Aristotle on the Good: A Formal Sketch

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ARISTOTLE ON THE GOOD :
A FORMAL SKETCH

This paper attempts a simple formal treatment of Aristotle's discussion
of the good in *Nicomachean Ethics* I 1-7 (1094 a 1 - 1098 a 20). Its aim is
to distinguish some of the leading concepts used by Aristotle, and to examine
some of the logical relations between them ; with the particular purpose of
establishing what premisses, granted the formal apparatus, are sufficient or
necessary for some of the main conclusions supposedly established by
Aristotle in this passage.

We shall use first-order predicate calculus with identity and with the
modal operator " N " = " it is necessary that ". Variables " x ", " y ", etc.,
will range over the arts, enquiries, actions, etc. (1094 a 1 seq) which can in
the broadest sense be pursued, and which Aristotle regards as the subject
matter of his enquiry into the good. We shall use one primitive predicate
letter " P ", which will however have two rôles, as standing for a one-place
and for a two-place predicate. " Px " is to be interpreted " x is pursued " ;
" Pxy " is to be interpreted " x is pursued for the sake of y ". In these
interpretations, the terms " pursued " and " pursued for the sake of " are
intended to cover a number of expressions used by Aristotle, but which,
at least in the discussion under consideration, he seems to use interchange-
ably, such as " aims at ", " is desired for the sake of ", etc. Some further
remarks about the interpretation of " P " will follow the next paragraph.

1094 a 1 seq gives us, a little tentatively, a statement which is pre-
supposed throughout the discussion : that everything that is pursued is
pursued for the sake of something ; we may trivially add that everything
pursued for the sake of something is pursued :

$\quad$ (1) (x) [Px $\longleftrightarrow$ (∃y) Pxy].

We may add further that if x is pursued for the sake of y, y is itself pursued :

$\quad$ (2) (x) (y) [Pxy $\rightarrow$ Py].

The discussion of the architectonic relations in cc. 1, 2 init. make it clear that the relation " Pxy " must be *transitive* and, for different values of x and y, *asymmetrical* :

$\quad$ (3) (x) (y) (z) [Pxy & Pyz $\rightarrow$ Pxz].

$\quad$ (4) (x) (y) [Pxy & x $\neq$ y $\rightarrow$ $\sim$ Pyx].

The restriction of asymmetry to non-identical values of x and y in (4) is necessary because, as will be seen, " Pxy " is not irreflexive.

$\quad$ (3) and (4) clearly raise certain questions about the interpretation of " P ". If, as the previous sketch of an interpretation might well suggest, it is a sufficient condition of the truth of " Pa " that a is at some time pursued by somebody, (3) and (4) will almost certainly under interpretation come out false, since it is clearly possible that Pab by somebody at some time, and Pbc by somebody else at some time, without Pac by anyone at any time. Again, it seems that it might well be the case that Pab by someone at some time, and Pba by someone else at any time, or by the same person at a different time. These difficulties, however, lie not so much in the formalization of Aristotle's discussion by means of the predicate " P ", as in Aristotle's discussion itself ; exactly these questions are bound to arise in any consideration of his very general method of argument. The most effective ways to avoid these difficulties are perhaps : (1) to restrict interpretation of the variables to actions *as characterized by* certain preferred descriptions—for instance by selecting characterizations of a and b in " Pab " such that it will be obvious how someone can pursue a for the sake of b, which will in general rule out " Pba " for the same characterizations of a and b ; (2) to take as interpretation the dispositional pursuit-structure of one fully rational and consistent agent with a settled pattern of desires. ' Fully rational ' here must include an evaluational element, which is certainly inherent in Aristotle's treatment ; his discussion, for instance, of things which can be pursued for their own sake is confined to things which a sane and rational man would pursue for their own sake. (This accords with his remarks on the requirements for intelligent study of his theory, 1095 b 3 seq.) It will be safe to say that Aristotle supposes all his propositions to hold for the pursuits of a thoroughly consistent *phronimos* ; some for wider classes of agents ; some, being, or supposedly being, logical truths (such as (1) and (2)), for any conceivable agent. The same must hold for the interpretation of " P ".

$\quad$ Granted that something is pursued, (1) and (2) can generate an infinite series of P-statements :

$\quad$ Pa ;

$\quad$ Pab, for some b (by (1)) ;

$\quad$ Pb $\qquad$ (by (2)) ;

$\quad$ Pbc, for some c (by (1)) ; etc.

1094 a 20 (whatever else it states) states that such a series cannot be continued indefinitely, for the good reason that the notion of ' pursuit for the sake of ' would then lack content (' our desire would be empty and vain '). Nor can we stop its infinite prolongation by making it circular, i.e. by introducing some earlier term into the second argument place of some later statement. For suppose such a series

  (i) Pab
  (ii) Pbc . . .
  (n) Pna.

Repeated applications of (3) will give from (ii) . . . (n) : Pba ; but this, by (4), is inconsistent with (i). Hence we may infer, as Aristotle infers,

  (5) $(\exists x)$ Px $\to$ $(\exists y)$ Pyy.

From this, of course, we cannot infer that there is only *one* x such that Pxx, i.e. :

  *(6) $(\exists x)$ Px $\to$ $(\exists y)$ [Pyy & $\sim$ $(\exists z)$ (z $\neq$ y & Pzz)] ;

nor can we infer

  *(7) $(\exists x)$ Px $\to$ $(\exists y)$ [(z) (Pz $\to$ Pzy)] ;

for there is nothing in what has gone before to show that we may not have independent chains Pab Pbc . . . Pkk, Pmn Pno . . . Puu, with no common members.

It may be tempting to think that the two statements *(6) and *(7) are equivalent. This is not so, and since *(7), but not *(6), will be of interest in the later argument, it is worthwhile to distinguish them. The following argument will illustrate the issue :

  Suppose (i) Paa.
  Suppose further that for some d
          (ii) (x) [Px $\to$ Pxd]
  Then   (iii) Pa   (i, by 1)
          (iv) Pad   (ii, iii)

Thus, it may be thought, from (i) and (iv)

          (v) a = d.

But since the argument applies equally well to any a such that Paa, we shall have derived *(6) from *(7).

The foregoing argument, however, is invalid, since it requires an extra premiss, that if something is pursued for its own sake, it cannot be pursued for the sake of something else as well :

  *(8) (x) [Pxx $\to$ $\sim$ $(\exists y)$ (x $\neq$ y & Pxy)]

*(8) is rejected by Aristotle, 1097 a 34. If *(8) were accepted, *(6) and *(7) would indeed be equivalent, since (*a*) the foregoing argument, with the extra premiss, would be valid, so that *(6) could be derived from *(7) ; (*b*) *(7) can be derived from *(6) even without the extra premiss, as follows :

  (i) Suppose *(6), and $(\exists x)$ Px ;
then   (ii) there is just one thing, say d, such that Pdd ;
but   (iii) everything that is pursued is pursued either for its own sake, or

for the sake of something pursued for its own sake (by the
argument of (5)) ;

(iv) in either case, it is pursued for the sake of d (by (ii)) ;

so   (v) there is something for which everything is pursued.

Thus *(6) implies *(7).

Even the addition of *(8), however, would not make either *(6) or *(7)
derivable from any of (1) - (5).

The notion contained in *(7), viz. that of something for the sake of which
everything is pursued, is important for Aristotle : it is his notion of " the
good, or the supreme good " (1094 a 22). We may therefore define :

(9) SGx $\longleftrightarrow$ (y) [Py $\rightarrow$ Pyx] Def.,

and the consequent of *(7) may be written

(10) ($\exists$x) SGx.

Aristotle is not yet in a position to prove (10). It might be suggested that
1094 a 1 seq. shows Aristotle to have derived (10) invalidly from (1) ; but
this is an implausible, as well as uncharitable, interpretation of his intention,
since he clearly regards (1) as certain and presupposes it throughout, but
(10) is later introduced as uncertain (1097 a 22 seq.). Nor, I think, does the
difficult passage 1094 a 18 seq. (the first sentence of c. 2) *have* to be taken
as an attempt to infer (10) from (5) and the considerations that lead up to
it. The passage runs :

> If, then, there is some end of the things we do, which we desire
> for its own sake (everything else being desired for the sake of this),
> and if we do not choose everything for the sake of something else
> (for at that rate the process would go on to infinity, so that our
> desire would be empty and vain), clearly this must be the good and
> the chief good. [tr. Ross]

Here we have two antecedents for the same consequent, each containing
a parenthesis. Let the antecedents with their parentheses be signified by
' A(a) ', and ' B(b) '. Clearly (b) is offered as a *reason* for B, and (a) as a
specification of the ' end ' mentioned in A. If now B(b) were offered as a
reason for A(a), we should have the invalid inference referred to. However,
it is possible to take B, not as a reason for A(a), but as a consequence of it,
brought in by Aristotle as suggested by the larger hypothesis A(a), and
suggesting in its turn a reason for itself (b) ; in this case the invalid inference
will not be committed. The passage is in any case confusedly expressed,
and it is perhaps impossible to say exactly what it means. If so, the former
interpretation is certainly not obligatory. Even it it is accepted, however,
it is clear that Aristotle lays no great weight on the invalid argument.

We shall return to (10). Meanwhile, it is to be noted that Aristotle
believes that there are at least four things pursued for their own sake
(pleasure, honour, reason, virtue 1097 b 2, cf. 1096 b 17) :

(11) ($\exists$x) ($\exists$y) ($\exists$z) ($\exists$w) [x $\neq$ y $\neq$ z $\neq$ w & Pxx & Pyy & Pzz & Pww]

Of any such thing, it is evident that there does not have to be anything else
for the sake of which it is pursued, although (*(8) being rejected) there

could be. Of any such thing, Aristotle says that it is *final* (*teleion*) :

(12) $TLx \longleftrightarrow Px \& \sim N (\exists y) [y \neq x \& Pxy]$ Def.

1097 a 34 seq. In the same passage, Aristotle goes on to introduce the notions, at first sight surprising, of " more final " (*teleioteron*) and " most final " (*teleiotaton*). He gives in fact two applications of the former, the first to things not pursued for their own sake, the second to things that are pursued for their own sake ; of these the former seems of little use, and involves the awkward consequence that something can be more final without being final. Omitting this application, his accounts of the two notions may be represented, respectively :

(13) $TLRxy \longleftrightarrow TLx \& TLy \& \sim N \sim (Pyx) \& N \sim (Pxy)$ Def. ;

(14) $TLTx \longleftrightarrow TLx \& (y) [(TLy \& y \neq x) \rightarrow TLRxy]$ Def.

We may infer

(15) $(x) [TLTx \rightarrow (y) \{y \neq x \rightarrow N \sim (Pxy)\}]$

For   (i) $(x) [TLTx \rightarrow (TLx \& (y)\{TLy \& y \neq x \rightarrow \sim N \sim (Pyx) \& N \sim$

$(Pxy)\})]$ (by Defs.)

so   (ii) $(x) [TLTx \rightarrow (y)\{TLy \& y \neq x \rightarrow N \sim (Pxy)\}]$ (by dropping

consequents)

but the restriction to y's that are TL in the consequent of (ii) can be dropped, the consequent holding for non-TL y's as well : for, for any non-TL y such that Pxy, there would have to be (by the argument of (5) and by (12)), some TL z such that Pyz ; whence by (3), there would be a TL z such that Pxz, contrary to (ii).

We may further infer

(16) $(x) [TLTx \rightarrow (y) (TLTy \longleftrightarrow y = x)]$

i.e. that there is at most one TLT thing. For

suppose (i) $TLTa \& TLTb \& a \neq b$;

then   (ii) $TLRab \& TLRba$, (by (14)) ;

so   (iii) $\sim N \sim (Pba) \& N \sim (Pab) \& \sim N \sim (Pab) \& N \sim (Pba)$,

(by (13)),

which is absurd.

We cannot infer, however, that there is at least one TLT thing :

(17) $(\exists x) TLTx$.

(It is worth noting, in passing, that we could *not* derive this by adopting the premiss, nowhere offered by Aristotle,

*(18) $(x) (y) [TLx \& TLy \& x \neq y \rightarrow TLRxy \vee TLRyx]$

Since the TLR-relation is asymmetrical, as seen in the proof of (16), *(18) would allow us to derive (17) if the TLR-relation were also transitive, but it is not.)

More importantly for Aristotle's argument, we could not derive (17) from (10), if we independently accepted that. From (9) and (10) we should have

(i) $(\exists x) (y) [Py \rightarrow Pyx]$; from this *a fortiori*

(ii) $(\exists x) (y) [TLy \& y \neq x \rightarrow Pyx]$.

From (ii), by the acceptable principle *ab esse ad posse valet consequentia*, we could infer

(iii) $(\exists x)\ (y)\ [\text{TLy} \ \&\ y \neq x \rightarrow\ \sim \text{N} \sim (\text{Pyx})\ ]$.

(17), however, by (13) and (14), requires

(iv) $(\exists x)\ (y)\ [\text{TLy}\ \&\ y \neq x \rightarrow\ \sim \text{N} \sim (\text{Pyx})\ \&\ \text{N} \sim (\text{Pxy})\ ]$,

and it is clear that the second conjunct of the consequent of (iv) cannot be provided from (i) - (iii). Moreover, (iv), and hence (17), does not imply (10), since *a posse ad esse non valet consequentia*. Thus we have the interesting negative result that the existence of an SG thing and of a TLT thing do not imply one another. Nor is this surprising, if one considers the interpretation of " N " in (12) - (14). It is clear that Aristotle regards the necessity and possibility involved in the notions there defined as intensionally connected with the specifications of the things that are final, most final, etc. Thus happiness (*eudaimonia*), which is said in fact to be most final, is so because it makes no sense, according to Aristotle, to ask of happiness, so specified, for what it is being pursued. This being so, we can see that merely from the fact that a certain thing was that for which everything was done, it would not follow that it had the further intensional property of being TLT ; or conversely.

What we can show, however, is that if there is a SG thing, and there is a TLT thing, they must be one and the same :

(19) $(x)\ (y)\ [\text{SGx}\ \&\ \text{TLTy} \rightarrow x = y]$.

For suppose   (i) SGa

             (ii) TLTb

             (iii) $a \neq b$

Then, from (i), (ii) (which implies Pb), and (10),

             (iv) Pba ;

but from (ii), (iii), and (15),

              (v) $\text{N} \sim (\text{Pba})$.

Aristotle does in fact assert both (10) and (17). What are his reasons ? For the latter, two reasons can perhaps be found. The first consists in the mere observation that happiness does possess the intensional characteristic in question, and is in fact pursued. The second is a more *a priori* reason. It seems to be Aristotle's belief that a reason or justification can be given or found for a given pursuit only in terms of some end served by the pursuit.[1] This end may, of course, lie in the pursuit itself, as has been seen (5). This can be the case with any TL thing. But Aristotle remarks that there is more than one TL thing (11). If we were merely left with the set of TL things as each providing reasons, there could be ultimately reasonless choices *between* pursuits, since no reason could be given for choosing what conduced to one TL thing rather than what conduced to another. If this is to be avoided, there must be some TL thing that stands in a special relation to

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[1]Cf. e.g. the account of deliberation, 1112b12 seq. and its rôle in *phronesis* 1140a24 seq.

the others, as something for which they can be pursued, but not it for them, and this will be the TLT thing.

From neither of these reasons does (10) follow, because it does not follow from the fact that there is one special TL thing for which the others *can* be pursued that they always are or should be pursued for it. It may be that Aristotle overlooked the lack of implication between (17) and (10), and falsely supposed that an argument for the first was an argument for the second. There is, however, no necessity to think this. For Aristotle gives at least one quite independent argument for (10), viz. the argument from function, 1097 b 25 seq. Moreover, even omitting this as coming rather late in the discussion to be more than a confirming argument, (10 might be supported merely *a posteriori* (thus 1094 a 1 seq., granted it is not an invalid derivation of (10) from (1), may be the tentative acceptance of an *endoxon* or received opinion in favour of (10)). If this is so, we may suppose Aristotle to have had independent reasons for believing in an SG thing, and in a TLT thing, and then (validly) have inferred their identity.

We may end by considering one further characterisation that Aristotle applies to the good, viz. that it should be *autarkes*, or self-sufficient. There are two senses of this term, which, though connected, are distinct, and are sufficiently distinguished by Aristotle at 1097 b 14. In the first sense, the good is *autarkes* if its possession is to the greatest extent secure, and not subject to the whims of others and similar contingencies not in the possessor's power. This is the sense that Aristotle also, and better, expresses by saying that the good should be " proper to a man and not easily taken from him " (*oikeion ti kai dusaphaireton*, 1095 b 25). In the second sense, the *autarkes* is " that which when isolated makes life desirable and lacking in nothing . . . the most desirable, not counted as one good thing among others " (1097 b 14 seq.). This account at least involves the thought that what is *autarkes* is a TL thing, and that no other TL thing can be pursued independently of it ; for TL things are things pursued for their own sake by reasonable men, i.e. are worth pursuing, and besides the *autarkes* there is nothing worth pursuing. Thus we can say, as a possible definition :

(20) $AUTx \longleftrightarrow TLx \ \& \ (y)\big\{(y \neq x \ \& \ Pyy) \rightarrow Pyx\big\}$ Def. 1.

But by transitivity (3) we can conclude that anything pursued for the sake of any of the things pursued for their own sake is also pursued for the sake of the AUT. Moreover, the AUT is pursued for its own sake ; since nothing else is left, we have

(21) $(x) \ [AUTx \rightarrow (y) \ (Py \rightarrow Pyx) \ ]$.

The implication, moreover, obviously holds in the opposite direction *a fortiori* ; so, by (9)

(22) $(x) \ [AUTx \longleftrightarrow SGx]$.

In this case, by previous arguments, being AUT will be neither a necessary nor a sufficient condition of being TLT.

However, an alternative definition might be suggested in which " AUT " was expressed in terms of necessity. Thus it might be suggested that any-

thing that satisfied Aristotle's description of AUT would be something other than which there could not *conceivably* be anything higher, i.e.

*(23) $AUTx \longleftrightarrow TLx \,\&\, N \sim (\exists y)\{y \neq x \,\&\, Pxy\}$ Def. 2.

This, however, would not by itself be adequate, since it leaves open the possibility, excluded by the first definition and by Aristotle, that there should be some TL thing independent of, though not superior to, the AUT. Thus what is needed is rather a combination of (20) and *(23). If we allow the principle of inference

(n) $Np \rightarrow p$,

this can be economically effected by merely adding " N " to (20) :

(24) $AUTx \longleftrightarrow TLx \,\&\, N \,(y)\{(y \neq x \,\&\, Pyy) \rightarrow Pyx\}$ Def. 3.

From this, using (n), we can infer as before (21) ; but we cannot derive the converse implication to give (22). We can infer the universal closure of *(23) by a simple argument :

Suppose that AUTa and that, contrary to *(23), for some x, say b, such that $b \neq a$, Pab. Then, by the argument of (5), there is some y, say c, such that Pcc, and Pac, either directly if $c = b$, or by transitivity. Then by (24), since Pcc, Pca. But this, by (4), is impossible, since we have Pac.

Moreover, we can clearly derive

(25) $(x) [AUTx \rightarrow (y)\{Py \rightarrow \sim N \sim (Pyx)\}]$

from (21), (which we have seen to be derivable from (24)), by the principle *ab esse ad posse*. Putting together (25) and the universal closure of *(23), obtained by the last result, we can reach

(26) $(x) [AUTx \rightarrow TLTx]$,

though the converse implication will not of course hold. Thus if the attractive definition (24) is accepted, we shall have the result that something's being AUT will imply both its being SG and its being TLT ; while, as we have seen before, its being SG neither implies nor is implied by its being TLT ; the most that can be established in respect of the two latter concepts alone being that if there is an SG thing and there is a TLT thing, they are one and the same.

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