

INTRODUCTION TO MEDIEVAL LOGIC

Second Edition

ALEXANDER BROADIE

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INTRODUCTION TO MEDIEVAL LOGIC

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PREFACE TO THE SECOND EDITION

The opportunity to prepare a second edition has enabled me to make substantial improvements in the light both of my own subsequent misgivings and of criticism by reviewers, particularly E. J. Ashworth, Peter King, Norman Kretzmann, Stephen Read, and J. A. Trentman. I am happy to acknowledge their help.

Peter T. Geach has sharpened up my thinking on a number of medieval logic matters and I am grateful to him for that as for much else besides.

Among the major changes, I have extended my account of the different sorts of supposition, and of the logical problems relating to intentional contexts. The discussion of the most elementary part of medieval syllogistic is now not quite so breathless; the Conclusion also is more substantial, as is the Bibliography. The chapter 'Inference Theory: Medieval and Modern' has been deleted, as I no longer think that I can say anything useful about the topic without taking the matter a good deal further than would be justified in a book of the kind I had it in mind to write. Except where otherwise stated, the translations from Latin are my own.

This new *Introduction to Medieval Logic*, though appreciably longer than the first edition, has the same shape as before, and also the same character. Other books of a very different character could be written under that title, and I hope that some of them will see the light of day.

Glasgow 1992

A.B.

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I

Introduction

There can be no doubting the central role accorded logic in the educational scene in the Middle Ages. There are two related aspects to this role, one institutional and the other scientific. The first is that at the heart of the medieval educational system were the seven liberal arts, divided into the trivium, three arts of language, and the quadrivium, four mathematical sciences. The arts of the trivium, the 'trivial' arts, were grammar, rhetoric, and logic, and during a period of several centuries practically every university graduate received a training in those arts.

The second aspect of the role of logic explains the first. Logic was considered as a propaedeutic to the remaining sciences. Robert Kilwardby¹ states the position in his great work *The Rise of the Sciences*, where he writes:

The origin of this science, as was mentioned before, was as follows. Since in connection with philosophical matters there were many contrary opinions and thus many errors (because contraries are not true at the same time regarding the same thing), thoughtful people saw that this stemmed from a lack of training in reasoning, and that there could be no certainty in knowledge without training in reasoning. And so they studied the process of reasoning in order to reduce it to an art, and they established this science by means of which they completed and organized both this [science] itself and all others; and it is the science of the method of reasoning on all [subject] matters.²

¹ English Dominican c.1215–1279. Taught at Paris and Oxford. Became Prior Provincial of the English Dominicans 1261. Archbishop of Canterbury 1272–c.1277. Cardinal Bishop of Porto 1278.

² *De Ortu Scientiarum*, ch. 53, para. 494. Trans. taken from *The Cambridge Translations of Medieval Philosophical Texts* (hereinafter *CTMPT*) (Cambridge, 1988), ed. N. Kretzmann and E. Stump, vol. i, 265.

In a similar spirit Kilwardby's contemporary Peter of Spain³ begins his *Summule Logicales* with the words: 'Logic is the art which provides the route to the principles of all methods, and hence logic ought to come first in the acquisition of the sciences.'⁴ William of Sherwood⁵ said of grammar, rhetoric, and logic that they teach us respectively to speak correctly, ornately, and truly.⁶ As regards logic, part of what he had in mind was that logic is a tool with whose aid we can reach the truth by a rational investigation of what is already known to be true. That is, logic can prevent us from slipping from truth to falsehood. Considered in this light it was bound to be concluded that there was no person who could not benefit from a training in logic, for truth is a goal that we all by our very nature seek.

Medieval logic is of course a vast field, involving a wide diversity of subjects investigated over a very long time span. Within it are several subjects which were perceived as sufficiently distinct to merit separate treatises. Fallacies were studied as a distinct area,⁷ as were so-called insolubilia.⁸ The latter were characteristically propositions whose truth value is problematic in virtue of a self-referential element. The 'liar paradox' in its basic form, namely that my proposition 'I am saying something false' appears to be false if it is true, and true if it is false, is the most famous insoluble, though very many more were discussed.

There was also an extensive literature on obligations exercises.⁹ In these exercises, which are in the form of a disputation, an opponent seeks to manoeuvre a respondent into

³ Born c.1205. Studied arts at Paris. Professor of medicine at Siena 1246-50. Dean of Lisbon and Archdeacon of Braga 1250. Archbishop of Braga and Cardinal Archbishop of Tusculum 1273. Elected Pope John XXI 1276. Died the following year.

⁴ *Summule Logicales*, ed. L. M. de Rijk, 1. Cf. Aristotle, *Topics*, 101 b 3-4.

⁵ c. 1200/10-between 1266 and 1272. Student at Oxford. Master at Oxford 1252. Treasurer of Lincoln from 1254/8.

⁶ *Introductiones in logicam*, ed. M. Grabmann, 30.

⁷ See *CTMPT*, 245-61: 'The fallacy of composition and division', for part of Peter of Spain's contribution to the field.

⁸ See *The Cambridge History of Later Medieval Philosophy* (hereinafter *CHLMP*), ed. N. Kretzmann, A. Kenny, J. Pinborg, ch. 12; also P. V. Spade, *The Medieval Liar: A Catalogue of the Insolubilia-Literature*, also *CTMPT*, 338-68, for Albert of Saxony on insolubilia.

⁹ See *CHLMP* ch. 16; also *CTMPT*, 370-412 for Walter Burley on obligations; also Paul of Venice, *Logica Magna*, Pt. II, fasc. 8, ed. E. J. Ashworth.

assenting to a contradiction, while the respondent seeks to avoid this outcome. The precise purpose of these exercises is not yet definitely understood, though they may well have formed part of the training of students in which they had the opportunity to display, and also extend, their competence at logic. But such exercises also provided a context within which a wide range of logical and philosophical problems could be investigated. For example, the literature of obligations is a major source for medieval discussions of insolubilia, and of counterfactual inferences.

A further area of logic to which medieval logicians paid close attention was the logic of dialectical inference, which took its starting-point from Aristotle's *Topics*.¹⁰ Additionally the field of exponible terms was extensively investigated.¹¹ These terms, which include 'only', 'except', 'in so far as', 'begins', 'ceases', and 'differs'—terms thought to be obscure in various ways and thus in need of exposition (hence the word 'exponible')—were of great interest to logicians and philosophers.

An all-inclusive account of medieval logic would include the foregoing areas, and more besides. However, in a brief work, especially one written for those with no previous acquaintance with the subject, choices of various sorts have to be made, both in respect of the topics covered, and in respect of the logicians to whom particular attention will be given. I have elected to concentrate, first, upon aspects of the large area that went under the heading of 'properties of terms', thus providing an opportunity for discussion of the signification, supposition, and ampliation of terms; and, secondly, upon aspects of the theory of consequences. The latter theory deals most especially with the identification of rules of valid inference, though in the course of our discussion it will prove necessary to attend also to various examples of fallacious reasoning. Without doubt these two areas, properties of terms and the theory of consequences,

¹⁰ See *CHLMP*, ch. 14; *CTMPT*, 226–45, for Peter of Spain on topics.

¹¹ See *CHLMP*, ch. 11; also *CTMPT*, 163–215.

are crucially involved in all aspects of medieval logic, and the study of them can readily be seen as a valuable preliminary to the study of the various fields mentioned earlier. Thus, for example, the nature of the signification of routinely discussed syncategorematic terms, such as 'every', 'no', 'and', and 'or', has an immediate bearing on what should be said about the signification possessed by exponible terms such as 'only', 'except', and 'in so far as'. And approaches to the discussion of fallacies are determined in detail by the account of the nature of valid reasoning.

In a comprehensive guide it would be necessary to deal with the logic, and therefore with the logicians, of many centuries, perhaps starting with Abelard (1079-1142); perhaps even, and with good reason, starting with Boethius (c.480-524). For despite the fact that Boethius belonged to the late Roman Empire rather than to the Middle Ages his writings, which include not only the *Consolation of Philosophy* but also translations of most of the logical works by Aristotle and commentaries on a number of books, are, for logicians no less than for philosophers, a crucial bridge between the ancient world and the medieval.

However, this Introduction is not intended as a survey of ten centuries. I have chosen instead to attend mainly to the period from the mid-thirteenth century to the earlier part of the fifteenth. For the purposes of an Introduction a different period could no doubt have been chosen, but the period just mentioned is one of intense activity by men with as strong a claim as any to being included among the greatest of the medieval logicians. Certainly that period yielded an extraordinarily rich harvest. The logicians upon whom I shall be drawing most heavily are Peter of Spain, Walter Burley¹², William Ockham¹³, John

¹² Walter Burley (or Burleigh) c.1275-1344/5. MA, Oxford, by 1301. Fellow of Merton College, Oxford, for at least four years till 1305. Theology student at Paris from before 1310. Doctor of Theology, Paris, c.1320. Fellow of Sorbonne by 1324.

¹³ William Ockham c.1285-1347. Franciscan. Student at Oxford. Lectured at Oxford 1317-19. Summoned to Papal Court at Avignon 1324, charged with heresy. Fled from Avignon 1328, and was given asylum by German Emperor, Ludwig of

Buridan¹⁴, Albert of Saxony¹⁵, and Paul of Venice¹⁶. But my net will be cast sufficiently wide to take in the writings of earlier logicians and also of later ones, even considerably later ones. For in the decades before the Reformation important discoveries were being made in logic by men who were very much part of the medieval logical world.

A distinction commonly drawn now, though not explicitly drawn by medieval logicians, is that between philosophical and formal logic. The distinction is however useful for present purposes, since each of those two heads of division picks out a great deal of material found in the medieval logic textbooks. Philosophical logic is a philosophical enquiry which takes as its subject certain concepts of particular concern to logicians. A comprehensive exposition of the science of logic must include a study of valid inference. Let us say, provisionally, that an inference is valid if it is impossible for its premisses to be true without the conclusion being true. That definition naturally prompts certain questions. For example, can any light be shed on the concept of truth invoked in our definition? And in ascribing truth, what is it to which it is ascribed? Is it perhaps the proposition considered as an utterance or inscription, or is it the proposition considered as the sense of the utterance or inscription? Or is the bearer of truth value something different from any of these things? These questions, and others also prompted by the above definition of validity, are philosophical and are not to be answered by a logician working solely within the bounds, however loosely conceived, of formal logic.

Bavaria. Ockham excommunicated. Shortly before his death he apparently sought to have the excommunication rescinded, but it is unknown whether he signed the formula of submission.

¹⁴ John Buridan c.1295/1300–c.1360. MA, Paris, c.1320. Lecturer in Arts Faculty, Paris. Rector at Paris 1328, 1340.

¹⁵ Albert of Saxony d. 1390. Student at Prague, and at Paris where his master was John Buridan. MA, Paris, 1351. Taught at Paris 1351–62. Rector at Paris 1353. Co-founder and first rector of Vienna University 1365. Bishop of Halberstadt 1366–90.

¹⁶ Paul of Venice c.1369–1429. Studied at Padua. At Oxford 1390–c.1393. Doctor of Arts and Theology at Padua 1408. Rector and Vicar-General of Augustinians 1409–10. Ambassador of Venice on several occasions. Rector of University of Siena 1428. His *Logica Magna* (1397–8) is an encyclopaedia of logic.

Formal logic, on the other hand, presupposes either that we already know the answers to the above questions, or at least that we have an insight into the answers which is sufficient for the needs of formal logic, and proceeds to the question of the identification of the rules of inference which can then be used to test inferences for validity.

Most of those who made a significant contribution to logic in the Middle Ages were philosophers just as much as they were logicians, and in most cases they were as much at home on the philosophical as on the formal frontier of logic. Their philosophical researches illuminated the logic, and their logical researches underpinned the philosophy. The philosophy led naturally to an investigation of the rules governing valid inference; for philosophers, committed as much as anyone could be to rational enquiry, had to be able to defend their arguments against charges of invalidity. And some of their philosophical enquiries were themselves prompted by consideration of the rules of logic. In the following pages I shall be attending to their ideas on both philosophical and formal logic. And since they conceived of logic as a science of language I shall begin by considering certain units of language basic to logic.

Aspects of Language

I. TERMS, PROPOSITIONS, INFERENCES

William of Sherwood said that logic teaches us to speak truly; and perhaps, in accordance with that dictum, we should start by examining the concept of truth. But if he was speaking about the fact that logic teaches us to infer truths on the basis of truths already known, then perhaps it is preferable to start by examining the concept of an inference. Now, any inference has a characteristic complexity. It must contain at least two propositions, one of which is presented as following from the others. In that case an inference must contain a further level of complexity, for propositions also are complex; as Aristotle said, a proposition must have at least two parts, a noun and a verb. Nouns and verbs were traditionally classed as terms. There are thus at least three levels with which the logician is concerned, first, terms; next, propositions, which contain terms; and finally, inferences, which contain propositions. This way of putting the matter suggests an obvious order of exposition for the logician. Since terms are elements out of which propositions are composed, they should be examined before propositions, and inferences, as composed of propositions, should be examined last.

There are grounds, however, for considering inferences before terms. At least as regards some terms it is commonly held that their sense should be given by showing how they, as elements in propositions, contribute to the validity or otherwise of inferences. Thus a standard way to expound 'and' is to say that given two propositions, P and Q , the

conjunctive proposition 'P and Q' follows; and given 'P and Q', P follows and Q follows. Likewise a standard way to expound 'or' is to say that given a proposition P, the proposition 'P or Q', formed by disjoining any other proposition to P, follows; and given that from P plus a set of propositions S^1 there follows R, and given also that from Q plus a set of propositions S^2 there follows R, then R follows from 'P or Q' plus the two sets of propositions S^1 and S^2 . It might indeed be impossible to expound 'and' and 'or' without describing their role in valid inferences. According to this approach logicians should start not with the least complex, the term, but with the most complex, the inference.

However, though it is I think important to notice the grounds just mentioned for starting with terms and alternatively for starting with inferences, there is clearly the basis here for an unhelpful oscillation. For even if we allow that the exposition of certain terms requires a display of the contribution that those terms make to the validity or otherwise of inferences containing them, those very inferences can only be constructed out of terms, and hence the terms must already be to hand if the inferences which illuminate the sense of the crucial terms are to be set out. We need the terms if we are to construct the inferences, and we need the inferences if we are to expound the terms.

It was not uncommon for medieval logicians to begin their logic textbooks, at least those of their textbooks containing comprehensive accounts of logic, by considering terms first, and then reaching their study of inferences by way of an analysis of propositions. Thus, for example, Part I of William Ockham's three-part *Summa Logicae* is entitled 'Concerning terms' and deals with the definition of 'term', with different kinds of term, and with properties of terms. Part II, 'Concerning propositions', deals with singular, indefinite, particular, and universal propositions, with tensed, modal, exclusive, exceptive, and reduplicative

propositions,¹ and with molecular propositions. And Part III is divided into four sections dealing, in order, with the syllogism, simply understood, with demonstrative syllogisms, with inferences, and with fallacies. Similarly the *Perutilis Logica* of Albert of Saxony begins with two treatises on kinds of terms and properties of terms, followed by a treatise on propositions, dealing with propositions as such, and then with modal, molecular, and explicable propositions. The fourth treatise deals with inferences, the fifth with fallacies, and the final treatise, which is in two parts, deals with insolubles and then with obligations exercises.

But the fact that certain logicians adopted this order of exposition should not be taken to signify that they would have rejected the notion that terms, or at least some terms, should be expounded by reference to the role they play in valid inferences. On the contrary, their practice shows that they accepted this point.

The order of exposition I have just been describing is, in a sense, reflected in modern axiomatic formal logic. For modern axiomatic systems characteristically begin by setting out the elements out of which propositions can be composed, and classifying those elements. They then lay down rules for combining those elements into well-formed formulae, that is, into propositions, and finally they lay down the rules by which a proposition may be inferred from a given set of propositions. The difference between medieval practice and modern is therefore, at least in this respect, not very great.

¹ Exclusive propositions have the basic form 'Only A is B'. Exceptive propositions have the basic form 'Every A except B is C'. Reduplicative propositions have the basic form 'A in so far as it is B is C' or 'A *qua* B is C' (as in 'being *qua* being is the subject-matter of metaphysics'). For recent discussions on these see e.g. J. Pinborg, 'Walter Burley on Exclusives', in *Medieval Semantics* (ed. S. Ebbesen). For subsequent developments in the theory of explicables see A. Broadie, *George Lokert*, 102-108, and *The Circle of John Mair*, 172-205; also E. J. Ashworth, 'The Doctrine of Exponibilia in the Fifteenth and Sixteenth Centuries', *Vivarium* 11 (1973), reprinted in E. J. Ashworth, *Studies in Post-Medieval Semantics*; also A. Bäck, *On Reduplication*.

The practice is traceable to Aristotle, and indeed the traditional ordering of the books which constitute his contribution to logic reflects the ordering: terms, propositions, inferences. For of that set of books, known as the *Organon* (= 'tool' or 'instrument'), the first, the *Categories*, is concerned with terms and their classification. The second, the *De Interpretatione*, deals with propositions and their classification, and the remaining books—the *Prior* and *Posterior Analytics*, the *Topics*, and the *Sophistical Refutations*—deal with the rules of valid inference and with the classification of arguments good and bad.

II. TERMS

In this book I shall follow the order of exposition which has just been described, and shall therefore begin with the notion of a term. Since a term is a kind of sign let us consider the concept of a sign. Near the start of the *Summa Logicae* William Ockham presents two accounts of a sign. Regarding the first, he writes: '[A sign is] anything which, when grasped, makes something else come to mind, though what is brought to mind is not in the mind for the first time but is actually in the mind after being known dispositionally.'² Thus, for example, a barrel hoop outside a tavern is a sign of wine, and the utterance 'William' is a sign of William. In each case the sign's being a sign for someone depends upon his prior knowledge of an association between two things, in the one case it is between the hoop and the wine, and in the other it is between the name and its bearer. The person's knowledge is said to be dispositional, and to be exercised when the person reads the sign on the basis of his prior knowledge of the relation between sign and thing signified. This sense of 'sign' is said to be a wide sense.

The second sense, presumably narrower, is this: 'A sign is that which makes something come to mind and is fitted

² *Summa Logicae*, Pt. I, ch. 1, pp. 8-9.

by its nature to stand for that thing in a proposition, or to be added to what stands for that thing in a proposition . . . or is fitted by its nature to be composed of such things'.³ Ockham is thinking here of nouns and nominal phrases, which stand for things in the context of a proposition, and of words such as 'is', 'every', and 'not', which can be added to such expressions, and finally, of propositions, which are composed of expressions of the kind just mentioned.

It is not clear precisely what Ockham took to be the relation between the wide and the narrower senses of 'sign', but to call them wider and narrower is probably itself misleading since it is not certain that everything which is a sign in the narrower sense is also a sign in the wider sense. For it is probable that any sign in the wider sense is something which stands for something—certainly Ockham's examples suggest this interpretation. But he frequently said of many expressions which are signs in the second sense, for example, 'is' and 'every', that they neither do nor can stand for anything, in which case it might be better to think not of one sense as wide in relation to the other, but rather of two sets of signs that overlap each other. However, whatever the nature of the relation between these two senses of 'sign' it was with the second, narrower, sense that Ockham was chiefly concerned, and it is with that sense that we shall be concerned hereafter.

Granted that terms are a kind of sign, as also are propositions, and as also are inferences, let us enquire into the kind of sign that a term is. The word 'term' (*terminus*) was defined in a variety of ways by medieval logicians, and indeed it was not rare for a logician to offer several definitions on a single page. Perhaps the commonest definition was 'proximate part of a proposition' (*pars proxima propositionis*). Elucidation of this mysterious phrase is provided by reference to Aristotle: 'I call a term that into which a proposition is resolved, namely, a predicate and that of which the predicate is predicated, when it is affirmed or denied that something is

³ Ibid. 9.

or is not the case.⁴ According to this account a term is a subject or a predicate of a proposition, whether an affirmative or a negative proposition. Thus, for example, in 'A young man is reading a book' the subject 'young man' and the predicate 'reading a book' are both terms.

The subject and predicate terms in the above sample proposition do not constitute the proposition, for there is, in addition, a coupling device, a 'copula', which links the subject to the predicate. A proposition composed of subject, predicate, and copula was called a 'categorical proposition', and those three parts were commonly referred to as the principal parts of a categorical proposition. This notion, that is, 'principal part of a categorical proposition', was also used as the definition of 'term'. Thus in 'A young man is reading a book' there are, on this latter definition, three terms.

One further definition might usefully be mentioned here. It received particularly clear expression from the late scholastic logician David Cranston. He says that the proximate part of a proposition is 'a part of a proposition from the signification of which part there arises partially the signification of the whole proposition'.⁵ Thus if a part of a proposition lacks signification it is not a term. Hence in the proposition 'Walter is dreaming' the 'alter', which appears in the subject term, is not itself a term. For though 'Walter' has signification, 'alter', in so far as it is a part of 'Walter', does not have a signification which contributes to the signification of the whole proposition. The reason for this is simply that it does not, in so far as it is a part of 'Walter', have signification at all. This is not to say that 'alter' does not contribute to the signification of the proposition. It clearly does make such a contribution, since if the 'alter' were not there, then neither would 'Walter' be, in which case the signification of the proposition would not be the same. The point

⁴ *Prior Analytics*, 24 b 16-18, and e.g. Ockham, *Summa Logicae*, Pt. I, Ch. 1, p. 7.

⁵ D. Cranston. *Tractatus terminorum*, sig. d 2; for discussion of this see A. Broadie, *The Circle of John Mair*, 28-30. Cranston c.1479-1512 was a pupil of John Mair at the Collège de Montaigu. Began to teach Arts at Paris 1499. Doctor of Theology, Paris, 1512.

is that 'alter' does not make its contribution by itself signifying in the proposition. It seems to follow from this account of 'term' that a negative particle in a word, for example, 'un' in 'unhelpful', as that word occurs in 'The example is unhelpful', is a term. For 'un' does signify, and of course its signification contributes to the signification of the whole proposition.

The definitions so far presented were by no means the only ones canvassed, nor did any one of them acquire a particularly widespread acceptance. This fact might be thought a matter of some embarrassment to the very logicians who, on account of their deep investigations into the properties of terms, came to be known as 'terminists' (*terministi*). All the same, this wealth of definitions of 'term' rarely caused confusion, since it was in general quite clear which concept of 'term' the logician had in mind when he predicated the word of a linguistic expression. It is, however, plain that whatever concept the logicians were seeking to capture they were not primarily, if at all, aiming to encapsulate in a definition of 'term' our concept of 'word'. But all the definitions of 'term' have this much in common, that they all imply that a term is a significative part of a proposition, even if they do not all imply that every significative part of a proposition is a term.

Sufficient for immediate purposes has now been said about the nature of a term, and I should like to turn to a consideration of a crucial principle of division of terms.

III. THOUGHTS, UTTERANCES, INSCRIPTIONS

In a passage of great importance for the development of medieval logic, Aristotle writes:

Now spoken sounds are symbols of affections in the soul, and written marks symbols of spoken sounds. And just as written marks are not the same for all men, neither are spoken sounds. But what these are in the first place signs of—affections of the

soul—are the same for all; and what these affections are likenesses of—actual things—are also the same.⁶

In due course these words were appropriated for a wide range of purposes. St Thomas Aquinas, for example, used the first sentence of the foregoing passage as a starting-point for his argument for the claim that words do not signify entirely differently when applied to God and to his creatures.⁷ Something should be said here about that sentence, for it was given an interpretation that at first sight seems ill suited to the words. Aristotle appears to be envisaging a quadrilateral of relations between an inscribed word, an uttered word, a thought (an 'affection of the soul'), and a thing: say, the inscription 'man', the utterance 'man', the thought of a man, and a man, and to be saying that the inscription 'man' signifies the utterance 'man', the utterance signifies the thought of a man, and the thought is a likeness of a man. On this interpretation the utterance directly signifies not a man but a thought of a man. For example, Aquinas tells us that a word of the intellect (*verbum intellectus*) or a conception of the intellect, is signified by the physical utterance.⁸ He is here making the point that a person's speech is taken to signify that he is thinking and what he is thinking. This is not to deny that on occasion—though of course it cannot become the norm amongst us—we can recognize a person as mouthing words unthinkingly. Elsewhere, in the same vein, Aquinas affirms: 'This thought (*intentio*) is called an internal word (*verbum interius*), which is signified by an external word.'⁹ What directly signifies the external object, a man, is the thought. Thus for Aquinas speech points in two directions at once, inward to the thought and outward to what the thought is about.

This interpretation of Aristotle does indeed seem to fit Aristotle's words well, but it was not the only interpretation.

⁶ *On Interpretation*, 16 a 3–8.

⁷ *Summa Theologiae*, 1a, 13, 1 c.

⁸ *De Veritate*, 4, 2 c.

⁹ *Summa Contra Gentiles*, IV 11.

Ockham's was a dissenting voice.¹⁰ He held that Aristotle's position is that an inscription and an utterance signify a thing no less immediately than does a thought. But the thought signifies primarily what is signified secondarily by the utterance or inscription. For example, on this view the relation between the utterance 'man' and the thought of a man is not the relation of significans to significate but of subordinate to superordinate. We see something, and in so doing we naturally form a thought of that thing. Until we have the thought we cannot use any sound to signify the thing. Once we have the thought a convention can be established whereby a sound has the signification that the thought has by nature. Thus what the utterance signifies depends on our thought, but the signification of the thought does not depend on an utterance. Hence thoughts have a kind of priority over utterances and inscriptions; but if we choose to call this a priority of signification, that is not to be understood to mean that the inscription and the utterance do not signify the thing as directly as does the thought. They do signify as directly, but are nevertheless subordinate to the corresponding thought.

In his commentary on the Aristotelian passage under discussion, Boethius asserts that there are three kinds of speech (*oratio*), namely, written, spoken, and conceived or thought speech, and that there are correspondingly three kinds of term. Written speech is visible, spoken speech is audible, and conceived or thought speech is not available to any of the five external sensory modalities but exists in the intellect only and therefore exists only as thought.¹¹ We have observed that there is a certain relation of subordination which can be seen to hold between written and spoken terms on the one hand and thoughts on the other, and staying with Boethius's tripartite classification of language I should like to specify some of the chief differences between these sorts of language. Aristotle will, on this matter as on many others, be our guide.

¹⁰ *Summa Logicae*, Pt. I, Ch. 1, pp. 7-8.

¹¹ *In Librum De Interpretatione*, 2a, I, cap. *De Signis*. See *Patrologia Latina*, 64, 407B.

He asserts that 'affections of the soul' are the same for all men, but utterances and inscriptions are not. Let us stay with this point. Spoken and written signs are in an obvious sense conventional. That we use the sounds or marks we do use in order to communicate is not a fact of our nature, for we could have used other signs, and other nations do use other signs. But what I think of when I think of what I call a 'man' is the same as what a Frenchman thinks of when he thinks of an 'homme', and as what a Greek thinks of when he thinks of an 'anthropos'. The thought is the same though the conventional expression of it differs. Thus the language of thought is universal in contrast to what we may term the 'parochiality' of conventional language. Indeed the intertranslatability of conventional languages is due precisely to the fact that, different as they are in respect of many of their characteristics, they can all be used to express the same set of thoughts. In contrast with the conventionality of written and spoken languages, medieval logicians spoke of the language of thought as a natural language, a language we have by our nature. The term 'natural' is misleading for us in this context since it is customary to speak of English, Latin, and so on, as natural languages, as contrasted with artificial languages such as Esperanto. But I shall continue to use the terminology of the medieval logicians. My practice in this matter will be constant, and so should not lead to misunderstanding.

Each of us has a cognitive faculty, that is, a set of abilities, to understand, to calculate, to intuit, and these abilities were thought of as part of our natural endowment. A change occurs in us when we think, for we are different, at least in respect of the thoughts that we have that we had previously not had. A change, however, to what? The commonest answer was that the cognitive faculty undergoes modification. Let us suppose that you say to me 'A man is reading'. My cognitive faculty is modified by the very fact of my grasping what you have said. But that modification is itself an act of understanding. And it should be said that my

cognitive faculty is modified in exactly the same way when someone says to me 'Homo legit' or 'Un homme lit'.¹² What this suggests, as medieval logicians saw clearly, is that when we think, we do not think in any of the conventional languages, even though we cannot express what we think without using one of those languages.

If the foregoing points are correct a problem arises concerning how, if at all, we spell the terms in mental language. I think that a man is reading, and I say what I think in English. How many letters has the mental term which corresponds to the spoken term 'man'? The answer cannot be three, since that mental term is identical to the mental term corresponding to the spoken Latin term 'homo' and to the French term 'homme'. But if a term has any letters it surely has a determinate number. The conclusion drawn was that mental terms differ from conventional terms in this, among other things, that they are not composed of letters.

An important corollary of this is that a conventional term can change its signification but a mental term cannot. Change implies a permanent underlying the change. In the case of conventional terms the permanent is the string of letters in the inscription or the string of sounds in the utterance. But a mental term, conceived of as a modification of a cognitive faculty, cannot change its signification for there is nothing to it over and above its signification. The mental term can cease to exist and does so when the mental act, which is what the term really is, ceases. But the mental term cannot lose one signification and gain another, for there is nothing by which it could be identified as the same mental term again, lacking as it does anything corresponding to letters and sounds.

¹² It is modified in exactly the same way in respect of what I understand by the two propositions, but it is modified in different ways in respect of the auditory experience I have when I hear these propositions uttered. The concept of the proposition considered merely as an utterance and not as significative was sometimes said to be 'non-ultimate', for the mind does not rest at that concept but goes on to a grasp of the signification of the utterance. On grasping it, the mind has an ultimate concept of the utterance. See A. Broadie, *The Circle of John Mair*, 43-5, for a discussion of this distinction.

Several logicians raised the question of the extent to which mental language corresponds to conventional languages. William Ockham, for example, held that mental language contains or features nouns, verbs, and prepositions, singularity and plurality, verb moods, tenses, and voices (active and passive), but that it does not distinguish between the various declensions of nouns and the various conjugations of verbs.¹³ It seems to have been the opinion of Albert of Saxony that mental language does not make a distinction between nouns and pronouns.¹⁴ Also it was commonly held that mental language does not contain synonymous terms, that is, distinct terms with the same signification.¹⁵ The last point is readily understandable, for there is no way to distinguish mental terms which do not differ in respect of their significations if in fact there is nothing to them except those very significations. That mental language does not distinguish between the declensions of nouns is also readily understandable, for declensions are distinguished by systematic differences in spelling. And by the same token there is, at the level of mental language, no distinction in the conjugation of different verbs. Indeed it makes no sense to ascribe declensions and conjugations to mental nouns and verbs, given that mental terms contain no letters.

Those elements in a conventional proposition to which there are corresponding elements in the corresponding mental proposition are the significative elements which make a contribution to the truth value of the conventional proposition. That is the chief reason logicians were interested in mental language, the language of thought. A distinction was commonly made between those features of language of interest to the logician and those of interest to the grammarian. Some grammatical features of a proposition interest the logician, for instance the distinction between subject and predicate, the tense of verbs, the voice of the verb, the person of the verb (whether first, second,

¹³ *Summa Logicae*, Pt. I, Ch. 3.

¹⁴ *Perutilis Logica*, 3^{rb}.

¹⁵ *Summa Logicae*, Pt. I, Ch. 3.

or third person), the singularity or plurality of nouns, and the case of nouns, for all of these have an effect on the truth or otherwise of the propositions containing the terms in question. But that the subject term of a proposition is a first-declension rather than a second-declension noun can make no contribution to the customary signification. Thus when the logician set out to determine whether a given inference was valid, his procedure in effect was to determine how the inference would appear in mental language, and then to establish the validity or otherwise of that mental inference. The mental inference would be the conventional inference shorn of all those of its features that made no contribution to the truth value of the premisses and conclusion or to the validity or otherwise of the inference.

Medieval logicians did not devise a symbolic logic, but in the main investigated inferences which were couched in Latin, a living language. They wanted to know which inferences expressed in that language were valid, and to do this they had first to lay bare the logical forms of the propositions in the inferences, the logical, not the grammatical forms. To this end they asked themselves what it was that they thought when they thought the inferences which were then expressed in conventional language. In other words, they investigated the natural language of thought as the only means available to them to discover the rules of valid inference. It is, then, not at all surprising that the concept of mental language was such a central concept for the medieval logicians.

IV. THE TEMPORALITY OF PROPOSITIONS

So far propositions have been classed under one or other of three headings, as mental, spoken, or written. As we saw, the difference between spoken and written propositions is that the former are audible and the latter visible. It is possible, as some noted, to conceive of terms which are none of

these three kinds, for signification can be imposed upon qualities other than visible or auditory ones. Indeed Braille is a tactile language, and its terms are tactile terms. In addition one might even conceive of an olfactory or gustatory language. But let us stay for the moment with utterances, inscriptions, and thoughts. It was a characteristic doctrine of medieval logic, and one upon which a great deal hinged, that propositions are either thoughts, or are visible or audible expressions of thoughts. For from this it follows that it makes sense to speak of a proposition as coming into existence and ceasing to exist, as for example when someone thinks something and then ceases to think it, or when someone writes something down and the inscription is erased.

There is a modern view that a proposition is the sense of a sentence, perhaps specifically an indicative or an assertoric sentence. And some hold that the sense of a sentence has a life of its own which is quite independent of the sentences which express it. This view is associated in particular with Gottlob Frege.¹⁶ If a proposition is considered as the sense of a sentence, as Frege understands the phrase 'sense of a sentence', then a proposition as so conceived can exist even if the corresponding thought, utterance, or inscription does not exist and has never existed. There is room for doubt over whether this is a viable theory, for any sense is the sense of an expression, and if there is no expression for the sense to be a sense of we might wonder whether the sense itself can exist. But however this difficulty should be resolved, we should note that this way of considering propositions is as far removed as could be from the way revealed in very many medieval logic textbooks.

The medieval view that a proposition has a time-span and also, in the case of inscriptions, a spatial location, plays a role right at the heart of medieval logic in discussions

¹⁶ See especially 'The Thought: A Logical Enquiry', trans. A. M. and Marcelle Quinton, *Mind*, 65 (1956), 289-311; and M. Dummett, *Frege: Philosophy of Language* (London, 1973), ch. 11.

about the nature of valid inference. For if we say that an inference is invalid if it is possible for the premisses to be true without the conclusion being true, then many, perhaps all, inferences which we should regard as valid would have to be classed as invalid on the grounds that the premisses might be true at a time when the conclusion does not exist and in that case is not at that time true.

I shall not take this problem further here; it will be considered again later.¹⁷ I wish at this stage merely to stress that for the rest of this book when I speak of propositions, I mean propositions understood as having the kind of existential status ascribed to them by medieval logicians. And given the relations between propositions and terms, it follows that whatever existential status is ascribed to propositions must also be ascribed to the terms out of which the propositions are composed.

V. TERMS: CATEGOREMATIC AND SYNCATEGOREMATIC

I have spoken of medieval logicians as engaged, at least as an important part of their task, in identifying the logical form implicit in propositions. It was recognized that terms of a certain class played a particularly important role in this task of identification, and in this section I should like to focus on that class of terms.

Medieval logicians were accustomed to make a distinction between significative and non-significative terms. The latter are what we should now call nonsense words. The former are all the other terms, that is, every term which has a signification. Within the class of significative terms a distinction was made. Paul of Venice writes: 'A term significative *per se* is one which, taken by itself, represents something, for example "man" or "animal". A term which is not significative *per se* is one which, taken by itself, represents nothing, for example, "every", "no", and such

¹⁷ Ch. 5, Sect. I.

like.’¹⁸ This passage sheds light on a further passage which occurs some lines later:

Some terms are categorematic, some are syncategorematic. A categorematic term is one which, by itself, and also with another term, has a proper significate, for example ‘man’. Whether it is placed in a sentence (*oratio*) or outside one it always signifies a man. A syncategorematic term is one with a function, which taken by itself is significative of nothing, for example, universal signs such as ‘every’, ‘no’, and the like; and particular signs, for example, ‘some’, ‘a certain’, and so on, prepositions, adverbs, and connectives.¹⁹

One point made by Paul of Venice is that the fact that a term is significative does not imply that it signifies something, though it is true of some terms that they are fitted by their nature to signify things. For example, the term ‘man’ signifies something, namely, a man. And even if, as a matter of fact, no man exists the term is still fitted by its nature to signify something. It is not, as it were, the term’s fault that there is no man for it to signify. A term with such a nature is called a categorematic term. A syncategorematic term is a significative term which is not categorematic. Paul instances ‘every’, ‘no’, and several others. We could add ‘and’, ‘or’, ‘if’, and numerous other terms of special interest to logicians. When Paul says that such terms have a function he has in mind the fact that they play a distinctive role in the context of a proposition, and that their signification is to be defined in terms of the distinctive role. For example, William of Sherwood says that the function of the word ‘every’ is to divide the subject in relation to the predicate, so that in the proposition ‘Every man is an animal’ the ‘every’ functions in such a way that the proposition implies that this man is an animal, and that man is an animal, and so on for all men.²⁰ Further syncategorematic terms are ‘and’, ‘or’, and ‘if’. It is evident that the class of syncategorematic terms contains almost all, if

¹⁸ *Logica*, 1-2.

¹⁹ *Logica*, 2.

²⁰ *Syncategoremata*, 48.

not all, the terms that we should think of as the special preserve of the logicians.

Some of the terms classed as syncategorematic and extensively investigated were those called *exponible* terms.²¹ We have already made brief reference to them. Such a term is, very roughly, one of interest to the logician and in need of exposition or clarification. The reason commonly given for the need was the 'obscurity' of the term. Not all logicians included 'every' in their list of *exponible* terms, though some did. But among terms which were generally included were 'begins' and 'ceases'. In one respect it is plain that these terms should be classed as syncategorematic, for they are auxiliary verbs *exponible* in terms of tensed copulas. Thus 'A begins to be B' means 'Immediately before now A was not B and now A is B, or now A is not B and immediately after now A will be B'. But the matter is not entirely plain sailing. David Cranston describes the terms 'begins' and 'ceases' as *categorematic* *exponibles*.²² What he appears to have in mind is that the terms 'begins' and 'ceases' are grammatically simple but logically complex,²³ and in particular that our conventional language conflates terms of logically quite distinct natures. For 'begins' and 'ceases' can be seen to contain a part of the verb 'to be', which in Cranston's view was, when taken copulatively, syncategorematic, plus a *categorematic* term. Thus 'begins' is subordinate to a mental expression which is rendered more perspicuously by 'is beginning' or 'is a beginner' (*incipiens* can equally well be rendered by 'beginning' and 'beginner'). And one might then say that 'begins' is *categorematic* in virtue of implicitly containing 'beginner' and is syncategorematic in virtue of implicitly containing the copula 'is'.

²¹ e.g., see William Ockham, *Summa Logicae*, Pt. II, Ch. 19; Walter Burley, *De Puritate Artis Logicae*, 191-7; and Albert of Saxony, *Perutilis Logica*, 22^v-23^f.

²² *Term.*, sig. b vii.

²³ He writes: 'The grammarian and the logician have different views on the complexity of a term. The grammarian attends to the complexity or incompleteness of an expression in respect of that which, in speech or writing, has several parts of different natures, whether or not it has them in the mind, but logicians take the compositeness (or complexity) or incompleteness from the concept' (*Term.*, sig. g ii).

This last example illustrates the way that medieval logicians set about their task of investigating the logical form implicit in conventional propositions. 'A man begins' appears to have two parts, a noun and a verb, and grammatically speaking it does. But logically it was taken to have three parts, a subject, copula, and predicate. And the principal reason why the form had to be made explicit was that the rules of valid inference were so formulated that they could be applied only to inferences whose premisses and conclusion had all been written in a form which was an adequate representation of the form of the mental propositions to which they were subordinate. Thus, given the rule that if every B is C and every A is B it follows that every A is C, we can argue that 'Every beginner is a substance and every man is a beginner, therefore every man is a substance' is a valid inference, but we cannot even apply the rule to 'Every beginner is a substance and every man begins, therefore every man is a substance', and certainly we cannot on the basis of the rule judge the inference valid, for the second premiss is of the wrong form.

3

Categorical Propositions

I. PROPOSITIONS: CATEGORICAL AND MOLECULAR

'Proposition' was commonly defined in terms of truth.¹ Paul of Venice, for example, following a long tradition, said that a proposition is 'indicative speech signifying something true or something false'.² For an item of speech to signify something true or something false that item must have an appropriate logical form. Such forms were extensively investigated, and were generally expounded in a recursive manner. That is, a given form was specified as minimally sufficient if the item of speech was to be able to signify something true or something false, and other forms were described in terms of operations carried out on items of speech which could signify something true or something false.

As regards the minimal form, we have already met that. It is the categorical proposition, composed of just three parts, subject, copula, and predicate, for example 'A man is reading'. Operations can be carried out on such a proposition transforming it into a more complex item of speech still able to bear a truth value. For example, it can be universalized by prefixing 'every' to the subject: 'Every man is reading'. It can also be negated by placing 'not' before the subject or before the predicate: 'Not [= it

¹ The classic investigations of this are by G. Nuchelmans in *Theories of the Proposition: Ancient and Medieval Conceptions of the Bearers of Truth and Falsity*, and *Late-Scholastic and Humanist Theories of the Proposition*; also E. J. Ashworth, 'Theories of the Proposition: Some Early Sixteenth Century Discussions', *Franciscan Studies* 38, 1978 [1981], reprinted in E. J. Ashworth, *Studies in Post-Medieval Semantics*.

² *Logica*, 4.

is not the case that] a man is reading' or 'A man is not reading'. The propositions resulting from such transformations are as categorical as the proposition thus transformed by the addition of the syncategorematic terms. Thus 'Not only every man is not reading' is categorical.

But there are other propositions, described as molecular, which do not have the basic logical form: subject + copula + predicate. A molecular proposition is any pair of propositions connected by a syncategorematic term taken from a precisely enumerated list of terms. But though many logicians gave a list, they did not all list the same terms. However, every list included at least 'and', 'or', and 'if'. Two propositions connected by 'and' form a conjunctive proposition, two connected by 'or' form a disjunctive proposition, and two connected by 'if' form a conditional proposition. Each proposition thus connected could be categorical, or one or both could be molecular. Hence a single molecular proposition could contain one or more occurrences of 'and', 'or', and 'if', all connecting propositions. Whether the resulting proposition is conjunctive, disjunctive, or conditional depends on what the principal connective is. If, say, the proposition is composed of a disjunction connected to a conditional by 'and', then the proposition is conjunctive. It was well recognized that where a molecular proposition contains several propositional connectives of distinct kinds it is the order of construction of the proposition that determines what kind of molecular proposition it is. Thus, given three propositions P, Q, and R, the molecular proposition 'P and Q or R' was said to be conjunctive if it was formed by placing 'and' between the two propositions 'P' and 'Q or R'. And it was said to be disjunctive if it was formed by placing 'or' between the two propositions 'P and Q' and 'R'.³

At the beginning of his treatise on molecular propositions Paul of Venice refers to the disparity in the length of the lists of propositional connectives drawn up by different

³ See Burley, *De Puritate*, 108.

logicians.⁴ Some, he reports, list five. He may have had in mind William Ockham who, in addition to the three so far mentioned, listed causal and temporal connectives.⁵ 'Since' is a sign of causality. Thus 'Since John does not wish to be ignorant John philosophizes' is given as an example of a causal proposition. 'When' and 'while' are temporal connectives, as in 'When John speaks, Peter listens' and in 'Peter listens when John speaks' (which are not, according to Paul of Venice, different ways of saying the same thing). Likewise 'While John speaks, Peter listens' and its converse are temporal propositions. Ockham adds, though not in his list, the connective of locality, that is, 'where', and Albert of Saxony's list of connectives consists of the five in Ockham's plus 'where'.⁶ Another source, reports Paul, lists seven connectives. Since he says that he himself is going to subsume the sign of a rational proposition, namely 'therefore', under the heading 'conditional',⁷ he no doubt had in mind the foregoing list of six connectives to which 'therefore' is added as an independent seventh. Other signs also were canvassed, for example, the sign of adjunction, namely, 'in order that'. And Paul discusses the question of whether the concessive sign 'though' should count as a propositional connective.⁸

Paul eventually accepts the view, accepted also by Peter of Spain,⁹ that there are just three kinds of molecular proposition: namely, conjunction, disjunction, and conditional. It might be said, for example, that temporal and local propositions are to be classed as conjunctions, on the grounds that, for example, 'A exists when B exists' and 'A exists where B exists' are equivalent to 'A exists and at the same time B exists' and 'A exists and in the same place B exists' respectively. But whether or not Paul's arguments for rejecting

⁴ *Logica Magna*, 124^{va}. See A. Broadie (1990), 2.

⁵ *Summa Logicae*, Pt. II, Ch. 30.

⁶ *Perutilis Logica*, 19^{ra}.

⁷ *Logica Magna*, 124^{va}. See A. Broadie (1990), 2.

⁸ *Logica Magna*, 127^{ra}. See A. Broadie (1990), 40 ff.

⁹ *Tractatus*, 9.

the claims that there are more than three kinds of molecular proposition are sound, I shall restrict myself to consideration of the three kinds sanctioned by Peter of Spain and Paul of Venice. Those three kinds received much the greatest coverage in medieval discussions of molecular propositions.

II. SUPPOSITION: PERSONAL, SIMPLE, MATERIAL

Supposition is the signification that a certain kind of term has in the context of a proposition. The definition that John Buridan gives is representative of an important school of thought on this matter:

Supposition is the taking of a term in a proposition for some thing or things such that when that thing or those things are pointed to by the pronoun 'this' or 'these' or their equivalent, then that term is truly affirmed of that pronoun with the copula of the proposition mediating.¹⁰

It is clear from this that only a categorematic term can have supposition, for no term can supposit that is not fitted to be a predicate. Let us say that a given categorical proposition contains a term *T*. That term supposits for something such that we can pick the thing out with the demonstrative pronoun 'this' and say truly 'This is *T*'. Wherein, then, lies the difference between supposition and the signification of a categorematic term, for the latter seems to answer to the description just given of supposition? An important part of the answer can be given by reference to the example of the term 'man'. According to Ockham's account, this term signifies everything such that we can point to it and say truly 'This is a man', and as regards that term as it occurs in the proposition 'A man is reading' its supposition is the same as its signification. In the context of that proposition the term 'man' stands for something such that we can point to the thing and say truly 'This is a man'.

¹⁰ *Sophismata*, 50.

The situation is, however, quite otherwise as regards the proposition 'Man is a species', for in the context of that proposition, according to Ockham, 'man' does not stand for what it stands for in 'A man is reading'. A species is not the kind of thing that can read even if the members of it are of that kind. That the term 'man' has different significations in the context of the two propositions is made plain by the fact that this argument 'Every man is seated and a man is reading, therefore a reader is seated' is valid, but this 'Every man is reading and man is a species, therefore a species is reading' is invalid. What has gone wrong in the second of these inferences is that the so-called fallacy of equivocation has been committed, for in the first premiss the term 'man' signifies an individual man and in the second it does not. There is considerable room for dispute regarding the manner of existence of a species as opposed to the members of the species, but in the view of Ockham a species is a concept (or mental term) under which we bring the members of the species. This is a version of the theory known as nominalism, as contrasted with realism which states that species are real things which exist apart from thought. Whichever formulation of the nominalist position we adopt, the outcome is that the term 'man', used to signify a species, does not signify what it customarily signifies, namely, individual men.

Ockham appropriated, though for a distinct purpose, terminology already in use. When a term *T* in the context of a proposition signifies what it customarily signifies so that *T* stands for or, so to say, personates the thing, then it is said to have *personal* supposition. If, in a given proposition, *T* does not have its customary supposition but instead signifies a mental term, in particular the mental term under which fall things which are, in the customary sense, *T*, then *T* is said to have *simple* supposition.

It should be said that Ockham's position on this matter was itself a rejection of views of earlier thinkers, and Ockham himself was duly challenged. As regards his rejection

of earlier views, one point concerns the question of whether proper names have personal supposition. Ockham, of course, thought that they did, but some of his predecessors did not. For example, Peter of Spain gave the following definition: 'Personal supposition is the acceptance of a common term for its inferiors. For example, when "A man is running" is said, the term "man" supposits for its inferiors'.¹¹ This is not to say that Peter denied that proper names can have supposition—on the contrary he says that they can.¹² His point is that their supposition is not personal.

As regards those who rejected Ockham's view let us for the moment attend to Walter Burley. Burley, almost certainly with Ockham in mind, says of 'some people':

They say that personal supposition is when a term supposits for its significate or for its significates; and simple supposition is when a term supposits for an intention of the mind or for intentions of the mind. Hence they say that in this: 'Man is a species', the term 'man' has simple supposition and does not supposit for its significate, since the significates of the term are this man and that man. But in this: 'Man is a species', the term 'man' supposits for an intention in the mind, which indeed is the species of Socrates and of Plato.¹³

But Burley adds:

Without doubt this is indeed an irrational thing to say, for in this: 'Man is a species', in so far as it is true, the term 'man' stands for its significate. Proof: it is certain according to Aristotle that 'man' is the name of a secondary substance;¹⁴ therefore the term 'man' signifies a secondary substance. And it does not signify a secondary substance which is a genus, therefore it signifies a species. Therefore, taking 'man' for that which it signifies, this will be true: 'Man is a species', for the noun 'man' is the name of a species and signifies a species.¹⁵

¹¹ *Tractatus*, 82.

¹² *Ibid.* 80.

¹³ *De Puritate*, 7.

¹⁴ *Categories*, 2 a 15 ff.

¹⁵ *De Puritate*, 7.

In support of his case he invokes Priscian's dictum that the noun 'man' is the name of a species.

A third kind of case of supposition was distinguished. In the proposition "'Man" is trilateral' the term 'man' is evidently not intended to stand for an individual man, for it is senseless to say that a man, say, Peter, is trilateral. And neither does the term stand for the species man, for the species is a mental term, and we have already observed that mental terms are not composed of letters and hence cannot be trilateral. Clearly 'man' is intended to stand for the very word 'man' itself, considered as an inscription. The three letters m, a, and n are the material out of which the inscription is composed. And since in "'Man" is trilateral' the subject stands for man in respect of the material out of which it is composed, the term is said to have material supposition in the context of that proposition.

It was sometimes said that a term with material supposition stands for itself in the context of the proposition, and sometimes it was added that it also stands for anything equiform with itself (*consimilis sibi*). The point is that the term 'man' in "'Man" is trilateral' does not stand only for the very occurrence of 'man' in the subject place of the inscription just inscribed. It stands for any occurrence of the inscription 'man', that is, for the occurrence of any term equiform to the term in the subject place of our sample proposition.

Burley has a subtle and interesting discussion of material supposition in connection with the question of the material supposition of syncategorematic terms.¹⁶ We have already identified 'every' as a syncategorematic term. Consider the proposition "'Every" is a syncategorematic term taken syncategorematically'. Is the term 'every' in that proposition being taken categorically or syncategorematically? If the latter then the proposition is false. For when the term is taken categorically (as it is here for something is being said about the term) it is not a syncategorematic term taken syncategorematically. If, on the other hand, it is being taken syncategorematically then the proposition is incoherent,

¹⁶ *De Puritate*, 5-6.

for in so far as it is a syncategorematic term taken syncategorematically it is not the kind of term which can occupy the subject place of a proposition. Burley's solution to this problem is this:

In so far as 'every' is taken materially and in the manner of a categorematic term, "Every" is a syncategorematic term taken syncategorematically' is true. But it supposits for itself taken syncategorematically, and hence is true, although the predicate does not inhere in that which supposits in so far as it supposits here. For it is sufficient for the truth of the affirmative proposition that the predicate inheres in that for which it supposits; and that is true because it is certain that 'every' in some proposition is a syncategorematic term taken syncategorematically.¹⁷

The bare bones of this answer are that if "Every" is a syncategorematic term' is so interpreted as to be true, then the first word in the proposition does not supposit for itself considered as in that proposition. Instead it supposits for equiform words elsewhere, for example, in the proposition 'Every man is running'. 'Every' is, after all, a syncategorematic term, but it cannot be being used as a syncategorematic term in a proposition which states that it is such a term. This doctrine is close to Carnap's account of the autonymous use of terms, but is possibly deeper.¹⁸

In the fourteenth century most logicians recognized the three kinds of supposition that I have just listed, personal, simple, and material, though some, including Buridan, reduced the list to two by conflating simple and material supposition.¹⁹ An obvious reason for such a conflation is that in each case what the term in question stands for is not what the term customarily signifies. Instead it stands for the term itself (or a term equiform to it), whether considered as a mental term or as an utterance or an inscription.

¹⁷ *De Puritate*, 6.

¹⁸ See 'Is it Right to Say Or is a Conjunction?', in P. T. Geach, *Logic Matters*, 204-5.

¹⁹ *Sophismata*, 51.

So far we have considered only the supposition of subjects of propositions. Whether predicates have supposition was a matter for dispute. This fact is clearly signalled by Ockham. He writes:

There is therefore a general rule that a term in a proposition, at any rate when it is taken significatively, only supposits for something of which it is truly predicated. It follows from this that it is false what some ignorant people say, namely that a concrete noun on the side of the predicate supposits for a form, for example, that in 'Socrates is white', 'white' supposits for whiteness. For 'Whiteness is white' is simply false howsoever the terms supposit. Hence according to Aristotle a concrete term of such a kind never supposits for the form which is thus signified by the corresponding abstract noun.²⁰

However, it should be noted that among the 'ignorant people' berated by Ockham are Thomas Aquinas, Peter of Spain, and Walter Burley. Thus, for example, Aquinas affirms: 'A term placed in the subject position is taken materially, that is, is taken for a subsisting subject; but placed in the predicate position it is taken formally, that is, for the nature signified.'²¹ Aquinas is using different terminology from that which we have so far been using, but it is plain that what he has in mind is that the predicate term in 'Socrates is white' has what Ockham would call simple supposition, and not what he would call personal supposition. This is not to say that for Aquinas 'white' stands for the form of whiteness as such since in Aquinas's view the form of whiteness as such does not exist. What does exist is the whiteness of whatever is white. Hence the form in question here is best represented not by 'whiteness' but by 'whiteness of—' as in 'whiteness of Tom', and hence the form which exists is, in Peter Geach's phrase an 'individualized form'; the whiteness is Tom's whiteness, which is as individual as Tom; or is Dick's whiteness, which is as individual as Dick, and so on.²²

²⁰ *Summa Logicae*, Pt. I, Ch. 63, p. 194.

²¹ *Summa Theologiae*, 3, 16, 7 ad 4.

²² See P. T. Geach, 'Form and Existence'.

One further view should here be noted, no less in conflict with Ockham's than is that of Aquinas. The logician in question is Vincent Ferrer, who saw himself, correctly, as a Thomist; and certainly his teaching on supposition is in many ways close to that of Aquinas. Ferrer rejects the claim that supposition is a possible property of predicates, and this rejection is implicit in his definition. He writes: 'Supposition is a property of a subject in relation to the predicate in a proposition. And supposition is said to be the property of a subject . . . For every subject supposits, and only a subject does, and always.'²³ For Ferrer, then, there can be no supposition except in relation to a predicate; and indeed the kind of supposition that the subject has is known principally and primarily through the predicate. He gives as his examples the three propositions: 'A man is an animal', 'Man is a species', and "'Homo" is a bisyllable'. Of course, that there can be no supposition without predication does not for a moment imply that a predicate can have supposition. It does however imply that supposition is essentially a syntactic property. And that is a first move towards fixing the distinction between supposition and signification. For, as Ferrer sees the matter, 'From this it follows that supposition is distinct from signification, for the latter does not belong to a subject in so far as it is related to a predicate. Rather it belongs to the term considered in itself'.²⁴ As already indicated this position is far removed from Ockham's. Supposition for him is the signification that a term has in the context of a proposition. It is the 'taking the place [*positio*] of another thing'²⁵ in the context of a proposition. And in Ockham's view this is a role which is played by predicates no less than by subjects.²⁶ But whereas a subject can have personal, simple, or material supposition, a predicate has just personal supposition. For example, in

²³ *Tractatus de Suppositionibus*, 93

²⁴ *Ibid.*

²⁵ *Summa Logicae*, Pt. I, Ch. 63, p. 193.

²⁶ *Ibid.*

“‘Man’ is trilateral’ the predicate stands in the context of that proposition for what it customarily signifies, namely, something which is trilateral. Assuming that the proposition is true, then which trilateral thing the predicate stands for in that context is given by the subject. And in ‘Man is a species’ the predicate stands for what it customarily signifies, namely, a species. Assuming that the proposition is true, then which species it is for which the predicate stands in the context of that proposition is given by the subject.

As to the problem of determining the kind of supposition possessed by the subject the solution is to consider the predicate with which the subject is coupled. Ockham²⁷ lays down the general rule that whatever the proposition in which a given term is placed, that term can have personal supposition unless those who use the term restrict it to a different sort of supposition. But a term cannot have simple or material supposition in every proposition, but only in a proposition where the term is linked to an extreme (presumably the predicate extreme rather than the subject) which refers to a mental term or to an utterance or inscription. For example, in ‘A man is running’ the subject term must have personal supposition since ‘running’ cannot refer to a mental term or to an utterance or inscription. And since ‘species’ signifies a mental term, ‘man’ can have simple supposition in ‘Man is a species’. If it does have simple supposition then the proposition is true. If it has personal supposition the proposition is false.

For the remainder of this chapter our attention will be focused on personal supposition. This was the kind of supposition in which medieval logicians were especially interested, and their writings on that topic are particularly rich in logical insights.

²⁷ *Summa Logicae*, Pt. I, Ch. 65, p. 197

III. PERSONAL SUPPOSITION

Categorematic terms were considered under two headings: some are discrete, or singular, and some are common. The distinction is of great importance for the development of the theory of supposition. Peter of Spain gives this definition: 'A singular term is one which is fitted by its nature to be predicated of only one thing.'²⁸ 'Tom', for example, is a singular, or discrete term. That a given term signifies only one thing is not by itself proof that the term is discrete. The term 'sun' was not considered discrete, for though as a matter of fact there was (as it was thought) only one sun, that fact was taken to be one we could discover only by looking at the world; it was not thought to be a fact deducible from a consideration of the mode of signification of the term. In dealing with the question whether 'sun' is a discrete or singular term, therefore, the question is not: Is there one and only one sun? but rather: Is the term 'sun' fitted by its nature to signify only one thing? Proper names are not the only kind of discrete term. Though a given common term is not discrete, that term prefixed by a singular demonstrative term is discrete. Thus 'man' is not a discrete term but 'this man' is.

Peter of Spain affirms, 'A common term is one which is fitted by its nature to be predicated of many things; for example, 'man' is naturally fitted to be predicated of Socrates, of Plato, and of every other man'.²⁹ As with the preceding definition of 'singular term', the phrase 'fitted by its nature [*aptus natus*]' is important here also; for even if there is only one man the term 'man' is common since that there is only one man cannot be learned from a consideration of the mode of signification of the term. Indeed, a term may be common though there is in reality nothing of which it can be predicated. The stock medieval example was 'chimera'. The chimera is a mythical beast whose existence was thought a physical impossibility. Nevertheless 'chimera'

²⁸ *Tractatus*, 5.²⁹ *Ibid.* 4.

was classed as a common term, for it was fitted by its nature to be predicated of many things. That fact about its mode of signification is the only relevant fact in establishing whether the term is common.

Supposition was considered under two heads, discrete and common. Ockham gives the following definition:

Discrete supposition is the supposition in which a proper name taken significatively, or a demonstrative pronoun taken significatively, supposits; and this supposition renders a proposition singular, as here: 'Socrates is a man', 'This man is a man', and so on.³⁰

Why 'taken significatively'? In 'Tom is reading' the subject has discrete supposition, since 'Tom' signifies what it was imposed to signify, namely Tom. In "'Tom" is a proper name' the subject does not have discrete supposition, since here 'Tom' is not being taken significatively, that is, it is not being taken to signify what it was imposed to signify, which is what it customarily signifies. Though 'Tom' taken in isolation is fitted by its nature to stand for one and only one thing, nevertheless in the context of "'Tom" is a proper name' the term has material supposition. It stands for the name 'Tom' and there can be many occurrences of that name. In 'This man is reading', 'man' has discrete supposition because in the context of that proposition it stands for one and only one man, namely, this one. Thus whether the supposition of a term is discrete does not depend simply on whether the term is in any case a discrete term, it depends on whether it is discrete in the context of the proposition.

As regards common supposition, Ockham gives this definition: 'Common personal supposition is when a common term supposits, as here: "A man runs", "Every man is an animal".'³¹ And it was to this latter type of supposition rather than to the discrete type that medieval logicians paid special attention. It was not in the least that they considered discrete supposition to be comparatively unimportant,

³⁰ *Summa Logicae*, Pt. I, Ch. 70, p. 209.

³¹ *Ibid.* 210.

but rather that there was simply much more to be said about common supposition. Let us, then, turn to the notion of common supposition in order to see what the notion yields up about the logical form of propositions.

Two kinds of common supposition were identified, determinate and confused. We shall attend first to determinate. Consider the proposition:

- (1) Some man is a logician.

It was standardly held that an affirmative categorical proposition is false unless the subject and predicate stand for something, not in the sense that they are fitted by their nature to stand for something, but in the sense that there actually is something such that the subject can be truly predicated of a demonstrative pronoun indicating that thing, and there actually is something such that the predicate can be truly predicated of a demonstrative pronoun indicating that thing. Thus we should have to say that (1) is false unless a man exists, for if no man exists there is no man to be a logician in which case it is not true that some man is a logician. Likewise if no logician exists there is no logician for some man to be, in which case it cannot be true that some man is a logician. This position concerning the existential import of affirmative categorical propositions seems sound. But as regards the truth conditions of (1) it has to be added that there should be some particular, *man*ⁿ, such that the predicate of (1) can be truly predicated of '*man*ⁿ'. Hence, given (1) this follows:

- (2) *Man*¹ is a logician or *man*² is a logician, and so on for every man.

So (1) implies a disjunction of singular propositions, each disjunct of which is like (1) except that the quantifier is deleted and the common term in the subject place of (1) is replaced by a singular term of which the common term is truly predicated. Medieval logicians spoke of this relation between (1) and (2) as a descent under the subject to a

disjunction of singular propositions. Descent was classed as a form of valid inference. It can easily be shown that descent can also be made under the predicate of (1) to a disjunction of singular propositions. Additionally (1) follows from any one of the disjuncts in (2). That is, ascent can be made from any one of the singulars to the original proposition. A term under which such descent and such ascent can be made was said to have determinate supposition.

We turn now to confused supposition. Ockham tells us that confused personal supposition is every personal supposition of a common term which is not a determinate supposition.³² There are two kinds, confused distributive and merely confused. We deal with the first kind first. If the following proposition:

- (3) Every man is mortal

is true then this proposition:

- (4) Man¹ is mortal and man² is mortal, and so on for every man

is also true. Here descent is made under the subject of (3) to a conjunction of singular propositions. But ascent cannot validly be made from any one of the conjuncts to the original proposition. Where descent can be made under a given term to a conjunction of singular propositions but ascent cannot be made from any of the conjuncts then the term is said to have confused distributive supposition.

Finally we turn to merely confused supposition. Let us stay with (3). (3) implies the following:

- (5) Every man is mortal¹ or mortal², and so on for every mortal.

That is, descent can be made under the predicate term of (3) to a proposition like (3) except that the predicate of (3) is replaced by a disjunction of singular terms of each of which

³² Ibid. 211.

the predicate in (3) can truly be predicated. And additionally if it were true that every man was mortalⁿ, as would be the case if there were only one man and he was mortal, then (3) would be true. Here ascent is made from one of the singular terms under the original term. Where such descent and such ascent can be made under a given term then that term is said to have merely confused supposition.

Numerous rules were given for determining the kind of personal supposition that terms possessed. We shall briefly rehearse certain of the more important of these rules. They were hedged about with qualifications, but I shall pay little attention to those. One rule is that a term covered by no syncategorematic term has determinate supposition, as also has a term immediately covered by 'some' and not also covered by a sign of negation. Secondly, a term covered immediately by a sign of universality, for example, by 'all' or 'every', has distributive supposition, and one covered mediate by a sign of affirmative universality has merely confused supposition. A term is mediate covered by a given sign if the term comes at the predicate end of a proposition whose subject is immediately covered by the sign. Thirdly, a term covered, whether immediately or mediate, by a sign of negation has confused distributive supposition (hereinafter just 'distributive supposition'). Thus in the universal negative proposition 'No man is immortal', both the subject and the predicate have distributive supposition, and in the particular negative proposition 'Some man is not a logician', the predicate has distributive supposition and the subject has determinate supposition.

I should like now to turn to rules of descent and ascent which were well studied in the early sixteenth century, and were a common feature of logic treatises of that period. The rules in question concern the order in which descent should be made under subject and predicate. To see what is at stake here we can note briefly the following example. In:

- (6) Some man is not a logician

the subject has determinate supposition and the predicate distributive supposition. If we descend first under the predicate and assume there to be not more than two logicians, we reach the proposition:

- (7) Some man is not logician¹ and some man is not logician².

It is clear that (7) is consistent with every man being a logician. Indeed, so long as there are at least two men, (7) must be true because it cannot be that each man is every logician. Hence (6) does not imply (7), and yet it should do if the descent is correctly made under the predicate term. The error consisted in descending under the term with distributive supposition before descending under the term with determinate supposition. Assuming there to be not more than two men, what (6) implies is this:

- (8) Man¹ is not a logician or man² is not a logician.

We can now descend under the distributed predicate to reach the following:

- (9) Man¹ is not logician¹ and man¹ is not logician², or man² is not logician¹ and man² is not logician².

Stated briefly the rules said to govern the order of descent are as follows: determinate supposition has priority over distributive and merely confused supposition, and distributive supposition has priority over merely confused.

Given the syntactic rules presented earlier for determining the kind of supposition possessed by a given term, it follows that changing the position of a term in a proposition can have an effect on the truth value of that proposition. In:

- (10) Every teacher has a pupil

'pupil' has merely confused supposition, and consequently the proposition says that this teacher has some pupil or

other and that teacher has some pupil or other, and so on for every teacher. But in:

(11) A pupil every teacher has

‘pupil’ has determinate supposition, and since ‘teacher’ has distributive supposition descent must be made first under ‘pupil’ and then under ‘teacher’. Assuming there to be just two teachers and two pupils, the first stage of descent takes us to:

(12) Pupil¹ every teacher has or pupil² every teacher has.

The next stage takes us to:

(13) Pupil¹ teacher¹ has and pupil¹ teacher² has, or pupil² teacher¹ has and pupil² teacher² has.

(13) implies that some one pupil is shared by all the teachers, and that is plainly not implied by (10), though it does imply (10).

Concern for order of terms in a proposition emerges frequently in medieval logic textbooks, and the chief point at issue is the effect that a change in order might have on the kind of supposition a term has. Thus Albert of Saxony quotes the rule: ‘A term which includes a negation in itself gives distributive supposition to the following term’,³³ and instances the term ‘differs’, presumably because it has the same signification as ‘is not the same’. Hence in:

(14) Socrates differs from a man

‘man’ is distributed, and consequently (14) signifies that Socrates differs from every man, i.e. that there is no man that Socrates is. Albert contrasts this with:

(15) Socrates from a man differs

for in (15) ‘man’ has determinate supposition, and hence (15) implies that it is from man¹ that Socrates differs or from

³³ *Perutilis Logica*, 13^{ra}.

man², and so on for all men, from which it follows that (15) is true so long as Socrates is not the only man. It is in virtue of the different kinds of supposition possessed by 'man' in (14) and (15) that the former proposition is false and the latter true. On the basis of the same kind of considerations it can be shown (to adapt one of Albert's examples) that:

(16) Socrates differs from Plato

implies (15) but not (14) even though Plato is a man, for in (14) 'man' signifies every man and in (15) it signifies this man or that man, etc.

Likewise it was commonly held that terms expressing comparison have a similar effect to negation signs.³⁴ Let us say that in a proposition of the form 'A is Xer than B', 'A' is the excedent and 'B' the excessum. Then it was held that the term expressing comparison gives distributive supposition to the following excessum. Thus in:

(17) Tom is stronger than a man

'man' has distributive supposition, and hence, assuming that Tom is a man, (17) is false since it implies that Tom is stronger than every man—including himself. But the rule applies to the following excessum, not to a preceeding one. If, therefore, we wish to say that Tom is stronger than a man, i.e. than at least one man, without wishing to imply that he is stronger than every man, the way to do this is to transfer the excessum to a position before the comparative term, for example:

(18) Tom than a man is stronger.

We have considered cases of terms giving distributive supposition to a succeeding term. I should like now to consider a different sort of case. There was great interest in verbs which give, or appear to give, merely confused distribution to the following common term, and there was a

³⁴ Ibid. 13^{rb}.

rather wide divergence in the line taken on such verbs. I shall begin with the discussion by William Ockham. He raises a question about the supposition of the subject term in such propositions as 'A horse to you is promised', 'Twenty pounds to you are due'.³⁵ In the case of 'I promise you a horse' the term 'promise' is taken by Ockham to give merely confused supposition to the immediately following common term,³⁶ and hence though it is not possible to descend under 'horse' to a disjunction of singular propositions (as one could if the term had determinate supposition), it is possible to descend under that term to a disjoint predicate of the form 'horse¹ or horse² . . .', and so on for every horse. Thus, 'I promise you a horse' implies 'I promise you this horse or that horse, and so on for all horses'. Here, Ockham claims, the list of horses enumerated must include not only present horses but also future ones. The reason is that verbs of the kind here considered include in effect future-tensed verbs. Thus 'I promise you a horse' is equivalent to this: 'You will have a horse from me as a gift', and hence in 'I promise you a horse' 'horse' can supposit for future horses, as it does in 'You will have a horse'.

Of course, strictly speaking it is not correct to say that in 'I promise you a horse' 'horse' has merely confused supposition, for since 'horse' is not a complete extreme of the proposition, but is only a part of the predicate, it does not have supposition at all. Ockham acknowledges this fact, but adds:

Nevertheless by extending the term it can be said that 'horse' has merely confused supposition, and this is because it follows that kind of verb. And hence it is universally the case that a common term which follows such a verb always has personal, merely confused supposition, and not determinate, even though it is only a part of an extreme.³⁷

Ockham adds a twist to this account. Sometimes a proposition of the kind here considered, where the distinctive

³⁵ *Summa Logicae*, Pt. I, Ch. 72, pp. 214-15.

³⁶ *Ibid.* 219.

³⁷ *Ibid.* 220.

kind of verb is past-, or present-, or future-tensed, and where a common term is part of the predicate, implies that a given proposition will be true or ought to be true. For example, in virtue of the term 'promise', 'I promise you a horse' denotes that 'I am giving you a horse' will be true, or ought to be true; and it does not denote that 'I am giving you *this* horse' (pointing to some horse or other), will or ought to be true.

The horse promised to someone was indeed a popular inhabitant of medieval logic textbooks. In his discussion of this horse, Walter Burley begins with a puzzle: if I promise you a horse then surely this is true: 'A horse is promised to you'. But does 'horse' have personal, material, or simple supposition?³⁸ If the proposition is true and the term has personal supposition, then there must be some horse which has been promised to you. But that is clearly false. There is also no ground whatever for thinking that the term has material supposition. Has it then simple supposition? Burley replies:

Taking 'horse' for the significate [of the term], whether it signifies a common thing or a concept in the mind 'A horse is promised to you' is always false, because neither a concept in the mind nor a common thing is promised to you. Therefore a term with simple supposition does not supposit for its significate, and yet the contrary of that has already been stated.³⁹

But Burley does not settle for this. He goes on:

Granted that someone promises you a horse, and holding on to the fact that outside the mind there is a unity other than numerical unity, one would have to say that this is true: 'A horse is promised to you', in so far as the subject has absolute simple supposition; for I do not promise you this horse, or that one, but simply a horse. And since a universal cannot exist by itself and hence cannot be given up except in a singular thing, it follows that someone who promises you a horse is bound to give you a singular horse. Otherwise it is not possible to give you what has been promised.⁴⁰

³⁸ *De Puritate*, 13.

³⁹ *Ibid.*

⁴⁰ *De Puritate*, 14.

In Burley's view, those who say that only singulars exist outside the mind have to say that 'A horse is promised to you' is true, where the subject term has personal supposition. And from that it follows that whoever promises you a horse by saying 'I will give you a horse', promises you, though disjunctively, every actual and possible horse, for whatever horse he gives you he fulfils the promise.

Let us turn now to Albert of Saxony's exposition. He makes the familiar assertion that some terms have the power to give merely confused supposition to terms following them, and gives as examples such terms as the verbs 'seek', 'desire', 'promise', and 'owe'.⁴¹ Hence in :

(19) I promise you a penny

'penny' has merely confused supposition, as it has in:

(20) I owe you a penny

and as has 'horse' in:

(21) For riding there is required a horse.

Let us stay with (19) and consider the effect of placing 'penny' before the verb, as in:

(22) A penny I promise you.

Here 'penny' has determinate supposition. That is, (22) implies:

(23) This penny I promise you or that penny I promise you

(assuming, for the sake of simplicity, that there are only two pennies in existence). That is, there is some particular penny that I promise you. Yet that is not at all what (19) says, for (19) says that I promise you some penny or other, though no penny in particular, and hence I discharge my debt to you by giving you a penny, no matter which of the available pennies it is. If in fact the truth is sufficiently expressed by (19), then

⁴¹ *Perutilis Logica*, 13^{vb}.

if I give you a penny in order to discharge my debt, you cannot say truly that I have not discharged my debt for I have given you the wrong penny. Nothing counts as the wrong penny, for any penny will do as well as any other. As Albert of Saxony puts the point, (19) is consistent with:

(24) No penny I promise you.

But this matter is not entirely plain sailing. Albert argues as follows: Let us suppose that Tom promises to pay Dick a penny and that Tom then gives him a penny. We can fairly ask whether Tom gave to Dick what he had promised him. If the answer is affirmative then since it was penny A that Tom gave him it must have been penny A that Tom promised him, and therefore not only did Tom promise Dick a penny but also there was a penny that Tom promised Dick. If, however, the answer is negative, then Tom still owes Dick what he promised him, and he might give Dick a hundred or a thousand pennies and still not fulfil the promise. It seems, then, that whenever A promises B an X it is some X in particular that A promises B.

However, Albert does not accept this. He takes the view that in promising Dick a penny Tom does not promise him some penny in particular. Though of course the penny he gave was a particular penny, it was not that penny in particular that was promised. That penny in particular is used to discharge the debt, but it is not in virtue of being that penny in particular that it can discharge the debt, but in virtue of its being any penny. It is only if Tom gives neither penny A nor penny B, and so on for all pennies, that the debt would not be discharged. 'And so', concludes Albert, 'it is possible that someone owes another person something and there is nothing that he owes him.'⁴²

Albert adds the following parallel for good measure:

Likewise it follows that I need an eye for seeing, and yet there is not some eye that I need for seeing. For I do not need the right

⁴² Ibid. 14^{ra}.

eye since I can see with the left, nor the left eye since I can see with the right. But that I need an eye to see with is true, since if I had only one eye then there is an eye I would need for seeing.

The foregoing discussion about verbs such as promising should be read in the light of the distinction made by William of Sherwood to which reference was made near the start of Chapter 1, namely, that between rhetoric and logic. Latin is highly flexible as regards word order, very much more so than English, and a Roman orator or poet could pick one word order rather than another on the basis of likely psychological impact. But the medieval logicians were forging a scientific Latin, a Latin much freer from ambiguity than ordinary everyday Latin was. They had no interest in elegance, only in truth and in whatever serves the truth. And truth is well served by a language in which the truth can be unambiguously expressed. One solution that the logicians adopted in dealing with the problem of ambiguity was to fix on certain word orderings as having a specific logical significance. In doing this they were not reporting established linguistic practice. They were laying down rules which, if followed by philosophers and theologians, would contribute to a greater mutual understanding, and hence would provide conditions in which the truth was more likely to emerge from the dialectical process.

A final point is in order here. The preoccupation with the ordering of terms in a proposition was no greater among medieval logicians than it is among their modern successors. In general, whether an existential quantifier precedes or follows a universal quantifier makes a difference to the sense of the propositions containing those quantifiers. In an idiom appropriate to their epoch medieval logicians made a very similar point, made it with great clarity, and pursued its implications with rigour.

IV. NEGATION

While on the topic of categorical propositions I should like to make certain points about negation. It is appropriate to

consider the matter here since the negation sign functions, among other things, as an operator forming a categorical proposition out of a categorical proposition. But a distinction has to be made here, for the two principal kinds of linguistic unit with which we have so far been concerned are the term and the proposition, and medieval logicians recognized that the negation sign can serve as an operator covering each of these kinds of sign. Let us consider briefly the concept of term negation.

Such negation was termed 'infinetizing negation'. The 'not' or 'non' infinetizes the term which it covers. We shall use 'non' as our term negating sign. Thus, given the categorematic term *T*, another term 'non-*T*' can be formed. 'Non-*T*' is also categorematic, and stands for everything for which *T* does not stand and for nothing for which *T* does stand. The number of non-chairs in a given room is the number of things in the room minus the number of chairs in it. Prefixing such a negation to a singular term results in a common term; 'non-Tom' is common since it is fitted by its nature to stand for many things—namely for everything which is not Tom.

Whether that negation added to a common term results in a common or a singular term depends on what the negation is added to. Added to 'chair' the result is another common term. But there is a law of double negation for terms. What it states is that given any term *T*, the negated negation of *T* signifies precisely what *T* signifies. Consequently the negated negation of a singular term must signify precisely what the singular term signifies. 'Non-non-Tom' is fitted by its nature to signify just Tom. Hence a doubly negated singular term is itself a singular term. But what the first negation in the string negates is a common term. It must be concluded therefore that an infinetized common term is not necessarily a common term. Whether it is common or not depends on whether the categorematic term to which the string of negation signs is prefixed is common or not. If it is common its double negation is common, and if singular its double

negation is singular. To appreciate the significance of certain rules of valid inference, it is important to note that the presence of a negation sign in a proposition is not sufficient to justify classifying the proposition as negative. For the presence of an infinitizing negation does not result in a negative proposition. Hence the rule of syllogistic inference, that at least one premiss must be affirmative, is not violated by a syllogism merely in virtue of its having a negated term in each premiss.

Nevertheless it was commonly held that a proposition containing a negated term is equivalent to a proposition composed of two or more categorical propositions none of which contains a negative term. For example, Ockham asserts that 'A donkey is a non-man' is equivalent to 'A donkey is something and a donkey is not a man'.⁴³ The reason why 'A donkey is a non-man' is not said to be equivalent to 'A donkey is not a man' is that an affirmative categorical proposition is not true unless both the subject and the predicate stand for something, and hence 'A donkey is a non-man' implies the existence of a donkey and of a non-man. But it was held that a negative categorical proposition is true if either the subject or the predicate does not stand for anything. Hence 'A donkey is not a man' is true if there are no donkeys. In the light of these considerations it is not surprising to find Ockham denying that 'A chimera is a non-man' is equivalent to 'A chimera is not a man'.⁴⁴ For him the former proposition is false and the latter true. Indeed, it has to be concluded, as Ockham notes, that a chimera is no more a non-man than it is a man. Put otherwise, 'A chimera is a man or a chimera is a non-man' is false (necessarily false given the view that it is impossible for there to be any chimeras), but 'A chimera is a man or a chimera is not a man' is necessarily true.

Medieval logicians developed a number of sophisms based on the ambiguity that arises when it is possible that a given negation negates either the proposition or the term. Thus

⁴³ *Summa Logicae*, Pt. II, Ch. 12, p. 283.

⁴⁴ *Ibid.* 284.

'Non terminus est terminus' can mean either 'It is not the case that a term is a term' or "'Non-term" is a term' or 'A non-term is a term'. We shall not delay over these puzzles, whose flavour it is in any case difficult to recreate in English, but shall focus on an offshoot of them.

When the negation sign occurs at the start of a proposition and is followed immediately by a common noun it is often difficult to determine, at any rate in the case of a Latin proposition, which of the two roles, outlined above, the negation sign has in that context. But it should be noted that a negation sign does not infinitize a term unless it is prefixed immediately to that term, whereas a proposition can be negated by a negation sign which does not occur at the start of a proposition, but occurs in the middle of it, or even at the end. All of the following three propositions are negative:

- (a) Not every man is a logician
- (b) Every man is not a logician
- (c) Every man a logician is not.

In accordance with medieval practice, (a) should be classified not as a universal proposition but as a particular—the result of negating a universal affirmative proposition was said to be a proposition neither universal nor affirmative. But (b) and (c) are both universal. More important, however, is the fact that the three propositions all differ in respect of the conjunction or disjunction of singular propositions that they imply and that are reached by application of the rules of descent. In (a) the subject has determinate and the predicate distributive supposition; in (b) both extremes have distributive supposition; and in (c) the negation sign has no effect on the supposition of either extreme since it follows both of them. The subject has distributive supposition since it is covered immediately by the universal quantifier, and the predicate, which is not covered immediately by the 'every' and is not covered at all by the negation sign, has determinate supposition. Assuming there to be just two men and two

logicians, and using A and B as abbreviations for 'man' and 'logician', and the cancelled identity sign ' \neq ' for 'is not', application of the rules of descent brings us to the following three propositions respectively:

- (a1) $(A^1 \neq B^1 \text{ and } A^1 \neq B^2) \text{ or } (A^2 \neq B^1 \text{ and } A^2 \neq B^2)$
- (b1) $(A^1 \neq B^1 \text{ and } A^1 \neq B^2) \text{ and } (A^2 \neq B^1 \text{ and } A^2 \neq B^2)$
- (c1) $(A^1 \neq B^1 \text{ and } A^2 \neq B^1) \text{ or } (A^1 \neq B^2 \text{ and } A^2 \neq B^2)$

Put otherwise, (a) implies that there is something for which 'man' stands for which 'logician' does not stand. (b) implies that there is no one thing for which 'man' and 'logician' both stand. (c) implies that there is something for which 'logician' stands for which 'man' does not stand, i.e. not every logician is a man.

On the basis of the principles so far outlined it is easy to work out the implications of the foregoing three sample propositions if 'every' is replaced by 'some'. Since no new issue of principle is involved I shall not give the details here, but shall instead turn to an examination of the implications of propositions where new issues of principle do arise.

V. PAST, PRESENT, FUTURE

We have so far considered a highly restricted range of categorical propositions and I should like now to extend that range considerably. For, first, we have considered only the analysis of present-tensed propositions and there is good reason to suppose that additional considerations have to be brought into play in dealing with past- and future-tensed propositions. And secondly, we have restricted our attention to so-called 'propositions of inherence' (*propositiones de inesse*). A proposition of inherence is a non-modal proposition. A modal proposition is one expressing possibility, necessity, impossibility, or contingency. Some logicians would add to that list, but I shall for the present deal only with the foregoing modalities. We should therefore ask what the truth conditions are for modal propositions.

Let us deal first with propositions about the past and the future. It should be said at once that logical discussions about tensed propositions occurred against a background of lively, indeed intense, debate concerning a number of physical, philosophical, and theological issues relating to time, all of them traceable at least partly to Aristotle. Here I shall mention two major sources.

First there was the extensive discussion on time in the *Physics*,⁴⁵ where Aristotle attempts a definition, focusing especially upon time considered as a measure of motion. This definition itself gives rise to problems concerning the fact that if time is indeed a measure, then this perhaps implies that where there is no measurer there is no time, and hence in a world where there were no minds to take notice of motion, there would be no time either. A closely related question concerns the concept of a now. If time is a succession of nows, then it is indeed difficult to see how a world containing no knowers could be in time. For any now is someone's now.

This is not the place to enter into discussion of the meta-physical questions that lie behind these points. I merely observe here that medieval thinkers were fascinated by these issues and some wrote extensive commentaries on the part of Book IV of the *Physics* containing Aristotle's discussion of time. Among the commentators are Roger Bacon (d. 1248), Albert the Great (c.1200–1280), Aquinas, William Ockham, John Buridan, and Albert of Saxony. And there were also works dedicated to the subject of time, of which the *De Tempore* of Robert Kilwardby, which seeks a balance between Aristotle and St Augustine, is an important example.⁴⁶ Two further major sources of inspiration to the medieval thinkers are Aristotle's *De Interpretatione*, Ch IX, where Aristotle raises questions relating to the truth values of propositions about future, supposedly contingent, events, and his *Metaphysics* VI 3.⁴⁷

⁴⁵ Bk. IV, 217 b 29–224 a 16. For trans. and helpful commentary see E. Hussey, *Aristotle's Physics*, Books III and IV.

⁴⁶ See Robert Kilwardby, *On Time and Imagination*, ed. P. Osmund Lewry (Oxford, 1987); and trans. A. Broadie (Oxford, 1993).

⁴⁷ See e.g. Aquinas, *In Perihermeneias*, Bk. I, lectiones 13–15.

Time was also of interest to theologians because they were, naturally, interested in such questions as the relation between eternity and time, and especially between the timeless acts of God and the temporal events in the natural order, and between God's foreknowledge of the future and our supposedly free future acts.⁴⁸ And finally mention should be made of the extensive medieval discussions on physics relating to velocity, acceleration and deceleration, instantaneous change, and the whole idea that motion, like time, is continuous (if time is). There can, then, be no doubt that the concept of time exerted a very strong pull indeed on medieval thinkers, and some who played an important role in the development of medieval logic contributed importantly to the literature on time. I have in mind such thinkers as Thomas Bradwardine (c.1295–1349), Richard Kilvington (d. 1361), Roger Swineshead (d. c.1365), and William Heytesbury (d. 1372/3).⁴⁹

I shall now say something about questions concerning the analysis of non-present-tensed propositions; in Chapter 8 I shall raise certain issues about the role of such propositions in syllogistic inference. Given that 'A man is a logician' implies that there is something for which 'man' and 'logician' both stand, it might seem that 'A white thing was black' implies that there is something for which 'white' and 'black' both stood, or perhaps that there *was* something for which 'white' and 'black' stood. But neither of these suggestions is plausible, and neither was canvassed by medieval logicians. 'A white thing was black' might be true because what is now for the first time white had up to this moment been black and not because there is something for which 'white' and 'black' stood, for by our hypothesis 'white' did not stand for what 'black' stood for, though it now stands for what 'black' stood for. By

⁴⁸ The central medieval discussion is Aquinas, *Summa Theologiae*, 1, 14, 13. For modern discussion see e.g. A. Kenny, *The God of the Philosophers*, ch. 4.

⁴⁹ See M. Clagett, *Science and Mechanics in the Middle Ages*, pt. II; N. Kretzmann (ed.) *Infinity and Continuity in Ancient and Medieval Thought*; also *CHLMP*, chs. 25, 26; also N. and B. E. Kretzmann, (Oxford, 1990, and Cambridge, 1990).

the same token the claim that it is true is not justified on the ground that there *was* something for which 'white' and 'black' stood. Of course, 'A white thing was black' might be true though there is now nothing that is white. It would be sufficient for the truth of the proposition that there was something for which 'white' stood and 'black' stood. It is evident from these points that in the context of the proposition, 'white' cannot be taken to signify simply something which is white or simply something which was white. The tense of the copula causes an extension—medieval logicians called it an 'ampliation'—of the signification of the subject term to that which is or that which was. So Albert of Saxony, following a common line, asserts: 'This proposition "A white thing was black" signifies that that which is white or that which was white was black.'⁵⁰

There is not, however, a corresponding extension or ampliation of the predicate. Its relation to the past-tensed copula ensures that the predicate signifies what was black. The implication of this is that the truth conditions of our sample proposition can be given as follows: 'This is white' is or was true, and 'This is black' (said indicating the same thing as that indicated by the demonstrative pronoun in 'This is white') was true. There is ample evidence that this represents the kind of approach taken by many medieval logicians to the question of the truth conditions of past-tensed propositions. For example, Ockham raises the question of the truth conditions of 'A white thing was Socrates', given that 'white thing' supposits for what is white and he answers:

It is not necessary that this was at some time true: 'A white thing is Socrates', but it is necessary that this was true: 'This is Socrates', said while indicating that for which the subject supposits in 'A white thing was Socrates'.⁵¹

⁵⁰ *Perutilis Logica*, 15^{rb}.

⁵¹ *Summa Logicae*, Pt. II, Ch. 9, p. 270.

The reason why it is incorrect to say that the truth condition of 'A white thing was black' is given by 'This was true: "A white thing is black"' is that the latter proposition implies that something was white and black at the same time, whereas the sample proposition does not have that implication. But a distinction should be made here between a past-tensed categorical proposition whose subject is a common term and one whose subject is a proper name. For in contrast with the previous sample proposition, Walter Burley states that the proposition 'Socrates was white' has a truth condition which can be stated simply as this: "'Socrates is white" was true'.⁵² For whereas if a white thing was black, it could not have been white while it was black, if Socrates was white then he must have been Socrates while he was white. He need not have been *called* 'Socrates', but as John Mair points out: 'Socrates is not Socrates because he is called "Socrates". He was in fact Socrates before that name was imposed to signify that thing.'⁵³

It does not follow from this that in order to state the truth conditions of a past-tensed proposition whose subject is a proper name it is sufficient to replace the past-tensed copula by the corresponding present-tensed copula and then say of the duly transformed proposition that it was true. One obstacle to such a manoeuvre is that the original past-tensed proposition might contain a time specification besides the pastness of the copula. For example, as regards the proposition 'Tom was busy yesterday' it is clearly unacceptable to give its truth conditions as: "'Tom is busy yesterday" was true'. The obvious tactic in dealing with such a proposition is to remove the time specification from the original proposition and place it in the form of words used to make an ascription to the duly rewritten past-tensed proposition. In accordance with that prescription the truth condition of 'Tom was busy yesterday' is given by "'Tom is busy" was true yesterday'. In all this there is clear evidence of the adoption of a recursive procedure in giving the

⁵² *De Puritate*, 48.

⁵³ *Term.*, 12^{ra}.

truth conditions of propositions. Present-tensed non-modal propositions were dealt with first. And once the means for identifying the truth conditions of such propositions had been established, the means for establishing the truth conditions of past-tensed propositions could be specified. Such means involved, essentially, rewriting the past-tensed proposition in the present tense and placing the past-tensed features of the original proposition in a predicate within whose argument place the rewritten present-tensed proposition was placed. In general, the question of the kind of supposition possessed by terms was not raised for terms as they occurred in the past-tensed propositions but only as they occurred in the present-tensed rewrites of the past-tensed propositions.

The identification of the truth conditions of future-tensed propositions was not on the whole thought to raise problems of a different kind from those involved in identifying the truth conditions of past-tensed propositions. With obvious changes the account to be given can be read off the account given in this section of the truth conditions of past-tensed propositions. I shall therefore not discuss future-tensed propositions here, but shall turn briefly, in the next section, to the interesting question of the truth conditions of modal propositions. The excursion into modal logic will have the additional advantage of introducing us to terminology which will be of importance in the next chapter.

VI. MODAL PROPOSITIONS

Two statements by Aristotle lie behind a good deal of medieval theorizing about truth conditions. He affirms: 'To say of what is that it is not, or of what is not that it is, is false, while to say of what is that it is, and of what is not that it is not, is true',⁵⁴ and 'The true judgment affirms where the subject and predicate really are combined, and denies where they are separated.'⁵⁵ It should be plain that these two dicta, both of which concern present-tensed

⁵⁴ *Metaphysics*, 1011 b 26

⁵⁵ *Ibid.* 1027 b 20.

propositions, cannot be applied without modification to either past-tensed or future-tensed propositions. The point I wish to attend to here is that, unless adapted, they also fail if applied to propositions expressing possibility, necessity, impossibility, or contingency. For example, the proposition 'A man can be a logician' is true, but it is not thereby true that, to use Aristotle's phrase, 'the subject and predicate really are combined'. For if they are really combined then a man is, and not merely can be, a logician. A common medieval interpretation of Aristotle's accounts of truth was this: 'Every true proposition is true because, howsoever the proposition signifies, so it is in the thing signified or in the things signified.'⁵⁶ And Buridan, noting the need for an adaptation, affirms: 'This is true: "Something which never will be can be", not because things are as the proposition signifies, but because things can be as the proposition signifies they can be.'⁵⁷ For the remainder of this section we shall be considering the implications of this position.

It was common to distinguish between a categorical proposition and the dictum of the proposition. In Latin the dictum of a proposition is formed by replacing the subject of the proposition by its accusative form, and replacing the finite main verb by its infinitive form. I shall use the standard 'that' clause construction to render the Latin accusative plus infinitive construction. Thus the dictum of 'A man is an animal' is 'that a man is an animal'. Plainly what I have been calling 'the dictum of a proposition' is what many modern logicians would call a 'proposition' *simpliciter*, but I shall continue my practice of using the term 'proposition' (= *propositio*) as the medieval logicians themselves used it. One way to construct a modal proposition is to predicate a modal term of a dictum. Ockham gives the example: 'That every man is an animal is necessary',⁵⁸ and he argues that its truth condition is that the

⁵⁶ e.g. Buridan, *Cons.*, 17.

⁵⁷ Ibid.

⁵⁸ *Summa Logicae*, Pt. II, Ch. 9, p. 273.

proposition corresponding to the dictum is necessary. That is, 'That every man is an animal is necessary' is true just if the proposition 'Every man is an animal' is necessary. And similarly for the dictum of which 'possible', 'impossible', or 'contingent' is predicated. But Ockham adds an important rider:

As regards a necessary proposition it should be noted that it is not because the proposition is always true that it is necessary, but because if it exists it is true and cannot be false. For example, this mental proposition 'God exists' is necessary, not because it is always true—for if it does not exist it is not true—but because if it exists it is true and cannot be false.⁵⁹

He held likewise that an impossible proposition is impossible not because it is always false, but because if it exists it is false and cannot be true. The implication, though not stated explicitly by Ockham, is that a proposition is possible not because it is sometimes true but because its existence does not imply its falsity.

Where a modal operator includes within its scope, or covers, an entire proposition or dictum, the proposition containing the modal operator is said to be a modal proposition 'with composition' (*in sensu composito*). The examples we have so far considered are of this kind. Where a modal operator covers a part, but not the whole, of a proposition then the proposition is said to be a modal proposition 'with division' (*in sensu diviso*).⁶⁰ In such cases the modal term divides the proposition into two parts, the part not covered by the term, and the part covered by it. In the light of our earlier discussion on the way the position of a negation in a proposition was held to affect the truth conditions of the proposition, it comes as no surprise to discover that Ockham and others held that the fact that a modal term occupies a given position in a proposition can

⁵⁹ Ibid. 275.

⁶⁰ See e.g. William Heytesbury on the compounded and divided sense in CTMPT, trans. N. Kretzmann and E. Stump, ch. 13, pp. 415–34.

have an effect on the truth conditions of the proposition. As we observed, it was said that 'That every man is an animal is necessary' is true if the proposition 'Every man is an animal' is necessary. But 'Every man is necessarily an animal' is true if there exists something for which 'man' stands, and 'This is an animal' is necessary, for every man indicated by 'this'.⁶¹

The general rule is this: 'A is modally B', where 'modally' holds a place for 'necessarily', 'possibly', and so on, is true just if the mode expressed in such a proposition is truly predicated of a non-modal proposition in which B is predicated of a pronoun indicating that for which A stands. For example, of 'Every truth is necessarily true', Ockham gives the following truth condition: 'This is true' is necessary, for every truth indicated by 'this'. And in that case, he concludes, 'Every truth is necessarily true' is itself false. There are, after all, propositions which, though true, are only contingently so. In contrast Ockham does not argue against 'That every truth is true is necessary', presumably because he saw it as an instance of the law of identity, that is, that everything is itself.⁶²

Because Ockham accepts this account of the truth conditions of propositions which are modal with division, he accepts that 'A white thing can be black' is true. For the latter proposition is true if the following condition is satisfied: there is something for which 'white' stands, and 'This is black' is possible where 'this' indicates something for which 'white' stands. Were the sample proposition to be understood to mean 'That a white thing is black is possible' it would, of course, have to be rejected as false, since the

⁶¹ Ockham did not accept that 'Every man is necessarily an animal' is true, for he held that 'This is an animal' is not necessary, whatever is indicated by 'this'. For whatever is an animal has merely contingent being, and therefore this (which is an animal and which is contingently) is contingently whatever it is, and hence it is contingently an animal.

⁶² The common rejection of 'Every chimera is a chimera' does not imply a rejection of the law of identity formulated as above, for though everything is itself, no chimera exists and hence no chimera is anything—and hence no chimera is itself.

proposition corresponding to the dictum 'that a white thing is black' is contradictory.

'Can' and 'possible' are ampliative. They extend the supposition of the subject to cover what is and what can be. But if the subject in 'A white thing can be black' stands for what is or can be white, then the truth condition given in the preceding paragraph is not the only one that the proposition would satisfy. For the earlier truth condition specifies that there is something for which 'white' stands, whereas the sample proposition is consistent with there being nothing for which 'white' stands so long as there can be such a thing. Assuming that white things do not exist though they can, then 'A white thing can be black' is true if the following condition is satisfied: this is possible: 'A white thing can be black' where 'white' stands for something which is white. And this condition is itself satisfied by the truth condition given in the previous paragraph.⁶³

There is a metaphysical aspect to the above account of the truth conditions of propositions of possibility. 'A can be B' does not imply the attribution of B to something with possible being, as though possible being is a way of being, and is a way of being which involves having sufficient being to have the attribute B. The Ockhamist line is that when it is said that A is possibly B, or can be B, where A does not but can exist, the modality must be understood to be predicated of one or more non-modal propositions, and should not be understood to qualify the existence of A itself. To Ockham, at least, it seemed plain that if A does not exist, then its existence does not have the quality of possibility.

⁶³ See *Ockham's Theory of Propositions*, trans. A. J. Freddoso and H. Schuurman, 58.

4

Molecular Propositions

I. CONJUNCTION

At the start of Chapter 3 brief reference was made to molecular propositions (= *propositiones hypotheticae*), and I should now like to examine that topic in detail. It was stated earlier that there was little agreement on the question of how many kinds of molecular propositions there are. The lack of agreement however was not a reflection of disagreement about the basic conditions that have to be satisfied if a proposition is to be classed as molecular. A molecular proposition is one containing several categorical propositions. Such a proposition also contains, whether explicitly or implicitly, a connective (often called a copula), or even several connectives if the connective after which the particular molecular proposition takes its name connects propositions of which at least one is also molecular.

A molecular proposition was said to be 'simple' if it contained just two categorical propositions, and 'composite' if at least one of the propositions connected by the principal connective was itself molecular. We shall for the most part attend to simple molecular propositions. While 'and', 'or', and 'if' were the most commonly investigated propositional connectives, it should be said that each one of these terms was also examined in respect of its role as a connective between categorematic terms as well as between propositions. Let us take the term 'and' first.

Paul of Venice writes:

A connective of conjunction is sometimes taken conjunctively and sometimes conjointly. It is taken conjunctively when it connects categorical propositions, and it is taken conjointly when it unites terms only. An example of the first kind is 'Socrates is running and Plato is moving'. An example of the second kind is 'Socrates and Plato are running'.¹

In addition, a connective of conjunction taken conjointly can itself be distinguished in terms of what follows from the proposition containing it. The connective 'is taken divisively when from a proposition of which it is a part there follows a conjunctive proposition composed of equiform terms. It is taken collectively when no such conjunctive proposition follows'.² For example, 'and' occurs as conjoint and divisive in 'Socrates and Plato are running', since (a) the 'and' connects two terms, and (b) the proposition implies the conjunctive proposition 'Socrates is running and Plato is running'.

Paul's example of a proposition containing 'and' functioning collectively is 'Socrates and Plato are sufficient to lift stone A'. As he says, the fact that together they can lift the stone does not imply that each alone can do so. A good deal of effort went into the identification of the rules by which on syntactic grounds it could be established whether a given conjoint 'and' was functioning divisively or collectively. We shall not pursue that tortuous line of enquiry here, but shall instead attend to 'and' where it is conjunctive.

The truth conditions of a conjunction are easily stated: 'For the truth of a conjunction it is sufficient and necessary that all its principal parts, between which the conjunction is the connective, are true',³ and correspondingly it is false if either principal part is false.⁴ The truth conditions of a

¹ *Logica Magna* 127^{vb}. See A. Broadie (1990), 54.

² Ibid.

³ Burley, *De Puritate*, 110.

⁴ Albert of Saxony, *Perutilis Logica*, 19^{fv}.

proposition in which possibility is predicated of a conjunction can be stated almost as simply: that a given conjunction of propositions is possible is true if each of the conjuncts is possible and the two conjuncts are mutually compossible. Hence for a conjunction to be impossible it is sufficient that it have two conjuncts, both possible, which are mutually impossible, as in Burley's example: 'Socrates is white and Socrates is black'. It is also sufficient that either of the parts be impossible. These two conditions together form the necessary and sufficient conditions for the impossibility of a conjunction. The notion of impossibility is immediately invoked by Burley to resolve a problem about valid inference. It is a rule of valid inference that the impossible does not follow from the possible, and yet the impossible 'A black thing is white' follows from the two premisses 'Socrates is white' and 'Socrates is black'. As Burley points out, each of the premisses is possible, but they are not compossible, and from impossible premisses an impossible proposition can be concluded without infringing any law of logic.⁵

Paul of Venice considers a more complicated kind of case than Burley's. Paul gives as a sufficient condition for the possibility of an affirmative conjunction that each principal part be compossible with each, or with all the others at the same time, if there are more than two. He gives two examples, of which the first is obvious enough: 'Some man is every runner, and Socrates is a runner and Plato other than Socrates is a runner.' As Paul points out, the first part is compossible with the second and with the third, and the second is compossible with the third, but the first is not compossible with the other two combined, nor the second with the first and third combined. The other, rather more interesting example, is: 'Into these parts continuum A is divided and into these parts continuum A is divided and so on to infinity.' Each part of this conjunction is compossible with each other part. But they are not all

⁵ *De Puritate*, III.

mutually compossible, because of the impossibility of an actually infinitely divided continuum. Had the sample proposition contained 'divisible' rather than 'divided' the proposition would not have been found unacceptable.⁶

In the light of remarks made in the previous chapter about the truth conditions of past-tensed propositions, a word should be said about a doubt that Burley raises in connection with the identification of the truth conditions of conjunctions. It was commonly held that a proposition, containing a conjoint term followed by a part of the verb 'to be' and no predicate, was equivalent to a conjunctive proposition with parts equiform with the first proposition, for example, 'Tom and Dick are' and 'Tom is and Dick is'. But Burley argues that whereas 'Adam was' and 'Noah was' are both true, 'Adam and Noah were' is false, and therefore 'Adam was and Noah was' does not imply 'Adam and Noah were'. His argument for the claim that the latter proposition is false is that 'every true past-tensed proposition at some time was true in the present tense'.⁷ If 'Adam and Noah were' is true then at some time in the past 'Adam and Noah are' was true. But the latter proposition never was true since Adam and Noah were never alive at the same time. This position may seem odd. Burley himself offers the counter-argument that 'Adam and Noah were' is true since Adam is dead and Noah is dead also. Neither of them, therefore, is, though both were—that is, Adam and Noah were. He appears to accept this latter argument. At any rate he accepts the conclusion, and to the former argument he offers the following objection:

For the truth of a past-tensed proposition where an act is signified by a plural verb, it is not necessary that it have some present-tensed version which at some time was true, but it is sufficient that it have several present-tensed versions which at some time were true. So 'Adam and Noah were' has these two

⁶ *Logica Magna*, 129^{vb}. See A. Broadie (1990), 93.

⁷ *De Puritate*, 111–12.

present-tensed versions 'Adam is' and 'Noah is', which were at some time true.⁸

Such a set of truth conditions for a proposition consisting of a conjunction of proper names followed only by a past-tensed part of the verb 'to be' clearly has the desirable consequence of ensuring that if the bearer of each of the names was, then both bearers were.

II. DISJUNCTION

There was no dispute that the connective of disjunction should count as a propositional connective, though there was certainly dispute regarding the truth conditions of disjunctive propositions. We shall reach the nub of the dispute shortly. First it should be noted that a certain distinction we made regarding the function of 'and' should also be made regarding 'or'. In each case what has to be noted is that the connective can connect either two terms or two propositions. Following our earlier distinction between 'and' in its conjoint and its conjunctive employment, we must make a distinction between 'or' in its disjoint and its disjunctive employment. In 'You are a man or a donkey', 'or' is disjoint. In 'You are a man or you are a donkey', 'or' is disjunctive.

Furthermore just as 'and' can be either divisive or collective, so 'or' can be one or other of these. Paul of Venice explains: 'It is taken divisively when the argument from any part of the disjoint term to the disjoint term, and from the disjoint term to a disjunctive proposition with equiform terms is sound.'⁹ He gives two examples: since 'You are a man, therefore you are a man or a donkey' is sound, the disjoint 'or' in the conclusion is divisive. And likewise, since 'Socrates or Plato is running, therefore Socrates is running or Plato is running' is sound, the disjoint 'or' in the premiss is divisive. A disjoint 'or' occurs collectively when neither of

⁸ Ibid. 112.

⁹ *Logica Magna*, 131^{rb}-va. See A. Broadie (1990), 119-20.

these conditions is satisfied. Paul's examples are 'or' in 'I am different from you or from me', since the proposition does not follow from 'I am different from you', and the 'or' in 'You know that a king is seated or that no king is seated', since that proposition does not imply 'You know that a king is seated or you know that no king is seated'. The disjoint 'or' will be of concern to us chiefly in so far as it is used divisively, that is, in so far as the proposition containing the disjoint 'or' implies a disjunctive proposition.

As regards the truth conditions of a disjunctive proposition there were two main opinions. Walter Burley attributes to the great Augustinian theologian Giles of Rome (c.1234-1316) the view that a necessary condition for the truth of a disjunctive proposition is that one of the disjuncts be true and the other false. For, as Boethius says, the connective of disjunction between the disjuncts does not permit them to be (i.e. to be true) simultaneously.¹⁰ Giles's conception just outlined is of what we should now call 'exclusive disjunction', a disjunction in which affirmation of either disjunct excludes affirmation of the other, or, put otherwise, in which each disjunct implies the negation of the other. But Giles's was a minority opinion. The standard position was that the sufficient and necessary condition for the truth of a disjunction is that at least one of the disjuncts be true. The falsity of either disjunct does not imply the falsity of the disjunction, but the truth of a disjunct does imply the truth of the disjunction. The way the matter was commonly stated was that the truth of either part of the disjunction is by itself a sufficient cause of the truth of the disjunction, and therefore the truth of the two disjuncts together is a cause of the truth of the disjunction. This conception of 'or', the 'inclusive' conception, is the one we shall assume to be employed in subsequent examples of propositions with disjunctive occurrences of the term.

Peter of Spain adopted an awkward compromise position between the minority and the majority views. He asserts that

¹⁰ Burley, *De Puritate*, 115.

for the truth of a disjunction it is sufficient that one or the other part be true, and adds: 'It is permitted that both parts be true, though this is a less stringent use'.¹¹ But this statement of position is unsatisfactory from the logical point of view, as the following generation of logicians recognized. For they were engaged in the construction of a scientific Latin in which, most especially, the role of every logical term was precisely defined so that the truth conditions of propositions containing those terms could be stated precisely. Thus, for example, a logician wished to know precisely what the truth conditions of a disjunction were. He was not helped by knowing what Peter of Spain told him, namely, that there was a more stringent and a less stringent use of 'or', even if he knew what the more and the less stringent uses were. For additionally he needed to know, for each occurrence, how 'or' was being used.

Given the truth conditions stated above, the falsity conditions for disjunctive propositions can easily be deduced. For granted that the necessary and sufficient condition for the truth of a disjunctive proposition is the truth of at least one disjunct, the necessary and sufficient condition for the falsity of a disjunctive proposition is the truth of neither disjunct. Peter of Spain himself states the falsity conditions in exactly the terms I have just used, and hence he must have had in mind the less stringent use of 'or'. For his more stringent use is the exclusive use, and a disjunction of propositions connected by the exclusive 'or' is false when both propositions have the same truth value, whether they are both true or both false. Evidently, then, in stating the falsity conditions for disjunctive propositions, Peter of Spain had in mind the inclusive 'or'.

Formally the possibility conditions of a disjunctive proposition are the same as the truth conditions. A disjunctive proposition is possible just if at least one of the disjuncts is possible, and it is impossible if neither disjunct is possible. The impossibility of just one disjunct is insufficient for the

¹¹ *Tractatus*, 9-10.

impossibility of the disjunction. For if the other disjunct is not impossible it is possible, and if it is possible it might be true. And if it might be true then so might the disjunction, in which case the disjunction is possible. Hence just as a disjunct implies a disjunction of which it is a principal part, so the possibility of a disjunct implies the possibility of a disjunction of which it is a principal part.

But the necessity conditions of disjunctive propositions are more complicated. Burley affirms that 'for the necessity of a disjunction it is sufficient that one of its parts be necessary'.¹² The argument for this position is that a disjunction follows from each of its parts. However, from a necessary proposition there follows only another necessary proposition, for otherwise if the conclusion of the inference is merely contingent then it might be false in which case what is necessarily true would imply what is false and therefore the inference would be invalid. Consequently if the disjunction has a necessary disjunct the disjunction itself is necessary.

Yet Burley states this as a sufficient, not a necessary condition, because a disjunction might be necessary though neither disjunct is necessary. The general rule is that if two propositions are so related that either is implied by the denial of the other then the disjunction of those two propositions is necessary. As Burley states the matter:

It is sufficient for the necessity of a disjunction that its parts be contradictory, for it is impossible for contradictories to be false simultaneously. Hence it is necessary that one or other contradictory always be true. And so it is necessary that a disjunction composed of contradictories be necessary.¹³

It should be added, however, that two propositions may, disjoined, form a necessary disjunction even though neither is necessary and neither contradicts the other, for it is possible for two propositions to be so related that either is implied by the negation of the other, even though the two propositions

¹² *De Puritate*, 117.

¹³ *Ibid.* 117-18.

are not contradictory, but instead are subcontrary. Two propositions are subcontrary if they cannot be false together though they can be true together. Thus 'Some man is a logician' and 'Some man is not a logician' are subcontraries. They cannot both be false. If the first of the two is false it follows that no man is a logician from which, in turn, it follows that some man is not a logician. For if no man is a logician then this (said while pointing to a man) is not a logician, in which case some man (for example, this one) is not a logician. And if it is said that where no man exists neither of the subcontrary propositions is true, it should be recalled that a negative proposition with a subject which stands for nothing is true.

A plausible line of argument can be brought to bear on the points just made. It was commonly held that there are no degrees of truth. Or perhaps it should be said, instead, that logicians had in general no use for such a concept. There are to be found occasional lapses into modes of expression that suggest that a theory of degrees of truth was held, as for example when Burley asserts that 'The impossible seems to be less true than anything else'.¹⁴ But in general any proposition that was false was considered to be exactly as false as any other proposition that was false. Indeed, some went further. Paul of Venice asserts:

One proposition is not more true than another, nor more false, nor more possible, nor more impossible, nor more necessary, nor more contingent, but truth, falsity, possibility, and impossibility consist of something indivisible as do straightness and other relations. For just as it is not said that one father is more a father than another father is, or that one equal is more equal than another, so also one thing which is true or necessary is not more true or necessary than another.¹⁵

Let us attend here to the particular point that truth is without degrees. It might be argued that 'You are or you are not' cannot, therefore, be more true than its first disjunct.

¹⁴ Ibid. 61.

¹⁵ *Logica Magna*, 132^{ra}. See A. Broadie (1990), 129.

But the first disjunct is only contingently true, and hence the disjunction is only contingently true. And yet it was argued that a disjunction whose disjuncts are mutually contradictory is necessary and not contingent.

But against this argument it has to be said that even if a disjunction is exactly as true as one of its disjuncts and that disjunct is contingently true, it does not follow that the disjunction itself is contingently true. For 'contingent' and 'necessary' are not different calibrations on a scale of truth. A truth is not somehow more true for being necessary than it would be if it were merely contingent. That a proposition is necessary has an immediate implication for its truth value. For if it is necessary it is true. But it is judged necessary in virtue of features it possesses other than its truth value. In particular it is of the essence of a necessary proposition that there is no proposition from which it does not follow, and that every proposition which follows from it is a proposition which follows from any proposition. And it is the essence of a contingent proposition that it follows from some propositions and does not follow from others.

That a necessary proposition and a contingent proposition can both be true is, of course, not in question. The point is, instead, that given that two propositions are both true, questions can then be asked about them in respect of their modal status, and these questions are not about how true the propositions are, for example, about whether one of them is truer than the other. Paul of Venice compares the relation between the truth and the contingency of a proposition with the relation between the straightness and length of a line. He argues that 'A is exactly as true as B is, and B is a contingent truth. Therefore A is a contingent truth' is invalid in much the same way that this is: 'Line A is exactly as straight as line B, and B is a straight line one foot long. Therefore A is a straight line one foot long.'¹⁶

¹⁶ *Logica Magna*, 132^{ra}. See A. Broadie (1990), 129.

III. CONDITIONALITY

We have already noted that the terms 'and' and 'or' have this in common, that each can function as a propositional connective, and also as a connective connecting other than propositions. The same point can be made about the third variety of connective which we shall be investigating, namely, the connective of conditionality. Two such connectives were commonly invoked, 'if' and 'unless', the latter being treated as equivalent to 'if not'.¹⁷ But most attention was given to 'if', and we shall restrict ourselves to that connective. When 'if' plays the role of forming a categorematic term out of other categorematic terms, it is said to function 'conditionally'. A term thus formed can occur as an extreme in a categorical proposition, as for example, in 'Every animal, if it is a brayer, is a donkey'. When 'if' plays the role of forming a molecular proposition out of two propositions it is said to function 'conditionally', as it does in 'If a donkey is walking an animal is moving'.

A great deal was written on the question of the rules for determining the relations of validity between propositions containing 'if' functioning conditionately and 'if' functioning conditionally. For example, is this argument valid: 'If the Antichrist is a man, the Antichrist is an animal. Therefore the Antichrist, if he is a man, is an animal'? Paul of Venice, who discusses this example among many others, argues that it is not. One reason is that the conclusion is an affirmative categorical proposition, and, as we have already observed, such a proposition implies that there is something for which its subject stands. The subject of the proposition in question is 'the Antichrist, if he is a man'. But the Antichrist, if he is a man, cannot exist unless the Antichrist exists. But the Antichrist does not now exist. Hence, argues Paul of Venice, the conclusion of our sample argument is false. And yet the premiss is true. Therefore the argument is invalid.¹⁸

¹⁷ Ockham, *Summa Logicae*, Pt. II, Ch. 31, p. 347.

¹⁸ *Logica Magna*, 134^{va}. See G. E. Hughes (1990), 5.

I am not sure that new life can be breathed into the notion of the conditionate 'if', and I would prefer to devote space instead to the undoubtedly important concept of 'if' in its conditional role, that is, as a connective which forms molecular propositions out of propositions. The conditional, then, has three parts, the connective of conditionality, and the two propositions which it connects. Of those two propositions the one which immediately follows the 'if' is called the antecedent, and the other proposition is called the consequent. The latter proposition can occur either after the antecedent or before the 'if'. There are, therefore, two basic forms of the conditional proposition, (1) If P, Q, and (2) Q if P. The antecedent and the consequent can both be categorical propositions, and one or both can be molecular. That is, to use Burley's terminology explained earlier, a conditional can be either a simple or a composite molecular proposition.

This account of what a conditional proposition is is purely syntactic. A further syntactic point should here be added. It was recognized that a proposition might count as a conditional even though it did not consist of 'if' linking two propositions but instead consisted of 'if' linking what were sometimes called propositional complexes. Such conditionals were a matter of deep interest, and continued to be discussed into the period of late flowering of terminist logic. For example, Robert Galbraith (c.1483-1544) paid close attention to them. He gave this definition: 'A propositional complex is an expression [*oratio*] which taken by itself does not signify what is true or what is false, but added to something else it makes the resulting aggregate represent what is true or what is false. And changing the verb into one in the indicative mood produces a proposition.'¹⁹ In his example, a stock one, 'If a donkey were to fly a donkey would have wings', 'a donkey were to fly' and 'a donkey would have wings' lack a truth value, though the entire conditional can take one. There is here, as Galbraith notes, the basis of a

¹⁹ *Quadrupentium*, 68^{ra}.

distinction between conjunctions and disjunctions on the one hand, and conditionals on the other; for connecting propositional complexes by 'and' or 'or' does not result in a well formed proposition, whereas, as we see, connecting two such expressions by an 'if' does.

The concept of a subjunctive conditional was of much more than merely logical interest. It was central to a major controversy, especially featuring Jesuits on the one side and Dominicans on the other, concerning the doctrine of middle knowledge (*scientia media*), a doctrine particularly associated with the sixteenth-century Jesuit Luis de Molina, who was concerned to elucidate the concept of divine knowledge.²⁰ Molina's highly controversial doctrine was that God knows not only all necessary truths and his intentions, but also all the things that would have happened if something else had happened, and knows all the things that would happen if something else were to.

Despite the very great interest attached to the concept of a conditional in which the 'if' connects propositional complexes rather than propositions, I shall for the remainder of this chapter focus on indicative, rather than subjunctive, conditionals. In my characterization of such conditionals I spoke only of their syntactic features, and said nothing about the truth conditions of such a proposition; and there is indeed no suggestion that a proposition is not really conditional if the consequent does not truly follow from the antecedent. For a proposition to be a conditional it is sufficient that it consist of two propositions (or propositional complexes) related, in the way already described, by a connective of conditionality. But we should now try to identify the truth conditions of conditional propositions.

Ockham, in line with many others, says simply that a conditional is equivalent to an inference, and he will

²⁰ For modern discussion of this concept see A. Kenny, *The God of the Philosophers*, ch. 5; also R. M. Adams, 'Middle Knowledge and the Problem of Evil', *Am. Phil. Q.*, 14 (1977), reprinted in his *The Virtue of Faith and Other Essays in Philosophical Theology*. (Oxford, 1987).

therefore defer discussion of the matter till he reaches the topic of inferences.²¹ But Ockham is wrong about this. No conditional is equivalent to any inference. The most that can be said is that similar considerations must be brought to bear in determining whether a conditional is true and an inference is valid. Other logicians did not simply refer their readers to their discussion of validity conditions of inferences. Paul of Venice lists ten accounts, canvassed at one time or another, of the truth conditions of conditionals.²² Some, for example, argued that a necessary condition for the truth of a conditional is that it should not be possible for the antecedent to be true without the consequent being true. But this was commonly held not to be a necessary condition. The ground for the rejection of this account was the doctrine that propositions are not timelessly existing entities, but things that exist only when they are being thought or uttered, or while they exist as inscriptions, and therefore they are things which can come into existence and can cease to exist. For it might be said that 'If a man exists, an animal exists' is true, and yet it is possible for the antecedent to be true without the consequent being true. For 'A man exists' might exist at a time when the proposition 'An animal exists' does not exist. And at that time, since 'An animal exists' does not exist, it is not anything, and therefore is not true either. At that time therefore the antecedent of the conditional is true (assuming that a man then exists) and the consequent is not, from which it follows that it must be possible for the antecedent to be true without the consequent being true. And in that case, given that the conditional really is true, it follows that the impossibility of the truth of the antecedent without the truth of the consequent cannot be a necessary condition of the truth of conditionals.

Albert of Saxony reports²³ that, in the face of this consideration, some revised the earlier account to read: 'For the truth of a conditional it is necessary that the antecedent

²¹ *Summa Logicae*, Pt. II, Ch. 31, p. 347.

²² *Logica Magna*, 134^{va}–135^{ra}. See G. E. Hughes (1990), 6–11.

²³ *Perutilis Logica*, 19^{va}.

not be able to be true unless the consequent be true, if formed.' We are, then, to assume that antecedent and consequent both exist, and to ask of these two propositions whether the antecedent can be true without the consequent being true. Clearly this manoeuvre avoids the earlier difficulty. But another obstacle remains. Consider 'If no proposition is negative no donkey exists'. This conditional is false since the antecedent and the opposite of the consequent can be true together. That is, 'No proposition is negative' is compatible with 'A donkey exists'. But we are to suppose that the conditional cannot be false unless it is possible for the antecedent to be true and the consequent false. But it is not possible for the antecedent to be true when the antecedent does not exist, so we are to suppose that the antecedent exists. But when it exists it signifies that no proposition is negative. Yet it itself is negative, and hence so long as it exists it is false. And in that case it is not possible for the antecedent to be true without the consequent being true, because it is impossible for the antecedent, when it exists, to be true. Therefore the conditional is after all true. Evidently the revised account of the truth conditions of conditionals is not free from difficulties.

In the light of the difficulties raised, Albert offers, and apparently accepts, another account of the truth conditions of conditionals. He states the account as if it is only of the necessary condition, but it seems in fact to be intended as the necessary and sufficient condition: 'It is impossible for things to be in whatever way the antecedent signifies, and yet not to be in the way the consequent signifies, if [each] is formed.'²⁴ The advantage of this formulation is that in order for things to be in the way a given proposition signifies them to be, the proposition does not itself need then to exist, whereas a proposition has to exist if it is to be true. So things can be in the way signified by 'No proposition is negative' even if (or only if) there is no such proposition, but 'No proposition is negative' cannot be true unless it exists; and, as noted, if it exists it is false.

²⁴ Ibid.

For all the differences between these, and other, accounts given of the truth conditions of conditionals, one feature they all share is the requirement that conditionality be understood modally. If a given conditional is true then the conjunction of the antecedent and the negation of the consequent is not merely false but impossible. So given the truth of the conditional, either the antecedent is to be denied or, if not, then the consequent must be affirmed, that is, necessarily if the antecedent is to be affirmed the consequent is to be affirmed. It comes, then, as no surprise to find Albert asserting:

For the necessity of a conditional the same thing is required that is required for its truth, and for its impossibility the same thing suffices as is required for its falsity, because every true conditional is necessary and every false one is impossible.²⁵

This is a view in which Albert is preceded in the thirteenth century by Peter of Spain,²⁶ and followed in the fifteenth by Paul of Venice's pupil Paul of Pergola (d. 1451/5), who writes: 'Note that every true conditional is possible and necessary and every false one is impossible, and there is none which is contingent.'²⁷ The kind of conditionality at issue here, which a later generation of medieval logicians termed 'illative conditionality', corresponds closely to the concept of strict implication, though detailed comparison of the two notions is not appropriate here.²⁸ But it is at any rate plain that illative conditionality is not the same thing as material implication. For if of two propositions the first materially implies the second, then either the first is false or the second is true, but from the material implication it does not follow that necessarily either the first is false or the second true. Thus, Paul of Venice asserts:

²⁵ *Perutilis Logica*, 19^{va}.

²⁶ *Tractatus*, 9.

²⁷ *Logica*, 17.

²⁸ See e.g. M. M. Adams, 'Did Ockham Know of Material and Strict Implication? A Reconsideration', *Franciscan Studies*, 36, 5-37.

The argument from an affirmative conditional . . . to a disjunction made up of the contradictory of the antecedent, and the consequent of the same conditional, is a formal inference. This is formal: 'If you are man you are an animal. Therefore you are not a man or you are an animal'.²⁹

His argument is that if P is a condition of Q, then the conjunction of P and not-Q is impossible, in which case the conjunction is to be denied. Therefore one or other of the conjuncts is to be denied. Therefore P is to be denied or not-Q is to be denied. And if not-Q is to be denied Q is to be affirmed. Therefore the disjunction of not-P and Q follows from the conditional affirming that if P then Q.

The preceding discussion gives the main outlines of one concept of conditionality investigated in the fourteenth century. As we have seen, the concept closely resembles that of strict implication, and a question naturally arises as to whether medieval logic contains a concept of material implication. The answer is that this concept, or one very like it, is to be found in medieval logic. Promises, and reports to a third person of the content of a promise, are often couched in conditional terms, and for that reason the kind of conditional now in question was called the promissory conditional. Thus, I say to Socrates: 'If you come to me I will give you a horse', and I say to a third person: 'If Socrates comes to me I will give him a horse.' What are the truth conditions of such conditionals? Robert Galbraith discussed this question in his *Quadrupertitum*:

For the truth of a promissory conditional it is not necessary for it to be impossible for things to be as they are signified by the proposition immediately following the 'if' but not be as they are signified by the proposition mediately following the 'if'. But it is necessary and sufficient that if things are as they are signified by the proposition immediately following the 'if' then they are as they are signified by the mediately following proposition.³⁰

²⁹ *Logica Magna*, 136^{vb}. See G. E. Hughes (1990), 40-1.

³⁰ *Quadrupertitum*, 71^{vb}.

For example, 'If Socrates comes to me I will give him a horse' implies neither 'Socrates will come to me' nor 'I will give him a horse'. Neither does it imply that 'Socrates will come to me and I will not give him a horse' is impossible. The necessary and sufficient condition for the truth of the promissory conditional is that if this is true: 'Socrates will come to me', then this is true: 'I will give him a horse'.

Galbraith asks what form the contradictory of a promissory conditional takes. The answer reveals a good deal about his conception of a promissory conditional. The contradictory of 'If Socrates comes to me I will give him a horse' is, we are told, 'Socrates will come to me and I will not give him a horse'. Now, any proposition is equivalent to the negation of its contradictory. So 'If Socrates comes to me I will give him a horse' is equivalent to 'It is not the case that (Socrates will come to me and I will not give him a horse)'. But the latter proposition is equivalent to: 'Either Socrates will not come to me or I will give him a horse'. Evidently, then, promissory conditionality is truth functional. That is, to know whether such a conditional is true or not it is sufficient to know the truth values of the antecedent and the consequent. The conditional is true so long as it is not the case that the antecedent is true and the consequent false.

There is some reason to doubt that Galbraith in fact gave a perfectly unambiguous description of material implication in describing promissory conditionals.³¹ But certainly the concept of promissory conditionality is a good deal closer to that of material implication than is that of illative conditionality. This is made especially plain when Galbraith enquires into the form taken by the contradictory of an illative conditional. He asserts that the contradictory of 'If Socrates comes to me I will give him a horse', where the 'if' is illative, is 'It is possible that (Socrates will come to me and I will not give him a horse)'. Given the aforementioned principle that a proposition is equivalent to the negation of its contradictory, it follows that the illative conditional

³¹ See A. Broadie, *The Circle of John Mair*, 147-51.

implies: 'It is not possible that (Socrates will come to me and I will not give him a horse)'. And that last proposition is equivalent to 'Necessarily if Socrates will come to me I will give him a horse'. We have here a route to the statement quoted earlier from Albert's *Perutilis Logica*: 'Every true conditional is necessary'.

5

Valid Inference

I. INFERENCE

Let us say that two propositions are in 'logical sequence' if one follows from the other, and that they are signified to be in logical sequence if one is signified to follow from the other. Two propositions may, of course, falsely be signified to be in logical sequence, and much medieval discussion, some of it to be noted shortly, stems from this obvious distinction between what is and what is signified to be. But for the present it will be sufficient if certain important terminology for dealing with the relation of logical sequence can be established. Of two propositions signified to be in logical sequence let us reserve the term 'antecedent' to stand for the proposition from which the other is signified to follow. And 'consequent' will stand for the proposition which is signified to follow from the antecedent. We can also say that of two propositions which actually are in logical sequence—whether signified so to be or not—the antecedent and the consequent are, respectively, the proposition from which the other follows and the proposition which follows from the other.

Two propositions related in the way just described were said to stand in the relation of 'consequence'. Under this general heading medieval logicians listed two relations which we should now regard as of logically quite distinct kinds. In fact, despite their practice, it is plain that the logicians were well aware that the two relations are of very different kinds, and their use of the one term 'consequence' should not be allowed to conceal that fact. The two

relations in question are that of antecedent to consequent in a conditional proposition, and that of premiss to conclusion in an inference or argument. The relation of consequence was indicated by two connectives, 'if', which is the connective of conditionality, and 'therefore', which connects a premiss to a conclusion in an inference. Thus 'If P, Q' and 'P, therefore Q' were both said to have the form of a consequence.

At least four distinctions were drawn between conditionals and inferences. First, 'if' can connect propositional complexes and 'therefore' cannot. 'If a man were to have wings, a man would be flying' is a well-formed proposition, but 'A man were to have wings, therefore a man would be flying' is not well formed. It was allowed that an inference could have, as a premiss, a conditional which contains two propositional complexes. But that is a different matter. For in such a case the problem is not that of coping with the ungrammaticality of a premiss, but of specifying the truth conditions of the premiss. And that was not considered a serious problem 'If a man were to have wings, a man would be flying' is true if it is impossible that things would be as signified by 'A man has wings' without being as signified by 'A man is flying'.

The second difference between 'if' and 'therefore' is, like the first, a syntactic one. In a conditional the proposition or propositional complex immediately following the connective of conditionality is the antecedent; in an inference the proposition immediately following 'therefore' is the consequent. In an inference the antecedent always precedes the 'therefore'; in a conditional the consequent can either precede the 'if' or follow the antecedent. Thus, as earlier observed, 'If P, Q' and 'Q if P' both give a form of the conditional.¹ However, the fact that a conditional has these two forms does not mark any serious distinction between conditionals and inferences, for if we take 'since' as the inference connective, as it was often said to be, then it can

¹ See above, Ch. 4, Sect. III.

be pointed out that 'Since P, Q' and 'Q since P' are both grammatically sound forms. We shall, however, retain 'therefore' as the inference connective for the remainder of this book.

Thirdly, the antecedent of a conditional consists of either a single proposition, whether or not molecular, or a single propositional complex. But the antecedent of an inference can be composed of several unconnected propositions, each of which is classed as a premiss. It is not however clear that any important logical truth underlies this fact about the way inferences are set out. Many logicians, relying on the distinction between what is explicitly or expressly contained in a proposition, and what is implicitly or virtually contained, spoke of the premisses of an inference as a conjunction of propositions. Certainly no difference is made to the inferential power of the antecedent of an inference by treating the several premisses as a single conjunction, for even if two premisses are not explicitly a conjunction, their conjunction follows directly from the two premisses, and each of the two premisses follows from the conjunction.

The first three differences between conditionals and inferences or arguments concern syntactic matters. The fourth does not; it relates to semantic considerations. Truth was in general understood, as we have already seen, in Aristotelian terms, for a proposition was said to be true if things were as the proposition signified them to be. Qualifications and further elaborations of this account of truth do not here concern us. The point we have to attend to is that logic was regarded as the science that teaches us to speak truly, for by the application of that science we can, starting with true propositions, reach other true propositions. That is how a science, a systematically ordered body of knowledge, is constructed. However brilliantly we reason, it is to no avail if we start from falsity, except where we use the fact that we have reached a false conclusion as itself proof that the starting-point was false and therefore can be transformed into a truth by being negated. But even

then the fact that we could start with a false proposition and from it reach a true one does not show that we value falsity as much as we value truth. On the contrary, we started with a false proposition in order to prove its falsity, and thereby demonstrate our entitlement to replace it by its negation. And that negative proposition could not, on grounds of its truth value, be debarred from a place in a science.

But though, being curious creatures, our interest in propositions is really an interest in *true* propositions, and though we use inferences to reach true propositions from other propositions already known to be true, it does not follow that the inferences by which we extend our knowledge themselves have a truth value. Inferences were, indeed, not generally regarded as bearers of truth value, but as bearers of what might instead be called 'validity value'. Among the standard phrases were *Argumentum est bonum* ('The argument is good') and *Argumentum valet* ('The argument is valid'). Just as, in respect of the development of science, our interest in propositions is really an interest in true ones, so also, in respect of the development of science, our interest in inferences is really an interest in valid ones. What is wrong with invalid ones is, precisely, that using them we have no guarantee that starting with truth we shall reach truth. The regular ascription of validity or invalidity (or soundness or unsoundness, or goodness or badness) to inferences, and of truth or falsity to conditional propositions, indicates quite clearly that, though medieval logicians classed conditionals and inferences equally under the heading 'consequence', they saw that conditionals and inferences were, in respect of a fundamental principle of classification, in logically quite different categories.

Hereafter I shall use the term 'inference' when I am speaking about that kind of consequence whose distinctive connective is 'therefore', and I shall continue to speak of conditionals when referring to expressions whose distinctive connective is 'if'. Attending then, for the present, to

inferences I shall begin by asking how we are to recognize an inference as one. Two views were held on this matter. One was based on purely syntactic considerations. Something is an inference, according to this first view, if it is composed of a set of propositions (perhaps a one-membered set), followed by 'therefore' or one of its synonyms, followed by another proposition. Buridan, however, mentions this view only to dismiss it for his immediate purposes. By 'inference' he evidently means 'sound inference', for he says that by 'antecedent' and 'consequent' he understands propositions of which one follows from another in a sound inference.² But a piece of speech cannot be an inference unless it has an antecedent and a consequent, and if those two terms are described in the way Buridan has chosen it follows that a piece of speech cannot be an inference unless one part of it actually does follow from the other.

Yet a question may certainly be raised about whether an inference is an inference only if it is valid. It is of course common enough to meet with an inference which is so bad that it is a travesty of an inference rather than a real one. The fact that it contains a 'therefore' is surely not, we are tempted to think, sufficient to justify its classification as an inference. What, though, of other cases where we do, perhaps only eventually, detect an error in the chain of reasoning? Should such a case not be brought under the heading of 'inference'? It might be replied that if in fact a given proposition *Q* does not follow from some other given proposition *P*, then the relation of *P* to *Q* is not an inferential relation. But there is more to inference than a relation between propositions. Inferring is a cognitive act in which, starting with a set of propositions, we draw a conclusion from that set. We infer *Q* from *P*. Whether we ought to have reached that conclusion, or ought not, we have all the same inferred *Q*. If, despite our signifying, by the use of 'therefore', that *Q* follows from *P*, *Q* in fact does

² *Cons.*, 21.

not follow from P, then we are said to have inferred invalidly, and the inference 'P, therefore Q' is said to be invalid or unsound. According to this view, we do not require to know whether Q follows from P in order to know whether P is the antecedent and Q the consequent in an inference. We need only know whether Q is signified to follow from P, and for that it is sufficient to observe that P is connected to Q by 'therefore'.

The price we have to pay for using 'inference' in this latter way, is that what we should perhaps prefer to regard as a travesty of an inference has nevertheless to be classified as an inference. But this is a small price to pay, for the alternative is to say that there is no such thing as an invalid inference, only a valid one, and that therefore in order to establish whether something is an inference or not we would first have to establish whether it is valid. Indeed, establishing that it is an inference and establishing that it is valid would be precisely the same thing. But such a line would be very similar to the claim that something should not be called a conditional unless it is true, and that therefore to establish whether something is a conditional it is not enough to observe that it consists of two propositions or propositional complexes connected by 'if'. We have further to establish whether it is true or not. But, so far as I know, it was not suggested by any medieval logicians that a conditional is a conditional only if it is true. And I shall follow the spirit of this view by accepting the general position that whether or not something is to be called an inference must depend only on certain syntactic considerations and not at all on semantic ones.

Accepting, therefore, that a piece of speech is an inference if it consists of a set (perhaps one-membered) of propositions, followed by 'therefore' or one of its synonyms, followed by another proposition, I shall turn now to the question of the validity conditions of such a piece of speech. It was commonly stated that an inference is valid if it is impossible for the antecedent to be true without the consequent also being true. The point can also be made in terms of

incompatibility: an inference is valid if the truth of the antecedent is incompatible with the falsity of the consequent. Two propositions were said to be incompatible if it is impossible for them to be true together, that is, if the conjunction of them is impossible.

However, this account of validity meets with precisely the same objection that was levelled at the corresponding account of the truth of a conditional. For we have to remember that propositions were not thought of as timeless entities, but as temporal, coming into existence when they are thought or uttered or inscribed, and going out of existence when the thought or utterance or inscription ceases. In that case, faced with an inference which we should otherwise wish to accept as valid, we would still have to recognize the possibility that the antecedent might exist at a time when the consequent does not exist. And if the antecedent can ever be true it might be true then, but the consequent would not then exist, and therefore would not then be anything, and therefore would not then be true. In that case the condition for validity is not met.

We might yield to pressure from this direction and amend our account of validity by requiring that the antecedent cannot be true without the consequent also being true when the antecedent and consequent are both formed. We are, then, to ignore what might be the case at those times when one or other of the antecedent and consequent does not exist. But this modification is susceptible to an attack familiar from the literature of sophisms. Let us consider the inference: 'No proposition is negative. Therefore no donkey is running.' Application of the revised account of validity yields the conclusion that the model inference is valid. But in fact it is invalid, and hence the revised account must be rejected. That it is, on the revised account, valid is easily shown. The antecedent has only to exist to be false, for it is a self-falsifying proposition. Since it is impossible for it, while existing, to be true, it is impossible for it, while existing, to be true without every other proposition which exists at the

same time as it also being true. Hence the antecedent cannot, while existing, be true without the consequent, assuming it exists, also being true.

That the inference is in fact invalid can be shown by invoking the following rule of valid inference: If a given inference is valid then the negation of the antecedent follows from the negation of the consequent. The negation of the consequent of our model inference is 'It is not the case that no donkey is running', which is equivalent to 'Some donkey is running'. The negation of the antecedent of that inference is 'It is not the case that no proposition is negative', which is equivalent to 'Some proposition is negative'. Hence if the model inference is valid then 'Some proposition is negative' follows from 'Some donkey is running'. Yet there is no ground whatsoever for supposing that those latter two propositions stand in the relation of consequent to antecedent in a valid inference.

In the light of considerations such as the one just outlined, a number of logicians adopted a different approach to the question of the validity conditions of inferences. The third approach was characterized by a reluctance to invoke the concepts of truth or falsity. One advantage is the avoidance of embarrassing problems arising from the non-existence of truth-value bearers. The approach was in terms of signification. According to the view now under consideration, an inference is valid if it is impossible for things to be as signified by the antecedent but not be as signified by the consequent. This account has at least the merit that application of it to the model inference we have been discussing does not force us to the conclusion that the manifestly invalid inference is in fact valid. For it is possible for things to be as signified by the antecedent while not being as signified by the consequent. That things could be as signified by the antecedent would not be disputed by our logicians, for there was not considered to be something peculiar about the class of negative propositions such that at any time at least one member of the class has to exist.

Indeed it was not supposed that there was any proposition of such a nature that it had ever to exist. And at that time, if any, when no negative proposition existed it could be that things were not as signified by 'No donkey is running'. And in that case 'No proposition is negative, therefore no donkey is running' is not valid.

Even this account of validity was not accepted exactly as formulated above. Buridan, for example, argues that that formulation is too narrow. The formulation refers to things being as signified by the antecedent and the consequent. But such a formulation leaves out of account the fact that a valid inference may contain past-tensed or future-tensed propositions, and, as Buridan reminds us, a past-tensed proposition is true if things were as the proposition signifies that they were, and a future-tensed proposition is true if things will be as it signifies that they will be.³

It is not difficult to see the way we should approach the problem of how to amend the third account in order to deal with this criticism. Thus, for example, let us suppose that the inference which is to be tested for validity consists of a future-tensed antecedent and a future-tensed consequent. As a first stage in the modification of the account of valid inference we could say that that inference is valid if it is impossible that things will be as it is signified by the antecedent that they will be, and yet will not be as it is signified by the consequent that they will be. Inferences which contain past-tensed propositions can be dealt with in a corresponding way. And the fact that an inference may contain propositions of possibility or necessity must also be taken into account. If, for example, the inference to be tested for validity contains an antecedent which is a proposition of necessity and a consequent which is a proposition of possibility, then its validity condition is this: It is impossible that things necessarily are as the antecedent signifies that they necessarily are without it being the case that things can be as the consequent signifies that they can be.

³ Ibid. 17-18.

However, though taking great pains to be precise about the identity of validity conditions, and as a result being forced to abandon the view that an inference should be called valid merely because it is impossible for the antecedent to be true without the consequent being true, this last account of validity was in fact quite often used, since 'it has counter-examples in few instances'.⁴ However, some logicians may have spoken of an inference as having an antecedent and a consequent so related that it was impossible for the antecedent to be true without the consequent being true, as a shorthand way of expressing the point that the antecedent and consequent were so related that it was impossible for things to be as signified by the former without being as signified by the latter. Be that as it may, the fact remains that as regards the 'official' account of validity-conditions a consensus formed round the third of the accounts given above.

II. KINDS OF VALID INFERENCE

We turn now to distinctions which were commonly drawn between kinds of valid inference. The first of these distinctions is that between inferences which are valid formally and those which are valid materially. A formally valid inference is valid, and every inference equiform to it is also valid.⁵ To understand this we have to bear in mind that inferences were thought of as being informed matter. The matter of an inference is every categorematic term in the inference. The form, simply stated, is everything else.⁶ Connectives, negation signs, signs of quantity ('every', 'some', and so on), and other syncategorematic signs belong to the form.

It is not only syncategorematic terms which should be counted as part of the form. Other aspects of a proposition, which are better thought of as features than as terms, are

⁴ Ibid. 22.

⁵ Ibid. 22-3; c.f. Albert of Saxony, *Perutilis Logica* 24^{ra-rb}.

⁶ Buridan, *Cons.* 30-1.

part of the form. Thus the order of terms is part of the form. As noted in Chapter 3, if the position of the predicate is altered so that instead of being the last term in the proposition it precedes a negation sign which had previously covered it or precedes a universal quantifier which covers the subject, the proposition thus altered may have a different truth value. And the fact that the order of the categorematic terms is what it is in relation to the syncategorematic terms is a formal feature of the proposition, though not a feature expressed by any syncategorematic term or set of syncategorematic terms in the proposition. Consequently the proposition before the change in the order of terms is said to have a different form from the proposition after that change in order. The proposition has been *transformed*.

A further feature of the form of a proposition is the number of tokens of a given categorematic term. Thus it was held that 'A man is a man' has a different form from 'A man is an animal', because a given categorematic term occurs twice in the first proposition and in the second proposition no categorematic term appears more than once. And two inferences do not have the same form if a categorematic term of a given type appears in more than one proposition in one of the inferences, but there is no corresponding recurrence of a categorematic term of a given type in the other inference. Thus in 'Every man is an animal. Therefore some man is an animal', it is a feature of the form of the inference that the subject in the antecedent is equiform to the subject in the consequent, and that the predicate in the antecedent is equiform to the predicate in the consequent. Hence 'Every man is an animal. Therefore some donkey is an animal' is not of the same form as the preceding model inference. That is, the two inferences are not equiform, and hence the fact that the first is formally valid provides by itself no ground for concluding that the second also is valid. Had 'donkey', and not 'man', been the subject of the antecedent of the second model inference, the situation would have been different, for then the two

inferences would have been equiform. In that case we could have argued that since the first inference is formally valid and the second inference is equiform to the first, the second inference is valid—indeed formally valid—also.

A materially valid inference is an inference which is valid though not formally so. That is, other inferences equiform to it are not valid.⁷ For example, 'A man is running. Therefore an animal is running' is materially valid. But it is not formally valid, for other inferences of that logical form are not valid, for example, 'A man is running. Therefore a donkey is running'. But a question is to be asked as to how it comes about that such an inference is valid at all, given that it is not valid in virtue of its form. One answer that was canvassed was that materially valid inferences are 'reducible' to inferences which are formally valid by the addition of a premiss. For 'A man is running. Therefore an animal is running' is valid because it is impossible to be a man without being an animal. And if the proposition 'Every man is an animal', which is a necessary proposition, is added to the inference as a premiss, then the inference is transformed from one valid materially into one valid formally. It might, in that case, be said that the original inference was recognized to be valid because the premiss 'Every man is an animal' was, so to say, read into the inference, so that that additional proposition was seen as virtually or implicitly present. And if that is said then perhaps the conclusion to be drawn is that so-called materially valid inferences are formally valid inferences one of whose premisses is unstated. Most of our arguments in ordinary life are like that. Some of the most hard-working premisses are not stated because they are so obviously true, and so obviously at work, that there is no need to state them.

We turn now to a further distinction between inferences. Some are valid simply, and some valid *ut nunc*, that is, as of

⁷ Ibid. 23; cf. Albert of Saxony, *Perutilis Logica* 24^{rb}.

now, or at present.⁸ A simply valid inference is one which, without anything added to it, satisfies the earlier account of validity conditions, namely: it is impossible for things to be as signified by the antecedent and not be as signified by the consequent. On these grounds a materially valid inference can be simply valid even if it is interpreted as an inference which has a necessarily true premiss left unstated. Thus, it is impossible for things to be as signified by 'A man is running' yet not be as signified by 'An animal is running', and hence 'A man is running. Therefore an animal is running' is a simply valid material inference.

Part of the reason for the fact that 'A man is running. Therefore an animal is running' is valid is that 'Every man is an animal' is a necessary truth. That proposition is not just true of the world as at present constituted, but for all time and all space if there exists anything that the term 'man' signifies, then that very same thing is an animal. Put otherwise, whenever 'A man is running' is said truly, at that same time 'An animal is running' would, if said, be true. But there are other inferences which are valid, not because of something which is always and necessarily so, but because of something which is so now. Thus 'Tom is talking. Therefore a pianist is talking' is valid, not because of something which is always and necessarily so, but because Tom is a pianist, which is now true and certainly is not always true—for when he does not exist he is not then anything and therefore is not a pianist. An inference which is valid given the way things are now, is said to be valid *ut nunc*.

Inferences valid *ut nunc* have an important feature in common with materially valid inferences, namely, that by the addition of an appropriate premiss they can be 'reduced' to formally valid inferences. The appropriate premiss in the case of the inference valid *ut nunc* is a proposition stating a relevant fact about the way things are now. Thus if we add the premiss 'Tom is a pianist' to the premiss 'Tom is

⁸ Buridan, *Cons.* 23-4.

talking', the conclusion 'A pianist is talking' follows as the conclusion of a formally valid inference.

As we have already noted, it was generally held that an inference is valid if it contains as a premiss an impossible proposition. A parallel situation arises in the case of inferences valid *ut nunc*. For it can be shown that an *ut nunc* inference is valid if it contains as a premiss a false proposition, and its validity does not depend in any way on the truth value of the conclusion. Let us take two letters P and Q to be abbreviated propositions. It does not matter what exactly these propositions signify, but let us say that, things being as they are now, P happens to be false. We can now set up the following line of argument:

- (1) P This is false now.
- (2) P or Q From 1, for a disjunction is implied by each disjunct.
- (3) Not P This describes how things are now.
- (4) Q From 2 and 3, for the denial of a disjunct implies the other disjunct.

It follows from this that in an *ut nunc* inference anything whatever can be proved to follow from a proposition false *ut nunc*. And the underlying reason for this is closely allied to the fact that from an impossible proposition anything follows. For let us assume a proposition which is false *ut nunc*. Since it is false its negation is true. And since its negation is true let us assume that negation. In that case we have now assumed a proposition and its negation, and from those two propositions there follows their conjunction. But that conjunction is a contradiction, and a contradiction is a paradigm case of an impossible proposition. From an impossible proposition, however, anything follows. In particular, where a proposition is impossible because it is a contradiction it is easy to show that from it anything whatever follows in a formally valid argument. Thus, just as an *ut nunc* inference which is valid can be transformed into a formally valid inference by the addition of the relevant *ut*

nunc true proposition, so also where a proposition is known to be *ut nunc* false, an inference in which a conclusion is drawn from that false proposition can be transformed into a formally valid inference by the addition of the relevant *ut nunc* true proposition.

Buridan points out that in a sense propositions which are true *ut nunc* are a special group within a wider class of propositions. Starting with our perceptions as they are now (*ut nunc*), we can ask what follows about what else must be the case now (*ut nunc*). But we can address ourselves to the past or future state of things also. Starting from our anticipation of how things will be, we can ask what follows about what else will be the case then (*ut tunc*). And starting from our memory of how things were, we can ask what follows about what else must have been the case then (*ut tunc*). Hence as well as *ut nunc* inferences Buridan speaks of inferences which are valid *ut tunc* ('as of then' as opposed to 'as of now').⁹

Burley, however, in his discussion of inferences valid *ut nunc* gives an account of such inferences which applies equally well to inferences valid *ut nunc* and *ut tunc*. He writes that such an inference 'holds for a determinate time and not always, for example: "Every man is running. Therefore Socrates is running". For this inference does not hold for all time, but only while Socrates is a man."¹⁰ On the basis of this account we should have to call the following an inference valid *ut nunc*: 'John will run. Therefore a man will run'. It is not the case that, things being as they are now, the conclusion follows from the premiss (for John—let us suppose—is not now alive); but given that things will be as they will be, and in particular given that John will be a man, the conclusion follows from the premiss. There is here a difference in terminology between Burley and Buridan, for Buridan would not class the foregoing model inference as valid *ut nunc*. But nothing of logical, as opposed to terminological, significance appears to be at stake in this

⁹ Ibid. 24.

¹⁰ *De Puritate*, 61.

difference of opinion. For both men would classify the inference as valid, and would do so for the same reason. And each would say that by the addition of the appropriate contingent proposition the inference could be transformed into one that is formally valid.

Validity Conditions and Unanalysed Propositions

I. ANALYSED AND UNANALYSED PROPOSITIONS

In our investigation of the way various syncategorematic terms signify, we began by investigating syncategorematic terms which appear characteristically within a categorical proposition, or at least which can appear within or as a prefix to a categorical proposition. Then, in Chapter 4, we turned to a consideration of syncategorematic terms which characteristically connect categorical propositions, so forming molecular propositions out of categoricals. In the earlier phases of medieval logic the theory of inference was treated as a theory applying specifically to propositions in so far as those propositions displayed the form of categoricals. I shall say that rules of valid inference designed to deal with the inferential power of propositions which are specifically categorical are rules for analysed propositions.

But there are also rules of inference which, though applicable to categorical propositions, are applicable independently of the internal structure of those propositions. Instead they apply to categorical propositions merely as propositions, and they apply to molecular propositions merely as molecular. For such rules it is the propositional connectives and the sign of negation that are important, and the quantifiers are of no importance whatever. Rules of the latter kind are rules for what I shall term 'unanalysed propositions'. The expression is not entirely satisfactory,

for the rules do deal with propositions which are analysed in so far as they are identified as molecular propositions, though they do not deal with propositions which are analysed in so far as they are categoricals containing given quantifiers.

The distinction I am making here corresponds to that between inference rules for the predicate calculus and for the propositional calculus. It is well recognized that in the correct order of exposition the propositional calculus comes first, but that this is the correct order was not always obvious. It was not until the fourteenth century that there came to be a reasonably widespread recognition of the importance of expounding the logic of molecular propositions before the logic of categorical propositions. Recognition of its priority appears to have given great impetus to the study of rules of inference for molecular propositions. Certainly in that century great strides were made in that area of logic. Conspicuous amongst the logicians who recognized the priority of propositional logic were Burley, Buridan, and Albert of Saxony. In their writings, that part of the theory of valid inference which has special reference to the kind of quantifier expressions propositions contain is discussed only after a preliminary discussion of that part of the theory of valid inference which has special reference to negation signs and to the kinds of connectives that form molecular propositions. I shall begin my examination of individual rules of valid inference by considering rules applicable to unanalysed propositions. The following symbols will be employed: '¬' (a minus sign), read as 'It is not the case that' or 'not', to symbolize negation; an ampersand, '&', read as 'and', to symbolize conjunction; 'v' (the initial letter of the Latin *vel* = 'or'), read as 'or', to symbolize disjunction; '→' to symbolize illative conditionality; '↔' to symbolize illative equivalence; 'pos' to symbolize 'It is possible that'; and 'nec' to symbolize 'It is necessary that'.

II. DOUBLE NEGATION

Most systems of logic are constructed on the assumption that there are just two truth values, true and false. Other systems, as is well known, are constructed on the assumption that there are more. One logical rule at stake here is that of double negation. Put in semantic terms, this rule states that the truth of a proposition is equivalent to the falsity of its negation. A bivalent system, one assuming that there are just two truth values, accepts this rule. But if there are three values, say, true, false, and undecidable, then the rule is at risk; that the negation of a proposition is false does not imply that the proposition itself is true, for two values remain, not one. It might after all be an undecidable proposition rather than a true one. Medieval logic was not free from disputes about the number of truth values there are. And some logical speculations, prompted by Aristotle's account of the sea battle tomorrow, focused on the idea that a third truth value, specially reserved for propositions about future contingent events, may have to be countenanced.¹

In the past century pressure for a three-valued logic has been thought to arise from the fact that it is possible to construct propositions containing a referring expression which fails to refer. We have noted in an earlier chapter that medieval logicians faced the problem of the ascription of a truth value to propositions of the kind just described. And their conclusion left intact the fundamental assumption of the bivalence of logic, for they held that if an affirmative proposition contains a subject or a predicate which stands for nothing, then that proposition is false. If no chimera exists then no chimera is anything, and therefore no chimera is white, and in that case the proposition that a chimera is white is false; and for the same reason no chimera is a chimera—despite Boethius' dictum that no proposition is a truer predication than one in which something is predicated

¹ See e.g. L. Baudry, *La Querelle des futurs contingents*; also *CHLMP* ch. 18.

of itself.² The point here is that an affirmative subject-predicate proposition was held to make two existential assumptions, namely, that there is some thing for which the subject stands, and something for which the predicate stands. If either of these existential assumptions is false for a given affirmative proposition then that proposition is false. Buridan lined himself up very firmly with this approach. Nor does he seem tempted by any other consideration to abandon the principle of bivalence. Hence he introduces his list of rules of inference with a statement which proclaims without qualification the principle of bivalence:

Of every contradiction one of the contradictories is true and the other false, and it is impossible for both to be true together or false together. Also, every proposition is true or false, and it is impossible for the same proposition to be true and false at the same time.³

The proposition that every proposition is true or false could be symbolized as: $T(P) \vee F(P)$. I shall however present it shorn of its semantic garb:

$$(1) P \vee \neg P.$$

The proposition that it is impossible for the same proposition to be true and false at the same time could be symbolized as: $\neg \text{pos}[T(P) \& F(P)]$. I shall however present it shorn of its semantic garb:

$$(2) \neg \text{pos}(P \& \neg P).$$

The first part of the above quotation from Buridan can be seen to embody the two parts of the rule of double negation. Let us take P and $\neg P$ as our two contradictory propositions. If P is true then $\neg P$ is false, and if $\neg P$ is false then P is true. Expressed syntactically these yield the two rules:

² Boethius, *In Librum de Interpretatione*, *Patrologia Latina*, 64, 577; and Buridan, *Soph.*, 36⁶-7, 46⁶-7.
³ *Cons.*, 31.

(3) $P \therefore \neg \neg P$.

(4) $\neg \neg P \therefore P$.

Rules 3 and 4 combine to sanction the rule that a proposition is equivalent to itself twice negated, that is:

(5) $P \leftrightarrow \neg \neg P$.

This rule is stated most simply by Burley as: 'Two negations make an affirmation'.⁴ He understands this rule to be saying that 'two negations, of which one is so related to the other that it negates the other, makes an affirmation, even if the two are entirely related to the same thing'. Burley's exposition is intended to focus on the fact that the rule does not apply to every proposition containing two negation signs, but only to those propositions in which there is what he terms a 'negation of a negation'. For it is not true of all propositions containing two negation signs that the first negates the second; the scope of the first may fall short of the second. Burley gives the example:

(i) A man, who is not moving, is not running.⁵

The logically significant feature of this example is that the power of the first negation sign does not extend beyond the relative clause. And since that first negation sign does not cover the second, rule 5 cannot be applied to draw the conclusion that the example is equivalent to

(ii) A man, who is moving, is running.

If there are just two men of whom one is not moving and the other is moving though he is not running, then (i) is true and (ii) false. The point here is that the non-equivalence of (i) and (ii) is not a reason for rejecting rule 5; it merely shows that (i) does not feature a negation of a negation. Consider now

⁴ *De Puritate*, 226.

⁵ *Ibid.* 227.

- (iii) No man, who is moving, is not running.

This features a negation of a negation, and hence (iii) is equivalent to a purely affirmative proposition, namely,

- (iv) Every man, who is moving, is running.

It is with such examples in mind that Burley refers to the fact that the negation signs must be 'related to the same thing', for where there is a negation of a negation the scope of the second negation is included in the scope of the first. In the extreme case, also noted by Burley, the scope of the two negations is identical, except of course for the fact that the scope of the first negation sign includes the second sign, as in

- (v) It is not the case that no man is running.

By rule 5 example (v) must be equivalent to a purely affirmative proposition. The proposition in question is

- (vi) Some man is running.

The nature of the equivalence relation between a proposition and that proposition doubly negated must be made plain, given that medieval logicians distinguished between two sorts of equivalence, equivalence (*a*) in inferring, and (*b*) in signifying. Two propositions are equivalent in inferring if whatever follows from either follows from the other. Two propositions are equivalent in signifying if they have the same signification. Two propositions are equivalent in the second sense if they are subordinate to the same mental proposition. But in that sense a proposition and itself doubly negated are not equivalent, for a certain concept, that of negation, expressed twice in the one proposition is not expressed at all in the other. If a proposition prefaced by two negation signs were equivalent in signifying to that same proposition but lacking the two negative prefixes, then we could not even form a concept of a doubly negated, as opposed to an unnegated, proposition. And in that case we could not conceive the rule of double negation.

III. SOME BASIC RULES OF INFERENCE

In this section we shall consider certain rules of inference considered basic by the logicians with whom we are chiefly concerned. The order is dictated largely by the order that Buridan adopted in his *De Consequentis*. After laying down the rules, considered in the preceding section, in which he establishes the bivalence of his logic, Buridan turns to a consideration of certain rules of valid inference, and he starts by discussing rules containing the modal operators 'It is possible that' and 'It is necessary that'. The first two rules are as follows: 'From every impossible proposition every other follows, and every necessary proposition follows from every other.'⁶

(6) $\neg\text{pos}P \therefore P \rightarrow Q$.

(7) $\text{nec}P \therefore Q \rightarrow P$.

Buridan takes these rules to be apparent from a consideration of the definitions of 'antecedent' and 'consequent'. Howsoever an impossible proposition signifies things to be, it is impossible that they be so. And therefore it is impossible that things be as that proposition signifies them to be without also being as any other proposition signifies them to be. Put plainly, given that a certain proposition is impossible, then if its being impossible is not a barrier to its being true then anything can be true. And likewise, howsoever a necessary proposition signifies things to be it is impossible that they not be so. Therefore, howsoever any other proposition signifies things to be it is impossible that they be so without also being as the necessary proposition signifies them to be. These two rules correspond to the so-called 'paradoxes of strict implication', but expounded as Buridan expounds them no air of paradox lingers.

Modern logic also has paradoxes of material implication—that from a false proposition any proposition follows, and that a true proposition follows from any

⁶ *Cons.*, 31.

proposition. Medieval logic has analogues to these so-called paradoxes. But they are to be found in inferences valid *ut nunc* rather than valid simply. Buridan writes: 'From every false proposition every other proposition follows in an *ut nunc* inference. And every true proposition follows from every other proposition in an *ut nunc* inference.'⁷ We saw in the preceding chapter that any conclusion can be drawn from a false proposition in an *ut nunc* inference. That a true proposition follows from any proposition in such an inference can also readily be shown. For the premiss is either true or false. If false, then, as shown earlier, the true proposition follows since any proposition follows. And if the premiss is true, then given that the conclusion also is true, it is clearly impossible that, things being as they are now, the premiss can be true without the conclusion being true, for as things are now the conclusion is true.

The next two rules are simply stated: 'From every proposition there follows every other whose contradictory cannot be true together with the first; and from no proposition does there follow another whose contradictory can be true at the same time as the first.'⁸ In symbols:

$$(8) \neg \text{pos} (P \& Q) \therefore P \rightarrow \neg Q$$

$$(9) \text{pos} (P \& Q) \therefore \neg (P \rightarrow \neg Q).$$

The arguments for these two rules are as follows: let us assume that P and Q are incompatible—a relation signified by $\neg \text{pos}(P \& Q)$. As regards rule 8, either P is impossible or it is not. If it is impossible then, in accordance with rule 5, it implies anything, and in that case implies $\neg Q$. Alternatively P can be true. If it is true then at the same time either Q or $\neg Q$ is true, in accordance with rule 1. But we have assumed that it is impossible for P and Q to be true together. And hence, if P is true then Q is not. Regarding rule 9, if P and Q are compatible then they can be true at the same time. And in that case the truth of P cannot imply the falsity of

⁷ *Cons.*, 32.

⁸ *Ibid.*

Q. Therefore, given the compatibility of P and Q, P does not imply the contradictory of Q.

The reverse of rule 8 was also accepted: 'For a conjunctive proposition to be impossible it is sufficient that its parts be incompatible.'⁹ One way to expound incompatibility semantically is this: two propositions are incompatible if they cannot be true together. The syntactic version of this account is: two propositions are incompatible if one implies the negation of the other. Let us follow this latter mode of expression and symbolize 'P is incompatible with Q' as $P \rightarrow \neg Q$. Albert's rule, just quoted, can now be symbolized as:

$$(10) P \rightarrow \neg Q \therefore \neg \text{pos} (P \& Q).$$

Replacing Q by $\neg Q$ systematically in rules 8 and 10 we reach:

$$(11) \neg \text{pos} (P \& \neg Q) \therefore P \rightarrow \neg \neg Q.$$

$$(12) P \rightarrow \neg \neg Q \therefore \neg \text{pos} (P \& \neg Q).$$

But rule 5 states that any proposition is equivalent to its double negation, and hence in 11 and 12 $\neg \neg Q$ is replaceable by Q, yielding:

$$(13) \neg \text{pos} (P \& \neg Q) \therefore P \rightarrow Q$$

$$(14) P \rightarrow Q \therefore \neg \text{pos} (P \& \neg Q).$$

It is routine in modern logic to introduce certain logical operators as primitive and to use them to define others which will then be derivative operators. Such an approach to the ordering of operators was not characteristic of medieval logic, though it is clear that it had ample resources to proceed in that way. One move in this direction could have been based on rules 13 and 14. For those rules suggest the possibility of illative conditionality being defined in terms of possibility, conjunction, and negation:

$$\text{Df. } 1: \rightarrow P \rightarrow Q = \text{df } \neg \text{pos} (P \& \neg Q).$$

⁹ Albert of Saxony, *Perutilis Logica*, 19^{rb}.

Replacing Q by P throughout rule 13 we reach:

$$(15) \neg \text{pos} (P \& \neg P) \therefore P \rightarrow P.$$

But the premiss in 15 is rule 2. Hence the conclusion of rule 15 can be presented as a rule:

$$(16) P \rightarrow P.$$

Since rule 16 is the conclusion of 15, the denial of 16 implies the denial of the premiss in 15. The denial of the premiss is, however, equivalent to the affirmation of the possibility of a contradiction. A contradiction is the paradigm case, for medieval as for modern logicians, of an impossibility. Rule 15 is therefore perfectly secure. But medieval logicians were interested in the question of what, if anything, followed from the conditional $P \rightarrow \neg P$. Evidently it cannot be laid down as a rule that every proposition implies its own negation. But that is not at issue. The question is only whether something can be learned about any given proposition if its negation follows from its affirmation. There are such propositions, in particular, those which are said to 'include opposites'. Burley discusses the curious proposition: (a) 'You know that you are a stone.'¹⁰ The proposition implies (b) 'You are a stone', since what is known is true. It also implies (c) 'You are a knower'. But stones are not knowers, and therefore (a) implies (d) 'You are not a stone'. Since (a) implies the mutually contradictory (b) and (d), (a) is said to 'include opposites'. Given (d), which plainly follows from (a), we can conclude (e) 'You do not know that you are a stone', for given the truth of 'You are not a stone', 'You are a stone' must be false, and what is false cannot be known to be true. Hence (a) implies its own negation. Clearly (a) cannot be true. It is, in medieval jargon, 'virtually contradictory', and hence, howsoever it signifies things to be, they cannot be so. The conclusion to be drawn is that since (a) implies its own negation it must itself be denied.

¹⁰ *De Puritate*, 70.

Burley formulates the underlying rule of inference in this way: 'Every proposition which includes opposites implies its own contradictory.'¹¹ The kind of proposition which most obviously includes opposites is an explicitly contradictory proposition, one of the form $P \& \neg P$. But propositions which are implicitly contradictory do not any the less contain opposites, and Burley's rule must apply to them no less than to those which are explicitly contradictory. It therefore applies to the proposition 'Every proposition is true', for if that proposition is prefaced by a negation sign then the proposition thus formed is true, and if that is true then the first is false, and should be denied. And the proof that it should be denied is precisely that its very affirmation implies its negation. Burley's rule can be symbolized:

$$(17) P \rightarrow \neg P \therefore \neg P.$$

Replacing P by $\neg P$ in (17), and then applying the rule of double negation to the result, we reach:

$$(18) \neg P \rightarrow P \therefore P$$

Jan Łukasiewicz titles this rule, or rather the corresponding implication $(\neg P \rightarrow P) \rightarrow P$, 'the law of Clavius', after a sixteenth-century Jesuit who commented on Euclid's use of the law in his proof of the theorem 'If a^2 is divisible by a prime number n , then a is divisible by n .'¹²

We should also bear in mind here the definition, given above, of illative conditionality. $P \rightarrow Q$ is a shorthand form of $\neg \text{pos}(P \& \neg Q)$. Replacing Q by $\neg P$ in the definiendum, $P \rightarrow Q$, and in the definiens, we reach $P \rightarrow \neg P$ as a shorthand form of $\neg \text{pos}(P \& \neg \neg P)$, which itself is equivalent to $\neg \text{pos} P$. By this means the equivalence of 'P implies its own negation' and 'P is impossible' is easily established. Put otherwise, given that a conditional is true if the antecedent is incompatible with the negation of the consequent, it follows that if P implies $\neg P$ then P is incompatible with $\neg \neg P$, that is, with P , and hence P must be incompatible with itself. But any proposition

¹¹ Ibid.

¹² See J. Łukasiewicz, *Aristotle's Syllogistic*, 50-1, 80.

incompatible with itself is impossible. So the affirmation $P \rightarrow \neg P$ permits the inference of $\neg \text{pos}P$, and therefore of $\neg P$. John Mair discusses the example 'A man is a donkey. Therefore no man is a donkey'.¹³ The point he has in mind is that 'A man is a donkey' contains opposites, since it implies 'A rational animal is a non-rational animal'. 'A man is a donkey' is therefore incompatible with itself since it affirms of something both that it is rational and that it is non-rational. And such a proposition, since it cannot be truly affirmed, must be denied.

Since we have been employing a concept of illative conditionality defined in terms of the impossibility of a conjunction, certain rules should here be added in clarification of the modal concept in question. Ockham writes: 'For a [conjunction] to be possible it is necessary that each part be possible.'¹⁴ This yields three rules:

- (19) $\text{pos}(P \& Q) \therefore \text{pos}P$
- (20) $\text{pos}(P \& Q) \therefore \text{pos}Q$
- (21) $\text{pos}(P \& Q) \therefore \text{pos}P \& \text{pos}Q$.

However, for the impossibility of a conjunction it is not necessary that each part be impossible, for, as Ockham points out, a conjunction of mutually contradictory contingent propositions is impossible and yet, by definition of 'contingent', each of the parts is possible. Ockham adds: 'For a conjunction to be impossible it is necessary either that one or other of the parts be impossible or that one be impossible with the other.'¹⁵ This may be expressed as:

- (22) $\neg \text{pos}(P \& Q) \therefore \neg \text{pos}P \vee \neg \text{pos}Q \vee P \rightarrow \neg Q$.

In fact, as Ockham was aware, each of the three disjuncts in the conclusion of rule 22 is by itself sufficient as a premiss for which $\neg \text{pos}(P \& Q)$ is a validly drawn conclusion. We can therefore add the following three rules:

¹³ *Introductorium*, 59^v.

¹⁴ *Summa Logicae*, Pt. II, Ch. 32, p. 348.

¹⁵ *Ibid.*

- (23) $\neg \text{pos}P \therefore \neg \text{pos}(P \& Q)$
- (24) $\neg \text{pos}Q \therefore \neg \text{pos}(P \& Q)$
- (25) $P \rightarrow \neg Q \therefore \neg \text{pos}(P \& Q)$

To complete this section on inferences involving modal operators, certain commonly formulated rules involving modal operators operating on disjunctions will here be listed. Albert writes:

For the possibility of [a disjunctive proposition] it is sufficient that either part be possible, for if a disjunctive proposition is possible it can be true but not without either of its parts [being true]. But for the impossibility of a disjunctive proposition it is necessary that each of its parts be impossible, for the disjunction follows from each of its parts.¹⁶

There are at least four rules of inference encapsulated in this brief passage:

- (26) $\text{pos}P \therefore \text{pos}(P \vee Q)$
- (27) $\text{pos}Q \therefore \text{pos}(P \vee Q)$
- (28) $\neg \text{pos}(P \vee Q) \therefore \neg \text{pos}P$
- (29) $\neg \text{pos}(P \vee Q) \therefore \neg \text{pos}Q$

It is notable that the rules concerning conjunctions and disjunctions are symmetrical. The possibility of a conjunction implies the possibility of each conjunct (rules 19 and 20), and the possibility of a disjunction is implied by the possibility of each disjunct (rules 26 and 27). The impossibility of a conjunction is implied by the impossibility of each conjunct (rules 23 and 24), and the impossibility of a disjunction implies the impossibility of each disjunct. Not surprisingly, the rules concerning the necessity of conjunctions and disjunctions show a like symmetry. The necessity of a conjunction implies the necessity of each conjunct, and the necessity of a disjunction is implied by the necessity of each disjunct. But it is not the case that if a disjunction is necessary one or other of its disjuncts is necessary. Both

¹⁶ *Perutilis Logica*, 19^{va}.

Ockham and Albert make the point that if the disjuncts are mutually contradictory the disjunction is necessary. And neither of the contradictory propositions need be necessary. Their mutual contradictoriness is, by itself, sufficient to secure the necessity of the disjunction. But there is another relation which can obtain between the disjuncts and which would secure the necessity of the disjunction, namely, subcontrariety. Thus the disjunction of the propositions 'Some man is disputing' and 'Some man is not disputing' is necessary because the negation of either of these propositions implies the other proposition. Contradictories and subcontraries share the feature that the negation of either proposition implies the other proposition (the law of contradiction adds that either proposition implies the negation of the other). We can therefore add a further rule:

$$(30) \neg P \rightarrow Q \therefore \text{nec}(P \vee Q).$$

IV. *MODUS PONENS* AND *MODUS TOLLENS*

In this section we shall mainly be concerned with a number of rules bearing a more or less close relation to two rules familiar to us under the names *modus ponens* and *modus tollens*. We shall deal first with *modus ponens*. Ockham writes: 'From a conditional and the antecedent . . . of that conditional the consequent always follows.'¹⁷ Paul of Venice adds a detail: 'If the antecedent of a sound inference is true, then the consequent likewise is true. For from something false something true can follow, but from something true nothing except something true follows.'¹⁸ An obvious way to symbolize this rule is:

$$(31) P \rightarrow Q, P \therefore Q.$$

If, therefore, it could be the case that *P* were true without *Q* being true, then *Q* does not follow from *P*; that is, does not

¹⁷ *Summa Logicae*, Pt. III-1, Ch. 68, p. 501.

¹⁸ *Logica*, 68.

ever follow from P, for here we are dealing with simple, as opposed to *ut nunc* inference. As regards rule 31, if the arrow is interpreted as 'implies *ut nunc*' then $P \rightarrow Q$ is false if, things being as they are now, P is true and Q false.

A stretched version of rule 31 was frequently invoked. For given that a certain inference is valid, then not only does the consequent follow from the antecedent, but additionally (a) whatever follows from the consequent follows from the antecedent. Or, to consider the same point from the opposite direction, (b) whatever is antecedent to the antecedent is antecedent to the consequent.¹⁹ These rules differ only in the order of the premisses. For given a certain conditional $P \rightarrow Q$, anything, say R, which follows from Q follows also from P, and given $Q \rightarrow R$, then anything, say P, which is antecedent to Q is antecedent to R. Thus (a) and (b) can be represented as:

$$(32) P \rightarrow Q, Q \rightarrow R \therefore P \rightarrow R.$$

$$(33) Q \rightarrow R, P \rightarrow Q \therefore P \rightarrow R.$$

Rule 31 is the limiting case of a general rule, and in relation to that general rule the neighbour of rule 31 is 32. The general rule is as follows. Given a series of conditionals so related to each other that, except in the case of the first conditional, the antecedent in each conditional is equiform to the consequent in the immediately preceding conditional, then given the antecedent in the first conditional the consequent in the last can be inferred. Or, to be as precise as our logicians were, from a proposition equiform to the antecedent in the first conditional there follows a proposition equiform to the consequent in the last. This form of argument was known as 'inference from first to last', and it was recognized that in a single inference many conditionals could occur as premisses between the first antecedent and the last consequent. The following, therefore, is a rule:

¹⁹ For (a) and (b) see Burley, *De Puritate*, 200.

(34) $P \rightarrow Q, Q \rightarrow R, R \rightarrow S \therefore P \rightarrow S$.

But it should be noted that, given rule 32, 34 is redundant. For Rule 32 can be invoked to infer $P \rightarrow R$ from the first two premisses in 34, and from $P \rightarrow R$ plus the third premiss in 34 the conclusion in 34, $P \rightarrow S$, can be drawn, again by rule 32. The point that it was unnecessary to employ more than two conditionals as premisses in order to prove any valid argument that relies on the rule 'from first to last' was known, though not considered of great importance. Much more importance was attached to the requirement that where rule 32 is applied to a pair of conditionals, the consequent in the first conditional and the antecedent in the second should be subordinate to the same mental proposition. That is, physical equiformity is not sufficient to ensure the preservation of truth from premisses to conclusion.

Burley gives a number of examples in illustration of the need to ensure that the consequent in the one conditional and the antecedent in the next one should be not only physically equiform but also subordinate to the same mental proposition. His examples include:

- (a) The more you are ugly the more you adorn yourself. The more you adorn yourself the more you are beautiful. Therefore, from first to last, the more you are ugly the more you are beautiful.
- (b) The more you are thirsty the more you drink. The more you drink the less you are thirsty. Therefore, from first to last, the more you are thirsty the less you are thirsty.²⁰

Evidently (a) and (b) are to be understood as containing conditional premisses. They could be rewritten so that each begins with 'if', but I shall retain Burley's mode of expression. His point is that both examples commit the fallacy of equivocation by virtue of trading on an ambiguity

²⁰ For (a) and (b) see Burley, *De Puritate*, 70.

in 'the more'. This ambiguity is brought out if the examples are rewritten to display more perspicuously the relations between 'the more' in the antecedent and the consequent in each conditional. Let us attend to (b) — the same considerations apply to (a). (b) signifies the same thing as:

- (c) By however much more you are thirsty, by that much more you drink. By however much more you drink, by that much more you are less thirsty. Therefore by however much more you are thirsty, by that much more you are less thirsty.

And when (b) is written in this way, rule 32 cannot be applied to it, since (b) is the wrong shape. That is, the consequent in the first conditional is not equiform to the antecedent in the second conditional.

A point of criticism is in order here. It is true, for the reason given by Burley, that inferences of the kind that may be called 'comparative molecular syllogisms' are not simply molecular syllogisms with the consequent of the one premiss equiform to the antecedent of the other. But this does not supply a rationale for their invalidity, for many comparative molecular syllogisms are valid. One such is the following:

- (d) The more people come to the party, the more noise there is. The more noise there is, the madder the neighbours will get. Therefore the more people come to the party, the madder the neighbours will get.

What underlying difference there is between valid and invalid cases is still obscure.

A further apparent counter-example to rule 32, discussed by Burley, involves interesting features not displayed by examples (a) or (b). This is the example:

- (e) If I say that you are a donkey I say that you are an animal. If I say that you are an animal I say the truth. Therefore if I say that you are a donkey I say the truth.

For good measure Burley adds that therefore this is the truth: 'You are a donkey.'²¹ The first premiss must, it seems, be accepted. If I say that you are a donkey I say that you are an animal. (I say in fact that you are an animal of the donkey kind.) The second premiss also seems acceptable. Given that you are a human being you are an animal, and therefore in saying that you are an animal I say the truth. Arguing from first to last in accordance with rule 32 we reach Burley's conclusion.

Now, in (e) the first premiss is ambiguous. It can be understood to affirm: 'If I say "You are a donkey" I say "You are an animal"', and in that case the premiss is false. And in that case the inference does not proceed from truth to falsity. Or it can be understood to affirm that if I say of you that you are a donkey then I am saying of you that you are some kind of an animal. In that case the premiss is true. But it might then be argued that the conclusion is after all true. That is, if I say that you are a donkey I do say the truth. The truth I say, however, is not 'You are a donkey'. It is instead something implied by 'You are a donkey', namely, that you are an animal. That is, if in saying that you are a donkey I am understood to be saying, amongst other things, that you are an animal, then in so far as I say that you are an animal I am saying the truth. Of course I am not saying nothing but the truth. But it is not stated that I am saying nothing but the truth. Clearly the reason why it seems that the conclusion in Burley's example must be false is that we naturally understand it to be saying not only that I say the truth but also that the truth I say is 'You are a donkey'.

The point emerges plainly if we take the example of the affirmation of a contradiction in which the second conjunct is true. In such a case, when I say the contradiction I say the second conjunct. And in saying the second conjunct I say the truth. Therefore in saying the contradiction I say the truth. What truth? The truth which forms one principal part of the contradiction. In that case what looks like a

²¹ *De Puritate*, 203 4.

counter-example to rule 32 is seen to be no counter-example at all but to be fully and properly sanctioned by the rule.

Further rules related to those just formulated will now be given. 'Whatever follows from the antecedent and the consequent follows from the antecedent by itself',²² that is:

$$(35) P \rightarrow Q, (P \& Q) \rightarrow R \therefore P \rightarrow R.$$

The justification of rule 35 is as follows. By rule 16, every proposition implies itself. Therefore if P implies Q it implies itself and Q .²³ Hence, given $P \rightarrow Q$, this follows: $P \rightarrow (P \& Q)$. But $P \rightarrow (P \& Q)$ plus the second premiss in 35 jointly yield the conclusion of 35, by rule 32.

'From the antecedent with something added there follows the consequent with the same thing added':²⁴

$$(36) P \rightarrow Q \therefore (P \& R) \rightarrow (Q \& R)$$

The rule can also be expressed as:

$$(37) P \rightarrow Q, P \& R \therefore Q \& R.$$

The rule can easily be justified. P follows from $P \& R$. And from P plus the first premiss of 37, Q can be derived. R follows from $P \& R$. Hence both Q and R , and therefore also $Q \& R$, follow from the premisses of 37.

'Whatever follows from the consequent with something added follows from the antecedent with the same thing added':²⁵

$$(38) P \rightarrow Q, (Q \& R) \rightarrow S \therefore (P \& R) \rightarrow S.$$

Burley bases his argument for rule 38 on rule 37. From the antecedent with something added there follows the consequent with the same thing added. But (rule 33) whatever

²² Burley, *De Puritate*, 62.

²³ See Ockham, *Summa Logicae*, Pt. II, Ch. 32, p. 349: 'If part of a conjunction implies the other part then from that first part to the whole conjunction is a sound inference.'

²⁴ Burley, *De Puritate*, 62.

²⁵ Ibid. 62.

follows from the consequent follows from the antecedent. Therefore whatever follows from the consequent with something added follows from the antecedent with the same thing added.

‘Whatever is compatible with the antecedent is compatible with the consequent’:²⁶

$$(39) P \rightarrow Q, \text{pos}(P \& R) \therefore \text{pos}(Q \& R).$$

Suppose that P is true. In that case, given the first premiss of 39, Q is true. And if R, which (by the second premiss) can be true at the same time as P, is in fact true at the same time, it will be true when Q is true. Therefore Q and R can be true together, that is, $\text{pos}(Q \& R)$.

‘If antecedents are compatible, consequents also are compatible’:²⁷

$$(40) P \rightarrow Q, R \rightarrow S, \text{pos}(P \& R) \therefore \text{pos}(Q \& S).$$

For let us suppose that P and R are mutually compatible. That is, they can be true at the same time. If they ever are in fact true together, at that time by rule 31 Q must be true and S also. Therefore Q and S are compatible.

We turn now to the rule commonly named *modus tollens* (= ‘the denying way’ or ‘the way of denial’). Paul of Venice states it as follows: ‘From an affirmative conditional along with the contradictory of the consequent, to the contradictory of the antecedent is a sound inference’:²⁸

$$(41) P \rightarrow Q, \neg Q \therefore \neg P.$$

This rule was recognized as standing in a close relation to another: ‘Of every sound inference, from the contradictory of the consequent there follows the contradictory of the antecedent’,²⁹ which can be symbolized as:

$$(42) P \rightarrow Q \therefore \neg Q \rightarrow \neg P.$$

²⁶ Burley, *De Puritate*, 63.

²⁷ Ibid. 63.

²⁸ *Logica*, 80.

²⁹ Buridan, *Cons.*, 33.

This, in turn, is closely related to: 'Every proposition, formed as an inference, is a sound inference if from the contradictory of a proposition equiform to the consequent there follows the contradictory of a proposition equiform to the antecedent',³⁰ which I shall symbolize as:

$$(43) \neg Q \rightarrow \neg P \therefore P \rightarrow Q.$$

My formulations 42 and 43 may not represent Buridan's intentions with complete accuracy. His wording suggests that he may have in mind the justification of the claim that an inference of one kind is valid by reference to the fact that an inference of another kind is valid. For example, the rule I symbolize as 43 should perhaps be taken to say that, given the validity of $\neg Q \therefore \neg P$, it follows that $P \therefore Q$ is also valid. But I shall stay with the formulations 42 and 43.

A proof of 42 was given, which proceeded by assuming the rule unsound and deriving an absurdity from that assumption. Let us assume $P \rightarrow Q$ and also $\neg(\neg Q \rightarrow \neg P)$. Now, by rules 12 and 13, $\neg Q \rightarrow \neg P$ is equivalent to $\neg \text{pos}(\neg Q \& P)$, and thus the denial of that, which we are assuming, is equivalent to $\neg \neg \text{pos}(\neg Q \& P)$, and hence to $\text{pos}(\neg Q \& P)$. By rule 39 whatever is consistent with the antecedent of $P \rightarrow Q$ is consistent with the consequent. Therefore, if $\neg Q$ is consistent with the antecedent of $P \rightarrow Q$ (which is what our second assumption implies, since $\neg(\neg Q \rightarrow \neg P)$ implies $\text{pos}(\neg Q \& P)$), then $\neg Q$ also is consistent with the consequent of $P \rightarrow Q$. That is, $\neg Q$ is consistent with Q . But that is impossible. Given that our two assumptions yield a contradiction, if we retain the first assumption the second must be denied. The denial is $\neg \neg(\neg Q \rightarrow \neg P)$, that is, $\neg Q \rightarrow \neg P$. Q.E.D. The proof of rule 43 proceeds in the same way.

'Whatever follows from the opposite of the antecedent follows from the opposite of the consequent':³¹

$$(44) P \rightarrow Q, \neg P \rightarrow R \therefore \neg Q \rightarrow R.$$

Burley derives this rule from two other rules, listed above as 32 and 42. The proof is as follows.

³⁰ Ibid.

³¹ Burley, *De Puritate*, 65

- (i) $P \rightarrow Q$ = first assumption
- (ii) $\neg P \rightarrow R$ = second assumption
- (iii) $\neg Q \rightarrow \neg P$ from (i) by rule 42
- (iv) $\neg Q \rightarrow R$ from (iii) and (ii) by rule 32. Q.E.D.

‘Whatever is antecedent to the opposite of a consequent is antecedent to the opposite of the antecedent’:³²

$$(45) P \rightarrow Q, R \rightarrow \neg Q \therefore R \rightarrow \neg P.$$

The proof of rule 45 is similar to that for rule 44. It runs as follows:

- (i) $P \rightarrow Q$ = first assumption
- (ii) $R \rightarrow \neg Q$ = second assumption
- (iii) $\neg Q \rightarrow \neg P$ from (i) by rule 42
- (iv) $R \rightarrow \neg P$ from (ii) and (iii) by rule 32. Q.E.D.

Rule 42 is also employed in the proof of a rule commonly invoked, and formulated by Burley as: ‘Every proposition which includes opposites implies its own contradictory.’³³ The simplest case of a proposition which includes opposites is the explicit contradiction. Let us take $P \& \neg P$ as our ‘proposition which includes opposites’. In that case Burley’s rule can be expressed as:

$$(46) P \& \neg P \therefore \neg(P \& \neg P).$$

Let us assume (i) $P \& \neg P$. A conjunction implies each conjunct, that is, (ii) $(P \& \neg P) \rightarrow P$ and (iii) $(P \& \neg P) \rightarrow \neg P$. Applying rule 42 to (ii) and (iii) respectively yields (iv) $\neg P \rightarrow \neg(P \& \neg P)$ and (v) $\neg \neg P \rightarrow \neg(P \& \neg P)$. Applying the rule of double negation to (v) yields (vi) $P \rightarrow \neg(P \& \neg P)$. Assumption (i) is the conjunction of the antecedents, respectively, of (vi) and (iv). Hence, by application of rule 31, the consequents of (vi) and (iv) can be asserted. But those consequents are equiform with (vii) $\neg(P \& \neg P)$. Hence, given assumption (i), (vii) can be deduced. Q.E.D.

Rule 46 might be thought of as a special case of a rule expressed by Buridan as: ‘From every conjunction composed

³² Burley, *De Puritate*, 65.

³³ Ibid. 70.

of two mutually contradictory propositions there follows any other proposition, also in a formal inference':³⁴

(47) $P \& \neg P \therefore Q$.

This rule is itself a more specific case of our rule 6, that from an impossible proposition anything follows. Since it is impossible for things to be as signified by $P \& \neg P$, it is impossible for things to be as signified by $P \& \neg P$ without at the same time being as signified by proposition Q , no matter what Q signifies.

The obverse of rule 47 is the rule that, from any proposition whatsoever, a disjunction of mutually contradictory propositions follows:

(48) $Q \therefore P \vee \neg P$.

Since it is impossible for things not to be as signified by $P \vee \neg P$, then no matter what Q signifies, it is impossible for things to be as Q signifies without also being as $P \vee \neg P$ signifies. From rules 47 and 48 it follows, by the rule 'from first to last', that a conjunction of contradictories implies a disjunction of contradictories.

I shall end this section by considering a small group of rules concerned with conjunction and disjunction. First the rule of 'conjunction elimination': 'From every conjunction there follows each of its parts.'³⁵ This gives rise to two rules:

(49) $P \& Q \therefore P$

(50) $P \& Q \therefore Q$.

Ockham, who discusses this rule,³⁶ adds that the converses of these rules do not work, except sometimes for 'material' reasons. It is possible that what he has in mind here is that an inference such as 'Brownie is a donkey. Therefore Brownie is a donkey and Brownie is an animal' is valid, though not formally, because of the relation between the

³⁴ Buridan, *Cons.* 36

³⁵ *Ibid.* 80.

³⁶ *Summa Logicae*, Pt. II, Ch. 32, p. 348.

concepts of 'donkey' and 'animal'. But if this is Ockham's position it is incorrect. An argument of the form:

(i) $P \therefore P \& Q$

may be valid for formal reasons. For example, where the first conjunct in a conjunction is a contradiction, the conjunction follows from that first conjunct, and does so for formal and not material reasons. For an inference whose premiss is of the form $P \& \neg P$ is formally valid. Propositions can be placed on a line whose termini mark points of greatest strength and greatest weakness. Let us say that of two propositions one is formally stronger than the other if, for formal reasons, it implies the other but is not, for formal reasons, implied by the other. The principle at issue here can now be expressed as follows: given a conjunction of which one conjunct is formally stronger than the other, then not only does each conjunct follow formally from the conjunction but the conjunction itself follows formally from the formally stronger of the two conjuncts. As regards a conjunction of which one principal part is a contradiction, the contradiction formally implies the other part, since a contradiction formally implies anything whatever—it has the greatest strength that any proposition can have. Hence not only does each conjunct follow from that conjunction, but the conjunction itself follows from the part which is a contradiction.

In fact I have not stated the only circumstance in which it is permissible on formal grounds to deduce a conjunction from one of its conjuncts. Suppose that two propositions, P and Q , for formal reasons imply each other. In that case, of course, on a given line calibrated in terms of formal strength P and Q must occupy the same point. Where P and Q are thus related, their conjunction follows formally from each of P and Q . To take the simplest kind of case, if two propositions are mutually equiform then a proposition equiform with them implies their conjunction. Thus $P \& P$ follows from P . To state the rule more generally than I have

done so far, if two propositions, P and Q, are so related that P is not less strong than Q, formally speaking, then the conjunction of the two propositions follows formally from P. This discussion suggests that one further rule can now be stated, specifying the condition in which the 'fallacy of the consequent' is not committed when arguing from a conjunct to a conjunction:

$$(51) P \rightarrow Q, P \therefore P \& Q.$$

Ockham was familiar with the rule just stated. He writes: 'If one part of a conjunctive proposition implies the other part then from that part to the whole conjunctive proposition is a sound inference.'³⁷

Let us now turn to the rule of 'disjunction introduction'. Albert writes: 'From each part of an affirmative disjunctive proposition to the affirmative disjunctive proposition of which it is a part, is a sound inference.'³⁸ Buridan states the matter rather differently: 'From every proposition there follows itself disjoined from any proposition.'³⁹ This gives rise to two rules:

$$(52) P \therefore P \vee Q$$

$$(53) Q \therefore P \vee Q.$$

Ockham, also, states these rules. He adds: 'The reverse inference involves the fallacy of the consequent, though sometimes there is some special obstacle to that fallacy.'⁴⁰ The fallacy of the consequent 'arises because people suppose the relation of inference to be reciprocal. For whenever, suppose *this* is, *that* necessarily is, they suppose that if the latter is, the former necessarily is.'⁴¹ The point here is that the fact that a given argument, say one from a disjunct to a disjunction, is valid, does not by itself justify the claim that the reverse argument, say from the disjunction to a disjunct,

³⁷ Ibid. 349.

³⁸ *Perutilis Logica*, 19^{ra}.

³⁹ *Cons.*, 80.

⁴⁰ *Summa Logicae*, Pt. II, Ch. 33, p. 350.

⁴¹ Aristotle, *Sophistical Refutations*, 167 b 1 ff.

is also valid. The reverse argument might in fact be valid, but not because the original one is. Ockham claims that in the case of rules 52 and 53 the reverse argument is indeed sometimes valid, that there is sometimes 'some special obstacle to the fallacy of the consequent'. Consideration of our discussion following rules 49 and 50 reveals what the special obstacle is. Let us suppose that P is, in the sense defined above, formally stronger than Q . In that case Q follows formally from the disjunction of P and Q . Thus, for example, a contingent proposition follows formally from any disjunction in which that contingent proposition is disjoined from an explicit contradiction. For an explicit contradiction is formally stronger than any contingent proposition. Likewise a formally necessary proposition follows from any disjunction in which that necessary proposition is disjoined from a contingent proposition. For every contingent proposition is stronger than any necessary proposition.

But again, it should be said that the foregoing conditions in which a disjunct follows formally from a disjunction are not the only conditions. It is sufficient that of two propositions each is precisely as strong as the other, formally speaking. Thus, P follows from $P \vee P$, and from $P \vee \neg P$. Therefore, to state the rule more generally than I have done so far, if two propositions P and Q , are so related that P is not less strong than Q , formally speaking, then Q follows formally from the disjunction of P and Q .

We can now state a further rule specifying the 'special obstacle' which allows us to argue from a disjunction to a disjunct without committing the fallacy of the consequent:

$$(54) \quad P \rightarrow Q, P \vee Q \therefore Q.$$

We turn to a further rule involving disjunction. Buridan invokes the concept of a 'sufficient division' and says that if one of its heads of division is denied then the other should be inferred. In illustration he offers the inference: 'Every A is B or every A is C . And an A is not B . Therefore every A is

C.⁴² Evidently, then, Buridan's inference can be seen as illustrating the rule formulated by Ockham as: 'From a disjunctive proposition, along with the negation of one of its parts, to the other part, is a sound inference'.⁴³

$$(55) \text{P} \vee \text{Q}, \neg \text{P} \therefore \text{Q}$$

$$(56) \text{P} \vee \text{Q}, \neg \text{Q} \therefore \text{P}$$

However, as it stands Ockham's example is less helpful than Buridan's. For Ockham writes: 'Socrates is a man or a donkey. Socrates is not a donkey. Therefore Socrates is a man.' But the first premiss is not in fact a disjunctive proposition and in that case rules 55 and 56 are not applicable to it. The point is that the disjoint 'or' in the first premiss is to be understood divisively, so that the premiss should be taken as equivalent to the disjunctive proposition 'Socrates is a man or Socrates is a donkey'. And when the first premiss is replaced by its disjunctive equivalent then the rule can indeed be applied to the two premisses to yield the conclusion Ockham specifies.

A set of rules, commonly known as De Morgan's laws, after the nineteenth-century mathematician and logician Augustus De Morgan, was part of the repertoire of all the medieval logicians with whom I have been concerned. Albert of Saxony writes:

The contradictory of a conjunctive proposition is a disjunctive proposition composed of parts which are the contradictories of the parts of the conjunctive proposition . . . The contradictory of an affirmative disjunctive proposition is a conjunctive proposition composed of parts which are the contradictories of the parts of the disjunctive proposition.⁴⁴

Two propositions are contradictories if each implies the negation of the other and the negation of each implies the other. The above two rules therefore yield the following four rules:

⁴² *Cons.*, 81.

⁴³ *Summa Logicae* Pt. II, Ch. 33, p. 350.

⁴⁴ *Perutilis Logica*, 19^{rb}.

$$(57) P \& Q \therefore \neg(\neg P \vee \neg Q).$$

$$(58) P \vee Q \therefore \neg(\neg P \& \neg Q).$$

$$(59) \neg(P \& Q) \therefore \neg P \vee \neg Q.$$

$$(60) \neg(P \vee Q) \therefore \neg P \& \neg Q.$$

The validity of rules 58 and 60 is guaranteed by the validity of rules 57 and 59 respectively. Proof:

$$(57a) \neg \neg(\neg P \vee \neg Q) \therefore \neg(P \& Q)$$

is valid given rule 57, since if a given inference is sound, then from the negation of the conclusion there follows the negation of the premiss.

$$(57b) \neg P \vee \neg Q \therefore \neg(P \& Q)$$

is valid given 57a, by application of the rule of double negation.

$$(57c) \neg \neg P \vee \neg \neg Q \therefore \neg(\neg P \& \neg Q)$$

is valid given 57b, replacing P by $\neg P$ and Q by $\neg Q$ systematically. Rule 58 is valid given 57c by application of the rule of double negation. Q.E.D. Rule 60 is derivable from rule 59 by the same means.

It can also be shown that each of these rules is valid if reversed. For example, if in rule 60 P is replaced by $\neg P$ and Q by $\neg Q$ systematically, and the rule of double negation is then applied, the result is the converse of 57. By similar means the converse of 58 is derived from 59, of 59 from 58, and of 60 from 57.

It follows from rule 57, or rather from 57 expressed as a mutual inference, that the conjunction sign need not be treated as a primitive sign, but can instead be defined in terms of disjunction and negation. And likewise it follows from rule 58 expressed as a mutual inference that the disjunction sign can be defined in terms of conjunction and negation. But our logicians did not give such definitions. It was sufficient for their purposes that they were aware of the logical relations between the concepts of conjunction and disjunction. No goal,

worthwhile in relation to their brief, would have been secured by defining '&' as:

$$P \& Q = \text{df } \neg(\neg P \vee \neg Q)$$

or by defining ' \vee ' in a parallel way. It is not that medieval logicians were loath to give definitions. They gave a great many. But definition had a purpose, to clarify the comparatively obscure. Conjunction and disjunction were simply not obscure enough to merit definition, and certainly neither could reasonably be claimed to be more obscure than the other.

On the basis of rule 59 let us argue as follows:

- (i) $\neg(P \& Q)$ = first assumption
- (ii) P = second assumption
- (iii) $\neg P \vee \neg Q$ from (i) by rule 59
- (iv) $\neg \neg P$ from (ii) by rule 3
- (v) $\neg Q$ from (iii) and (iv) by rule 55.

Therefore (v) is deducible from (i) and (ii). This inference can be set out as follows:

$$(61) \neg(P \& Q), P \therefore \neg Q$$

It is easy to show that the following also is valid:

$$(62) \neg(P \& Q), Q \therefore \neg P.$$

The rule here is: From a negated conjunction, along with one of the conjuncts, to the negation of the other conjunct, is a valid inference.

Among the many modes of valid inference given above are four forming a neat group. They, with their Latin names, are as follows:

- (M1) $P \rightarrow Q, P \therefore Q$ = *modus ponendo ponens*, that is, by affirming (the antecedent), affirming (the consequent)—see rule 31.
- (M2) $P \rightarrow Q, \neg Q \therefore \neg P$ = *modus tollendo tollens*, that is, by denying (the consequent),

- denying (the antecedent)—see rule 41.
- (M₃) $P \vee Q, \neg P \therefore Q$ = *modus tollendo ponens*, that is, by denying (one disjunct), affirming (the other disjunct)—see rule 55.
- (M₄) $\neg(P \& Q), P \therefore \neg Q$ = *modus ponendo tollens*, that is, by affirming (one conjunct), denying (the other conjunct)—see rule 61.

I shall complete this section by establishing a rule which will be invoked in the discussion of syllogistic theory in chapter 8.

- (i) $(P \& Q) \rightarrow R$ = first assumption
 (ii) $\neg R \& P$ = second assumption
 (iii) $\neg R \rightarrow \neg(P \& Q)$ from (i) by rule 42
 (iv) $\neg R$ from (ii) by rule 49
 (v) P from (ii) by rule 50
 (vi) $\neg(P \& Q)$ from (iv) and (iii) by rule 31
 (vii) $\neg P \vee \neg Q$ from (vi) by rule 59
 (viii) $\neg \neg P$ from (v) by rule 3
 (ix) $\neg Q$ from (vii) and (viii) by rule 55
 (x) $(\neg R \& P) \rightarrow \neg Q$ from (ii) and (ix) on assumption (i).

Therefore from first to last, that is, from (i) to (x):

$$(63) (P \& Q) \rightarrow R \therefore (\neg R \& P) \rightarrow \neg Q.$$

By a similar line of reasoning:

$$(64) (P \& Q) \rightarrow R \therefore (P \& \neg R) \rightarrow \neg Q$$

$$(65) (P \& Q) \rightarrow R \therefore (\neg R \& Q) \rightarrow \neg P$$

$$(66) (P \& Q) \rightarrow R \therefore (Q \& \neg R) \rightarrow \neg P$$

Validity Conditions and Analysed Propositions

I. THE SQUARE OF OPPOSITION

In the previous chapter we attended to rules of valid inference for unanalysed propositions. That is to say, although propositions are either categorical or composed of categoricals, the distinctive features of categorical propositions, in particular their subject-predicate structure and their quantity, that is, their universality, particularity, or singularity, were not relevant to the rules. For example, the rule sanctioning the move from a proposition to the negation of its negation holds for all categorical propositions whether universal or otherwise, and holds also for all non-categorical propositions. In this chapter attention will be paid to the inferential power of categorical propositions where the structure of those propositions is taken into account.

At the heart of the medieval theory of valid inference for analysed propositions lies an account of three ways in which two categorical propositions with the same categorematic terms may be related to each other. They may be related by opposition, equipollence, or conversion. Propositions related by opposition or equipollence have the same categorematic terms in the same order; where the relation is that of conversion the order is not the same. Equipollence is a relation of equivalence between two propositions structurally related to each other in a certain quite specific way. Opposition is not a relation of equivalence. These are the

main differences between these kinds of relation. We are concerned here with the relations because they all give rise to rules of inference. We shall start, where most medieval discussions in this area started, with the notion of opposition.

Perhaps the best-known notion of medieval logic is that of the square of opposition, a square that medieval logicians liked and of which they drew a considerable variety. We shall start by considering the simplest. At different places in the course of this book, aspects of the simplest square have been invoked, but now we shall bring the various threads together. Let us for the time being ignore categorical propositions whose subject is a proper name and focus instead on categorical propositions which are either universal or particular. Such propositions can be either affirmative or negative. We have to deal, therefore, with four kinds of proposition.

Using *a*, *e*, *i*, and *o* as signs of universal affirmation, universal negation, particular affirmation, and particular negation respectively, the four kinds of proposition can be symbolized as *AaB*, *AeB*, *AiB*, and *AoB*. There is no obvious sense in which each proposition of these four is opposed to all the others, but there is a reasonably obvious sense in which *AaB* and *AiB* are each opposed to both *AeB* and *AoB*, namely in the sense that *AaB* and *AiB* are related to *AeB* and *AoB* as affirmation to negation. The relation between *AaB* and *AiB*, and also the relation between *AeB* and *AoB*, were called relations of opposition because of their place in the square of opposition, rather than because of any obvious sense in which those propositions were opposed. It might be said that *AaB* and *AiB* are opposed in that the first is universal and the second particular, but if so then any difference might have to be called an opposition, and then the concept of opposition would lose its point. We shall start by considering the relation between *AaB* and *AeB*.

Two universal propositions equiform except that one is affirmative and the other negative are related as contraries. Peter of Spain writes: 'The law of contraries is that if one

[proposition] is true, the other is false, and not vice versa.”¹ Peter expounds the relation of contrariety in semantic terms, but a syntactic account can be drawn from the semantic version. Peter’s law thus yields three rules:

- (1) $AaB \therefore \neg (AeB)$
- (2) $AeB \therefore \neg (AaB)$
- (3) $\neg \text{pos}(AaB \ \& \ AeB)$.

The underlying logical relation between contrary propositions is that expressed in one of the De Morgan rules discussed in Chapter 6. Rule 6o sanctions the move from the denial of a disjunction to the affirmation of a conjunction whose principal parts are the contradictories of the principal parts of the disjunction. And it was stated that the reverse inference also holds. In the light of this equivalence in inferential power, we can argue as follows:

- (i) $\neg (PvQ) \& \neg (RvS)$ = first assumption
- (ii) $\neg (PvQ)$ from (i) by rule 49 (Ch. 6)
- (iii) $\neg (RvS)$ from (i) by rule 50 (Ch. 6)
- (iv) $\neg P \& \neg Q$ from (ii) by rule 6o (Ch. 6)
- (v) $\neg R \& \neg S$ from (iii) by rule 6o (Ch. 6)
- (vi) $(\neg P \& \neg Q) \& (\neg R \& \neg S)$ from (iv) and (v) since from two propositions to their conjunction is a valid inference.

Let us now make certain replacements in (vi). P is to be replaced by $A^1=B^1$. Since in (vi) P is negated, $A^1=B^1$ will have to be negated also. That negation can be expressed as $A^1 \neq B^1$. So instead of $\neg P$ we write $A^1 \neq B^1$. Likewise let us replace Q by $A^1=B^2$ (which will be duly negated and expressed as $A^1 \neq B^2$). We shall replace R by $A^2=B^1$, and S by $A^2=B^2$, and negate them likewise. The result of these replacements in (vi) is:

- (vii) $(A^1 \neq B^1 \ \& \ A^1 \neq B^2) \ \& \ (A^2 \neq B^1 \ \& \ A^2 \neq B^2)$.

¹ *Tractatus*, 7.

Now, in AeB the subject and predicate both have distributive supposition, and hence descent is to be made under each of them to a conjunction of singular propositions. Since they have the same kind of supposition the order of descent is immaterial. Let us, therefore, assume that there are just two things, A^1 and A^2 , which are A , and just two things, B^1 and B^2 , which are B , and we shall descend first under A and then under B . The conjunction of singular propositions reached by this procedure is (vii).

We turn now to AaB , and argue as follows:

- (viii) $(PvQ) \& (RvS)$ = second assumption
- (ix) PvQ from (viii) by rule 49 (Ch. 6)
- (x) RvS from (viii) by rule 50 (Ch. 6)

In (ix) let us make the same replacements that were made for step (vii):

- (xi) $(A^1=B^1)v(A^1=B^2)$ from (ix), replacing P by $A^1=B^1$,
 Q by $A^1=B^2$.

But if A^1 is identical with B^1 or identical with B^2 , then it is identical with B^1vB^2 . Therefore:

- (xii) $A^1=B^1vB^2$ from (xi)
- (xiii) $(A^2=B^1)v(A^2=B^2)$ from (x), replacing R by $A^2=B^1$,
 S by $A^2=B^2$

But if A^2 is identical with B^1 or is identical with B^2 , then it is identical with B^1vB^2 . Therefore:

- (xiv) $A^2=B^1vB^2$ from (xiii)
- (xv) $(A^1=B^1vB^2)\&(A^2=B^1vB^2)$ from (xii) and (xiv),
 since from two propositions to their conjunction is a valid inference.

Now, in AaB the subject has distributive supposition and the predicate merely confused supposition. Descent should therefore be made first to a conjunction of singular

propositions under A, and then to a disjunction of singular terms under B. Let our domain, once again, be A^1 , A^2 , B^1 , B^2 . The two stages of descent take us to (xv). And hence, for the domain just described, (xv) is equivalent to:

(xvi) AaB .

The point to note here is the relation between (i) and (viii). Both are conjunctions, and the conjuncts affirmed in (viii) are negated in (i). Clearly, of two such conjunctions if one is true the other is false, and it does not follow from the fact that one is false that the other is true. That is, the law of contraries applies to them. What has just been demonstrated is that it is because the law of contraries applies to two such conjunctions that it applies to AaB and AeB .

We turn now to the second of the relations of opposition represented in the square of opposition, namely, contradiction. Peter of Spain writes: 'The law of contradictories is such that if one [proposition] is true the other is false, and vice versa.'² This relation holds between two propositions which differ in both quantity and quality. Thus AaB and AoB are contradictories, as are AeB and AiB . Since the truth of each member of the pair implies the falsity of the other, and the falsity of each implies the truth of the other, each member of the pair is equivalent to the negation of the other. However, in order to prepare the ground for proofs to be presented later in this chapter, I shall set out the inferences as one-way:

- (4) $AaB \therefore \neg (AoB)$
- (5) $AeB \therefore \neg (AiB)$
- (6) $AiB \therefore \neg (AeB)$
- (7) $AoB \therefore \neg (AaB)$
- (8) $\neg (AoB) \therefore AaB$
- (9) $\neg (AiB) \therefore AeB$
- (10) $\neg (AeB) \therefore AiB$
- (11) $\neg (AaB) \therefore AoB$.

² *Tractatus*, 7.

Rule 7 holds if 4 does, in accordance with the rule that if an inference is valid then from the negation of the conclusion to the negation of the premiss is a valid inference. That yields $\neg \neg (AoB)$ as the premiss of $\neg (AaB)$. And the law of double negation can then be applied to that premiss, thus yielding rule 7. In much the same way, it can be shown that 6 holds if 5 does. Likewise 11 holds if 8 does, and 10 if 9 does.

The logical basis of the contradictoriness of AaB and AoB can be displayed as follows:

- | | |
|--|---|
| (i) $(PvQ) \& (RvS)$ | = first assumption |
| (ii) AaB | from (i); see steps (viii) \neg (xv) above and the paragraph following (xv). |
| (iii) $\neg (PvQ) v \neg (RvS)$ | = second assumption |
| (iv) $(\neg P \& \neg Q) v (\neg R \& \neg S)$ | from (iii) by replacing each disjunction in (iii) by its De Morgan equivalent (see rule 6o (Ch. 7)) |

Replacing P by $A^1=B^1$, Q by $A^1=B^2$, R by $A^2=B^1$, and S by $A^2=B^2$, in (iv), and expressing the negation of those identities by the sign of non-identity (\neq), we reach:

$$(v) (A^1 \neq B^1 \& A^1 \neq B^2) v (A^2 \neq B^1 \& A^2 \neq B^2).$$

Given that there are just two things, A^1 and A^2 , which are A , and just two things, B^1 and B^2 , which are B , descent to singulars under AoB takes us first to: A^1 is not B or A^2 is not B . And the second stage of descent takes us to (v). Therefore, for the domain just described (v) is equivalent to:

$$(vi) AoB.$$

The point to note here is the relation between (i) and (iii). It is that between a conjunction of propositions and a disjunction whose principal parts are the negations of the principal parts of the conjunction. (i) and (iii), being so related, cannot both be true and cannot both be false. That is, the law of contradictories applies to them. What has just

been demonstrated is that it is because the law of contradictories applies to (i) and (iii) that it also applies to AaB and AoB. By a similar line of reasoning it can be shown that it is because the law of contradictories applies to $(PvQ)v(RvS)$ and $-(PvQ)\&-(RvS)$, that it also applies to AiB and AeB.

The third variety of opposition is that of subcontrariety. Peter of Spain writes: 'The law of subcontraries is such that if one [proposition] is false the other is true, and not vice versa.'³ This yields three inferences:

(12) $-(AiB) \therefore AoB$

(13) $-(AoB) \therefore AiB$

(14) $\text{pos}(AiB \& AoB)$.

Rules 12 and 13 each hold if the other does. For given rule 12, from the negation of its conclusion there follows the negation of its premiss. And by applying the rule of double negation to the resulting conclusion we reach rule 13. In the same way 12 can be derived from 13. The fact that AiB and AoB can be true together might call in question the propriety of speaking of the relation of subcontrariety as a form of opposition. But there is an opposition, though a syntactic rather than a semantic one. It lies, as was stated earlier, in the fact that one is affirmative and the other is exactly like the first except for containing a negation sign.

We turn now to the fourth and last variety of opposition, that of subalternation. Peter of Spain writes: 'The law of subalternates is such that if a universal is true a particular is true, and not vice versa. For a universal can be false while its particular is true. And if a particular is false its universal is false, and not vice versa.'⁴ This yields two rules of inference:

(15) $AaB \therefore AiB$

(16) $AeB \therefore AoB$.

³ Ibid. 7.

⁴ Ibid. 7.

Rule 15 is derivable from the rule (itself readily derivable from rules 49 and 52, Ch. 6) that from a conjunction to a disjunction whose parts are the same as those of the conjunction is a valid inference. The proof is as follows:

(i) AaB = assumption.

For the domain we have been assuming, (i) is equivalent to:

- (ii) $(A^1=B^1 \vee A^1=B^2) \& (A^2=B^1 \vee A^2=B^2)$
 (iii) $(A^1=B^1 \vee A^1=B^2) \vee (A^2=B^1 \vee A^2=B^2)$ from (ii) by the rule that from a conjunction to a disjunction with the same parts is a valid inference.

For the domain we have been assuming, (iii) is equivalent to:

(iv) AiB .

Therefore from first to last: $AaB \therefore AiB =_{15} \text{Q.E.D.}$

Rule 16 can be established by similar means.

A later generation of logicians did not tie the notion of subalternation exclusively to the relation between a universal and a particular proposition. Thus George Lokert asserts: 'An affirmative disjunctive proposition is subalternate to an affirmative conjunctive proposition composed of the same parts.'⁵ Such an extension of the notion of subalternation leads to the notion being indistinguishable from that of a one-way valid inference.

Rule 15 reminds us of the 'existential import' of universal affirmative propositions as these were understood by medieval logicians. A universal affirmative proposition, say 'Every whale is a mammal', would be said by most modern logicians to have a form more perspicuously represented by 'For every x, if x is a whale then x is a mammal', that is, it has the form:

⁵ *De Oppositionibus*, 35^{ra-rb}.

$$(x) (Wx \rightarrow Mx).$$

As so understood, the proposition could be true though no whale existed. But on the modern interpretation of particular affirmative propositions, say 'Some whale is a mammal', the logical form of this proposition is more perspicuously represented by 'For some x , x is a whale and x is a mammal', that is:

$$(Ex) (Wx \& Mx).$$

On this interpretation the proposition does imply that a whale exists. On the modern view, therefore, the universal affirmative proposition in one respect makes a stronger claim, and in another a weaker, than the particular affirmative. For the universal affirmative makes a claim about everything but without implying the existence of anything, whereas the particular affirmative both makes a claim about something, and also implies the existence of that thing. For this reason the inference from the universal affirmative to the particular is invalid, on the modern interpretation.

But, as we have seen, medieval logicians invoked rules of descent, rules which were regarded expressly as rules of valid inference. Thus every affirmative categorical proposition, whether universal or particular, implies at least one singular proposition affirming that something signified by the subject is identical with something signified by the predicate. Given AaB , it follows that there is something which is both A and B . AaB , therefore, has the existential import which it must have if AiB is to be deducible from it.

A closely related point can be made about negative categorical propositions. 'No A is B ' has two mutually equivalent modern interpretations, (i) 'It is not the case that, for some x , x is A and x is B '; and (ii) 'For every x , if x is A then it is not the case that x is B ', in symbols respectively:

$$(i) \neg(Ex)(Ax \& Bx)$$

$$(ii) (x)(Ax \rightarrow \neg Bx).$$

'Some A is not B' is interpreted as 'For some x, x is A and x is not B', in symbols:

$$(iii) (Ex)(Ax \& \neg Bx).$$

From (iii) there follows:

$$(iv) (Ex)Ax.$$

But (iv) does not follow from (i). That it does not is perhaps clearer in the light of the consideration that (i) and (ii) are equivalent. All that (ii) says is that if anything is A it is not B. And this carries no implication as to whether there is or is not something which is A. But though (iii) is not subalternate to (i), AoB is subalternate to AeB. This is so because neither AeB nor AoB implies that the subject has a significate. But each implies that if the subject has a significate then at least one of those significates is not also a significate of the predicate. The ground principle here is that a negative categorical proposition is true if either of its extremes has no significate. It is easy to see this point as regards universal negative propositions. No A is B if there is no A, for if there is no A then there is no A to be anything, and therefore none to be B. As regards particular negative propositions, we should bear in mind here rules 7 and 11 above, according to which $\neg(AaB)$ and AoB follow from each other. If there is nothing which is A then it is not the case that every A is B. For, as was just observed, if nothing is A no A is B. That is, from 'There is no A', $\neg(AaB)$ follows, and therefore its equivalent, namely AoB, follows also.

Thus, although there are two basic principles of division for a, e, i, and o propositions, namely quantity (universal or particular) and quality (affirmative or negative), as regards ontological significance it might seem that the more basic division is that into affirmation and negation, because that principle divides propositions according as they do or do not imply the existence of significates of their terms. But that point should be held lightly, for the division into universal and particular also has existential implications. AaB and

AeB both imply a non-existence, namely of an A which is not B, and of an A which is B, respectively. And AiB and AoB do not imply the non-existence of an A which is not B, and of an A which is B, respectively.

Of the sets of rules concerning contrariety, contradiction, subcontrariety, and subalternation, none of these seems to have logical priority over the others. That is, there seems no good logical reason why certain of these sets should be treated as primary and the rest as derivative. Certainly, three of the sets can be derived from the others. Let us begin with the rules of subalternation. These can be derived via the rules of contrariety and contradiction. For example:

- (i) AaB = assumption
- (ii) $\neg(\text{AeB})$ from (i) by rule 1
- (iii) AiB from (ii) by rule 10

Therefore from first to last: AaB \therefore AiB (= rule 15).

Likewise the rules of subcontrariety can be derived via the rules of contradiction and of subalternation. For example:

- (i) $\neg(\text{AoB})$ = assumption
- (ii) AaB from (i) by rule 8
- (iii) AiB from (ii) by rule 15

Therefore from first to last: $\neg(\text{AoB}) \therefore$ AiB (= rule 13).

Likewise the rules of contrariety can be derived via the rules of contradiction and subalternation. For example:

- (i) AeB = assumption
- (ii) AoB from (i) by rule 16
- (iii) $\neg(\text{AaB})$ from (ii) by rule 7

Therefore from first to last: AeB \therefore $\neg(\text{AaB})$ (= rule 2).

But none of the rules of contradiction can be derived from any combination of the others. Given AaB, AiB can be derived by subalternation, but from AiB it is not possible to reach $\neg(\text{AoB})$ because AiB and AoB are subcontraries. Likewise, from AaB, $\neg(\text{AeB})$ can be derived, by contrariety.

But from $\neg(AeB)$ it is not possible to reach $\neg(AoB)$. This does not imply that the rules of contradiction in any sense have logical priority over the other rules, but it certainly points to a distinctive logical feature of the relation of logical contradiction.

This distinctive feature is itself connected with a further feature of contradiction which it is appropriate to mention here. The four relations in the square of opposition can be presented in syntactical terms. P and Q are contraries if each implies the negation of the other. They are contradictories if each implies, and is implied by, the negation of the other. They are subcontraries if each is implied by the negation of the other, and P is subalternant to Q if P implies Q and not vice versa. Given any proposition there is an indefinite number of other non-equivalent propositions which are contraries of the first proposition. Thus, for example, $P \& Q$ has as contraries $\neg P$, $\neg Q$, $\neg P \& \neg Q$, $(\neg P \& \neg Q) \& R$. Likewise a proposition is subalternant to an indefinite number of non-equivalent propositions. For example, $P \& Q$ is subalternant to P , Q , $P \vee Q$. Likewise a proposition is subcontrary to an indefinite number of non-equivalent propositions. For example, P is subcontrary to $\neg P \vee Q$, $(\neg P \vee Q) \vee R$. But there cannot be two non-equivalent propositions each of which is the contradictory of a given proposition. The contradictory of $P \& Q$ is $\neg(P \& Q)$, or $\neg P \vee \neg Q$, or any other proposition equivalent to these. Thus we could form a square of opposition as follows:

$$P \& Q \qquad \neg P$$

$$P \qquad \neg P \vee \neg Q$$

If we start to construct a square of opposition by placing $P \& Q$ as one of the intended subalternants, we have a choice as to which proposition to place under it as its subalternate, and

which to place as contrary to it. But the choice of either fixes what the other will be, because whatever proposition is picked as the contrary of $P \& Q$ the negation of that proposition, or an equivalent of that negation, is then the subalternate of $P \& Q$. And whichever proposition is chosen as the subalternate of $P \& Q$, the negation of that subalternate, or a proposition equivalent to that negation, is then the contrary of $P \& Q$. It is the fact that a proposition does not determine its own contrary, subcontrary, subalternant, or subalternate, whereas it determines its own contradictory, that prevents any rules of contradiction being derived from any combination of the rules governing the other sorts of relation. Such determinateness cannot be derived from such indeterminateness.

The Spanish-Jewish humanist Juan Luis Vives (1492–1540) wrote influential treatises which were highly critical of medieval logic in general and of contemporary innovations in particular.⁶ Vives, who set as much store by correct Latin as the late John Austin set by correct English, was hostile to the scholastic ideal of a Latin forged as a scientific language. Such a Latin must be in some measure artificial, and for Vives and the other Renaissance humanists schooled in the Roman orators it was even barbaric. One area of medieval logic singled out by Vives for special censure is a late scholastic development of quantification theory in which artificial quantifiers are introduced.⁷ Their artificiality would of course be sufficient by itself to provoke Vives. But all the same they have features of considerable interest, and it is a particular pity that the ‘traditional’ logic that developed out of medieval logic should have left them entirely out of account. The artificial quantifiers were represented by single letters from the beginning of the alphabet, and the first four letters, used as quantifiers, were common currency in the decades before the Reformation. Since those quantifiers

⁶ See esp. his *In Pseudodialecticos*.

⁷ Ibid. 47 ff. Vives’s editor C. Fantazzi is thus wrong in claiming that the artificial quantifiers were Vives’s own invention.

played a special role in the development of the theory of opposition, this is an appropriate place to consider them. I shall deal in turn with the quantifiers represented by the letters a , b , c , and d .⁸

The letter a placed immediately before a categorematic term gives merely confused supposition to that term, and its power to give such supposition overrides the power of any other syncategorematic sign to give that term any other sort of supposition. Thus if a term is within the scope of a negation sign, and an a is placed immediately before the term, the a removes the term from the scope of the negation sign. In the plainly false

- (i) A man is not an animal

(where the 'A' is the indefinite article) the subject has determinate, and the predicate distributive, supposition. The final descendant of (i) is therefore:

- (ii) $(M^1 \neq A^1 \ \& \ M^1 \neq A^2) \vee (M^2 \neq A^1 \ \& \ M^2 \neq A^2)$.

In this proposition:

- (iii) a man is not an animal

(where a is the artificial quantifier) 'man' has merely confused supposition. Descent is therefore made first under the distributed 'animal', taking us to:

- (iv) a man is not A^1 & a man is not A^2

and then under 'man', taking us to:

- (v) $M^1 \vee M^2 \neq A^1 \ \& \ M^1 \vee M^2 \neq A^2$

which is equivalent to

- (vi) $(M^1 \neq A^1 \ \vee \ M^2 \neq A^1) \ \& \ (M^1 \neq A^2 \ \vee \ M^2 \neq A^2)$.

⁸ For further details concerning these quantifiers see E. J. Ashworth, 'Multiple Quantification and the Use of Special Quantifiers in Early Sixteenth Century Logic' in *Notre Dame Journal of Formal Logic* 19 (1978), 599-613, reprinted in E. J. Ashworth, *Studies in Post-Medieval Semantics*, ch. 10; see also A. Broadie, *George Lakert*, 50-4, 170-9.

(vi) states that there is some animal that some man is not, which is true on the assumption that there are at least two men. Given those assumptions, therefore, (iii) is true, though (i) is false. Likewise:

(vii) *a* man is not a man

is true, given that more than one man exists. For the two stages of descent under (vii) are:

(viii) *a* man is not M^1 & *a* man is not M^2

(ix) $M^1 \vee M^2 \neq M^1$ & $M^1 \vee M^2 \neq M^2$.

(ix) is equivalent to:

(x) $(M^1 \neq M^1 \vee M^2 \neq M^1)$ & $(M^1 \neq M^2 \vee M^2 \neq M^2)$.

(x) is true, since the second disjunct in the first disjunction and the first disjunct in the second disjunction are both true. Therefore, contrary to first appearances, (vii) is true also.

In some kinds of case *a* has to be introduced at the second stage of descent. For instance, in the stock example:

(xi) Of every man an eye is not an eye

(since of every man his right eye is not his left eye) 'man' and the second 'eye' have distributive supposition, and the first 'eye', being indirectly covered by 'every', has merely confused supposition. Descending first under 'man' we reach:

(xii) Of M^1 *a* eye is not an eye & Of M^2 *a* eye is not an eye.

In (xii) the *a* is introduced to indicate that the first 'eye' in each of the two conjuncts has merely confused supposition, for the singular M^1 and M^2 cannot by themselves ensure that the 'eye' which they immediately precede has merely confused supposition. Descent is next made under the predicate 'eye' in each conjunct of (xii), and lastly descent is made under the determinable 'eye'. The final descendant of (xii) is:

- (xiii) (Of $M^1 E^1 \vee E^2 \neq E^1$ & Of $M^1 E^1 \vee E^2 \neq E^2$) &
(Of $M^2 E^1 \vee E^2 \neq E^1$ & Of $M^2 E^1 \vee E^2 \neq E^2$).

On this analysis (xi) is true so long as each man has two eyes.

So far in this section we have considered the relation of subcontrariety as holding between a particular affirmative and a particular negative proposition. But it might be asked whether a universal proposition can have a subcontrary. The question was duly raised, and one answer given was that:

- (xiv) a A is not B

is subcontrary to:

- (xv) Every A is B.

As we know, the final descendant of (xv) is:

- (xvi) $A^1 = B^1 \vee B^2$ & $A^2 = B^1 \vee B^2$

which is equivalent to:

- (xvii) $(A^1 = B^1 \vee A^1 = B^2)$ & $(A^2 = B^1 \vee A^2 = B^2)$.

Descent under (xiv) takes us to:

- (xviii) a A is not B^1 & a A is not B^2

and thence to:

- (xix) $A^1 \vee A^2 \neq B^1$ & $A^1 \vee A^2 \neq B^2$

which is equivalent to:

- (xx) $(A^1 \neq B^1 \vee A^2 \neq B^1)$ & $(A^1 \neq B^2 \vee A^2 \neq B^2)$.

(xvii) and (xx) are subcontraries. If (xvii) is false, (xx) is true. For if (xvii) is false one of its disjuncts is false. Suppose it to be the first that is false. Its denial is equivalent to:

- (xxi) $A^1 \neq B^1$ & $A^1 \neq B^2$.

The first conjunct of (xxi) implies the first conjunct of (xx), and the second conjunct of (xxi) implies the second conjunct

of (xx). Hence (xxi) implies (xx). Similarly it can be shown that denial of the second conjunct of (xvii) also implies (xx). Therefore the denial of (xvii) implies (xx), and therefore the denial of (xv) implies (xiv). It can be shown by similar means that the denial of (xiv) implies (xv). Additionally, (xvii) and (xx) can be true together, and are on the assumption of

$$(xxii) (A^1=B^1 \ \& \ A^1 \neq B^2) \ \& \ (A^2 \neq B^1 \ \& \ A^2=B^2).$$

Therefore (xiv) and (xv) also can be true together, and therefore (xiv) and (xv) are subcontraries.

We turn now to the second of the artificial quantifiers. *b* gives determinate supposition to the immediately following categorematic term, and its power to give that supposition cannot be overridden by any other sign. Introducing *b* into a proposition can change its truth value. William Manderston⁹ and others gave the example:

$$(xxiii) \text{ Every man is } b \text{ animal.}$$

Since the predicate has determinate supposition, descent must be made under 'animal' before being made under the distributed subject. Descent takes us to:

$$(xxiv) \text{ Every man is } A^1 \vee \text{ Every man is } A^2.$$

Descending now under 'man', we reach:

$$(xxv) (M^1=A^1 \ \& \ M^2=A^1) \vee (M^1=A^2 \ \& \ M^2=A^2)$$

which says that some animal is every man, and which therefore is false except on the assumption that there is only one man.

Once *a* has been introduced into the system there is reason to introduce *b*, for medieval logicians recognized the importance of being able to specify the contradictory of any given proposition. They wished to know, therefore, what the contradictory of (xiv) was, and they found that the quantifier *b* helped them to solve this problem. It helped in this way. Since (xiv) was in the canonical form: subject+copula+

⁹ *Tripartitum*, sig. g vii.

predicate, they wished its contradictory to be specified in the same form. We have observed that (xiv) is equivalent to (xx). Hence the contradictory of (xx) will also be the contradictory of (xiv). Let us, then, contradict (xx). (xx) negated is equivalent to:

$$(xxvi) \neg(A^1 \neq B^1 \vee A^2 \neq B^1) \vee \neg(A^1 \neq B^2 \vee A^2 \neq B^2)$$

which is equivalent to:

$$(xxvii) (A^1 = B^1 \ \& \ A^2 = B^1) \vee (A^1 = B^2 \ \& \ A^2 = B^2).$$

(xxvii) states that there is some B that every A is. But that is precisely what is stated by:

$$(xxviii) \text{ Every } A \text{ is } b \ B$$

as can be verified by observing that (xxviii) and (xxvii) are formally the same as (xxiii) and (xxv) respectively. Therefore the contradictory of (xiv) is (xxviii).

Having fixed the contradictory of (xxviii), let us consider the question of what is contrary to (xxviii). The Spanish Dominican philosopher Domingo de Soto¹⁰ gave the answer:

$$(xxix) \text{ Some } A \text{ is not } B.$$

Descent first under A and then under B takes us to:

$$(xxx) (A^1 \neq B^1 \ \& \ A^1 \neq B^2) \vee (A^2 \neq B^1 \ \& \ A^2 \neq B^2).$$

The relation between (xxvii) (which spells out (xxviii)) and (xxx) is the relation between:

$$(xxxi) (P \ \& \ Q) \vee (R \ \& \ S)$$

and

$$(xxxii) (\neg P \ \& \ \neg R) \vee (\neg Q \ \& \ \neg S)$$

respectively. Inspection shows that each of (xxxi) and (xxxii) implies the negation of the other. Therefore the two schemata are contraries, and therefore so also are (xxviii) and (xxix).

¹⁰ *Introductiones Dialectice*, 48^{r-v}.

With the artificial quantifiers to hand, further squares of opposition can be constructed. I shall here describe just one, taking as my starting point:

(xxxiii) No A is b B.

In the absence of b , B would have distributive supposition, but the b overrides the power of the negation sign to give such supposition. The b gives determinate supposition to B, and descent must therefore be made under B before being made under the distributed A :

(xxxiv) No A is $B^1 \vee$ No A is B^2

(xxxv) $(A^1 \neq B^1 \ \& \ A^2 \neq B^1) \vee (A^1 \neq B^2 \ \& \ A^2 \neq B^2)$.

To find the contradictory of (xxxiii) we shall deny (xxxv). Its denial is equivalent to:

(xxxvi) $\neg(A^1 \neq B^1 \ \& \ A^2 \neq B^1) \ \& \ \neg(A^1 \neq B^2 \ \& \ A^2 \neq B^2)$

which is equivalent to:

(xxxvii) $(A^1 = B^1 \vee A^2 = B^1) \ \& \ (A^1 = B^2 \vee A^2 = B^2)$.

This formula spells out the truth conditions of:

(xxxviii) a A is every B

for under this last formula descent must be made first under the distributed B and then under the merely confused A, taking us to:

(xxxix) $(A^1 \vee A^2 = B^1) \ \& \ (A^1 \vee A^2 = B^2)$

which is equivalent to (xxxvii). Therefore the contradictory of (xxxiii) is (xxxviii).

One formula which implies (xxxvii) in a one-way implication is:

(xl) $(A^1 = B^1 \ \& \ A^2 = B^1) \ \& \ (A^1 = B^2 \ \& \ A^2 = B^2)$

which gives the truth conditions of:

(xli) Every A is every B.

Therefore (xli) is subalternant to (xxxviii).

Let us now look for the contradictory of (xli), and approach the problem by an examination of (xl). The negation of (xl) is equivalent to:

$$(xlii) \neg(A^1=B^1 \ \& \ A^2=B^1) \vee \neg(A^1=B^2 \ \& \ A^2=B^2)$$

which is equivalent to:

$$(xliii) (A^1 \neq B^1 \vee A^2 \neq B^1) \vee (A^1 \neq B^2 \vee A^2 \neq B^2).$$

This gives the truth condition of:

$$(xliv) \text{ Some } A \text{ is not } b \ B$$

for in (xliv) A and B each have determinate supposition. Let us descend first under B:

$$(xlv) \text{ Some } A \text{ is not } B^1 \vee \text{ Some } A \text{ is not } B^2.$$

And descent under A takes us to (xliii). Hence (xli) and (xliv) are contradictories.

Additionally (xli) and (xxxiii) are contraries, as can be seen by inspection of (xl) and (xxxv) which give their respective truth conditions. (xxxviii) and (xliv) are subcontraries, as inspection of (xxxvii) and (xliii) reveals. And finally, (xxxiii) is subalternant to (xliv). These relations are represented in the following square of opposition:

$$(1) \text{ Every } A \text{ is every } B \qquad (3) \text{ No } A \text{ is } b \ B$$

$$(2) \text{ } a \ A \text{ is every } B \qquad (4) \text{ Some } A \text{ is not } b \ B$$

Other such squares can be constructed. But we shall turn now instead to an examination of more complex artificial quantifiers, quantifiers which in their turn led to the construction of squares of opposition appreciably more complex than the one just described.

The more complex artificial quantifiers were particularly prominent in discussions concerning the possessive construction. Late-scholastic logicians had a lively interest in possessives. For example, Lokert specified the rules which permit such inferences as: 'Some man's donkey is running and every donkey is a quadruped. Therefore some man's quadruped is running', and 'Every donkey is a quadruped and some runner is a man's donkey. Therefore some runner is a man's quadruped'.¹¹ Possessives, however, give rise to logical problems. For example, the following two arguments seem to have the same form: 'Brownie is yours and Brownie is a donkey. Therefore Brownie is your donkey' and 'Brownie is yours and Brownie is a father. Therefore Brownie is your father'. Yet the first appears to be valid whereas the second is invalid. Or is the first not in fact valid? What, then, is wrong with it? At any rate it is plain that the first is not formally valid, since the second argument has the same form and yet is invalid. Problems concerning possessives are not restricted to the employment of possessive pronouns. For example, what should be said about this inference: 'Any mother-in-law is a parent. Jane is Mary's mother-in-law. Therefore Jane is Mary's parent'? I give these examples to illustrate the point that the logic of possessives is only very imperfectly understood. Medieval logicians seem not to have noticed some problems which now trouble us, though they certainly faced up to others. For the remainder of this section I shall deal with aspects of that corner of their theory of possessives which involves complex artificial quantifiers.

c and *d* are quantifiers conferring mixed supposition. They can occur only in categorical propositions containing at least three categorematic terms. We are concerned here, therefore, with propositions displaying such forms as:

(xlvi) Every A of some B is C

(xlvii) Of some A every B is C

¹¹ *Sill.* 4^{vb}.

- (xlviii) Every A is some B's C
 (xlix) Some A is every B of every C.

The letter *c* immediately preceding the third categorematic term indicates that that term should be taken to have merely confused supposition in relation to the first term and to have determinate supposition in relation to the second. This has an immediate effect on the order in which descent is to be made under the three terms. In (xlvii) the order of descent is this: first under A, then under B, and finally under C, since those terms have respectively determinate, distributive, and merely confused supposition. On the other hand, in:

- (l) Of some A every B is *c* C

C has merely confused supposition in relation to A which is determinate, and it has determinate supposition in relation to B which is distributed. The order of descent is therefore as follows: first under A which has determinate supposition, then under C which is determinate in relation to B, and finally under B which is distributed. The three stages of descent under (l) are therefore these:

- (li) Of A^1 every B is *b* C \vee Of A^2 every B is *b* C.

(Here *b* occurs rather than *c*, to indicate that C has determinate supposition in relation to B.)

- (lii) (Of A^1 every B is $C^1 \vee$ Of A^1 every B is C^2) \vee
 (Of A^2 every B is $C^1 \vee$ Of A^2 every B is C^2).
 (liii) [(Of A^1 $B^1=C^1$ & Of A^1 $B^2=C^1$) \vee (Of A^1 $B^1=C^2$
 & Of A^1 $B^2=C^2$)] \vee
 [(Of A^2 $B^1=C^1$ & Of A^2 $B^2=C^1$) \vee (Of A^2 $B^1=C^2$
 & Of A^2 $B^2=C^2$)].

We turn lastly to the quantifier *d*. When it immediately precedes the third categorematic term in a categorical proposition it indicates that that term has determinate supposition in relation to the first term and has merely confused supposition in relation to the second. Thus the *d* in:

(liv) Every A of every B is *d* C

indicates that C has determinate supposition in relation to A (and therefore descent should be made under C before being made under A), and has merely confused supposition in relation to B (and therefore descent should be made under B before being made under C). Descent, therefore, is to be made under B, C, and A, in that order. The three stages of descent are as follows:

(lv) Every A of B^1 is *b* C & Every A of B^2 is *b* C

(Here *b* occurs at the first stage of descent to indicate that C has determinate supposition in relation to A.)

(lvi) (Every A of B^1 is $C^1 \vee$ Every A of B^1 is C^2) &
(Every A of B^2 is $C^1 \vee$ Every A of B^2 is C^2)

(lvii) [$(A^1 \text{ of } B^1 = C^1 \text{ \& } A^2 \text{ of } B^1 = C^1) \vee (A^1 \text{ of } B^1 = C^2 \text{ \& } A^2 \text{ of } B^1 = C^2)$] &
 $[(A^1 \text{ of } B^2 = C^1 \text{ \& } A^2 \text{ of } B^2 = C^1) \vee (A^1 \text{ of } B^2 = C^2 \text{ \& } A^2 \text{ of } B^2 = C^2)]$

In the light of this explanation of *d* let us now search for contraries as the late scholastics did, and in particular let us examine:

(lviii) Of every A every B is not C.

A, B, and C all have distributive supposition. Descent should therefore be made under A before being made under B, in accordance with the rule that where a complex phrase consists of a determinant and a determinable which each have the same kind of supposition, the determinant has priority over the determinable in the order of descent. We shall descend first under A, then under B, and finally under C:

(lix) Of A^1 every B is not C & Of A^2 every B is not C

(lx) (Of A^1 B^1 is not C & Of A^1 B^2 is not C) & (Of A^2 B^1 is not C & Of A^2 B^2 is not C)

- (lxi) $[(\text{Of } A^1 B^1 \neq C^1 \ \& \ \text{Of } A^1 B^1 \neq C^2) \ \& \ (\text{Of } A^1 B^2 \neq C^1 \ \& \ \text{Of } A^1 B^2 \neq C^2)] \ \& \ [(\text{Of } A^2 B^1 \neq C^1 \ \& \ \text{Of } A^2 B^1 \neq C^2) \ \& \ (\text{Of } A^2 B^2 \neq C^1 \ \& \ \text{Of } A^2 B^2 \neq C^2)]$.

A contrary of (lviii) is the following:

- (lxii) Of every A every B is c C.

The c indicates that C has merely confused supposition in relation to A and determinate supposition in relation to B. The order of descent is therefore as follows: first under the distributed A, then under the determinate C, and finally under the distributed B. The stages of descent under (lxii) are:

- (lxiii) Of A^1 every B is b C & Of A^2 every B is b C

(The b indicates that C is determinate in relation to B.)

- (lxiv) $(\text{Of } A^1 \text{ every B is } C^1 \vee \text{Of } A^1 \text{ every B is } C^2) \ \& \ (\text{Of } A^2 \text{ every B is } C^1 \vee \text{Of } A^2 \text{ every B is } C^2)$

- (lxv) $[(\text{Of } A^1 B^1 = C^1 \ \& \ \text{Of } A^1 B^2 = C^1) \vee (\text{Of } A^1 B^1 = C^2 \ \& \ \text{Of } A^1 B^2 = C^2)] \ \& \ [(\text{Of } A^2 B^1 = C^1 \ \& \ \text{Of } A^2 B^2 = C^1) \vee (\text{Of } A^2 B^1 = C^2 \ \& \ \text{Of } A^2 B^2 = C^2)]$.

Each of (lxi) and (lxv) implies the negation of the other. And additionally the two can be false together, as for example when this holds:

- (lxvi) Of $A^1 B^1 = C^1 \ \& \ \text{Of } A^1 B^2 \neq C^1 \ \& \ \text{Of } A^1 B^1 \neq C^2$.

Therefore (lxii) is a contrary of (lviii). By similar means it can be shown that

- (lxvii) Of a A every B is not C

and

- (lxviii) Of every A every B is d C

are contraries.

These points complete my exposition of the four relations of opposition. I shall turn now to a consideration of the various relations of equipollence. As will quickly become obvious, the notions of opposition and equipollence are very closely related.

II. EQUIPOLLENCE

Given that of two propositions one is contradictory, contrary, subaltern, or subcontrary to the other, the question can be raised: by what (if any) placing of negation signs in the first proposition is it transformed into a proposition equivalent to the second? The rules of equipollence provide an answer to this question. There are four sets of rules of equipollence, corresponding to the four kinds of relation exhibited in the square of opposition.

Peter of Spain writes: 'If to some [universal or particular] sign a negation is prefixed then [the proposition] is equipollent to its contradictory.'¹² That is, if two propositions are contradictories, then by placing a negation in front of the sign of quantity in one of the propositions those propositions become equipollent. Therefore 'Every A is B', which is the contradictory of 'Some A is not B', is equipollent to 'Not some A is not B' (= 'It is not the case that some A is not B'). 'Some A is B', which is the contradictory of 'Every A is not B' (= 'No A is B'), is equipollent to 'Not every A is not B'. Using \leftrightarrow to symbolize 'is equivalent to', the full list of rules of equipollence, as applied to contradictories, is as follows:

$$(17) AaB \leftrightarrow \neg(AoB)$$

$$(18) AeB \leftrightarrow \neg(AiB)$$

$$(19) AiB \leftrightarrow \neg(AeB)$$

$$(20) AoB \leftrightarrow \neg(AaB).$$

The negation sign here is straightforwardly a sign negating the whole proposition, and there is no impropriety in placing

¹² *Tractatus*, 10.

it at the start of the proposition. But where a negation sign renders contraries equipollent, the matter is not quite so straightforward. The rule is: 'If a negation sign is placed after some universal sign then [the proposition] is equipollent to its contrary.'¹³ Thus, 'Every A is B', which is contrary to 'No A is B', is equipollent to 'No A is not B'. Here we are to understand 'not' as including in its scope 'is B'. The view that medieval logicians took of a negation sign so placed in a proposition was that by operating on the copula it reversed the quality of the proposition, but that since it did not operate on the initial quantifier it did not affect the quantity. Thus, inserting 'not' after the subject in 'No A is B' transforms the original proposition into one which retains the original quantity (which is universal) but reverses the quality (which had been negative). Hence 'No A is not B' is equipollent to the universal affirmative 'Every A is B'.

The negation placed in the proposition after the subject is a sign of propositional negation, since its insertion in an affirmative proposition transforms it into a negative proposition, and its insertion into a singly negative proposition transforms it into a proposition equivalent to one which is affirmative. Certainly there is no question that the negation placed after the subject is of the infinitizing variety. For this reason it is permissible to use the same sign to symbolize 'not' after the subject as is used to symbolize a negation at the beginning of a proposition whose next term is a quantifier. The rules of equipollence as applied to contrary propositions can therefore be expressed as follows:

$$(21) \text{ AaB} \leftrightarrow \text{Ae}\neg\text{B}$$

$$(22) \text{ AeB} \leftrightarrow \text{Aa}\neg\text{B}.$$

Both kinds of negation just discussed are invoked in specifying the rules of equipollence as applied to subaltern propositions: 'If a negation sign is placed both before and after the universal or particular sign, [the proposition] is equipollent to its subaltern.'¹⁴ Thus, 'Every A is B', which is

¹³ *Tractatus*, 10.

¹⁴ *Ibid.*

subalternant to 'Some A is B', is equipollent to 'Not some A is not B' (= 'It is not the case that some A is not B'). 'Not some A is B' is equivalent to 'No A is B'. To insert a predicate negation in 'No A is B' transforms that proposition, as we have seen, into one equivalent to a universal affirmative. Likewise 'Some A is not B', which is subalternate to 'Every A is not B', is equipollent to 'Not every A is not not B', that is, 'Not every A is B', that is, 'Some A is not B'. The rules of equipollence, as applied to subalterns, can be expressed as:

$$(23) AaB \leftrightarrow \neg(Ai\text{-}B)$$

$$(24) AeB \leftrightarrow \neg(Ao\text{-}B)$$

$$(25) AiB \leftrightarrow \neg(Aa\text{-}B)$$

$$(26) AoB \leftrightarrow \neg(Ae\text{-}B).$$

During the fourteenth century, discussion of rules of equipollence did not include an account of how subcontraries are to be transformed into equipollent propositions. It is possible that it was taken for granted that the rules given for transforming contraries would be seen to apply to the rules for transforming subcontraries. It is also possible, though less likely, that it was not realized that subcontraries could be transformed into equipollents. If it was thought that they could not be so transformed, this would surely have prompted the question of why this was so. But in any case, on the basis of the foregoing discussion it is easy to construct the rules of equipollence as applied to subcontraries.

$$(27) AiB \leftrightarrow Ao\text{-}B$$

$$(28) AoB \leftrightarrow Ai\text{-}B.$$

As can be seen, the rules concerning the transformation of contraries apply in exactly the same way to subcontraries. It is difficult to say rule 28 aloud in such a way as to present the logical point being made. AoB and $Ai\text{-}B$ are both read most naturally as 'Some A is not B'. The same problem occurs in Latin, and this fact about the Latin rendering may

have contributed to the lack of discussion on the relation between equipollence and subcontrariety.

Before leaving the topic of equipollence, I should like to deal with the following objection: Section 1 of this chapter contains a discussion concerning existential import. It is stated there that an affirmative categorical proposition, whether universal or particular, implies the existence of a significate of the subject and of the predicate, but a negative categorical does not. Indeed a negative categorical is true if either the subject or the predicate does not have an existing significate. How, then, can two propositions be equipollent if one is affirmative and the other negative? For example, how can rule 27 be justified given that if no A exists, then AiB , being affirmative, is false, and $Ao\text{--}B$, being negative, is true? The answer is to be derived from Burley's discussion concerning categorical propositions with two negation signs of which one is within the scope of the other.¹⁵ Such negative propositions are equivalent, he argues, to affirmative propositions. If a categorical proposition is negative in virtue of containing a single negation sign, then it is true if either its subject or its predicate does not signify any existing thing. If a categorical proposition is negative in virtue of containing two negation signs of which one is within the scope of the other, then its equivalence to an affirmative proposition ensures that if either extreme does not signify any existing thing the proposition is false. Hence $Ao\text{--}B$ is false if there exists no A or no B, just as AiB (which is equipollent to $Ao\text{--}B$) is false if either of those conditions is satisfied.

III. CONVERSION

A conversion is a valid inference consisting of two propositions plus a sign of inference. Both propositions are categorical, one the premiss, called the 'convertend', and the other the conclusion, called the 'converse'. The subject and predicate in the convertend recur as the predicate and

¹⁵ See above, Ch. 6, Sect I.

subject respectively in the converse. The theory of conversion sets out to answer two main questions: (a) Given a true categorical proposition, does replacement of the subject by the predicate and the predicate by the subject result, for logical reasons, in another true proposition? (b) If not, then what other changes need to be made to the first proposition to ensure, on logical grounds, the preservation of truth?

Three kinds of conversion were discussed—simple, accidental, and contrapositive—though as we shall see there are reasons, recognized by some medieval logicians, for denying that contrapositive conversion is, strictly speaking, a variety of conversion. But contrapositive conversion was always classed as a kind of conversion, even if only a rather degenerate kind, and for that reason I shall examine it after dealing with the simple and the accidental varieties.

Simple conversion first. This is a conversion in which the converse has the same quantity and quality as the convertend. Two kinds of proposition were each said to be simply convertible, the particular affirmative and the universal negative. The following are, therefore, rules of valid inference:

$$(29) \text{AiB} \therefore \text{BiA}$$

$$(30) \text{AeB} \therefore \text{BeA}.$$

In the light of the doctrine of supposition it is possible to pinpoint the underlying logical features of those kinds of proposition that ensure their simple convertibility. Let us assume, as usual, that A^1 and A^2 are the only things that are A, and B^1 and B^2 the only things that are B. Descent to singulars under first the subject and then the predicate in the premiss of rule 29 takes us to:

$$(i) (A^1=B^1 \vee A^1=B^2) \vee (A^2=B^1 \vee A^2=B^2).$$

Descending to singulars first under the predicate and then under the subject of the conclusion of rule 29 takes us to:

$$(ii) (B^1=A^1 \vee B^2=A^1) \vee (B^1=A^2 \vee B^2=A^2).$$

To each disjunct in the one disjunction there corresponds one disjunct in the other, differing only in the order of the extremes. But the relation of identity is commutative, that is, for any x and any y , if $x=y$ then $y=x$, a point Ockham puts by saying that a singular affirmative proposition can be converted into a singular affirmative proposition.¹⁶ The example he gives is 'Socrates is Plato. Therefore Plato is Socrates'. Hence (a) and (b) are deducible from each other. It is clear that however large the domain, the descendants of AiB and BiA will be mutually deducible. And it is this logical feature of the identity relation that ultimately underlies the simple convertibility of particular affirmatives.

Precisely the same point can be made about the simple convertibility of universal negatives. Assuming the same domain as before, descent first under the subject and then under the predicate of the premiss in rule 30 takes us to:

$$(iii) (A^1 \neq B^1 \ \& \ A^1 \neq B^2) \ \& \ (A^2 \neq B^1 \ \& \ A^2 \neq B^2).$$

Descent under first the predicate and then the subject in the conclusion in rule 30 takes us to:

$$(iv) (B^1 \neq A^1 \ \& \ B^2 \neq A^1) \ \& \ (B^1 \neq A^2 \ \& \ B^2 \neq A^2).$$

Once again there is a one-to-one correspondence of the descendants of the two categorical propositions. And since non-identity is commutative (for any x and any y , if $x \neq y$ then $y \neq x$), it follows that (iii) and (iv) follow from each other. Whatever the size of the domain the descendants of AeB and BeA will be mutually deducible. AeB is therefore simply convertible, and it is the commutativity of the non-identity relation that underlies this feature of universal negatives.

Each of rules 29 and 30 can be derived from the other along with certain other rules already accepted. Let us assume rule 29. If an inference is valid the negation of its premiss follows from the negation of its conclusion.

¹⁶ *Summa Logicae*, Pt. II, Ch. 21, p. 319.

Therefore the following must also be valid:

$$(29a) \neg(\text{BiA}) \therefore \neg(\text{AiB}).$$

But by rules 5 and 9 (Ch. 7), $\neg(\text{BiA})$ and $\neg(\text{AiB})$ are equivalent respectively to BeA and AeB . By substitution of equivalents for equivalents in rule 29a we reach:

$$(29b) \text{BeA} \therefore \text{AeB}.$$

Since 29b is formally valid, its validity is preserved if categorematic terms are replaced systematically. Replace B by A, and A by B. Then this is valid:

$$(29c) \text{AeB} \therefore \text{BeA} = \text{rule 30 Q.E.D.}$$

Rule 29 can be derived from rule 30 in the same way.

The two kinds of conversion just described are both simple and mutual; simple in that convertend and converse have the same quality and quantity, mutual in that convertend and converse follow from each other. Ockham mentions a wider sense of 'simple conversion', namely 'mutual conversion'.¹⁷ For a conversion can be mutual without being simple in the narrow sense. He has in mind a singular proposition and a particular proposition with which it is convertible. 'John is a man' converts mutually with 'Some man is John'. But the conversion is not 'simple' in the original sense, since convertend and converse do not have the same quantity. In Ockham's view, at any rate, singularity is not the same quantity as particularity. In so far as he did not think of singularity as a third kind of quantity alongside the other two he thought of it as identifiable with universality rather than with particularity—if John is a man then everything which is John is a man.

We turn next to accidental conversion. Here the converse has the same quality as the convertend but not the same quantity. Both a universal affirmative and a universal negative proposition can be converted in this way. Thus the following are rules of valid inference:

¹⁷ Ibid. 318.

(31) $AaB \therefore BiA$

(32) $AeB \therefore BoA$.

The soundness of these two rules can be displayed by descending to singulars under the premiss and conclusion of each of them, and considering the logical relations between the descendants. But rules 31 and 32 can in any case be derived from rules already established. The derivations are as follows:

(i) AaB = assumption

(ii) AiB from (i) by rule 15 (Ch. 7)

(iii) BiA from (ii) by rule 29 (Ch. 7).

Therefore from first to last: $AaB \therefore BiA$ = rule 31 Q.E.D.

(iv) AeB = assumption

(v) BeA from (iv) by rule 30 (Ch. 7)

(vi) BoA from (v) by rule 16 (Ch. 7)

Therefore from first to last: $AeB \therefore BoA$ = rule 32 Q.E.D.

Unlike simple conversion, accidental conversion is not mutual. Consideration of the foregoing two proofs reveals that the reason for this is that the inference from subalternant to subalternate (steps (ii) and (vi) above) is one-way only.

We have not so far identified any categorical proposition with which 'Every A is B' is mutually convertible, but one such proposition is readily to hand if use is made of one of the artificial quantifiers introduced in Section I of this chapter. The predicate in

(i) Every A is B

has, as we know, merely confused supposition. In Section I the quantifier *a* was introduced. Its function was to confer merely confused supposition on the immediately following categorematic term. Reversing (i) by transforming it into:

(ii) *a* B is every A

results in a proposition whose subject has the same kind of supposition as the predicate of (i), and whose predicate has

the same kind of supposition as the subject of (i). Since in (ii) A is distributed, descent must be made under that term before being made under the merely confused B. The final descendant is therefore:

$$(iii) B^I \vee B^2 = A^I \ \& \ B^I \vee B^2 = A^2$$

which is equivalent to:

$$(iv) (B^I = A^I \vee B^2 = A^I) \ \& \ (B^I = A^2 \vee B^2 = A^2).$$

Since identity is a commutative relation (iv) is equivalent to:

$$(v) (A^I = B^I \vee A^I = B^2) \ \& \ (A^2 = B^I \vee A^2 = B^2).$$

But (v) also gives the truth conditions of (i), and hence (i) and (ii) are mutually convertible. But the conversion, though mutual, is not strictly speaking simple, since convertend and converse, though the same in quality, are different in quantity. Medieval logicians did not indeed provide a word to describe the quantity of (ii). It is certainly not universal since the subject is undistributed. Neither is it particular or indefinite since the subject is not determinate. Perhaps we should say, as Vives no doubt would, that (ii) is merely confused. But whatever name we use, it is clear that on the standard account of 'simple conversion' the conversion with which we are here dealing is not simple. If however we revised the original conception of simple conversion, and said instead that a conversion is simple if the supposition of the subject and predicate in the convertend is the same as the supposition of the predicate and subject respectively in the converse (as is the case with AiB and AeB , each of which is simply convertible), then (i) and (ii) would, after all, be simply as well as mutually convertible.

It should be added that even without the use of the artificial quantifier a , we can construct a converse of 'Every A is B' which is mutually convertible with the convertend. The converse in question is 'Only B is A'. But in this case, also, the conversion is not simple if a simple conversion is one in which the convertend and converse have the same

quantity. For it was held¹⁸ that exclusive propositions (that is, roughly, those of the form 'Only A is B', 'Only A is not B', and their negations) have no quantity.

Finally, we must consider contrapositive conversion. Such conversion was investigated in the first place in an attempt to solve the problem of how particular negative propositions are to be converted. AoB is not convertible into BeA, as is obvious. Neither is it convertible into BoA, for 'Some logic book is not a book' is not a valid converse of 'Some book is not a logic book'. Hence AoB is neither simply nor accidentally convertible. The problem was solved, at least provisionally, by the invention of contrapositive conversion. Peter of Spain writes:

Contrapositive conversion is making the predicate out of the subject and the subject out of the predicate, while keeping the quality and quantity, but changing finite terms into infinite terms. Universal affirmatives and particular negatives are converted in this way.¹⁹

Where $\neg(T)$ signifies the negation of the term T, the two rules indicated by Peter of Spain can be expressed as:

$$(33) AaB \therefore \neg(B)a\neg(A)$$

$$(34) AoB \therefore \neg(B)o\neg(A).$$

For example, given that some animal is not a man it does not follow that some man is not an animal, but it does follow that some non-man is not a non-animal. If some animal, say the donkey Brownie, is not a man then some non-man, namely Brownie, is not a non-animal. And if every man is an animal then every non-animal is a non-man.

Peter of Spain expressed no qualms about this type of conversion, but some later logicians were less happy about the validity of rules 33 and 34, and I should like to comment here about the grounds for their hesitation. When turning from a consideration of general rules of inference to those

¹⁸ e.g. Paul of Venice, *Logica*, 7.

¹⁹ *Tractatus*, 8.

involving inference of one categorical proposition from another, Albert of Saxony lays down, as his first rule, that no contrapositive conversion is a formal inference. In justification of his rule, Albert furnishes counter-examples to rules 33 and 34 above. First, as regards 33, let us, at Albert's suggestion, consider:

- (i) Every man is an entity.

According to 33 this should imply:

- (ii) Every non-entity is a non-man.

But unlike (i), (ii) is false. Since there are no non-entities there is no non-entity which is a non-man. (ii) is false, therefore, in accordance with the principle that every affirmative proposition with a subject which does not supposit for anything is false.

Secondly, as regards rule 34, Albert gives the example:

- (iii) Some chimera is not a man.

According to rule 34 this should imply:

- (iv) Some non-man is not a non-chimera.

But (iii) is true and (iv) false. That (iii) is true follows from the principle that every negative proposition with a subject which does not supposit for anything is true. Since no chimera exists there is no chimera to be a man, and therefore, by subalternation, some chimera is not a man. (iv) is false because its contradictory is true. Its contradictory is:

- (v) Every non-man is a non-chimera.

Since everything is a non-chimera, every non-man is one.

Conversions are not merely valid inferences, they are formally valid and therefore their validity is invariant through systematic replacement of categorematic terms in the convertend and the converse. It follows from this that: 'Every man is an animal. Therefore every non-animal is a non-man', even if valid, is not formally so, and neither is:

'Some animal is not a man. Therefore some non-man is not a non-animal'. But if we wish to insist that, formally or not, these two last sample arguments are valid, it follows that they have unstated premisses. What, then, are those premisses?

Attempts to convert (i) and (iii) contrapositively fail for opposite reasons. In the case of (i) the predicate is a transcendental term, that is, one which is truly predicable of anything whatever that exists, and infinitizing it results in a term which, placed as a subject in an affirmative proposition, ensures the falsity of the proposition, whereas in the case of (iii) the subject stands for nothing, and therefore infinitizing it results in a transcendental term. But any negative proposition, in which a transcendental term is predicated of a term signifying something that exists, must be false. Contrapositive conversion fails, therefore, because no restriction is placed on the categorematic terms in the convertend. Since in the case of (i) the problem arises because the predicate when infinitized stands for nothing, along with the universal affirmative proposition an additional premiss must be placed in the inference, affirming that there exists something of which that infinitized predicate is truly predicable. Thus we can argue validly: 'Every man is an animal. There is a non-animal. Therefore every non-animal is a non-man'. This is formally valid. Hence this also is valid: 'Every man is an entity. There is a non-entity. Therefore every non-entity is a non-man'. It is granted that the conclusion is false, but so also is the second premiss. Consequently nothing false is being inferred from something true.

Since in the case of (iii) the problem arises because the subject stands for nothing, along with the particular negative proposition there must be placed an additional premiss affirming that there exists something for which the subject of the particular negative proposition stands. Hence this is a valid argument: 'Some animal is not a man. An animal exists. Therefore some non-man is not a non-animal'. The

foregoing argument is formally valid. Consequently this argument also is formally valid: 'Some chimera is not a man. A chimera exists. Therefore some non-man is not a non-chimera'. The conclusion is false, as was shown earlier. But so also is the second premiss. Consequently, the false conclusion has not been drawn from two true premisses. Albert of Saxony was well aware of these moves. He writes: 'Contrapositive conversion is a formal inference on the hypothesis or assumption that every one of its terms stands for something.'²⁰ But it has to be added that what he here describes as a formal inference is not a contrapositive inference in the original sense of the phrase. For contrapositive conversion has, by definition, a single premiss, the convertend, and contrapositive conversion, if presented as a real conversion, that is, with only the convertend as premiss, is not formally valid but is instead an inference whose suppressed premiss asserts the existence of something for which an extreme of the convertend stands.

It might seem that instead of adding a premiss, and thereby constructing an inference which is not strictly speaking a conversion at all, one could employ the tactic of restricting the list of categorematic terms available to logicians, so that they do not have access either to transcendental terms or to terms which do not stand for anything. Given such restrictions, any inference from a universal affirmative or particular negative to its contrapositive converse would go through smoothly. But two criticisms should be made against this tactic. First, it runs counter to the entire spirit of medieval logic, for logicians took as their object language the whole of natural language, not just the parts or features of it that did not cause problems for logicians—they had, of course, a particular interest in the parts or features that *did* cause them problems. Secondly, the proposal would require very drastic reduction indeed in the resources available to logicians. For, as we have seen, they investigated the logic of compound

²⁰ *Perutilis Logica*, 26^{ra}.

terms, terms such as those constructed by placing a disjunction or conjunction sign between categorematic terms. And given this use of disjunction and conjunction signs, and the use of an infinitizing negation sign, it was easy for them to construct, as they duly did, transcendental terms and terms which cannot stand for anything, for example, 'man or non-man' and 'man and non-man', the former of which is predicable of everything and the latter of nothing.

I should like to make a further point about the conversion of particular negative propositions. In:

- (i) Some A is not B

A has determinate supposition and B is distributed. It is possible to reverse (i) in such a way that B occurs as a distributed subject and A as a determinate predicate, while the transformed proposition is, like (i), negative:

- (ii) Every B is not *b* A.

In (ii) descent should be made first under A and then under B. The two stages of descent under (ii) are:

- (iii) Every B is not $A^1 \vee$ Every B is not A^2
 (iv) $(B^1 \neq A^1 \ \& \ B^2 \neq A^1) \vee (B^1 \neq A^2 \ \& \ B^2 \neq A^2)$.

Since the non-identity relation is commutative, (iv) is equivalent to:

- (v) $(A^1 \neq B^1 \ \& \ A^1 \neq B^2) \vee (A^2 \neq B^1 \ \& \ A^2 \neq B^2)$.

(v) also gives the truth conditions of (i). Hence (i) and (ii) are mutually convertible. But if we retain the conception of simple conversion as a conversion whose convertend and converse have the same quantity and quality, then the conversion with which we are here dealing is not simple, since (i) is a particular proposition whereas (ii) is universal. If however we adopt a suggestion made earlier in this section, and say instead that a conversion is simple if the supposition of the subject and predicate in the convertend is the same as the supposition of the predicate and subject respectively in

the converse, then (i) and (ii) are simply convertible with each other. Considered on that basis, the relation between them is the same as the relation between 'Every A is B' and 'a B is every A'.

All the rules of conversion so far given relate to propositions containing no ampliative terms, and it cannot be assumed that all, or even any, of those rules apply to propositions whose copulas are non-present-tensed or whose predicates have ampliative power. Examples should make it plain that propositions with ampliative terms provide exceptions to our rules. Particular affirmative propositions convert simply. Let us therefore consider this proposition:

(i) A man is dead.

'Dead', used as a predicate in an affirmative proposition with a copula in the present tense, ampliates the subject to supposit for what does or did exist, and indeed the subject in (i) does not supposit for what is a man, since, according to a standard doctrine inherited from Aristotle, dead men are not men. Hence (i) implies:

(ii) What is or was a man is dead.

It follows that (i) cannot be converted simply into:

(iii) A dead thing is a man

for (i) is true and (iii) is false—necessarily false since a man is a rational animal, and nothing can be both dead and rational.

Again, as we have seen, universal negatives were said to be simply convertible. Let us, then, consider this example:

(iv) No white thing was a man.

It might seem that a converse of (iv) is:

(v) No man was white.

In (iv) the subject is amplified to stand for what is or was white. Let us suppose that it stands for what *is* white. Let us

suppose in addition (a) that only two white things W^1 and W^2 have ever existed of which at past time t^1 both existed, (b) that W^1 is the only presently existing white thing, (c) that only one man M ever existed, and he exists now, (d) that M was W^2 , and (e) that M was not and is not W^1 . On this set of assumptions (iv) is true since each of the singular propositions 'This white thing was not a man' is true, for nothing we can now point to while saying truly 'This is white' was a man. But (v) is false, for it is laid down in the hypothesis that M was W^2 .

It follows that the rules for conversion require to be modified to deal with propositions containing ampliative terms. Let us stay with past-tensed propositions in which the subject is a common term. The subject can supposit for what does exist or for what did exist. The rules for conversion include the following: if the subject supposits for what does exist then the proposition should not be converted into a past-tensed proposition but into a present-tensed proposition in which the subject is taken with the verb 'was' and the pronoun 'which'.²¹ Therefore (iv) converts into:

(vi) Nothing which was a man is white.

This rule does not apply to (i) since there the subject cannot be taken to supposit for what now exists. If, on the other hand, the subject is taken to supposit for what did exist, then the proposition is convertible simply into a past-tensed proposition. Let us suppose, for example, that in (iv) 'white' is taken to supposit for what was white, then (iv) converts into:

(vii) No man was white.

Since in (i) the subject must be taken to supposit for what was a man, (i) converts into:

(viii) A dead thing was a man.

²¹ Ockham, *Summa Logicae*, Pt. II, Ch. 22, p. 322.

So far we have considered amplified propositions whose subject is a common term. A different, and rather simpler, account of conversion can be given for such propositions whose subject is a proper name. The crucial difference between, say, 'A white thing was a man' and 'John was a man' is that what we now point to and call 'John' always was John so long as he existed. But what we now point to and call 'a white thing' might have existed in the past without then being white. Hence a past-tensed singular proposition, say:

(ix) John was not white

is convertible into a proposition in which the subject is taken for what was:

(x) Nothing which was white was John.

The subject must be taken for what did exist, for otherwise fallacies occur. If (x) were replaced as the converse of (ix) by:

(xi) Nothing which is white was John

then the conversion is invalid. For if John had never been white but is now white for the first time, then (ix) is true and (xi) false.

The points just made about the convertibility of past-tensed propositions apply, with obvious adaptations, to future-tensed propositions also. No important new principle arises with the necessary adaptations. For the present we shall leave the topic of conversion and shall turn next to the large subject of syllogistic inference. But rules stated in this chapter will remain much to the fore since, as we shall see, the proof procedures for syllogisms include rules of subalternation and conversion, as well as other rules we have discussed.

Syllogistic Tense Logic

I. ELEMENTARY SYLLOGISTIC

The term 'syllogism' was used in a wide sense to signify any piece of reasoning, theoretical or practical. It is the theoretical syllogism, in the broad sense of 'syllogism', that Aristotle has in mind when he gives this definition:

A syllogism is a form of words in which, when certain assumptions are made, something other than what has been assumed necessarily follows from the fact that the assumptions are such.¹

Within the area of theoretical reasoning a distinction was drawn between categorical and molecular syllogisms. A molecular syllogism is distinguished by the presence of at least one molecular proposition occurring as a premiss. In a categorical syllogism, on the other hand, each proposition, whether premiss or conclusion, is categorical. In the *Prior Analytics* Aristotle made a systematic study of categorical syllogisms, focusing there on syllogisms containing just two premisses, the first, the 'major' premiss and the second the 'minor', where the categorical conclusion relates an extreme of one premiss to an extreme of the other. The two extremes could be thus related in the conclusion because of the role played by a term which occurs twice in the premisses, once in each premiss. This term, the 'middle term', mediates between the two other extremes in the premisses.

The theory of the syllogism expounded by Aristotle was taken up by medieval logicians and extended in a variety of

¹ *Prior Analytics*, 24 b 19-21.

directions. The largest part of what is now commonly thought of as 'traditional' logic is a small fragment of medieval syllogistic. I do not wish to say a great deal about 'traditional' logic, for many expositions are available.² My chief concern here with medieval syllogistic is with areas of that theory which did not find their way into the subsequent 'traditional' logic. But I shall first set out some of the elementary parts of the medieval account. I shall however set it out in the light of what the medieval logicians, rather than their successors, said. The brief description of certain of the elementary parts should provide a basis, sufficient for immediate purposes, on which to construct an account of the role played in valid syllogisms by propositions with ampliative terms. In particular I shall attend to the question of whether there can be valid syllogisms containing non-present-tensed premisses.

Ockham gives the following description:

Only two categorical premisses and a conclusion should be placed in the syllogism, and only three terms, a major extreme, a minor, and a middle term. It is the middle term which is placed in each premiss. It is the major extreme which is placed along with the middle term in the major premiss. It is the minor extreme which is placed along with the middle term in the minor premiss, that is, in the second proposition.³

Thus the first proposition in the syllogism, that is the first premiss, is the major premiss, and the second proposition or premiss is the minor premiss. The major term is the extreme, other than the middle term, which occurs in the major premiss, and the minor term is the extreme, other than the middle term, which occurs in the minor premiss. It should be noted that the major and minor terms are here identified without any regard to the position of those terms in the conclusion, but instead solely on the basis of the premiss in which they occur. From the Renaissance onwards, however, it was common to class the major and minor

² See e.g., J. N. Keynes, *Studies and Exercises in Formal Logic* (London, 1894).

³ *Summa Logicae*, Pt. III 1, Ch. 2, p. 362.

terms on the basis of their position in the conclusion, the major term being the predicate in the conclusion, and the minor term the subject.

Peter of Spain defines 'syllogistic figure' as 'the ordering of three terms in respect of being subject and being predicate'.⁴ And he holds that there are just three syllogistic figures, for as regards the middle term: 'it is the subject in one premiss and predicate in the other, or it is predicate in both, or it is subject in both'.⁵ It seems that he has here described just three figures, but in fact the first description covers two possibilities, (i) the middle term as subject in the major premiss and predicate in the minor, and (ii) the middle term as predicate in the major premiss and subject in the minor. It appears to follow that there are after all four syllogistic figures. Symbolizing the major, minor, and middle terms as A, B, and C, respectively, the premisses can evidently have one or other of the following four forms or figures:

- | | | | |
|--------|--------|--------|--------|
| (1) CA | (2) AC | (3) CA | (4) AC |
| BC | BC | CB | CB |

But the fourth figure, in so far as its existence was noted at all, was commonly regarded as redundant. Thus, for example, Ockham argues as follows:

A fourth figure should not be counted in, because if the middle term is predicate in the first proposition and subject in the second, there will be merely a transposition of the propositions posited in the first figure, and therefore no conclusion will follow other than the conclusion which follows from the premisses laid down in the first figure . . . For if the argument proceeds as follows: 'Every man is an animal, every animal is a substance', the conclusion which follows primarily is this: 'therefore every man is a substance', and this follows from the same premisses, laid out as in the first figure, thus: 'Every animal is a substance, every man is an animal; therefore every man is a substance'.⁶

⁴ *Tractatus*, 44.

⁵ *Ibid.* 43.

⁶ *Summa Logicae*, Pt. III-1, Ch. 2, p. 362. Cf. Albert of Saxony, *Perutilis Logica*, 29^{ra}.

But Ockham does not acknowledge a crucial distinction. Given a first-figure syllogism of the form: 'CA, BC \therefore BA', the figure that Ockham alleges to be the fourth figure is 'BC, CA \therefore BA'. But there is another figure like the one just mentioned though with subject and predicate of the conclusion reversed. And that last figure is logically quite distinct from the one Ockham describes, and is a real fourth-figure syllogism. It is in fact one of the two coming under Peter of Spain's description of what he took to be just one of the figures, namely the one in which the middle term is subject in one of the premisses and predicate in the other.

This leaves us with four distinct syllogistic forms:

(1) CA	(2) AC	(3) CA	(4) AC
BC	BC	CB	CB
\therefore BA	\therefore BA	\therefore BA	\therefore BA

In each of these the predicate of the conclusion is the same as the non-middle term in the major premiss, and the subject of the conclusion is the same as the non-middle term in the minor premiss. Such a syllogism was called a direct syllogism. If however we say that the major term is the predicate of the conclusion and the minor term is the subject of the conclusion without specifying in which premiss those terms occur, the possibility is opened up that the major term occurs in either the first premiss or the second, and likewise with the minor term. This permits us to describe a set of syllogistic forms different from those just displayed. For there are also syllogistic forms in which the major term, namely the predicate of the conclusion, also occurs in the second premiss, and the minor term, namely the subject of the conclusion, also occurs in the first premiss. There are, therefore, the following four forms:

(5) CA	(6) AC	(7) CA	(8) AC
BC	BC	CB	CB
\therefore AB	\therefore AB	\therefore AB	\therefore AB

These are forms of 'indirect syllogisms', and are logically distinct from direct syllogisms. In what follows I shall focus upon the forms of the direct syllogism.

Of valid syllogistic forms four had a special status, for on their basis the validity of every valid syllogism, of whatever form, was to be established. The four valid forms were themselves established on the basis of two 'regulative principles' of syllogistic, namely, the rules *dici de omni* [= 'to be said of everything'] and *dici de nullo* [= 'to be said of nothing']. There is a great deal of logic to be coaxed out of these principles. The first asserts that what is said of a distributed subject is said of everything of which that subject is truly predicated. The second asserts that what is denied of a distributed subject is denied of everything of which that subject is truly predicated.⁷

In AaB the subject is distributed. Hence, by *dici de omni*, if A is truly predicated of every C then what is said of every A, namely B, is truly predicated of every C, and if A is truly predicated of some C then what is said of every A, namely B, is truly predicated of some C. In AeB the subject is distributed. Hence, by *dici de nullo*, if A is truly predicated of every C then what is denied of every A, namely B, is truly predicated of no C, and if A is truly predicated of some C then what is denied of every A, namely B, is truly denied of some C. The four syllogistic forms thus generated are as follows (I add their medieval names in brackets):

- (i) AaB & CaA \therefore CaB (= *Barbara*)
- (ii) AaB & CiA \therefore CiB (= *Darii*)
- (iii) AeB & CaA \therefore CeB (= *Celarent*)
- (iv) AeB & CiA \therefore CoB (= *Ferio*)

In addition to these four syllogistic forms, rules of immediate inference, in particular those of conversion, equipollence, and subalternation, were employed in proving the validity of syllogisms, as also were four rules (63–66 in Chapter 6) which were used in the demonstration *per impossibile* of the

⁷ Cf. Aristotle, *Prior Analytics*, 24 b 28–32.

validity of syllogisms, that is, a demonstration which proceeds by assuming one of the premisses and the negation of the conclusion, and proving the negation of the other premiss. I shall set out some proofs in illustration of the medieval method.

To prove: $AeB \ \& \ CaB \ \therefore \ CoA$ (= *Cesaro*)

- (i) $AeB \ \& \ CaB$ = assumption
- (ii) AeB from (i) by rule 49 (Ch. 6)
- (iii) CaB from (i) by rule 50 (Ch. 6)
- (iv) BeA from (ii) by simple conversion
- (v) $BeA \ \& \ CaB$ from (iv), (iii), from two propositions to their conjunction
- (vi) CeA from (v) by *Celarent*
- (vii) CoA from (vi) by subalternation

Therefore from first to last: $AeB \ \& \ CaB \ \therefore \ CoA$ Q.E.D.

To prove: $AaB \ \& \ AaC \ \therefore \ CiB$ (= *Darapti*)

- (i) $AaB \ \& \ AaC$ = assumption
- (ii) AaB from (i) by rule 49 (Ch. 6)
- (iii) AaC from (i) by rule 50 (Ch. 6)
- (iv) CiA from (iii) by accidental conversion
- (v) $AaB \ \& \ CiA$ from (ii), (iv), from two propositions to their conjunction
- (vi) CiB from (v) by *Darii*

Therefore from first to last: $AaB \ \& \ AaC \ \therefore \ CiB$ Q.E.D.

To prove: $AeB \ \& \ BaC \ \therefore \ CoA$ (= *Fesapo*)

- (i) $AeB \ \& \ BaC$ = assumption
- (ii) AeB from (i) by rule 49 (Ch. 6)
- (iii) BaC from (i) by rule 50 (Ch. 6)
- (iv) BeA from (ii) by simple conversion
- (v) CiB from (iii) by accidental conversion
- (vi) $BeA \ \& \ CiB$ from (iv), (v), from two propositions to their conjunction
- (vii) CoA from (vi) by *Ferio*

Therefore from first to last: $AeB \ \& \ BaC \ \therefore \ CoA$ Q.E.D.

This completes my exposition of the most elementary part of medieval syllogistic. In the next section we shall deal with complicating factors.

II. SYLLOGISTIC TENSE LOGIC

Syllogistic as developed along the lines pursued in the preceding section can be tucked into a corner of the lower predicate calculus, and cannot therefore be expected to arouse much interest among modern logicians looking for new ideas (new to us!) from their medieval forebears. However the chief purpose of Section I was to place us in a position to examine certain aspects of medieval syllogistic which did not, unfortunately, find their way into the 'traditional' account of logic and which might reasonably be expected to interest not only antiquarians.

In Chapter 3, Section VI, attention was directed to the fact that the present tense was not the only tense of interest to medieval logicians. Neither, as we saw, was that interest prompted by purely logical considerations. For example, future contingent propositions were seen to generate problems both philosophical and theological. One such problem concerned the question of whether God's present knowledge about future human actions implies that no future human action will be performed freely. In the light of such concerns it is not surprising that medieval logic textbooks routinely included discussion of future-tensed propositions. Our earlier discussion focused on the appropriate way to set out the truth conditions of future- and past-tensed propositions, and in that respect our concern was primarily semantic. But of course nothing tells us more about the logic of non-present-tensed propositions than their role in valid inferences. It is to this syntactic topic that I wish to turn.

In Chapter 7, Section III, we examined immediate inferences containing non-present-tensed propositions, for there we were concerned with rules of conversion for propositions whose subjects are amplified to the past. In this

section our concern is with mediate inference, and in particular with rules for determining the validity or otherwise of syllogisms containing past- or future-tensed propositions. And since this was a matter of lively concern to medieval logicians it can come as no surprise that they were interested in the rules of conversion for such propositions, given the use to which, as we have seen, rules of conversion were put in establishing the validity or otherwise of syllogisms. The first detailed discussion of syllogisms with propositions amplified to the past and the future was that by Ockham in his *Summa Logicae*. What is said in the remainder of this section is derived from that discussion. I shall attend to the first three figures only. No interesting additional logical principles appear to be involved in the fourth figure, in respect of premisses of which at least one is not present-tensed.

I shall deal with the three figures in turn. First the first. We have noted four first-figure syllogisms, *Barbara*, *Celarent*, *Darii*, and *Ferio*, each being immediately sanctioned by either the *dici de omni* or *dici de nullo* rule. There are two other valid first-figure syllogisms. For since the premisses of *Barbara* support a universal affirmative conclusion, they also, by a rule of subalternation, support a particular affirmative conclusion, and likewise the premisses of *Celarent* support not only a universal negative conclusion but also, by subalternation, a particular negative one. Each one of these six valid syllogisms remains valid if one or more of the propositions in each syllogism is transformed by replacing the present-tensed copula by a copula in another tense. But not every such transformation preserves validity. Our question now therefore concerns the identity of those rules of transformation that do preserve it.

The ampliative power of a non-present-tensed copula has to be taken into account here. Where the copula is past-tensed the subject supposits for what is or for what was, and where it is future-tensed the subject supposits for what is or for what will be. The rules of transformation that we are

seeking specify how the subject is to be taken in a non-present-tensed proposition. For taken in one way a syllogism may be valid, but invalid if taken otherwise. Let us put some flesh on these bones. Assuming the middle term to be common rather than singular, this rule holds: 'If the subject of the major premiss supposits for things which are, then the minor premiss should be neither future- nor past-tensed'.⁸ The reason for this, briefly, is that otherwise the syllogism would not be regulated either by *dici de omni* or *dici de nullo*. Let us restrict consideration to the past tense (examples concerning the future tense can be dealt with along exactly the same lines), and use *Barbara* as an example. We shall consider why no conclusion can be drawn syllogistically from:

- (i) Every (present) A was B & Every C was A.

The copula in the second conjunct ampliates the subject, but not the predicate, for the signification of the latter is restricted by the copula to what was A. Therefore the range of values of A in the putative major premiss is a set of present objects and the range of values of A in the putative minor premiss is a set of past objects. Hence no grounds are provided for concluding that anything whatever for which the first A supposits is identical with anything whatever for which the second A supposits. In effect the past-tensed copula in the second conjunct ensures that any syllogism of which (i) constitutes the set of premisses does not have a middle term, since two terms are middle terms in a given syllogism only if the ranges of values of the two terms are identical. The point can be brought out by writing the subject of the first conjunct as 'present-A' and the predicate of the second conjunct as 'past-A'. To conclude 'Every C was B' or 'Some C was B' from (i) is therefore to commit the fallacy of equivocation.

But if the minor premiss is present-tensed then no such equivocation is committed. In each premiss A signifies what is A, and the rule *dici de omni* can be applied to

⁸ Ockham, *Summa Logicae*, Pt. III-1, Ch. 17, p. 406.

draw the conclusions 'Every C was B' and 'Some C was B'. That is:

- (ii) Every (present) A was B & Every C is A. Therefore every C was B

is valid because it falls under the rule: What is said (namely that it was B) of a distributed subject (namely what is now A) is true of everything (and therefore of every C) of which that distributed subject is truly predicated.

I have taken as my example an affirmative syllogism (that is, one with two affirmative premisses), but the same considerations apply to negative syllogisms—(that is, syllogisms with a negative premiss). Thus:

- (iii) No (present) A was B & Some C is A. Therefore some C was not B

is valid since it falls under the regulative principle that what is denied (namely that it was B) of a distributed subject (namely what is now A) is denied of everything (and therefore of some C) of which that distributed subject is truly predicated. And once again, and for the same reason as before, the syllogism is rendered invalid if the copula of the minor premiss is replaced by one which is not present-tensed.

Nothing said so far, however, permits the inference that unless the minor premiss of a first-figure syllogism is in the present tense, such a syllogism cannot have a past-tensed conclusion. The rule that for a past-tensed conclusion to be drawn the minor premiss must be in the present tense is applicable only to those cases where the past-tensed major premiss has a subject which is taken to stand for what is. For if the subject of the past-tensed major premiss is taken to stand for what was, then a conclusion can be drawn syllogistically if the minor premiss is past-tensed. The basic consideration here is the same as that invoked earlier; the range of values of the subject of the major premiss must be identical with the range of values of the predicate of the minor. If, therefore, the subject of the major premiss is taken

to signify what was, then the predicate of the minor must also be taken to signify what was, and that signification is contrived by placing the copula of the minor premiss in the past tense.

This prompts a question concerning the subject of the minor premiss, for given that the copula is past-tensed it follows that the subject is amplified to signify either what is or what was. And we have to ask how that subject has to be taken if any syllogistic conclusion is to be drawn, or alternatively if one syllogistic conclusion rather than another is to be drawn. As we shall see, given that the subject of the past-tensed major premiss is taken to stand for what was, then a syllogistic conclusion can be drawn whether the subject of the minor premiss is taken to stand for what is or for what was. But how we take that minor subject certainly affects how we can take that term when it recurs in the conclusion, since it must be taken in the same way, whatever that way is, on both its occurrences. Thus this is valid:

- (iv) Every (past) A was B & Every C was A. Therefore every C was B.

The *dici de omni* rule sanctions (iv) since in (iv) what is said (namely that it was B) of the distributed subject (namely what was A) is said of everything (and therefore of every C) of which that subject is truly predicated. If the subject of the minor premiss is taken to signify what was C, then that is how the subject in the conclusion should be taken. Otherwise the conclusion is not warranted by the premisses. Thus this is invalid:

- (v) Every (past) A was B & Every (past) C was A.
Therefore every (present) C was B.

It is clear that the premisses do not warrant any affirmative conclusion about a present C, for they do not imply even that any C now exists. And if (consistently with the premisses) no C exists, then, in accordance with the rule that an affirmative proposition with a subject which signifies

nothing is false, the conclusion of (v) is false. And under certain conditions under which that conclusion is false the premisses are true. Hence the invalidity of (v). Similarly, this is invalid:

- (vi) Every (past) A was B & Every (present) C was A.
Therefore every (past) C was B.

In each of the kinds of case so far considered, from a pair of premisses of which at least one is not present-tensed a conclusion is drawn which has the same tense as the premiss which is not present-tensed. Let us now ask whether there can be a valid first-figure syllogism with a present-tensed conclusion though one of the premisses is not present-tensed. Let us suppose the major premiss present-tensed and the minor past. And we shall suppose that the subject of the major premiss is taken for what was, and the subject of the minor is taken for what is. Then a present-tensed conclusion can be drawn, as in this syllogism in *Barbara*:

- (vii) Whatever was A is B & Whatever is C was A.
Therefore whatever is C is B.

(vii) can readily be seen to be sanctioned by the rule *dici de omni*. Additionally if the subject of the minor premiss is a singular term instead of a common term taken to signify what is, the inference is valid. Thus this is valid where S is a singular term:

- (viii) Whatever was A is B & S was A. Therefore S is B.

S might of course signify something which no longer exists, in which case the conclusion would be false. That would not affect the truth value of the minor premiss, since there the 'was' ampliates the subject to the present or the past. But it does affect the truth value of the major premiss. If S, which no longer exists, was A then it is not true that whatever was A is B, for S, not being anything, is not B. In that case the inference is not to a false conclusion from two premisses

both of which are true. And therefore we have not set up a model which demonstrates the invalidity of (viii).

It has to be added, however, that even though the conclusion of a first-figure syllogism might be in the present tense while the minor premiss is not present-tensed, nevertheless if the major premiss is not present-tensed then neither can the conclusion be. The reason for this is that if the major premiss is past- or future-tensed then the predicate P of that premiss signifies what was or what will be. But if the conclusion is present-tensed then the predicate P in the conclusion must signify what is, and yet the premisses say nothing that permits any conclusion about what is P, even about whether any P exists. Thus this is invalid:

- (ix) Whatever is A was B & Whatever is C is A.
Therefore whatever is C is B.

It is plain that the conclusion of (ix) should be:

- (x) Whatever is C was B.

We turn now to a consideration of second-figure syllogisms. Certain rules which are inapplicable to first-figure syllogisms are applicable to those of the second figure. For example, as regards first-figure syllogisms, we have just noted that if the major premiss is past-tensed then so also must be the conclusion. But in the second figure there are valid syllogisms containing two past-tensed premisses and a present-tensed conclusion. Ockham writes: 'When both premisses in the second figure are past-tensed and the subject of each of these supposits for things which are, there always follows a present-tensed conclusion.'⁹ Thus this is valid:

- (xi) No (present) A was B & Some (present) C was B.
Therefore some C is not A.

Ockham's argument for such syllogisms is as follows: 'From the major premiss and the opposite of the conclusion there

⁹ Ockham, *Summa Logicae*, Pt. III-1, ch. 18, p. 408.

follows the opposite of the minor premiss in the first figure."¹⁰ The rule he here invokes is based on our rule 64 (Ch. 6). (xi) is equivalent to:

- (xii) No (present) A was B & Every C is A. Therefore
no (present) C was B.

And (xii) is a valid first-figure syllogism in *Celarent* as can be demonstrated by using the rule *dici de nullo*. That is, what is denied (namely that it was B) of a distributed subject (namely what is A) is denied of everything (and therefore of every present C) of which that subject is truly predicated. The argument for (xi) may be presented as follows:

- (a) No (present) A was B & Some (present) C was B
= assumption
(b) No (present) A was B
from (a) by rule 49 (Ch. 6)
(c) Some (present) C was B
from (a) by rule 50 (Ch. 6)
(d) Nothing which was B is A
from (b) by simple conversion (see Ch. 6, Section III)
(e) Nothing which was B is A & Some (present) C was B
from (d), (c), from two propositions to their conjunction
(f) Some C is not A
from (e) by Ferio

Therefore from first to last: (a) \therefore (f) = (xi) Q.E.D.

But, as just mentioned, from the premisses of (xi) no past-tensed conclusion follows. Ockham's argument in support of this claim is this:

If it [the past-tensed conclusion] followed then from the opposite of the conclusion along with the major premiss there would follow the opposite of the minor premiss in the first figure, and

¹⁰ Ockham, *Summa Logicae*, Pt. III-1, Ch. 18, p. 408.

consequently in the first figure under a major premiss in which the subject supposits for things which are there would be a minor premiss in the past tense.¹¹

We have already in this section discussed the fallacy to which Ockham here alludes. Let us follow the argument through in relation to a suitably modified (xi). That is, we shall wrongly suppose this valid:

- (xiii) No (present) A was B & Some (present) C was B.
Therefore some (present) C was not A.

Applying rule 64 (Ch. 6) to (xiii) we derive this inference:

- (xiv) No (present) A was B & Every (present) C was A.
Therefore no (present) C was B.

This appears to have the form of a syllogism in *Celarent*, but the putative middle term does not in fact mediate between the other extremes. For A in the major premiss signifies what is, and in the minor what was. It is easy to construct counter-examples to (xiv). Let us suppose a set of two objects, O^1 , O^2 . At past time t^1 O^1 was grey and O^2 black. At t^2 , between t^1 and the present, both objects were grey. Now O^1 is still grey but O^2 is white. We can now assert the following truths about our two objects: (a) No present grey object was black, (b) every present white object was grey, and (c) some present white object was black. (xiv), therefore, is invalid.

If however, the subject in each premiss is taken to supposit for what was, then a past-tensed conclusion can validly be drawn. Thus this is valid:

- (xv) No (past) A was B & Some (past) C was B.
Therefore some (past) C was not A.

Applying rule 64 (Ch. 6) we derive:

- (xvi) No (past) A was B & Every (past) C was A.
Therefore no (past) C was B.

¹¹ Ibid.

(xvi) is a valid syllogism in *Celarent* and can readily be derived from the rule *dici de nullo*. (xv) can be proved in the following way also:

- (a) No (past) A was B & Some (past) C was B
= assumption
- (b) No (past) A was B
from (a) by rule 49 (Ch. 6)
- (c) Some (past) C was B
from (a) by rule 50 (Ch. 6)
- (d) No (past) B was A
from (b) by simple conversion
- (e) No (past) B was A & Some (past) C was B
from (d), (c), from two propositions to their conjunction
- (f) Some (past) C was not A
from (e) by *Ferio*

Therefore from first to last: (a) \therefore (f) \equiv (xv) Q.E.D.

Indeed even if the subject of the major premiss supposits for what was, and the subject of the minor supposits for what is, then, again, a past-tensed conclusion may follow. For example, this is valid:

- (xvii) Every (past) A was B & No (present) C was B.
Therefore no (present) C was A.

The proof of (xvii) is as follows:

- (a) Every (past) A was B & No (present) C was B
= assumption
- (b) Every (past) A was B
from (a) by rule 49 (Ch. 6)
- (c) No (present) C was B
from (a) by rule 50 (Ch. 6)
- (d) No (past) B is C
from (c) by simple conversion
- (e) No (past) B is C & Every (past) A was B
from (d), (b), from two propositions to their conjunction

(f) No (past) A is C

from (e) by *Celarent*

(g) No (present) C was A

from (f) by simple conversion

Therefore from first to last: (a) \therefore (g) = (xvii) Q.E.D.

We turn now to a consideration of third-figure syllogisms,¹² and shall begin by supposing that both premisses are past-tensed and that the subjects are taken uniformly, that is, both are taken to supposit for what is or both for what was. In that case a past-tensed conclusion can validly be drawn. If the subjects of the premisses are not taken uniformly then no conclusion can be drawn syllogistically. An example should clarify this rule. The following is valid:

(xviii) No (present) A was B & Every (present) A was C.

Therefore some (past) C was not B.

Applying rule 63 (Ch. 6) to (xviii) we reach:

(xix) Every (past) C was B & Every (present) A was C.

Therefore some (present) A was B.

(xix) is a first-figure syllogism whose premisses are those of a syllogism in *Barbara*, and whose conclusion follows, by subalternation, from the conclusion of the syllogism in *Barbara*.

An alternative proof of (xviii) is the following:

(a) No (present) A was B & Every (present) A was C
= assumption

(b) No (present) A was B

from (a) by rule 49 (Ch. 6)

(c) Every (present) A was C

from (a) by rule 50 (Ch. 6)

(d) Some (past) C is A

from (c) by accidental conversion

¹² See *Summa Logicae*, Pt. III-1, Ch. 19, pp. 409-11.

- (e) No (present) A was B & Some (past) C is A
 from (b), (d), from two propositions to their conjunction
 (f) Some (past) C was not B
 from (e) by *Ferio*

Therefore from first to last: (a) \therefore (f) = (xviii) Q.E.D.

But it is clear that if the subject of the first premiss in (xviii) were taken for what was, then no conclusion could be drawn concerning the relation between B and C.

There are, also, valid third-figure syllogisms with a past-tensed conclusion, which do not have two past-tensed premisses. We can, for example, suppose the major premiss past-tensed and the minor present-tensed, with the subject of the major being taken for what is. Then a past-tensed conclusion can be drawn whose subject, like that of the major premiss, is taken for what is. Thus this is valid:

- (xx) Every (present) A was B & Every (present) A is C.
 Therefore some (present) C was B.

This can be proved as follows:

- (a) Every (present) A was B & Every (present) A is C
 = assumption
 (b) Every (present) A was B
 from (a) by rule 49 (Ch. 6)
 (c) Every (present) A is C
 from (a) by rule 50 (Ch. 6)
 (d) Some (present) C is A
 from (c) by accidental conversion
 (e) Every (present) A was B & Some (present) C is A
 from (b), (d), from two propositions to their conjunction
 (f) Some (present) C was B
 from (e) by *Darii*

Therefore from first to last: (a) \therefore (f) = (xx) Q.E.D.

It can also be shown that where the major premiss is present-tensed and the minor is past, a conclusion follows syllogistically. The reason for this can be displayed by 'reducing' such a third-figure syllogism to one in the first figure. Let us take as our third-figure syllogism:

- (xxi) No (present) A is B & Some (present) A was C.
Therefore some (past) C is not B.

In the conclusion of (xxi) a past C is specified, for in the minor premiss the predicate stands for what was. Replacing the major premiss by the negation of the conclusion, and replacing the conclusion by the negation of the major premiss (see rule 65, Ch. 6), we reach:

- (xxii) Every (past) C is B & Some (present) A was C.
Therefore some (present) A is B.

The validity of (xxii) can be demonstrated by reference to the rule *dici de omni*. We can also prove (xxi) in the customary way:

- (a) No (present) A is B & Some (present) A was C
= assumption
(b) No (present) A is B
from (a) by rule 49 (Ch. 6)
(c) Some (present) A was C
from (a) by rule 50 (Ch. 6)
(d) Some (past) C is A
from (c) by simple conversion
(e) No (present) A is B & Some (past) C is A
from (b), (d), from two propositions to their conjunction
(f) Some (past) C is not B
from (e) by *Ferio*

Therefore from first to last: (a) \therefore (f) = (xxi) Q.E.D.

But the premisses of (xxi) do not support a past-tensed conclusion, for if the copula were past-tensed the predicate would stand for what was, though in the major premiss it

stands for what is. The invalidity of (xxi) when thus modified is clearly displayed by reference to its equivalent first-figure syllogism. This latter is reached by replacing the major premiss of (xxi) by the negation of the modified conclusion, and replacing the modified conclusion by the negation of the major premiss. The result of this transformation of (xxi) is:

(xxiii) Every (past) C was B & Some (present) A was C.
Therefore some (present) A is B.

It is plain that the premisses support the conclusion 'Some (present) A was B'. But that they do not support 'Some (present) A is B' follows from the fact that neither premiss in (xxiii) implies the present existence of anything which is B. And if, consistently with the premisses, no B exists, then the conclusion of (xxiii) is false. And on some consistent assumptions which are incompatible with the conclusion of (xxiii) the premisses are true. Therefore (xxiii) is invalid.

9

Conclusion

It will be clear from the sources quoted in this book that what I have presented is largely logic of the fourteenth century. The masters of that century were widely studied during the following 150 years, and numerous books were written transmitting their ideas and adding to them. Many of the additions were made when inferences were investigated with which the previously accepted rules of inferences seemed unable to cope, unable either because the new inferences, if valid, were in conflict with old rules, or because the new inferences contained features and elements that placed them beyond the jurisdiction of the old rules. Other additions were made when questions arose concerning the application of accepted rules. For example, given rules involving conjunctive terms, questions were naturally raised about how a given occurrence of ‘and’ between categorematic terms is to be recognized as divisive or collective. Are there any ways of determining, on the basis of purely syntactic considerations, whether an ‘and’ is to be taken divisively or collectively?

Not surprisingly, medieval logic has been criticized for its ‘damnable particularity’. But I should like to make two points in reply. First, concern for the particular is a price that is inevitably paid by any logician who takes the whole of a natural language as his object language, rather than taking an artificial language whose elements are introduced systematically item by item with a duly assigned role. Secondly, medieval logic was not concerned solely with the particular. The interest in the particular was, after all, a

consequence of the desire to refine the presentation of the universal. Of course, this led to a multiplication of low-level general rules. But there were also high-level rules of which the low-level ones were seen as so many specifications forced by the exigencies of the object language. In all this we should not lose sight of the fact that our logicians formulated and used a number of rules of a very high level of abstraction. We looked at a number of such rules in the course of our discussion of molecular propositions. And as regards the logic of terms, the basic rules of descent under terms covered by universal or particular quantifiers, or by signs of negation, are at a high level of logical abstraction and work well for a large proportion of the cases we are liable to encounter.

Nevertheless, to many it seemed as though the logic of the late medieval period was running practically out of control. There were simply too many rules, and no assurance that new ones might not be introduced indefinitely. The time was becoming ripe for change. The change came under the banner of the new humanism which was, by the late fifteenth century, beginning to take deep root in the universities. It is appropriate here to say something about the new humanism, and I shall focus in particular upon Juan Luis Vives, whom we met briefly in the course of our discussion of artificial quantifiers. In 1509 he went up to the Collège de Montaigu in the University of Paris, a college which was at that time profoundly influenced by John Mair. Mair, described by his pupil and colleague Antonio Coronel as 'prince of philosophers and theologians at the University of Paris',¹ had taught a generation of late scholastic logicians their trade. He numbered among his many pupils at Montaigu the Belgian logician John Dullaert and Gaspar Lax from Aragon, who in their turn were teachers of Vives. There is therefore no doubt that by the time the thoroughly disenchanted Vives left Paris in 1512, he had become steeped

¹ In Antonio Coronel, *In Posteriora Aristotelis* (Paris, 1510). See prefatory letter to Coronel's brother Francisco.

in the thought-world he was to spend the rest of his life attacking. To understand the sheer ferocity of his attack it is essential to recognize that he was not attacking an abstraction, but a living reality awesome in its power, as he saw it, to do damage, and even to corrupt.

Vives was, to use scholastic terminology, *removens prohibens*, the clearer of an obstacle. John Mair and his circle were an obstacle to the reception of the new learning and Vives saw it as his task to destroy their influence; it is the principal, perhaps sole purpose, of his *In Pseudodialecticos*. As regards the so-called new learning what is meant of course is very old learning. And there are two aspects to that, first the languages and secondly the ideas expressed in those languages. Vives, spellbound by Cicero's Latin, criticizes the logic of Mair and his school partly on the grounds that Cicero would not have understood the language they used in order to present their logic. Sorbonnic Latin was not real Latin, and by implication a logic that required Sorbonnic Latin for its expression was not real logic either.

From Vives's point of view medieval logicians were mistaken over the relation between language and grammatical rules, and the mistake could be seen to vitiate the logic. At issue is the order of priority. To Vives it was plain that linguistic common usage comes first, that is, first in time, and rules of grammar are to be gleaned from that usage; a rule not sanctioned by the consensus of the people as displayed in their linguistic practice is a bad rule. But it was his contention that the late medieval logicians got things the wrong way round. For on his account of the matter they began by constructing grammatical rules, and then wrote propositions whose grammatical correctness they assessed in terms of whether or not they conformed to the antecedently devised rules. Thus it was not linguistic practice that sanctioned the rules of grammar but the rules of grammar that sanctioned linguistic practice. Since the rules led and the language followed, it was not to be wondered at that the logicians peddled in grammatical absurdities.

A similar point can be made about the signification of terms and propositions. Signification is fixed by the consensus of the people as displayed in their linguistic practice. It cannot be fixed by an individual person who takes it upon himself to decide what a term or proposition is to signify. Once it is recognized as permissible for an individual to prescribe signification, then who is to say what anything means? With the appropriate prescription anything could signify anything, and therefore anything could follow from anything. And then there would no longer be any consensus about what follows from what. That was precisely the situation that obtained in the University of Paris, according to Vives. He speaks about a John Dullaert logic and a Gaspar Lax logic, and a Vives logic for good measure. Of course, once logicians are permitted to prescribe significations, there could then be at least as many logics as there are logicians.

All this is an absurd state of affairs, and what is really needed is not a Dullaert logic or a Lax logic but a Latin logic, by which of course is meant a logic written in Ciceronian Latin using words with the signification that Cicero would have reported them as having. Greek will do equally well, so long as we are clear about what Greek is at issue. As Vives says: 'Is anyone of the opinion that Aristotle fitted his logic to a language which he had invented for himself, instead of to the current form of Greek which everyone spoke?'² However, Vives's persistent references to linguistic ordinary usage are seriously misleading, for what he had in mind was the usage of a tiny handful of humanists like himself who were modelling themselves on Cicero, who himself wrote a Latin that was no doubt never spoken by more than a small minority of Romans even in Cicero's day. Hence when Vives refers to 'the common speech that everyone uses [*qui est omnium in ore sermo*]',³ his description of the language he has in mind is false. Indeed it might well be the case that far more people spoke the despised Latin of the schools than spoke the Latin of Cicero.

² In *Pseudodialecticos*, 36, 37, trans. C. Fantazzi.

³ Ibid.

But no doubt Vives would reply that a head-count was irrelevant, for it was really quality that was at issue, two kinds in particular. First there was the quality of the Latin, the bewitching style of Cicero as opposed to that favourite target of the humanists, the crabbed artificiality of Sorbonnese. It is almost as if Cicero's Latin constituted a Platonic form of the language and the further removed any Latin was from that ideal the more corrupt it was. And of course a Latin falling so far short that Cicero would be unable to understand it would be corrupt indeed. Secondly there was the quality of Ciceronian logic, a logic owing a great deal to courtroom procedures and argumentation, in which evidence is brought under the investigative spotlight in order to determine the soundness of the claims made on behalf of defence or prosecution. And here we have to keep in mind Cicero's role as trial lawyer, arguing over substantive issues, and dealing in probabilities rather than focusing upon timeless necessary truths and upon arguments so rigorous as to satisfy Aristotelian canons of demonstrative inference. For arguments of the latter sort cannot bring anything genuinely new to light, but only lay bare what is already implicit in the starting-point. Cicero, on the contrary, was interested in reaching truths which were not deductively inferable from the premisses, but could be reached only by arguing probabilistically on the basis, for example, of analogies of one sort or another.

The context in which such exercises of rationality were mounted was the rhetoric of the law courts, and to a substantial extent also the rhetoric of political debate, and there need be no surprise in the enthusiasm with which Renaissance thinkers took up the idea of logic as a branch of rhetoric, the art of persuasion by speech, where of course the quality of the language employed could itself be such a powerful weapon. But the logic itself was not a negligible element in rhetoric. On the contrary what more persuasive instrument could one employ than a clearly stated valid argument? The point I wish to stress here is that the

Renaissance humanist thinkers were far from being despisers of logic, even given their espousal of rhetoric; but they did emphasise a rather broad concept of logic, one sufficiently broad to encompass the probabilistic arguments of politicians and lawyers. Thus the humanists emphasised such forms of rational persuasion as the *sorites*, whose persuasiveness can be very great even though it cannot be displayed as deductively valid.⁴

We should not lose sight of the fact that we hardly ever reason syllogistically. The usual exercises of logic that we engage in consist of the presentation of evidence which supports a given proposition sufficiently well to make it more reasonable to accept that proposition than not to accept it. It is within the context of rational discourse of that nature that we occasionally meet with examples of demonstrative reasoning. But since, overwhelmingly, logic as practised in the real world is non-deductive, and since the humanists sought to reflect in their writings our actual practices, the heart of their logic was non-deductive, and demonstrative inference was peripheral. This was not just a matter of emphasis, but a revolution with wide ramifications.

Indeed it has been persuasively argued that with Lorenzo Valla, if not before him, we find the beginnings of the attempt to move away from the abstract logical questions raised by the scholastic philosophers and towards the development of genuine scientific interrogation of the natural order, closely akin to the interrogation of a witness in a court of law conducted in order to coax out of him whatever pertinent information he is capable of yielding up.⁵ And there is a great deal to be said for the view that it was precisely the humanists' interest in the Ciceronian non-deductive logic, as based upon juridical practice, that led them to think in terms of nature as itself something to be put in the witness box and questioned in the same sort of way. Some would think that there could hardly be a greater

⁴ The 'heap' argument. A heap of sand is a heap. Minus one grain it is still a heap. Therefore minus two grains it is still a heap. Therefore . . . Finally there is only one grain left. But that is not a heap.

⁵ See e.g. T. F. Torrance, *Juridical Law and Physical Law*, esp. 39-40.

contrast than that between on the one hand this exercise of rationality which led to the rise of empirical scientific enquiry, and on the other those supposedly arid scholastic abstractions purveyed as logic.

Elsewhere I have complained that logic was a casualty of the Reformation, and in consequence have been thought by some not to be well-disposed to the sixteenth-century humanism that undermined late scholastic logic. Of course I am not hostile to the positive achievements of humanism.⁶ However, not everything which occurred under the banner of humanism should be counted as progress, and in particular while the rather more open-textured concept of logic no doubt contributed to the rise of empirical scientific enquiry and to other priceless things also, its attacks upon the logic of the schools were altogether too successful; the good was thrown out with the bad, and some marvellous logical ideas which should never have been allowed to disappear have had to be rediscovered. It could be that the science of formal logic would be far in advance of where it is now if the best that scholastic logic had had to offer had been given room to develop.

Yet there were indeed many things said by the late scholastics that seemed utterly absurd, and Vives gives numerous examples. Here are two: 'Varro, though he is a man, is not a man [*hominem non esse*], because Cicero is not Varro', and 'A head no man has, but no man lacks a head'.⁷ We might suppose, from brief exposure to such propositions as these, that late scholastic logic was indeed, as Vives thought, guilty not merely of invalidity but even of sheer absurdity, and that in contrast to humanist logic, this chief target of Vives was on the way to nowhere, indeed had already arrived. Vives says of the above two propositions that they would have brought Cicero to a standstill (*haereat*), stopped him dead in his tracks.

⁶ For an account of those achievements see *The Cambridge History of Renaissance Philosophy*, ed. C. B. Schmitt, Q. Skinner, E. Kessler, esp. ch. 7, 'Humanistic Logic', by Lisa Jardine.

⁷ In *Pseudodialecticos*, 41, 43.

In fact, on the basis of logical positions developed earlier in this book we can see that the above exemplars of absurdity are not absurd. They make good sense, and it is possible to construct contexts within which it is appropriate to assert them. I shall now seek to justify this claim, and shall start by attending to the first of the exemplars, the one concerning Varro and Cicero. What is centrally at issue is the effect upon the truth value of a proposition of the precise position in it of the negation sign, a topic of as much interest to present-day logicians as it was to the late scholastics.

We are then attending to the argument: 'Varro, though he is a man, is not a man, because Cicero is not Varro'. It might seem at first sight that this is an invalid argument. The premiss is 'Cicero is not Varro', and the conclusion is 'Varro is a man and is not a man'. The premiss is true and the conclusion, being a contradiction, is necessarily false. But a valid argument with a true premiss has a true conclusion. Vives's exemplar therefore is invalid. Yet there were logicians who would have said that it was valid. But how could it be? The answer is that it is true that Cicero is not Varro, that Varro is a man, and in addition that there is another man, Cicero, who Varro is not. So there is one man who Varro is (namely, himself) and another who Varro is not. In that case, we should surely accept the argument: 'Varro, though he is a man, is not a man, because Cicero is not Varro'.

I have just argued that the argument is invalid and also that it is valid. How can it be both? The obvious explanation is that 'Varro is not a man' is ambiguous; it can be taken in either of two ways, first as denying that Varro is any man at all, and secondly as denying, of some man in particular, that Varro is he, which is the sense in which we take it when we argue validly: 'Cicero is not Varro, and Cicero is a man, therefore Varro is not a man.' Can I name a man who Varro is not? Yes—Cicero.

It is plain to us that 'Varro is not a man' ordinarily means the same as 'Varro is not any man at all', and that it takes a

special context to get us to read the sentence as meaning the same as 'There is some man (say, Cicero) who Varro is not'. Nevertheless it seems that the proposition can bear both interpretations. Furthermore it is plain that the proposition's role in an inference is affected by the particular interpretation it is given. Thus as regards 'Cicero is not Varro and Cicero is a man, therefore Varro is not a man', the inference is valid on one of those interpretations and invalid on the other.

Two questions can be asked about the logical situation just discussed. First, should anything be done about it? And secondly, if anything should be, then what? Let me take the first one first. The short answer is that something should be done about it, because we all have a sovereign interest in the truth and therefore in unambiguous discourse. The connection between truth and ambiguity is obvious; an ambiguous proposition might be true on one interpretation and false on another. The fact that two disputants are in dispute might then be due simply to the fact that they are interpreting the same proposition in different ways, whereas if they could agree on interpretation they would find that they agreed on substance as well. And contrariwise the fact that two people are interpreting the same proposition in different ways might lead them to see themselves as in agreement with each other, whereas if each realized how the other was interpreting the proposition they would realize that they disagreed with each other substantively. In general it is better for people to understand each other correctly and so it is better for ambiguities to be disambiguated.

As regards the medieval logicians, we have to remember that many of them were also theologians whose attention was focused upon the truths that save, and who disputed with each other about those truths. They did so with passion because what was at stake was nothing less than human salvation. Out of their disputes would arise a clear understanding of the saving truths, and then the Church could do its best possible to save souls. It was therefore of the utmost importance that those men understand each other.

The particular ambiguity we are dealing with here, that illustrated by the inference involving Varro and Cicero, centres on the role played by the negation sign, for it is unclear what is being denied by the problematic proposition. Given the pervasiveness of negation signs in our language, and particularly in the language of dispute, there is good reason to seek to disambiguate the ambiguity we are facing—not because anything hangs on the truth value of the proposition ‘Varro is not a man’, but because the ambiguity of other negative propositions might well have harmful consequences. We have observed that medieval logicians had ways of dealing with certain sorts of ambiguities. The means included the application of rules for interpreting propositions on the basis of word-order. The methodology here is important. There was little point in basing the rules simply upon ordinary usage, for it was precisely ordinary usage that was the cause of the trouble. We do ordinarily say things which are ambiguous, and we rely on all sorts of signals, some more linguistic than others, in order to work out what is meant. And even so, we are often left uncertain as to which of several interpretations accords with the speaker’s intentions. So rules were devised that were based largely on fiat, though of course ordinary usage was one guiding consideration. The result was a Latin which was to a certain extent artificial, a scientific Latin appropriate for scientific discourse. In virtue of its artificiality, as well as of other things, it was a target of humanist abuse, and the proper response to the abuse is to say that this careful refinement of the language was necessary as a means to reducing the level of ambiguity.

Let us now turn to my second question, namely, what should be done about the ambiguity in question. The proposition which I have translated ‘Varro is not a man’ does not have that word order in Vives’s Latin. Vives writes ‘Varro homo non est’ (I have ironed out Vives’s accusative and infinitive construction). He does not write ‘Varro non est homo’. Vives, taking his cue from humanist usage, would

have said that these propositions have the same signification. But he must have known, from his training under Dullaert and Lax, that late scholastic logicians would have said that they do not have the same signification.

The difference as regards the structure of the proposition lies in the position of the negation sign. In 'Varro non est homo' the sign precedes 'homo', and 'homo' is therefore within the scope of the sign. In 'Varro homo non est', 'homo' precedes that negation sign and therefore lies outwith its scope. How does this come to make a difference? The answer is that the late scholastic logicians legislated the difference. They may indeed have been responding sympathetically to a clue provided by ordinary usage but, whatever ordinary usage might have dictated, the logicians devised rules for interpreting a proposition depending on whether a given term was or was not within the scope of a negation sign. And if the community of scholars knew the rules and followed them so that there was uniform practice on this matter, then the presence of the rule was justified as leading to the ironing-out of a common ambiguity concerning the word 'non'.

The distinction drawn by assigning a given logical significance to the order of terms in a proposition, and in particular to the placement of the terms in relation to the negation sign, is a distinction which can easily be made within the confines of modern logic, that is, the logical tradition started by Frege. It is important to be able to make the distinction in question, and the late scholastics made it rather well while staying within the limits set by Latin grammar.

A point should be added here about the inference involving Varro and Cicero. It was characteristic of the late scholastics to take as their exemplars propositions which were, in respect of their information content, very dull indeed if not plain silly. Humanists mocked these exemplars, such as the ones about the donkeys Brownie and Honey (Brunellus and Favellus) who belonged to Socrates and

brayed their way through many a medieval logic textbook. The humanists believed that subject matter of greater weight and dignity was required for so important a task as the exposition of logic. And they were heavily influenced in this belief by such writings as the *De Inventione* and the *Topica* of Cicero, where serious legal problems are routinely aired in illustration of logical points. Thus for example, we find the following arguments in the *Topica* in illustration of various argument types: 'So-and-so is not a free man unless he has been set free by entry in the census roll, or by touching with the rod, or by will. None of these conditions has been fulfilled, therefore he is not free';⁸ 'If a man has bequeathed to his wife all the money that is his, he has not, therefore, bequeathed what is owed him; for it makes a great difference whether the money is stored in a strong-box or is on his books'.⁹ And in illustrating the 'argument by similarity', Cicero writes: 'If honesty is required of a guardian, a partner, a bailee, and a trustee, it is required of an agent. This form of argument which attains the desired proof by citing several parallels is called induction.'¹⁰ Such illustrations are typical of Cicero, but logicians such as Mair, Dullaert, and Lax preferred in general to tell us about Brownie running and Honey braying.

Yet the silly examples used by medieval logicians are not so silly. The logicians were interested in the role that certain terms played in valid inferences, terms such as 'every', 'some', 'no', 'and', 'or', 'if', 'only', 'except', and 'in so far as'. They therefore constructed propositions containing such terms as these and showed how from such propositions, singly or conjointly, others could be deduced. The other terms in the propositions were of no interest, only the logical terms, for it was the logical form of the propositions that was of interest, not the particular point being made by those propositions. Indeed a proposition which said something striking might be a pedagogical hindrance in so far as the

⁸ *Topica*, II 10, trans. H. M. Hubbell.

⁹ *Ibid.* III 16.

¹⁰ *Ibid.* X 42.

point made tended to distract attention from the logical form of the proposition used to make that point. Hence a feature of the exemplar propositions that drew humanist mockery in fact made a useful contribution to those exemplars considered as pedagogical devices.

It is worth comparing these uninformative or even plain silly propositions with the kinds of propositions used in modern logic textbooks. There we find that names, predicate terms, and other terms that are not logical constants, are replaced by letters of the alphabet, with the result that the propositions have no information content whatsoever. Thus instead of 'Brownie is not a donkey' we have ' $\neg Db$ '. Instead of 'A donkey is braying' we have $(Ex)(Dx \ \& \ Bx)$. The letters of the alphabet are to be seen as holding the place for ordinary proper names, predicates, and so on. Thus the propositions in question are really propositional schemata rather than propositions as such. As nearly as they could manage, medieval logicians approached this situation also. The information content of 'Socrates is running and Plato is disputing', a standard example of a conjunctive proposition, is totally irrelevant. In effect we learn from it no more than we learn from ' $P \ \& \ Q$ '. It is presented as the premiss in relation to the conclusion 'Socrates is running', and all this is an illustration of the rule: 'From a conjunctive proposition to each of its principal parts is a formally valid inference.' In the light of these considerations it is plain that to object to the lack of weightiness in the content of the exemplars used by our logicians is to miss the point.

I should now like to turn briefly to the second example of propositions given above which, as Vives says, would bring Cicero to a standstill. What Vives produces in his *In Pseudodialecticos* is a garbled version of an illustration of a logical distinction familiar to students at Paris in the early 1500s.¹¹ The distinction, briefly treated in Chapter 3 above, was routinely illustrated by the two propositions: 'Omnis homo habet caput' and 'Caput omnis homo habet'—'Every

¹¹ See A. Broadie, *The Circle of John Mair*, 59–61.

man has a head' and 'There is a head that every man has'. Apparently to Vives's humanistically refined ear there is no logical difference between these two propositions, and perhaps Cicero would not have said they meant different things. But Vives's teachers at Paris were in fact discussing, in the only way that was then available to them, something we now recognize to be of great logical importance, namely the logical significance of the order of quantifiers in a multiply general proposition.

The late scholastics had, as I have already indicated, rules for determining the kind of supposition that terms have, given the position of those terms in relation to quantifiers. On the basis of those rules they said that in 'Omnis homo habet caput', 'homo' has distributive supposition and 'caput' has merely confused supposition. The result of this is that the proposition signifies exactly what one would expect, namely that each man has a head, with each presumably having a different head from every other man. But since in 'Caput omnis homo habet', 'caput' has determinate supposition while 'homo' has distributive supposition, the proposition signifies that there is some one head that every man has. That is, every man has one and the same head (which would not be true unless there were only one man).

It was easy for our late scholastics to show that there is a one-way implication relation from 'There is a head that every man has' to 'Every man has a head'. They could do this by application of their highly developed theory of supposition. The humanists, who had little good to say about supposition theory, perhaps because Aristotle did not employ the concept of supposition, could not themselves make the distinction in question and settled for abusing those who did. Thereafter tools for making the distinction remained unavailable to logicians until Frege managed it with the aid of his quantifier and bound-variable notation.

The fallacy committed when making a move such as the following: 'Every man has a head, therefore there is a head that every man has' (*Omnis homo habet caput, ergo caput omnis*

homo habet) has been christened 'the quantifier shift fallacy' by Peter Geach.¹² The phrase 'quantifier shift fallacy' is wholly appropriate. What is being said is that the truth of a doubly quantified proposition, with the first quantifier universal and the second existential, may not be preserved if the order of the quantifiers is reversed. Thus this is an invalid inference: $(x)(\exists y)Fxy$, therefore $(\exists y)(x)Fxy$. To take Geach's example: From 'Every boy loves some girl' we do not validly deduce 'There is some girl whom every boy loves'. At the start of the *Nicomachean Ethics* is Aristotle arguing that since every act aims at some good, there must be some one and the same good at which every act aims? If he is arguing thus, then he is committing the quantifier shift fallacy expounded by Geach on the basis of the epochal logical discoveries of Frege, and previously expounded with great subtlety by John Mair and others who contributed to the late flowering of terminist logic in the early decades of the sixteenth century. And it has to be noted that just as the distinction is made in modern logic by the order of the quantifiers, so in late scholastic logic the distinction is made by the order of the terms, in particular, the quantifiers, in the proposition.

In short what happened under the onslaught of the humanists was that invaluable discoveries made by the medieval logicians disappeared into oblivion. To be sure, Fregean logic has provided us with a vastly more powerful weapon of logical analysis than anything available in the earlier period, but it is plain that medieval logicians were, in many cases, struggling with the same problems as their modern counterparts and were trying to reach solutions within the limits set by the language in which they wrote. Their Latin was of course far from Ciceronian, but it had special merit in so far as it enabled them to say unambiguously the kinds of things that they thought it was important to be able to say, and that we can now say by courtesy of Frege.

No doubt the Renaissance humanist logicians were well pleased with themselves. They were in the business of

¹² P. T. Geach, *Logic Matters*, 1. See also his *Reference and Generality*, 134-5.

rhetoric, the art of persuasion by speech, and on that basis they could congratulate themselves on achieving a fair measure of success. For they had succeeded in persuading the multitude of the merits of their concept of logic as a branch of the art of rhetoric. In that sense, their work constitutes an illustration of the unity of theory and practice. But we should not lose sight of the less happy part of the picture. I could have attended here to other areas of medieval logic where valuable advances were made which disappeared from view and which are now back on the agenda of philosophers and logicians. I hope to have shown that many things in medieval logic are of great value. The best that the medieval logicians had to offer should never have been allowed to slide into oblivion, and our acknowledgement of this fact is an act of justice long overdue.

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