THE PLACE OF GEOMETRY: HEIDEGGER'S MATHEMATICAL EXCURSUS ON ARISTOTLE

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Where is geometry? This is the key question this article seeks to address. It is part of a wider enquiry which would look at the *place* of geometry in a number of texts, dating from antiquity through to the modern age. I am intending to raise the question of when the notion of *space – spatium, Raum, espace –* first arose. There are a number of questions related to this. Did the Greeks have a word for 'space'? Where did they see geometric shapes existing? Is Plato's *Timaeus* predicated upon an understanding of 'space'? How did Descartes's work differ from that of, say, Euclid? What role does Galileo play? How does this relate to wider metaphysical issues, such as Kant's transcendental aesthetic, or to modern understandings of science? What political implications follow from these questions?

In order to impose some kind of order on these wide-ranging – and no doubt, overly ambitious questions – I intend to use the work of Heidegger as a kind of guiding clue. Heidegger makes a number of references to issues pertaining to these questions in his works, some of which have only recently been published. In part here, but more thoroughly in the wider project, I read Heidegger in relation to the work of Edmund Husserl, for whom geometry was a central concern, and to two other thinkers – Jacob Klein and David Lachterman – whose work can be seen to be continuing within a Heideggerian milieu. This enquiry therefore seeks to address these questions, to open up new avenues of understanding the philosophy of mathematics within the Continental tradition, and to shed some light on the development of Heidegger's own thought.

I. GEOMETRY AND SPACE

Geometry occupies a peculiar place in Heidegger's *Being and Time*: peculiar as both strange and special. Though it is mentioned explicitly only twice (GA2, 68; 112), it is behind the critique of Descartes's view

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of the world and space in §§18–24, indeed throughout the analytic of *Dasein*, and would have been explored in more detail in the promised but never published division (the second of Part Two) that was to treat Descartes explicitly (see GA2, 40). As Heidegger notes, his preliminary remarks in the first division 'will not have been grounded in full detail until the phenomenological de-struction of the "*cogito sum*" ' (GA2, 89).[1]

In brief, Heidegger believes that we encounter space in a way that is very different from Cartesian notions. Descartes sees space in terms of extension, measurable by co-ordinates and mathematical. For Heidegger, rather than encountering a room in a geometrical spatial sense, we react to it as *Wohnzeug*, equipment for dwelling (GA2, 68; see GA24, 414). Our mode of reaction to space is much closer to notions of near/far or close/distant, not *primarily* determined by geometry and measurable distance, but by the more prosaic notions of closeness or nearness (*Nähe*), de-distancing (*Ent-fernung*) and directionality (*Ausrichtung*).[2] Space is encountered in everyday life, and lived in, not encountered in geometrically measurable forms and shapes. It is part of the structure of our being-in-the-world. And the critique of Descartes, which Rockmore calls a 'violent attack',[3] is pursued in the treatment of Kant made at around the same time. As Heidegger notes, in the *Groundwork for the Metaphysic of Morals* Kant suggests 'the idea of a twofold metaphysics: a metaphysics of nature and a metaphysics of morals'.[4] Heidegger interprets these as an ontology of *res extensa* and an ontology of '*res cogitans*' (GA24, 197–8) – precisely the Cartesian formulation he is trying to escape, through a rethinking of space without a basis in extension, and through a rethinking of *Dasein* that is not reducible to a subject (see GA20, 237ff., 322).

Now a number of points can be made here as summary. Heidegger wants to free 'space' (*Raum*) from the Cartesian understanding, where it is based on the notion of extension. But he still retains a notion of 'space'. Equally, geometry can be used to make some sense of 'space', but our lived experience of 'space' is more fundamental – the notions of nearness, directionality and de-distancing. Geometry is an abstraction from the world, but the results of this abstraction are taken by Descartes and Kant as fundamentals of our way of being.[5] Elsewhere in Heidegger's corpus these themes return again and again.

One of the most detailed critiques of Descartes is found in the Nietzsche lectures. Here, in what is almost certainly a glimpse of what would have gone on in *Being and Time*, Part Two, Heidegger makes the link between the Cartesian view of space as extension and the wider Cartesian method.

> The certitude of the principle *cogito sum* (*ego ens cogitans*) determines the essence of all knowledge and everything knowable; that is, of *mathesis*; hence, of the mathematical … The mathematically accessible, what can be securely reckoned in a being that humans themselves are not, in lifeless nature, is *extension* (*Ausdehnung*)

(the spatial), *extensio*, which includes both space *and* time. Descartes, however, equates *extensio* and *spatium*. In that way, the nonhuman realm of finite beings, 'nature', is conceived as *res extensa*. Behind this characterisation of the objectivity of the nature stands the principle expressed in the *cogito sum*: Being is represent-edness (*Vorgestelltheit*) (GA6.2, 145–6; N IV, 116).

Heidegger has spent a number of pages within the text criticizing various aspects of the *cogito*, as did Nietzsche before him, and it appears, given the problems he has identified with this thought, that he wishes to discard it and its results. Undermining the *cogito ergo sum* and with it the opposition between *res cogitans* and *res extensa* clearly furthers the purpose of thinking space experientially rather than geometrically. As Heidegger notes, the view of space as uniform extension, determined by Galileo and Newton, is the correlate of modern European subjectivity (GA13, 205). Moreover, Heidegger continues to suggest that though interpreting nature as *res extensa* is one-sided and unsatisfactory, when thought and measured metaphysically 'it is the first resolute step through which modern machine technology, and along with it the modern world and modern mankind, become metaphysically possible for the first time' (GA6.2, 146; N IV, 116). In Heidegger's terms modern technology is the inevitable result of the world made picture, the Cartesian objectification of the world. In the *Beiträge*, he suggests that calculation (*die Berechnung*) is one of the three concealments (*Verhüllungen*) of the forgetting of being. Calculation is set into power by the machination of technology and is grounded by the science or knowledge of the mathematical (GA65, 120).

However in Heidegger's work there is the potential not simply to see that a particular conception of space was held in the seventeenth century, but that something far more fundamental is at stake. In *An Introduction to Metaphysics* Heidegger makes the following somewhat inchoate remark:

> The Greeks had no word for 'space'. This is no accident; for they experienced the spatial on the basis not of extension (*extenso*) but of place (*Ort*) (*topos*), as *khora*, which signifies neither place nor space but that which is occupied by what stands there … the transformation of the barely apprehended essence of place (*topos*) and of *khora* into a 'space' (*Raum*) defined by extension (*Ausdehnung*) was initiated (*vorbereitet*) by the Platonic philosophy, i.e. in the interpretation of being as *idea* (GA40, 71; IM 66; see GA55, 335–6).[6]

This raises a number of important questions. *Where* then is Greek geo-metry? How then did their geometry work with no conception of space? When did the term space arise? What implications does this have?

Geometry is explicitly discussed in Heidegger's 'Building Dwelling Thinking' and 'Art and Space', as well as other late essays. Several other places – notably the lectures on Hölderlin – are central to understanding what Heidegger is doing.[7] But the most detailed treatment of these issues, to my knowledge, is found in an excursus in Heidegger's revered course

on Platonic dialogues in 1924–25, published as Volume 19 of the *Gesamtausgabe* as *Plato's 'Sophist'*.[8] Delivered during the time *Being and Time* was being composed, this is an important text for a number of reasons. Despite its title, the course actually spends a great deal of time on Aristotle, particularly Book VI of the *Nicomachean Ethics*. The discussions of *phronesis*, the relation of being to non-being, and the question of *aletheia* are interesting and illuminating, as is the engagement in detail with the motto from *The Sophist* that served as an epigraph for *Being and Time*. A few weeks into the course however, Heidegger turns aside from his main aim at that point, which is the meaning of *sophia*, and discusses mathematics in some detail. It is this seemingly tangential discussion or excursus (*Exkurs*) that will be the central focus of this article.

II. THE PLACE OF GEOMETRY IN §15 OF *PLATO'S 'SOPHIST'*

The purpose of this excursus is first to examine *mathematike* in general, and second *arithmetike* and *geometria*.[9] The mathematical sciences have as their theme *ta eks aphaireseos*, that which shows itself as withdrawn from something, specifically from what is immediately given – *physika onta*. In other words, mathematics is an abstraction from being. This is generally accredited to Thales, who took the Egyptian geometry of empirical measurement, and turned it into an abstract and deductive process.[10] This abstraction is recognized by Aristotle when he speaks of *khorizein*, a separating, which links to the important word *khora*, which Heidegger here translates as 'place' (*Platz*). For Heidegger therefore, mathematics takes something away from its own place. But mathematics itself does not have a *topos*. This might have the ring of a paradox, as *topos* is often translated as 'space' (*Raum*): Heidegger prefers 'place' (*Platz*). (We should note here that Heidegger therefore sees both *khora* and *topos* as *Platz*, though he clarifies the latter with the additional word *Ort* which is usually translated as 'locale' or 'place'.) Heidegger suggests that the *khorizein*, the separating, is for Aristotle the way in which the mathematical becomes objective. This is clearly linked to the *khorismos* of Plato's ideas, where the ideas have their *topos* in the *ouranos*, the heavens (GA19, 100–1).

In the *Physics* (II, 2), discussing the scope of natural science, Aristotle examines the mathematical objects of *stereon* and *gramme* – solids and lines. Whilst these can be considered as *physika*, with a surface as *peras*, the limit of a body (*als Grenze eines Körpers*), the mathematician considers them purely in themselves.[11] Heidegger suggests that this negative description of the mathematical in Aristotle – 'that it is *not* the *peras* of a *physikon soma*' – means that 'the mathematical is not being considered as a 'place' (*Ort*). Therefore this abstracting, this extraction (*Heraussehen*) of the essence of the mathematical from the realm of *physikon soma*, is

essential, but *oyden diaphora*, it makes no difference (*macht das keinen Unterschied*). By this, Aristotle means that the abstracting does not turn them into something else, but the 'what' of the *peras* is simply taken for itself. The *khorizein* therefore, this extracting, does not distort. Such an extracting is at play in the ideas generally. Now *khorismos* has a justifiable sense in mathematics, but not where *beings* are concerned. For the *physika onta* are *kinoumena*, related to motion, and hence cannot be removed from their *khora*, their place (*Platz*). Heidegger therefore stresses the *topos* where being and presence (*das Sein und die Anwesenheit*) are determined (GA19, 101–2).

Heidegger then moves on to distinguishing between arithmetic and geometry – the former is concerned with *monas*, the unit; the latter with *stigmé*, the point. *Monas* is related to *monon*, the unique or the sole, and is indivisible according to quantity. *Stigmé* is, like *monas*, indivisible, but unlike *monas* it has the addition of a *thesis* – a position, an orientation, an order or arrangement. *Monas* is *athetos*, unpositioned; *stigmé* is *thetos*, positioned.[12] This addition – this *prosthesis* – is crucial in understanding the distinction between arithmetic and geometry. Tantalizingly, Heidegger asks 'what is the meaning of this *thesis* which characterizes the point in opposition to the *monas*?' He recognizes that a 'thorough elucidation of this nexus would have to take up the question of place and space', but at this point can only look at what is necessary to describe mathematics (GA19, 102–4).

In doing so, Heidegger clarifies the distinction (*Unterschied*) between *thesis* and *topos*, position (*Lage*) and place. Mathematical objects are for Aristotle, Heidegger says, '*ouk en topo*',[13] 'not any place (*nicht an einem Platz sind*)'. Heidegger warns us that 'the modern concept of space (*Begriff des Raumes*) must not be at all allowed to intrude here', so he turns to *Physics* IV, which provides an outline of the concept of *topos*. He suggests that these 'assertions of Aristotle's are self-evident, and we may not permit mathematical-physical determinations to intrude'. And then, in one of the most striking passages in this course, he discusses Aristotle's suggestion that place has a *dynamis*. Rather than opting for the standard translation of this as force or power (*Kraft*), Heidegger argues it must be understood in an ontological sense: it

> implies that the place (*Platz*) pertains to the being itself, the place constitutes precisely the possibility of the proper presence (*eigentlichen Anwesendseins*) of the being in question … every being has *its* place (*Jedes Seiende hat seinen Ort*) … Each being possesses in its being a prescription toward a determinate location or place. *The place is constitutive of the presence of the being* (*Jedes Seiende hat in seinem Sein die Vorzeichnung auf einen bestimmten Platz, Ort. Der Ort ist konstitutiv für die Anwesenheit des Seienden*) (GA19, 105–6).[14]

According to Aristotle, above–below, front–back and right–left are crucial to determining a place. But these determinations are not always

the same, i.e., though they are *absolute* within the world, they can also change in relation to people. This change is one of *thesis*, orientation, and therefore *topos* is not the same as *thesis*. Geometrical figures have *thesis*, they can have a right or a left for us, but they do not occupy a place.[15] Now if geometry does not have a place, what indeed is place? It is only because we perceive motion that we think of place, therefore only what is moveable (*kineton*) is in a place. Glossing two lines of the *Physics*, Heidegger contends that 'place is the *limit* (Grenze) *of the periekon*, that which delimits (*umgrenzt*) a body, not the limit of the body itself, but that which the limit of the body comes up against, in such a way, specifically, that there is between these two limits no interspace, no *diastema*'. Heidegger admits the difficulty of this determination, saying it requires an absolute orientation of the world. He then quotes lines he will use over forty years later (with a slightly different translation) to preface his essay 'Art and Space': 'It appears that to grasp place in itself is something great and very difficult.'[16] Heidegger admits a temptation: to take the extension of the material (*die Ausdehnung des Stoffes*) or the limit of the form as the place. And equally it is difficult to see place as such, because place does not move, and what is in motion has a privilege in perception (GA19, 106–8).

The first understanding of the concept of place is therefore summarized by remembering that place has a *dynamis*. *Dynamis* is a basic ontological category. Place is something belonging to beings as such, it is their capacity to be present, it is constitutive of their being. 'The place is the ability a being has to be there (*Dortseinkönnen*), in such a way that, in being there, it is properly present (*dortseiend, eigentlich da ist*)' (GA19, 109). In respect of this, we should note two things. 'Being there' (*dortsein*) in this phrase *can* be understood in the concrete sense of being in place, being somewhere. It is *not* the same as *Dasein*, which has been translated as 'being there'. *Dasein* more properly understood is 'being-the-there', the clearing.[17] Second, and again a point of translation, *eigentlich* is not here seen as 'authentic'. Such a reading, like 'being there' for *Dasein*, is heavily influenced by Sartre. Rather, following the rethinking of this sense of this key notion, we need to bear in mind its links to *Ereignis*, the propriating event.

When geometry intervenes, what it extracts from the *aistheta* in order for it to become the *theton* is precisely the moment of place (*Ortsmomente*). These moments of place are the *perata* of a physical body, and in their geometrical representation acquire an autonomy over and against the physical body. So geometrical objects are not in a place, but have directions – above/below, right/left, etc. We can use this to give us insight into the positions as such, an *analysis situs*, even though geometry does not possess the same determinations. Every geometrical point, line, surface is fixed through a *thesis*, they are therefore *ousia thetos*. The *monas* does not bear an orientation, therefore they are *ousia*

athetos.[18] Geometry therefore has a greater proximity to the *aistheta* than does arithmetic. The basic elements of geometry – point, line, surface – are the *perata* for the higher geometrical figures. But for Aristotle, in opposition to Plato, such higher geometrical figures are not *put together* out of such limits.[19] A line will never arise out of points, nor a surface from a line, nor a body from a surface, for between any two points there is again and again a *grammé*. Heidegger takes this forward by discussing the unity that must exist in order for lines to be made of points, surfaces from lines, etc. He relates these questions to arithmetic too, asking what is the mode of manifoldness of number? (GA19, 110–12).

In investigating this manifold [*Mannifgaltigen*], Heidegger reminds us of the link between geometry and the *aistheta*. 'Everything in *aisthanesthai* possesses *megethos*, everything perceivable has stretch (*Erstreckung*). Stretch, as understood here, will come to be known as continuity.' Aristotle derives this notion of continuity (*synekhes*) not from his work on geometry, but on physics. This occurs in *Physics*, Book V in the discussion of co-being, being-together (*Miteinanderseins*), the *physei onta*. There are seven forms of co-being:

1. the *hama*, the concurrent – understood as something concerning place, not temporality. The *hama* is that which is in one place.
2. *khoris*, the separate – that which is in another place.
3. *haptesthai*, the touching – that whose ends are in one place (*hama*).
4. *metaksu*, the intermediate – that which something, in changing, passes through. Such as a boat moving in a stream, the stream is the *metaksu*, the medium.
5. *ephekses*, the successive – where something is connected to something else, and between them there is nothing 'of the same lineage of being'. There might be something else, but not another of the same.
6. *ekhomenon*, the self-possessed – a *ephekses* determined by *haptesthai*. In other words a succession where the ends meet in one place, the *hama*.
7. *synekhes, continuum* – a complicated form since it presupposes the other determinations. It is a *ekhomenon*, but more, a *hoper ekhomenon* – more originary, not only do the ends of the elements of a succession meet in the same place, but the ends of one are identical with the other.

'These are the determinations of co-being. The *synekhes* is the structure that makes up the principle of *megethos*, a structure which characterizes every stretch.' *Monas* and *stigmé* cannot be the same, Aristotle shows, for the mode of their connection is different. For points are characterized by *haptesthai*, by touching, indeed they are *ekhomenon* – a *ephekses*

determined by *haptesthai*. But the units (of arithmetic) have only the *ephekses*. The mode of connection of the geometrical, of points, is characterized by the *synekhes*; the series of numbers – where no touching is necessary – by the *ephekses*. To consider geometrical figures therefore, we must add something over and above the *ephekses*. These additions – *megethos, pros ti, thesis, topos, hama, hupomenon* – ensure that the geometrical is not as original as the arithmetical (GA19, 113–16).

Heidegger is now steering his excursus back to the issue of *sophia*. But before he moves to its fourth and last moment, he briefly discusses his contemporary Hermann Weyl's *Raum–Zeit–Materie* (*Space–Time–Matter*), and then makes a number of points through reference to the *Categories*, 6. Weyl's work is interesting, contends Heidegger, because it looks at the *continuum* not as something that can be resolved analytically, but something that must be thought as pre-given. This brings it closer to Aristotle, in part a result of the interest in the theory of relativity. Unlike Newtonian physics, the notion of the field in the theory of relativity is normative (GA19, 117–18).[20]

The genuineness of the *Categories* is in dispute, but Heidegger considers it to be by Aristotle – 'no disciple could write like that'. Here there is a discussion of *poson*, quantity. Heidegger claims that what is posited in the *thesis* is nothing else than the continuum itself. 'This basic phenomenon is the ontological condition for the possibility of something like stretch, *megethos*: position and orientation are such that from one point there can be a continuous progression to the others; only in this way is motion understandable' (GA19, 118–19). The line, which is continuous, can have points extracted from it, but these points do not together constitute the line. The line is more than a multiplicity of points, it has a *thesis*. But with numbers there is no *thesis*, so the series of numbers has a constitution only by way of the *ephekses*. Because a *thesis* is not required to understand arithmetic, number is ontologically prior: it seeks to explain being without reference to beings. Which is why Plato begins with number in his 'radical ontological reflection'. But Aristotle does not claim this. Instead he shows that the genuine *arche* of number, the unit, *monas*, is no longer a number, and therefore a more fundamental discipline is discovered, that which studies the basic constitution of beings, namely *sophia* (GA19, 120–1).

This incredibly rich exegesis of Aristotle raises any number of interesting and challenging points. Let me try to summarize some of the ones I find most important or intriguing.

1. Mathematics is an abstraction, an extraction from, an extractive looking at (*Heraussehen*) being. There is therefore a *khorizein*, a separating, between mathematics and being.
2. Arithmetic's *monas*, the unit, is *athetos*, unpositioned; geometry's *stigmé*, the point, is *thetos*, positioned.

3. Mathematical objects are positioned but do not have a place. For the Greeks, the objects they are abstractions from have a place. The modern concept of space is not present in either.
4. Place has a *dynamis*. This should be understood ontologically: every being has *its* place. Place is something belonging to beings as such: it is their capacity to be present.
5. The extension of material is not sufficient to understand place.
6. Motion is tied up with place. Only what is movable is in a place, but place itself does not move.
7. Everything perceivable has stretch, size, *megethos*. This is understood as *synekhes*, the *continuum*. This is a succession, not only where the ends meet in one place, but where the ends of one are identical with the next.
8. This is the crux of the difference between arithmetic and geometry: the mode of their connection is different. Arithmetic – succession where between the units there is nothing of the same lineage of being. Geometry – succession where the ends of one point are the ends of the next.
9. Therefore, though points can be extracted from a line, these points do not constitute the line. The line is more than a multiplicity of points, the surface more than a multiplicity of lines, the solid more than a multiplicity of surfaces.

For the early Heidegger this is by far the most explicit treatment of the question of place. And yet, there is a clear sense that he could have said more. As I have already quoted, he recognizes that a thorough elucidation of the issue of the *thesis* 'would have to take up the question of place and space', but here he can only indicate what is necessary for the issue of mathematics (GA19, 104). Similarly, in the following semester's course, published as *History of the Concept of Time*, he suggests that he has thought the notions of de-distancing, region and orientation in a way that suffices 'in relation to what we need for *time* and the *analysis of time*' (GA20, 322). Other concerns take precedence over this issue in Heidegger's thinking of the twenties.

The excursus on mathematics in the course on Platonic dialogues therefore helps us to understand Heidegger's work and development much more clearly. In particular it makes the moves he makes post *Being and Time* much more of a development than a radical break. As I have shown elsewhere, the lectures on Hölderlin are crucial to Heidegger's understanding of place.[21] In particular, this excursus make the remarks in *An Introduction to Metaphysics* on *khora* much clearer. In that light, it is worth discussing a related work, David Lachterman's *The Ethics of Geometry*, which makes a number of important points in this regard.

III. THE ETHICS OF GEOMETRY

By 'ethics', Lachterman is not talking about morals in the conventional sense. Rather by thinking ethics in terms of the Greek *ta ethe*, he is able to discuss the different ways (*mores*) and styles in which the Euclidean and Cartesian geometers *do* geometry, and comport themselves to their students – which will not concern us here – and the nature of those learnable items (*ta mathemata*) – which is the very focus of this study. Because *ta ethe* can be thought of as the way we have of acting in the world, his study is of *ethical* difference.[22]

Lachterman sees his work explicitly as a foil to Husserl's 'Origin of Geometry', as he argues that there are *origins* rather than an origin to geometry. Rather than a transcendental or historical origin, there are ethical origins.[23] Indeed, Lachterman argues that 'the institution of geometry among the Greeks is not, in my account, retained in the intentions of Descartes and his successors'.[24] There is therefore for Lachterman a discontinuity, rather than a clear development *ab origine*.

Lachterman's study has four closely allied enquiries:

1. How do techniques (constructive or otherwise) stay within or stray beyond the boundaries prescribed by the implicit ontology of Greek mathematics?
2. What does 'construction' involve for the Greeks, and how is its contribution to the articulation of geometry *as a science* evaluated?
3. A question of analysis: how did these techniques work?
4. A complicated question, essentially 'Where do these constructions take place?'

Naturally I am most interested in the fourth enquiry, as it seems at root the central question behind the role of geometry in yoking modern philosophy to an understanding of space on the basis of extension. The question can be better clarified by asking *where* the lines, planes and points of geometry are actually found or installed. Lachterman suggests that the conventional answer – 'Euclidean space' – has become so in-stalled, so unrevolutionary, that we find it self-evident that some conception of 'space' *must* lie in the background of Greek geometry. But such an answer is so close to the need for such a 'space' in a modern mathematical physics of extended corporal entities and their motions that we should guard against accepting it as ahistorical. 'The locale of Greek geometry may be foreign to the modern conceptions of extension and space.'[25]

Indeed this is precisely what Lachterman argues. There is, he suggests, no term corresponding to or translatable as 'space' in Euclid's *Elements*.[26] *To khorion* 'is the area within a perimeter of a specific figure, while *topos* and *thesis* in the *Data* have functions determined by the contextual aims of that work as a "dialectical" foil to the *Elements*, not

by a physics of space hidden in the background'.[27] The reason behind the conventional answer is no doubt the traditional scientific training lauded by Sokal and Bricmont in *Intellectual Impostures*:

> The author or the literality of the text have, in literature or even in philosophy, a relevance they do not have in science. One can learn physics without ever reading Galileo, Newton or Einstein, and study biology without reading a line of Darwin. What matters are the factual and theoretical arguments these authors offer, *not the words they used*. Besides, their ideas may have been radically modified or even overturned by subsequent developments in their disciplines (emphasis added).[28]

We can clearly see how what Lachterman suggests has happened. Following Sokal and Bricmont, it would appear that one can learn geometry without reading Euclid. And what matters are the factual and theoretical arguments he offers – who decides what these are? – not the words he used. Even if a word attributed to him – space – does not appear in his work! And we can clearly see how this might have arisen, because subsequent developments in geometry have modified his ideas, even if, retrospectively, they bear his name. I will return to this point below.

Now such an analysis certainly bears definite comparison with Heidegger. Unlike Husserl, who predicates this question – as so many others – as essentially ahistorical, for Lachterman this is a genealogical question. Indeed the subtitle of *The Ethics of Geometry* is *A Genealogy of Modernity*. Like Heidegger, Lachterman argues that the Greeks had no word for space. He notes the importance of *khora*, and places the origin of extension with Descartes. But in one of the few references to Heidegger in his work, he suggests he is explicitly countering the view expressed in *An Introduction to Metaphysics*. He claims it is frequently asserted, *for example by Heidegger*, that *khora* in the *Timaeus* is identified with Cartesian extension.[29] John Sallis makes a similar point in his otherwise illuminating *Chorology*, adding 'there is little to suggest any originary engagement with the Platonic discussion on the *khora*'.[30] As I have shown, this is not what Heidegger is asserting and there is certainly a thoughtful engagement. He is not saying that *topos*, which he translates as *Ort*, and *khora* are 'space' (*Raum*) – far from it. What he does say is that the shift from *topos* and *khora* to a 'space' defined by extension was *initiated* by Platonic philosophy. Initiated, *vorbereitet*, that is begun or prepared for, because of its interpretation of being as *idea*. If Plato's view of being initiates this conception of space, why is it not at play in the *Timaeus*, which is a late text of Plato's? It appears that although Plato himself did not have this conception, his thought paved the way, prepared the ground, for such a conception. Heidegger sketches this out in summary fashion:

> By a certain interpretation of being (as *idea*) the *noein* of Parmenides becomes the *noein* of *dialegesthai* in Plato. The *logos* of Heraclitus becomes the *logos* as

statement (*Aussage*) and becomes the leading theme (*Leitfaden*, textbook) of the 'categories' (Plato's *Sophist*). The combining of both into *ratio*, that is the related comprehension of *nous* and *logos*, is prepared in Aristotle. With Descartes *ratio* becomes 'mathematical'; only possible because since Plato this mathematical essence has been the focus, and is *one* possibility grounded in the *aletheia* of *physis* (GA65, 457).

Lachterman's work is most valuable in showing that Euclid, who wrote in the wake of Plato's thought (though he never references him),[31] did not rely on this understanding of space defined by extension: indeed not on a view of 'space' at all.

IV. RENÉ DESCARTES

If the Greeks did not have a word for space we would be forced to ask *when* it was introduced into Western thought. It seems that it was tied up with the translation, the trans-lation, the carrying across, of Greek to Latin thought, with the word *spatium*. The original translation of *topos* in Latin was with the word *locus*; only later did the word *spatium* arise.[32] As Heidegger surmises with his note that the *ratio* becomes 'mathematical', no one more than Descartes contributes to this. His understanding of space in terms of extension, in terms of mathematical co-ordinates is a radical break with Greek thought. I can only make a number of initial remarks here: this is a topic for further investigation.

Geometry occupies a central place in Descartes's work. In his division of mind and body, mind is *res cogitans*, matter *res extensa*. This division, found for example in the *Meditations on First Philosophy*,[33] places the notion of extension at the heart of his project. Extension is the central characteristic of nature, and geometry the science that allows us best access to it. Descartes's *Discourse on the Method* is followed by three examples – the *Dioptrics, Meteorology*, and *Geometry*. With the first two Descartes is merely trying to persuade us that his method is better than the ordinary one. But with the *Geometry* he claims to have 'demonstrated it'.[34] Descartes's geometry is certainly one of the great events in the history of mathematics, but what is actually at stake here?

In reply to a letter from Mersenne, which mentions that the mathematician Desargues has heard Descartes is giving up geometry, Descartes says:

> I have only resolved to give up abstract geometry, that is to say, research into questions which serve only to exercise the mind; and I am doing this in order to have more time to cultivate another sort of geometry, which takes as its questions the explanation of the phenomena of nature.[35]

What we find here is in some ways a reversal of the move made by Thales. Geometry is no longer the Platonic ideal of mental exercise,

but a science of the real world. Geometry and physics have the same *objectum*, 'the difference consists just in this, that physics considers its object not only as a true and real being, but as actually existing as such, while mathematics considers it merely as possible, and as something which does not actually exist in space, but could do so'.[36] For example, in the *Discourse on the Method*, Descartes says that the 'object dealt with by geometricians' is '*like* [emphasis added] a continuous body or a space indefinitely extended in length, breadth, and height or depth, divisible into various parts which could have various shapes and sizes and be moved or transposed in all sorts of ways'.[37] Geometry is no longer simply an *abstraction* from being, but is seen as a generalization of being. What Descartes does is to see geometry as equivalent to algebra. Just as algebra is symbolic logistic, geometry is a symbolic science. It is this, rather than the simple equation of arithmetic and geometry that is his most radical break with the past.[38]

As Jacob Klein has shown, 'extension has, accordingly, a *twofold* character for Descartes: It is "symbolic" – as the object of a "general algebra", and it is "real" – as the "substance" of the corporal world.' So, not only is Descartes moving geometry from abstract mental exercise to practical science – the foundation of physics, a study of the world – he assumes that the insights of geometry can tell us about the world. The concept of extension is not simply a geometrical property, *but a physical property*. Indeed, as Heidegger recognizes, it is for Descartes 'the fundamental ontological determination of the world' (GA2, 89). The reason is a critique: at once negative and positive. It criticizes the position of scholasticism and provides the foundation for scientific knowledge (GA33, 94). It is the *symbolic* objectivity of extension within the framework of the *mathesis universalis* that allows it to explain the being of the corporal world. 'Only at this point has the conceptual basis of "classical" physics, which has since been called "Euclidean space", been created. This is the foundation on which Newton will raise the structure of his mathematical science of nature.'[39]

What this means, and this is the crucial point, is that not only is the understanding of space as 'non-Euclidean' possible, but there is no such thing as Euclidean space. What we call Euclidean space is actually a seventeenth-century invention, based no doubt on the postulates of Euclid's *Elements*, but crucially introducing the idea that this is constituent of reality, whatever that might mean. Euclid, like Plato, sees his geometry as a mathematical system. It is the generalization of this to explain the world that is the crucial element introduced in the seventeenth century.[40] Now, not only does this introduce this word 'space' but, by conceiving of geometrical lines and shapes in terms of numerical co-ordinates, which can be divided, it turns something that is *thetos* into *athetos*, positioned into unpositioned. Indeed for Descartes, it is the very nature of a body, *res extensa*, that it is divisible.[41] At the very beginning of the

Geometry Descartes boasts that 'all problems in geometry can be simply reduced to such terms that a knowledge of the lengths of certain straight lines is sufficient for their construction'.[42] That is, geometric problems can be reduced to equations, the length (i.e., quantity) of lines: a problem of number. The *continuum* of geometry is transformed into a form of arithmetic. The mode of connection of the geometrical for the Greeks is characterized by the *synekhes*; the series of numbers – where no touching is necessary – by the *ephekes*. Descartes's geometry, because of its divisibility, can only be *ephekes*. Geometry loses position just as place is transformed into space.[43]

V. CONCLUSIONS

My conclusions here are necessarily openings rather than formal statements. My feeling is that we can take this forward in a number of ways. First it gives us the potential for thinking about the philosophy of mathematics within the Continental tradition in new ways. It is clear that Heidegger understands Greek mathematics in a detailed and intimate way (he was registered with the Freiburg Faculty of Natural Science and Mathematics between 1911 and 1913,[44] and later used to examine mathematics PhD students). Clearly Lachterman brings the same kind of critical acumen to bear. Contrary to the *impression* given by Sokal and Bricmont's *Intellectual Impostures*, there are those within the Continental tradition who not only work with the philosophy of science, but understand it deeply. Second, it helps us understand the development of Heidegger's own thought. The *Gesamtausgabe* regularly provides this sort of insight, and it was to be expected that *Plato's 'Sophist'* would – because of the revered way it was spoken about by those such as Hannah Arendt – do this. But in a seemingly marginal excursus, Heidegger opens up hugely important avenues of research.

One of these avenues is that Greek geometry – and therefore the foundation of modern geometry – does not require a concept that is equivalent to the modern notion of 'space'. We can therefore conceive of an understanding of geometry without Cartesian extension. We can conceive of place without space. This would enable us to see that Descartes's move to 'Cartesian space' is not necessary, which allows fundamental insights into the notion of the world made picture. Given that, in Heidegger's view, this scientific, (modern) mathematical understanding of space paves the way for modern technology, we can potentially begin to perceive a way out of the problem, or at least, to understand it in a much more detailed way.

But the avenue I am most interested in is its major political consequences. Modern technology requires a view of space as mappable, controllable and capable of domination. This is not found in Greek

thought. The modern state system of bounded geographical territories arises from the Peace of Westphalia in 1648. This is some eleven years after Descartes's *Discourse*. Though the two are not directly linked, it is symptomatic that a philosophical justification for demarcated, controllable, calculable space is made at the same time this is put into practice. The modern concept of the state is as remote from the Greek *polis* – a site – as 'space' is from *topos* and *khora*.[45]

Much work remains to be done, certainly. But the central conclusion that this reading gives is that the *place* of geometry has not always been the same. This is an historical enquiry. Geometry does not, *contra* Husserl, have *an* origin (*Ursprung*), but rather a descent (*Herkunft*), an emergence (*Entstehung*). Husserl's *Ursprung* would be the *Evidenzen* or conditions of possibility for geometry – a putatively historical examination that is essentially anti-historical. But *this* enquiry would be genealogical. It is not a question of (fundamental) ontology, but of historical ontology.[46]

Abbreviations to works by Martin Heidegger

GA   *Gesamtausgabe*, Frankfurt am Main: Vittorio Klostermann, 1975ff.

GA2   *Sein und Zeit*, 1977. Translated by Edward Robinson & John Macquarrie as *Being and Time* (Oxford: Blackwell, 1962). References are to the marginal pagination found in the English *and* German, which refer to the original text, *Sein und Zeit* (Tübingen: Max Niemeyer, eleventh edition, 1967).

GA6   *Nietzsche*, 2 vols., 1996. Translated by David Farrell Krell, Frank Capuzzi & Joan Stambaugh as *Nietzsche* (N) (San Francisco: Harper Collins, 4 vols., 1991).

GA13 *Aus der Erfahrung des Denkens*, 1983.

GA17 *Einführung in die Phänomenologische Forschung*, 1994.

GA19 *Platon: Sophistes*, 1992. Translated by Richard Rojcewicz and André Schuwer as *Plato's 'Sophist'* (Bloomington: Indiana University Press, 1997).

GA20 *Prolegomena zur Geschichte der Zeitbegriffs*, 1979. Translated by Theodore Kisiel as *History of the Concept of Time: Prolegomena* (Bloomington: Indiana University Press, 1985).

GA24 *Die Grundprobleme der Phänomenologie*, 1975. Translated by Albert Hofstader as *The Basic Problems of Phenomenology* (Bloomington: Indiana University Press, 1982).

GA33 *Aristoteles, Metaphysik Θ 1–3: Von Wesen und Wirklichkeit der Kraft*, 1981.

GA40 *Einführung in die Metaphysik*, 1983. Translated by Ralph Mannheim as *An Introduction to Metaphysics* (IM) (New Haven: Yale University Press, 1959).

GA41 *Die Frage nach dem Ding. Zu Kants Lehre von den transzendentalen Grundsätzen*, 1984.

GA55 *Heraklit: (1) Der Anfang des abendländischen Denkens; (2) Logik: Heraklits Lehre Vom Logos*, 1979.
GA65 *Beiträge zur Philosophie (Vom Ereignis)*, 1989.

The majority of *Gesamtausgabe* translations have the German pagination at the top of the page, allowing a single page reference. Exceptions noted above.

Notes

1   The most thorough treatment of Descartes from this time is found in GA17.

2   Important discussions of space in Heidegger are found in Maria Villela-Petit, 'Heidegger's Conception of Space' in Christopher Macann (ed.), *Martin Heidegger: Critical Assessments* (London: Routledge, 4 vols., 1992), vol. IV; Edward S. Casey, *The Fate of Place: A Philosophical History* (Berkeley: University Presses of California, 1997); and Didier Franck, *Heidegger et le problème de l'espace* (Paris: Les Éditions de Minuit, 1986).

3   See Tom Rockmore, *Heidegger and French Philosophy: Humanism, Antihumanism and Being* (London: Routledge, 1995).

4   Immanuel Kant, *Grundlegung zur Metaphysik der Sitten* in *Werke*, edited by Ernst Cassirer (Berlin: Bruno Cassirer, 11 vols., 1912–22), vol. IV, p. 286; translated by James W. Ellington as *Grounding for the Metaphysic of Morals* in Stephen M. Cahn (ed.), *Classics of Western Philosophy* third edition (Indianapolis: Hackett, 1990), p. 1010.

5   This bears comparison with the late work of Edmund Husserl, particularly *Die Krisis de europäischen Wissenschaften und die tranzendentale Phänomenologie: Eine Einleitung in die Phänomenologische Philosophie* in *Husserliana: Gesammelte Werke*, edited by Walter Biemel (Den Haag: Martinus Nijhoff, 1950), vol. VI; translated by David Carr, *The Crisis of European Sciences and Transcendental Phenomenology: An Introduction to Phenomenological Philosophy* (Evanston: Northwestern University Press, 1970).

6   See Plato, *Timaeus and Critias*, translated by Desmond Lee (Harmondsworth: Penguin, 1971). On Plato and mathematics generally, see François Lasserre, *The Birth of Mathematics in the Age of Plato* (London: Hutchinson, 1964). For a critical discussion of Heidegger's reading, see Jacques Derrida, *Khora* (Paris: Éditions Galilée, 1993). See John Sallis, *Chorology: On Beginning in Plato's 'Timaeus'* (Bloomington: Indiana University Press, 1999). Alternative readings – noting the implications they have for feminist thought – are found in Luce Irigaray, *Speculum de l'autre femme* (Paris: Éditions de Minuit, 1974); Julia Kristeva, *La Révolution du langage poétique: L'avant-garde à la fin du XIXᵉ siècle: Lautréamont et Mallarmé* (Paris: Éditions du Seuil, 1974); and Judith Butler, *Bodies That Matter: On the Discursive Limits of Sex* (London: Routledge, 1993). On space and place in Greece generally, and the change from religious, qualitative, differentiated space to homogeneous geometric space, see ch. 3 of Jean Pierre Vernant, *Mythe et pensée chez les Grecs* (Paris: François Maspero, 2ᵉ édition, 1969).

7   See Stuart Elden, 'Heidegger's Hölderlin and the Importance of Place', *Journal of the British Society for Phenomenology*, vol. 30 no. 3 (October 1999).

8   On GA19 generally, see Richard Polt, 'Heidegger's Topical Hermeneutics: The *Sophist Lectures', Journal of the British Society for Phenomenology*, vol. 27 no. 1 (January 1996); Ingeborg Schüssler, 'Le *Sophiste* de Platon dans l'interprétation de M. Heidegger' in Jean–François Courtine (ed.), *Heidegger 1919–1929: De l'herméneutique de la facticité à la métaphysique du 'Dasein'* (Paris: J. Vrin, 1996). On related questions more generally, see Ted Sadler, *Heidegger and Aristotle: The Question of Being* (London: Athlone, 1996); Catherine H. Zuckert, *Postmodern Platos: Nietzsche, Heidegger, Gadamer, Strauss, Derrida* (Chicago: University of Chicago Press, 1996); William McNeill, *The Glance of the Eye: Heidegger, Aristotle and the Ends of Theory* (Albany: State University of New York Press, 1999).

9   On this see also GA41.

10 See Edward A. Maziarz and Thomas Greenwood, *Greek Mathematical Philosophy* (New York: Frederick Ungar, 1968), p. 7. The notion of geometry as a land measuring device by the Egyptians is described in Herodotus, *The Histories*, translated by Robin Waterfield (Oxford: Oxford University Press, 1998), p. 109.

11 Aristotle, *Physics* (Oxford: Clarendon Press, 1936); translated by Robin Waterfield (Oxford: Oxford University Press, 1996), 193b32. See *The Meno of Plato*, edited by E. Seymer Thompson

(London: Macmillan, 1937); translated by W. K. C. Guthrie, *Meno* (Harmondsworth: Penguin, 1956), 76a: 'shape is that in which a solid terminates (*perainei*, i.e., comes to a limit) … shape is the limit of a solid (*stereon peras skhema einai*)'. Compare Euclid, *The Thirteen Books of Euclid's Elements*, with introduction and commentary by Thomas L. Heath (New York: Dover, three volumes, second edition, 1956), vol. I, p. 153: 'a limit (*horos*) is that which is an extremity (*peras*) of anything … a shape (*skhema*) is that which is contained by any boundary or boundaries (*periekhomenon*)'. Heath provides the Greek in his commentary on pp. 182–3. I have altered the translation.

12 See *Aristotle's Metaphysics*, introduction and commentary by W. D. Ross (Oxford: Clarendon Press, 1924) 2 vols.; translated as *Metaphysics* in W. D. Ross (ed.), *The Works of Aristotle: Volume VIII Metaphysica*, (Oxford: Clarendon Press, 1930, 1016b29–31).

13 Aristotle, *Metaphysics*, 1092a18–20. The full sentence reads 'for place is peculiar to the individual things, and hence they are separate in place (*khorista topo*); but mathematical objects are not anywhere [*pou*]'. Heidegger is therefore not quite justified in his interpolation of '*ouk en topo*' to Aristotle: such a formulation is not found in the text.

14 For a discussion of *dynamis* in relation to geometry, see GA33, 58–61. This course as a whole provides Heidegger's most sustained treatment of *dynamis* in Aristotle.

15 Aristotle, *Physics*, 208b22.

16 Aristotle, *Physics*, 212a7. See Jean Beaufret, *Dialogue avec Heidegger IV: Le Chemin de Heidegger* (Paris, Les Éditions de Minuit, 1985), p. 94, where he suggests that this phrase of Aristotle's is the 'guiding thread' of all of Heidegger's work after *Being and Time*. See also Polt, 'Heidegger's Topical Hermeneutics', pp. 72–3.

17 See, among many other discussions, Beaufret, *Dialogue avec Heidegger IV*, pp. 113; 115.

18 See Maziarz and Greenwood, *Greek Mathematical Philosophy*, p. 23, where they suggest this was found in the Pythagoreans, who saw 'the unit as a "point without position", and the point as "a unit having position" '.

19 Aristotle, *Physics* 231a24ff.

20 See Hermann Weyl, *Space – Time – Matter* (New York: Dover, 1950); *Philosophy of Mathematics and Natural Science* (Princeton: Princeton University Press, 1949). I am not convinced that Heidegger's reading of Weyl holds.

21 See Elden, 'Heidegger's Hölderlin'; and 'Rethinking the *Polis*: Implications of Heidegger's Questioning the Political', *Political Geography*, vol. 19 no. 4 (May 2000). This argument is greatly developed in *Mapping the Present: Heidegger, Foucault and the Project of a Spatial History* (London: Athlone Press, 2001).

22 David Rapport Lachterman, *The Ethics of Geometry: A Genealogy of Modernity* (New York: Routledge, 1989), p. xi.

23 Lachterman, *The Ethics of Geometry*, pp. xi–xii.

24 Lachterman, *The Ethics of Geometry*, p. xii.

25 Lachterman, *The Ethics of Geometry*, pp. 27–8. For good examples of this assumption, see Jeremy Gray, *Ideas of Space: Euclidean, Non-Euclidean and Relativistic* (Oxford: Clarendon Press, second edition, 1989); and Weyl, *Space – Time – Matter*.

26 This is confirmed by the work of Thomas Heath, both in his translation of the *Elements*, and his *A History of Greek Mathematics* (New York: Dover, 1981) 2 vols. Although Heath occasionally uses the word 'space' in his translations, his glossary to the latter work (vol. II, pp. 563–69) includes no word that is the equivalent of 'space', though the entry for '*khorion*' reads 'area … *khorein apotome, sectio spatii*'. This would seem to bear out my suggestion that the shift is made in the transition from Greek to Latin thought.

27 Lachterman, *The Ethics of Geometry*, p. 80.

28 Alan Sokal and Jean Bricmont, *Intellectual Impostures: Postmodern Philosophers' Abuse of Science* (London: Profile Books, 1998), p. 185.

29 Lachterman, *The Ethics of Geometry*, p. 219 n. 88.

30 Sallis, *Chorology*, p. 111 n. 22.

31 Maziarz and Greenwood, *Greek Mathematical Philosophy*, p. 242: '[Euclid's] works make no allusion to Plato or Aristotle, or even to their strictly methodological views.'

32 See Jacob Klein, *Greek Mathematical Thought and the Origin of Algebra* (New York: Dover, 1992), p. 256 n. 209.

33 René Descartes, *Meditationes de Prima Philosophia*, in *Oeuvres de Descartes*, publiées par Charles Adam et Paul Tannery (Paris: Vrin, 1964ff.) 13 vols., vol. VII, p. 44; translated by Donald

A. Cress in *Discourse on Method* and *Meditations on First Philosophy* (Indianapolis: Hackett, 1980), p. 73.

34   Descartes, letter to Mersenne, December 1637, *Oeuvres de Descartes*, vol. I, p. 478.

35   Descartes, letter to Mersenne, 27 July 1638, *Oeuvres de Descartes*, vol. II, p. 268.

36   Descartes, *Oeuvres de Descartes*, vol. V, p. 160; *Conversation with Burman*, translated by John Cottingham; Oxford, 1976, p. 23.

37   Descartes, *Discours de la méthode* in *Oeuvres de Descartes*, vol. VI, p. 36; *Discourse on Method*, p. 19.

38   Klein, *Greek Mathematical Thought*, p. 206.

39   Klein, *Greek Mathematical Thought*, pp. 210–11.

40   See Maziarz and Greenwood, *Greek Mathematical Philosophy*, p. 256. It would be instructive to compare this with the birth of the classical episteme as outlined in Michel Foucault, *Les Mots et les choses* (Paris: Gallimard, 1966).

41   Descartes, *Meditationes* in *Oeuvres de Descartes*, vol. VII, pp. 85–6; *Meditations*, p. 97.

42   *The Geometry of René Descartes*, French–Latin–English edition, translated by David Eugene Smith and Marcia L. Latham (New York: Dover, 1954), p. 2/3. Translation modified.

43   This is developed from a valuable e-mail correspondence with Michael Eldred on heidegger@lists.village.virginia.edu.

44   John van Buren, *The Young Heidegger: Rumor of the Hidden King* (Bloomington: Indiana University Press, 1994), pp. 58–9.

45   This argument is greatly developed in Stuart Elden, 'The Geometry of the Political', forthcoming.

46   See Stuart Elden, 'Reading Genealogy as Historical Ontology' in Alan Milchman and Alan Rosenberg (eds.), *Foucault and Heidegger: Critical Encounters* (Minneapolis: University of Minnesota Press, 2001); and *Mapping the Present*.