UM_R, while GM interacts with local SM_R and UM_R, forming the broad-spectrum SM and UM.

The GM that obeys the regularities of (4.1)-(4.9) minimizes a difference between the total supply S_Σ and the total demands $D_\Sigma : \Delta_\Sigma$. A regulation takes place (by changing cr_{in}^k) if the equilibrium condition $\min \Delta_\Sigma \to 0$ is not fulfilled automatically. This regulation can first be applied to a regional market to minimize its potential unbalance on the GM.

Information that has been sold and exchanged on the above information markets can lose it value if it does not hold the equivalent of standard information measure. In this case, the uncontrollable market's information exchanges lead to a collapse of the entire baking, market and a production systems, because, without an active standard equivalent, both quantity and quality of information are vanished in the circulating information flows (each such a market sells "nothing").

An integration of the interacting commodities for the graph structures RM and GM (Fig. 4.3) can be evaluated by a multiplicative integral along the graph [22]. The random contributions to the graph's multiplicative integral are modeled by the conditional probabilities of a discrete Markovian process, distributed along the graph. Considering the Markov model with a recurring state, returning to the graph's starting node after the

completion of the process, we can determine the product (path) integral along the close graph loop by the entropy of the Markov model, which is defined by a logarithmic measure of the multiplicative path integral (ch.1.1). Finally, we get an analog of the integral information measure (ch.1.3), which is able to evaluate any of the RM, GM inbalances, generated by the total contributions in the graph's loop. The stability of the market structure can be estimated by the Lyapunov function, determined by the information path functional (ch.1.3). Modern communication networks, including WAN, INTERNET, E-COMMERCE, and others, provide the RP^p's and GM's connections, in which a coding language of different transactions plays an important role. Multiple information exchanges join different businesses into various forms of information organizations, whose cooperation keeps until the connecting controls are able to overcome environmental randomness (Ch.1.7).

2.4.6. Example

The examples of a practical implementation of the macrodynamic model, considered in [23, 24], involves the prognosis of an optimal tax policy (Hu) for the cooperating LP^p producers (n=50), which continually consolidate their volumes V of production, starting with a basic volume Vb.

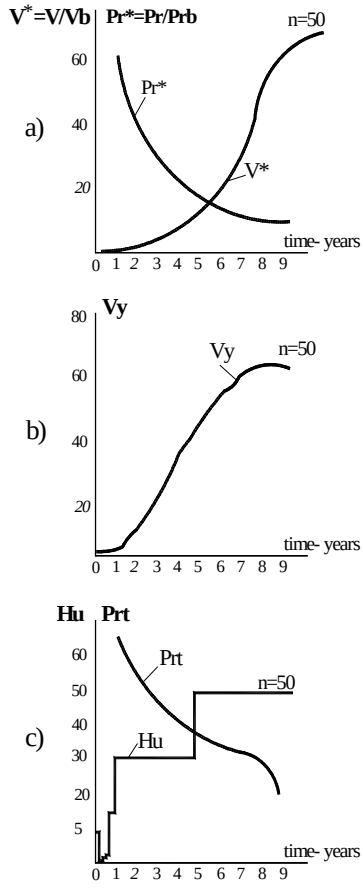


Figure 4.4.

The dynamic prognosis of the optimal development and cooperation of the LP^p producers.

The computed dynamics (Fig. 4.4) characterize an increase of the relative volume of production V*=V/Vb, and a decrease of the relative price Pr*=Pr/ Prb, where Pr is a current price, Prb is a basic price.

The Pr*, as a function of demands, is proportional to time (in years), with increasing the price from the volume of productions (in the price's form: Vy=V*• Pr*).

The optimal controlling tax policy is decreasing (at the beginning), and then is increasing with growing the discrete intervals and the volume production.

The joint relative price Prt (for the cooperating producers) has a tendency of decreasing at a continuation of the increasing the tax policy Hu.

The modeling results show that involving new producers into the cooperative process will positively change this tendency.

The methodology of the spatial-time synthesis was implemented on computers by the IMD software package.

2.4.7. Summary

Compared to the existing modeling methods, which have never used the information path functional, the IMD formalism, and the IN, this approach has the following advantages for the economic applications:

- Unified information description of a variety of economic processes (including human interactions) by the two-levels *systemic* information model with stochastic processes at microlevel and dynamic processes at macrolevel, identifiable on a real object;
- The IMD microlevel models *stochastic dynamics* using Markovian random processes, stochastic differential equations and Chapman-Kolmogorov equation, which essentially covers approach [6] dealing with related stochastic phenomena;
- The minimax variation principle, applied to the introduced integral entropy measure, allows us to model, in addition to the stochastics, the *dynamic macroequations with optimal control functions*, which, via the found punched localities, unveil the dynamic regularities and cooperative phenomena of the random microlevel; while an interaction of the stochastics and macrodynamics reveals the chaotic phenomena, singularities, and specific resonances;
- The model allows extracting a maximum of a minimum of available information from an economic object, while the optimal control functions determine the necessary actions, minimizing uncertainty of the systems;
- The macrodynamic equations automatically allocate the discrete process' intervals with the local reversible processes within each interval, the irreversible processes out of them, and the jump-wise stochastic and dynamic phenomena, memorized at the interval's border; this allows revealing and minimizing the most *essential systems uncertainties*;
- The model cooperative information is sequentially arranged, ordered, and enclosed into the *hierarchical structured space distributed dynamic network* (IN), where the information MC-function measures the *complexity* at each hierarchical level, and the IN node's information is encoded into the double spiral model's *genetic code* (DSS).

- The DSS accumulates and compresses *both* the micro- and macromodel's information allowing its algorithmization, electronic transmission, and the systemic models' communication using this code;
- The model enables automatic filtration, adaptation with an improvement under a mutation and self-organization;
- The information model of the local production systems LP^p *hierarchically organizes* into the IN, cooperating various labor and production processes (delivered by materials, machines, goods, human being, etc.), with the quantitative and qualitative measurement of produced structural information in a common scale, convenient for communication, trades, and exchanges on market;
- The information model of a market establishes the *information dynamic producer-supply relations*, described by the time-spatial balance equation, which determines the condition of *equivalent exchanges* and a possibility to reach a common price in a *cooperative resonance* process. The developed market's master information equations determine both the *equilibrium and the moving economic forces of self-organization and evolution*;
- The information models of banking and stock market's processes are *connected to the optimal LP's decisions in a profit's management*. The found *optimal human's contributio*n in the synthesis-production satisfies the LP stability with a market's feedback and a human's optimal information capability;
- The information exchanges between varieties of markets, described by the master equations, are *unified* by a market, exchanging currently admitted units of information; the market periodically adjusts and validates a common information measure. An *integration* of the interacting markets, represented by the map-graph structures, allows us to evaluate the map's nodal connections and the graph loop by the integral information measure and to find the equilibrium conditions;
- The model's algorithms and codes enable the *direct* communication and information transmission between all elements of market economy (in the terms of *cooperative macrodynamics*).

The considered model's features not only generalize economic information description but also detect unknown phenomena and new possibilities of managing and improving the economic objects, which none of existing models can reveal.

In a "New Economy" [25], "the cost of information determines the size and shape of firm", but this is not just the cost of computers, but rather a firm's *structural and operational organization*, supported by modern computer applications and electronic communications.

The economy is an *uncertain* system, whose complex processes' improvement consists of *minimizing* this uncertainty by finding a dynamic *tendency* which this uncertain covers.

The obtained chain of the equations minimizes the process' uncertainty, providing the methodology and mathematical tools for a dynamic information modeling of a given economic system. The IMD software implements the developed methodology.

The IPF variation principle, as a law of minimum uncertainty, is implemented for economic system via the market balance relations. When this law is violated, the system experiences some essential changes, perturbations, associated with transferring to stochastics and chaotics; for complex multi-dimensional interactions, such changes could be dramatic, bringing catastrophic cataclysms (See also ch.1.7).

Chapter 2.5

AN OUTLINE OF THE COMPUTER BASED METHODOLOGY

2.5.1. The Hierarchy of the Model's Micro-and Macrovariables and Their Identification

The model has the following hierarchy of micro-macrovariables:

- Random variables at the statistical microlevel, defined by a statistical ensemble $\{\tilde{x}_i\}$;
- Dynamic variables x_i, defined on the extremal trajectory and by the model functions of the microlevel's statistical ensemble;
- Information flows $I=\dot{x}_i$ and information forces X_i, defined by the macromodel's variables and their derivations;
- Macrovariables, defined by the model eigenvalues and eigenfunctions, which determine the model's self–frequencies as the spectral model characteristics, and the information wave functions;
- Integrated macrovariables, defined via assembling of the macrovariables into the model's cooperative units (doublet, triplet) composing the IN nodes, which are able to successively consolidate in the cooperative dynamics;
- The information code of the integrated macrovariables.

These variables are

- identifiable automatically during the macrodynamic movement at the identified discrete points (DP) and through the model's consolidation process;
- measured in the unit of quantity of information, and for specific objects-in the unit of quality of information, determined by the variable's hierarchy on their space-time locations in the cooperative dynamics.

This comprehensive system's hierarchy of the information variables is detailed for a particular object by the object's specifics, its dimension, and the peculiarities of the cooperative dynamics.

Space boundaries between the variables are defined by the geometrical surfaces, separating the every formed macrostructure. Some of the boundaries limit a particular macrolevel hierarchy and the number of the corresponding macrovariables.

The *mechanism of the IN's building* includes an *automatic ordering* of the model segments' evaluated by the information quantity of the segment's eigenvalues (and the eigenvectors) in the processing of the segment's sequence.

The proceeding of these information quantities involves the *automatic generation* of the triplet's structures with the corresponding space movement of the *local space coordinate systems, and forming* ***t****he IN's collective large-scale coordinate system*, which are determined by the eigenvectors' inputs.

This automatic procedure not only transforms the spatial-temporal input to its spatial form and finally to spatial-temporal output, but also establishes an ordered geometrical mapping relationship between them, allowing the exact localization of the transformed inputs.

These transformations are carried by the model's mechanism of space- time consolidation of the multiple spirals trajectories on the cones into the IN.

The elements of the identification procedure include

- Measuring the object's covariation matrix (r_{ik}) and calculating the rank of this matrix (as a maximal number of the nonzero matrix's determinants)**,** which determines dimension n of specific object;
- Finding the macroequation's spectrum of the initial eigenvalues**:** $(\lambda_{io})_{i=1}^{n} = sign(1/2\dot{r}_{ik}r_{ik}^{-1})_{i,k=1}^{n}$, where n is the rank of covariation matrix (r_{ik});
- Applying the diagonal form of macroequation $\dot{x}_i = \lambda_{io}\, x_i$ for selecting the independent macrocoordinates, ordering their speeds $\dot{x}_i = I_i$ as the information flows, and setting up the corresponding information forces:

  $X_i = 1/2 \sum_{k=1}^{n} r_{ik}^{-1} x_k$;
- Establishing the specific physical meanings of the information flows and forces using their connections to related physical variables (analogous to secs.1.7.5, 1.9.4).

2.5.2.The Computer's Restoration of the IMD Model

The diagrams, implementing the procedures of the model restoration and simulation of its performance, are shown on Figs.5.1a, 5.1b, and 5.1c.

On Fig.5.1a, the statistical data from the microlevel process $\tilde{x}_t = \tilde{x}(\tilde{x}(0), t)$ are used to identify matrix *A* of the macrolevel equation by computation of the correlation function and its derivative during each discrete interval t_i, which compose the computed invariant $\mathbf{a}(\gamma)$**.**

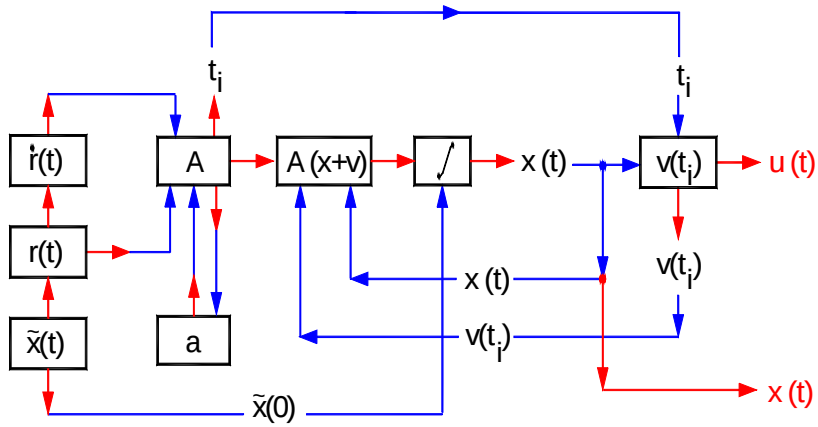


Figure 5.1a. Diagram of computation of the optimal model's process $x(t) = x_i(t, l, u(t_i))$, using the microlevel's random process $\tilde{x}(t)$ by calculating the correlation function $r(t)$ its derivative $\dot{r}(t)$; the object macrooperator A, invariant a, discrete interval t_i; these allow simulating the optimal macroprocess $x(t)$, the inner $v_i(t_i)$ and output $u_i(t_i)$ optimal controls.

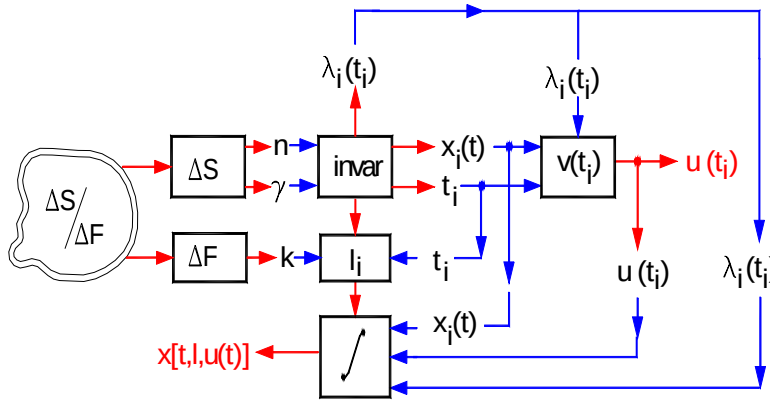


Figure 5.1b. See the text.

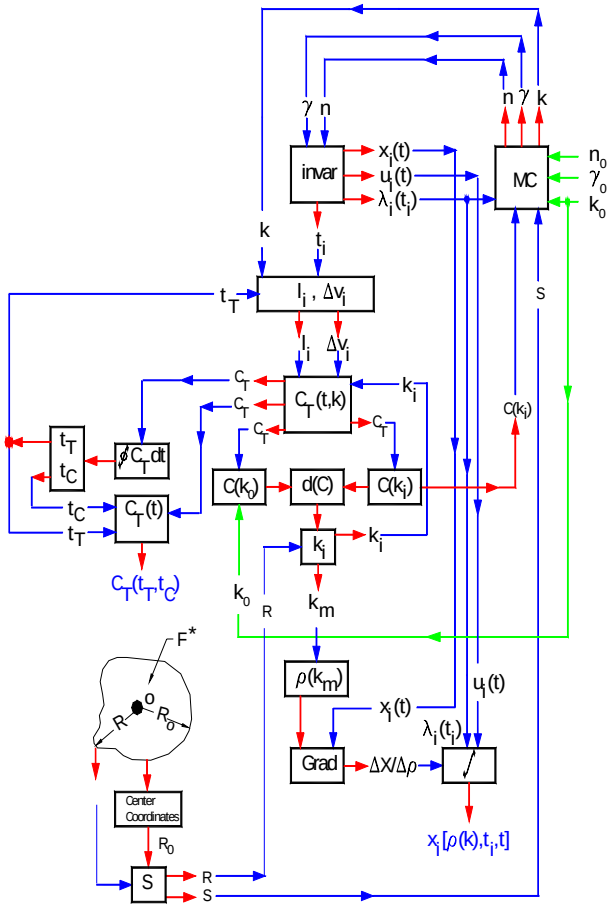
The computed variables allow to simulate the macroprocess $x_t = x(x(0), t)$, synthesize the inner control $v(t_i)$ and the output optimal control $u(t_i)$, using the calculation of the optimal model's process $x_o(t) = x(v(t_i), t)$, current feed-back input, and t_i.

Fig.5.1b illustrates the scheme of *computation* of the optimal model's process $x(t) = x_i(t, l, u(t_i))$, using a given space distributed information ΔS per cross-section ΔF, the model's invariants INVAR, the time t_i and space l_i discrete intervals, eigenvalues $\lambda_i(t_i)$ of the model differential operator, and *simulates* the inner $v_i(t_i)$ and the output $u_i(t_i)$ optimal controls.

The methodology is based on the connection of the model macrodynamics with the corresponding information geometry (chs.1.4-1.6).

In this case, the microlevel stochastics are not used for the macromodel's restoration. Instead, the restoration requires the computation of the model's basic parameters: dimension n, uncertainty γ, and the curvature's indicator k; which allow finding the model optimal macroprocess, the synthesized optimal control, as well as the model's hierarchy.

The computation uses the primary parameters of a basic model (n_o, γ_o, k_o) and the known parameters of the object's geometry.



invar	a=d|o|t|i	a_0=d|t|i	$x_i/x_0, x_i<x_0$	$x_i/x_0, x_i>x_0$	t_2/t_1	t_3/t_1	$X_{L1}=\alpha_{(2k+2),0} \bullet x_{(2k+2),0}/x_{(2k+1),0}$	$X_{L2}=\alpha_{(2k+3),0} \bullet x_{(2k+3),0}/x_{(2k+2),0}$

Figure 5.1c. See the text.

Diagram Fig.5.1c. presents the functional schema of the IMD software operations: computing invariants INVAR, discrete moments t_i, space coordinates l_i, increment of volume Δv_i, MC-function, speeds C_T, C and their difference $d(C)$, the current space

parameters k_i, polar coordinates ρ, and gradients GRAD $\Delta X / \Delta \rho$ for a given space distribution's cross-section F*; with calculating its radius R, coordinates of center O- R_o, and a square S, which are used to compute the object space model's minimal (optimal) parameter k_m.

The output variables are: optimal dynamic process $x_i(t)$, optimal controls $u_i(t)$, eigenvalues $\lambda_i(t_i)$ of the model differential equation, distributed space-time process $x_i(\rho(k), t_i, t)$, space's current speed $C_T(t_T, t_c)$ with the intervals of moving t_T and stopping t_c, which are computed by averaging a speed $\oint C_T dt$.

An estimated time of computation for each of the diagrams is approximately 3-5 minutes on conventional PC.

The computation can be performed during a real–time movement of the object's cross section (Fig.5.1b), or through an input of the calculated object's current statistics (Fig.5.1a).

Solving the considered complex problem in a real-time by *traditional computation* methods requires the developing of mathematical methodology and the software, which are able to overcome the method's high computational complexity.

For solving even a part of the problem, the existing techniques require many hours' computation on the modern main frames.

2.5.3. The Structure of the IMD Software Package

The software package transfers the IMD analytical methodology into the numerical procedures, computer algorithms and programs.

The packet (consisting of 35 programs) includes the following modules for computation of:

- the identification procedure for the restoration of the object's equations;
- the parameters of space–time transformations and a time-space movement;
- the OPMC parameters, processes, controls, and the IN structure;
- the function of macrosystemic complexity;
- the transformation of the informational macromodel's characteristics into the appropriate physical and technological variables (using the particular applied programs).

The *main software modules* compute:

- the basic optimal macromodel parameters (n, γ, k);
- the spectrum of the model's eigenvalues $\{\lambda_{io}\}, \lambda_{io} = \alpha_{io} \pm j\beta_{io}, i = 1, ..., n$;
- the macromodel informational invariants $\mathbf{a}_o(\gamma) = \alpha_{io} t_i$, $\mathrm{b}_o(\gamma) = \beta_{io} t_i$;
- the time-space intervals (t_i, l_i);

- the distribution of the optimal eigenvalues $\lambda_i(t_i, l_i)$ and the optimal controls $v_i(t_i, l_i)$;
- the geometrical macromodel's coordinates and the space distributed macroprocesses $x_i(t_i, l_i)$;
- the procedure of the macrocoordinates' cooperation and aggregation;
- the IN hierarchical macromodel structure and its macrocomplexity.

The formulas, algorithms, and numerical computation's equations are given in the program descriptions (not included in this book).

The IMD software programs have been used for the practical solutions of the different applied problems including chs.2.1-2.4 and other [25].

CONCLUSION

The book introduces the following main math results:

- the entropy integral functional (EF) and its connection to a stochastic differential equation, describing the model random process at the microlevel;
- the dynamic approximation of the EF by the information path functional (IPF) defined at the model macrolevel;
- the solution of variation problem for IPF with establishing the IPF constraint equation, connecting the microlevel randomness with the macrolevel dynamics;
- the basic equations of informational macrodynamics (IMD), determined on the IPF extremal's segments and the small windows between the segments, belonging to the microlevel;
- the solution of IMD control problem with finding the specific needle controls, connecting the extremal segments through the window, where the information from the microlevel is extracted and the macrodynamics emerge;
- finding a quantum nature of the extracted macrolevel's information and its processing during each following extremal segment, as a basic link from the microlevel to the macrolevel information;
- the solution of a cooperative dynamic problem by incorporating a set of the extremal segments into an assembled information structure;
- finding the IPF information invariants and using them for a simplification of the obtained solutions;
- building a cooperative ordered information network (IN) with a hierarchy of the elementary cooperative triplet's nodes;
- finding an IN genetic information code that is able to encode and decode the IN assembled information;
- finding the IN information geometry;
- finding the complexities of informational dynamics, their invariant information forms, limitations, and the connection to Kolmogorov's complexity;
- finding the math forms of evolution laws for the evolutionary macrodynamics;

- finding a dynamic limitation (constraint), imposed on a diffusion process, which forms a bridge to Schrödinger's information equation (aimed at the cooperating information wave in a formal "observer").

Book considers *information as an abstract entity* (state, process), defined in *math terms of the IPF certainty* as an opposite of the related EF uncertainty.

A random process eigenfunctional EF is transformed in the corresponding dynamic process eigenfunctional IPF, having the invariant information measure in terms of the Hamiltonian, eigenfunctions, and the IN triplet code.

The transformation of randomness to the dynamics models a formal acceptance of the *information regularities* by a complex dynamic system.

The IPF complex information in a form of information waves satisfies the information *Schrödinger's equation*, which allows revealing this information to an observer.

Such an observer self-transforms an initial information measure (EF) into its dynamic form (IPF) and then to the information algorithm and code.

Revealing the dynamic regularities of a random process by applying a variation principle (VP) to the process' information functional (as a universal attribute for any natural process) *automatically brings the dynamic constraint, imposed discretely* on the random process, which allows selecting (with a maximal probability) the process *quantum states* that represent *both the process' dynamics and randomness.*

The VP creates the dynamic equivalents for both EF and related IPF–at one side, and for random process and the IPF extremals' segments (between the selected quanta) as the related intervals of a dynamic process– at another side.

The IMD *regularities* are emerged in the cooperative dynamics, governed by the IPF variation *law*.

At the variation law's violation, the macrodynamics become random, losing its regularities with the potential chaotic and catastrophic cataclysms.

In the IPF-IMD *approach*, an object is represented by *multiple interactive processes*, which are sources of random processes and uncertainty, and the approach aim is to reveal this object's information in a form of its *genetic code.*

This approach differs from both the Shannon information of an object *random events' observation* and the Kolmogorov encoding of *an individual* object's description by a shortest algorithm (this algorithmic complexity does not require the probability function in the object description).

The IPF-IMD is able to implement the notion and measure of information independent on the probability measure by using directly the informational invariants and/or the IN information code.

Actually, the IPF uses Shannon's *definition* of quantity of information, which we apply to math expectation of the *functional* logarithmic probability on the *process (dynamic) trajectories*, whereas the Shannon's quantity of information in the traditional information theory is applicable to the process' *static* information.

While Shannon's information measure provides an optimal code for the process *states* (as the instants of the process cross-sections), the IPF, in addition to the *process* optimal encoding, allows building the *process information structure* by the *dynamic* IN and the *hierarchy* of its optimal codes.

Studying *dynamics of evolution* of information process, the IPF regularities is useful for the comparison, evaluation and design of a variety of *information systems* with complex information dynamics.

Other than Markovian measures of the uncertainty (for example, fuzzy set's integral uncertainty measure and many others) essentially extends the formulation and application of the information functional and IMD methodology.

Uncertainty is a component of physical reality, but is also the basic element of computer coding language which creates a virtual reality, going beyond physics: as the real and imaginary informational images, algorithms, and software.

The introduced *unified* math-information formalism for *information modeling*, which includes a common computer-based modeling *methodology*, *algorithms, and the computer software*, builds a *bridge* connecting the formalism to the world of information, intelligence, and information technologies.

Book *integrates* the IPF theory, the IMD foundation, and a broad scope of their applications.

The provided applications demonstrate just *starting* but also the productive steps toward utilizing these results in wide area of information science and technology.

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Prof. Lerner's biography is listed in "*Who is who in America", "Who is who in the World*".

INDEX

A

ABC, 225
academic, xiv, xv
access, 94, 96, 210, 320, 400, 407
accuracy, 23, 56, 81, 177, 178, 179, 189, 197, 199, 293, 361, 395, 423
achievement, 356
adaptation, xix, 225, 285, 289, 290, 294, 295, 301, 308, 310, 377, 378, 427, 429, 439, 444
adaptive control, 289, 337, 370, 408
adjustment, 431
age, 460
aggregate demand, 441
aggregate supply, 441
aggregation, ix, 141, 336, 428, 450
aging, 399
agricultural, 441
aid, 5, 57, 87, 143, 350
algorithm, 202, 205, 210, 269, 270, 309, 320, 332, 372, 400, 422, 452
alloys, 421
alternative, 224, 313, 373
alternative behaviors, 313
amplitude, 295, 318, 319
Amsterdam, 459, 461, 465, 466
analog, xviii, xix, 55, 62, 83, 125, 128, 239, 288, 315, 382, 442
ants, 90
application, xiii, xiv, 26, 48, 92, 97, 103, 104, 107, 167, 175, 211, 226, 244, 246, 252, 383, 410, 452
argument, 18, 19
arithmetic, 20, 21
Arizona, 464
assimilation, 242, 275
assumptions, 75, 80, 139, 272, 402
asymmetry, 141, 192, 193, 241, 280, 289, 299, 306, 309, 328, 372, 402, 405, 406
asymptotic, 312
attractors, 283, 302, 306, 401, 404, 406, 408
automatization, 465, 469
autonomy, 405
averaging, 338, 393, 439, 449
axon, 403, 404
axonal, 404

B

BAC, 225
baking, 441
Banach spaces, 460
banking, 429, 444
banks, 427
barriers, 328, 402
BCA, 225
behavior, 3, 241, 245, 295, 404, 406, 408, 429
bifurcation, 129, 148, 408
binding, xii, 172, 178, 236, 240, 259, 266, 375, 401
biological processes, 373, 399, 429
biophysics, 462
blocks, 212
borrowing, 434
boson, 266
bosons, 266
Boston, 456, 460, 463, 466
bottleneck, 370
boundary conditions, 51, 343
boundary value problem, 63, 458
brain, 394, 399, 401, 402, 406, 440, 464
branching, 239, 404
Brownian motion, 121
Brownian particle, 10
buyer, 433, 434, 435, 439

C

calculus, 4
capacity, 230, 233, 278, 281, 282, 283, 284, 296, 306, 377, 401, 406, 464, 468
carbon, 412, 413
carrier, 201, 221, 223, 269, 305, 332, 402
cast, 422, 423, 465
casting, 409, 412, 420, 422, 423, 424, 425, 465, 469
Cauchy problem, 356

cell, 205, 206, 208, 210, 211, 213, 215, 216, 267, 326, 327, 399, 440, 460
cell division, 399
channels, 239, 459
chaos, 253, 279
Chapman-Kolmogorov, 428, 443
chemical kinetics, 245
chemical reactions, 244, 245, 421
classical, 211, 366, 401, 405
classical mechanics, 401
classical physics, 401
classification, 237, 250, 322, 332, 439
codes, 169, 210, 216, 224, 225, 263, 269, 296, 332, 377, 378, 392, 399, 401, 432, 433, 444, 452
coding, 210, 224, 225, 296, 300, 305, 371, 373, 402, 406, 407, 442, 452, 459, 462
codon, 258
cognition, 226, 306, 397, 398, 402, 406
cognitive, xiv, 226, 373, 374, 396, 398, 405, 408
cognitive function, 373
cognitive process, 398
cognitive system, xiv
coherence, 281, 406
collective production, 227
Columbia, 464
Columbia University, 464
commodity, 432
communication, xix, 171, 202, 210, 216, 230, 233, 235, 239, 259, 266, 269, 300, 304, 305, 309, 372, 408, 432, 433, 435, 439, 442, 443, 444
compatibility, 406
compensation, 328, 422, 440
competition, 283, 300, 306, 307, 378, 428, 432, 433
complex interactions, 335
complex systems, xiv, xviii, 103, 230, 468, 469
complexity, x, xi, xiii, xviii, xix, xx, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 242, 243, 245, 246, 249, 250, 251, 252, 254, 255, 256, 257, 258, 259, 261, 262, 263, 268, 269, 270, 299, 300, 301, 310, 314, 332, 376, 377, 388, 396, 398, 401, 402, 405, 437, 439, 443, 449, 451, 452, 458, 459, 460, 461, 462
components, ix, x, xiii, 4, 60, 61, 70, 97, 101, 130, 145, 149, 158, 169, 173, 200, 212, 244, 245, 261, 265, 267, 271, 292, 298, 302, 310, 318, 324, 325, 326, 341, 354, 357, 388, 398, 411, 412, 416, 421, 424, 428, 429, 430, 432, 433, 435, 436, 437, 439
composition, 257, 421
computation, 63, 97, 100, 121, 199, 200, 229, 258, 259, 297, 320, 322, 348, 361, 409, 420, 422, 446, 447, 448, 449, 450
computer science, xv, xix
Computer simulation, 250, 461, 469
computing, xi, xii, 5, 355, 367, 368, 448
concentrates, 421
concentration, 230, 231, 232, 234, 264, 265, 374, 375, 378, 398, 403, 410, 412, 413
concrete, xiv, 121, 126, 242, 418, 433, 440
conductance, 404
conductivity, 105, 265, 325, 326, 367, 403, 404, 409, 410, 421, 424, 425
conjugation, 240, 323
connectivity, 404, 405
consciousness, 401, 464
conservation, 108, 112, 227, 322, 460, 465
consolidation, ix, x, xiv, xviii, xix, 129, 141, 142, 147, 148, 156, 164, 176, 177, 181, 185, 188, 189, 191, 192, 247, 259, 291, 293, 300, 303, 326, 327, 328, 336, 338, 354, 356, 358, 367, 370, 371, 445, 446, 469
constraints, xii, 5, 47, 51, 57, 67, 104, 114
construction, 441
consumers, 235, 237, 432
consumption, 257, 394, 437
continuity, 73, 381
control, x, xi, xii, xiii, xiv, xv, xvii, xviii, xix, 3, 4, 5, 6, 9, 11, 13, 15, 16, 17, 42, 47, 48, 52, 55, 57, 59, 61, 62, 67, 68, 69, 71, 75, 77, 78, 79, 81, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 97, 98, 100, 104, 105, 106, 117, 118, 119, 121, 129, 130, 131, 132, 137, 138, 141, 142, 143, 145, 147, 148, 150, 151, 152, 161, 162, 165, 166, 168, 169, 170, 171, 173, 179, 195, 196, 197, 198, 199, 200, 201, 202, 205, 208, 209, 210, 212, 213, 214, 216, 217, 220, 221, 222, 223, 224, 240, 241, 243, 244, 245, 246, 247, 248, 251, 253, 258, 262, 266, 279, 281, 283, 284, 285, 286, 287, 288, 289, 291, 292, 295, 296, 298, 303, 305, 306, 307, 308, 309, 310, 312, 313, 314, 320, 321, 322, 323, 324, 325, 327, 328, 335, 336, 337, 338, 341, 342, 344, 345, 350, 351, 352, 354, 355, 356, 357, 359, 360, 361, 365, 366, 367, 368, 369, 370, 371, 372, 398, 399, 403, 406, 407, 408, 409, 416, 420, 421, 422, 423, 424, 425, 428, 430, 431, 433, 437, 440, 443, 447, 448, 451, 456, 462, 463, 465, 466, 467, 468, 469
convergence, 312, 456, 458, 460
conversion, 402
convex, 18, 19, 20
cooling, 422, 424
coordination, 53, 56, 190, 274, 275, 284, 289, 309, 372, 404
correlation, xii, 5, 16, 63, 87, 89, 98, 117, 119, 120, 121, 137, 148, 320, 329, 340, 342, 368, 393, 404, 409, 446, 447
correlation function, 63, 87, 89, 137, 320, 368, 393, 404, 409, 446, 447
correlations, 17, 90, 93, 297, 313, 328, 337, 341, 344, 350, 357, 367, 404
counterbalance, 278, 281, 296
couples, 175, 289
coupling, 289, 307, 312, 404, 405, 407
covering, 271
creativity, 305, 398, 408
credit, 434, 438
cross-sectional, 423
CRR, 9
crystallization, 409, 410, 411, 413
crystals, 327, 411

customers, 427, 435
Cybernetics, 455, 458, 463, 465, 466, 467, 468
cycles, 295, 300, 381, 382, 389

D

Darwinian evolution, xiii, 310
Darwinian evolutionary theory, xiii, 310
decay, 178, 253, 293, 294, 297
decision making, 398
decisions, 444
decoding, 296, 373, 374, 378, 382, 383, 385, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 400
decomposition, 110, 153, 341
decompression, 392, 393, 400
decoupling, 93, 98, 253, 279, 341, 342, 344, 350, 357
defects, 375, 380, 417
definition, xvii, 14, 15, 18, 37, 54, 66, 82, 99, 119, 132, 186, 190, 211, 230, 301, 452, 458
deflation, 439
demand, 432, 433, 435, 436, 437, 441
dendrites, 464
density, 14, 29, 81, 82, 135, 208, 209, 263, 264, 265, 285, 295, 343, 346, 347, 410, 411, 412, 413, 414, 417, 422
derivatives, 9, 10, 13, 103, 108, 123, 137, 189, 410
destruction, 394, 398, 399
determinism, 141, 148, 269, 405
deviation, 25, 26, 106, 297
differential equations, xvii, 49, 103, 126, 131, 183, 443, 457, 463
diffusion, xvii, xviii, 4, 5, 8, 12, 14, 15, 16, 17, 33, 55, 65, 81, 82, 83, 84, 89, 90, 93, 94, 101, 105, 107, 119, 120, 121, 129, 130, 148, 230, 240, 243, 246, 273, 275, 276, 277, 280, 301, 306, 309, 313, 315, 317, 320, 323, 325, 326, 327, 328, 337, 343, 345, 354, 367, 371, 404, 409, 410, 413, 416, 421, 422, 423, 452, 458, 462
diffusion process, xvii, xviii, 4, 5, 14, 15, 16, 17, 65, 82, 83, 84, 89, 94, 243, 315, 317, 320, 404, 413, 452, 458, 462
direct measure, 409, 421
disclosure, ix, xi
discontinuity, 13, 48, 52, 71, 463
discretization, 162, 176, 194, 282, 351, 353, 354, 356, 403, 405, 456
discrimination, 239
dislocations, 411
disorder, 269
dispersion, 57, 87, 89, 91, 121, 180, 240, 350, 354
displacement, 261
dissipative structure, 241, 245, 323, 460
dissipative structures, 241, 245, 323
dissipative system, 119, 456
distribution, 50, 63, 84, 99, 128, 138, 152, 155, 156, 209, 210, 215, 239, 316, 327, 344, 345, 346, 348, 411, 419, 420, 433, 437, 448, 449
divergence, 110, 118, 263, 264, 312, 345, 366, 459
diversity, x, xiii, xiv, 103, 209, 213, 227, 271, 272, 277, 280, 284, 295, 296, 297, 300, 301, 310, 335, 395, 402, 405, 406, 427, 428, 429
division, 399
DNA, 201, 224, 258, 259, 266, 309, 373, 396, 399, 459
duality, 35
duplication, xix, 210, 289, 303, 307, 309, 372
duration, 166, 226, 355, 374, 422
dynamic systems, xix, 4, 322
dynamical system, 313, 459
dynamical systems, 313, 459

E

economic policy, 428
economic problem, 428
economics, xiv, xv, xviii, 427, 428, 429, 437
eigenvector, ix, 142, 143, 144, 145, 146, 147, 148, 158, 186, 187, 194, 216, 260, 261, 262
elastic deformation, 422
elasticity, 411
electrical conductivity, 326, 421, 425
electrical power, 465
electrolysis, 425
electromagnetic, 421, 425
electron, xvii, 3, 218
electron charge, 218
electronic communications, 432, 437, 439, 444
emission, 403
employment, 428
encapsulated, 210, 376
encapsulation, 376
encoding, ix, x, 100, 101, 201, 224, 226, 259, 270, 302, 314, 322, 373, 374, 377, 378, 382, 384, 385, 386, 388, 389, 391, 392, 393, 395, 396, 397, 398, 399, 400, 404, 406, 452, 459
encryption, 399
energy, xix, 219, 220, 227, 241, 264, 313, 319, 321, 322, 330, 331, 421, 422, 441
enlargement, 232
entanglement, 332
entropy, ix, xi, xii, xiii, xvii, xix, 4, 5, 14, 15, 16, 17, 18, 20, 34, 37, 42, 62, 63, 66, 67, 80, 81, 82, 83, 84, 86, 90, 91, 92, 98, 99, 100, 104, 106, 118, 119, 137, 148, 175, 176, 196, 197, 200, 202, 211, 230, 231, 232, 240, 244, 245, 246, 251, 257, 263, 269, 271, 276, 282, 284, 285, 291, 295, 296, 298, 299, 300, 301, 311, 312, 313, 314, 316, 317, 320, 321, 322, 323, 325, 326, 331, 332, 337, 350, 354, 357, 366, 374, 376, 377, 380, 381, 382, 393, 394, 395, 396, 397, 399, 400, 401, 402, 411, 417, 428, 432, 442, 443, 451, 458, 459, 460, 465

environment, xix, 4, 214, 215, 216, 227, 236, 242, 271, 273, 281, 282, 289, 291, 295, 300, 306, 322, 372, 378, 438
equality, 14, 17, 20, 28, 29, 38, 54, 57, 65, 73, 74, 76, 77, 78, 93, 95, 108, 110, 111, 114, 115, 116, 135, 139, 154, 178, 183, 186, 187, 191, 197, 218, 233, 235, 255, 286, 287, 299, 339, 343, 344, 346, 350, 353, 356, 360, 363, 366, 380, 383, 387, 390, 391, 393, 394
equilibrium, 148, 165, 176, 220, 289, 297, 298, 300, 308, 313, 323, 330, 331, 372, 415, 431, 433, 436, 439, 440, 441, 444, 458, 460
equilibrium price, 433
erosion, 398, 399
estimator, 39
estimators, 33
evolution, x, xiii, xviii, xix, 55, 121, 226, 271, 272, 274, 276, 277, 279, 280, 281, 282, 283, 284, 285, 289, 290, 291, 295, 296, 299, 300, 301, 305, 306, 307, 308, 309, 312, 313, 314, 371, 372, 378, 399, 402, 406, 408, 416, 427, 429, 437, 444, 451, 452, 459, 460, 461
evolutionary process, 272, 275, 300, 301, 309, 314, 439
excitability, 464
excitation, 306
execution, 193
exploitation, 440
external environment, xix, 284, 405
external influences, 283
extraction, 289, 341, 344, 372, 406

F

fabric, 210, 262
family, xii, xv, 8, 14, 31, 104, 105, 107, 141, 212, 361, 366
feedback, xix, 9, 78, 83, 89, 93, 203, 223, 245, 259, 289, 298, 308, 309, 321, 367, 371, 372, 398, 439, 444, 447
feeding, 420, 422, 424
Fermi, 456, 459
fermions, 266
Feynman, x, xi, xiii, xvii, 3, 15, 83, 251, 314, 315, 337, 455, 460, 462
filters, 393, 463
filtration, 100, 281, 337, 339, 340, 354, 357, 371, 414, 421, 444
firms, 427
fitness, xiii, 275, 285, 460
flexibility, 408
flow, 10, 11, 55, 105, 118, 119, 120, 214, 230, 231, 232, 233, 234, 235, 236, 240, 264, 265, 269, 298, 323, 325, 376, 395, 410, 411, 412, 414, 415, 422, 441
flow rate, 422
fluctuations, 241, 272, 281, 292, 295, 298, 307, 313, 460
focusing, 272, 338, 427
Fourier, 39, 121, 411
Fourier transformation, 39
fractal geometry, 216
free market economy, 429
freedom, 392
fulfillment, 10, 24, 33, 37, 38, 42, 44, 51, 52, 56, 62, 63, 71, 73, 74, 76, 78, 79, 82, 85, 86, 87, 89, 90, 93, 95, 96, 97, 108, 109, 112, 113, 114, 115, 116, 123, 125, 127, 128, 130, 137, 145, 146, 159, 164, 166, 174, 175, 176, 179, 183, 188, 191, 197, 244, 260, 272, 275, 277, 316, 317, 319, 320, 338, 350, 353, 363, 369, 381, 390, 394, 415, 436
furnaces, 424, 425, 465, 466, 469

G

gas, 421
Gaussian, 260, 393
gene, 460
generalization, 229, 462
generation, 90, 202, 212, 224, 225, 227, 258, 282, 286, 294, 295, 300, 301, 307, 317, 322, 323, 326, 331, 377, 378, 394, 401, 403, 404, 405, 406, 446
genes, 271
genetic code, x, xiii, xix, 169, 201, 210, 213, 216, 218, 270, 271, 285, 296, 310, 406, 443, 452
genetic information, xiii, xviii, 101, 259, 309, 331, 378, 451
genetics, 216, 295, 401, 406, 407, 460
geometrical parameters, 250
gluons, 266
gold, 252, 441
gold standard, 441
government, 429
graduate students, xv
graph, 441, 442, 444
graphite, 420, 465
gravitation, 266
gravity, 414
grouping, ix, 164
groups, 164, 404
growth, 10, 249, 253, 273, 297, 307, 313, 378, 428, 437, 440, 465

H

Hamiltonian, xix, 3, 4, 56, 69, 71, 72, 73, 84, 86, 88, 91, 100, 102, 119, 140, 175, 179, 194, 219, 221, 244, 247, 248, 263, 277, 314, 316, 318, 319, 322, 326, 330, 366, 409, 417, 420, 421, 452, 456, 463
harm, 226, 227, 396, 397, 407
harmonics, 226
harmony, 226, 227, 396, 397, 407
healthcare, 441
heat, 64, 343, 345, 346, 410, 411, 421, 425
heat transfer, 64, 343, 345

height, 414
helix, 182, 184, 186, 191, 203, 204, 423
high temperature, 421
high-speed, 425
hippocampal, 464
Holland, 459, 461, 465, 466
homogenous, 119, 338, 417
House, 462, 466
human, 227, 374, 394, 395, 397, 401, 402, 427, 430, 431, 432, 439, 440, 443, 444
humans, 435
hydrodynamic, 413, 414, 417, 419, 421
hydrogen, 459
hyperbolic, 210, 213, 214, 216, 218, 227, 258, 411

I

identification, ix, x, xi, xii, xiv, xviii, xix, 4, 5, 6, 55, 57, 59, 63, 67, 74, 75, 78, 87, 90, 93, 94, 96, 97, 103, 104, 107, 117, 118, 120, 121, 141, 142, 145, 210, 225, 320, 335, 336, 337, 338, 339, 340, 341, 343, 345, 348, 350, 351, 354, 355, 357, 358, 359, 361, 367, 368, 370, 371, 374, 377, 382, 389, 409, 424, 425, 446, 449, 457, 463, 468, 469
identification problem, xviii, 337, 338, 343, 357, 359
identity, 49, 113, 351
ideology, 3
images, 374, 452
implementation, xiv, xix, 52, 56, 63, 86, 97, 112, 131, 179, 194, 251, 289, 321, 323, 371, 389, 420, 422, 425, 431, 436, 442, 455
income, 437, 438, 440
incomes, 428
incompatibility, 203
incompressible, 377, 396
independence, 26, 30, 459
independent variable, 142
indication, 174, 248, 417
indicators, 229, 240, 246, 253, 262, 279, 366
indirect measure, 421, 425
individual character, 282
individual characteristics, 282
individuality, 300, 439, 460
induction, 458
industrial, xiv, 422, 424, 425, 428
industrial application, 424
industry, 424
ineffectiveness, 466
inequality, 5, 18, 19, 25, 28, 29, 31, 35, 36, 37, 38, 64, 66, 67, 220, 370, 400, 458
infinite, 139, 179, 291
inflation, 439
information exchange, 282, 428, 430, 432, 441, 444
information processing, 373, 374, 394, 399, 401, 402
information production, 196, 282, 430, 431, 432
information retrieval, 397
Information System, xv, 427, 429, 456, 457, 459, 460, 462, 466, 468, 471
information systems, xi, 104, 267, 282, 332, 427, 452, 458, 466, 468
Information Theory, 458, 463, 466
inherited, 307, 378
inhibitory, 464
initial state, 66, 357, 359
initiation, xiv, 299, 306, 398, 402
insight, 272
instabilities, 148, 241, 404, 407
instability, 148, 241, 242, 245, 279, 284, 293, 311, 323, 405
instruments, 10, 11
insurance, 441
integrated unit, 397
integration, 60, 230, 293, 295, 332, 373, 401, 429, 441, 444
intellect, 227
intelligence, xix, 266, 322, 373, 398, 453
intensity, xiii, 209, 230, 261, 269, 402, 405
interaction, xix, 174, 218, 220, 231, 233, 242, 245, 266, 282, 305, 320, 331, 332, 391, 398, 403, 405, 421, 427, 429, 430, 443, 464
interactions, x, 103, 104, 128, 209, 210, 212, 213, 214, 227, 229, 230, 241, 266, 272, 281, 285, 289, 296, 309, 320, 323, 326, 328, 331, 335, 372, 374, 379, 382, 388, 396, 397, 402, 407, 416, 421, 425, 427, 428, 429, 432, 439, 443, 444
interdisciplinary, xiv, xv, xvii, 336
interference, 318, 319, 320
interpretation, 11, 366
interval, ix, 17, 66, 78, 95, 97, 99, 100, 101, 147, 148, 165, 166, 169, 170, 173, 174, 175, 176, 179, 180, 181, 182, 186, 187, 188, 190, 196, 197, 198, 199, 200, 205, 209, 211, 212, 220, 223, 234, 242, 243, 244, 246, 247, 248, 249, 251, 255, 256, 257, 260, 268, 273, 274, 276, 282, 285, 291, 297, 298, 300, 301, 315, 320, 322, 327, 329, 336, 352, 354, 355, 357, 358, 359, 361, 362, 363, 365, 366, 368, 369, 370, 378, 394, 399, 420, 422, 423, 431, 432, 433, 443, 446, 447
intrinsic, xix, 194, 285, 300, 389, 396, 397, 406, 419, 421
invariants, ix, x, xiii, xix, 6, 80, 95, 96, 97, 100, 104, 142, 146, 165, 168, 169, 170, 171, 172, 175, 176, 179, 181, 187, 200, 202, 205, 208, 209, 211, 212, 217, 247, 248, 249, 250, 251, 255, 256, 257, 259, 269, 279, 281, 282, 284, 285, 290, 294, 295, 296, 307, 314, 320, 321, 332, 336, 376, 378, 382, 392, 439, 447, 448, 449, 451, 452
inventions, 471
inversion, 158
investment, 438, 439
investors, 427, 438
ions, 245, 253, 394, 432
iron, 420, 422, 423, 424, 465
Israel, 460, 465
iteration, 337

J

Jung, 457

K

Keynesian, 439
kinematics, 417
kinetic instability, 246
kinetics, 104, 119, 129, 130, 230, 240, 244, 245, 246, 313, 323, 327, 328, 416, 459
Kolmogorov, x, xi, xiii, 31, 50, 84, 203, 269, 270, 311, 315, 316, 366, 376, 396, 409, 451, 452, 456, 458, 461, 463, 464, 465
Kolmogorov complexity, x

L

labeling, 397, 400
labor, 427, 430, 440, 444
Lagrange multipliers, 48
Lagrangian, xviii, 20, 48, 50, 54, 55, 56, 59, 85, 106, 107, 109, 112, 118, 120, 130, 135, 136, 137, 326, 457, 462
LAN, 432
language, xvii, xix, 104, 202, 210, 224, 229, 239, 266, 267, 272, 296, 300, 305, 309, 328, 336, 372, 373, 394, 395, 396, 397, 399, 406, 428, 442, 452
large-scale, 446
law, x, xiii, 9, 103, 130, 210, 226, 232, 251, 264, 271, 272, 275, 291, 295, 296, 301, 310, 322, 391, 408, 411, 412, 420, 422, 444, 452, 455, 460, 461
laws, xix, 104, 107, 108, 112, 138, 227, 307, 405, 419, 451
lead, 5, 19, 42, 59, 66, 70, 82, 118, 174, 219, 265, 300, 302, 315, 328, 349, 383, 441
learning, 404, 438
lifetime, 299, 300
limitation, 19, 37, 83, 282, 295, 296, 317, 384, 400, 452
limitations, xii, 10, 24, 33, 39, 56, 80, 89, 104, 128, 203, 251, 252, 258, 277, 299, 300, 301, 373, 374, 385, 392, 429, 433, 451
linear, 8, 10, 26, 51, 73, 103, 131, 153, 186, 202, 204, 215, 222, 223, 269, 292, 313, 323, 327, 339, 341, 389, 411, 412, 413, 422, 463
links, 269
liquid phase, 417
loans, 434, 438
local order, 172
localization, 405, 446
location, 183, 201, 202, 205, 206, 208, 209, 210, 211, 216, 220, 259, 262, 267, 268, 297, 302, 303, 394, 405, 407, 430, 440
London, 455, 458, 462, 464, 466
long-term, 397, 406
long-term memory, 397, 406
Los Angeles, xv, 466, 471
love, xv
lover, 137
LTD, 455
Lyapunov, 311, 312, 314, 366, 442
Lyapunov function, 442

M

machinery, 464
machines, 430, 444
macroeconomic, xiv
macroeconomics, 429, 466
macrosystem, 79, 93, 190, 277, 281, 291, 293, 294, 296, 298, 300, 307, 327, 332, 351, 404, 468, 469
magnetic, 194
magnetic field, 194
management, 427, 428, 437, 444
manifold, 174, 216
manufacturing, 425
mapping, 321, 405, 446
market, xiv, 427, 428, 429, 430, 432, 433, 434, 435, 437, 438, 439, 440, 441, 442, 444, 467
market economy, 444
market period, 444
market structure, 442
markets, 428, 438, 441, 444
Markov, xvii, xviii, xix, 3, 4, 5, 15, 27, 67, 82, 83, 93, 99, 121, 128, 337, 428, 441, 442, 455, 456, 458
Markov model, 441, 442
Markov process, 3, 4, 5, 67, 99, 428, 456
Markovian, 8, 105, 121, 357, 441, 443, 452
Marx, 440, 466
mass transfer, 410, 412
mathematical biology, 272
mathematical logic, 396
mathematical methods, 467
mathematics, xv, 3, 463, 464
Matrices, 460
matrix, 15, 16, 49, 55, 56, 57, 60, 62, 63, 69, 73, 74, 76, 87, 88, 89, 90, 91, 93, 94, 95, 97, 99, 101, 102, 107, 112, 115, 117, 118, 119, 120, 122, 125, 128, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 157, 158, 160, 163, 164, 174, 182, 185, 186, 194, 214, 240, 243, 245, 261, 318, 323, 326, 327, 338, 339, 340, 342, 345, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 366, 368, 371, 397, 415, 420, 437, 446
MCS, 250
meanings, 446
measurement, 3, 4, 63, 239, 259, 320, 335, 409, 421, 425, 444, 462
measures, ix, xiii, 8, 15, 18, 29, 34, 100, 176, 220, 229, 230, 232, 235, 236, 237, 238, 242, 243, 250, 257, 259, 269, 277, 280, 299, 313, 314, 322, 331, 388, 399, 424, 431, 432, 443, 452, 459

melting, 421, 424, 465, 466
memorizing, 191, 303, 306, 354, 397, 398
memory, 322, 397, 401, 406, 408
memory formation, 401, 408
messages, 266, 406
metals, 421, 465
metric, 7, 81, 141, 206, 260, 263, 267, 269, 322, 402, 462
microstructures, 468
microwave, 425
military, 429
minors, 122
MIT, 464
modeling, ix, x, xi, xii, xiv, xv, xvii, xviii, xix, xx, 4, 5, 103, 104, 142, 241, 246, 257, 323, 325, 335, 354, 370, 373, 374, 378, 409, 410, 420, 424, 428, 429, 432, 443, 444, 453, 463, 464, 465, 469
models, xi, xii, xiv, 5, 7, 13, 81, 103, 104, 121, 128, 141, 194, 210, 216, 226, 227, 244, 250, 260, 272, 275, 282, 300, 302, 303, 313, 328, 331, 335, 336, 378, 382, 408, 409, 427, 428, 429, 432, 433, 443, 444, 452, 467, 468
modernization, 431, 435, 437
modules, 186, 449
mold, 424
Moldova, 467, 468, 469
molecules, 241
momentum, 220
money, 427, 429, 439
Moscow, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469
motion, ix, 78, 107, 121, 164, 181, 187, 190, 240, 260, 336, 354, 357, 371, 396, 401
motivation, 226, 433, 436
movement, ix, xiii, xvii, xix, 3, 67, 86, 93, 115, 116, 117, 133, 138, 141, 142, 144, 145, 147, 152, 156, 160, 173, 177, 182, 186, 187, 190, 191, 194, 204, 206, 211, 215, 216, 233, 238, 259, 260, 290, 291, 293, 297, 298, 303, 311, 312, 313, 320, 326, 337, 345, 394, 395, 396, 405, 412, 416, 417, 421, 422, 445, 446, 449
multidimensional, ix, 172, 240, 272, 289, 336
multiple nodes, 164, 404
multiplication, 18, 109, 194, 220, 248, 257, 264, 294, 299, 392, 431, 432
multiplier, 28, 34, 205, 431
music, 226
mutation, xix, 295, 444
mutations, 225, 279, 281, 282, 283, 284, 285, 298, 299, 307, 309, 377, 378, 406

N

Nash, 436
natural, xii, xiii, 3, 4, 32, 78, 104, 105, 107, 130, 134, 138, 176, 268, 300, 301, 328, 389, 396, 397, 399, 401
network, ix, xiii, xiv, xviii, xix, xx, 101, 142, 164, 177, 201, 202, 204, 216, 238, 239, 251, 252, 272, 289, 305, 306, 308, 336, 338, 367, 371, 373, 374, 375, 392, 394, 401, 404, 405, 406, 407, 408, 427, 429, 443, 451, 468
neurobiological, 465
neurodynamics, 401, 402, 405, 407
neurons, 404, 406, 407
neuroscience, 403, 407, 408, 464
New York, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467
Newton, 411, 417
nodes, x, xiii, xviii, 164, 167, 169, 177, 180, 182, 191, 200, 201, 202, 204, 205, 206, 208, 209, 212, 216, 226, 227, 233, 238, 256, 259, 289, 302, 304, 305, 306, 307, 309, 328, 331, 370, 372, 374, 375, 376, 377, 379, 380, 382, 388, 397, 403, 404, 405, 406, 445, 451
noise, 81, 84, 281, 335, 405, 408
nonequilibrium, 119, 242, 272, 289, 308, 372, 405, 411
nonlinear, 55, 103, 168, 240, 292, 305, 306, 312, 313, 323, 326, 340, 341, 344, 354, 357, 358, 367, 370, 381, 383, 388, 389, 459, 463
nonlinear dynamics, 312
nonlocality, 464
normal, 123, 128, 139, 146, 147, 184, 189
novelty, 306, 307, 400
nucleotides, 258
numerical computations, 211, 431

O

observations, ix, xi, 259, 270, 357
occupational, 428
one dimension, 268
online, 461, 464
operator, ix, xii, xix, 4, 5, 8, 31, 32, 33, 37, 52, 55, 57, 63, 73, 74, 78, 79, 89, 90, 91, 92, 94, 95, 96, 103, 104, 117, 119, 121, 164, 173, 212, 223, 243, 246, 251, 261, 273, 274, 277, 284, 299, 307, 312, 313, 314, 317, 320, 323, 324, 325, 326, 328, 330, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 350, 354, 355, 357, 358, 367, 371, 374, 378, 409, 419, 447, 463
Operators, 345
opposition, 269
optimization, 104, 119, 314, 335, 337, 354, 356, 409, 419, 421, 422, 424, 425, 465, 466, 467, 468, 469
ordinary differential equations, 116, 407
organ, xviii, 180, 205, 338, 441, 444
organism, 271, 399
organization, 164, 168, 206, 210, 216, 239, 289, 294, 296, 299, 300, 308, 371, 378, 392, 404, 406, 408, 427, 428, 429, 430, 438, 439, 440, 444, 466
organizations, 428, 429
orientation, 205
orthogonality, 184

oscillations, 129, 245, 292, 293, 305, 306, 457, 459
oscillatory activity, 406

P

paper, 402, 466
parameter, 105, 106, 108, 117, 145, 168, 176, 180, 183, 192, 193, 200, 205, 218, 247, 248, 249, 257, 266, 267, 279, 282, 284, 292, 293, 294, 297, 299, 300, 313, 329, 332, 340, 366, 376, 379, 391, 392, 418, 424, 439, 440, 449
parents, 294, 295, 307
Pareto, 437
PART, 1, 333
partial differential equations, 463
particles, 10, 266, 412
partition, 311
perception, 226, 397
performance, 93, 104, 285, 336, 337, 395, 409, 421, 446
periodic, 407
periodicity, 393, 400
permittivity, 218
perturbation, 281
perturbations, xvii, 3, 13, 272, 278, 281, 284, 288, 296, 297, 300, 303, 304, 307, 444
phase space, 210, 261, 299
phase transformation, 241, 327, 413, 422, 461
phase transitions, 411
physical properties, 251
physical sciences, 459, 461
physics, 3, 218, 229, 251, 323, 335, 343, 354, 401, 452, 458, 463, 464
Plank constant, 218, 316
Poisson, 455
polar coordinates, 448
polymers, 328
poor, 309
population, 275, 276, 277, 280, 281, 295, 309, 407, 460
Portugal, 467
positive feedback, 298
power, 264, 265, 269, 299, 424, 465
prediction, 295, 408, 463, 466
preference, 306
pressure, 192, 330, 410, 414, 417, 419, 420, 421, 462
prices, 429, 433
probability, ix, xi, xvii, xix, 5, 7, 8, 10, 14, 15, 17, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 36, 37, 39, 42, 44, 45, 65, 73, 81, 82, 83, 84, 86, 93, 99, 105, 117, 121, 135, 241, 270, 314, 315, 317, 318, 319, 320, 331, 336, 337, 338, 343, 346, 347, 348, 354, 359, 428, 452
probability density function, 338
probability distribution, 14, 99, 348, 359, 428
probability theory, xi
producers, 427, 432, 442, 443
production, 62, 231, 232, 244, 245, 251, 252, 263, 282, 284, 291, 299, 301, 311, 312, 314, 321, 322, 325, 326, 381, 399, 402, 407, 417, 427, 428, 429, 430, 431, 434, 435, 440, 441, 442, 443, 444
productivity, 421, 437
profit, 437, 438, 439, 444
profits, 435, 439
prognosis, x, xiii, 55, 79, 93, 271, 272, 295, 301, 310, 351, 416, 424, 442
program, 202, 203, 205, 210, 269, 296, 309, 332, 372, 383, 419, 450
programming, 79, 93, 336, 351, 370, 395
propagation, 305, 306, 403
property, 8, 27, 105, 121, 210, 321
proportionality, 256
proposition, 31
public, 429
punctuated equilibrium, 272

Q

quanta, 220
quantum, xvii, 3, 90, 221, 317, 318, 320, 337, 401, 402, 451
quantum mechanics, xvii, 3, 337, 402
quantum state, 320
quarks, 266

R

radio, 332
radius, 146, 182, 183, 185, 194, 204, 205, 224, 249, 267, 417, 448
random, ix, x, xi, xii, xvii, xviii, xix, xx, 3, 4, 5, 7, 8, 13, 14, 17, 29, 45, 47, 55, 63, 66, 67, 76, 80, 83, 84, 90, 92, 94, 99, 100, 102, 103, 104, 105, 106, 107, 119, 120, 121, 128, 129, 131, 163, 211, 212, 270, 272, 274, 275, 281, 282, 283, 295, 303, 304, 307, 309, 310, 311, 313, 314, 315, 316, 317, 320, 329, 335, 336, 337, 338, 339, 342, 343, 355, 357, 371, 391, 392, 393, 396, 397, 399, 400, 428, 429, 440, 441, 443, 447, 451, 452, 456, 457, 458, 463, 469
randomness, ix, xviii, 54, 90, 269, 273, 274, 280, 281, 289, 295, 309, 335, 372, 377, 393, 396, 405, 451, 452
range, 200, 227, 249, 258, 332, 389, 400, 431
raw material, 424
raw materials, 424
reading, xv, 392, 393
real time, 101, 148, 322, 395, 420
reality, 222, 321, 452
reception, 398
recognition, 306, 328, 398
reconstruction, 210, 389, 395
reduction, 354, 419, 422
redundancy, 284, 377

reflection, 32, 122, 193, 226, 305, 321, 402
regional, 438, 441
regular, xii, 5, 10, 11, 15, 31, 67, 79, 90, 94, 105, 126, 129, 131, 156, 196, 197, 200, 201, 202, 203, 208, 209, 217, 221, 223, 247, 250, 253, 259, 279, 286, 302, 303, 305, 350, 371, 405, 406, 421
regulation, 441
regulators, 424
relationship, 405, 435, 446
relationships, 15, 405
replication, 307
reproduction, 295, 296, 300, 301, 305, 399
research, xi, xiv, xv, 401, 408, 440, 471
researchers, xv
resistance, 265, 266, 285, 402, 466
returns, 435
rhythm, 226
rhythms, 226, 404, 407
RNA, 399, 464
robustness, 279, 281, 288, 289, 296, 297, 300, 308, 309, 371, 406, 407, 408
rotations, 144, 145, 147, 205, 326, 354, 357
Royal Society, 461
Russian, xiv, 467

S

sales, 439
satisfaction, 86, 113, 226, 433
scalar, 109, 194, 205, 206, 208, 212, 261, 407, 415
schema, 254, 302, 304, 308, 378, 448
schemas, 282
Schrödinger equation, 3, 317, 318
scientists, xv, 401
search, 174, 306
searching, xv, 373
second Fick's law, 412
seeds, 266
segregation, 420
selecting, 74, 96, 168, 211, 240, 281, 303, 317, 318, 357, 371, 398, 446
self-assembling, 306
self-control, 79, 93, 179, 274, 275, 285, 288, 289, 294, 309, 351, 372, 406, 427
self-organization, x, xiii, 246, 271, 284, 298, 299, 300, 301, 310, 313, 328, 405, 406, 429, 437, 444
self-organizing, 250, 295, 299, 300, 306, 307
self-regulation, 405
self-reproduction, 301
semantics, 396, 397
sentences, 396, 397
separation, 311, 312
series, 110, 153, 253, 374, 395, 423
services, 439, 441
set theory, 35
shape, 204, 211, 213, 216, 321, 328, 444
Shell, 461
shortage, 436
short-term, 397, 406
short-term memory, 397, 406
sign, 62, 78, 88, 98, 129, 138, 148, 149, 170, 171, 222, 274, 286, 299, 303, 314, 349, 357, 384, 385, 433
signals, 427
signs, 150, 189, 191, 222, 235, 305, 306, 380, 381, 427
similarity, 250, 424
simulation, xv, 165, 176, 250, 272, 299, 300, 301, 332, 420, 423, 424, 425, 446, 461, 469
Singapore, 455, 461
singular, xiii, 104, 121, 122, 123, 125, 128, 129, 130, 131, 135, 137, 138, 168, 183, 306, 361, 365
singularities, 104, 121, 122, 128, 129, 148, 269, 331, 404, 407, 416, 443
skills, 430
skin, 422
SMR, 438, 441
social relations, 427
social systems, 428, 440
software, x, xviii, 194, 307, 322, 332, 423, 443, 444, 448, 449, 450, 452, 453
solid phase, 411, 412, 417
solidification, 409, 414, 415, 416, 420, 422, 423, 465
solidification processes, 414, 422
solid-state, 107
solutions, xii, xiv, xv, xvii, xix, 4, 5, 7, 32, 39, 42, 43, 53, 55, 60, 61, 63, 81, 84, 89, 90, 92, 96, 101, 102, 104, 126, 130, 131, 138, 149, 153, 159, 162, 165, 175, 222, 246, 274, 287, 291, 317, 318, 319, 320, 325, 336, 337, 343, 345, 348, 349, 350, 352, 353, 359, 361, 367, 384, 385, 419, 433, 435, 450, 451, 460
sounds, 226, 396, 408, 429
Soviet Union, xv
space-time, xii, 104, 106, 142, 156, 181, 204, 209, 210, 213, 252, 297, 402, 416, 445, 449
spatial, 107, 122, 145, 147, 148, 170, 182, 203, 205, 206, 209, 212, 223, 224, 285, 298, 303, 374, 378, 404, 405, 406, 408, 423, 433, 446
spatial location, 408
spatiotemporal, 407
spectrum, ix, 142, 164, 166, 167, 170, 171, 172, 173, 174, 175, 176, 177, 178, 180, 181, 201, 202, 203, 209, 212, 226, 227, 246, 251, 275, 277, 280, 281, 282, 283, 284, 285, 289, 292, 293, 297, 299, 301, 308, 371, 432, 434, 446, 449
speed, x, xiii, 10, 55, 115, 117, 145, 147, 151, 152, 186, 190, 192, 215, 218, 222, 223, 230, 231, 232, 233, 234, 237, 246, 249, 260, 261, 262, 263, 264, 265, 268, 269, 271, 275, 276, 277, 280, 282, 291, 301, 309, 310, 314, 354, 366, 367, 411, 412, 413, 416, 421, 422, 423, 449
speed of light, 218, 264, 265, 269
stability, ix, x, xiii, xix, 54, 129, 148, 173, 174, 223, 227, 241, 271, 277, 280, 281, 284, 289, 295, 296, 300, 306, 308, 310, 311, 312, 314, 332, 371, 388, 405, 439, 440, 442, 444, 467

stages, 313
state control, 303
statistical mechanics, 3, 313
statistics, 449, 458
steady state, 312
stimulus, 401, 402, 404, 407
STM, 406
stochastic, xi, xii, xvii, xviii, 3, 4, 5, 7, 10, 11, 12, 15, 31, 33, 37, 81, 82, 101, 103, 105, 106, 129, 156, 240, 241, 242, 269, 272, 273, 277, 281, 282, 307, 313, 320, 335, 337, 338, 354, 357, 358, 374, 404, 405, 407, 408, 428, 433, 443, 451, 456, 457, 458, 461, 462, 463, 466
stochastic processes, 3, 443
stock, 428, 434, 437, 438, 439, 440, 441, 444
stock markets, 428
storage, 397, 398, 401, 406
strains, 410
strategies, 180, 436
strength, 192, 193, 208, 209, 226, 408
structuring, 210, 259, 429
students, xv
subdomains, 132
substances, 272, 296, 401
substitutes, 427
substitution, 36, 60, 70, 75, 76, 85, 151, 161, 178, 267, 287, 339, 341, 344, 383, 388
superimposition, xiii, 148, 220, 221, 241, 323, 326, 328, 331, 416, 425
superposition, 26, 51, 129, 240, 292, 323, 327, 378, 461
suppliers, 435, 436
supply, 256, 282, 432, 433, 435, 436, 437, 439, 441, 444
surplus, 217, 275, 298, 394, 431, 435, 440
survival, 282, 300
surviving, 306, 439
switching, 13, 17, 100, 179, 180, 186, 188, 190, 205, 208, 209, 210, 212, 213, 214, 216, 221, 223, 240, 291, 292, 296, 355, 356, 359, 365, 422
symbolic, 322, 459, 462
symbols, 34, 210, 225, 258, 259, 266, 305, 322, 374, 375, 376, 377, 379, 386, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 400
symmetry, 60, 141, 146, 148, 193, 284, 323, 327, 328, 338, 340, 343, 346, 347, 349, 405, 406
symptom, 399
synapses, 464
synchronization, 227, 258, 306, 404, 405, 406, 407, 408, 457
synchronous, 306
syntax, 395
synthesis, x, xii, xviii, 4, 5, 6, 47, 67, 79, 93, 118, 130, 131, 142, 323, 351, 371, 373, 374, 375, 378, 388, 397, 398, 424, 430, 431, 432, 439, 440, 443, 460, 463, 464, 467, 468
systems, ix, x, xi, xiv, xvii, xviii, xix, 3, 65, 78, 103, 104, 107, 109, 119, 127, 137, 144, 147, 148, 163, 188, 193, 205, 226, 227, 230, 250, 253, 267, 282, 289, 300, 309, 313, 320, 332, 335, 337, 366, 372, 389, 399, 404, 405, 424, 425, 427, 428, 429, 439, 440, 441, 443, 444, 446, 452, 455, 456, 458, 461, 462, 463, 464, 465, 466, 467, 468, 469

T

tax policy, 442, 443
technology, xiv, xviii, 142, 409, 412, 420, 421, 424, 425, 453, 467, 469
telomerase, 399, 464
telomere, 464
temperature, xix, 329, 330, 331, 410, 411, 412, 413, 421, 422, 423
temperature gradient, 411, 412
temporal, 170, 209, 313, 405, 406
tension, 417
theory, xi, xiv, xv, xvii, xix, 3, 4, 84, 105, 107, 119, 211, 229, 259, 267, 271, 272, 322, 335, 336, 337, 401, 411, 414, 427, 428, 429, 452, 453, 456, 458, 461, 462, 463, 464, 465, 466, 468, 469
thermodynamic, 119, 232, 239, 241, 251, 259, 313, 328, 329, 330, 331, 410, 414, 415, 425
thermodynamic function, 330
thermodynamics, xviii, 4, 104, 119, 121, 251, 313, 323, 336, 354, 405, 461
third order, 157, 355
three-dimensional, 142, 145, 146, 147, 152, 182, 193, 204, 212, 345, 368, 403, 407
three-dimensional space, 142
threshold, 217, 218, 279, 288, 297, 300, 304, 305, 306, 403, 406, 408, 439
thresholds, 297, 406
time, ix, x, xiii, xix, 7, 9, 10, 17, 18, 63, 78, 90, 95, 96, 98, 99, 101, 103, 108, 121, 122, 138, 139, 141, 148, 160, 165, 168, 169, 170, 171, 172, 173, 174, 176, 177, 179, 181, 195, 196, 198, 199, 200, 210, 211, 212, 220, 222, 223, 226, 234, 243, 244, 245, 246, 247, 248, 249, 251, 253, 255, 256, 259, 264, 268, 269, 273, 274, 276, 279, 280, 286, 291, 292, 293, 295, 296, 297, 300, 301, 302, 305, 306, 309, 310, 311, 313, 317, 320, 322, 329, 338, 341, 344, 345, 351, 353, 354, 356, 357, 358, 359, 368, 370, 374, 378, 391, 393, 394, 395, 398, 399, 401, 402, 404, 406, 407, 408, 415, 417, 419, 420, 422, 428, 431, 432, 433, 435, 439, 440, 442, 443, 446, 447, 449, 459, 461
Timmer, 457, 463
tolerance, 271
topological, 262, 322, 402
topological structures, 262, 402
topology, 402, 404
trade, 441
trading, 429
trajectory, xiii, 24, 94, 128, 142, 172, 182, 183, 186, 203, 206, 210, 216, 221, 222, 223, 267, 275, 314, 315, 318, 337, 356, 416, 445
trans, 469

transactions, 412, 442
transfer, 62, 64, 103, 182, 235, 265, 296, 343, 345, 346, 356, 381, 410, 413, 421, 424, 435
transformation, 14, 15, 26, 31, 34, 35, 39, 74, 81, 102, 104, 107, 108, 109, 110, 113, 114, 115, 141, 143, 144, 145, 147, 148, 149, 151, 155, 160, 161, 162, 183, 193, 203, 209, 212, 240, 241, 242, 245, 246, 262, 269, 295, 296, 320, 321, 322, 326, 327, 345, 351, 354, 356, 357, 373, 375, 383, 397, 399, 402, 405, 449, 452, 461, 464
transformation matrix, 115
transformations, xii, 37, 51, 73, 104, 107, 108, 144, 145, 147, 150, 158, 181, 193, 242, 243, 246, 262, 311, 322, 327, 332, 362, 368, 381, 388, 392, 410, 411, 421, 428, 446, 449, 462
transition, 14, 232, 234, 236, 244, 269, 313
transitions, 230, 234, 237, 244, 245, 262, 311, 460, 461
translation, 327, 374, 378, 397
transmission, 234, 239, 268, 305, 403, 408, 439, 443, 444
transportation, 441
trend, 279, 284, 295, 300
triggers, 403
two-dimensional, 148

U

uncertainty, xviii, 4, 106, 142, 156, 178, 191, 203, 208, 210, 211, 216, 218, 220, 251, 263, 265, 269, 279, 282, 284, 290, 297, 300, 314, 322, 331, 335, 336, 337, 395, 396, 399, 402, 428, 437, 443, 444, 448, 452
uniform, 342, 420
universality, 229, 239
USSR, 461, 463, 465, 467, 469, 471
uti, 447

V

validity, 71, 77
values, xix, 8, 10, 14, 17, 18, 26, 39, 68, 78, 93, 98, 101, 139, 142, 146, 155, 167, 176, 178, 179, 185, 189, 190, 191, 197, 199, 206, 209, 225, 226, 238, 242, 245, 249, 252, 264, 284, 291, 297, 298, 299, 300, 301, 314, 318, 327, 332, 337, 366, 367, 377, 382, 392, 396, 427, 430, 435, 436, 437, 441
variable, 29, 110, 132, 220, 339, 346, 415, 417, 445, 461
variables, 7, 8, 9, 10, 50, 61, 76, 83, 84, 105, 118, 122, 131, 164, 181, 202, 219, 220, 221, 249, 316, 353, 354, 355, 357, 381, 410, 414, 415, 416, 417, 423, 445, 446, 447, 449, 461
variation, ix, xi, xii, xiii, xvii, xviii, 3, 4, 5, 15, 45, 47, 48, 50, 53, 56, 62, 73, 82, 83, 84, 85, 86, 88, 97, 99, 100, 102, 104, 106, 110, 112, 113, 114, 120, 130, 138, 141, 164, 167, 174, 175, 227, 230, 240, 246, 247, 269, 271, 272, 277, 280, 283, 316, 317, 322, 325, 326, 331, 337, 392, 401, 407, 408, 411, 415, 422, 443, 444, 451, 452, 457, 463, 468
vector, 7, 9, 10, 11, 13, 16, 50, 52, 53, 54, 55, 69, 71, 79, 84, 86, 87, 94, 101, 105, 107, 108, 117, 118, 142, 144, 145, 148, 150, 151, 152, 162, 182, 183, 185, 186, 188, 194, 204, 206, 212, 224, 240, 260, 261, 262, 263, 267, 268, 273, 323, 338, 341, 342, 354, 355, 357, 358, 359, 367, 368, 414
velocity, 114, 306, 417, 419, 420, 422
virtual reality, 452
viscosity, 414
visible, 227

W

WAN, 442
water, 422, 423
web, 297
windows, xii, xviii, 5, 92, 104, 371, 405, 451
Wisconsin, 463
withdrawal, 422
working hours, 440
working memory, 406
writing, 76, 160, 352