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NEW STUDIES IN DEONTIC LOGIC

Norms, Actions, and the Foundations of Ethics

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PREFACE

The present volume is a sequel to *Deontic Logic: Introductory and Systematic Readings* (D. Reidel Publishing Company, Dordrecht 1971): its purpose is to offer a view of some of the main directions of research in contemporary deontic logic. Most of the articles included in *Introductory and Systematic Readings* represent what may be called the *standard modal approach* to deontic logic, in which deontic logic is treated as a branch of modal logic, and the normative concepts of *obligation*, *permission* and *prohibition* are regarded as analogous to the "alethic" modalities *necessity*, *possibility* and *impossibility*. As Simo Knuuttila shows in his contribution to the present volume, this approach goes back to late medieval philosophy. Several 14th century philosophers observed the analogies between deontic and alethic modalities and discussed the deontic interpretations of various laws of modal logic. In contemporary deontic logic the modal approach was revived by G. H. von Wright's classic paper 'Deontic Logic' (1951). Certain analogies between deontic and alethic modalities are obvious and uncontroversial, but the standard approach has often been criticized on the ground that it exaggerates the analogies and tends to ignore those features of normative concepts which distinguish them from other modalities. Throughout its history deontic logic has been plagued by various (apparent or real) "paradoxes", which may be regarded as evidence of the peculiar, "nonstandard" character of deontic modalities. The following characteristics of normative discourse seem especially striking in this respect: (i) Normative concepts are usually applied to human actions: they belong to *practical* discourse, and the logic of action concepts should therefore form an essential part of the logical study of normative statements. (ii) It has often been argued that unlike "natural" necessities, obligations can conflict with one another, and the resolution of such normative conflicts is an important part of moral discourse. (iii) If deontic sentences are used for the purpose of directing and controlling people's conduct, they are essentially "forward-looking"; thus there appears to be an interesting connection between the concept of obligation (or the concept of ought) and *temporal* modalities. It has been suggested that many problems and paradoxes of deontic logic can be solved by studying the temporal relativity of the concept of obligation.

The papers included in this volume show how recent research has extended deontic logic in the three directions mentioned above, and thus increased its relevance to moral philosophy and the philosophy of law. The book is divided into four parts, the first three of which are concerned with topics (i)–(iii); the fourth part discusses the history of deontic logic. None of the papers included in this collection has been published before.

In the first part (entitled 'Deontic Logic, the Logic of Action, and Deontic Paradoxes') Georg Henrik von Wright discusses three ways of interpreting and applying normative concepts: they can be regarded as predicates of individual actions, as (second-order) predicates of act-categories or generic actions, and as operators on actions sentences. Hector-Neri Castañeda argues that many problems and paradoxes of deontic logic can be solved once we accept the distinction between an action considered as a circumstance and an action deontically or *practically* considered (an action in deontic "focus"), or the semantic distinction between *propositions* and *practitions*, and supports his view by numerous examples. David Makinson's paper 'Quantificational Reefs in Deontic Waters' discusses the interpretation of quantifiers in systems of deontic logic containing variables for individual actions, and shows that such systems give rise to considerable interpretational difficulties.

The main topic of the second part is the problem of normative conflict. In 'The Expressive Conception of Norms' Carlos E. Alchourrón and Eugenio Bulygin distinguish two general views of the nature of norms, the *hyletic* conception and the *expressive* conception, and analyse different types of normative change and normative conflict. The latter topic is investigated in greater detail and on a more technical level by Carlos E. Alchóurron and David Makinson (in 'Hierarchies of Regulations and Their Logic'), who show how an ordering of a code of regulations can resolve inconsistencies within the system determined by the code, and confer uniqueness on the result of removing some proposition from a given normative system. (In general, the elimination of a proposition from a normative system or the *derogation* of a norm need not transform the system into another well-defined system.) In 'Non-Kripkean Deontic Logic' Peter K. Schotch and Raymond E. Jennings present a new approach to deontic logic which allows the possibility of genuine moral conflicts. They argue that deontic logic should be regarded as a generalization rather than application of modal logic, and show how their approach to deontic logic can be developed into a general theory of inference from inconsistent premise sets.

The third part contains two papers by Richmond Thomason, 'Deontic

Logic as Founded on Tense Logic' and 'Deontic Logic and the Role of Freedom in Moral Deliberation', in which he presents a semantical theory of deontic modalities based on tense logic, and discusses some philosophical implications of the temporal character of the concept of 'ought'. The former paper has already reached the status of a classic in its field: various mimeographed versions of it have been circulating among deontic logicians for years. The latter paper discusses (among other things) the dependence of moral judgments on judgments of possibility and freedom. Lennart Åqvist and Jaap Hoepelman discuss the problem of "contrary-to-duty obligation" and present a system of deontic tense logic with deontic, temporal and historical modalities.

Part IV contains a paper by Simo Knuuttila on a hitherto overlooked and almost unknown period in the history of deontic logic, viz. medieval deontic logic. Leibniz and Bentham are often mentioned as precursors of contemporary deontic logic, but Knuuttila shows that the history of the subject goes farther back. Some fourteenth century philosophers and logicians treated the logic of normative concepts as a variety of modal logic, and studied problems similar to those discussed in contemporary deontic logic, e.g. the paradox of the Good Samaritan and problems of conditional obligation. Knuuttila also argues that the development of deontic logic in the fourteenth century was related to a new conception of ethics as a system of rules (normative system). His paper shows that the history of deontic logic deserves a great deal of further investigation.

I wish to thank the authors of the papers, Professor Jaakko Hintikka, Managing Editor of *Synthese Library*, and Ms. J. C. Kuipers of D. Reidel Publishing Company for cooperation and support.

RISTO HILPINEN

PART I

DEONTIC LOGIC, THE LOGIC OF ACTION, AND DEONTIC PARADOXES

GEORG HENRIK VON WRIGHT

ON THE LOGIC OF NORMS AND ACTIONS

I. DEONTIC LOGIC AS MODAL LOGIC – ANALOGIES AND DISANALOGIES

1. In what may be called the prehistory of modern deontic logic one can distinguish two main traditions. The one goes back (at least) to Leibniz, the other (at least) to Bentham.

Bentham entertained a grand idea of a Logic of Imperation or of the Will. It was going to be a new branch of logic, "untouched by Aristotle". Bentham did not develop it systematically. This was left to the Austrian Ernst Mally in his work *Grundgesetze des Sollens, Elemente der Logik des Willens* (1926). Mally seems not to have been aware of Bentham's pioneer work which remained practically unnoticed until the late mid-twentieth century. As an aftermath to Mally's work one may regard discussions in the 1930's and early 1940's on the logical nature of imperatives – including some constructive efforts at developing a Logic of Imperatives and of Optatives.

The discipline which now goes under the established name Deontic Logic did not evolve in the tradition of Bentham and Mally. It was born as an offshoot of Modal Logic. None of its founding fathers, however, seems to have been aware that their leading idea had been anticipated, quite explicitly, by Leibniz who in the *Elementa juris naturalis* (1672) wrote: "Omnes ergo Modalium complicationes, transpositiones, oppositiones ab Aristotele et Interpretatibus demonstrate, ad haec nostra Iuris Modalia non inutiliter transferri possunt". With these words the birth of deontic logic can truly be said to have been announced.

2. By the *Iuris Modalia* Leibniz meant the deontic categories of the obligatory (*debitum*), the permitted (*licitum*), the prohibited (*illicitum*), and the facultative (*indifferentum*). And by saying that to the deontic modalities may be transferred all the "complications, transpositions, and oppositions" of Aristotelian modal logic, Leibniz was in the first place thinking of the relations of interdefinability which obtain between the traditional (alethic) modalities.

R. Hilpinen (ed.), New Studies in Deontic Logic, 3–35

I shall refer to these observations by Leibniz as *analogies of interdefinability* between alethic and deontic modalities. The analogies can be exhibited in the following table:

it is possible, M	it is permitted, P
it is impossible, $I = \sim M$	it is forbidden, $F = \sim P$
it is necessary, $N = I \sim\ = \sim M \sim$	it is obligatory, $O = F \sim\ = \sim P \sim$

We shall omit from special consideration here the category of the facultative, *i.e.* the neither-obligatory-nor-forbidden, answering to the alethic category of the contingent. Then interdefinability, as shown by the table, means that one can, taking *one* of the concepts as basic or primitive, through a process of "double negation" define or engender the other concepts of the triad. Which of the three one regards as *Grundbegriff* is indifferent.

As far as the interrelatedness of the basic deontic categories is concerned, Bentham seems to have been of the same opinion as Leibniz. But Bentham did not note the analogies with the modal concepts. The first author to study in detail both the analogies and the interdefinability relations seems to have been the Austrian Alois Höfler in a paper written in the 1880's but not published until 1917.[1]

3. The analogies of interdefinability do not, by themselves, suffice for the construction of an (interesting system of) deontic logic. To this end some logical laws or principles governing the deontic notions must be found or suggested.

The additional observation which gave a decisive impetus to my efforts in the area, concerned the distributive properties of the alethic and deontic modal operators. For the notion of possibility we have the distribution law $M(p \vee q) \leftrightarrow Mp \vee Mq$. It seemed to me then that an analogous principle holds for the notion of permission (permittedness). Accepting this *and* the analogies of interdefinability gives us the following *analogies of distributivity*:

$M(p \vee q) \leftrightarrow Mp \vee Mq$	$P(p \vee q) \leftrightarrow Pp \vee Pq$
$I(p \vee q) \leftrightarrow Ip \& Iq$	$F(p \vee q) \leftrightarrow Fp \& Fq$
$N(p \& q) \leftrightarrow Np \& Nq$	$O(p \& q) \leftrightarrow Op \& Oq$

The first to pay attention to the distributive properties of the deontic concepts was, as far as I know, Mally. In Mally's *Deontik* the law (answering to) $O(p\&q)\leftrightarrow Op\&Oq$ holds. The analogy with modal logic, however, passes unnoted.

4. By the *Minimal System* of deontic logic, I shall understand a calculus which can be characterized syntactically as follows:

Every tautology of propositional logic (PL) is a valid formula of the system when the propositional variables are replaced by deontic formulas. The sole (additional) axiom of the system is the formula $P(p \vee q)\leftrightarrow Pp \vee Pq$. The definitions $F =_{df} \sim P$ and $O =_{df} \sim P \sim$ are accepted. In addition to the usual inference rules of substitution and detachment we have a transformation principle to the effect that provably equivalent PL-formulas are intersubstitutable *salva veritate* in deontic formulas. This last principle may be regarded as a version of what is sometimes also called Leibniz's Law.

Bentham regarded it as a law of his Logic of the Will that if something is obligatory (Bentham says "commanded") then it is not also prohibited. In our symbolism above: $OP \rightarrow \sim Fp$. In the minimal system this is equivalent with the formula $\sim(Op\&O \sim p)$ which is equivalent with $OP \rightarrow Pp$ which again is equivalent with $Pp \vee P \sim p$. By virtue of the distribution axiom, finally, this last is equivalent with Pt where "t" stands for an arbitrary tautology of PL.

Bentham's Law is also valid in Mally's *Deontik*. Mally, moreover, recognized the rôle Leibniz's Law plays in the proofs of deontic theorems. His system, one could say, has all the ingredients of a "sound" deontic logic, but also contains additional ingredients which, unfortunately, from a formal point of view vitiate the whole undertaking. The "unsound" features of Mally's system have to do with his way of treating *conditional* obligations.

The system of deontic logic which I presented in my 1951 paper[2] was the Minimal System embellished with one additional axiom. This was the formula $Pp \vee P \sim p$. I coined for it the name Principle of Permission. Accepting the relations of interdefinability this, as we have just seen, is but another form of what above I called Bentham's Law.

For the Minimal System embellished with Bentham's Law I shall, *faute de mieux*, propose the name the *Classical System* of deontic logic.

Any normal modal logic accepts as valid the formulas $Mp \vee M \sim p$ and $\sim(Np\&N \sim p)$ and their "contracted" form Mt. Thus Bentham's Law too has an analogue in modal logic.

From the Classical System we reach what I shall, following Bengt Hansson[3], call the *Standard System* by making the following two modifications: The deontic operators are understood as operating on and resulting in propositions. Leibniz's Law is replaced by a stronger inferential principle which says that, if f is a valid formula of the deontic system, then Of is so too.

Both modifications strengthen the analogy with modal logic. The analogue of the inferential principle allowing the inference from f to Of is in modal logic known as the Rule of Necessitation.

The Standard System may be said to stretch the analogy between modal and deontic logic to its utmost limit. The only significant deviation lies in the fact that whereas traditional modal logic accepts as valid the formulas $p \to Mp$ and $Np \to p$, a "sound" deontic logic must reject their analogues $p \to Pp$ and $Op \to p$. It has to rest content with the weaker forms of those analogical formulas which are represented by the Principle of Permission and by Bentham's Law respectively.

When building the Classical System I took the view that the deontic operators operated on names of (categories or types of) action, and not on propositions. In the Classical System, therefore, "mixed" formulas, such as *e.g.* $p \to Oq$, or "higher order" formulas such as *e.g.* OPp, were not regarded as well-formed. To logicians these restrictions may seem impediments to the development of deontic logic. Their removal within the Standard System has gained more or less universal acceptance. There may nevertheless have existed some good and serious reasons against taking the step from the Classical to the Standard System – and even against the step from the Minimal to the Classical System.

5. Suggestive and, from a formal point of view, fertile as the analogies between the alethic and the deontic modalities may be, they are also open to doubts. The more I have reflected on the nature of norms and normative concepts, the stronger have these doubts grown with me. I shall next mention some points on which one may focus such doubts.

a. Disanalogies of interdefinability. It seems much more plausible to regard the operators (concepts) O and F as being interdefinable than to regard P and O, or P and F, as being so. One can ask: is permission to do something simply the absence of prohibition to do this same thing? That permission should entail the absence of a "corresponding" prohibition seems clear. But does the reverse entailment hold? Is not permission something "over and above" mere absence of prohibition?

This question is in fact a classic problem of legal philosophy and theory.

Do permissions (rights) have an independent status in relation to prohibitions (obligations), or not? I think it is correct to say that opinions continue to be *very much* divided on this issue.

To say that prohibition to do is tantamount to obligation to forbear (omit) one and the same thing, and to say that obligation to do is tantamount to prohibition to forbear, seems uncontroversial. What is not clear and uncontroversial, however, is whether the relation between doing and forbearing (omission of action) is simply the relation between something and its *negation*. This is a grave problem for a "logic of action".

b. Disanalogies of distributivity. Doubts concerning the analogies of interdefinability may, but need not, affect the analogies of distributivity. One might, for example, reject the formulas $\sim Fp \rightarrow Pp$ and $\sim O \sim p \rightarrow Pp$ and yet regard disjunctive permissions as being disjunctively distributable and conjunctive obligations as conjunctively distributable. In building a calculus or system of deontic logic one would then have to lay down independently in axioms the distributive properties of permissions and obligations respectively.

One can, however, for other reasons doubt the analogies of distributivity. Since its beginnings, deontic logic has been beset by some "anomalies" or "paradoxes". The best known and most discussed one is Ross's Paradox. Two others are the Paradox of Derived Obligation and the Paradox of the Good Samaritan. The two last ones may be regarded as variant formulations of the first. And all three have their roots in the formulas $O(p \& q) \rightarrow Op$ or, alternatively, $Pp \rightarrow P(p \vee q)$ of "traditional" deontic logic – whether in the "minimal", the "classical", or the "standard" version. Therefore, in a deontic logic which rejects the implication from left to right in the equivalence $O(p \& q) \leftrightarrow Op \& Oq$ while retaining the implication from right to left, the "paradoxes" would not appear[4].

Analogous "paradoxes" are known from modal logic. The Paradox of Derived Obligation, for example, is an analogue in deontic logic to what is known as the Paradox of Strict Implication in modal logic. But the conflict between "intuition" and "formalism" of which the paradoxes are symptomatic seems to be much more serious in deontic than in modal logic. In this fact I would see an indication that the analogy between the two logics is not as perfect as many people have thought.

On a normal understanding of the word "or" in normative language, disjunctive permissions are *conjunctively*, and not *disjunctively*, distributable. If someone is told that he may work *or* relax this would normally be understood to mean that he is permitted to work but also permitted to relax: it is up to him to *choose* between the two alternatives. Disjunctive permissions of this character I have called Free Choice Permissions. Opinions on their logical status differ considerably. Some logicians think that they only apparently conflict with the distribution law $P(p \vee q) \leftrightarrow Pp \vee Pq$. Another attitude is to reject, at this point, the analogy with modal logic and build a deontic logic which incorporates a distribution principle $P(p \vee q) \leftrightarrow Pp \& Pq$. Such a deontic logic, however, will have to differ in many other features as well from the traditional systems.

c. Disanalogies in the rules of inference. In a "normal" modal logic the contradiction is pronounced impossible and the tautology necessary. A "normal" modal logic, moreover, accepts the Rule of Necessitation (above p. 6) and all tautologies of PL. From the Rule of Necessitation and the distribution principles of the modal operators one easily derives Leibniz's Law (above p. 5). Since $p \vee \sim p$ is a theorem, $N(p \vee \sim p)$ is a theorem, too. By virtue of Leibniz's Law and the interdefinability of the modal operators, Nt and $\sim M \sim t$ are theorems. However, taking $N(p \vee \sim p)$ or $\sim M(p \& \sim p)$ as *axioms*, one can, with the aid of Leibniz's Law, derive the Rule of Necessitation.

What I have called the "standard" system of deontic logic accepts the deontic analogue of the Rule of Necessitation. That which is, in deontic logic, provably true is also obligatory. This always seemed to me highly counterintuitive, sheer nonsense. Most logicians, however, seem willing to swallow the absurdity – presumably for reasons of formal elegance and expediency. I cannot regard this as an acceptable ground. The "classical" system, therefore, did not accept the necessitation rule and, since it accepted Leibniz's Law, it did not regard Ot, the deontic analogue of Nt, as logically true. This still seems to me a sound attitude.

d. In standard modal logic, the operators operate on propositions. The expressions are read "It is possible (impossible, necessary) *that* ...". The place of the blank is taken by a descriptive *sentence*. The deontic phrase "it is permitted (forbidden, obligatory)" is sometimes, in ordinary parlance, continued "that ...". Equally often, however, or maybe more often, it is continued "to ...".

In "it is permitted to..." the place of the blank is taken by a *verb* (or verb-phrase) for (a category or type of) *action* or *activity*. For example: "to smoke" or "to walk on the grass".

As mentioned above (p.6), in my first effort to build a deontic logic I regarded the variables in the deontic formulas as standing for *names* of actions. This suggests yet a third reading of the deontic operators. By names of action one could understand nouns such as "smoking" or "trespassing". On this conception, *Pp* might be a schematic representation for "smoking permitted", and *Fp* for "trespassing prohibited". It is feasible to think, however, that such phrases can be translated into the idiom using verbs for actions. "Smoking (is) permitted" and "it (one) is permitted to smoke" seem to say very much the same thing.

The readings of the deontic operators with "that" and with "to" respectively may be said to answer to two different types of deontic logic. The one is a logic of that which ought to, may or must not *be*, and the second a logic of that which ought to, may or must not *be done*. To use a terminology which has become established in German, it is a difference between a deontic logic of the *Sein-Sollen* (*-Dürfen*) and the *Tun-Sollen* (*-Dürfen*) type. The Classical System was intended to be a logic of the *Tun-Sollen-*; the Standard System is by its very nature a logic of the *Sein-Sollen-*type. It follows from what has been said above that only a deontic logic of the second type *can* preserve a perfect analogy with modal logic.

It is problematic whether deontic sentences prefixing the operators to verbs of actions can be "translated" into sentences prefixing the operators to sentences. Consider, for example, the sentences "it is permitted to smoke" and "it is permitted that everyone smokes". It is doubtful whether they mean the same. Another rendering of the first sentence might be "everyone is permitted to smoke" – replacing the impersonal "it" by the universal quantifier "everyone". This sentence then says that permission is given to everyone. But the sentence "it is permitted that everyone smokes" seems to say that a certain state of affairs is allowed, *viz.* the one when everyone is (maybe at the same time) smoking. This is something different from permission given to everyone.

There is of course no objection to thinking that the variables *p*, *q*, *etc.* of the Standard System represent sentences describing actions ("action sentences"). But this by itself, as seen from the above considerations, does not mean that the *Sein-Sollen* nature of the Standard System could capture and do justice to the *Tun-Sollen* logic which the Classical System intended to formalize. As a formal system, the Classical System is much poorer than the Standard System. But from the point of view of intended content the former aims at embracing something which seems out of reach of the latter.

In the third part of the present paper I shall try to sketch a new type of deontic logic which I hope will do justice to the intentions implicit in my first venture into the subject. But first we must say something more about action.

II. ON ACTION SENTENCES AND THE LOGIC OF ACTION

1. As the basic type of action sentence one may regard one which says that an agent *a* on an occasion *o* does a certain thing *p*. The content of such a sentence can often be viewed, alternatively, under two aspects. I shall call them the aspect of *achievement* and the aspect of *process*. The two aspects are related, loosely, to the ideas of *making* and *doing* respectively. They are also related to the distinction between *act(ion)* and *activity*.

a on *o* opens a door, say. By his activity he achieves the opening of the door and it is, at least for a short time, open. What he thus achieves is the *result* of his action in opening the door. He makes the result come about, happen. (Whether we think of the result as the *event* of the door's opening or as the *state* of its being open is, for present purposes, immaterial.)

The connection betweeen an action and its result is *intrinsic*. Had the door not opened, the agent would not have opened the door; this is "logically true".

The opening of the door makes a creak and the noise wakes a sleeping child, say. These effects of the result of the action are also called *consequences* of the action. The connection between an action and its consequences is *extrinsic*.

The phrase "the opening of the door" is ambiguous. The process denoted can be *the door's opening*, the fact that the door opens. This can be the achieved result of an action. But it may also come about independently of action, as when the wind blows the door open. The process, however, can also be *the agent's opening the door*. This consists, for example, in his seizing the handle, pressing it down, and pushing. These are *bodily movements* and *muscular activity* displayed by the agent on the occasion of his acting. I shall call them, for short, *bodily activity*. By "the action process" I shall mean the bodily activity involved in the performance of the action.

Every action which can be viewed under the aspect of achievement also presents an aspect of process. But some actions seem to consist solely in bodily activity, for example running or walking. They need not "result" in anything, produce any state in the world which remains, at least for a short time, once the activity has ceased.

It may be argued, however, that also such "pure activities" can be viewed under an aspect of achievement. The activity of running, for example, manifests itself in the transportation of a human body through a stretch of space. That a body was thus transported is an achievement which may be said to have resulted from the activity. Moving a limb, *e.g.* raising one's arm, results in a change of position of the limb in question. In the activity of moving a limb "back and forth" this position is restored but the (repeated) transportation of the limb through a stretch of space has been achieved. These observations seem to support a view according to which "pure activity" also presents an aspect of achievement and result. But whether this is a correct view or not I shall not try to decide here.

I shall use the symbol $[p]$ (a, o) as a schematic representation of the sentence that a certain agent on a certain occasion does a certain thing. The symbol "p" will have a different significance, depending upon whether we view what is being said in the sentence under the aspect of achievement or under that of process. When the sentence is viewed under the aspect of achievement, "p" is a schematic representation of a *sentence* describing either some state of affairs or some event, for example that the door is open or that it is opening. When again the sentence is viewed under the aspect of process, "p" is a schematic representation of a *verb* or verb-phrase denoting some type of action or activity, for example door-opening.

Adopting the achievement point of view, the schema $[p]\,(a, o)$ may be read "a on o makes (it so) that p". Adopting the process point of view, the reading could be "a on o is p'ing".

It should be noted that the sentence represented by "p" in our schema does not express a true or false proposition, but describes something which I propose to call a *generic* state or event. A generic state (event) is one which may or may not obtain (occur) on a given occasion o. That the state, *e.g.* that this door is open, obtains on the occasion o is a true or false proposition.

2. The meaning of $\sim [p]\,(a, o)$ is obvious, both on the achievement and the process view of action sentences. If, for example, $[p]\,(a, o)$ says that a on o opens the door, then $\sim [p]\,(a, o)$ says that a on o does *not* do this. On the achievement view this means that the agent does not make the door open – and on the process view that he does not engage in the bodily activity of door-opening.

Negation, however, need not be of the whole schema. It can also be of the part "p" in it. If "p" stands for "the door is open" (or for "the door is opening"), then "$\sim p$" stands for "the door is not open(ing)", *i.e.* it says that the door is (stays) closed. This is entirely obvious. If, however, "p" stands for "opening the door", how shall we then understand "$\sim p$" or the phrase "not-opening the door"? This is not immediately clear since the phrase can hardly be said to have a settled place within ordinary usage. We need not, however, reject it as meaningless. It can be understood as another, somewhat "primitive", way of saying that the agent in question abstains from or *omits* opening the door. As far as he is concerned, he leaves the door closed, lets it remain closed.

One could say that, on the achievement conception, the symbolic form $[\sim p](a, o)$ signifies the un-doing of a certain existing state of affairs or its suppression on an occasion when otherwise it would come to be, and that, on the process conception, $[\sim p](a, o)$ signifies omission to engage in a certain activity.

As seen, it is possible to give a sense to the negation-sign when it stands in front of a verb or verb-phrase. It is obvious that "and" and "or" may be used for joining verbs (for example, "read and write", "read or write"). Junctors applied to sentences expressing true or false propositions, junctors applied to sentences describing generic states or events, and junctors applied to verbs or verb-phrases should be distinguished from one another. It cannot be taken for granted that they all behave in the same way logically. But we shall nevertheless use here *the same symbols* for the three kinds of junctor.

3. The notion of *omission* of action is notoriously tricky. Omission is a non-action – and yet it is, at the same time, a "mode of action or of conduct". It is something for which an agent can be held responsible. Omissions are *imputed* to agents. A logic of action, clearly, has to take this into account and treat omission as something different from mere *not* doing something.

It may be suggested that omission could be defined in terms of not doing and the notions of *ability* ("can do") and *opportunity*. On this view, a omitted to do a certain thing on o if he could have done this thing but did not do it. The expression of this view in a symbolic language requires some kind of "modal operator". If $M[p](a, o)$ means that a can do p on o, then $M[p](a, o)$ & $\sim [p](a, o)$ says that a on o omits doing it.

The notion of omission thus defined may be called "omission in the widest sense". In ordinary language, the word "omission" would hardly be used for a good many omissions in this widest sense. On most occasions there are innumerable things which I could do then but which I do not do simply because it does not occur to me to do them. (They do not fall within what may be called my "horizon of intentionality" on o.) Normally we should not say that I omitted to do these things then. But if I had a *reason* for doing or was *expected* to do some of the things, for example because it was my duty or because I had promised to, then we may say that I omitted what I did not do. Such omission is often called *neglect*. If again, upon deliberation, I decide not to do some action, we call the omission *forbearance*, sometimes also *refrainment* or *abstention*.

4. For sentences of the schematic type $[p](a,o)$ one can build a calculus, "Logic of Action". Such calculus is "based" on ordinary propositional logic (PL) in the sense that all tautologies of PL are theorems of the calculus when action sentences are substituted for the variables (in the formulas of PL). The inference rules are those of PL, *i.e.* Substitution and Detachment, *and no others.*[5]

When action sentences are being viewed under the aspect of process the following four principles intuitively seem valid:

A1. $[\sim p](a,o) \rightarrow \sim [p](a,o)$

A2. $[\sim\sim p](a,o) \leftrightarrow [p](a,o)$

A3. $[p\&q](a,o) \leftrightarrow [p](a,o)\ \&\ [q](a,o)$

A4. $[\sim(p\&q)](a,o) \leftrightarrow [p\&\sim q](a,o) \vee [\sim p\&q](a,o) \vee [\sim p\&\sim q](a,o)$

The question may be raised whether one could not replace the fourth axiom by a weaker distribution principle to the effect that $[\sim(p\&q)](a,o) \leftrightarrow [\sim p](a,o) \vee [\sim q](a,o)$. But consider what it means that an agent *omits* engaging in two different activities on one and the same occasion. The answer most in agreement with intuition seems to be that this is to omit engaging in both or omit engaging in one of them *while engaging in the other*. For example: What is to it to omit (abstain from) reading-and-writing? The best answer seems to be that one reads but neglects to write or writes but neglects to read or neglects the one as well as the other. Why not simply say that it is to omit at least one of the two activities? One could say this – but it may strike one as "unnatural". For it would mean that if an agent on some occasion omitted to read, which, say, he was expected to be doing, then one could say *a fortiori* that he omitted to read-and-write although perhaps he was not expected to be writing or he cannot write or could not have written on that occasion. We need not try to decide which one of the two views on the nature of a "conjunctive omission" is the right one. If one takes the more restrictive view which also strikes me as the "natural" one, then one would have to accept for action sentences when viewed under the aspect of process the principle A4 above.

We may define the notion of a "disjunctive activity" as follows:

$$[p \vee q](a,o) =_{\mathrm{df}} [\sim(\sim p \& \sim q)]\,(a,o)$$

An agent is engaged in the disjunctive activity of, say, reading or writing, if, and only if, he omits the conjunctive omission of both.[6] By virtue of A4 and A2 this is equivalent to saying that the agent engages in both or engages in the one while omitting to engage in the other.

The calculus of action sentences with the axioms A1–A4 is decidable and semantically complete.[7] Every formula of the calculus may be shown to be provably equivalent with a formula which is a truth-functional compound of "constituents" of the simple types $[\quad](a,o)$ or $[\sim\quad](a,o)$ where the place of the blank is held by a single variable *p*, *q*, *etc.* Truth-values may be distributed over the constituents in a truth-table subject to the sole restriction that constituents $[\quad](a,o)$ and $[\sim\quad](a,o)$ of the *same* variable cannot both be given the value "true". This is a simple consequence of A1 or of the truth that one and the same agent cannot on one and the same occasion both commit and omit the same thing. If, observing this restriction, a formula gets the value "true" for all distributions of truth-values over its constituents, it will be said to be an action(-sentence) *tautology*. All such tautologies are provable in the calculus and provable formulas of the calculus are action(-sentence) tautologies.

We may introduce quantification into our action logic. This can happen in steps. We can quantify the sentences with regard to agents and let the sentences refer to the same (arbitrary) occasion *o*; or we can quantify them with regard to the occasion and let the sentences refer to one and the same (arbitrary) agent *a*; or we can combine these two modes of quantification. Finally, the calculus may also become quantified in the "proposition-like" variables *p*, *q*, *etc.*

For action sentences when viewed under the aspect of achievement one can also build a logic. This will have to have a somewhat more complex structure than the above "logic of action as process". In its fully developed form the variables *p*, *q*, *etc.* would stand not only for sentences describing results of action but also for sentences describing states which are, or are not, transformed through the action. Only then can one, for example, express in the formal system the important distinction between *productive* and *preventive* action.[8]

5. One can distinguish between act-*categories* or *generic* actions, such as door-opening, murder, or smoking and act-*individuals* or *individual* actions, such as for example the murder of Caesar by Brutus.

Opinions differ on the question whether the deontic attributes *primarily* apply to generic or to individual actions. If one takes the view that they apply primarily to act-individuals, then the question will arise: Do they apply as operators to action-sentences or are they genuine attributes or properties of some individual things ("logical individuals")? Those deontic logicians who have opted for the second alternative in answering the first question have almost invariably opted for the first alternative in answering the second.

The question whether one can make good sense of the conception of actions as logical individuals is not uncontroversial.

Consider the schematic form of a sentence describing an individual action "*a* on *o* does *p*". (The action could be, for example, that of opening a door.) One cannot individuate this as "the action performed by *a* on *o*". It is logically possible to do more than one action on one and the same occasion.[9] From the name of an individual action it must also be plain *what* it was that *a* did on *o*, *i.e.* we must mention a generic characteristic of the action. The phrase "the opening of the door by *a* on *o*" names an individual action or, in pure schematic form, "the doing of *p* by *a* on *o*".

Individual actions have various properties (attributes, features). The individual action of *a*'s opening a door on *o* has the "property" of (being a case of) door-opening. This is trivial. But when an action is being individuated or identified as an act of a certain category or kind the question will sometimes arise whether it may not also be classified as an action of a certain other category or kind. This question is often a preliminary to evaluating the action or to qualifying it deontically. For example: A child has been ordered to stay at home for the afternoon studying, perhaps as a punishment for a minor offense. It stays at home reading a book. Is this studying? If the child in reading was doing its homework for the school, its activity would probably count as studying. If the reading was of a novel, the child's activity would probably not count as studying. A person spat. Was what he did perhaps an act of insulting somebody? Was the killing of *b* by *a* a case of murder? When such questions are considered and decided, properties are in a nontrivial sense being attributed to individual actions.

The property which, as we said above, "trivially" belongs to an individual action I shall also call its *essential* property or characteristic. It is the property which we use for *identifying* ("picking out") the act-individual under consideration. Some of the properties which an action may have in addition to its essential characteristic belong to it by virtue of *the way in which* the action was performed.[10] Suppose that the agent opens a door by pressing a button and pulling. Then his action, *viz.* his action of opening the door, is also a case of button-pressing and of pulling. It has these two additional characteristics.

Further additional properties may belong to an action by virtue of its consequences. Let us assume that the agent by opening a door lets cool air into the room. His action is thus also one of cooling the room. It has the property of being a "room-cooling action".

A non-essential property of an action is not necessarily either a causal prerequisite or consequence of its performance. Suppose *a* on *o* is waving his arms. In doing so he might be giving a signal. His action is thus also a case of signalling. It is this because there is a convention giving a "meaning" to the arm-waving.

The two events of a button sinking down and a room getting cool are different events from the event of a door opening. But the event of a pair of arms moving in a certain way is not a different event from the appearance of a certain signal.

Unintended consequences of an action may also constitute properties of the action. The agent who lets cool air into the room by opening the door may, as a consequence, catch a cold. His action is then a cold-giving action.

Which property of a given individual action is singled out as essentially belonging to it, is to a large extent a matter of choice. The choice may depend on our *interest* in the action, on what is *important* about it. The person who opens a window may "primarily" be ventilating a room. The essential property of his action is then that it is a case of room-ventilation. But because of the way the action was done, it was also an action of opening a window – and because of its consequences perhaps also an action of making a person sneeze.

The (causal) consequences of an action will normally materialize some time after the action was performed. At the time of performance it may therefore not be clear (known) which (all) the consequences will be. *a* runs over *b* with his car in the street. *b* is badly injured and dies soon after. Medical expertize attributes the death of *b* to the injury. It may be a matter of decision for a court whether *a*'s action of running over *b* should be deemed the *cause* of *b*'s death. But if it is thus regarded, *a* can correctly be said to have killed *b*; *a*'s action of running over *b* was a case of killing a man.

If the running over is regarded as the essential property of *a*'s action under consideration, then this action can be said to acquire a property, *viz.* that of being a killing, which initially it did not possess. If again the killing is regarded as the essential property, then this action may be said initially to have consisted in (causing) a car accident.

Is *a*'s running over *b* and *a*'s killing *b* one and the same action? The result of the action of running over *b* is the event that *b* gets under a car, and the result of the action of killing *b* is that *b* dies. Getting under a car and dying are two different events (even if they take place at the same time). But *a*'s action of running over *b* and his action of killing *b* are one and the same action. Some philosophers would say that they are one and the same action "falling under different descriptions".

At the generic level, *i.e.* as act categories, running over and killing, door-opening and room-ventilation, *etc.* are, of course, *different* (types of) *actions.*

Could we not say, therefore, that *e.g.* the person who ran over a man in the street thereby causing his (later) death performed *two* actions? We can do this if thereby we mean that his (one) individual action on a certain occasion exemplifies two (or more) generic actions. But it is important to distinguish this from the case when a man actually performs two individual actions on one and the same occasion, for example, opens a window and closes a door. Even if those two actions take place simultaneously and not successively they would be two different *individual* actions.

Once it is accepted that actions may be regarded as logical individuals there seem to exist no obstacles of a conceptual nature to regarding also deontic attributes as properties of individual actions. One such property would be permittedness. Actions of a certain category are, let us assume, permitted. Then the performance of an action of this category by a certain agent on a certain occasion may (but need not) have been a permitted individual action. (Cf. below p. 24f.)

Not all properties of individual actions, it seems, mark generic actions. Let it be granted that deontic status, *e.g.*, permittedness can be a genuine property of an individual action. This seems plausible. But it does not seem plausible to say that there is a generic action "doing the permitted". One cannot *identify* an individual action as being a case of doing the permitted. It must be identified as a case of doing such and such, *the doing of which is permitted.* Since permittedness cannot be used for identifying individual actions, it cannot be an essential property of an action either. Essential properties can only be those which name act-categories.[11]

As noted above (p. 15f), an individual action may be identified, now by one, now by another essential property. It may, for example, be identified as a case of flipping a switch or as a case if illuminating a room or as a case of alerting a prowler who was about to enter the room – to allude to a famous example from the literature in the philosophy of action.[12] Depending upon which property is used for identifying the action, the set of its properties is differently divided into a subset of prerequisites and consequences. If we identify the action as one of illuminating a room, then its being prowler-alerting is a "consequential" property of the action. If again we identify it, as the prowler himself may do, as a prowler-alerting action, then both its being a room-illuminating and a switch-flipping action are accidental properties which belong to the action as its causal prerequisites.

If "two" actions have all their properties in common but different essential properties, are they then "the same" action, or not? It seems to me that we are free to mould our criteria of identity so as to answer the question either by Yes or by No. But I should prefer to answer Yes, and I have a surmise that those who prefer to say No are misled by the fact that the *individual* action under consideration exemplifies several (different) *generic* actions.

6. Omissions too can be individuated and treated as logical individuals. The individuation of an omission is the identification (labelling) of the conduct of an agent on a certain occasion as an omission to do a certain thing. How is such an identification done? We may verify that *a* on *o* did *not* do *p*. The occasion was one when one could have done *p*; the occasion in other words provided an opportunity for doing the action. We know, *e.g.* from previous experience, that *a* can perform actions of the kind in question, that he has the required ability. If these are established facts, then it is also established that he omitted to do *p* – in the widest (weakest) sense of "omission" (above p. 12). We can now speak of "the omitting by *a* on *o* to do *p*" as of a logical individual. If "omission" is not understood in the weakest sense but in some stronger sense, such as not doing what one is expected or has a duty to do (above p. 13), then these additional criteria too will have to be taken into consideration in determining whether the agent should be said to have omitted this or that action on such and such occasion.

An omission of an action is usually "constituted" by the performance of some other action. For example, an agent is engaged in reading and thereby omits turning off the tap from which water is pouring into the bath-tub. As a consequence there is a flood in the bathroom, let us assume. We do not say that by reading he flooded the bathroom. But by omitting to turn off the tap he did so.

The agent's omission to turn off the tap does not "consist" in the *tap* being on (and the water pouring into the bath-tub). It "consists" in *his* reading in combination with the fact that he *could* have turned off the tap on that occasion – maybe even had a reason or was expected to do so ("instead of reading" as we should say).

There is a sense in which omissions can be called causes. What this means is that something, *e.g.* the tap remaining on, that happens because something else is omitted, *e.g.* the tap being turned off, causes a third thing, *e.g.* a flood, to take place.

An omission may have a less definite dating than a "corresponding" action. But in principle actions and omissions are on a par in this regard. The window was closed and *a* opened it at 11 : 15 a.m. On another occasion, the window remained closed the whole morning and *a* did not open it although he was there and could have opened the window, maybe even was expected to do so. The occasion for his omission to open the window was that (whole) morning.

It does not follow that the agent's opening, say, a door at 11 : 15 is identical with his omission that whole morning to open the window in the room. But his opening the door at 11 : 15 also constituted an omission to open the window at 11 : 15. And this "bit" of his failure to open the window in the course of the entire morning is, as an action individual, identical with his opening the door then.

Omissions can have further properties in addition to being omissions to do so and so. Someone stands by and sees another person drown. The first could have saved the second but omitted to do so. By his omission he became responsible for the death of a person. Depending upon the circumstances, a court may even pronounce his omission a case of murder.

The question may be raised whether an omission must always be "constituted" on the basis of some *other action* which the agent performs. Perhaps it is usually the case that one omits to do something because one is engaged in doing something else. But I do not think that it must be so. An agent need not do anything at all on a given occasion, he may stay completely passive. Then his passivity is omission to do every one of the things which he is able to do and for the doing of which the occasion of his passivity affords an opportunity.

7. The Logic of Action which was described in outline in Sect. 3 is a logic of *action-sentences* of the schematic prototype form "*a* on *o* does *p*". The actions described in such sentences, we have seen, may be regarded as logical individuals, the prototype name form of which is "the doing of *p* by *a*

on *o*". Under this conception of actions we get yet another type of Logic of Action. Its objects of study are sentences attributing properties to individual acts. The prototype form of such sentences is "the doing of *p* by *a* on *o* is *A*" where "is *A*" is a schematic representation of such phrases as, for example, "is (a case of) murder" or "is ventilating a room". One can, if one wishes, call such sentences action-sentences too. But then it should be remembered that, unlike the above prototype form of such sentences, they do not say that something or other is being done, but that something or other which was done has a certain characteristic or property.

Similarly, we shall have to count with sentences attributing properties to individual omissions. "The omission of *p* by *a* on *o* is *A*" might say that *a*'s omission to save a person from drowning was, on that occasion, a case of murder.

I shall use *x*, *y*, *etc.* as variables for individual actions or omissions and *A*, *B*, *etc.* as schematic representations for names of properties. Names of properties will also be called *predicates.*

The logic of the sentences now under consideration could be regarded as simply a fragment of "classical" (monadic) predicate logic and quantification theory. Then it is of no independent interest as a "logic of action".

There is, however, good reason for studying sentences attributing properties to individual actions within a more "refined" calculus than the traditional predicate calculus. I have elsewhere described this more refined calculus and coined for it the name Logic of Predication.[13] Its characteristic feature is that it allows us to make a distinction between denying that an individual has a certain property and affirming that it lacks a property. The distinction, in other words, is between two kinds of negation, an external negation which is of sentences (propositions) and an internal negation which is of properties. To use Aristotle's example,[14] between something not being white and something being not-white.

What then is the difference between (simply) not having a property and lacking a property? Roughly speaking: the things which lack a given property fall within the "range" of that property: they *could* have the property in question although in fact they have not got it; things outside the range neither have nor lack the property in question.

This is a rough characterization only and its application to specific properties is, often at least, a matter of decision. It is for us to give a meaning to the distinction in question. This, however, can often be done in a way which seems both enlightening and natural. Thus, for example, that an action is not permitted can be taken to mean that it is forbidden, or it can mean that it simply has no deontic status at all. What is forbidden "lacks" permittedness; an action void of deontic status neither has nor lacks permittedness.

If there are several ways of doing an action and the action is performed in one of the ways to the exclusion of the other, then it lacks the characteristic of being an action of the second kind. For example: Let it be that one can open a door either by pressing a button or by turning a key. Then an individual act of opening this door may (accidentally) have the property of being an act of button-pressing and lack the property, which it could have possessed, of being an act of key-turning.

When an action lacks a property which an action performed by that agent on that occasion could have had, it is, normally, "constitutive" (above p. 18) of an omission. If a child is reading a novel instead of the text he is supposed to be studying (and which he could have read then), his action of reading also constitutes an *omission* of his to study and can therefore be said to lack the property of being a case of studying.

8. The same device as before, square brackets [ ], will enable us to mark the distinction between not having a property and lacking it. Thus $\sim [A]x$ says that x is *not* A, and $[\sim A]x$ says that x is not-A. The axioms of a Logic of Predication are, with minor notational difference, the same as those of our above Logic of Action. One can debate whether a weaker version (cf. above p. 13) of A4 is valid for predications. Ordinary usage of the negation words is hardly settled, so the answer to the question is a matter of decision. On the whole it seems to me more natural to opt for the strong version. This would mean that a thing is said to lack the conjunction of two properties if, and only if, it belongs in the range of both but has at most one of the two. The axioms are then:

A1. $[\sim A]x \rightarrow \sim [A]x$
A2. $[\sim \sim A]x \leftrightarrow [A]x$
A3. $[A \& B]x \leftrightarrow [A]x \& [B]x$
A4. $[\sim (A \& B)]x \leftrightarrow [A \& \sim B]x \vee [\sim A \& B]x \vee [\sim A \& \sim B]x$

We can now also define the notion of a "disjunctive property": $[A \vee B]x =_{df} [\sim (\sim A \& \sim B)]x$. By virtue of A4 and A2 it follows that, for example, something has the property "red or round" if it has the one but lacks the other or has both the "simple" properties. But if "red" denotes the colour and "prime" a characteristic of some numbers, then nothing has the property "red or prime". There simply is no such disjunctive property because the range of things which are possibly red and of those which are possibly prime (numbers) have no common member.

The rules of inference are the usual ones of Substitution and Detachment. For quantified sentences one would have two additional axioms:

A5. $(Ex)([A]x \vee [B]x) \leftrightarrow (Ex)[A]x \vee (Ex)[B]x$

A6. $\sim (Ex)([A]x \,\&\, \sim [A]x)$

and an additional inference rule (Leibniz's Law) to the effect that formulas which are provably equivalent on the basis of A1–A4 are interchangeable *salva veritate* in quantified formulas.

III. DEONTIC LOGIC – A NEW APPROACH

1. Let it be agreed that deontic status can, in the genuine sense, be predicated of *individual* actions. I shall use "F" for the property of being forbidden, "O" for that of obligatoriness, and "P" for permittedness.[15] "$[F]x$" may be read "x is forbidden". "$\sim [F]x$" says that x is not forbidden, and "$[\sim F]x$" that x is not-forbidden, that it lacks the property of being forbidden. In what way the second is a stronger statement than the first will be discussed presently.

Undeniably, deontic status is often also attributed to generic actions or categories of action. I shall use the letters "$\mathscr{F}$" to stand for "forbidden", "$\mathscr{O}$" for "obligatory", and "$\mathscr{P}$" for "permitted" when this kind of attribution of deontic status is in question. Under this use, the deontic words are not predicates, but operators. About the difference more will have to be said later.

Let "$[A]x$" say, for example, that x, an individual action, is a case of murder. "$\mathscr{F}A$" then says that murder is forbidden or that it is forbidden to (commit) murder. The *kind of action* called "murder" is forbidden.

The expressions formed by deontic operators followed by a predicate or a molecular compound of predicates denoting generic actions may be used to express *norms* (of action). Norms are given to agents acting on certain occasions. Norms can be either for named individual agents, or for agents of a certain category, or for agents unrestrictedly. Analogously, they can be either for specified individual occasions, or for occasions satisfying certain conditions, or for occasions unrestrictedly (which provide opportunities for doing the actions in question).

2. The attribution of deontic status to individual actions will be called *deontic predication.*

What does it mean that an individual action x is a forbidden action? As was indicated above (p. 17), one cannot pronounce an individual action forbidden unless one has first identified it as an action of a certain category or kind. Assume that A is the essential property used for identifying the action. If there is a norm $\mathscr{F}A$ prohibiting actions of this kind, then x is (was) a forbidden action. Let us think, however, that there is no such norm. It does not follow that the action then is not forbidden. For x may possess some other property beside A, say B, such that actions of *that* category are forbidden. Then, obviously, x was a forbidden action (to the agent who on some occasion did it).

We can now define what it is for an individual action x to be a forbidden action, as follows: $[F]x =_{\mathrm{df}} (EX)([X]x \& \mathscr{F}X)$. In short: an action is forbidden if, and only if, it falls under some forbidden category of action. Or, in other words: an agent's action on some occasion is forbidden if, and only if, in performing this action he does something forbidden.

The commission of an individual action is obligatory if, and only if, the action is of a kind such that it is forbidden to omit actions of this kind. Conversely, the omission of an individual action is obligatory if, and only if, the omission is of a kind such that it is forbidden to commit actions of this kind.[16]

This interrelatedness between obligation and prohibition in the terms of commission and omission of actions calls for some further comments.

Consider the following example. The agent a enters a garden on an occasion o. The action is thus the entering of the garden by a on o. This action, let us assume, can be performed in three different ways. One can enter the garden either through one of two gates, g_1 and g_2, or by jumping the fence surrounding it (which is low). It is, however, forbidden to jump the fence. (There are flowers at the foot of the fence.) The agent entered the garden through g_1. His action of entering the garden was thus also a case of passing through g_1. If $x =$ the entering of the garden by a on o, and A is the "property" of being a passing through g_1, then the action x is A. The action *could* have been a passing through g_2 (B) or a jumping the fence (C). But it *lacked* (p. 20) these two properties. In our symbolism: $[\sim B]x$ and $[\sim C]x$. By virtue of lacking the property C, the action x also constitutes (p. 21) an omission on the part of a to jump the fence on o. If the fact that the action x lacked the property C is considered sufficient ground for saying that a omitted to jump the fence on o, then this *omission* on a's part was obligatory.

His action, what he *did*, was not, as such, obligatory. But in entering the garden a behaved *in accordance with duty* since he passed through the gate and observed the prohibition to enter by jumping the fence.

Obligatory omissions of action, *i.e.* the observance of prohibitions, could also be called "*negative*" obligations.

Assume next that our agent had been commanded to enter the garden and to do so through gate g_1. He was, in other words, not only forbidden to jump the fence but also to pass through g_2. Then his individual action x was an obligatory action by virtue of the fact that it had the property A, *viz.* that of being a case of passing through g_1. Had it lacked this property, it would have been a forbidden action.

Obligatory commissions of action might also be called "*positive*" obligations. Positive obligations often have the character of fulfilling *commands* (orders, imperatives).

As seen the *predicates* "forbidden" and "obligatory" can both be defined in terms of the operator "forbidden". The *operator* "obligatory" again can be defined in terms of the operator "forbidden", thus:

$$\mathcal{O}X =_{\text{df}} \mathcal{F} \sim X.$$

The two operators are interdefinable. This is in agreement with the "traditional" view of the matter.

Neither "in logic" nor "in real life" is there anything to prevent one and the same individual action (or omission) from being both obligatory and forbidden. If Jephthah had sacrificed his daughter, his action would have been obligatory because it was the fulfilment of a promise to the Lord, and forbidden because it was homicide. $[F]x$ & $[O]x$ is not a contradiction. It says that the individual action x is of a kind which is forbidden but also of a kind which is obligatory. It is forbidden by virtue of one of its characteristics and obligatory by virtue of another.

The *predicates* "forbidden" and "obligatory", be it observed, are *not* interdefinable. This is a simple consequence of the fact that individuals cannot be "negated". "$[F] \sim x$" and "$[O] \sim x$" are meaningless signs.

3. To deny that an individual action is forbidden is to affirm that it does not fall under any kind of action which is forbidden or, in other words, that *all* its features signify not-forbidden properties (of individual actions). In symbols:

$$\sim [F]x \leftrightarrow \sim (EX)([X]x \& \mathcal{F}X) \leftrightarrow (X)([X]x \rightarrow \sim \mathcal{F}X).$$

Shall we say that an action which is not forbidden is thereby permitted? This is an aspect of the much debated question whether permission is anything "over and above" the absence of prohibition. I think we are well advised to distinguish between things being permitted in the weak sense of simply not being forbidden and things being permitted in some stronger sense. Exactly in what this stronger sense "consists" may be difficult to tell. That which is in the strong sense permitted is, somehow, expressly permitted, subject to norm – and not just void of deontic status altogether.

The *predicate* "strongly permitted" we can define as follows: $[P]x =_{\text{df}} (X)([X]x \rightarrow \sim\mathscr{F}X) \;\&\; (EX)([X]x \;\&\; \mathscr{P}X)$. The (in the strong sense) permitted individual action does not fall under any forbidden kind of action but falls under at least one (in the strong sense) permitted one. This definition, of course, does not say anything about the meaning of the (strong) permission-*operator*.

As easily seen from the above, we have $[P]x \rightarrow \sim[F]x$.

4. Nothing has so far been said to give meaning to the *lack* of the properties F, O, and P. An action x of which it is true that $[F]x$ falls under some norm prohibiting a certain kind of action. An action x for which it is true that $\sim[F]x$ *need not* fall under any norm at all. But it *may* fall under a permissive or obligating norm. And similarly for the expressions $[O]x$ and $[P]x$ and their negations.

The following suggestions therefore appear natural: That an action *lacks* the property of being forbidden means that it is *not* forbidden but *is* either obligatory or (in the strong sense) permitted. That an action *lacks* the property of being obligatory means that it is *not* obligatory but *is* either permitted or forbidden. That an action *lacks* the property of being permitted, finally, means that it is *not* permitted but *is* either forbidden or obligatory. Thus we have the following three identities:

$$[\sim F]x =_{\text{df}} \sim[F]x \& ([O]x \vee [P]x)$$
$$[\sim O]x =_{\text{df}} \sim[O]x \& ([P]x \vee [F]x)$$
$$[\sim P]x =_{\text{df}} \sim[P]x \& ([F]x \vee [O]x)$$

The identities imply equivalences which may then be distributed into conjunctions of (two) implications. Since, in the Logic of Predication, lack of a property in a thing entails that the thing in question has not got this property, we also have the following relations: $[\sim F]x \rightarrow [O]x \vee [P]x$ and $[\sim O]x \rightarrow [P]x \vee [F]x$ and $[\sim P]x \rightarrow [F]x \vee [O]x$. Moreover, since we already proved $[P]x \to \sim [F]x$ we can now from the first of the above three identities derive the stronger formula $[P]x \to [\sim F]x$.

5. I have thus taken the view which seems to be reasonable, that when deontic status is predicated of an individual action, this predication is grounded in the deontic status of some category or kind of action under which this individual action falls. As a consequence, we have to regard the deontic predicates as *secondary* to the deontic operators.

By *normative sentences* I shall understand expressions of the forms $\mathcal{O}-$, $\mathcal{F}-$, and $\mathcal{P}-$ and their molecular compounds, the place of "–" being taken by an atomic or molecular predicate (of actions).

Normative sentences will be called *norm-formulations*. A characteristic use of them is for giving (issuing, laying down) norms or rules for human agents. When this use is in question, the normative sentences may be said to *express norms* (cf. above, p. 22).

Normative sentences, however, can also be used for making statements to the effect that there are (have been given or issued) such and such norms or rules. When used in this way, normative sentences express what I propose to call *norm-propositions*.

This ambiguity of usage is a very characteristic and important feature of atomic norm-formulations. Also molecular normative sentences can be used either for expressing norms or for expressing norm-propositions. But their use in the second way seems much more common.

It is a much discussed question whether a "deontic logic" is a logic of norms or of norm-propositions or maybe of both. My own position on this question has been wavering. It is probably a right characterization to say that I have more and more tended to think of deontic logic as a logic of propositions to the effect that *there are* (in some normative code or order or system) such and such norms.

My own efforts in the past were to a large extent to build axiomatic systems of (different) deontic logics. A few principles which seemed intuitively palatable were chosen as axioms and part of the interest in the choice was to study what did, or did not, follow logically from them.

I now prefer a very different approach. Its nature can best be seen from considering a few examples concerning possible relationships between norm-propositions.

Let there be a prohibition with a disjunctive content (action), $\mathcal{F}(A \vee B)$. Individual actions with the generic characteristic "$A \vee B$" are actions which either have both the characteristics or have the one but lack the other.

$[A \vee B]x \leftrightarrow [A \& B]x \vee [A \& \sim B]x \vee [\sim A \& B]x$ is a logical truth in the Logic of Predication. This fact will also be expressed by saying that the predicates $A \vee B$ and $A \& B \vee A \& \sim B \vee \sim A \& B$ are logically equivalent (predicates). We shall lay down the following

Principle of Deontic Equivalence: Logically equivalent predicates are intersubstitutable (*salva veritate*) in norm-sentences (expressing norm-propositions).

By virtue of this principle we may regard $\mathscr{F}(A \vee B) \leftrightarrow \mathscr{F}(A \& B \vee A \& \sim B \vee \sim A \& B)$ as a "truth of deontic logic". Now consider the following: Any individual action by the performing of which an agent may observe or violate this prohibition is an action which can have or lack the characteristic A *and* can have or lack the characteristic B. For this reason it appears natural to say that a prohibition of actions with a disjunctive characteristic is logically equivalent with a conjunction of prohibitions of actions of any one of the various kinds (the individual members of) which fall under the disjunctive kind. Thus we may regard as a logical truth about norms the formula

(1) $$\mathscr{F}(A \vee B) \leftrightarrow \mathscr{F}(A \& B) \& \mathscr{F}(A \& \sim B) \& \mathscr{F}(\sim A \& B).$$

Let there be a norm to the effect that actions of the category A are forbidden. Then an action x with the property A is a forbidden (individual) action: $\mathscr{F}A \& [A]x \rightarrow [F]x$. This can also be written: $\mathscr{F}A \rightarrow ([A]x \rightarrow [F]x)$. Since this holds for any arbitrary individual action, we also have $\mathscr{F}A \rightarrow (x)([A]x \rightarrow [F]x)$.

The first implication in the formulas, be it observed, does not hold in the reverse direction. It might be the case that all actions of the category A actually are forbidden actions, though not on the ground that they have the property A, but because every one of them happens to belong to *some* (not necessarily the same) forbidden kind of action. We are thus not suggesting, which would be quite wrong, that norms are logically equivalent with general statements about the deontic character of individual actions of certain kinds. (Deontic attributes of individual actions, be it remembered, were defined with the aid of deontic operators.)

From the fact that actions of a certain type are forbidden it thus follows that all individual actions of this type are forbidden; but from the fact that actions of a certain type are permitted it does not follow that all individual actions which are of this type are permitted individual actions. In doing something which is, "as such", permitted an agent may also be doing something which is forbidden. He may, for example, do what he does *in a manner* which is forbidden. Or his action may *lead to* something forbidden.

If a disjunctive action, for example reading or writing, is (generically) permitted, then any individual action which has both the permitted features and no forbidden feature is permitted; and so is also any individual action which has one of the permitted features, is lacking the other, and has no forbidden feature. This seems as good a ground as could possibly be required for holding that

$$(2) \qquad \mathscr{P}(A \vee B) \leftrightarrow \mathscr{P}(A \& B) \& \mathscr{P}(A \& \sim B) \& \mathscr{P}(\sim A \& B)$$

is a norm-logical truth.

We already noted that, if an individual action is permitted, then, by definition, it cannot be a forbidden action. (Above p. 25.) On this basis we may hold that a norm permitting actions of the type A excludes a norm prohibiting actions of that same type:

$$(3) \qquad \mathscr{P}A \rightarrow \sim \mathscr{F}A.$$

What shall we think about the principle $\sim(\mathscr{F}A \& \mathscr{F} \sim A)$ and its equivalent form $\sim(\mathscr{O}A \& \mathscr{O} \sim A)$?

First we must warn against a misunderstanding. $\mathscr{F}A \& \mathscr{F} \sim A$ would not have as a consequence that an agent could not perform anything but forbidden actions, so that whatever the poor man does he sins against the law (norms). Because not all individual actions need be such that they either have or lack the feature A. They may not fall within the range of actions of this kind at all. (An agent who does *not* do a certain thing omits doing it, we have said (p. 12), only if, on the occasion in question, he *could* have done it.) Why does nevertheless $\mathscr{F}A$ & $\mathscr{F} \sim A$ strike us as absurd? Obviously because, for reasons of logic, an agent who is in position to do an action of the type A will, whether he does it or not, commit an offense. Is this a *logical* impossibility? Rather than calling it a logical impossibility we should, I think, say that a normative order which happens to contain those two prohibitions is "irrational" – and that therefore the legislator should, "in the name of rationality", lift at least one of the two prohibitions or make them conditional upon different types of situation when they apply.

A "rational" normative order should therefore satisfy the principle

$$(4) \qquad \sim(\mathscr{F}A \& \mathscr{F} \sim A).$$

6. The formula $\mathscr{P}(A \& B) \rightarrow \mathscr{P}A$ cannot be accepted as generally valid in a sound logic of norms. From the permittedness, in the strong sense, of the kind $A \& B$ of actions, one cannot conclude to the permittedness, in the strong sense, of the kind A of actions. But, as we shall see presently (below p. 30), the permittedness of the kind $A \& B$ is "rationally incompatible" with the forbiddenness of the kind A (and the kind B). Hence one may from the permittedness, in the *strong* sense, of the kind $A \& B$ of actions conclude to the permittedness, in the *weak* sense, of the kind A.

What then of the distribution formula $\mathscr{O}(A \& B) \leftrightarrow \mathscr{O}A \& \mathscr{O}B$?

Consider first the implication from left to right. This clearly – although contrary to what might be expected from knowledge of "traditional" deontic logic – cannot be a truth of logic. From the fact that an agent is under an obligation to perform actions which exhibit two characteristics, it does not follow that he is under an obligation to perform actions which have (only) one of the characteristics. From the fact that he has been ordered to enter a garden through a certain gate, it does not follow that he ought to enter the garden regardless of how he does it. We need not think that he is forbidden to jump the fence. But if he finds the gate locked and cannot open it, then he cannot conclude "logically" that, because of the order given, he must now jump the fence – a feat which, let us assume, he could perform.

Consider next the implication from right to left. The agent has two obligations. It is not certain that he can satisfy both by performing just one individual action. But it may be that, on some occasions, it is possible to satisfy both obligations by doing just one thing *and in no other way*. Then clearly he is obligated to do this conjunctive action. If, however, he can, on that same occasion, do something which satisfies the one and something else which satisfies the other of his two obligations, then there is no need for him to do the *one* action which satisfies both.

Example: An agent has been asked to see to it that the window and the door in a certain room are shut. If he finds the window open, he ought to shut it – and similarly with the door. If he finds both open, he has to shut both. Perhaps he can do this by operating a mechanism, say by pressing a button. If that is the *only* way this can be done, he ought of course to press the button. But if one can also shut the window and the door separately he is not obligated to do this by pressing the button.

Thus there is not a relation of entailment either way between the conjunction $\mathscr{O}A \& \mathscr{O}B$ of two norms and the conjunctive obligation $\mathscr{O}(A \& B)$.

7. Let it be that $\mathscr{F}A$. In the Logic of Predication we prove $(x)([A\&B]x \rightarrow [A]x)$. This means that if an agent performs an action with the two characteristics A and B he will necessarily disobey the norm $\mathscr{F}A$. Must we not therefore say that if there is a prohibition to the effect that $\mathscr{F}A$ then there is also (implicitly at least) a prohibition to the effect that $\mathscr{F}(A\&B)$? So that $\mathscr{F}A \rightarrow \mathscr{F}(A\&B)$ would be a "law of deontic logic"?

I do not think that there is any clearcut answer Yes or No to this question. From the fact that there are individual actions of the kind A it does not follow that there are any of the kind $A\&B$. Maybe it is quite impossible, either for reasons of logic or for reasons of human ability, to perform actions with these two characteristics, although it is possible and maybe even easy to perform actions with either one of the characteristics. It seems silly then to say that actions of the (empty) kind $A\&B$ *are* forbidden, on the grounds that actions of the kind A are forbidden. Maybe the lawgiver would even permit actions of the kind $A\&B$, if they could be done. (Perhaps possessing the characteristic B would "make good" for the bad which actions with the characteristic A do and which motivated the prohibition $\mathscr{F}A$.)

Assume now that actions of the kind $A\&B$ actually are permitted. We thus have a norm $\mathscr{P}(A\&B)$. Must the lawgiver then repeal the norm $\mathscr{F}A$ if there was one before? If there are the two norms $\mathscr{P}(A\&B)$ and $\mathscr{F}A$, then no agent could avail himself of the permission without breaking the prohibition. This is logically true. And this fact would make it, if not "illogical" at least "irrational" to let permission $\mathscr{P}(A\&B)$ and prohibition $\mathscr{F}A$ co-exist within the same code of norms. A rational code should therefore satisfy the principle

(5) $$\mathscr{P}A\&(x)([A]x \rightarrow [B]x) \rightarrow \sim\mathscr{F}B.$$

Since $(x)([A]x \rightarrow [A]x)$ is logically true, it follows from (5) that $\mathscr{P}A \rightarrow \sim\mathscr{F}A$ or that what is (generically) permitted cannot be (generically) forbidden.

Thus the principle (3) which was already accepted as a "truth of deontic logic" is seen to be a consequence of a more general deontic principle to the effect, loosely speaking, that "what follows from the permitted cannot be forbidden".

In a similar manner it may be shown that the principle (4), one of the traditional corner stones of a deontic logic, is but a special case of a more general principle which, in the name of "rationality" if not in that of "logic", seems acceptable, *viz.*

(6) $$\mathscr{F}\sim A\&(x)([A]x \rightarrow [B]x) \rightarrow \sim\mathscr{F}B.$$

This principle says that if all (individual) actions with an obligatory characteristic also have another characteristic, then this other characteristic cannot be (generically) prohibited. Let it be observed, however, that this other characteristic need not itself be (generically) obligatory; it might even be the case that all actions which are *B* without also being *A* are forbidden actions.

If in (6) we for "*B*" substitute "*A*" we obtain $\mathscr{F} \sim A \& (x)([A]x \rightarrow [A]x) \rightarrow \sim \mathscr{F} A$ which reduces to $\mathscr{F} \sim A \rightarrow \sim \mathscr{F} A$ which is the same as $\sim (\mathscr{F} A \& \mathscr{F} \sim A)$ or $\sim (\mathscr{O} A \& \mathscr{O} \sim A)$.

8. The facts upon which I have here based my arguments for accepting and for not accepting certain statements about the relations between norms are *logical truths*. They are derived from the definitions of the deontic predicates with the aid of principles of the Logic of Predication. The accepted statements themselves we might call truths of (a) Deontic Logic. But I feel a certain hesitation calling them "logical truths" at all. The reason for this is that it seems to be a matter of extra-logical decision when we shall say that "there are" or "are not" such and such norms. Shall we, for example, say that "there is" a $\mathscr{F}$-norm with a disjunctive norm-content, if there are (have been given, issued) norms concerning all the various ways in which this disjunctive norm-content may be realized through action? Perhaps the norm with the disjunctive norm-content was never formulated or even thought of. Yet it was there "implicitly" one could say. Had the norm been given in the disjunctive formulation, it would have imposed exactly the same demands and granted exactly the same freedom to agents as the norms about the disjuncts would have done jointly.

The distribution principles (1) and (2) are conceptually on a somewhat different footing from the principles (3) and (4) and the more general principles (5) and (6) from which (3) and (4) may be derived. (1) and (2) are in a sense "linguistic", concern the way "and" and "or" are used when speaking of prohibitions and permissions. The principles which say that the permitted cannot also be forbidden or that prohibitions (obligations) with contradictory contents cannot co-exist are more of the nature of requirements of rational legislation than of strictly logical thinking.

Thanks to the distribution principles any molecular compound of norm-formulations can be split up into atomic constituents consisting of the letters "$\mathscr{F}$" and "$\mathscr{P}$" followed by atomic predicates or conjunctions of predicates and their negations. Over these constituents we can distribute truth-values subject to the two restrictions imposed by the principles (5) and

(6). For example: If there are two constituents $\mathscr{F}A$ and $\mathscr{P}(A \& B)$ and the first is given the value "true", then the second must be given the value "false". If, under all permissible distributions of the truth-values, the molecular formula assumes the value "true", it might be called a "deontic tautology".

9. The deontic operators which we have been studying so far are prefixed to names of action-categories. The "deontic logic" which emerged from this study, could be called a logic of what one ought to, may or must not *do*. The things which tell what ought to, may, or must not be done, we called *norms*.

Deontic operators, however, can also be prefixed to action-sentences – or to sentences generally. For example: it ought to be the case that *a* on *o* does *p*.

In order to avoid confusion I shall introduce the symbol N_d for obligation ("deontic necessity") and M_d for permission ("deontic possibility").[17] No special symbol for prohibition will be needed now. For sentences (expressing true or false propositions) I shall employ symbols *s*, *t*, *u*,... from the end of the alphabet.

One can build a deontic logic for sentences of the form "$N_d s$", "$M_d s$" and their molecular compounds. Such sentences say that a certain thing, *e.g.* that so and so does that and that on such and such an occasion, ought to or may or must not *be*.

This deontic logic may rightly be regarded as an off-shoot of modal logic. Just as there are several modal logics, one may also construct several systems of such a deontic logic. But I see no particular reason why it should be constructed in a manner which deviates considerably from the well-known modal logics – except in that it rejects the formula $s \rightarrow M_d s$. I shall not here inquire into the interest of preserving within such a deontic logic the distinction between strong and weak permission and of having a permission operator which distributes conjunctively over disjunctions. Perhaps there is no good motivation for these peculiarities at all.

The more interesting variations of this type of deontic logic arise, I think, when instead of the variables *s*, *t*, *etc.* we employ action-sentences $[p](a, o)$, *etc.* and their compounds. Then the basis on which the deontic logic stands is not propositional logic alone but also a Logic of Action of the type sketched in Section II of this paper.

Of particular interest will now be problems connected with quantification.

There can be no obstacles of a logical nature to applying deontic operators also to quantified action sentences. For example: $N_d(a)(o)[p](a,o)$ says that such a state of affairs is obligatory that everybody on all occasions does p. Quantified sentences are simply a species of sentences expressing true or false propositions and it is for sentences of this latter kind that what above (p. 5) I called "standard" deontic logic has been constructed.

The logical situation changes radically when we consider quantification *into* deontically qualified contexts. Consider, for example, the expression $(a)(o)N_d[p](a,o)$. Here "$[p](a,o)$" is *not* a sentence which expresses a true or false proposition but a *generic* sentence (open sentence, propositional function). It cannot be regarded as at all obvious that it makes sense to prefix deontic operators to open sentences. It is not clear that the meaning of the deontic attribution does not change when we shift from propositions to propositional functions.

What could $(a)(o)N_d[p](a,o)$ possibly mean? It might be an attempt to say that a norm which makes obligatory actions of a certain *kind* is addressed to all agents on all occasions. Then we are not concerned with the obligatory character of a certain *state of affairs* but with a norm obliging *agents*. Instead of $N_d[p](a,o)$ we have a norm $\mathcal{O}A$ which is being given to every agent and is for all occasions. How this universal character of the norm shall be properly expressed in a symbolism is not easy to tell. Perhaps we should introduce a *normative relation*, R, which holds between an agent, an occasion, and a norm, $R(a,o,\mathcal{O}A)$. This relational sentence could then be quantified, *e.g.* so as to become $(a)(o)R(a,o,\mathcal{O}A)$.

In which relation do the expressions $(a)(o)R(a,o,\mathcal{O}A)$ and $N_d(a)(o)[p](a,o)$ stand to one another? We shall assume that the action which is the doing of p by a on o is an action of the category A. The answer to the question, I suggest, is as follows: The two types of deontic expression are not, "by themselves", related to one another in any way whatsoever. If it is a deontic necessity that all agents always do a certain thing it does not follow that there is any norm addressed to all agents ordaining them always to do this thing. But a lawgiver who is anxious to see to it that, if possible, all agents on all occasions behave in this way may issue a norm to the said effect. He may, however, issue such a norm also *without* deeming it necessary for any particular end of his that all agents always behave as he has ordained.

10. What is deontic necessity? I think the best way to view it is as a requirement for something (some end). It is that which must (ought to) be, *if*

--- or *in order to* ---. Under this conception of deontic necessity as a necessary condition for something, one can justify the laws of a deontic logic of the "traditional" type. The justification can also be described as a reduction of deontic logic to ordinary (alethic) modal logic.[18] The conception of deontic logic as a modal logic "in disguise" is no trivialization. But it shows, in my opinion, that traditional deontic logic is *not* a genuine "logic of norms" but a logic of structures resembling what Kant called hypothetical imperatives.

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NOTES

[1] 'Abhängigkeitsbeziehungen zwischen Abhängigkeitsbeziehungen'. *Sitzungsberichte der kaiserlichen Akademie der Wissenschaften in Wien, Philosophisch-historische Klasse*, 181, 1917.

[2] 'Deontic Logic', *Mind* **60**, 1951.

[3] 'An Analysis of Some Deontic Logics', *Noûs* **3**, 1969.

[4] A promising sketch of a system of deontic logic satisfying these conditions has recently been given by R. Stranzinger, 'Ein paradoxenfreies deontisches System'. *Forschungen aus Staat und Recht, Band* 43*: Strukturierungen und Entscheidungen im Rechtsdenken.* 1978.

[5] There is a somewhat fuller description of this type of action logic in my papers 'Deontic Logic Revisited', *Rechtstheorie* **4**, 1973 and 'Handlungslogik' in the anthology *Normenlogik*, ed. by H. Lenk, München-Pullach 1974.

[6] I am indebted to Professor Carlos Alchourron for a correction of a previous attempt of mine to define the notion of a disjunctive activity.

[7] Cf. the papers mentioned in Note 5.

[8] A logical study of action under the achievement aspect is found in my book *An Essay in Deontic Logic and the General Theory of Action.* (1968). At that stage, however, I did not see clearly the relevance to a logic of action of the distinction between the two aspects of achievement and of process, respectively.

[9] The notion of "occasion" is vague. In this there is nothing objectionable. One could make the notion sharper by stipulating that the occasion must be restricted to the time-span of one single action. This would exclude that an agent on some occasion *first* does a certain thing and *then* another. But this restriction would not remove the possibility that at the same time as the agent did a certain thing he also did a certain other thing.

[10] The "way" in which an action is performed here means an(other) action which is instrumental for the achieving of the result of the first action. This sense of "way" must be distinguished from adverbial modifiers such as (doing something) quickly or silently or well, *etc.* One could distinguish the two senses as "way" and "mode" (of acting), respectively.

[11] What is here said of deontic predicates is true also of "moral" predicates of individual actions such as, *e.g.*, an action being "good" or "evil", "courageous", "temperate", or "self-sacrifying" Cf. my work *The Varieties of Goodness* (Routledge & Kegan Paul, London 1963) p. 139ff.

[12] This much discussed example was introduced by Donald Davidson in his influential paper 'Actions, Reasons and Causes' in *The Journal of Philosophy* **60**, 1963.
[13] Cf. my paper 'Remarks on the Logic of Predication', *Ajatus* **35**, 1973.
[14] Cf. *Analytica Priora* 52a1-2 and 52a25-. Cf. also my paper 'On the Logic of Negation' (*Societas Scientiarum Fennica, Commentationes Physico-Mathematicae* **XXII** 4, 1959), in which originally I introduced and discussed the distinction between the two types of negation – weak and strong, external and internal – which here I distinguish with the aid of the symbols $\sim[\;\;]$ and $[\sim]$.
[15] This use of the three letters, *O*, *F*, and *P*, is different from the use made of them in the first part of the present study and thereby also different from their established use in writings on deontic logic.
[16] I am indebted to my Argentinian colleagues Carlos Alchourrón and Eugenío Bulygin for useful observations relating to these matters.
[17] These symbols N_d and M_d correspond to the symbols *O* and *P* respectively of traditional deontic logic.
[18] The *locus classicus* for this conception of deontic logic is A. R. Anderson's note in *Mind* **67**, 1958 entitled 'A Reduction of Deontic Logic to Alethic Modal Logic'. Basically the same "reductivist" idea had been propounded by Kanger somewhat earlier (1957), in a mimeographed essay 'New Foundations for Ethical Theory' (reprinted in *Deontic Logic: Introductory and Systematic Readings*, ed. by Risto Hilpinen, D. Reidel, Dordrecht 1971). Later it was taken up by myself and further elaborated in an essay 'Deontic Logic and the Theory of Conditions' (in *Deontic Logic: Introductory and Systematic Readings*, ed. by Risto Hilpinen).

HECTOR-NERI CASTAÑEDA

THE PARADOXES OF DEONTIC LOGIC: THE SIMPLEST SOLUTION TO ALL OF THEM IN ONE FELL SWOOP

A main cause of philosophical illness – one-sided diet: one nourishes one's thinking with only one kind of example [often a simplistic one].
Ludwig Wittgenstein, *Philosophical Investigations*, No. 593

... the simplicity of means is balanced with the richness of effects [in the order of the universe].

G. W. Leibniz, *Discourse on Metaphysics*, 5

CONTENTS

INTRODUCTION

Deontic logic, as a discipline of study, deals with the structure of our ordinary reasoning about obligations, ought's, interdictions, prohibitions, wrongs, rights, and freedoms to act. Thus, it is supposed to: (i) reveal and clarify the criteria for valid reasoning about those matters; (ii) illuminate

R. Hilpinen (ed.), New Studies in Deontic Logic, 37–85.

and give us understanding of the logical structure of the ordinary language through which we live our experiences of obligations, requirements, wrongs, rights, etc. Consequently, a formal calculus proposed as a deontic calculus together with its primary interpretation is a theory about the logical structure of our ordinary deontic language and about our ordinary deontic reasonings. Clearly, then, such a calculus should be proposed *after* a careful examination of the linguistic data pertaining to our deontic experiences.

Many deontic calculi have been proposed. Some of them have been carefully investigated both for their mathematical properties and for their linguistic and philosophical adequacy. The latter investigation has developed a series of criticisms of many calculi that have evolved into standard tests of adequacy for all known and all future deontic calculi with respect to their primary intended interpretation. Such tests are the so-called paradoxes of deontic logic.

The deontic "paradoxes" are powerful weapons that refute both certain proposed deontic calculi and certain naive interpretations and analyses of our ordinary deontic sentences. Some "paradoxes" – like Alf Ross's – are nothing but elementary confusions. The most important ones – like the Good-Samaritan, the Knower, the Contrary-to-duty paradox, the Second-Best Plan – are genuine refutations of certain deontic calculi, even some that contain interesting sophistications.

After three decades of reflecting on the logic of deontic reasoning I have come to the conclusion that the so-called deontic paradoxes are not genuine paradoxes – or should no longer be considered such. The only genuine paradox I can find is the steady development of calculi, or systems of "deontic possible worlds", that do so little, when a cursory inspection of the structure of the deontic sentences involved in the "paradoxes" reveals that the solution to most of them is very simple. The standard methodology of the solution to a chosen "paradox" consists of the erection of a system of symbols with axioms and an attached set-theoretical structure of deontic possible worlds, with restrictions of different sorts, so that the "paradox" under consideration does not arise. The resulting systems are typically very weak. What we need are, on the other hand, very comprehensive systems that can furnish illuminating accounts of both our ordinary deontic language and of our ordinary deontic reasonings. To nail this point down firmly, just consider the following reasonings that most existing deontic calculi are unable to represent!

(A) 1. Alchourrón is obliged (obligated) to do the following: Both
(a) if Bulygin sends him the second draft of their latest joint paper and he is not working in his farm, revise it, and
(b) write the outline of their next paper, if and only if Bulygin calls him up to report that he will be going to Copacabana.
2. Bulygin has both sent Alchourrón the second draft of their latest joint paper and called him up to report that he will be going to Copacabana, and Alchourrón is not working in his farm.

Therefore,
3. Alchourrón is obliged to do the following: both revise the second draft of their latest joint paper [with Bulygin] and write the outline of their next paper.

(B) 1. It is wrong that anyone humiliate a man he has defeated or defeat a man he has humiliated.

Therefore,
2. If one has defeated a man, it is wrong that one humiliate him.

(C) 1. It is obligatory that Carrió pay $1000 to the owner of the apartment he is renting.
2. The apartment Carrió is renting is precisely the residence with the famous portrait of Kelsen painted by Vernengo.

Therefore,
3. It is obligatory that Carrió pay $1000 to the owner of the residence with the famous portrait of Kelsen painted by Vernengo.

These examples provide a most significant test of adequacy for any deontic calculus already built or to be built. I submit that its effects are generally devastating: either a calculus lacks resources to even attempt to formulate those simple and straightforward reasonings; or the calculus has resources to represent those inferences, but represents them as invalid, or represents their sets of premises as inconsistent. The latter situation is often called a "paradox" – that is, a clear failure of the calculi under consideration.[1]

I said above that a cursory examination of ordinary deontic language reveals that the solution to all so-called paradoxes of deontic logic is stunningly simple and magnificently uniform: they can all be solved in one single stroke. I further claim, more specifically, that the simplest and uniform solution to all those "paradoxes" is staring at us right there in examples (A)–(C). That is exactly why (A)–(C) are such powerful tests of adequacy for deontic logic. The reader should find it a rewarding experience to test his favorite deontic calculi against (A)–(C). Then he should test his favorite deontic calculi against the so-called paradoxes of deontic logic. He should discover that a positive result concerning (A)–(C) goes hand in hand with a negative result concerning the appearance of the "paradoxes."[2]

My plan in this essay is as follows: in Part I crucial features of reasonings (A)–(C) will be gleaned, features that any deontic calculus worthy of its name must comply with; in Parts II–VI we shall discuss the so-called paradoxes and show how each one depends on the fatal neglect of some of those crucial features, in particular one of them. We have no room for examining how standard attempts at solving the "paradoxes" are *ad hoc* and fall prey to other paradoxes. The dispassionate reader will undoubtedly want to learn about the character and span of his favorite deontic calculus. In part VII we present the *simplest deontic calculus*, which is so *marvellously faithful to ordinary language*. Because of that it provides the easiest and most uniform way of preventing the alleged paradoxes from arising. The calculus is, furthermore, not only stunningly simple but also very comprehensive[3]. (See Leibniz's principle of good theorizing quoted above.)

I. CRUCIAL LINGUISTIC DATA

A deontic calculus, we said, is with its primary interpretation a theory about the logical form of ordinary sentences of natural language. Thus, unless we find a good unbeatable reason to do otherwise, we must consider the syntactico-semantical contrasts of ordinary deontic sentences as clues to important distinctions in logical structure. Let us, therefore, with scrupulous attention to the best methodology, examine the syntactical structure of the above examples.

1. *The basic structural duality*. Let us consider example (A), since it is the most complex of them and can, therefore, reveal more relevant structure. (See Wittgenstein's admonition quoted above.) The first premise of reasoning (A) is this:

4. Alchourrón is obliged (obligated, required) to do the following:
(a) if Bulygin sends him the second draft of their latest joint paper, REVISE IT, and

(b) WRITE THE OUTLINE OF THEIR NEXT PAPER, if and only if Bulygin calls him to report that he will be going to Copacabana.

Evidently, the overall structure of example 4 is that of an obligatoriness (requiredness) encompassing a conjunction. Now, each of the conjuncts, labeled (a) and (b), is of a conditional form, but of a very peculiar sort. Conditional (a) has a straight-forward conditional *indicative* clause, but its consequent is worthy of careful exegesis. It is an *infinitive* clause with a tacit subject, namely 'Alchourrón'. That this indicative-infinitive contrast is of the greatest importance is reinforced by (b). This is a biconditional with the same contrast. Semantically the distinction is clear. The indicative clauses are conditions and not actions that are considered as obligatory. The conditions are circumstances that determine whether Alchourrón has certain specific obligations or not. The conditions are as such not properly obligatory; so much so that, even though they describe possible actions by Bulygin, they do not present Bulygin as being under any obligation.

In short, unless a powerful overriding argument is to be found, *no deontic calculus of worthy richness can ignore the infinitive-indicative duality in the scope of deontic prefixes*, as 'Alchourrón is obliged (obligated, required) to do the following' in example 4. Therefore, any useful deontic calculus must contain two types of sentences and well-formed formulas, even at the purely sentential level, before the adjunction of quantifiers. This basic structural duality is fundamental for an understanding of the syntactical structure of our deontic language.

2. *Basic logical duality.* The ordinary infinitive-indicative contrast within the scope of deontic prefixes just noted is more than merely structural in the sense of well-formedness of sentences. It is logical also. It anchors crucial implications and non-implications. To show this let us continue our examination of the preceding data.

Clearly, premise 1 of argument (A), that is, *No.* 4 *above, implies* (5) *and* (6):

(5) If Bulygin sends him the second draft of their latest joint paper, and he is not working in his farm,
Alchourrón is obliged (obligated, required) to REVISE IT.

(6) Alchourrón is obliged (obligated, required) to WRITE THE OUTLINE OF THEIR NEXT PAPER, if and only if Bulygin calls him up to report that he will be going to Copacabana.

In these examples, then, the indicative clauses, which express circumstances, can be taken out of the scope of the deontic prefix. The infinitive clauses, capitalized above for facility of inspection, on the other hand, must remain inside the scope of the deontic prefix. The implication of (5) and (6) by (4) involves several principles: the distribution of the deontic operator through the conjunction of conditionals, and a passing through the conditions to the conditioned actions. A little reflection shows that these principles have to do with the semantical contrast between a *circumstance or condition* and an *action deontically considered as the focus of obligatoriness* (requiredness). The issue does not pertain to the contrast between the roles played by Alchourrón and Bulygin. Indeed, one and the same person may be involved in an obligation statement both as an agent of an action in a circumstance or condition and as agent of an action that is the focus of obligatoriness. An example is this:

(7) Alchourrón is obliged (obligated, required) by his work agreement with Bulygin to do the following: RE-WRITE THE FOOTNOTES, if and only if he mislays them, *AND* SEND THE FOOTNOTES TO CARRIO, only if *he re-writes them.*

Clearly, (7) implies (8):

(8) Alchourrón is obliged (obligated, required) by his work agreement with Bulygin to do the following: RE-WRITE the footnotes, if and only if he mislays them.

But neither (7) nor (8) implies:

(9) Alchourrón is obliged (obligated, required) by his work agreement with Bulygin to do the following: MISLAY the footnotes, if and only if he re-writes them.

The action of Alchourrón's mislaying the footnotes is simply a circumstance, and is not deontically considered. On the other hand, Alchourrón re-writing the footnotes is deontically considered, and not considered as a circumstance or condition.

Now, a sharp and short datum that nails the duality we are discussing very firmly is the *logical asymmetry* between (8) above and (10) below:

(10) If and only if he mislays them, is Alchourrón obliged (obligated, required) by his work agreement with Bulygin to re-write the footnotes.

Evidently, (8) *implies* (10), *but* (10) *does not imply* (8). This asymmetry highlights the contrast between an action considered as a deontic circumstance and an action considered deontically, by moving the former, but not the latter, outside the scope of the deontic operator. Go a step further and consider cases in which the same agent and action content are together treated both as a circumstance and as a deontic focus in the very same statement. This is precisely what (7) itself shows: it contrasts the infinitive clause in capital letters '[ALCHOURRÓN] RE-WRITE THE FOOTNOTES' with the indicative clause in italics '*he* [*Alchourrón*] *rewrites them* [*the footnotes*]'. The difference can be brought out further by observing that (7) *implies* (11) below, but does *not* imply (12) next:

(11) Alchourrón is allowed (permitted) by his work agreement with Bulygin to send the footnotes to Carrio, only if he re-writes them.

(12) If he sends the footnotes to Carrio, Alchourrón is obliged (obligated, required) to re-write them.

The action of Alchourrón's re-writing the footnotes is *not* deontically considered but is considered only as a circumstance in the second conditional in (7) ('SEND THE FOOTNOTES TO CARRIO, only if he re-writes them'.)

We have seen how deontic operators distribute through a conjunction, even if the conjunction in question is composed of mixed conditionals.

We have seen how mixed conditional practitions have a circumstance (or proposition) as a sufficient condition, as a necessary condition, or even as both necessary and sufficient. Argument (A) has an example of the third case in it is first premise, namely: "(Alchourrón ... to ...) write the outline of their next paper, if and only if Bulygin calls him up to report that he will be going to Copacabana." *Are there pure conditional practitions*? This is an interesting question that I have treated in *Thinking and Doing* in great detail. The question allows of verbal disputes, and it has to do with the *non-logical* roles (especially the ones I call *thematic roles*) of English logical words.[4] Thus, the following is of the greatest importance:

Crucial fact about English conditional particles

No conditional particle precedes an imperative sentence, or an infinitive or subjunctive clause that expresses a practition. In general, the same holds for all *subordinating conjunctions*, but not for so-called coordinating conjunctions.

Hence, given that fact of English we cannot express pure conditional practitions by means of the English conditioning particles, like 'if', 'when', 'only if', 'only in the case that', 'if and only if', 'just in case that', etc. Thus, if a conditional practition has to be expressed by means of a so-called conditioning particle, then there are no pure conditional practitions. But if we consider the *logical form*, not merely the grammatical form, and allow other variations, then we can have pure conditional practitions, like:

(*c*) Rabossi ought to do the following: not gamble or get a higher-paying job.

Clearly, (*c*) implies:

(*c*.1) If Rabossi ought to gamble, then he ought to get a higher-paying job.

The implication by (*c*) of (*c*. 1) shows that in some sense the practition (*Rabossi ... to*) *not gamble or get a higher-paying job* is a conditional one – and a pure conditional practition, since each infinitival clause is a deontic focus.

Perhaps some English speakers would take (*c*) to be equivalent to:

(*c*.2) Rabossi ought to do the following: if he gambles, get a higher-paying job.

But that would introduce a serious and abnormal ambiguity in the indicative 'if' clause. For then we must distinguish the sense of (*c*.2) in which it implies (*c*.1), but not (*c*.3) below, from that in which, conversely, it implies (*c*.3), but not (*c*.1):

(*c*.3) If Rabossi gambles, he ought to get a higher-paying job.

For my part, taking the language system (*la langue*, Saussure would say) of English at face value, I will ignore abnormal speech acts and dialectal ambiguities. Hence, I say that:

Fact about pure conditional practitions

In English (for good reasons explained in *Thinking and Doing*, Chs. 3 and 4) pure conditional practitions are expressed disjunctively (as shown in (*c*)). We can introduce a purely theoretical conditional sign (deprived of theoretic roles), say, '$\supset$', *not* translatable into ordinary conditioning particles, and formulate pure conditional practitions of the form $A \supset B$, where 'A' and 'B' are infinitive or subjunctive clauses (or imperatives). (See *Thinking and Doing*, Ch. 4 for a full study of this.)

Let us now consider a case of a mixed conjunction, for instance:

(13) It will rain tomorrow and Peter is obliged (obligated, required)

by the rules of his job to work outdoors tomorrow during the morning hours.

(14) Peter is obliged (obligated, required) by the rules of his job to do the following: it being the case that it will rain tomorrow, work outdoors tomorrow during the morning hours.

(14a) Peter is obliged (obligated, required) by the rules of his job to do the following: although it will rain tomorrow, work outdoors tomorrow during the morning hours.

(14b) Peter is obliged (obligated, required) by the rules of his job to do the following: while it is the case that it will rain tomorrow, nevertheless work outdoors tomorrow during the morning hours.

Concerning the logical character of the information provided, it seems that (14), (14a), and (14b) all convey the same proposition. On the other hand, (13) conveys a different proposition. However, it seems that the proposition formulated by (13) is logically equivalent to the one conveyed by (14).

The equivalence between (13) and (14) is of the greatest importance. It tells us that circumstances in conjunction with deontic foci within the scope of the deontic operators of the type being discussed can fall out of the operator – so to speak – in reality.

3. *Main conclusion.* We can multiply cases of the above sorts, in order to determine crucial features of compounds by means of the standard logical connectives of infinitive and indicative clauses within the scope of deontic prefixes. For instance, the validity of argument (A) in the *Introduction* shows how the indicative components, but not the infinitive ones, allow forms of internal *modus ponens.* The reader, anxious to understand the structure of ordinary deontic language and reasoning, can (and perhaps should) accumulate further examples, (following Wittgenstein's admonition) complex ones, in order not to miss the crucial patterns, and investigate other principles of inference. Here we must move on.

The preceding exegesis of examples suffices to show that no deontic logic worthy of its name that has some useful richness can afford to ignore the momentous duality within, and without, the scope of deontic operators, namely, the duality entrenched in the contrast between *an action considered as a circumstance* and *an action (perhaps the very same action) considered deontically as a deontic focus.*

4. *Some crucial results.* Lest we miss the most significant points we have gleaned from the preceding exegesis of examples, we must record them.

Primary distinction:

(a) circumstances – indicative elements – which can include actions performed, or

(b) foci of deontic operators – the infinitival elements performable by some agents who are subject to obligations, duties, etc.

Formation rules

(a) A connective compound of circumstance and deontic foci is also a deontic focus. Examples are provided above, especially in sentences (4) and (7) which contain conjunctions of mixed conditionals and biconditionals.

(b) Deontic operators apply to deontic foci, whether pure or mixed, but the results, i.e., the deontic statements that result, are not themselves deontic foci.

(c) Deontic operators need some further modalities to produce full statements that are true or false.

Examples of such further modalities are the adverbial qualifiers *by his work agreement with Bulygin*, which is explicit in sentence (7) above, and is tacit in (4) and in argument (A); *by the rules of his job*, which appears in (13) and (14); the adverb *morally*, which is implicit in argument (B), which has as its sole premise "It is wrong [morally] that anyone humiliate a man he has defeated or defeat a man he has humiliated."

Principles of implication. In order to be more perspicuous, let us use the abbreviation 'O_i' to represent a deontic operator oughtness, or O of the type we have been discussing (*viz.*, obligatoriness, requiredness, oughtness, or dutyhood), with the subscript '*i*' standing for an appropriate further adverbial modality, like those mentioned in formation rule c). We let 'P_i' represent the corresponding type of *permittedness*$_i$, and 'W_i' the corresponding type of *forbiddenness*$_i$, *wrongness*$_i$ or *unlawfulness*$_i$. Let us, moreover, represent a circumstance with a standard propositional variable and a deontic focus with a capital letter. Thus, we have uncovered the following implications, where the single-headed double arrow indicates implication, and the double-headed one, equivalence:

1. $O_i(p \supset A) \Leftrightarrow p \supset O_i A$ 2. $O_i(p \& A) \Leftrightarrow p \& O_i A$
3. $O_i(p \equiv A) \Rightarrow p \equiv O_i A$ 4. $O_i(A \supset p) \Leftrightarrow P_i A \supset p$
5. $O_i((p \supset A) \& (q \equiv A)) \& (p \& q) \Rightarrow O_i A \& O_i B$ This is argument (A)
6. $O_i(A \supset B) \Rightarrow (O_i A \supset O_i B)$, where '$\supset$' must be read as 'not ... or'

5. *Propositions and practitions.* We have seen that the contrast between infinitive and indicative clauses within the scope of deontic prefixes is of paramount importance. The contrast is, unfortunately, *not canonical* in English. On the one hand, the same contrast expresses in other contexts other important distinctions. On the other hand, in variant deontic constructions what the indicative-infinitive contrast expresses is expressed by the indicative-subjunctive contrast. Let us consider these two points in order.

The infinitive-indicative contrast also appears in psychological sentences. Compare, for instance:

(15) Simpson believes that the quasi-indexical mechanism of reference is a problem for Quine's views about belief sentences.

(16) Simpson believes the quasi-indexical mechanism of reference to be a problem for Quine's views about belief sentences.

Here the infinitive-indicative contrast has to do with what the speaker of (15) or (16) can attribute to Simpson. By using (15) one can attribute to Simpson a doxastic relation to the proposition expressed in the context of assertion by the subordinate clause 'the quasi-indexical mechanism of reference is a problem for Quine's views about belief sentences'. So used, (15) is *propositionally transparent* in that it reveals the proposition that is the accusative of Simpson's believing mentioned there. On the other hand, (16) does not attribute to Simpson the exact reference that the speaker makes, on his own responsibility, to the indexical mechanism of reference. Presumably the infinitive segment of (16) 'to be a problem for Quine's views about belief sentences' reveals the predicate of the proposition that is the accusative of Simpson's believing. Thus, (16) is *propositionally opaque* with respect to the subject of Simpson's doxastic accusative[5]. There are other aspects to the semantics of the infinitive-indicative contrast in (15)–(16). But the above is sufficient for our purpose here.

To see the deontic sense of the indicative-subjunctive contrast consider (7) again and compare it with (17) below:

(7) Alchourrón is obliged (obligated, required) by his work agreement with Bulygin to do the following: RE-WRITE THE FOOTNOTES, if and only if he mislays them, *AND* SEND THE FOOTNOTES TO CARRIO, only if he re-writes them.

(17) The following is obligatory (required): that Alchourrón RE-WRITE THE FOOTNOTES, if and only if he mislays them,

AND (Alchourrón) SEND THE FOOTNOTES TO CARRIO only if he re-writes them.

Naturally, each of these two sentences can be used to make different types of statements, but they can be used to make the same statement. In such a case, clearly, the infinitive-indicative contrast of (7) converges with the subjunctive-indicative contrast of (17). They both express the contrast between an action considered as a circumstance and an action considered deontically as a deontic focus[6].

Let us for convenience call the agent-action complexes that are deontic focus *practitions*. Let us further generalize the concept to refer to the mixed complexes of propositions or circumstances and practitions we have been considering in the preceding examples. We can say, then, that *a deontic operator applies to a practition and yields a propositional matrix* (if it lacks the proper adverbial modality) or *proposition or propositional function* (if it has its own adverbial modality).

6. *Laws of deontic English about circumstances, practitions, and quantifiers.* We have seen how some circumstances, or propositions, move in and out of the scope of some deontic operators, with preservation of logical equivalence, or sometimes only one-way virtue of a non-reversible implication. (See Section 4.) Now, I submit the following law holds:

First law about deontic English (concerning circumstances)

Let d be an English deontic statement within which circumstances $p_1, \ldots, p_n$ lie within the scope of some deontic operator, and d contains no quantifiers or any other modalities except deontic ones, so that $p_1, \ldots, p_n$ are related to each other and to the practitions in d only by means of the propositional connectives directly or through the deontic operators in which they lie. Then there is a deontic statement d^* in English so that: d and d^* are logically equivalent, and in d^* the circumstances $p_1, \ldots, p_n$ are not in the scope of any deontic operator.

Naturally, this law is very hard to prove. We have found examples of it, and *I theorize* that it is valid for English, subject to limitations of ambiguity and non-deontic uses of deontic sentences of English. Part of what I am claiming for English is that all the indicative components within the scope of a deontic sentence, if there are only deontic prefixes and propositional connectives involved, can be brought outside the scope of the deontic prefixes, so that these apply directly to infinitival clauses, or subjunctive clauses, deontically used. That Law is thus a guide to theory construction.

On the other hand, the combination of quantifiers and deontic operators create some new structures. In particular, not all circumstances within the scope of deontic operators can be taken without, in cases in which they are also within the scope of quantifiers in the scope of those operators. To show this we need but one example. Consider the following practition, with the crucial infinitive (or subjunctive) verb capitalized:

(18) Someone who chaired a meeting last year PRESIDE over the Executive Committee this year.

This practition is the core of the deontic statement

(19) It is obligatory that someone who chaired a meeting last year preside over the Executive Committee this year.

Let us abbreviate as follows, using round parenthesis to signal the propositional copulation of agent and action in a circumstance and square brackets to signal the practitional copulation of agent and action in a practition:

$C(x)$: x chaired a meeting last year,
$R[x]$: x (to) preside over the Executive Committee this year.

Then we have the following analyses of practition (18) and statement (19):

(18a) $\exists x(C(x) \& R[x])$.
(19a) $\text{Obligatory}_i(\exists x(C(x) \& R[x]))$.

Here the obligation applies to the whole class of relevant agents. Those who have never chaired any meeting last year, i.e. who lack the property $C(x)$, are of course free from the duty imposed by (19). On the other hand, those who have that property $C(x)$ are collectively under a duty to preside over the entire Executive Committee this year; but this duty is a *disjunctive* one: if one man who has the property $C(x)$ presides over the Executive Committee this year all of last year's meeting chairmen comply with their duty. Clearly, there is nothing anybody can do about having been a meeting chairman last year or not. But if anybody is to comply with the duty that (19) demands, then property $C(x)$ must be possessed by some of the relevant agents. For if $\mathit{Obligatory}_i(\exists x(C(x) \& R[x]))$ is both true and fulfilled, then $\exists x(C(x) \& R(x))$ is true; i.e., somebody both has *already* in fact chaired a meeting last year and presides or has presided this year over the Executive Committee. Therefore, somebody has already property $C(x)$. Thus, if $\mathit{Obligatory}_i$ $(\exists x(C(x) \& R[x]))$ is fulfillable, i.e., prescribes no empty duty, $\exists x(C(x))$ is true.

We can list, therefore, another principle of implication, to be added to those recorded in Section 4, to wit:

11 $\quad O_i(\exists x(C(x) \& B[x])) \Rightarrow \exists x(C(x))$

On the other hand, because of the disjunctive, or *Gestalt*, character of the obligation imposed by (19), it is incorrect to infer from (19) that *there is* someone who ought$_i$ to preside the Executive Committee this year. Clearly, then, the particular quantifier $\exists x$ must remain within the scope of the deontic operator *Obligatory*$_i$. Hence, the only initially plausible transformation of (19) so as to take the circumstance $C(x)$ outside the scope of *Obligatory*$_i$ would be to dislocate the particular quantifier in some form like this: $\exists x(C(x))$ & *Obligatory*$_i(\exists x(R[x])$. But this doesn't work. This is *not* equivalent to (19), because it does not connect the circumstance $C(x)$ with the practition $R[x]$.

I submit, in conclusion, that we have:

Second Law about deontic English (concerning circumstances)

Let d be a deontic statement having some quantifier q within the scope of a deontic operator and some circumstances in the scope of q. In general, for some such a deontic statement d there is no deontic statement d^* equivalent to d having no circumstance in the scope of the deontic operator in d^*.

This limiting result about the exportation of circumstances from deontic operators is of great moment. It establishes conclusively that the distinction between practitions and circumstances is fundamental – as if we didn't know this from the previous exegesis of data! The present point is this. In the sentential calculus one might be under the *impression* that the distinction is not needed because for every deontic statement d with the distinction inside the scope of deontic operators there is an equivalent statement d^* in which the deontic operators apply to mere practitions. Then one can adopt the view that those practitions are simply the propositions corresponding to them. This is a naive view, since there are differences between practitions and propositions or circumstances that go beyond deontic logic. But the view is in theoretical error in the field of deontic logic. In order to find the segregational equivalent d^* for a given deontic statement d containing both practitions and circumstances in the scope of deontic operators, we must go through the logical equivalences that permit the derivation of d^* from d. Hence, we must acknowledge the duality practition-proposition inside the scope of the deontic operators subjected to the exportation of circumstances. Of course, since equivalence is symmetric, it is naive to deny the distinction just because there is a d^* for every d in the sentential calculus. Because of those equivalences, we can go from d^* *back* to d.

There is the programmatic view that only the inferences involving segregational deontic statements d^* are to be counted within the canonical language of logic. But such a view is obscurantist. It would enthrone a core of statements as logical, and would regard the equivalences that connect each d^* with its corresponding d as non-logical. All those transformations required for the derivation of d^* from d would be non-logical, unofficial. The view simply does not provide an account of the structure of natural language; nor does it illuminate the nature of our ordinary reasoning, which makes essential use of those non-segregational deontic statements.

In any case, with the combination of deontic operators and quantifiers, not even the naive views mentioned above can be justified. We have, therefore, the following:

Third Law of deontic English

The distinction between an action considered as a circumstance and an action considered as a deontic focus is the most primitive and fundamental distinction in the syntax and the semantics of the deontic segment of English. Since our proposition-practition distinction is a theoretical generalization of the preceding distinction, *no deontic calculus of worthy richness can afford to ignore it.*

II. THE SO-CALLED DEONTIC PARADOXES

Most of the "paradoxes" that have been discussed, whether they have received special names or not, have to do with the circumstance-deontic focus, or proposition-practition distinction. We discuss here just a few of such "paradoxes" in order to provide them with the extraordinarily simple solution that the distinction affords. Yet the most impressive aspect of the situation is that the distinction alone can provide a unified solution to most of the deontic "paradoxes"; this unitary solution is the best reward of our attentive exegesis of the ordinary semantico-syntactical contrasts of our ordinary deontic language. The situation is even more satisfactory. The so-called paradoxes of deontic logic were formulated after the construction of the first deontic calculus containing the distinction. Even that calculus had thereby a built-in solution to the later puzzles.[7]

We shall also discuss Powers' justly celebrated Susy Mae example. This raises a serious and important problem that has to do with certain systemic aspects of deontic logic. And we will show that the so-called Alf Ross's paradox is self-refuting.

1. *The Good-Samaritan Paradox.* We all assume a principle of deontic logic naturally, though obscurely, put as follows:

(P) If X's doing A entails Y's doing B, then that X's doing A is —— obligatory entails that Y's doing B is —— obligatory,

where the blanks are to be filled in with exactly the same appropriate adverb, e.g., 'morally' and 'legally' or more stilted locutions like 'according to the rules of his job' or 'by his work agreement with Bulygin'. The first obscurity of (P) lies in 'doing'. If doings are events they do not have entailments or implications. The phrases 'X's doing A' and 'Y's doing B' must, then, formulate in (P) statements, or something like statements that can have implications. Thus, it is natural to interpret (P) as

(P′) If *X performs A* entails *Y performs B*, then *X is obligated*$_i$ *to do A* entails *Y is obligated*$_i$ *to do B*, where 'i' stands (as before) for an adverb denoting type of *obligation.*

But (P′) leads immediately to the so-called Good-Samaritan paradox. Suppose, for example, both that Arthur is today legally (morally) obligated to bandage his employer, Jones, and that a week from today Arthur will murder Jones. (It makes no difference whether Arthur or Jones knows today that the former will murder the latter.) Thus we have:

(1) Arthur is legally (morally) obligated to perform the act, call it C, of bandaging the man he will murder a week hence.

Clearly,

(2) Arthur's doing C entails his doing the act of murdering a man a week hence.

Hence, by (P′) from (1) and (2):

(3) Arthur is legally (morally) obligated to murder a man a week hence.

Evidently, then, at least one of (1), (2) and (P′) must, logically, be false. And the choice can only be (P′). Here most deontic philosophers take desperate measures. For instance, (P′) is restricted to actions that lie in the future, or actions performed by one agent, and not others. These measures work well for the original examples that justify calling the puzzle The Good Samaritan. The first example had to do with a good Samaritan tending the wounds of a robber's victim.

My Arthur example shows that the Good-Samaritan paradox cannot be resolved by insisting that (P) or (P′) must hold only for one and the same agent, or for future actions, etc. The difficulty has nothing to do with distinctions of agents, patients, times, or places[8]. The difficulty arises solely from taking the implicational links between deontic statements as patterned on the implication lines between the corresponding statements of fact, rather than on those between the corresponding practitions. Specifically, in our example, the trouble is this: in the deontic statement or proposition (1) *Arthur is legally (morally) obligated TO BANDAGE the man he* will murder *a week hence*, the act of bandaging is deontically (or practically) considered, while the act of murdering is considered in it as a circumstance. Relating the matter to the discussion in Part I note the infinitive-indicative contrast between 'TO BANDAGE' and 'will murder'. Thus, the principle that (P) hides is:

(P*) *Principle of dispersion of obligatoriness across practitional lines*

If practition *X to do A* entails practition *Y to do B*, then *It is obligatory*$_i$ *that X do A* entails *It is obligatory*$_i$ *that Y do B*.

Let us check the "paradox" against (P*). On the one hand, the practition *Smith to bandage the man he will murder a week hence* does entail the statement or proposition *Smith will murder a man a week hence*. But we cannot apply (P*) to it: the operator *It is obligatory that* applies to practitions only, as we noted in Part I Section. Hence, by this route we cannot derive *It is obligatory that Smith murder a man a week hence*. On the other hand, the *Smith to bandage the man he will murder a week hence* does not entail the practition *Smith to murder a man a week hence*. Thus, even though we do have the materials to infer *It is obligatory that Smith murder a man a week hence*, we do not have the appropriate premise. So, there is really no Good-Samaritan paradox. And there is a nice, simple principle, namely

(P*), without restrictions of any sort, that bridges the gap between practitional and deontic implication.

There are other solutions to the so-called paradox of the Good-Samaritan. Many of them are too awkward. Actually, in some cases the "paradox" can be solved by forcing distinctions in the scope of deontic operators. For instance, in our example above, we can interpret the sentence 'Arthur has a duty to bandage a man he will kill' as expressing (4) below, where '*a*' abbreviates 'Arthur':

(4) $(\exists x)$ (*a* will kill *x* & *a* has a duty to bandage *x*),

and not as:

(5) $(\exists x)$ (*a* has a duty to (bandage *x* (& kill *x*))).

It does not follow from (4) by (P′) that Arthur has a duty to kill, since the part '*a* has a duty to bandage *x*' of (4) implies nothing about killing.

There is, however, a reply to the preceding scope solution of the Good-Samaritan "paradox". It requires that (4) does not imply (6):

(6) There is a man such that Arthur has a duty to do the following: to bandage him while it is the case (or, it being the case) that Arthur will kill him.

As we noted in Part I, Section 2, and the equivalence of (3) to (6) evidences, the conjunct '*a* will kill *x*' may be moved in and out of the scope of the deontic 'Arthur has a duty to do the following'. Of course, (6) cannot be represented in a system that does not distinguish propositions (and propositional functions) from practitions (and practitional functions). In brief, the scope solution does dissolve the paradoxicality of certain examples; but it does that at the cost of mishandling the logical form of propositions like (6), and like the ones discussed in earlier sections, and, *a fortiori*, at the cost of proscribing the implication, indeed, equivalence, between propositions like (3) and those like (6).

2. *The psychological deontic paradoxes.* Several variants of the Good-Samaritan "paradox" have been proposed. For some of them scope distinctions, erroneous though they are, as noted above, look initially plausible. A beautiful case that cannot even initially be analyzed away by scope distinctions is Åqvist's "paradox" of the Knower[9].

Consider the case of a man, say Jones, whose job, in accordance with the rules R of the office in which he works, is to know what is done wrong, in accordance with the same rules, by other people in the same office. Suppose that Smith did A, which is wrong by the rules of the office. Thus, we have:

(7) It is $wrong_R$ that Smith do A;
(8) Jones $ought_R$ to know that Smith (does) did A;
(9) "Jones knows that Smith (does) did A" implies "Smith (does) did A."

By principle (P′) it follows:

(10) Smith $ought_R$ to do (have done) A.

Clearly, (10) contradicts (7).

Here the scope distinction does not help. For one thing, there is apparently no satisfactory analysis of knowledge for some conjuncts of the analysis to be considered to lie outside the scope of the deontic operator $ought_R$. But suppose that we can analyze "Jones knows that *p*" as "*p* and Jones believes that *p*, and Jones has excellent evidence for that *p*." Then the scope analysis of "Jones $ought_R$ to know that Smith did A" may yield "Smith did A and Jones $ought_R$ to both believe that Smith did A and have evidence for this." But this won't do. The fact is that a duty to know is *not* the same as a duty to believe and have evidence: surely one can have the latter without having the former[10].

There are psychological attitudes that one must acquire, and psychological acts that one must perform, that imply that something that happens to be wrong has occurred. Such cases give rise to troubles for (P′). The trouble is compounded in those cases in which there is no purely psychological content, that can be extracted, in the way believing is the pure psychological core of knowing. For instance, there is no purely psychological core that can be really $obligatory_i$ when one is obliged (obligated, required) to repent, to lament, or to apologize for, having done some action A which it is $wrong_i$ to do.

Åqvist's proposed solution consisted of distinguishing different types of duties. This proposal was shown by Lawrence Powers not to be adequate[11]. Of course, I do not object to distinguishing types of duties: I have done so in the logical exegesis of conflicts of duties, and I insist on attaching a systemic subscript to every deontic operator. Yet we do not have to resort to this to solve the "paradox" of the Knower, the Repenter, the Apologizer, the Wrongdoer, etc. In fact, we have already found, independently of (P), that deontic operators apply to practitions. Thus, we can, with great simplicity, introducing *no* new concept or restriction, recognize in (P) a practitional principle, not a propositional one, namely, (P*) above.

(P*) provides an immediate, sharp and unified solution to *all* the forms of the Good-Samaritan "paradox". In all those cases we have:

(11) It is *obligatory*$_R$ that Jones E (know, repent, regret, lament, etc.) that he himself (or Smith) did A.

This contains the practition *Jones* E *that you (or Smith) did A*. This practition implies neither the proposition *Jones* Es *that Smith did* A nor the practitions *Jones to do A*, or *Smith to do A*. Hence, from (11) by (P*) we cannot derive that it is *obligatory*$_R$ for Jones (or for Smith) to do (have done) A. Consequently, we may properly and consistently accept that the system of rules governing the tasks of all the people in Jones' office, including Jones, are all duties *in exactly the same sense*, and even of the same type, expressed by the subscript 'R'.

3. *Contrary-to-duty normatives.* Recall our discussion of pure practitional conditionals in Part I Section 2. We discussed:

(12) Rabossi ought$_i$ to do the following: not to gamble or to get a higher-paying job.

We noted that (12) implies:

(13) If Rabossi ought$_i$ to gamble, then he ought$_i$ to get a higher-paying job.

Recall that '$\supset$' is a pure logical connective with none of the thematic or dialectical properties of 'if' and 'only if'. Thus we can formulate (12) as:

(12a) Rabossi ought$_i$ to do the following: (he) to gamble $\supset$ (he) to get a higher-paying job.

This is just a special instance of the following principle of implication that governs deontic statements, already listed as No. 6 in Part I Section 4:

(P.*1) "It is obligatory$_i$ (A $\supset$ B)" implies "It is obligatory$_i$ A $\supset$ it is obligatory$_i$ B." [But remember that in 'A $\supset$ B' the sign '$\supset$' cannot be read by means of ordinary English conditional particles, but as 'not ... or'.]

The so-called Chisholm's paradox[12] of contrary-to-duty imperatives is just another variation of the problems created by the failure to see that deontic operators sometimes operate on practitions that have both propositions and simpler practitions as components. To illustrate consider the case of a university that has rules *r* governing commencements, and Name-professorships, such that:

(14) Mellon Professor Goldsmith ought$_r$ to attend the June commencement,

(15) Goldsmith ought$_r$ do the following: wear academic regalia if he attends the commencement; and

(16) (Since the only academic ceremony in June is the commencement) if Goldsmith does not attend it, it's not the case that he ought$_r$ to wear academic regalia in June.

If we do not distinguish between propositions and practitions, and take (P.*1) above to have A and B as expressions of propositions, ignoring our warning about 'if' and '$\supset$', we would derive (17) below from (15) above with the help of the misinterpreted (P.*1):

(17) If Goldsmith ought$_r$ to attend the June commencement, he ought$_r$ to wear academic regalia in June.

From (14) and (17) we may derive, by *modus ponens*:

(18) Goldsmith ought$_r$ to wear academic regalia in June.

Suppose that being tired of commencements:

(19) Goldsmith does not attend the June commencement.

From (19) and (16), we can infer also by *modus ponens*:

(20) It is not the case that (18).

Obviously, we cannot have both (18) and (20). We cannot any longer ignore the heterogeneity of the elements in (15). Thus, the simplest and most elucidatory course is to insist on that heterogeneity and insist that it is present in (P.*1), in which the components A and B are practitions, not propositions. This view also fits in harmoniously with the solution to the preceding paradoxes.

Keeping in mind that A and B in (P.*1) are *practitions*, not *propositions*, we conclude that (P.*1) does not apply to (15), which is of the form:

(15.a) Goldsmith ought$_r$ to do the following: ($p \supset$ B), where 'p' stands for the proposition *Goldsmith attends the June commencement* and 'B' for the practition *Goldsmith to wear academic regalia in June.* Hence, (15) does *not* imply (17). Therefore, we cannot validly derive (18) from (14)–(16). Hence, (20) is true and does not contradict any consequence of the given premises.

Clearly, one-sorted calculi that fail to distinguish between propositions and practitions fall before the Chisholm paradox, unless they complicate things beyond necessity, of course, such calculi cannot do justice to the linguistic data collected above in Part I.

4. *The paradox of the Second Best Plan.* The evidence for our old thesis that actions practically considered must be distinguished from actions considered as circumstances, as well as for the further thesis that it is the former that belong essentially and primarily in deontic judgements, comes from a consideration of the involvement of obligation and time. Examine the case of a person, or a group of persons, who are considering what to do during a certain period of time. It does not matter what sort of requiredness is involved in their considerations. For specificity we may suppose that they are operating under some type of utilitarian system u, and that they have, correctly, determined that there are several courses of action, pairwise incompatible, that would bring about the greatest value or utility:

$$A_1, A_2, A_3, \ldots$$
$$B_1, B_2, B_3, \ldots$$
$$C_1, C_2, C_3, \ldots$$

But things are not so simple that one course of these three is superior to the others at all times in all circumstances. The superior course, let us suppose, is the A-course, provided that every single A_i is performed in time. Suppose that as soon as one A-action A_i is not performed, then the agent ought$_u$ to shift to the B-series, and as soon as an action B_j is not performed, he ought$_u$ to shift to the C-series, and so on. This suggests that:

(21) The agents ought$_u$ to do A_1 and A_2 and A_3 and ...

and that, hence:

(22) The agents ought$_u$ to do A_2 at time t_2.

Suppose that

(23) The agents fail to do A_1.

Then:

(24) At time t_2 the agents ought$_u$ to do B_2, not A_2.

The contradiction between (21) and (24) is another "paradox". It can easily tempt a philosopher to take some drastic measures, for instance: (*i*) to reject the principle that "X ought$_i$ to do A&B" implies "X ought$_i$ to do B", which is a special case of (P*); or (*ii*) to put external *ad hoc*, not fruitful, restrictions on (P*) or some other deontic principle; (*iii*) to adopt the view that deontic operators have temporal parameters. Approach (*i*) would reject the derivation of (22) from (21) right away. But the view is *ad hoc*, i.e., purely local, and barren. We will say no more about it. Approach (*ii*) is also *ad hoc*. On the other hand, approach (*iii*) is intriguing and open-ended. It requires

(21*a*) The agents ought$_u$-at-time-t_1 to do A&B&C ...
(22*a*) The agents ought$_u$-at-time-t_2 to do A.

And the view would reject the derivation of (22*a*) from (21*a*).

View (*iii*) *deserves* to be developed in full detail, but we will not do so here. The view *can* be attached to the main theory put forward here. Indeed, that development could be a deepening of our theory. We must note, however, that the cases under consideration do *not* demand the tensification of 'ought' or the renunciation of (P*), (P*.1) or any of the principles listed in Part I Section 4. We can solve the "paradox" with materials already in hand, without introducing new principles, new primitives, or any other complications.

Propositions (21) and (22) do not describe the situation of our agents. Let us examine the situation in camera lenta. What happens is this:

(25) The A-series has more value *in toto* than the B-series, and (perhaps) the B-series has more value *in toto* than the C-series, etc.

(26) The agent's *obligation*$_u$ is the following:

(*a*) (*to do* A_1 *at* t_1)&(*to do* A_2 *at* t_2, IF they have done A_1 at t_1)&(*to do* A_3 *at* t_3, IF they have done A_1 at t_1 and A_2 at t_2)&...

(*b*) (*to do* B_2 *at* t_2, IF they have not done A_1 at t_1) & (*to do* B_3 *at* t_3, IF they have not done A_2 at t_2 but have done A_1 at t_1) & ...

(*c*) (*to do* C_3 *at* t_3, IF they have not done B_2 at t_2 but have done A_1 at t_1) & ...

In sentence (26) the italicized infinitive clauses formulae practitions, i.e., *actions deontically considered* as deontic foci; the non-italicized indicative clauses formulate propositions, i.e., *circumstances*. As we noted at the end of Part I Section 2, conditioning particles do *not* precede clauses expressing practitions. It is perfectly clear that (22), namely, "the agents ought_u to do A_2 at time t_2", does not follow from (26). But given the pairwise incompatibility of the courses of action, (24) *does* follow from (12) and (23). Evidently, then, *there is no paradox* and there is no need to construe $\textit{obligatoriness}_u$ as a generic one that is instantiated or specified differently at different times. Perhaps there are powerful reasons to temporalize the obligatorinesses determined by each normative systems, but the problems of the Second Best Plan are not such reasons: we *must* describe the situation of our agents as in (26), distinguishing between the actions that are foci of requiredness, and are, hence, practically considered, from the circumstances of that requiredness. This is precisely the distinction that cannot be maintained if we take deontic operators to operate on propositions. For if in (26) every clause expressed a proposition, (26) would become self-contradictory, and (26)(*a*) would become equivalent to (21). Clearly, "$p\&(\text{if } p, q)$" is equivalent to "$p\&q$."

Our ultimate point here is, therefore, that proposition (26)(*a*) is *not* equivalent to proposition (21). Hence, the duality expressed in (26)(*a*) by the contrast between subjectless infinitive clauses and full indicative clauses is irreducible.

5. *Ought and time.* According to (26) the temporal parameters $t_1, t_2, \ldots$ belong with the actions $A_1, B_1, \ldots$; $B_2 \ldots$ $\textit{Obligatoriness}_u$ *is timeless*. This timelessness is on a par with the timelessness of the possession by an object of a temporal property, e.g., being blue at 3 p.m. today. It is true, however, that deontic sentences do include a tensed verb in their expressions of deontic operators, for instance:

(27) John was required by the rules to retire at 65.
(28) It was obligatory_i that some men stayed behind.

These may suggest that obligations come and go. Yet this need not be any different from the way in which colors and shapes come and go leaving predication, on some views in any case, as timeless. In this respect, the English verb 'ought', inflexible and selfsame in all its constructions, seems to be philosophically the most perspicuous of all deontic words. Thus, (27) and (28) are better taken as:

(27a) John $\text{ought}_{\text{by the rules}}$ to have retired at 65.
(28a) It ought_i to have been that some men stayed behind.

There is, undoubtedly, an intimate connection between time and obligation. But we must be very careful to distinguish among: (a) the time of the action one ought to do; (b) the time of the oughtness; (c) the time of the truth of an ought-statement; and (d) the time of utterance or of the making of an ought-statement. Consider the following example:

(29) At 3 p.m. Pat ought to mail an apology to Mary.

What is 3 p.m. the time of? Obviously it is not the time of the utterance, nor is it the time of the truth of "Pat ought to mail an apology to Mary." What does it mean for this (sentence) to be true at some time or other? Obviously, 3 *p.m. is the time of the mailing*, not the time of the oughtness or obligatoriness.

By complicating our data somewhat we can establish the important datum:

(P.PO) *Principle of the present-tenseness of ought*

The English verb 'ought' is always in the present tense, so that: *Once an agent ought, according to rule R, to A, always thereafter he ought according to R to have Aed.* (Patently, the sense of 'ought' is the same in both occurrences.)

Consider the following array of examples in support of (P.PO):

(30) I ought to visit Mary next week.
(31) Next Sunday is when I now ought to visit Mary.
(32) Tomorrow is when I ought now to visit Mary.
(33) Today is when, now, I ought to visit Mary.
(34) Yesterday is when I ought, now, to have visited Mary.

Note that none of (30)–(34) implies that the visit took place, nor that it did not take place. Of course, given the inherent presentness of 'ought', the word 'now' is redundant. The chief point is that times and tenses change around 'ought', times and tenses that belong in the subordinate clause in the scope of 'ought', 'ought' remaining an unmovable bastion. The semantical unity of the array (30)–(34) requires a unitary account of the logic of ought that covers *all* of them, and respects the constant sense of 'ought' in them[13].

Perhaps we ought to distinguish the time of an action and the time of its *obligatoriness*$_i$. But we need more persuasive evidence than the mere phenomenon of *tense agreement* registered in (27) and (28).

6. *The Secretary or the Biconditional paradox.* We have considered "paradoxes" arising from conditionals. Let us now consider a biconditional example that is just a little complex, namely:

(35) Lydia ought_R to do the following:
(*a*) ARRIVE at her office at 8 a.m.;
(*b*) OPEN HER OFFICE TO THE PUBLIC AT 9 A.M.;
(*c*) just in case *she does not open her office to the public at* 9 *a.m.*, POST a note instructing the public to go to Room 311.

Note: 'ought_R' is short for 'ought according to rules R'.
The italicized antecedent of (*c*) is not the denial of clause (*b*). The former is a circumstance, the latter a practition.

a. Evidently the conjunction (*a*) (*b*) (*c*) lies within the scope of the deontic operator 'Lydia ought_R to do the following' in (35). This is a tall hurdle for the scope solutions to the deontic "paradoxes."

b. Undoubtedly, (35) is equivalent to the following conjunction:

(36) (*a*′) Lydia ought_R to (do the following): ARRIVE in her office at 8 a.m., and
(*b*′) Lydia ought_R to (do the following): OPEN her office to the public at 9 a.m., and
(*c*′) Lydia ought_R to do the following: just in case *she does not open her office to the public at* 9 *a.m.*, POST a note instructing the public to go to Room 311.

Furthermore, (36.*c*′) is equivalent to (37.*c*″) below, so that (36), and also (35), implies:

(37) (*a*″) = (*a*′)
(*b*″) = (*b*′)
(*c*″) just in case *she does not open her office to the public* 9 *a.m.*, Lydia ought_R to do the following:

POST a note instructing the public to go to Room 311.

c. Suppose, for the purpose of a *reductio*, that the italicized indicative clause ('*she does not open her office to the public at* 9 *a.m.*') in (*c*) and (*c*′) formulated an action of Lydia's in the same way in which the infinitive clause ('OPEN her office to the public at 9 a.m.) in (*b*) and (*b*′) do. Then it would be incorrect to derive (37) from (36). Furthermore, (36) would imply a falsehood. This is the Biconditional Paradox.

In the standard notation of one-sorted deontic calculi which do not distinguish between circumstances and actions deontically considered, (36) is represented in abbreviated version as:

(36′) O(Lydia arrives in her office) & O(Lydia opens her office) & O(Lydia does not open her office ≡ Lydia posts a note).

By standard principles of deontic logic we can derive a "paradoxical" result as follows:

(36′.1) O(Lydia opens her office ≡ Lydia does not post a note)
(From (36′) by simplification, and by exchanging negations in a biconditional.)

(36′.2) O(Lydia opens her office ⊃ Lydia does not post a note)
(From (36′.1) by propositional logic and distribution of 'O' through '&'.)

(36′.3) O(Lydia opens her office) ⊃ O(Lydia does not post a note)
(From (36′.2) by (P*.1) misinterpreted so as to apply to propositions, rather than to practitions.)

(36′.4) O(Lydia opens her office)
(From (36′) by simplification.)

(36′.5) O(Lydia does not post a note)
(From (36′.3) and (36′.4) by *modus ponens*.)

This is a paradoxical result. Clearly, (35), which is equivalent to (36), does *not* imply (36′.5), that Lydia is not to post a note instructing the public to go to Room 311. This "paradox" has *nothing* to do with whether Lydia's actions are unalterable or not, or with whether her actions are past, or future. The error lies in the identification of the deontically considered action of Lydia's mentioned in (*b*), Lydia TO OPEN HER OFFICE, with the circumstance mentioned in (*c*), Lydia's *opening her office*! The "paradox" can be solved very simply by respecting the indicative-infinitive contrast present in (35), and in (36). This respect for ordinary language can also lead us to recognize both the implication of (37) by (36), and the *non-equivalence* between (36) and (36′). Once again the simplest solution to one "paradox" is the solution to other "paradoxes".

7. *Alf Ross's "paradox."* This celebrated "paradox" was first posed for imperatives. It alleges that the inference "Mail the letter; therefore, mail the letter or burn the letter" is invalid. Naturally, since imperatives are closely related to deontic statements, many writers have thought that the following inference is invalid:

(D) Orayen ought, according to his contract with the Research Institute, to publish a yearly paper in the Bulletin. Therefore, Orayen ought, according to his contract with the Research Institute, do the following: publish a yearly paper in the Bulletin or burn all his previous publications.

It has never been entirely clear to me why (D) or its imperative counterpart is supposed to be paradoxical. The best I can make of the allegation is this: (i) the disjunctive order can be fulfilled by burning the letter, and likewise Orayen can fulfill his disjunctive obligation formulated in the conclusion of (D) by burning his publications; and (ii) the disjunctive order and the disjunctive obligation leave it *open* to the agent which disjunct to fulfill. But a little reflection suffices to show that a strong interpretation of (ii) is utterly incorrect. The adoption of a strongly interpreted (ii) is an erroneous form of *semantical atomism* – an atomism as erroneous here as anywhere. *No sentence is an island unto itself.* In particular, the members of an inference form a tightly knit community of thought contents. When one infers a conclusion one is considering one member of a related set – and one must remember the premises, or remember that the premises are still valid or true, or whatever property is supposed to be preserved in inference.

There are several reasons why a disjunctive order, or a disjunctive norm, may not *open* a genuine choice between alternatives. Not being an insular thing each order and norm must be related to the other orders or norms. Among such reasons we have: (*a*) one disjunct may itself be self-contradictory; (*b*) one disjunct may be physically impossible of realization; (*c*) one disjunct may be forbidden, wrong, or interdicted by another order or norm. Hence, *first*, presupposition (ii) is false, and, *second*, points (*a*)–(*c*) *must* be taken into account by any adequate theory of imperatives and norms.

Reason (*c*) is the crucial anti-atomistic principle we must fasten upon. Orders, requests, norms, etc., all come in families. The very same inference (D) shows this. It presents the norm "Orayen ought to publish a yearly paper in the Bulletin" as a primary norm, as the genitor of the norm "Orayen ought either to publish in the Bulletin or burn his previous publications." But *both* norms, or obligations, bind Orayen. He is to comply with *both* – and with whatever *other* obligations he has as well! Clearly, even if the conclusion of (D), to publish or to burn, gives Orayen a literal, genuine choice, this choice has to be paired with his obligation to publish. Obviously, he can satisfy both if and only if he publishes. There is, therefore, no paradox, only a confusion between the disjunctive character of a practical thought content and the existence of a genuine choice of courses of action. The disjunctive character of an imperative or a deontic judgment is a *local* matter pertaining to the imperative or judgment alone, by itself. The existence of a choice of courses of action is, on the other hand, a *structural* matter determined by *all* the orders and obligations that impinge upon the agent.

This is really all there is to the Ross's "paradox." Yet it is easy to construct a *reductio ad absurdum* of the 'paradox". Suppose it is paradoxical to infer as in (D). Hence, it is paradoxical to infer as in (D′) below:

(D′) Orayen ought, according to his contract with the Research Institute, to publish a yearly paper in the Bulletin. Hence, Orayen ought, according to his contract with the Research Institute, to publish a yearly paper in the Bulletin or NOT burn all his previous publications.

But then the conjunction of the conclusion of (D) with the conclusion of (D′) is also paradoxical. The conclusion is equivalent to "Orayen ought, according to his contract with the Research Institute, both to publish a yearly paper in the Bulletin or to burn his previous publications, and to publish a yearly paper in the Bulletin or NOT to burn his previous publications." Clearly, this is simply equivalent to the premise of both (D) and (D′), which is not paradoxical at all. Hence, given the obvious principles used in establishing the preceding equivalences, the supposition that (D) is paradoxical is contradictory.

Some writers have observed correctly that few persons would engage in reasoned commanding of the form "Do A, therefore, do A or B." Similarly, few would draw for others inference (D) or (D′). Yet this fact about *communication* or *speech* cannot tell against the implications behind the inferences in question. There are reasons pertaining to the transfer of information that explain why those inferences are not drawn. And the very same reasons apply to the corresponding indicative or propositional inferences. We must simply forget Ross's "paradox."

III. THE EXTENSIONALITY OF ORDINARY DEONTIC ENGLISH

The standard approach to deontic logic is to treat deontic operators on a par with the alethic modalities (necessarily true, possibly true, impossibly true). Often a deontic logic or calculus is said to be an alethic-modal calculus that lacks the reflexive, or consistency axiom "Obligatory $(p) \supset p$."

Of course, there is no point in objecting to the stipulation that some such calculus is to be called deontic. The only real issue is whether such so-called deontic calculi are adequate representations of our *ordinary deontic language* and of our *ordinary deontic reasonings* (about obligations, wrongs, interdictions, etc.) For many years now I have been claiming that such an approach to the logical structure of deontic English is seriously flawed. I have claimed that *deontic English is thoroughly extensional.* Just to mention the most crucial aspects:

1. English deontic operators with the same adverbial qualifier do not iterate;
2. English deontic operators do not create referential opacity;
3. English deontic operators do not create a breakdown of identity;
4. English deontic operators do not create by themselves variations in the domain of quantification.

The rationale for these extensional aspects is not difficult to fathom. Deontic language has to do with the production of conduct. Deontic language must be effective, and in order to gain its maximum of effectiveness it must be wholly extensional and behaviorist. Obligations are, thus, impervious to how one thinks of the agents involved in one's actions. Of course, *intentional action is intensional*, and obligations are indeed concerned with the production of intentional action, but the particular intentions with their intensional aspects are irrelevant to the obligatoriness of the action, unless the action itself brings in its own intensional aspects.

Let us discuss some of the preceding features of extensionality.

1. *English deontic quantification is extensional.* There are two types of proposition that must be carefully distinguished. To begin with:

(1) It is *obligatory*$_i$ for everybody to A.

Here the propositional function is its is *obligatory*$_i$ *(that) to A*, or, if you wish, it is *obligatory*$_i$ *(that) for x to A*. This function contains the practitional function *to A* or *x to A*. Propositions of form (1) contrast with the corresponding propositions of the form

(2) It is *obligatory*$_i$ that everybody *A*.

These propositions result from the application of the deontic operator *it is* *obligatory$_i$ that* to the universally quantified prescription *Everybody (to) A.* What is the difference between deontic statements (1) and (2)? The sentences (1) and (2) contain the syntactical contrast between practitional infinitive and practitional subjunctive clauses, discussed in Part I Section 5.

Reflexion does not reveal any difference in what (1) and (2) demand. They appear to be equivalent. In the case of a finite domain of agents they are both tantamount to a conjunction of the form agent 1 to do A agent 2 to do A ...

Propositions of forms (1) and (2) are alike in that both deal with allegedly real, not merely possible obligations. Real obligations belong, naturally, to real agents and demand real actions on real objects and patients. Thus, corresponding propositions of forms (1) and (2) deal with the same domain of agents. There is no reason at all to suppose that the exchange of the quantifier and the deontic operator, when we move from (1) to (2), or vice versa, alters the domain of the agents.

Here is a very important point in which deontic propositions differ from ordinary propositional modalities like necessity and possibility. A proposition of the form *Everybody is necessarily P* is clearly about all existing people, but the corresponding proposition of the form *Necessarily, everybody is P*, or better, *It is necessarily the case that everybody be P* is not clearly just about all existing persons; rather, it is about all *possible* persons. Thus, the interchange of modality and quantifier does clearly in this case reveal a change in the domain of the quantifier.

Consider a proposition of the form, like (1), $\forall x$ M(x is P), and the corresponding proposition of the form, like (2) M$\forall x$(x is P), where the quantifier has a different domain, depending on whether it precedes or follows the modality M. Then a subtle ambiguity appears in the sentential function 'M(x is P)'. If 'x' represents values of the external quantifier in $\forall x$ M(x is P), then the propositional function M(x is P), as well as each of its instances M(a is P), implies the existentially quantified proposition $\exists x$ M(x is P). But if the free occurrence of 'x' in 'M(x is P)' represents a value of the inside quantifier in M$\forall x$(x is P), then the existential implication is ruled out.

Now, there is *no* such ambiguity in the case of sentential functions of the form 'it is obligatory$_i$ that x do A'. Given the normal semantical conventions of English, that sentence schema expresses a propositional function from which, and from whose instances, we can derive the proposition *There is someone for whom it is obligatory$_i$ that he do A*, i.e., more idiomatically, *It is obligatory$_i$ for someone to do A*. Therefore, deontic operators do not in any way affect the range of the quantifiers.

Furthermore, there are no two kinds of positions in a deontic function, namely, those that do and those that do not allow of existential generalization. In short, we summarize all these points, by saying:

(E.Q*) Deontic propositional functions are extensional with respect to quantification.

Naturally, deontic logic does not preclude people from dying or getting born. At different times in the history of each institution there may be different sets of agents who have the corresponding institutional obligations or duties. This by itself imposes no need to alter the standard conception of quantification when this is combined with deontic operators. The phenomena of death and birth can be handled by either discussing the obligations of a type *i* at a given time, when the domain of agents is fixed, or by doing what is customarily done in the general theory of quantification, namely, to consider in the domain of a quantifier any entity that at some time or other exists. Clearly, then, existing at a time is a property of objects. The main point is, however, that temporal parameters belong in this context of quantification to agents and persons and objects. So here again is no need to tensify the deontic operators. This complements Part II Section 5.

In any case, it seems that:

> Propositions of the form *It is obligatory$_i$ for everybody to A* are equivalent to their corresponding propositions of the form *It is obligatory$_i$ that everybody A.*

2. *Deontic English is extensional with respect to Identity.* Obviously, identity statements are statements not practitions; therefore, they cannot give rise to deontic judgments. Thus, the only question about identity in the deontic logic of English is whether the standard principle of substitutivity of identity holds unrestricitedly for deontic propositions. This principle is:

(Id.) If a sentence of the form '$a = b$' expresses a true proposition and so does a sentence S, containing occurrences of the individual symbol *a*, then a sentence S′, obtained from S by replacing some occurrences of *a* with occurrences of *b*, also expresses a true proposition.

This principle fails in so-called intentional, or non-extensional, contexts. Among such contexts are both sentences containing words expressive of the ordinary propositional alethic modalities and sentences expressive of contemplative or theoretical – not practical – psychological attitudes: those that involve belief or cognition. Thus, 'Anthony believes that Napoleon was defeated at Waterloo' may express a truth while 'Anthony believes that Empress Josephine's divorced husband was defeated at Waterloo' expresses a falsehood.[14]

Obligation contexts are *not* intensional with respect to identity. While a man is the referent of another man's beliefs under some characterization or other, a man's obligations are *his* regardless of his characterizations, once he has got them. To be sure, obligations accrue to a man *because* of his circumstances and relationships to other people, but once they accrue they belong to the whole of him, so to speak. For example, let 'P' be any adjective such that Richard M. Nixon is the one and only man who in 1925 was P; then the sentence 'The man who in 1925 was P ought to finish the Viet Nam war in 1971' is as true, or as false, as the sentence 'Richard M. Nixon ought to finish the Viet Nam war in 1971'. The same happens with the patients of the actions that are *obligatory*$_i$. If a man has a duty$_i$ to do action A to a person or thing c, then that man has the same duty to do action A to entity d, if $c = d$. In short:

(E.Id*) Deontic propositional functions are extensional with respect to identity.

Palpably, the extensionality of deontic propositional functions with respect to quantification is intimately bound up with their extensionality with respect to identity. The fundamental thing is that the positions (represented by individual variables) in deontic functions are just of one type, namely, the type that allows quantification. Thus, a unique characterization of an individual inside the scope of a deontic operator is not prevented by that scope to refer to the entity it purports to refer. That is to say,

> Unique characterizations (i.e., what are expressed by uniquely referring descriptions) are not bound by the scope of the deontic operators in which they lie. More precisely, *Obligatory*$_i$ $\{\phi(\alpha)\}_\alpha$ is equivalent to $\{$*Obligatory*$_i$ $\phi(\alpha)\}_\alpha$, where '$\{\ \}_\alpha$' indicates the scope of α.

For instance, the three propositions below are equivalent:

(3) It is obligatory$_i$ that the one man who came late withdraw.

(4) The following is obligatory$_i$: that it being the case that just one man came late he withdraw.

(5) It is the case that just one man came late and it is obligatory$_i$ that he withdraw.

The pairwise equivalence of (3)–(5) is obviously dependent upon the fact that a unique characterization of an individual is a propositional complex, not a practitional one. Thus,

(E.D*) The extensionality of deontic propositional functions depends on the fundamental distinction between propositions and practitions as well as on the laws governing their transactions.

IV. THE DEFEASIBILITY CONDITIONS OF ENGLISH DEONTIC STATEMENTS

1. *Powers' Susy Mae Example.* Many powerful philosophers have rightly worried about the defeasibility of obligation claims. Lawrence Powers, in the paper mentioned earlier, has discussed a case that has been adduced in support of a so-called dyadic or conditional deontic logic. The idea of a special conditional has been strongly suggested by causal statements. Powers' example has been celebrated because it connects causality and obligation. Here is the example:

(1) John Doe has impregnated Susy Mae; so, he ought to marry her.

(2) But John kills Susy Mae and then it is not the case that he ought to marry her.

The conditional approach to deontic logic is so called because it builds the logic of *ought* on a special *if* connective called "conditional", which is not an extensional connective. In particular that connective is not governed by the principle

(C) *If p, then it is obligatory that A* implies *If p and q, then it is obligatory that A.*

But the situation is more complicated. First, it has been too quickly assumed that John Doe's killing Susy Mae automatically cancels his obligation to marry her – *if* he has one to start with. It is not out of order to suppose that some laws that demand marriage upon impregnation continue to require that the marriage be officiated with the dead Susy Mae, so that she goes away without the capital sin in question. The situation I am considering is in substance not different from the American practice of promoting George Washington posthumously to whatever may be the highest rank in the American Army, the latest one being to the rank of a five-star general when this rank was created during World War II. Note also that Susy Mae must be unmarried, unless we allow polyandria. It may be said that this quibbling is irrelevant, so that as long as *some* laws do not require John to marry a dead Susy, (C) fails. But this is not what the example establishes. The example with out commentary establishes that

> *Different* normative systems may have *different* conditions for the "cancellation" of obligations.

Now the question is: How are we to represent this in general deontic logic? Undoubtedly, the claim that (C) is false is one way, and it has several advantages.

Yet the denial of (C) is a negative solution that looks like overkill and leaves a converse problem unsolved. It tampers with the extensionality of ought discussed above. And it cannot account for the fact that only in *very* few cases we *seem* to have the situation of an obligation cancelled by the addition of another circumstance. The denial of (C) is a drastic reaction to the *exceptional* cases. Just that if John Doe ought *really* to marry Susy Mae because he got her pregnant, then he obviously ought to marry her regardless of most of what happens to them. For practically anything one can think of (C) holds obviously. *This too has to be explained.*

The loss of the extensionality is serious. A careful attention to Powers' example suggests that the 'if' in (1) is indeed more like the expression of a material conditional. For instance, it is governed by transposition:

(3) *Only if he didn't impregnate her, it is not the case that John Doe ought to marry Susy Mae*

is equivalent to (1). Furthermore, (1) also implies:

(4) (Either) John didn't impregnate Susy Mae, or he ought to marry her.

In general, since rules and norms are for the most part arbitrary stipulations, there is often *no* prior relevant or logical connection between conditions of obligation and obligatory actions. More deeply: the assumed freedom of practical thinking, which is presupposed in the creation of normative systems and institutions, suggests very strongly that the conditionality of rules and norms is an arbitrary and stipulative conditionality, one that can be fully grounded on the unrestricted conditionality of *material implication*. Normative "necessity" is to a large extent stipulative. And this is the profound rationale for the extensionality of the deontic operators.

In brief, I submit that the exegesis of Powers' powerful Susy Mae example establishes that:

(C.D). Each deontic operator D_i is paired with a characteristic set C_i of necessary conditions for *ought*$_i$-ness.

Thus, *in the normative system* j *that Powers is considering, where nobody marries dead people*:

(1*a*) At time *t* John Doe impregnated Susy Mae, and at time *t* (and later at t_1) John Doe *ought*$_j$ to marry Susy Mae *only if* Susy Mae is alive at *t* (or t_1). [Of course, similarly for the other cancellation conditions in C_j.]

We find, therefore, no need to leave the normal unproblematic cases unelucidated. Hence:

Deontic operators are governed by (C).

The preceding does not, of course, imply that the characteristic obligatoriness "cancellation" conditions C_i of a normative system N_i are always very simple and fully spelled out in some text publicly available and easy to read. In the very important case of a legal system there are wholly appropriate provisions that establish a fixed and clear procedure for determining the pervasive necessary conditions C_L of a given obligatoriness$_L$, without the procedure specifying in advance exactly what they are. For instance, judges or certain administrative officers have as one of their primary roles to determine some of the conditions C_L of obligatoriness$_L$. But the law, or the statutes of an institution, make the procedure final at a certain point. And the law of the land and the by-laws of institutions are more efficient by having both an open-ended set C_L of necessary conditions of obligatoriness$_L$ and a precise procedure for determining with *finality* certain element of C_L. They can cope with changes in circumstances.

2. *The dyadic or conditional approach to deontic logic.* Powers' Susy Mae example has been very influential, and rightly so. But most deontic logicians have simply taken it to mean that principle (C) above is false. They have

conceived that the 'if' in Powers' example is one that has some peculiar properties, including the property of not allowing the detaching of the clause that follows it. Thus, Powers' example (1) is parsed as follows:

(1.*d*) John Doe ought-if-he-has-impregnated-Susy-Mae to marry Susy Mae.

Then they proceed to formalize (1.*d*) as follows:

(1.*d.f.*) O(John Doe marries Susy Mae/John Doe has impregnated her).

This conception does solve the Powers' paradox, and it can be used to solve some of the other paradoxes. My complaint is that it accomplishes little at the cost of both neglecting crucial data – like those accumulated in Part I and the recent data about the material-implication properties of the 'if' in Powers' example – and has to introduce serious complications.

The approach neglects also the data displayed above about the fact that even if John Doe has killed Susy Mae he may still have$_k$ to marry her in some systems with different cancellation conditions C_k.

The dyadic approach neglects the abundant data that foster the practition-proposition duality. Those data, as we have noted establishes that we must distinguish, as a special case, between circumstances – which need not be obligations – that are necessary (or sufficient) conditions for obligations, and obligations that are conditions (necessary or sufficient) for other obligations. Thus, we cannot accept van Fraassen's guiding principle below if we want a deontic logic that illuminates more than the simplest deontic implications:

> As a further criterion, I propose that if something is a necessary *condition* of discharging an obligation then it is itself an obligation, given the same *conditions*[15]. [My italics.]

The so-called conditional deontic approach recognizes a duality of components in the scope of deontic judgment. A judgment like *If (given that) John Doe impregnated Susy Mae John Doe ought to marry Susy Mae* is taken by the "conditional" approach as having the connective 'if (given that) – ought' which treats the components *John Doe impregnated Susy Mae* and *John Doe marries Susy Mae* very differently. While I command this duality of roles, I find it too limited to do justice to the duality of circumstance and focus of obligation established in Part I. The pervasiveness of the latter duality demands full recognition.

The "conditional" approach recognizes also the relativity of duties or requirements to certain grounds or reasons. Again, this pleases me, but I am unhappy with the approach because it fuses several relativities and it fails to recognize a very crucial aspect of duties and requirements. *Duties are relative to circumstances*, and this relativity yields genuine conditional or disjunctive duties. *Duties are also relative to acts or processes of endorsement*, and, hence, to the reasons for such endorsements. But this is an entirely different matter. Here we do not have conditional duties, but the *adverbial qualifications* I have for over two decades been representing by means of subscripts attached to deontic words. The standard "conditional" approach to the logic of deontic judgments (as contrasted with the study of uninterpreted mathematical formalisms called deontic calculi or deontic logics) does not distinguish this. In fact the examples often given, like Powers' (1) above, "If John impregnated Susy, he ought to marry her", do not have a "condition" in the sense of the "conditional" approach. As we pointed out above, (1) is governed by transposition and has other properties of material implication. Thus, the "conditional" approach to deontic logic, which treats *ought*-ness as a form of alethic or intensional necessity seems *at initio* to be proceeding in the wrong direction. The "conditional" approach ignores the crucial aspect of deontic judgments of belonging into systems. But, as noted, this belonging is *not* a connective linking two propositions. Each system is an arrangement from a certain point of view of *all* propositions and *all* practitions[16].

V. THE SIMPLEST AND RICHEST DEONTIC CALCULUS IN WHICH THE SO-CALLED DEONTIC PARADOXES ABORT

We proceed now to formulate the solution to all the deontic "paradoxes" in a fell swoop, namely, the fell swoop of a calculus containing the proposition-practition distinction. The calculus includes also quantifiers and identity. It satisfies all the conditions of adequacy we have encountered in our exegesis of ordinary deontic English, but we leave to the anxious reader committed to philosophical knowledge to establish that by himself. We prove only one theorem, which has to do with the fundamental extensionality of deontic English.

1. *Formal deontic languages* D_i*. We start by constructing a large number of such syntactical structures, one for each prima facie obligatoriness and one for the overriding ought. These languages will be called D_i*, for $i = 1, 2, 3, \ldots$, where D_1* is the language of the pure overriding ought.

Primitive signs: Individual constants; individual variables; predicate constants; the connectives '$\sim$' and '&' expressing negation ('it is not the case') and logical conjunction ('and', 'but', 'although'), respectively; the square brackets '['and']' which paired constitute the sign of prescriptive copulation; the sign 'O_i' of oughtness, or obligatoriness$_i$; the identity sign '$=$', and the parentheses '('and')', which are both signs of grouping as well as of propositional copulation.

Rules of Formation: We use Quine's corners implicitly throughout. Let the small letters 'p', 'q', 'r', range over *indicatives* (i.e., expressions of propositions or propositional functions); let the capital letters 'A' and 'B' range over *practitives* (i.e., expressions of practitions or practitional functions); let 'p^*' and 'q^*' range over both indicatives and practitives; let 'C' range over predicates and 'x' and 'y' with subscripts or not range over individual variables, unless otherwise specified.

(a) The Indicatives of D_i^* are the sequences of signs of D_i^* having one of the forms:

1. $C(x_i, \ldots, x_n)$, where C is an n-adic predicate and each x_i is an individual constant or variable;
2. $(\sim p)$; 3. $(p \& q)$;
4. $((x)p)$; 5. $(O_i A)$;
6. $(x_i = x_j)$ where each of x_i and x_j is an individual constant or an individual variable.

(b) *Practitives* or *imperative-resolutives* of D_i^* are the sequences of signs of D_i^* having one of the following forms:

1. $C[x_i, \ldots, x_n]$, where C is an n-adic predicate, each x_i is an individual variable or constant;
2. $(\sim B)$; 3. $(p \& B)$;
4. $(B \& p)$; 5. $(B \& A)$;
6. $((x)B)$.

(c) The indicatives and the practitives of D_i* are all the wffs of D_i*.

Definitions. We adopt the usual definitions of 'bound variable' and the definitions of the other connectives and the existential quantifier but we generalize them so as to cover practitives. The occurrence of an individual variable x in a wff p^* is *bound* in p^* if and only if it is an occurrence in a wff which is a part of p^* and is of the form $(x)q^*$. An occurrence of an individual variable x in a wff p^* is *free* if and only if it is not bound in p^*. The *bound* [free] *variables* of a wff p^* are the variables which have bound [free] occurrences in p^*.

Def. 1 $(p^* \vee q^*) = (\sim((\sim p^*)\&(\sim q^*)))$

Def. 2 $(p^* \supset q^*) = (\sim(p^* \& (\sim q^*)))$

Def. 3 $(p^* \equiv q^*) = ((p^* \& q^*) \vee ((\sim p^*)\&(\sim q^*)))$

Def. 4 $((\exists x)p^*) = (\sim((x)(\sim p^*)))$

We introduce the following deontic definitions:

Def. 11 $(\mathrm{P}_i\mathrm{A}) = (\sim(\mathrm{O}_i(\sim \mathrm{A})))$

It is permitted$_i$ that A if and only if it is not obligatory$_i$ that not-A.

Def. 12 $(\mathrm{W}_i\mathrm{A}) = (\mathrm{O}_i(\sim \mathrm{A}))$

It is wrong$_i$ that A if and only if it is obligatory$_i$ that not-A.

Def. 13 $(\mathrm{L}_i\mathrm{A}) = ((\sim(\mathrm{O}_i(\sim \mathrm{A})))\&(\sim(\mathrm{O}_i\mathrm{A})))$

It is optional$_i$ (or there is a liberty$_i$ that A) if and only if neither is it obligatory$_i$ that not-A nor is it obligatory$_i$ that A.

We adopt the standard conventions on parentheses:
(1) we drop the pair of outermost parenthesis of each wff; (2) we associate chains of conjunctions, or disjunctions, to the left; (3) we rank the connectives in the following order of increasing scope or bindingness: deontic operators and quantifiers; $\sim$; $\&$; $\vee$; $\supset$; $\equiv$.

The desired primary interpretation for each language D_i^* is this: (1) indicatives should express propositions; (2) practitives should express practitions; (3) predicates should express properties or conditions. Patently, each language D_i^* satisfies some crucial data: (i) the difference between atomic propositions and corresponding atomic practitions is represented by the difference between the two expressions of copulation: '(...)' and '[...]'; (ii) deontic judgments are represented as propositions; (iii) mixed indicative-practitive compounds are practitive; (iv) the deontic operators expressed by 'O_i', 'P_i', 'W_i' and 'L_i' apply to practitions, not to propositions; (v) there are no iterations of deontic operators.

The part of each language D_i^* that includes neither quantifiers nor identity will be called D_i^{c*}. That is, D_i^{c*} is D_i^* without the wffs determined by rules (a)4, (a)6, or (b)6.

2. *The axiomatic systems D_i^{**}.* We build on each deontic language D_i^* an axiomatic system D_i^{**}, which is constituted by the axioms and rules of

derivation enunciated below. We shall take advantage of the above definitions and conventions on parentheses. We shall also use the following convention. Let X be a wff. Then an expression of the form '$X(a|b)$' stands for any wff resulting from X by replacing some occurrences of a in X with occurrences of b, where all occurrences in question of a or b (or both) are free if a or b (or both) are variables. $X(a\|b)$ is the wff $X(a|b)$ that results when *all* occurrences of (free) a in X are replaced with (free) occurrences of b.

AXIOMS. The axioms of D_i^{**} are all and only the wffs of D_i^* that have at least one of the following forms:

A0. $O_iA \supset C_i$, where C_i is the conjunction of the necessary or "cancellation" conditions of *ought*$_i$-ness.

A1. p^*, if p^* has the form of a truth-table tautology.

A11. $O_iA \supset \sim O_i \sim A$

A11a. $O_1A \supset A$

A111. $(x)p^* \supset p^*(x\|y)$

A1111. $(\exists y)(x = y)$

Note: A11a replaces A11 in the system $O_1(D_1^*)$ for the overriding *ought*$_1$. We adopt the standard definitions of 'proof' and 'theorem':

1. A *proof of* D_i^{**} is a sequence of wffs of D_i^* such that each member of it is neither (i) an axiom of D_i^{**} or (ii) a wff derivable from previous members of the sequence by one application of the derivation rules R1, R11, R111 or R1111 of D_i^{**}, formulated below.

2. A *theorem of* D_i^{**} is the last member of a proof of D_i^{**}. We write '$\vdash_i X$' to abbreviate 'X is a theorem of D_i^{**}'. We write '$\vdash_{i-a} X$' as short for 'X is a theorem of D_i^{**} provable without axiom A11a'.

3. $p_1^*, p_2^*, \ldots, p_n^* \vdash_i q^*$, if and only if $\vdash_i p_1^* \& p_2^* \; \& \ldots \& \; p_n^* \supset q^*$.

Rules of Derivation. The rules of derivation of D_i^{**} are:

R1. *Modus ponens*: From p^* and $p^* \supset q^*$, derive q^*.

R11. OG: If $\vdash_{i-a}(p \& A \;\& \ldots \& A_n \supset B)$, then $\vdash_i (x)(p \;\& \; O_iA_1 \;\& \ldots \& O_iA_n) \supset O_i(x)B \& (x)O_iB$, where $n \geq 0$.

R111. UG: If $\vdash_i p^* \supset q^*$, then $\vdash_i p^* \supset (x)q^*$, provided that p^* has no free occurrences of x.

R1111. ID: If $\vdash_i p^* \supset q^*$, then $\vdash_i x = y \supset (p^* \supset q^*(x|y))$

The axioms and the rules are labeled so that each pair $\{A_m, R_m\}$ characterizes one layer of implication:

1. $\{A_1, R_1\}$ is the *basic sentential calculus*, i.e., the propositional calculus generalized to practitions.

2. $\{A_1, A_{11}, R_1, R'_{11}\}$, where R′11 is R11 without quantifiers, is the *basic propositional-practitional deontic calculus.*

3. $\{A_1, A_{111}, R_1, R_{111}\}$ is the *calculus of quantification generalized to practitions.*

4. R11 is restricted to theorems not depending on A11a in order to allow the basic logic of conflicts of duties to be represented by the union of deontic calculi D_i^{**}. A conflict of duties is the truth of a conjunction of the form "$O_iA \& O_jB$," where A and B are at least causally incompatible and $i \neq j$. The solution to the conflict brings in the overriding *ought*$_1$ so that we come to discover, or postulate, the truth of a conjunction of the form "$O_iA \& O_iB \& O_1A \& \sim O_1B$." Thus, in general "$O_1A \supset O_iA$" is not a logical truth. This dreadful consequence would follow if we drop the restriction of R11 to $(i-a)$-theorems, thus:

1.	$\vdash_i O_1A \supset A$	A11a
2.	$\vdash_i O_1A \supset O_iA$	1; DR11.2 below.

3. *Some theorems and meta-theorems*

From the preceding axioms by means of rules R_1–R_{111} we can derive the following theorems and derived rules:

Th.1. $\vdash_i O_iA \& O_iB \equiv O_i(A \& B)$
Th.2. $\vdash_i O_i(A \supset B) \supset (O_iA \supset O_iB)$
Th.3. $\vdash_i p \& O_iB \equiv O_i(p \& B)$
Th.4. $\vdash_i O_i(A \supset p) \equiv (P_iA \supset p)$
Th.5. $\vdash_i O_i(p \supset A) \equiv (p \supset O_iA)$
Th.11. $\vdash_i (x)O_iA \equiv O_i(x)A$
DR11.1. If $\vdash_i A \supset B$, then $\vdash_i O_iA \supset O_iB$
DR11.2. If $\vdash_i p \supset B$, then $\vdash_i p \supset O_iB$.

These are the deontic axioms and rules stated at the end of §1 above. They together with A1, A11, A1111, R1, R111, and R1111 constitute a realization of the informal axiomatization discussed in §1.

The axiomatic system D_i^{c**} is the subsystem of D_i^{**} built on deontic language D_i^* constituted by axioms A1, A11 and rules R1, and R′11. It is decidable, e.g., by the techniques Quine has developed for monadic quantification with propositional variables[11]. We will not discuss this further here.

METATHEOREM I*. Let A wff of D_i^* containing no quantifiers. Then there is wff B of D_i^* such that $\vdash_i A \equiv B$, and no indicative occurs in B in the scope of O_i. *Proof* by induction on the length of A.

Meta-theorems representing the laws of deontic English (at the end of Part I) hold for D_i^{**}.

4. *An extensionality theorem in* D_i^{**}. It is easy to establish that identity has unrestricted substitutivity in the calculi D_i^{**}. We establish a more interesting result, namely, that *ought* does *not* introduce referential opacity in the sense that a definite description, analyzed a la Russell, that has the largest scope in the operand of O_i, automatically gains the largest scope including O_i. This result is included in the following theorem.

THEOREM.

$$O(\exists x)((y)(p(y) \supset x = y) \,\&\, p(y \| x) \,\&\, (B(x))$$
$$\equiv (\exists x)((y)(p(y) \supset x = y) \,\&\, p(y \| x) \,\&\, OB(x))$$

Proof. I will use 'PC' to signal a set of applications of A1 and R1. The sign '$\vdash$' is deleted before each formula for convenience. We take variables x and z that do not occur free in $p(y)$, so that $p(y \| x \| z)$ is $p(y \| z)$.

1.	$(y)(p(y) \supset x = y) \supset (p(y \| z) \supset x = z)$	A111
2.	$((y)(p(y) \supset x = y) \,\&\, p(y \| x) \,\&$ $B(x)) \supset (p(y \| z) \supset x = z)$	1; PC
3.	$((y)(p(y) \supset x = y) \,\&\, p(y \| x) \,\&$ $B(x)) \supset ((y)(p(y) \supset x = y) \,\&$ $p(y \| x) \,\&\, B(x))$	A1
4.	$x = z \supset ((y)(p(y) \supset x = y) \,\&$ $p(y \| x) \,\&\, B(x) \supset (y)(p(y) \supset z = y) \,\&$ $p(y \| z) \,\&\, B(x \| z))$	3; R1111
5.	$(y)(p(y) \supset x = y) \,\&\, p(y \| x) \,\&\, B(x)$ $\supset (p(y \| z) \supset ((y)(p(y) \supset z = y) \,\&$ $p(y \| z) \,\&\, B(x \| z)))$	2, 4; PC
6.	$\sim (p(y \| z) \supset ((y)(p(y) \supset z = y) \,\&$ $p(y \| z) \,\&\, B(x \| z))) \supset$ $(x) \sim ((y)(p(y) \supset x = y) \,\&\, p(y \| x) \,\&$ $B(x))$	5; PC; R111

7.	$(\exists x)((y)(p(y) \supset x = y)p(y \parallel x) \,\&\, B(x)) \supset (p(y \parallel z) \supset ((y)(p(y) \supset z = y) \,\&\, p(y \parallel z) \,\&\, B(x \parallel z)))$	6; PC; Def 4
8.	$p(y \parallel z) \,\&\, (\exists x)((y)(p(y) \supset x = y) \,\&\, p(y \parallel x) \,\&\, B(x)) \supset ((y)(p(y) \supset z = y) \,\&\, p(y \parallel z) \,\&\, B(x \parallel z))$	7; PC
9.	$p(y \parallel z) \,\&\, O(\exists x)((y)(p(y) \supset x = y) \,\&\, p(y \parallel x) \,\&\, B(x)) \supset O((y)(p(y) \supset z = y) \,\&\, p(y \parallel z) \,\&\, B(x \parallel z))$	8; R11, R111, PC
10.	(x) [9]	9; PC, R111: $z \rightarrow x$
11.	$(\exists x)((p(y \parallel x) \,\&\, O(\exists x)((y)(p(y) \supset x = y) \,\&\, p(y \parallel x) \,\&\, Bx)) \supset (\exists x)O((y)(p(y) \supset x = y) \,\&\, p(y \parallel x) \,\&\, B(x))$	10; PC, R111
12.	$(\exists x)(p(y \parallel x)) \,\&\, O(\exists x)((y)(p(y) \supset x = y) \,\&\, p(y \parallel x) \,\&\, B(x)) \supset (\exists x)O((y)(p(y) \supset x = y) \,\&\, p(y \parallel x) \,\&\, B(x))$	11; PC, R111
13.	$\sim (\exists x)(p(y \parallel x) \supset \sim (\exists x)((y)(p(y) \supset x = y) \,\&\, p(y \parallel x) \,\&\, B(x))$	PC, R111
14.	$\sim (\exists x)(p(y \parallel x) \supset O \sim (\exists x)((y)(p(y) \supset x = y) \,\&\, p(y \parallel x) \,\&\, B(x))$	13; R11, (n = 0)
15.	$O \sim (\exists x)((y)(p(y) \supset x = y) \,\&\, p(y \parallel x) \,\&\, B(x)) \supset \sim O \sim \sim (\exists x)((y)(p(y) \supset x = y) \,\&\, p(y \parallel x) \,\&\, B(x))$	A2
16.	$O(\exists x)((y)(p(y) \supset x = y) \,\&\, p(y \parallel x) \,\&\, B(x)) \supset (\exists x)(p(y \parallel x) \,\&\, O(\exists x)((y)(p(y) \supset x = y) \,\&\, p(y \parallel x) \,\&\, B(x))$	14, 15; PC
17.	$O(\exists x)((y)(p(y) \supset x = y) \,\&\, p(y \parallel x) \,\&\, B(x)) \supset (\exists x)O((y)(p(y) \supset x = y) \,\&\, p(y \parallel x) \,\&\, B(x))$	12, 16; PC
18.	$O((y)(p(y) \supset x = y) \,\&\, p(y \parallel x) \,\&\, B(x)) \supset ((y)(p(y) \supset x = y) \,\&\, p(y \parallel x)) \,\&\, O\,B(x)$	3; R11, R111, PC
19.	$(\exists x)O((y)(p(y) \supset x = y) \,\&\, (p(y \parallel x) \,\&\, B(x)) \supset (\exists x)((y)(p(y) \supset x = y) \,\&\, p(y \parallel x) \,\&\, O\,B(x))$	18; R111, PC, Def 4

*20.	$O(\exists x)((y)(p(y) \supset x = y) \& p(y \parallel x) \& B(x)) \supset (\exists x)((y)(p(y) \supset x = y) \& p(y \parallel x) \& O\,B(x))$	17, 19; PC
21.	$(y)(p(y) \supset x = y) \& p(y \parallel x) \& B(x) \supset (\exists x)((y)(p(y) \supset x = y) \& p(y \parallel x) \& B(x))$	3; A111, PC, Def 4
22.	$((y)(p(y) \supset x = y) \& p(y \parallel x)) \& OB(x) \supset O(\exists x)((y)(p(y) \supset x = y) \& p(y \parallel x) B(x))$	21; R11, R111, PC
*23.	$(\exists x)((y)(p(y) \supset x = y) \& p(y \parallel x) \& O\,B(x)) \supset O(\exists x)(p(y) \supset x = y) \& p(y \parallel x) \& B(x))$	22; R111, PC, Def 4
24.	Theorem follows from 20 and 23 by PC.	

5. *Models for the calculi* D_i^{**}. We proceed by parts.

1. D_i^{c**}. A model M for a system D_i^{c*} is an order triple $\langle W_0, \mathbf{W}, \mathbf{I} \rangle$, where $\mathbf{W}$ is a nonempty set of entities called *possible deontic worlds*, or just worlds, for short; W_0 is a member of $\mathbf{W}$ and is called the *real* or *designated world*, and $\mathbf{I}$ is a two-argument function that assigns to each pair of a world and a wff of D_i^{c*} one element of the set $\{\mathbf{1}, \mathbf{2}\}$, in accordance with the following rules, where W_j is a member of $\mathbf{W}$.

R1. $\mathbf{I}(p^*, W_j) = \mathbf{1}$ or $\mathbf{2}$, if p^* is atomic, i.e., p^* is a wff of D_i^* by formation rule (a) (1) or (b) (1).

R2. $\mathbf{I}(\sim p^*, W_j) = \mathbf{1}$, if and only if $\mathbf{I}(p^*, W_j) = \mathbf{2}$; otherwise $\mathbf{I}(\sim p^*, W_j) = \mathbf{2}$.

R3. $\mathbf{I}((p^* \& q^*), W_j) = \mathbf{1}$, if and only if both $\mathbf{I}(p^*, W_j) = \mathbf{1}$ and $\mathbf{I}(q^*, W_j) = \mathbf{1}$; otherwise, $\mathbf{I}((p^* \& q^*), W_j) = \mathbf{2}$.

R4. If there is a world W_j such that $\mathbf{I}(p, W_j) = \mathbf{1}$, then for every world W_h: $\mathbf{I}(p, W_h) = \mathbf{1}$.

R5a. $\mathbf{I}(O_i A, W_0) = \mathbf{1}$, if and only if for every world W_j in $\mathbf{W}$ different from W_0: $\mathbf{I}(A, W_j) = \mathbf{1}$.

R5b. $\mathbf{I}(O_1 A, W_0) = \mathbf{1}$, if and only if for every world W_j in $\mathbf{W}$: $\mathbf{I}(A, W_j) = \mathbf{1}$.

We define: p^* is *valid* in D_i^{c*}, $\models_{ic} p^*$, if and only if for every model M, $\mathbf{I}(p^*, W_0) = \mathbf{1}$, for $\mathbf{I}$ and W_0 in M. And p^* has a model if and only if for some model M, $\mathbf{I}(p^*, W_0) = \mathbf{1}$, for $\mathbf{I}$, and W_0 in M.

It is a simple thing to show that

MT1. If $\vdash_{ic} p^*$, then $\models_{ic} p^*$.

And the proof of the following proceeds along the lines of all proofs of Henkin completeness:

MT2. If p^* is consistent, p^* has a model.

Outline of proof. By standard procedures it can be shown that the set of wffs of D_i^{c*} is denumerable and that every consistent set can be extended to a maximal consistent set. Take any maximal consistent set of wffs of D_i^{c*} that includes p^*, and call it W_0. Take as **W** the set of maximal consistent sets W_j generated by W_0 as follows: every indicative p of W_0 is in W_j, and for every indicative of the form O_iA in W_0, A is in W_j; in the latter case A is also in W_0 if we are dealing with O_1A. We let **I** be the function such that $I(p^*, W_j) = \mathbf{1}$ if p^* belongs to W_j. It is clear from the construction that $\langle W_0, \mathbf{W}, \mathbf{I} \rangle$ is a model for p^*. We have, therefore, from MT1 and MT2, by standard reasoning, that:

MT3. $\vdash_{ic} p^*$, if and only if $\models_{ic} p^*$.

2. D_i^{**}. The models for the full systems D_i^{**} are ordered septuples $\langle W_0, \mathbf{W}, D, P, V, \pi, \mathbf{I} \rangle$, where W_0 and **W** are as above, D is a domain of persons and objects, V is a function assigning members of D to the primitive signs of D*, π is a function assigning members of P to the primitive predicates of D*, and **I** is as before except for conditions assigning **1** or **2** to quantified formulas. See the end of the penultimate paragraph of Section 2. P should be a domain of properties, both practical and contemplative, taking the practition copula as an operator on primitive predicates. We shall adopt an extensional model, taking P as a set of "practitional" objects and D as a subset of P. In modal propositional logic it is of great importance not to assume that the objects in the universe are necessarily fixed once and for all, i.e., regardless of the objects it has we must allow that the universe may have had more, or fewer, objects. On the other hand, for pure deontic logic we may assume without damage that the agents and objects are constant across all the practical worlds in **W** – if we are dealing with a system of duties and interdictions at a given time. This amounts to assigning D to each world in **W**. We need the following semantical rules:

V1. $V(x)$ is a member of D, for every individual constant or individual variable x of D*.

V2. $V(A^n)$ is a set of ordered n-tuples of D for every primitive n-adic predicate of D*.

π1. $\pi(A^n, W_h)$ is a set of ordered n-tuples of P, for every primitive n-adic predicate A^n of D*. Clearly $V(A^n)$ may turn out to be identical with $\pi(A, W_h)$ not only for some, but for all worlds W_h in **W**.

R6. $\mathbf{I}(A^n(x_1,\ldots,x_n), W_h) = \mathbf{1}$, if and only if $\langle V(x_1),\ldots, V(x_n)\rangle \in V(A^n)$

R7. $\mathbf{I}(A[x_1,\ldots,x_n], W_h) = \mathbf{1}$. if and only if $\langle V(x_1),\ldots, V(x_n)\rangle \in \pi(A^n, W_h)$.

By means of a standard Henkin-type proof it can be shown that

MT4. $\vdash_i p^*$, if and only if $\models_i p^*$.

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NOTES

[1] The situation nowadays with respect to deontic logic reminds me of Hume's celebrated challenge to the "vulgar system of morality" to be tested by the contrast between 'ought' and 'is'. Adopting his words, I hereby put my challenge to the deontic logicians:

> I cannot forebear adding [the] observations [that the contrast between actions practically considered and circumstances] should be observed and explained ... But as authors do not commonly use this precaution, I shall presume to recommend it to the readers; and am persuaded that this small attention would subvert all the [standard deontic] systems [whether ordinary or conditional]. (See, David Hume, *A Treatise of Human Nature*, III, i, ii; the bracketed expressions are mine.)

[2] The problems raised by the paradoxes should be placed in the larger context of practical thinking, including our use of imperatives and our adoption of intentions. For these matters and a comprehensive theory that deals with them, see Hector-Neri Castañeda, *Thinking and Doing* (Dordrecht: Reidel, 1975). The more comprehensive theory must also be based on a detailed exegesis of linguistic data. For a detailed discussion of the dialectics of philosophical exegesis and theorization, see Hector-Neri Castañeda, *On Philosophical Method* (Bloomington, Indiana: Noûs Publications, 1980).

[3] For the issues pertaining to simplicity and comprehensiveness, the former about the theory and latter mainly about the data, see *On Philosophical Method*, Ch. 4. For a detailed assessment of the role of Ockham's razor in the criticism of theories, see especially pp. 124–130.

[4] For a detailed characterization of the profile of the English logical words, including their logical roles and their thematic and dialectic aspects, see *Thinking and Doing*, Chs. 3 and 4. For pure conditional practitions, see pp. 111–115.

[5] The contrast between propositional opacity and propositional transparence, which contrasts with Quine's contrast between referential transparency and referential opacity, was introduced first in Hector-Neri Castañeda, 'On the Philosophical Foundations of the Theory of Communication: I. Reference', *Midwest Studies in Philosophy* **2** (1977), 165–186, reprinted in P. French, T. Uehling, and H. Wettstein, eds., *Contemporary Perspectives in the Philosophy of Language* (Minneapolis: University of Minnesota Press, 1979).

[6] For additional discussion of this contrast, and for a general discussion of the philosophical significance of syntactico-semantical contrasts see *On Philosophical Method*, especially Chs. 2 and 4.

[7] This was noted in a note by Roderick Chisholm in his justly famous 'Contrary-to-Duty Imperatives and Deontic Logic', *Analysis* **23** (1963), 33–36.
[8] Such solutions have been suggested, for instance, by H. P. Rickman, 'Escapism: The Logical Basis for Ethics', *Mind*, n.s., Vol. **72** (1963), 273–274, and John Robison, 'Who, What, Where, and When: A Note on Deontic Logic', *Philosophical Studies* **15** (1964), 89–91. P. H. Nowell-Smith and E. J. Lemmon wrote a nice discussion of the Good-Samaritan paradox in connection with a standard deontic calculus of Allan R. Anderson, in 'Escapism: The Logical Basis of Ethics', *Mind*, n.s., **69** (1960), 289–300.
[9] See Lennart Åqvist, 'Good Samaritans, Contrary-to-Duty Imperatives, and Epistemic Obligations', *Ibid.*: 361–379.
[10] For a view of knowledge as having a contextual relativization, analogous to the relativization of deontic operators to systems, see Hector-Neri Castañeda, 'The Theory of Questions, Epistemic Powers, and the Indexical View of Knowledge', *Midwest Studies in Philosophy* **5** (1980), 193–237. This view of knowledge was inspired in my view of deontic language. See the long Note 3 to this paper.
[11] Lawrence Powers, 'Some Deontic Logicians', *Noûs* **1** (1967), 361–400.
[12] See Note 7 above.
[13] Proposals to introduce a time parameter in the deontic operators have been made from time to time – and will continue to be made, undoubtedly. See, e.g., Wilfrid Sellars, 'Some Reflections on Contrary-to-Duty Imperatives', *Noûs* **1** (1967), 303–344, and Patricia Greenspan, 'Conditional Ought and Hypothetical Imperatives', *The Journal of Philosophy* **72** (1975), 259–276. For a discussion of the former see the essay by Powers mentioned in Note 11; for a discussion of the latter see Hector-Neri Castañeda, 'Ought, Time, and Deontic Paradoxes', *The Journal of Philosophy* **74** (1977), 775–791. The point I have made before, and want to insist upon here, is that a *careful exegesis of the relevant semantico-syntactical contrasts* be made before one introduces the tremendous complexity of tensing the deontic operators. One ought to see whether all the data can be illuminated by an account that includes all the tenses and time parameters in the actions and in the agents. In any case, I propose this approach as a sort of *Minkowskian or Kantian view of deontic logic*: all obligations, rights, and wrongs, are seen sub specie aeternitatis, *independently of whether or not one is metaphysically free, not causally determined, to choose*, therefore, independently of whether there are metaphysically open futures or not. In this regard, a very interesting paper on tense logic is J. F. A. K. van Benthem. 'Tense Logic and Standard Logic', L. Åqvist and F. Guenther, eds., *Tense Logic* (Louvain: Nauwelaerts, 1977) (*Logique et Analyse* **80**), pp. 41–83.
[14] For my own view of the Copernican – in Kant's sense of his Copernican Revolution – of the ontology and world-structure built in our natural language and ordinary references, see Hector-Neri Castañeda, 'Thinking and the Structure of the World', *Philosophia* **4** (1974), 4–40.
[15] Bas van Fraassen's in his 'The Logic of Conditional Obligation', *Journal of Philosophical Logic*, **1** (1972), 417–438, p. 424. Van Fraassen in this paper discusses the Good-Samaritan arguing that in modal contexts the substitutivity of identicals fails. This is not right, since what fails in general need not fail in particular cases. Furthermore, in ordinary deontic English ought contexts are extensional with respect to the substitutivity of identicals.
[16] For other difficulties of the dyadic treatment of deontic logic, see H-N. Castañeda, 'The Logic of Change, Action, and Norms', *The Journal of Philosophy*, **62** (1965), 333–334, which is a critical study of G. H. von Wright, *Norm and Action* (New York: The Humanities Press,

1963), who is the founder of this treatment, and does it in an impressive and large scale in this book. N. Rescher applied the "conditional" approach to imperatives in his *The Logic of Commands* (London, England: Routledge and Kegan Paul, 1966). For difficulties with this work see H-N. Castañeda's review of it in *The Philosophical Review*, (1970), 439–446. Some of these difficulties apply also to the conditional approach to deontic logic. An earlier study of the "conditional" systems of deontic logic by von Wright and Rescher is Bengt Hansson, 'An Analysis of Some Deontic Logics', *Noûs*, **3** (1969), 373–398. Recently, A. Al Hibri has defended the dyadic approach in the *Deontic Logic* (Washington: University Press of America, 1978). A critical discussion of this book and other dyadic systems appears in James Tomberlin, 'Contrary-to-Duty Imperatives and Conditional Obligation', forthcoming in *Noûs*.

17 Willard Van Orman Quine, *Methods of Logic* (New York: Holt, Rinehart, and Winston, revised edition, 1963), pp. 116ff.

DAVID MAKINSON

QUANTIFICATIONAL REEFS IN DEONTIC WATERS

ABSTRACT. This paper is concerned with the quantificational complexities that are implicit in even quite simple statements of prohibition, obligation, and permission in ordinary language. It has for some time been realized that quantifiers do lurk below the surface of such statements, but the intricacies of the situation seem to have been underestimated. In particular, we suggest, there has been a serious misidentification of the appropriate universe of discourse for the quantifiers; and moreover, there has been some oversimplification of the variety of roles which the quantifiers may play.

BACKGROUND

In a classic paper [1], Hintikka has observed that many of our statements pronouncing a permission, obligation or prohibition in natural language concern broad *kinds* of acts rather than isolated individual acts. For example, if in chess we say that it is forbidden to castle when the king is in check, then we are saying that *every* act of a certain kind, namely castling when the king is in check, is forbidden. If a regulation announces that in a certain estuary, it is permitted to gather and eat as many oysters as desired, but forbidden to take any away, then both the prohibition and the permission concern kinds of act, and have elements of generality within them. So far, so good; the question is, in what ways does the quantification occur?

A natural suggestion, also made by Hintikka [1], is to treat deontic operators as working primarily on statements about individual acts, and to *reduce* statements about the deontic status of kinds of acts to these. For example, the statement above, that it is forbidden to castle when the king is in check, might be paraphrased as "For every act x, it is forbidden that x be of kind C" (i.e. of the kind, castling when the king is in check), or, if it is preferred to take obligation as primitive, as "For every act x, it is obligatory that x not be of kind C". These would be symbolized as $\forall x[FC(x)]$ and $\forall x[O \neg C(x)]$ respectively. It should be observed that on this approach, obligation, prohibition and permission remain *propositional operators* and are *not* treated as *predicates of acts*, as they would be if we were to paraphrase our example as "For every act x, if x is of kind C, then x is forbidden". Such would be a quite different line of analysis, with its own attractions and difficulties, and which we shall leave aside here.

R. Hilpinen (ed.), New Studies in Deontic Logic, 87–91.

Hintikka's analysis raises several questions. Two which seem to be particularly important are: Is it really adequate to take the objects of the universe of discourse for the introduced quantifiers to be *individual acts*? And, are the particular quantificational reduction patterns suggested by Hintikka adequate? We shall argue that there are difficulties on both counts: that there is a serious misidentification of the universe of quantification, and that there is some oversimplification of the variety of roles played by the quantifiers.

THE UNIVERSE OF DISCOURSE

We begin with an example. Imagine that you are in a hospital waiting room, and on the wall is a sign saying that it is forbidden to smoke there. At a certain moment, you take a string of worry beads from your pocket, and begin to finger them. This, of course, is a particular action carried out at a specific time, and for cross-reference we shall call it *a*. Hintikka, who is sitting next to you, has read the sign, and in accord with his analysis interprets it as saying "For every act *x*, it is forbiddén that *x* be of the kind *S*"; he carries out in his head an instantiation to *a*, and leans over to comment "It is forbidden that what you did just now, toying with worry beads, be a case of smoking. In other words, $FS(a)$."

A most peculiar way of speaking! You would be tempted to reply "But it *wasn't* a case of smoking, it was a case of playing with worry beads, and it *couldn't* have been a case of smoking and still be the same individual act *a*; it would have been a different act, say *b*". And on a less metaphysical level you might well resume, "I see the sign; and I know that if a moment ago I had taken out a cigarette instead of worry beads, I would have been in infraction. The sign means to forbid *anyone* at *any time* to smoke in here, and that would have covered me, then. There *is* universal quantification in the rule, but surely it is odd to express it as quantification over acts, when what we are getting at is quantification over people and occasions".

Odd, yes; but is it incoherent? Our second example is meant to clinch the point. Imagine that a few days later you are in the waiting room of another, more easy-going, hospital. You ask a passing nurse whether it is forbidden to smoke. "No", she replies, "it isn't forbidden, though we don't actually encourage it." Hintikka, who is with you again, interprets: "She means", he confides, "that it is not the case that for every act *x*, it is forbidden that *x* be a case of smoking here." As you know at least a little *classical* logic, you reflect: "Aha. So there are *some* acts *x*, for which it is not forbidden that they be cases of smoking in here – in other words, $\exists x[\neg FS(x)]$." Then, carried away by your reflections, you suddenly ask, "But *which ones are they*? My present act of playing with worry beads – is *it* such an x? That fellow's chewing on gum, right now – is *it* one of the x's of which it is not forbidden that they be acts of smoking? And that actual case of smoking over there – does *it* provide an instance of our existential?"

Perhaps we can say: they are *all* such x's. But then the corresponding universal would seem to be true. Perhaps we can say: the acts of playing with worry beads and chewing gum are not among the privileged x's – it is forbidden that *they* be cases of smoking; but the actual act of smoking seen over there is one of the privileged x's – it is not forbidden that *it* be a case of smoking. But if it is not forbidden that an act of puffing on a cigarette be what it is, that is of the kind smoking, *why on earth* should it be forbidden that a neighbouring act, of chewing gum, be of the same category? There seems to be no ground for discrimination: if the act of chewing gum *were* an act of smoking, it would be well-nigh indistinguishable from its fellow act which already *is* an act of smoking. And neither the hospital nor the world at large would lose by the disappearance of a session of gum chewing. Whichever way we turn, we find no coherent way of distinguishing, in even the roughest of terms, between those privileged acts x that *do* satisfy the existential proposition in question (that is, those acts x of which it is not forbidden that they be smoking), and the remaining acts that *don't* satisfy it. It begins to look like a distinction *without even the vaguest criterion of discrimination*. By the same token, specific statements of the form "It is forbidden that the act a be of the kind A", seem to lack even the vaguest criteria for the determination of their truth-values, according to the choice of a, and so to be devoid of meaning, except as misleading ways of saying "It is forbidden that such and such a person do acts of kind A in such and such a specific situation".

SCHEMES OF QUANTIFICATIONAL REDUCTION

Given that statements of prohibition, obligation, and permission for kinds of act typically involve, in some way or other, quantification, and trying as far as possible now to abstract from the question considered above, of identifying the most appropriate universe of discourse for these quantifiers, the question remains of describing the particular patterns in which the quantifiers intervene. Hintikka [1] suggests standard schemes for understanding and representing statements of each of the three deontic varieties; our contention here is that in the cases of obligation and permission there may be considerable variation from example to example in the role played by the quantifiers. We begin with obligation.

Consider the statement "You must pay your membership fee". This can be rendered quite naturally in the manner suggested as a standard by Hintikka, as "It is obligatory that at some time within a certain limit you pay the fee" or, if we retain an act-universe, as "It is obligatory that one of your acts be a payment of fees": $O\exists x[Px]$. But the same cannot be said of "You must not commit adultery" or "In this country, you must drive on the right". These have an element of universality, though the former in a simpler way than the latter. The former means something like "Nobody should ever commit adultery", or, if we insist on an act-universe, "No act should be of adultery": $\forall x[O \neg Ax]$. In the latter example, the universality is a little more indirect, because conditional. To say that one must drive on the right is of course *not* to say that all one's acts should consist of driving on the right, but it is also much more than to say merely that it is obligatory that at least some acts be of driving on the right. Rather, common sense tells us, the injunction says something like "whenever one is driving in this country, one must do so on the right", or in the language of an act-universe, "All acts of driving in this country must be performed on the right", which is a conditionally restricted universal generalization. Note too (contrary to an idea that seems to be implicit in Hintikka's paper) that the statements of obligation taking a universal quantifier need not be particularly "negative" in their formulation. The statement saying that one must not commit adultery is indeed negative in its presentation, and that does seem to be responsible for the particularly simple and unconditional way in which the universal quantifier operates there; but the statement that one must drive on the right is not negative in its linguistic formulation in any interesting sense at all. We conclude, therefore, that statements saying that one must or must not do something may be interpreted existentially or universally, from case to case, although a little care is needed in deploying the universal quantifier in a not too simple-minded way. Grammar alone seems to give few clues as to which quantifier to choose; the selection depends on the entire intent of the statement, which involves context and background familiarities. For this reason, there is no single scheme for representing the logical force of such statements of obligation.

We now consider statements of permission. Here too there is variety, though of a rather different sort. It would seem that here the implicit quantifier is usually universal, as suggested by Hintikka, but that its application is often accompanied by implicit antecedent clauses. If a society of vegetarians, after careful study, declares that it is morally permissible to eat mussels, then it seems natural to read this universally: anybody may at any time eat mussels. But if an intending traveller is informed that it is permitted for women to drive cars in Oman, he or she will understand this as saying that any woman who satisfies suitable unspecified conditions (such as the possession of various papers) may do so. And in some cases, the antecedent conditions may be so stringent, and so difficult to fulfill, or so rarely satisfied, as to bring the *practical* import of the statement of permission very close to an existential. No merely grammatical features of the sentence will tell us whether such antecedent conditions are intended, and if so, how stringent or easily specifiable they may be; that depends on the context of utterance and background knowledge.

In conclusion: We have argued that when investigating the quantification that is implicit in statements of ordinary language, saying that certain kinds of act are or are not forbidden, obligatory, or permitted, it is a serious mistake to take the class of all actual acts as one's universe of discourse (people and places or times or circumstances would be better), and it is also something of an oversimplification to hanker after fixed symbolization schemes for the three kinds of statement (there is a variety of contextually determined ways in which the quantifiers may appear).

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PART II

NORMS AND CONFLICTS OF NORMS

CARLOS E. ALCHOURRÓN AND EUGENIO BULYGIN

THE EXPRESSIVE CONCEPTION OF NORMS*

1. TWO CONCEPTIONS OF NORMS

Questions concerning the ontological status and the logical properties of norms have been much debated in recent years, not only by legal and moral philosophers but also by a steadily increasing number of "deontic" logicians. In spite of this a whole number of very basic problems have apparently not been solved, and persist.

One such issue is the problem of the possibility of a logic of norms. Some authors think that there are logical relations between norms, and so favor the development of a specific logic of norms (sometimes called "deontic logic", though "normative logic" would perhaps be a more appropriate name).[1] Other writers deny the very possibility of such a logic because in their view there are no logical relations between norms. According to them deontic logic can only assume the form of a logic of normative propositions, i.e. (true or false) propositions about (the existence of) norms.[2]

Another fundamental problem, or perhaps another aspect of the same problem on which there is no consensus, is the relation of norms to truth: whereas some writers readily ascribe truth-values to norms,[3] others deny emphatically that norms could conceivably be true or false. This issue is related to the first one, but not in a very clear way. Those authors who believe that norms have truth-values will certainly accept the possibility of a logic of norms, but the converse does not hold: *accepting* that there are logical relations among norms does not commit one to the view that norms have truth-values.[4]

A third and apparently unrelated issue is the question concerning permissive norms. A great number of philosophers (especially philosophers of law) deny that there are permissive norms, admitting only one type of norms (mandatory norms, imperatives, commands). Logicians and lawyers – though probably for different reasons – feel less inclined to such a monistic conception and see no obstacle that would prevent them from speaking of permissive norms (independently of the question whether they are definable in terms of obligations or not).

These discrepancies are due to a large extent to the fact that authors often

R. Hilpinen (ed.), New Studies in Deontic Logic, 95–124.

start from two quite different and incompatible conceptions regarding the nature of norms, which are seldom – if ever – made explicit. It may be illuminating to characterize briefly these conceptions in order to see why different writers maintain conflicting and even diametrically opposed views on some very basic features of norms. These two conceptions will be called the *hyletic* and the *expressive* conception of norms.

For the *hyletic conception* norms are proposition-like entities, i.e. meanings of certain expressions, called normative sentences. A normative sentence is the linguistic expression of a norm and a norm is said to be the meaning of a normative sentence in much the same way in which a proposition is regarded as the meaning (sense) of a descriptive sentence. But normative sentences, unlike descriptive sentences, have *prescriptive meaning*: that something ought, ought not, or may be the case (or be done).

In this conception, norms are not language-dependent; they can only be expressed by linguistic means,[5] but their existence is independent of any linguistic expression. There are norms that have not yet been formulated in any language and that perhaps will never be formulated. A norm is, in this view, an abstract, purely conceptual entity.

But norms are not independent from descriptive propositions: they are the result of an operation on such propositions. So in a norm, say '$\mathrm{O}p$', we find two components: a descriptive proposition p and a normative operator O, both of them belonging to the conceptual import of the norm. In this sense the normative operators are similar to the modal alethic operators and a norm is a proposition in much the same sense in which a modal proposition like $\mathrm{N}p$ is said to be a proposition.

Norms must be distinguished from *normative propositions*, i.e. descriptive propositions stating that p is obligatory (forbidden or permitted) according to some unspecified norm or set of norms. Normative propositions – which can be regarded as propositions about sets (systems) of norms – also contain normative terms like 'obligatory', 'prohibited', etc., but these have a purely descriptive meaning.[6] In what follows the symbols '$\mathbb{O}$' and '$\mathbb{P}$' will be used to refer to these descriptive deontic operators.

For the *expressive conception*, instead, norms are the result of the *prescriptive use* of language. A sentence expressing the same proposition can be used on different occasions to do different things: it can be asserted, interrogated, commanded, conjectured, etc. The result of the performance of these actions will be a statement, a question, a command or a conjecture. It is only on the pragmatic level of the use of language where the difference between statements, questions, commands etc. arises: there is no such difference on a semantic level. For instance, the proposition expressed by the sentence "Peter puts the book on the table" can be used to make an assertion (Peter puts the book on the table.), a question (Does Peter put the book on the table?) or a command (Peter, put the book on the table!).

The signs '$\vdash$' and '!' will be used to indicate the kind of linguistic act (assertion or command) performed by an (unspecified) speaker. These signs are mere *indicators* of what the speaker does when uttering certain words, but they do not contribute to the meaning (i.e. the conceptual content) of the words uttered. They show what the speaker is doing, but in doing it he does not say what he is doing; so they are not part of what he says, or what his words mean. The expression '$\vdash p$' indicates that p is asserted and '$!p$' indicates that p is commanded, whereas 'Op' expresses a proposition that p ought be (done). So 'Op' is the symbol for a norm in the hyletic conception whereas '$!p$' symbolizes a norm in the expressive conception.

It is important to stress the fact that the expressions '$!p$' and '$\vdash p$' do not describe the fact that p has been commanded or asserted. The sentences "A asserts that p" and "A commands that p" certainly express propositions that describe certain speech acts, but they do not say what is done with them: they can in turn be asserted, questioned, commanded, etc. But '$!p$' and '$\vdash p$' do not express any proposition at all, although they are constructed by the help of the proposition p; so they have no truth-value and cannot be negated nor combined by propositional operators.[7] What a speaker *does* on a certain occasion cannot be said by him (on the same occasion): it can only be shown by a gesture, a certain inflexion of the voice or some special sign, but these devices only show the mood in which the sentence is used; they do not form part of what the sentence says (i.e. its conceptual content).

For the expressive conception, norms are essentially *commands*, but they must be carefully distinguished from propositions stating that there is a norm to such and such effect or that p is obligatory or prohibited, which are normative propositions. Normative propositions are related to the norms in the following way: if p has been commanded, then the proposition that p is obligatory is true. If $\sim p$ (the omission or forbearance of p) has been commanded, then it is true that p is prohibited or – what is the same – that $\sim p$ is obligatory.

The two conceptions of norms are radically different and incompatible; there is no room for any eclecticism. If norms are expressions in a certain pragmatic mood, then they are not part of the meaning; if they are meanings (propositions), they are independent of any use of language or pragmatic mood. And yet many authors do not clearly adhere to either of the two conceptions, or rather seem to adhere to both of them. It is symptomatic of the very intricate nature of the issue that among those who seem to oscillate between the two conceptions are those philosophers who have dug most deeply into these problems. Thus, C. I. Lewis appears to be a clear expressivist when he says: "... the element of assertion in a statement is extraneous to the proposition asserted. The proposition is something *assertable*; the *content* of the assertion; and this same content, signifying the same state of affairs, can also be questioned, denied, or merely supposed, and can be entertained in other moods as well." Among these moods Lewis explicitly mentions the "imperative or hortatory mood", including in his characterization the "modal statements of possibility and necessity".[8] But in his modal logic he treats the expression '$\Diamond p$' as a proposition, where the modal operator of possibility is part of the content of the proposition.

In von Wright we equally find arguments that would permit us to classify him according to either of the two conceptions. On the one hand, he speaks of "prescriptively interpreted deontic expressions" between which certain logical relations hold;[9] this seems to locate him among the adherents to the hyletic conception. On the other hand, he states that

> it would be misleading to conceive throughout of the relation between norms and their expressions in language on the pattern of the above two 'semantic dimensions' [sense and reference]. At least norms that are prescriptions must be called neither the reference nor even the sense (meaning) of the corresponding norm-formulations. ... the use of words for giving prescriptions is similar to the use of words for giving promises. Both uses can be called *performatory* uses of language.[10]

This seems to beckon more in the direction of expressivism.

What these quotations from the works of the founders of modal and deontic logic show is that both conceptions are plausible, with one perhaps more plausible than the other in some contexts and vice versa, so that it is not easy to take a decision concerning the two conceptions before exploring the whole range of their implications.

But as it stands, most legal and moral philosophers as well as deontic logicians share the expressive conception of norms; the most conspicuous and clear cases are those of Bentham, Austin, Kelsen, Alf Ross, Hare, Jörgensen, Moritz, Hansson, Åquist, Raz and von Kutschera.[11] Among the much less numerous representatives of the hyletic conception might be mentioned Kalinowski and Weinberger.[12]

It is not surprising that such antagonistic views on the nature of norms should lead to quite different answers to the three problems mentioned at the beginning of this paper. For the expressive conception there can be no logic of norms, because there are no logical relations among norms. Deontic logic can only assume the form of a logic of normative propositions.[13] For the hyletic conception, instead, there are two logics: a logic of norms and a logic of normative propositions.[14]

Regarding the second issue, the situation is less clear. Adherents to the expressive conception are committed to the view that norms lack truth-values; but among the representatives of the hyletic conception there are two tendencies. Some of them[15] believe that norms are true or false; others maintain that they lack truth-values.[16] This question will be not discussed here.

Most expressivists deny that there are permissive norms (which does not amount to denying the existence of permissible states of affairs), because they only accept one kind of normative (prescriptive) action: commanding. This version of the expressive conception is the *imperative theory of norms.* But there are exceptions even between clear expressivists; some of them accept a peculiar normative act, that of permitting.[17] We shall see later whether there are reasons for the expressive conception to accept other types of normative acts besides commands and to accept the existence of permissive norms.

No such problems arise for the hyletic conception; hence the authors that share this conception accept at least two kinds of norms: mandatory or O-norms and permissive or P-norms.

Our purpose in this paper is to examine in some detail the expressive conception. On close inspection it proves to be much more powerful than might appear at first sight. When duly enriched by some new concepts (compatible with its spirit though usually ignored by its adherents), it is capable of capturing most if not all important features of a normative phenomenon. But many expressivists, like Horatio, are bound to accept more things than are dreamt of in their philosophy.

In previous publications, especially in *Normative Systems*, we adhered to the hyletic conception. Norms were treated as abstract entities, as propositions with prescriptive meaning, capable of entering into logical relations. Since then, we realized that most writers share the expressive conception; so it seemed interesting to explore its possibilities in order to uncover its limitations and show the differences between the two conceptions. Such was the origin of this paper.

We now have the impression that the same conceptual distinctions appear in both conceptions, though, of course, expressed in different languages. The choice between them is motivated by ontological considerations regarding the nature of norms, but there seems to be no crucial test that would justify a decision in favor of one of them. So, after all, it looks more like a problem of philosophical style and even personal preference than a question of truth. As Carnap puts it: "Let us be cautious in making assertions and critical in examining them, but tolerant in permitting linguistic forms."

2. NORMS AND NORMATIVE SYSTEMS

The expressive conception is primarily concerned with norms issued by some agent (norm-authority) and directed to other agents (norms-subjects), i.e. norms that von Wright calls *prescriptions.*[18] So we shall take into account only this type of norm, of which many legal norms provide a clear example.[19]

We shall begin by examining the imperative theory of norms, which accepts only one kind of normative act, the act of commanding and therefore only one type of norm: mandatory norms. (It is immaterial whether they are conceived of as establishing obligations or prohibitions.)

Commanding is essentially a linguistic activity, a speech act. It consists of formulating certain words (or other symbols) with a certain meaning. A norm is the meaningful sentence in its imperative use ($!p$). The content of the norm is the proposition expressed by "p". Thus the act of commanding can be described as the act of *promulgating* a norm. The act of promulgating has temporal, but instantaneous existence. Yet norms are said to exist continually during a certain period of time (this is clearly so in the case of legal norms). How can this feature of norms be accounted for in the expressive conception?

In order to illustrate it we shall, following Hart, suppose a simplified situation in which a certain population living in a certain country is governed by an absolute monarch called Rex. Rex controls his people by general commands requiring them to do various things and to abstain from doing certain other things. Let us suppose further that Rex is the only legislative authority of that country.

From time to time Rex performs the action of commanding a certain proposition or a set of propositions. The propositions that have been commanded by Rex form a set, the *commanded set A*. Each time Rex issues a new command, this set is enlarged by the new proposition commanded by Rex, so that it becomes a new set, say A_1. Thus in the course of time we have not one set, but a *sequence* of sets (A_1, $A_2 \ldots A_n$). So far (i.e. as long as the imperativist variety of expressivism is accepted) these sets can only be increased by addition of new propositions, but there is nothing like subtraction.

A proposition becomes a member of some set of the sequence as a result of an act of commanding performed by Rex. Hence we can say that the norm $!p$ exists from the moment at which p has been commanded and so the proposition p has become a member of the corresponding set. This is, of course, only a mode of speech. In fact, the norm $!p$ has an instantaneous existence, exactly like the act of commanding p. The point however is, that all propositions that belong to the set A are regarded as obligatory in A. As one and the same proposition p can be a member of, e.g., successive sets A_2, $A_3 \ldots A_n$, but not of A_1, p is not obligatory in A_1, but is obligatory in A_2 etc. As long as the successive sets can only be increased by new commands, p commanded at t_1 belongs to all sets subsequent to the set corresponding to t_1.

Thus the existence of a norm (= the membership of the norm-content) is dependent on certain empirical facts (acts of promulgation in case of prescriptions; certain actions revealing dispositions in case of customary norms). Therefore, as there are no logical relations between facts, there is no room for a logic of norms.

But this does not preclude the possibility of a logic of normative propositions. Indeed, as we have already pointed out, the proposition that p is obligatory in A is true if p has been commanded by Rex and so is a member of the commanded set A. Yet this is a sufficient but not a necessary condition for the truth of "p is obligatory in A". It may occur that Rex has never commanded that p, but has commanded e.g. that p & q. This is a different proposition and so, according to our criterion, p would not belong to A. But as p is a consequence of p & q (for it is logically deducible from p & q), it is also true that p is obligatory in A. The obligatoriness of p is a consequence of the obligatoriness of p & q, because p is a consequence of p & q.

We can now define the concept of a normative system as the set of all propositions that are consequences of the explicitly commanded propositions.[20] (Though we use the traditional expression "normative system", it must be emphazised that for the expressive conception a normative system is not a set of norms, but a set of norm-contents, i.e. propositions.) This enables us to distinguish between the set A (formed by all explicitly commanded propositions) as the axiomatic *basis* of the system, and the normative system Cn (A), which is the set of all consequences of A.

We are in a position to correct our criterion for the truth of normative propositions: 'it is obligatory that p in A' is true if and only if p is a member of the system Cn (A) – i.e. if and only if p belongs to the consequences of A. This means that p is obligatory in A if and only if p has been commanded or p is a consequence of the propositions that have been commanded. In this last case we say that $\mathbb{O}_A(p)$ and that p is a *derived obligation.*

The notion of derived obligation is related to the notion of implicit command. This last concept, in its turn, is closely related to that of implicit assertion. Indeed, there are at least two different senses in which a person can be said to have made an assertion. In a psychological sense of "assertion", what is asserted in an act of assertion is the sentence uttered, not even the proposition expressed by that sentence. In this sense of "assert", if X asserted "John kissed Mary", he *did not assert* "Mary was kissed by John", because it is a different sentence, even if both sentences have the same meaning, i.e. express the same proposition. But in another, non-psychological, sense of "asserting" – if X asserted "John kissed Mary", he explicitly asserted the proposition expressed by that sentence, and so he also asserted that Mary was kissed by John, and moreover, he also (implicitly) asserted all those propositions – like "Somebody kissed Mary" – that are consequences of the propositions he asserted explicitly. This is a non-psychological sense of assertion, for it is clear that the person in question probably did not think at all of all such propositions and so had not the slightest intention of asserting them. It may even be the case that q is a consequence of p, and that the person who asserts p not only ignores this fact, but believes q to be false. If he is not prepared to assert q (e.g. because he believes it to be false), then we can show that his position is inconsistent by proving that q is a consequence of p. This is a very common way of arguing: we often try to refute our opponent showing that the propositions he asserts imply some proposition he is not willing to accept. This kind of argument is based on the notion of implicit assertion: in this sense, one asserts all the propositions that are a consequence of the explicitly asserted propositions.

A famous case might be mentioned in this connection. When Russell found a contradiction in Frege's system, this fact produced a terrible impact on Frege. Why? Frege certainly did not assert any inconsistent proposition; but Russell showed that the self-contradictory proposition was a theorem (a consequence) of Frege's system. Frege did assert it implicitly, by asserting the axioms of his system and he could not maintain the axioms and reject that theorem.

The same kind of observation can be made regarding the act of commanding. Here too we have a non-psychological sense of implicit commanding. If a person commands something, he also commands all the consequences of what he has explicitly commanded (even if he is not aware of them). For instance, if a teacher commands that all his pupils should leave the class-room, he also implicitly commands that John (who is one of his pupils) should leave the class-room, even if he is not aware of the fact that John is there.

These considerations show that there are logical relations between normative propositions. In addition to obligation, we can define also the concepts of prohibition and permission for normative propositions:

p is prohibited in A ($\mathbb{O}_A(\sim p)$) = df. the negation of p ($\sim p$) is a member of the system Cn (A).

p is permitted in A ($\mathbb{P}_A(p)$) = df. the negation of p ($\sim p$) is not a member of Cn (A).

Even if for the imperative theory of norms there are no permissive norms, there are permitted propositions or states of affairs. According to the definition, p is permitted if and only if p is not prohibited in A. This shows that permissions have a normative status which differs from that of obligations and prohibitions. The permission of p is given by the absence of certain acts (acts of prohibiting that p or – what is the same – of commanding that $\sim p$), whereas the prohibition (obligation) requires the existence of certain normative acts.

The analyses of this section show that a careful distinction must be made between: a) the act of promulgation of a norm (commanding), b) the operation of adding new elements to a system, as a result of such acts, and c) the criteria that govern such addition of elements. It is important to realize that what is added to the system A as a consequence of an act of promulgating a set of propositions B is not only the set B itself, but also all its consequences and moreover all those propositions that, without being consequences of B, nor consequences of A, are nevertheless consequences of A taken together with B. In other words, if to a set A we add a set B, the resulting system is not Cn (A) + Cn(B), but Cn ($A + B$). In most cases, this last set will be considerably larger than the first.

3. REJECTION

Suppose now that Rex realizes that the state of affairs p, that he prohibited some time ago, should not be prohibited now (perhaps because he committed a mistake in prohibiting p, or because the circumstances that made the prohibition of p convenient have changed). So he wants to permit p; how can he achieve it?

It is clear that once the act of commanding $\sim p$ has been performed, nobody can modify this fact. So it will always be true that p is forbidden in A. If Rex wants to permit p, he must change the system into a system where $\sim p$ does not obtain. But this change is impossible as long as there are only acts of commanding, which alone are accepted by the imperative theory of norms. By commanding, a permitted state of affairs may become prohibited, but not vice versa. The change from a prohibition to a permission requires an operation of *subtraction*; addition alone is clearly not sufficient. Therefore, in order to permit p, Rex must repeal or derogate the norm that prohibits p; more exactly, he has to eliminate $\sim p$ from the system. For this he must, first, identify what he wants to eliminate ($\sim p$) and, secondly, perform the operation of subtracting $\sim p$, so that as a result of this operation $\sim p$ will be eliminated from the system. Here again a distinction must be made between the act performed by Rex, that will be called *act of rejection*, the operation of eliminating certain propositions from the system and the criteria that govern such elimination.[21] We shall begin by the analysis of the act of rejection.

In the same way as there are (among others) two types of propositional attitudes: descriptive and prescriptive, i.e., in this context, asserting and commanding, there are two types of acts of rejecting that may refer to the same proposition. We shall call them the descriptive and the prescriptive rejection. The content of both types of acts is a proposition, but both acts reject it in a different way. The first act of rejecting is opposed to asserting, the second to a command. We shall use the signs '$\dashv$' and '¡' to symbolize the two kinds of rejection.

It is important to realize that rejecting is not the same as negating. When we negate a proposition, we assert another proposition that is the negation of the first. Thus to negate p is to assert $\sim p$. Similarly, to negate the command that p can be regarded as commanding that $\sim p$: in this case the negation of the command that p would be the prohibition that p. Hence if Rex in order to permit that p would negate the prohibition that p by commanding that p, the only thing he would achieve is to introduce a contradiction into the system: both p and $\sim p$ would belong to $\mathrm{Cn}(A)$ and both propositions "p is obligatory in A" and "p is prohibited in A" would be true, and neither p nor $\sim p$ would be permitted. This is not what Rex wants to do, if he wants to permit that p.

Therefore rejection is another type of speech act; he who rejects a proposition does not assert any proposition at all. It is the kind of difference that obtains between an atheist and an agnostic. The atheist negates the existence of God; he does it by asserting the proposition that God does not exist. The agnostic rejects the proposition that God exists without asserting the proposition that God does not exist. Incidentally, this shows also that the position of a skeptic need not be inconsistent. It would be self-refuting if the skeptic would assert that nothing can be known, for then he claims at least to know the proposition that nothing can be known. But if all he does is to reject all propositions, then he does not assert any proposition at all and his position becomes perfectly consistent.

Similarly, the (prescriptive) rejection of *p* is no prescription at all; in particular, it is not a prohibition of *p*. So the sign '¡' is a mere indicator of a certain speech act, and does not form part of the conceptual content of this act. ('¡*p*' like '!*p*' does not express a proposition, but only indicates what is done with the proposition *p*.)[22]

When lawyers speak of a derogation there is a rejection of a norm-content. No act of rejecting is required when what is derogated is not a norm-content, but a mere formulation of a norm (a sentence). When the legislator becomes aware that there are two or more redundant formulations, i.e. the same norm-content is expressed e.g. by different paragraphs of a statute, then he may be willing to derogate the redundant formulations, without eliminating the norm-content. In this case what he wants to do is to "efface" the redundant formulations, leaving only one of them. No rejection of the norm-content is required to achieve this aim. But removing of a norm-formulation should not be confused with the elimination of a norm-content. In this last case what the authority wants to eliminate from the system is a certain conceptual content (a proposition) and in order to achieve it, the performance of an act of rejection is necessary.

So expressivism must accept besides commanding another type of normative act: that of rejecting. The imperative theory of norms cannot account for the phenomenon of derogation, but expressivism is not bound to stand and fall with it. The acceptance of various types of normative acts and, in particular, acts of rejection, is perfectly compatible with the expressive conception.

If as a result of the rejection of a norm-content, it is eliminated from the system, the norm ceases to exist. Two important conclusions may be drawn from this fact: (1) norms not only begin to exist at a given time; they also cease to exist at a certain moment; (2) normative sets can not only be extended by addition of new elements; they can also be restricted by subtraction of elements.

Conclusion (1) is in need of some clarifying remarks. As we have already seen the temporal existence of norms is just a metaphor. What really happens is the performance of two types of acts (commanding and rejecting): these are the only empirical facts relevant for the existence of a norm. There is no need for the occurrence of any further fact that would make true the proposition that a norm exists.[23] On the other hand, the assertion that a given norm ceases to exist at a certain moment is misleading. All there is, is a sequence of different sets of propositions and a given proposition *p* may be a member of some of these sets and not of others. If it belongs to a certain set, it never ceases to belong to it; though it may occur that it does not belong to the next set. What we do, is to take at different times different sets as points of reference for our assertions that certain propositions or state of affairs are obligatory, prohibited or permitted: this gives the illusion of a temporal change. But in fact, the normative propositions are timeless, for they always refer to some definite system. Hence the proposition "*p* is obligatory in A_1" is either true or false, but if true, it is always true, even after the derogation of *p*. For if *p* is eliminated, we obtain a new system, say A_2. The proposition "*p* is obligatory in A_2" is, in this hypothesis, false, but it is a different proposition. The first proposition (*p* is obligatory in A_1) continues to be true, though one perhaps is no longer interested in it. In this sense normative systems are instantaneous;[24] when jurists speak of legal systems as persisting through time (as for instance the system of French Law) what they mean is not one system, but a sequence of systems.

4. CONFLICTS OF PROMULGATION AND REJECTION

If *X* asserts that *p* and *Y* asserts that $\sim p$, the two assertions are said to be incompatible, not in the sense that they could not coexist, but in the sense that the two propositions asserted by *X* and *Y* are contradictory, i.e. they cannot both be true (nor false). The fact that two persons assert two contradictory propositions is certainly possible (and moreover extremely frequent); it is even possible for one and the same person to assert two contradictory propositions. But such assertions conflict. If we want to integrate them in a coherent whole, we must first resolve the conflict.

Analogously, the *command* that *p* and the command that $\sim p$ conflict, because the norm-contents *p* and $\sim p$ are contradictory. This is the "classic" notion of *normative inconsistency*. It is an extension of the concept of contradiction between propositions to commands (norms), for it is based not on the criterion of truth (commands lack truth-values) but on the notion of fulfillment: it is logically impossible to fulfill or obey both commands $!\,p$ and $!\sim p$. Nevertheless it is surely possible for two persons or even for one person to issue two conflictive commands. As long as they belong to different systems there is no trouble; the need for resolving the conflict arises when they become members of the same system. It is the unity of the system that determines this need. A normative system that contains both p and $\sim p$ is *inconsistent* and this is regarded as a serious defect of the system, for relative to it the propositions that p is obligatory and p is prohibited are both true.

Consider now the kind of conflict that would arise not between a theist and an atheist (who assert two contradictory propositions "God exists" and "God does not exist"), but between a theist and an agnostic. An agnostic rejects the proposition that God exists, without affirming its negation. Here there is no inconsistency between two propositions, but a conflict between two propositional attitudes concerning the same proposition: assertion and rejection. In a certain sense, assertion and (descriptive) rejection are incompatible.

In a similar way promulgation of a norm and rejection of the same norm-content are incompatible: there is a kind of conflict between commanding that p and rejecting p. This conflict is different from that of commanding p and $\sim p$. In the latter case we have an agreement in attitude, but a disagreement in content; we call it *normative contradiction* or inconsistency. In the former case we have a disagreement in attitude and an agreement in content; this kind of conflict will be called, following Carnap,[25] *ambivalence*.

The need for resolving the conflict of ambivalence arises when the same proposition is (directly or indirectly) commanded and rejected by the same authority or by different authorities in the same system.

In order to resolve the conflicts of ambivalence certain criteria are used, which will be called *criteria or rules of preference*.[26] The rules of preference are designed to resolve the conflicts between acts of promulgation and acts of rejection referring (directly or indirectly) to the same norm-content. They stipulate which of the acts prevails over the other. That the act of rejection of p prevails over the act of commanding p means that the set which does not contain p is to be preferred to the set that contains p as the point of reference of normative judgments of the form $\mathbb{O}_A(p)$ or $\mathbb{P}_A(p)$, and vice versa.

The rules of preference are seldom if ever explicitly stated, but they are used in fact by lawyers and by all those who have to manipulate normative systems. Three such rules are commonly used in legal practice; we shall call them rules *auctoritas superior*, *auctoritas posterior* and *auctoritas specialis*. These names are an adaptation of certain other, analogous but different rules that lawyers explicitly use for resolving contradictions between norms (lex superior, etc.), of which we shall come to speak later (section 6).

The rule *auctoritas superior* stipulates that the act (be it promulgation or rejection) performed by an authority of higher hierarchical level prevails over the act performed by an authority of a lower level. This means that when a higher authority, e.g. a legislature, has promulgated a norm, it cannot be repealed by an inferior authority, e.g. by the executive. Even if it is rejected, the system does not change. On the other hand, when a higher authority rejects a norm-content, this act derogates it (i.e. leads to its elimination from the system) if it had been promulgated before by a lower authority and prevents its addition to the system by a later act of promulgation of an inferior authority. This last case is especially interesting: it shows that rejection need not be temporally posterior to the act of promulgation.

If we distinguish between the *operation of eliminating* norm-contents that have been rejected and the act of *rejecting* (which is frequently also called "derogation"), then we become aware that it makes perfectly good sense to reject the norm-content *p*, even if *p* is not a member of the system. Though such rejection does not lead to an elimination of any norm-content, it may produce the important result of preventing the addition of *p* if *p* is promulgated later by an authority of lower level. This is what happens with constitutional rights and guarantees: the constitution rejects in advance certain norm-contents (that would affect the basic rights), preventing the legislature from promulgating those norm-contents, for if the legislature promulgates such a norm-content, it can be declared unconstitutional by the courts and will not be added to the system.

The other two rules operate in a similar way. The rule *auctoritas posterior* stipulates that a temporally later act prevails over the former act, whether it be promulgation or rejection. Obviously, this rule only applies to acts performed by authorities of equal hierarchy; so it is supplementary to the first rule.

Finally, the rule *auctoritas specialis* stipulates that an act of promulgating (rejecting) a less general norm-content prevails over the act of rejecting (promulgating) a more general norm-content.

It is important to stress the fact that these rules do not resolve all possible conflicts between acts of promulgation and of rejection. It can very well occur that the same authority or two authorities of equal hierarchy perform simultaneously the acts of promulgating and rejecting the same norm-content. In such a case clearly none of the three rules is applicable; such cases, though rare, sometimes do occur in legal practice. If such conflicts are to be resolved, further criteria of preference must be introduced. But it would be a mistake to regard the rules of preference (traditional or not) as *logical* rules.

5. IMPLICIT REJECTION AND DEROGATION

When Rex rejects a norm-content (or a set of norm-contents), this act identifies what he wants to be eliminated (subtracted) from the system. The set of the explicitly rejected propositions will be called, accordingly, the *derogandum*.

But if only the derogandum is subtracted from the system, Rex may fail to achieve his purpose. Indeed, suppose that p has been rejected, but the system contains not only p, but also $p \& q$. Then to eliminate only p will simply not do, for as long as $p \& q$ is a member of the system, so is p. What the rejection of p would achieve in such a case, would be at most to change the status of p; if it was explicitly commanded and so a member of the basis, it will now be one of the consequences of the basis, but remain a member of the system. Hence p has not been derogated at all.

This argument makes it clear that the derogation of p requires not only the explicit rejection of p itself, but also the rejection of all those propositions of which p is a consequence. We shall say that these propositions are implicitly rejected by the act of rejecting p. Moreover, it may occur that two or more propositions imply (taken together) a rejected proposition, although none of them (taken alone) does it. Suppose e.g. that $q \supset p$ and q are members of the system and that p is rejected. The set $\{q \supset p, q\}$ implies p, so it must be (implicitly) rejected.

Generalizing this result we can state the following *general criterion for implicit rejection*: The rejection of a set of propositions B implicitly rejects all propositions and sets of propositions that imply some of the propositions belonging to B.

It is worth noting that what is rejected by an act of rejection is not a set of propositions, but a *family* of sets. This fact determines an important difference between promulgation and derogation: it is always a *set* of propositions that is promulgated, but it is always a *family* of sets that is rejected. ("Rejected" means here "explicitly or implicitly rejected".)

What effects does an act of rejection produce? We must distinguish two cases:

(i) If none of the explicitly rejected propositions is a member of the system Cn(A), then none of the rejected sets is included in A. Here the problem of subtraction does not arise. But if some of the rejected propositions or sets were promulgated later, this fact would give rise to a conflict of ambivalence. Such conflict can only be resolved by application of some rule of preference.

(ii) If some of the explicitly rejected propositions are members of the system Cn(A), then some of the rejected sets are included in A. As the members of Cn(A) are promulgated, we have a conflict of ambivalence and need some rule of preference to resolve it. If it is resolved in favor of promulgation, the rejection produces no effects whatsoever and no derogation takes place, and there is no change in the system. But if rejection prevails, certain propositions must be eliminated by subtraction from the system. Which are these propositions? What criteria determine the operation of subtraction?

It is clear that no rejected proposition, nor a rejected set can remain in A; for in that case some of the members of the derogandum (i.e. some of the explicitly rejected propositions) would continue to be members of the system Cn(A). In particular, if a set is rejected, at least one explicitly rejected proposition is a consequence of it. Therefore, all rejected sets must be eliminated from A. But what does it mean to eliminate a set? If one of its elements is removed from the set, the set disappears as such: what we have in its place is another, less numerous set. On the other hand, as long as all its members are there, it is the same set. So removing at least one of its elements is a sufficient and necessary condition for the elimination of a set.

Now if – as the hypothesis runs – at least one of the explicitly rejected propositions belongs to the system Cn(A), the set A (i.e. the basis of the system) is one of the rejected sets. Hence it must be eliminated; but if we remove all its elements, the whole system collapses. So by derogating one norm-content, we would succeed in derogating the whole system. This seems to be a little too drastic as a method of complying with the requirement that all rejected sets should be eliminated from A.

This observation suggests the following *adequacy conditions* for the operation of subtraction: (i) no rejected proposition or set of propositions shall remain in the system, and (ii) the set of the subtrahend shall be *minimal*, i.e. only those propositions shall be eliminated that it is strictly necessary to remove in order to comply with (i). In other words, the remainder of the operation must be the *maximal* subset of A consistent with the derogation.

A subset of A (i.e. the explicitly promulgated propositions) that fulfills the requirements (i) and (ii) will be called a *derogans*. To each non-empty derogandum corresponds at least one derogans.

In order to construct a derogans corresponding to a derogandum, we must take at least one proposition out of all rejected sets in A.[27] But as some of such sets may have several members (none of which is rejected), any of them can be used for the construction of a derogans; so there are several ways for constructing a derogans and consequently we have not one derogans but several derogantes. As each derogans is a set of propositions, the set of all derogantes is a family. But what we must subtract – if we want to satisfy the adequacy conditions – *is only one of them*, for if we remove one derogans, the remainder will contain no rejected set (and therefore no rejected proposition). On the other hand, if more than one derogans is removed, the remainder is no longer a maximal set and so condition (ii) fails to be fulfilled.

This shows that situations may arise where to one derogandum correspond several derogantes and therefore there are several different ways of operating the subtraction corresponding to the same act of rejection. And what makes things even worse, we may have no criteria to prefer one of them. In such situations, there are several possible remainders instead of one; the remainder is not a set, but a family of sets. This is what we have called elsewhere the *logical indeterminacy of the system*.[28]

The problem of indeterminacy does not arise if the explicitly rejected propositions (the derogandum) are independent members of A. Then it is sufficient to eliminate from A the derogandum alone. In general: the derogation is univocal if and only if there is only one derogans and therefore only one remainder.

It may occur that the subtraction of a derogans carries with it the elimination of some other propositions that are a consequence of A (i.e. they do not belong to A, but are members of the system $Cn(A)$), and are no longer consequences of A minus derogans. So the set of eliminated propositions may, after all, be larger than the set of subtracted propositions (a derogans). This makes it convenient to distinguish between subtraction and elimination.

To sum up: derogation has been analyzed into two components: the act of rejection and the operation of subtraction, that leads to a new system (the remainder). The act of rejection identifies a derogandum and the resulting system is the remainder after subtracting a derogans (corresponding to the derogandum) from the original system. It should be emphazised, finally, that this kind of subtraction is – as our informal analysis shows – a much more complicated operation than the ordinary set-theoretical subtraction.[29]

6. INCONSISTENCY

In section 4 we examined the conflicts of ambivalence, that arise between two propositional attitudes: promulgating and rejecting the same norm-content. The two acts are incompatible because they tend to achieve incompatible results: addition of a norm-content to a system and its subtraction from it. Our purpose in this section is to analyze the other kind of normative conflict: inconsistency between norm-contents (normative contradiction).

If both a proposition p and its negation $\sim p$ are members of a normative system, the system is said to be inconsistent. The trouble with inconsistent systems is that it is impossible, for logical reasons, to obey all its norms. At least, the norms $!p$ and $!\sim p$ cannot be complied with. Moreover, if the classic notion of consequence is accepted, the effects of a contradiction are even more disastrous: all propositions belong to an inconsistent system. This is so because according to the classic notion of consequence, from a contradictory pair of propositions any proposition whatsoever can be derived. So all inconsistent systems are equivalent: they contain the same consequences and are equally useless. Everything is obligatory according to such a system, and nobody can ever possibly comply with it and so it cannot guide any action.

And yet it is extremely important to realize that inconsistent normative systems are perfectly possible and their occurrence, at least in certain areas like law, is rather frequent. The reason for this fact is fairly clear. The selection of the propositions that form the basis of the system (the set A) is based on certain empirical facts: the acts of commanding or promulgating. Now, there is nothing extravagant about the idea that an authority commands that p while another authority (or the same authority perhaps on a different occasion) commands that $\sim p$. Even one and the same authority may command that p and that $\sim p$ at the same time, especially when a great number of norms are enacted on the same occasion. This happens when the legislature enacts a very extensive statute, e.g. a Civil

Code, that usually contains four to six thousand dispositions. All of them are regarded as promulgated at the same time, by the same authority, so that there is no wonder that they sometimes contain a certain amount of explicit or implicit contradictions.

Nevertheless, many authors are extremely reluctant to accept this relatively simple fact. Some of them (especially deontic logicians and moral philosophers) are perhaps influenced by their (direct or indirect) interest in moral discourse, for it seems hard to accept that the same action may be morally good and wrong (obligatory and prohibited) at the same time. There is some grain of truth in this thought. It is probably true of rational morality, but very likely not true of positive morality, and is plainly false of positive law. Strangely enough, there are also legal philosophers, i.e. people whose primary interests concern positive law, who share this antiseptic conception. Kelsen is – or rather was – perhaps its most prominent representative among legal philosophers. In his *Reine Rechtslehre* (1960) he does not deny that legislators can enact contradictory laws, but he firmly maintains that the *system* of law is always consistent. This "miracle" is achieved, according to Kelsen, by legal science; jurists eliminate all contradictions and so "the chaos becomes a cosmos", i.e. "the multiplicity of general and individual legal norms enacted by different legal authorities becomes a unitary and consistent system, a legal order".[30]

What Kelsen says here sounds perhaps a little too optimistic, but is substantially true. Yet far from supporting his contention that legal systems are always consistent, it proves it to be false. Indeed, if contradictions must be eliminated, then there is such a thing as a contradiction that must be eliminated. Q.E.D.

This result is corroborated even by Kelsen himself. Indeed, in his latest publications ('Derogation' and 'Law and Logic', both included in Weinberger's edition of Kelsen's *Essays in Legal and Moral Philosophy*) Kelsen changes radically his view concerning normative conflicts, a view that he maintained in all his previous writings.

Now, in 'Law and Logic' Kelsen clearly states that conflicts between norms are perfectly possible, where by "conflictive norms" he understands two norms that prescribe incompatible actions, e.g. p and $\sim p$. (So Kelsen's notion of conflict of norms corresponds exactly to our "inconsistency between norm-contents".) Such conflicts differ, according to the new doctrine, from logical contradiction insofar as two contradictory propositions cannot both be true, whereas two conflictive norms can both be valid, in the sense of their having been issued by competent authorities. And such a conflict can only be solved – on Kelsen's new view – by explicit or implicit derogation of one (or both) of the two conflictive norms. So Kelsen's new position is in complete agreement with the views put forward in this paper.

It is, of course, a purely terminological matter whether the term "system" will only be applied to sets of norm-contents, once they are purged of their inconsistencies, or to inconsistent sets as well. The important thing is to identify the inconsistencies and to examine the techniques used to remove them. This is what we propose to do in this section.

It is interesting to observe that lawyers (not contaminated by philosophy) readily accept the possibility of contradictions in law. This is shown by the fact that there are old, traditional principles designed to solve such conflicts. The principles lex posterior, lex superior and lex specialis would have no application at all if there were no inconsistencies in legal dispositions. The very fact that lawyers often resort to such principles shows at least that they believe that normative contradictions are quite possible. And this belief is not mistaken.

How are the cases of inconsistency treated in legal practice?

Two situations are to be distinguished: (a) When a legislative authority discovers a contradiction in a legal system, it may either derogate one or both of the two conflicting norm-contents, or leave the things as they are relying on the judges' ability to resolve the conflict. If it chooses to derogate one or both conflicting norm-contents this solves the problem. The curious thing about derogation is the fact that a solution of the conflict can be reached by a rather unexpected procedure (at least if the classic notion of consequence is accepted): by derogating any proposition you like! This can easily be proved. Suppose that p and $\sim p$ are members of $\mathrm{Cn}(A)$ and that the legislature rejects q; in this case $\{p, \sim p\}$ is one of the rejected sets (for from a contradiction any proposition, including q can be derived) and at least one of its members must be eliminated. It is enough that one proposition not be a member of a system for it to be consistent. Hence the derogation of any proposition ensures the consistency of the system. The only problem that may arise in this connection is the indeterminacy of the remaining system.

(b) The situation of the judge seems to be different. Judges are supposed to apply the law, not to modify it. They lack competence or power to derogate laws enacted by the legislature (except perhaps in the case of unconstitutional laws). What can judges do when faced with an inconsistent system? What methods do they really apply to handle with such situations?

We must remember at this point that legal systems are not just sets of norms, but hierarchical structures.[31] There are certain hierarchical relations among legal norms or, as we would say, between norm-contents belonging to a legal system. Such hierarchies may be established by the legislature (i.e. by laws themselves) or determined by some general criteria based on the date of promulgation (lex posterior), the competence of the promulgating authority (lex superior) or the degree of generality of norm-contents (lex specialis). They may even be imposed by the judge himself, using his personal criteria of preference.[32]

As in the case of ambivalence, the three well established traditional principles are not sufficient to resolve all possible contradictions. Sometimes, judges must resort to further criteria, based e.g. on considerations concerning justice or other values involved in the issue.

The hierarchical ordering of the system enables the judge to give preference to some norm-contents or sets of norm-contents over others and so to disregard the hierarchically lower sets. In such cases, lawyers tend to say that the conflict was an apparent one and that there was really no inconsistency at all.

This may be perfectly true, provided one understands by "normative system" not a set but an *ordered set* of norm-contents, the ordering relations being intrinsic to the concept of a normative system. This shows that lawyers tend to use – at least in some contexts – the term "normative system" in this special sense.

But if by "normative system" we understand an ordered set of norm-contents, then every modification of the ordering relations modifies *eo ipso* the system itself. The fact that as a result of a new ordering the system provides different solutions for the same specific cases shows that it is another system, not identical with the original one, *even if it contains the same elements* (norm-contents).

In spite of this, there is a widespread idea that derogation (that removes altogether certain norm-contents) is a much more fundamental operation than simple ordering, and that therefore the judge, though he can impose a new ordering or modify the existing one, cannot derogate legislated norm-contents for the same reasons that he cannot promulgate new norms. The idea is that as long as the system contains the same elements it remains substantially identical and so the judge who "only" orders the elements of the system does not change it and hence does not trespass beyond his powers. Consequently, ordering is regarded as a much more elastic and less permanent operation than derogation.

But this idea is wrong. The impression that the removing of one or more propositions by derogation is somehow more fundamental and permanent than the imposing of an ordering on a system proves to be a mere illusion. A modification of the ordering is as fundamental as the removing of elements; indeed *both procedures are substantially equivalent.*[33] Those norm-contents that are "put aside" or disregarded by an ordering have as little application (as far as this ordering is concerned) as when they are derogated. As to the alleged permanence of derogation there is no difference either. A derogation made by a legislature may last for a very short time if the legislature changes its mind and promulgates again the derogated norm-contents. On the other hand, an ordering imposed by a judge may enjoy a very long life, if other judges adopt it as well. So the question of temporal duration is quite irrelevant for this issue.

The much debated problem whether judges "create" law or only apply it, can be settled in favor of the first thesis, at least in the sense that they modify the legal system by imposing orderings on its elements when they have to resolve contradictions, disregarding some of the norm-contents (which amounts to derogating them).

Nevertheless, though these two methods lead to substantially identical results (and this is what justifies calling them equivalent), they are two distinct methods, applied by different kinds of authorities (legislative authority in the case of derogation, judicial authorities in the case of ordering). Both are designed to solve the same problem: the inconsistency of a normative system. This shows that inconsistency is indeed treated as a problem that calls for a solution and hence that there are contradictions and inconsistent systems.

7. PERMISSION

For the imperative theory of norms (which is the most popular version of the expressive conception) there is only one type of normative act (commanding); so there are only mandatory norms, prescribing acts and omissions and so giving rise to obligations and prohibitions. Permission appears to to be a purely negative notion; it is the absence of prohibition. So there may be permitted states of affairs, but so far there are neither permissive acts (i.e. acts giving a permission), nor permissive norms.

How can this theory explain acts granting a permission or authorization? When Rex says: "Hereby I allow (permit) that *p*", how is this speech act to be analyzed?

There seem to be two possible ways out of this difficulty. (i) One way is to describe this act as an act of lifting a prohibition, i.e. as derogation of the prohibition of *p*. (ii) An alternative way is to accept a new kind of normative act, the act of giving or granting a permission (for short: act of permitting). If this is accepted, then it must also be accepted that there are two kinds of norms, mandatory norms and permissive norms (in the sense in which an expressivist uses the term "norm"). A permissive norm is – like a mandatory norm – a meaningful sentence in its peculiar, i.e. *permissive*, use. So the act of granting a permission can be described as the act of promulgating a permissive norm.[34]

These two proposals will be examined separately.

(i) The second analysis entails the explicit acceptance of a new kind of normative act; this is probably the reason why it is less popular among expressivists who feel some affinity with Ockham. But as has been argued in section 3, the first analysis also leads implicitly to the acceptance of a new normative act: the act of rejection. But as philosophers and logicians have so far paid comparatively little attention to the concept of derogation, no full analysis of the act of rejection has been elaborated yet.[35] This is a serious shortcoming of current expressive theories. Only if the act of rejection is recognized as a fundamental and independent normative act, can the expressive conception give account of such important issues as derogation and permission. Once this is done, there are two different concepts of permission: *negative permission* (absence of prohibition) and *positive permission* (derogation of a prohibition).

Positive permission is linked to a positive act, the act of rejection, and so to a conflict of ambivalence. This conflict may be actual or merely potential, if *p* has not been so far prohibited. Once this conflict is resolved by giving priority to the rejection and the prohibition is eliminated (by subtraction), *p* is permitted in the positive sense.

The main difference between negative and positive permission (apart from their different origin) appears to be this: if *p* is negatively permitted, then if an authority prohibits *p* there is no conflict: $\sim p$ is added to the system and in the new system it is no more true that *p* is permitted. But if *p* is positively permitted, any act of prohibiting *p* gives rise to a conflict of ambivalence that calls for a solution. Only if this conflict is resolved in favor of the act of prohibiting, it will be true that *p* is prohibited (in the new system).[36]

(ii) We turn now to the second analysis of sentences granting permissions. For this analysis there are two different acts: commanding and permitting, promulgation of a mandatory norm and promulgation of a permissive norm. Consequently, there are also two kinds of permissions: negative or weak permission (absence of prohibition) and *strong permission*, given by a permissive norm. Strong permission, like positive permission, is incompatible with prohibition, but here the conflict seems to be not of ambivalence, but of contradiction between two norms. Yet it must be observed that this contradiction is not the classic contradiction where p and $\sim p$ are both members of the commanded set. In our hypothesis $\sim p$ has been commanded, so $\sim p$ belongs to the commanded set, but p has not been commanded; it has been permitted. What happens with p as a result of it being permitted? It certainly cannot belong to the commanded set, for in that case it would be true that p is obligatory. In other words: how are we to construct the system, once we accept two kinds of promulgation? We cannot put together all promulgated norm-contents, for then we could not distinguish between obligations and permissions. (For an expressivist the difference can only lie in the kind of act of promulgating, not in the conceptual content of the act; if there were a difference in the proposition this would mean the acceptance of the hyletic conception!) The only way out seems to be to form two sets: the set of commanded propositions (the commanded set A) and the set of permitted propositions (the permitted set B). But then we must unify somehow both sets, if we want a non-ambivalent system. It is clear that subtracting the permitted set from the commanded set would not do. What we want is not to remove obligations, but prohibitions; so if p is prohibited and hence $\sim p$ is a member of A, and p is permitted as well and so a member of B, what we must subtract from A is not p, but its negation ($\sim p$). Therefore the operation of unification requires subtracting from the commanded set the negations of the propositions that are members of the permitted set.[37] So if p is permitted, $\sim p$ must be subtracted (eliminated from A) and vice versa. Thus the permission of p gives rise to the same operation as the rejection of $\sim p$.

At this stage one feels tempted to ask: are there really two distinct analyses? What is the difference, if any, between promulgating a permission and derogating a prohibition? What is the difference between the act of permitting p and the act of rejecting $\sim p$?

There are indeed very strong analogies between the two concepts:

(1) Commanding a proposition is incompatible with permitting its negation, exactly in the same way as commanding that p is incompatible with rejecting p. In both cases we have a conflict of ambivalence (two incompatible attitudes regarding the same proposition).

(2) The set of the negations of permitted norm-contents (which is to be

subtracted from the commanded set) is formally identical to the set of rejected propositions, for it is constructed in the same way.

(3) The operation of subtraction is the same: the identity of the subtrahend determines the identity of the remainder.

(4) Strong permission proves to be the same as positive permission.

One has the impression that both analyses are substantially equivalent in the sense that they are two different descriptions of the same situation. If this were so, it would be a rather surprising result; it would show the fruitfulness of the concept of derogation and its importance for the theory of norms. The concept of a permissive norm could be dispensed with; a fact that would justify the position of those expressivists that only accept mandatory norms, provided they accept the existence of derogation.

8. CONCLUSIONS

We are in a position of drawing some conclusions from the preceding analyses; we shall do it by comparing the hyletic and the expressive conceptions of norms (henceforth HC and EC).

(1) HC rests upon a very strong ontological presupposition of platonic flavor: the assumption that there are prescriptive propositions. No such presupposition is needed for EC.

(2) The price that EC must pay for this advantage is the proliferation of illocutionary acts: it must distinguish between asserting and commanding, on the one hand; and between two kinds of rejecting (descriptive and prescriptive rejection) on the other. For HC there are only two types of acts: assertion and rejecting, because commanding is just asserting an O-norm, and permitting is asserting a P-norm. And there is only one kind of rejection; what varies is the content of this act; it may be a descriptive or a prescriptive proposition, i.e. a norm.

(3) EC can dispense with permissive norms, for it can give an account of acts granting a permission in terms of derogation (rejection and subtraction). For HC there can be permissive norms on the same level as mandatory norms (O-norms).

(4) For EC there are two kinds of incompatibility: conflicts between norm-contents (normative inconsistency: $!p$ and $! \sim p$) and conflicts between acts of promulgation and rejection (ambivalence: $!p$ and ¡p).

For HC there are two kinds of inconsistency between norms: the inconsistency between obligation and prohibition (Op and O $\sim p$) and the inconsistency between prohibition and permission (O $\sim p$ and Pp, or, which is the same, Op and $\sim Op$). Besides these two kinds of inconsistency between norms, there is the conflict of attitudes between promulgation and rejection (ambivalence). Whether the inconsistency between prohibition and permission is reducible to a conflict of ambivalence (as the analysis of Section 7 suggests) may be regarded as an open question.

(5) For HC there are two logics: a logic of norms and a logic of normative propositions (a logic of promulgation and derogation). The logic of norms is concerned with logical relations of prescriptive propositions (norms); it is a peculiar normative logic.[38] The logic of normative propositions is concerned with logical relations of descriptive propositions about normative systems. Its aim is the development of a comprehensive logic of normative systems, that may be regarded as a special case of Tarski's logic of systems. Especially interesting would be a logic capable of rendering the dynamic character of normative systems, i.e. their temporal development through acts of promulgation and derogation. (It need not be mentioned that at its present stage deontic logic is far from having reached this aim.)[39]

(6) For EC there is only one possible logic: the logic of (descriptive) normative propositions, in the same sense as for HC. This deontic logic looks very much like von Wright's "classic" deontic logic,[40] but with two important differences: (a) Normative propositions are always relative to a definite normative system. Hence the subscripts in formulae like $\mathbb{O}_A(p)$. (b) The law of deontic subalternation $\mathbb{O}_A(p) \supset \mathbb{P}_A(p)$ – analogous to von Wright's theorem $Op \rightarrow Pp$ – does not hold unrestrictedly.[41] It does not hold for inconsistent systems and one of the main contentions of this paper is that normative systems can be inconsistent. But from what has been said in sections 6 and 7 it follows that a system is consistent: (i) if there is at least one derogated proposition; (ii) if the notion of consequence is restricted by an ordering relation imposed on the system,[42] and (iii) if there is at least one positively permitted proposition. (In fact, the three conditions amount to the same: derogation of at least one norm-content.) So the conditions under which a system is consistent (and the law of deontic subalternation holds good) are extremely weak and easily obtainable.

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NOTES

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[1] Cf. Kalinowski's discussion of these terms in Kalinowski 1978.

[2] Cf. D. Føllesdal and R. Hilpinen, 'Deontic Logic: An Introduction' in Hilpinen 1971.

[3] Among others Kalinowski and Rödig.
[4] Cf. G. H. von Wright 1963, Weinberger 1977.
[5] The term "language" is to be understood in a wide sense; a gesture, a look, a traffic light are in this sense linguistic expressions.
[6] Cf. Alchourrón and Bulygin, 'Von Wright on Deontic Logic and the Philosophy of Law', in P. A. Schilpp (ed.), *The Philosophy of Georg Henrik von Wright*, Library of Living Philosophers, La Salle, Illinois (forthcoming).
[7] Cf. H. Reichenbach 1947, p. 337 ff.
[8] C. I. Lewis 1946, p. 49.
[9] G. H. von Wright 1963, p. 134; "The 'fully developed' system of Deontic Logic is a theory of descriptively interpreted expressions. But the laws (principles, rules), which are peculiar to this logic, concern logical properties of the *norms* themselves, which are then reflected in logical properties of norm-propositions. Thus, in a sense, the 'basis' of Deontic Logic is a logical theory of prescriptively interpreted O- and P-expressions."
[10] v. Wright 1963, p. 94.
[11] Cf. Bibliography. A less clear case – at least at first sight – is that of Castañeda, but one should not be misled by differences in terminology. What Castañeda calls 'norms' are normative propositions (in our sense); so his is a very interesting theory of normative propositions, but he does not analyze norms, that are referred to as 'regulations', 'ordinances' or 'rules'. Cf. Castañeda 1978.
[12] Cf. Kalinowski 1967 and 1978, Weinberger 1978.
[13] D. Føllesdal and Hilpinen, 1971. p. 7f.
[14] Cf. Alchourrón 1969 and 1972.
[15] Kalinowski 1967 and 1978.
[16] Cf. Weinberger 1978, von Wright 1963, 1968, Alchourrón-Bulygin 1971.
[17] Cf. Moritz 1963.
[18] *Norm and Action*, 1963, p. 7 ff.
[19] But the theory can also be adapted to customary norms. Their existence is dependent on certain dispositions, which are revealed by certain actions.
[20] On the notion of consequence see Tarski 1956, esp. III, V, XII and XVI.
[21] The literature on the concept of derogation is rather scarce. Cf. the excellent paper by Cornides who is a true forerunner in this field. Weinberger's distinction between 'Begrenzungssatz' and 'Tilgungsoperation (Streichung)' seems to reproduce our distinction between rejection and elimination. Cf. Weinberger 1978, p. 192.
[22] Hare describes the difference between negation and rejection saying that in a negation the term 'not' is part of the phrastic, but it can also occur in neustics: they become then 'not-yes' and 'not-please'. This seems to correspond to what we call rejection. Thus Hare's 'not-yes' is our descriptive rejection and 'not-please', the prescriptive rejection. Cf. Hare 1952, pp. 20–21.
[23] Cf. Alchourrón-Bulygin 1979; for a different view see v. Wright 1963, chapter 7.
[24] Cf. Raz 1970, who drew our attention to this fact. See Alchourrón-Bulygin 1976.
[25] Carnap 1942, p. 187.
[26] Here the term "rule" does not mean a norm (command or commanded content) but a purely conceptual criterion.
[27] We say 'at least one' instead of 'only one' because in the case of overlapping sets it is impossible to remove one and only one element of all of them. Consider e.g. the case of the three following sets: $\{x, y\}, \{y, z\}$ and $\{x, z\}$; if one element of two of them is removed, both elements of the third are removed as well.

[28] Cf. Alchourrón-Bulygin 1976, 1977, 1978. This problem has been already seen by Cornides, though he seems not to give it much importance. Cf. Cornides 1969, p. 1241.
[29] For a detailed analysis of the concept of derogation see Alchourrón-Makinson.
[30] Kelsen 1960, p. 74.
[31] This is emphasized by most legal philosophers. Cf. Kelsen, Alf Ross, Hart.
[32] From the logical point of view such an ordering is either partial ordering (reflexive, transitive and antisymmetric relation) or weak ordering (reflexive, connected and transitive, though not necessarily antisymmetric relation). The first alternative (partial ordering) is thoroughly studied in Alchourrón-Makinson.
[33] In the sense that to every derogation corresponds a (set of) ordering(s) and to every ordering corresponds a derogation. For a detailed proof see Alchourrón-Makinson. But they are not quite identical: a partial ordering imposed on a system confers uniqueness upon otherwise indeterminate derogations by means of a process of ranking the various remainders.
[34] There are relatively few expressivists who accept this second interpretation. Cf. Moritz 1963, who is one of the few.
[35] There are some valuable remarks on this subject. Cf. Hare 1952, p. 21, says: "Modal sentences containing the word 'may' could, it seems, be represented by negating the neustic; thus 'You may shut the door' (permissive) might be written 'I don't tell you not to shut the door' and this in turn might be rendered 'Your not shutting the door in the immediate future, not-please'." If the negation of the neustic is taken to be a rejection – as was suggested in Note 22 – Hare's proposal amounts to analyzing the act of permitting in terms of rejection.
[36] Some authors interpret permissions as exceptions in a prohibitive norm. So to permit would mean to introduce an exception in a prohibition. This can be explained as partial derogation of the norm-content, i.e. as a derogation of some of the consequences of the prohibitive norm.
[37] It would be pointless to form two sets, a set of permitted propositions and a set of their negations, as it would be pointless to separate the commanded from the prohibited propositions. In both cases we have the same attitude regarding two contradictory propositions.
[38] Cf. Alchourrón 1969, 1972, and Alchourrón-Bulygin 1971.
[39] Some hints in this direction are to be found in Alchourrón-Makinson.
[40] Von Wright 1951 and 1968.
[41] Cf. Lemmon 1965.
[42] Cf. Alchourrón-Makinson.

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HIERARCHIES OF REGULATIONS AND THEIR LOGIC*

ABSTRACT. We study some of the ways in which the imposition of a partial ordering on a code of laws or regulations can serve to overcome logical imperfections in the code itself. In particular, we first show how partial orderings of a code, and derivative orderings of its power set, may be used to confer uniqueness upon otherwise indefinite derogations by ranking remainders; and second, we show how such orderings may be used to resolve contradictions implicit in a code by a process which we shall call delivery. Finally, we investigate the relations between derogation and delivery, showing that although the two processes appear and are generally assumed to be quite different from each other, nevertheless for finite inconsistent codes, the composite process of derogating and then selecting a remainder turns out to be equipowerful with delivery. For consistent codes, where delivery reduces to its underlying consequence operation and so is of no special interest, the correspondence is with a more general process of 'relative delivery'. Sections 2 and 3, on derogation and the resolution of contradictions respectively, are written so that they may be read in either order. Section 4, on the relations between the two, depends on both. The study is carried out mathematically, and the reader is assumed to be familiar with elementary properties of partial orderings and consequence operations. Throughout, however, attention is also given to the realities of juridical practice.

1. HIERARCHIES OF REGULATIONS

When we consider the regulations in a legal or administrative code, we can often discern some kind of hierarchy among them. Some are regarded as more basic or fundamental than others. The ordering may be quite vague, or fairly well delineated; perhaps precise in some areas whilst vague in others. It may conceivably be quite far-reaching, so that almost every regulation in the code can be compared to almost every other; or it may be quite fragmentary, with only a few outstanding points of comparison made. It may be determined in part by considerations arising from the text of the regulations themselves, such as the existence of cross-references from one to another; and it may also be determined in part by factors of a more extrinsic kind, such as the powers and competences of the issuing bodies, dates of promulgation and amendment, and the degree of specificity or generality of the regulations made.

If we are to find some mathematical order in the affair, it is natural and, we hope to show, useful, to take the ordering to be reflexive, transitive, and antisymmetric, that is as a partial ordering. Rarely, however, would it get anywhere near being a chain. So let us define a *hierarchy of regulations* to be

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a pair $(A, \leq)$ where A is a non-empty set of propositions, called a *code*, and $\leq$ is a partial ordering of A. Given any set A, the strongest partial orderings of A will of course be the chains, and the weakest partial ordering will be the identity relation.

This definition, of course, already involves a choice of some consequence, for partial orderings are not the only kind of ordering that could be considered. One could for example work with weak orderings, that is, relations that are transitive, connected (and hence also reflexive) but not necessarily antisymmetric; and it seems that some writers on jurisprudence in the tradition of Kelsen are accustomed to viewing the relationship in such a way. But in this paper we have found it convenient to work with partial orderings, and we shall leave for a later occasion the question of how far a satisfying, and more or less analogous, theory can be constructed with weak orderings.

Now imagine a judge or administrative officer attempting to apply a code of regulations so as to reach a verdict on a particular case before him. It will sometimes happen that no one regulation by itself suffices to yield a verdict but two or more taken together do. Thus the judge, and we, need to consider *sets* of regulations. Of course, in practice, the entire code will be finite, and thus also each subset of it. But for the moment (sections 1 and 2) we shall proceed quite generally, imposing conditions of finitude only later when expressly needed (sections 3 and 4).

Just as the judge needs to consider questions of precedence between individual regulations, so too does he need to compare, whenever possible, one set of regulations with another. In other words, given a relation $\leq$ that partially orders A, we need to envisage ways in which $\leq$ induces some kind of ordering of 2^A. The difficulty here is that, from a mathematical point of view, there are very many ways in which we may define orderings (not necessarily partial orderings) over 2^A on the basis of $\leq$ over A, and it is not immediately obvious which among them are the most appropriate for the purposes in hand. Indeed, in the authors' view, there is no *a priori* way of determining what the most suitable definitions will be; we must be content to be guided in our trials by rough heuristic considerations, and corrected in our choices by seeing their results. A rough working principle that seems reasonable to begin with as a heuristic guide is that *the unchallengeability or security of a set of regulations is no greater than that of its most exposed members*; and two concepts that seem to the authors, after considerable experimentation, to be useful are the following. If $(A, \leq)$ is a hierarchy of regulations and $B, C \subseteq A$ we shall say that C *is at least as exposed as* B, and write $C \leq B$, iff for every $b \in B$ there is a $c \in C$ with $c \leq b$. For simplicity of notation we are here using the same symbol $\leq$ for the relation over 2^A as for the underlying relation over A; context or explicit reference will always make it clear which one we are dealing with. Notice that $\leq$ over 2^A agrees with $\leq$ over A for singletons: for all $b, c \in A$, $c \leq b$ iff $\{c\} \leq \{b\}$. If $B, C \subseteq A$ we shall say that C *is strictly more exposed than* B, and write $C \prec B$, iff $C \neq \varnothing$ and for all $b \in B$ there is a $c \in C$ with $c < b$ (i.e. $c \leq b$ and $c \neq b$). Clearly:

OBSERVATION 1.1. $\leq$ over 2^A is reflexive and transitive, but not in general antisymmetric.

A counterexample to antisymmetry is given by taking $B = \{b, c\}, C = \{c\}$, $c \leq b$, where b, c are distinct. Then we have $C \leq B$ and $B \leq C$, but $B \neq C$. Indeed, it is easy to show that $\leq$ over 2^A is never antisymmetric except when $\leq$ over A is the identity relation.

OBSERVATION 1.2. $\prec$ over 2^A is transitive. For finite A, $\prec$ is asymmetric and thus a strict partial ordering of 2^A.

OBSERVATION 1.3. For all $B, C \subseteq A$, $C \prec B$ implies $C \leq B$. For finite A we also have $C \prec B$ implies both $C \leq B$ and $B \nleq C$, but not always conversely.

The verification of asymmetry in 1.2 appeals to the condition that when $C \prec B$ then $C \neq \varnothing$, to rule out the case $\varnothing \prec \varnothing$. The condition of finitude in 1.2 and 1.3 cannot be dropped, as is easily shown by example. Finally we observe that in the special case where $\leq$ over A is chosen to be identity, then clearly $C \leq B$ iff $B \subseteq C$, whilst $C \prec B$ iff $C \neq \varnothing$ and $B = \varnothing$.

2. FIRST APPLICATION: CONFERRING UNIQUENESS ON A DEROGATION

The concept of derogation

Suppose that A is a set of regulations, y is some proposition that is implied by A, and that for some reason a legislative body wants to eliminate y. In such a situation, the body may decide to reject y, with the intention of thereby rejecting implicitly whatever in A implies y, retaining the remainder. This we shall call derogation.

However, such a procedure raises a serious logical difficulty, as was pointed out by C. E. Alchourrón and E. Bulygin, 'Unvollständigkeit, Widersprüchlichkeit und Unbestimmtheit der Normenordnung', in A. G. Conte, R. Hilpinen and G. H. von Wright, eds, *Deontische Logik und Semantik* (Wiesbaden, 1977), as also in their paper 'The Expressive Conception of Norms' in this volume. All goes well if whenever any subset C of A implies y then some element of C implies y; we may then simply choose from A those elements $a_1, \ldots, a_n$ that individually imply y, drop the set $D = \{a_1, \ldots, a_n\}$, and keep the remainder $A - D$. Then $A - D$ will not imply y (since no element of it does), and will in fact be the *greatest* subset of A with that property (since all such subsets are disjoint from D). But if there are subsets of A that imply y without any of their elements doing so, we no longer have such uniqueness: there may not exist any *greatest* subset of A that does not imply y, but only several *maximal* ones; and this even if, as in real life, A is finite. The rejection of y does not tell us which of these sets is to be retained, and which complement is to be eliminated.

To deal with this, we must put the situation more precisely. Let Cn be any consequence operation over the entire language of our regulations and verdicts – that is, any operation on sets of propositions such that $X \subseteq \mathrm{Cn}(X)$, $\mathrm{Cn}(X) = \mathrm{Cn}(\mathrm{Cn}(X))$, and $X \subseteq Y$ implies $\mathrm{Cn}(X) \subseteq \mathrm{Cn}(Y)$ for all sets X, Y of propositions. When $y \in \mathrm{Cn}(X)$ we shall say that X *implies* y. To simplify notation, we shall write $\mathrm{Cn}(\{x\})$ as $\mathrm{Cn}(x)$.

Now, generalizing a little upon the situation considered informally, let Y be any set of propositions; we define $(A \perp Y)$ to be the family of all maximal subsets $B \subseteq A$ that do not imply any element of Y. In other words, whenever $B \subseteq A$, we have $B \in (A \perp Y)$ iff both:

(1) $y \notin \mathrm{Cn}(B)$ for all $y \in Y$; i.e. $\mathrm{Cn}(B) \cap Y = \varnothing$

(2) whenever $B' \subseteq A$ and $\mathrm{Cn}(B') \cap Y = \varnothing$ then $B \not\subset B'$.

Each set $B \in (A \perp Y)$ will be called a *remainder* after rejecting Y. The process of passing from A to $(A \perp Y)$ will be called *derogation* of Y in A.

We have here first defined the notion of remainder, and thence indirectly that of its eliminated complement. It is also possible to do things in the reverse order, as follows:

OBSERVATION 2.1. Let A be any code of regulations, $B \subseteq A$, and Y any set of propositions. Then B is a remainder after rejecting Y iff $A-B$ is a minimal subset of A that contains at least one element from every subset of A that implies some element of Y.

However our initial characterization is simpler to formulate and usually easier to work with. As customary, we shall say that a consequence operation Cn is *compact over A* iff for all $D \subseteq A$ and every proposition x, $x \in \mathrm{Cn}(D)$ iff $x \in \mathrm{Cn}(D')$ for some finite $D' \subseteq D$. This condition will hold whenever Cn is compact *simpliciter*, or A finite. The following observation tells us, in effect, that in all real-life situations, $(A \perp Y)$ is non-empty.

OBSERVATION 2.2. Let A be any code of regulations, Y a set of propositions, and Cn a consequence operation compact over A. Then $(A \perp Y)$ is non-empty iff $\mathrm{Cn}(\varnothing) \cap Y = \varnothing$.

Proof. If $B \in (A \perp Y)$ then $\mathrm{Cn}(\varnothing) \cap Y \subseteq \mathrm{Cn}(B) \cap Y = \varnothing$. For the converse, if $\mathrm{Cn}(\varnothing) \cap Y = \varnothing$ then since Cn is compact over A we have by Zorn's Lemma that there is a maximal $B \subseteq A$ with $\mathrm{Cn}(B) \cap Y = \varnothing$, and thus $B \in (A \perp Y)$.

We now give a necessary and sufficient condition for the uniqueness of remainders, that makes precise some of the informal remarks made at the beginning of this section.

OBSERVATION 2.3. Let A be any code of regulations, Y a set of propositions, and Cn a consequence operation compact over A. Then $(A \perp Y)$ has a unique element iff whenever $C \subseteq A$ and $\mathrm{Cn}(C) \cap Y \neq \varnothing$ then for some $c \in C$, $\mathrm{Cn}(c) \cap Y \neq \varnothing$.

Proof. Suppose first that $(A \perp Y)$ does not have a unique element. If $(A \perp Y)$ is empty then by observation 2.2 $\mathrm{Cn}(\varnothing) \cap Y \neq \varnothing$, whilst of course there is no $c \in \varnothing$ with $\mathrm{Cn}(c) \cap Y \neq \varnothing$, and we are done. Suppose then that there are two distinct elements $B_1, B_2 \in (A \perp Y)$. By maximality, $B_2 \subsetneq B_1$, so $B_1 \subset B_1 \cup B_2$, so $\mathrm{Cn}(B_1 \cup B_2) \cap Y \neq \varnothing$. But for all $b \in B_1 \cup B_2$ we have $\mathrm{Cn}(b) \cap Y \subseteq \mathrm{Cn}(B_1) \cap Y = \varnothing$ or $\mathrm{Cn}(b) \cap Y \subseteq \mathrm{Cn}(B_2) \cap Y = \varnothing$, so that the condition of the observation fails.

For the converse, suppose that for some $C \subseteq A$, $\mathrm{Cn}(C) \cap Y \neq \varnothing$ whilst $\mathrm{Cn}(c) \cap Y = \varnothing$ for all $c \in C$. Since Cn is compact over A we may assume without loss of generality that C is finite; and so in turn assume without loss of generality that whenever $C' \subset C$ then $\mathrm{Cn}(C') \cap Y = \varnothing$. Now if $C = \varnothing$ then by observation 2.2, $(A \perp Y)$ is empty and we are done. If $C \neq \varnothing$ then clearly C is not a singleton, so that there are distinct elements c_1, c_2 in C. Put $C_1 = C - \{c_2\}$ and $C_2 = C - \{c_1\}$. Then $C_1, C_2 \subset C$, so $\mathrm{Cn}(C_1) \cap Y = \varnothing = \mathrm{Cn}(C_2) \cap Y$. Since Cn is compact over A, we have by Zorn's Lemma that there are B_1, $B_2 \in (A \perp Y)$ with $C_1 \subseteq B_1, C_2 \subseteq B_2$. It remains only to show that $B_1 \neq B_2$. Well, $c_1 \in C_1 \subseteq B_1$; but $c_1 \notin B_2$, for if $c_1 \in B_2$ then $C = C_2 \cup \{c_1\} \subseteq B_2$ so that $\mathrm{Cn}(B_2) \cap Y \neq \varnothing$ contradicting $B_2 \in (A \perp Y)$. This completes the proof.

In the special case where Y is a subset D of A, to derogate it, in the sense we have defined, is to do much more than merely rescind or abrogate it. For when we abrogate a subset D of A, we merely *drop* it from the code, leaving $A - D$ intact even if it implies some of the regulations in D. But when we derogate D, forming $(A \perp D)$, then no element of D is implied by any $B \in (A \perp D)$. Whenever $B \in (A \perp D)$ then clearly $B \subseteq A - D$ but not in general conversely. Clearly derogation coincides with abrogation just when no element of the set D rejected is implied by the complement $A - D$. That is:

OBSERVATION 2.4. Let $D \subseteq A$. Then the following conditions are equivalent:

$$A \perp D = \{A - D\},\ A - D \in A \perp D,\ \mathrm{Cn}(A - D) \cap D = \varnothing.$$

Proof. This can be obtained from observation 2.3 under the hypothesis that Cn is compact over A, but it is also easily verified from the definitions without needing that hypothesis.

A corollary of 2.4, of formal more than practical interest, is that when we iterate the process of derogation, we get nothing new:

OBSERVATION 2.5. Whenever $B \in (A \perp Y)$ then $A \perp (A - B) = \{B\}$.

Proof. When $B \in (A \perp Y)$ then clearly $\mathrm{Cn}(B) \cap (A - B) = \varnothing$, so we can apply 2.4.

The distinction between derogation and abrogation casts light on the process of amendment. For an amendment does not consist simply of *adding* a new piece of legislation, but rather of *replacing* one piece by another, and so may be seen as composed of two ingredients – a deletion and an addition. Now the deletion may sometimes be intended as a simple abrogation, so that only the explicitly rejected items are considered as eliminated; but it may sometimes be meant as a derogation, so that not only the explicitly rejected items, but also any others in the code that happen to imply them, are understood to be eliminated. In practice, both forms arise; in some cases the intention is left obscure; and of course as observation 2.4 tells us, there are many cases in which lack of clarity on the matter does not matter, as the result is the same.

Finally, we note in passing that serial derogation is not quite the same thing as simultaneous derogation. If A is a code of regulations and Y, Z are sets of propositions, then if we first choose a $B \in (A \perp Y)$ and then go on to choose a $C \in (B \perp Z)$, then C will be a *subset* of some $D \in A \perp (Y \cup Z)$, but as can be shown by a small finite example C need not itself be an element of $A \perp (Y \cup Z)$. On the other hand, every simultaneous derogation can be broken down into serial parts: if $C \in A \perp (Y \cup Z)$ then if Cn is compact over A, there is a $B \subseteq A$ with $B \in (A \perp Y)$ and $C \in (B \perp Z)$. Putting these two remarks together, we have that $A \perp (Y \cup Z) \subseteq \bigcup\{(B \perp Z) : B \in (A \perp Y)\}$ but not always conversely. We also note that serial derogation is not commutative, in the sense that if A is a code of regulations and Y, Z are sets of propositions, then $\bigcup\{(B \perp Z) : B \in (A \perp Y)\}$ need not equal $\bigcup\{(C \perp Y) : C \in (A \perp Z)\}$ as can also be shown by a small finite example. However these matters are not part of our main theme, which is the use of partial orderings of A to confer uniqueness on $A \perp Y$.

Conferring Uniqueness

Let us imagine the situation of a judge who is called upon to apply a code of laws upon which a non-unique derogation has been made. He needs to reach a verdict on some question before him, but he does not know *which* of the various remainders $B \in A \perp Y$ left after rejecting the set Y, he is free to use. The choice of B may make a material difference to this judgment on the question. Of course, if all the $B \in A \perp Y$ imply a given verdict, then he has no need to choose between them, and can leave that problem to the next judge. But in general, this will not be possible.

One way of dealing with this situation is to amplify the power of the derogation, by transforming Y into a set Y' such that $A \perp Y'$ is a singleton, whose only element is one of the elements of $A \perp Y$. If $A \perp Y$ is non-empty, this can always be done; for example, by observation 2.5 we have that for all $B \in A \perp Y, \{B\} = A \perp (A - B)$, so that we can put Y' to be $A - B$. And again, it is easy to verify that for all $B \in A \perp Y, \{B\} = A \perp (Y \cup B')$ where $B' = \bigcup(A \perp Y) - B$, so that Y' can also be chosen as an extension of Y.

However, another and very natural way of selecting a B from the various alternatives in $(A \perp Y)$ is by building up a relation $\leq$ over the set A of regulations (or extending an already existing such relation $\leq_0$) in such a way as to confer a preferred status on one of the sets $B \in (A \perp Y)$, or at least (and a judge may do less when less will do) isolate a subset $(A \perp Y)^*$ of $(A \perp Y)$ that is small enough for each set of regulations in $(A \perp Y)^*$ to suffice to resolve the case in hand. In other words, we may seek uniqueness of derogation, or at least mitigation of its multiplicity, by careful use of ordering.

Now usually there will be *many* ways in which this can be done (though, as we shall see after proving observation 2.7, there are some situations in which there is no way of doing it by an extension of a previously given $\leq_0$), and so the judge is faced with the responsibility of choosing which way to built up his relation $\leq$ over A. But the important advantage for judicial and legislative practice is that the building can be done by a series of small steps, and these steps need not be gratuitous. The procedure will be gradualist in that the judge will be able to built up his ordering by considering relevant pairs of elements of A one at a time, as contrasted with trying to decide between the bulky sets $B \in (A \perp Y)$ in one fell swoop. It need not be gratuitous, because the judge will usually be able to find some plausible reasons, intrinsic or extrinsic, for regarding one regulation in A as more or less solid than another.

Our problem now is to describe and analyse this in mathematical terms. First, we consider the simple case where the judge is facing virgin territory as far as ordering is concerned, in other words in which there is no given partial ordering $\leq_0$ of A (other than the identity relation) of which the judge's $\leq$ should be an extension. In this case, a suitable $\leq$ always exists, as our next observation shows.

OBSERVATION 2.6. Let A be any set of regulations and Y a set of propositions. Then for every $B \in (A \perp Y)$ there is a partial ordering $\leq$ of A such that for all $B' \in (A \perp Y)$, if $B' \neq B$ then $B' \prec B$ and $B \npreceq B'$.

Proof. Of course, when A is finite, then the assertion $B \npreceq B'$ follows from $B' \prec B$ (observation 1.3); but we have formulated the result so as to cover the infinite case as well. Let $B \in (A \perp Y)$, and write $\{B_i\}_{i \in I}$ for $(A \perp Y) - \{B\}$. Put $\leq$ to be the relation $[(A - B) \times B] \cup I$ where I is the identity relation over A. Clearly $\leq$ partially orders A. Now whenever $B_i \in \{B_i\}_{i \in I}$ then by maximality we have $B_i \nsubseteq B$, so there is a $c \in B_i - B \subseteq A - B$, so by the definition of $\leq$ we have $c \leq b$ and indeed $c < b$ for all $b \in B$, so that $B_i \prec B$ and $B \npreceq B_i$.

Sometimes, however, a judge is faced with a partial ordering $\leq_0$ of A, that has already been built up in some way, and which he prefers not to violate; any partial ordering that he uses should preferably be an extension of $\leq_0$. In this case the situation is a little more complex, as our next observation shows.

OBSERVATION 2.7. Let $(A, \leq_0)$ be any hierarchy of regulations, and Y a set of propositions. Then for every $B \in (A \perp Y)$ the following conditions are equivalent:

(1) There is a partial ordering $\leq$ of A with $\leq_0 \subseteq \leq$ such that for all $B' \in (A \perp Y)$, if $B' \neq B$ then $B' \prec B$ and $B \npreceq B'$,

(2) B is maximal modulo $\leq_0$ in $(A \perp Y)$; that is for all $B' \in (A \perp Y)$, if $B' \neq B$ then $B \npreceq_0 B'$.

Proof. Clearly (1) implies (2), for if $B \leq_0 B'$ and $\leq$ over A extends $\leq_0$, then $B \leq B'$. It remains to show that (2) implies (1). This can be done by deepening a little the proof of 2.6. As before, write $\{B_i\}_{i \in I}$ for $(A \perp Y) - \{B\}$. Suppose condition (2) holds. Put $\nabla_B = \{a \in A : b \leq_0 a$ for some $b \in B\}$, and choose $\leq$ to be the relation $[(A - \nabla_B) \times \nabla_B] \cup \leq_0$. Clearly this extends $\leq_0$, and it is easily verified that $\leq$ partially orders A. Now whenever $B_i \in \{B_i\}_{i \in I}$ we have by condition (2) that $B \npreceq_0 B_i$, so there is a $c \in B_i - \nabla_B$, so by the definition of $\leq$ we have $c \leq b$ and indeed $c < b$ for all $b \in B$, so that $B_i \prec B$ and $B \npreceq B_i$.

Note that there are simple examples of hierarchies $(A, \leq_0)$ and sets Y, with non-empty $A \perp Y$, such that for all $B \in (A \perp Y)$ conditions (1) and (2) of 2.7 both fail. For example, put $A = \{a_0, a_1, a_2\}$ where a_1 and a_2 are mutually inconsistent but each is consistent with a_0. Then the maximal consistent subsets of A are just $B_1 = \{a_0, a_1\}$ and $B_2 = \{a_0, a_2\}$, and so putting $Y = \{x \wedge \neg x\}$ we have $A \perp Y = \{B_1, B_2\}$. Put $\leq_0$ to be the linear ordering of A with $a_0 <_0 a_1 <_0 a_2$. Since $\leq_0$ linearly orders A, the only partial order of A that extends $\leq_0$ is $\leq_0$ itself. Moreover we have $B_1 \leq_0 B_2$ and $B_2 \leq_0 B_1$, and so conditions (1) and (2) of observation 2.7 both fail. Thus, as hinted in our earlier informal remarks, there are cases of a hierarchy $(A, \leq_0)$ and a non-empty derogation $(A \perp Y)$ such that there is no way in which we can select a unique element of $(A \perp Y)$ by means of the relation $\prec$ that corresponds to some extension $\leq$ of the given $\leq_0$. However, as 2.6 tells us, there is always a partial ordering $\leq$ of A, that is not necessarily an extension of the given $\leq_0$, such that the relation $\prec$ that corresponds to $\leq$ does select a unique element of $(A \perp Y)$.

3. SECOND APPLICATION: RESOLVING CONTRADICTIONS

The Problem

It sometimes happens that regulations *clash* in their application to a particular case. In other words, if A is a code of regulations, there may be subsets $B, C \subseteq A$ such that B together with some set of true (or reasonably well-supported) empirical facts about the world at large and the case in hand, implies a certain verdict x, whilst C, together with some corresponding set of facts, implies $\neg x$. In such a case, the code is of course inconsistent (modulo the empirical facts), and if we take our implication relation to include classical logic, the code will imply any and every proposition. Now imagine the situation of a judge or administrative officer who is called upon to apply an inconsistent code and reach a verdict on a specific question. What ways are open to him to mitigate or transcend the contradiction?

One idea is to distinguish between those parts of the code that are directly relevant to the case in hand, and those which are not. It may be, for example, that we are trying to settle on a verdict x or $\neg x$ in a certain case; certain regulations in A, recognized as relevant to the kind of case that we are considering, may imply x, whilst certain other regulations in A, which on the face of it have little to do with the case in hand, might imply a contradiction $y \wedge \neg y$, which by classical logic implies $\neg x$. In such a situation we would quite naturally choose the verdict x, as the one that is grounded directly upon a relevant portion of the code, rather than $\neg x$, which is grounded indirectly (via the contradiction $y \wedge \neg y$) on an apparently irrelevant part of the code.

In many circumstances, this solution may be not only natural but also quite satisfactory, though notoriously difficult to formalize. But there are other situations in which it is not adequate at all. It may happen, for example, that each of x and $\neg x$ is implied quite directly by a portion of the code that is generally recognized as relevant to cases of the kind in hand. What can be done then?

One approach would be to derogate, rejecting the proposition $x \wedge \neg x$ and thereby generating a family $A \perp \{x \wedge \neg x\}$ of remainders. The elements $B \in A \perp \{x \wedge \neg x\}$ are of course just the subsets of A that are maximally consistent under the consequence operation in use. The judge could then use some procedure, such as that of ordering described in the previous section, to select one remainder $B \in A \perp \{x \wedge \neg x\}$ from among the others.

However another approach, which on the face of it appears quite different, would be to 'temper' the consequence operation by an ordering $\leq$ of the code A. We might examine the various subsets of A that imply the verdicts x and $\neg x$ that we are interested in, to see which are more exposed, and which if any is least so; and if none comes forth as less exposed than all its rivals, we may extend our ordering of the code in such a way as to bring it forth. Once such a subset is found, we may select it for the nonce to yield a definite verdict, x or $\neg x$, for the case in hand.

This general idea, of resolving contradictions in a body of norms by imposing an order upon it, is far from new. It is used, for example, by Sir William David Ross in his discussion of conflicting *prima facie* moral duties in *The Right and the Good* (Oxford: Clarendon Press, 1930) and *Foundations of Ethics* (Oxford: Clarendon Press, 1939), and presumably goes back much further, perhaps even to antiquity. It has also been taken up in one way or another by several writers on moral philosophy influenced by Ross, but in almost all cases, the suggestion has remained at the level of vague generality. An exception to this is the work of Hector-Neri Castañeda in his paper 'A Theory of Morality' *Philosophy and Phenomenological Research* **17** (1957) 339–352 and his subsequent and more detailed book *The Structure of Morality* (Springfield, Illinois: Charles Thomas, 1974). However Castañeda's approach is quite different from ours. Quite apart from the technical point that he uses what is in effect a weak ordering (transitive, connected) induced by a function from the collection of all systems of norms into the positive integers, there is the more general point that he is interested in the *moral* ordering of systems of norms, and this interest leads his investigations in directions quite different from ours. His central concern is not with the logical structure generated by an ordering, but rather with the provision of a utilitarian-style recipe for calculating the position in the ordering that a system of norms has, given intrinsic moral values for each of the (indefinitely, perhaps infinitely, many) individual possible actions (both performed and unperformed) that the systems enjoin. This makes his work more of a venture in moral philosophy, and perhaps a rather quixotic one, than a study in logic. Our focus, on the other hand, is on the logical issues and relationships that arise from the use of ordering to resolve contradictions, and our principal application is to codes of law and regulations. Thus whilst there is similarity of spirit (our definition of the relation of delivery, in particular, is fed by the same underlying idea as Castañeda's postulate *E*), there is a considerable difference of execution.

In the remainder of this section we shall study the procedure mathematically, and establish some of its basic properties. In the following section we shall use those properties to show that although on the surface the process appears, and has generally been taken to be, quite different and more flexible than the composite process of derogating and selecting a remainder, nevertheless for inconsistent codes the two processes are essentially equipowerful (observations 4.1, 4.2).

Delivery

We shall need to be a little more restrictive in our assumptions than in the previous section. *We shall henceforth assume* that the code A of regulations is *non-empty* and *finite*, that the regulations are formulated in a language that contains the usual *truth-functional operators*, and that the consequence operation Cn *includes classical tautological implication* and is *non-trivial* in the sense that Cn $(\emptyset)$ is not the set of all propositions of the language – equivalently, for no x do we have $(x \wedge \neg x) \in \mathrm{Cn}(\emptyset)$. We now introduce the central notions. Let $(A, \leq)$ be a hierarchy of regulations, and Cn a consequence operation satisfying the conditions above. Let $B \subseteq A$ and let x be any proposition. We say that B *indicates* x and write $\leq : B \to x$ iff $x \in \mathrm{Cn}(B)$ and moreover for all $C \subseteq A$, if $\neg x \in \mathrm{Cn}(C)$ then $B \not\leq C$. We say that B *determines* x and write $\prec : B \to x$ iff $x \in \mathrm{Cn}(B)$ and moreover for all $C \subseteq A$, if $\neg x \in \mathrm{Cn}(C)$ then $C \prec B$. In the special case where A is a completely unordered code, that is, where $\leq$ is the identity relation, then these notions clearly collapse as follows: B indicates x iff $x \in \mathrm{Cn}(B)$ and B is consistent; and B determines x iff $x \in \mathrm{Cn}(B)$ and either $B = \emptyset$ or A is consistent. Note also that when $B \subseteq A$ is inconsistent, in the sense that for some x (and thus all x), $x \wedge \neg x \in \mathrm{Cn}(B)$, then B does not indicate or determine any proposition whatsoever, irrespective of the choice of $\leq$. In general, determination and indication are related to each other as follows:

OBSERVATION 3.1. Whenever $\prec : B \to x$ then $\leq : B \to x$, but not in general conversely.

Proof. The positive part is immediate from observation 1.3. For a counterexample to the converse, put $A = \{a_1, a_2\}$, $B = \{a_1\}$, $C = \{a_2\}$; $x \in \mathrm{Cn}(B)$ but $\neg x \notin \mathrm{Cn}(B)$, $\neg x \in \mathrm{Cn}(C)$ but $x \notin \mathrm{Cn}(C)$; and put $\leq$ over A to be identity. Then $\leq : B \to x$ but $\prec : B \not\to x$.

OBSERVATION 3.2. No set of regulations within any hierarchy $(A, \leq)$ of regulations indicates two incompatible verdicts. In other words, for no B, x do we have both $\leq : B \to x$ and $\leq : B \to \neg x$. However two distinct sets of regulations from the same code may sometimes indicate incompatible verdicts.

Proof. If $\leq :B \rightarrow x$ and $\leq :B \rightarrow \neg x$ then $B \nleq B$ which is impossible by observation 1.1. For an illustration of the negative observation, the same example as in the proof of 3.1 gives $\leq :B \rightarrow x$ and $\leq :C \rightarrow \neg x$.

OBSERVATION 3.3. Whenever a set of regulations within a hierarchy $(A, \leq)$ determines a verdict, then no set of regulations from the same hierarchy indicates an incompatible verdict. In other words, for no $B, D \subseteq A$ do we have $\prec :B \rightarrow x$ and $\leq :D \rightarrow \neg x$.

Proof. If $\prec :B \rightarrow x$ and $\leq :D \rightarrow \neg x$ then $D \prec B$ and $D \nleq B$, contradicting observation 1.3.

COROLLARY 3.4. No two sets of regulations from the same hierarchy determine incompatible verdicts.

Proof. Immediate from 3.3 using 3.1.

Observation 3.4 shows that determination is somewhat more regularly behaved than indication, and suggests naturally a further concept. We say that a hierarchy $(A, \leq)$ of regulations *delivers* a proposition x, and write $\prec :A \Rightarrow x$, iff some subset $B \subseteq A$ determines x. Delivery is a very well behaved relation. Indeed, its strength can be measured by an appropriate subset of A, as follows. For each $a \in A$, we say that a is *normal* iff for every inconsistent $C \subseteq A$ there is a $c \in C$ with $c < a$; and we put N to be the set of all normal elements of A. The identity of the set N thus depends upon the choice of A, $\leq$, and Cn. Clearly:

OBSERVATION 3.5. N is consistent, i.e. $x \wedge \neg x \notin \mathrm{Cn}(N)$.

Proof. If $N = \varnothing$ then the observation holds by the general conditions of the section. If $N \neq \varnothing$ then it holds by the definition of N and the irreflexivity of $\prec$.

FUNDAMENTAL THEOREM 3.6. Let $(A, \leq)$ be any hierarchy of regulations. Then for every proposition x, the following conditions are equivalent: $\prec :A \Rightarrow x$, $\prec :N \rightarrow x$, $x \in \mathrm{Cn}(N)$.

Proof. Suppose $x \in \mathrm{Cn}(N)$. To show $\prec :A \Rightarrow x$ it suffices to show $\prec :N \rightarrow x$. Let $C \subseteq A$ and suppose $\neg x \in \mathrm{Cn}(C)$. We need to show that $C \prec N$. Now $C \cup N$ is inconsistent, so using the definition of N we have $C \cup N \prec N$, and also using 3.5 we have $C \neq \varnothing$. But since A is finite, this implies that

$C \prec N$. For when $a \in N$ then by the finiteness of N there is a minimal element a' of N with $a' \leq a$; since $C \cup N \prec N$ there is a $c \in C \cup N$ with $c < a' \leq a$; and since a' is minimal in N we have $c \notin N$, so $c \in C$.

To complete the circle, suppose $\prec : A \Rightarrow x$. Then there is a $B \subseteq A$ with $\prec : B \rightarrow x$. That is, $x \in \mathrm{Cn}(B)$ and for every $C \subseteq A$, if $\neg x \in \mathrm{Cn}(C)$ then $C \prec B$. Hence in particular for every inconsistent set $C \subseteq A, C \prec B$, which by the definition of N gives $B \subseteq N$, and so $x \in \mathrm{Cn}(N)$ and we are done.

From theorem 3.6 it follows that if we want to find out whether a proposition x is delivered by a hierarchy $(A, \leq)$, one way of proceeding is to locate the set $N \subseteq A$ and then work out whether $x \in \mathrm{Cn}(N)$, where Cn is the underlying consequence operation. This method is particularly useful when working with small finite examples.

OBSERVATION 3.7. Let $(A, \leq)$ be any hierarchy of regulations. Then:

(1) For no proposition x do we have both $\prec : A \Rightarrow x$ and $\prec : A \Rightarrow \neg x$.

(2) Whenever $\prec : A \Rightarrow x$ and $y \in \mathrm{Cn}(x)$ then $\prec : A \Rightarrow y$.

(3) For all propositions x_1, x_2, $\prec : A \Rightarrow (x_1 \wedge x_2)$ iff $\prec : A \Rightarrow x_1$ and $\prec : A \Rightarrow x_2$.

(4) For every proposition x, $\prec : A \Rightarrow (x \vee \neg x)$.

Proof. Part (1) is immediate from observation 3.4. Parts (2), (3), (4) are immediate from the fundamental theorem. Part (4) can indeed be put more strongly: whenever $x \in \mathrm{Cn}(\varnothing)$ then $\prec : A \Rightarrow x$.

It is possible to use observation 3.7 as a basis for a model theory for the propositional deontic logic of von Wright, with Ox interpreted as $\prec : A \Rightarrow x$, yielding each of $\neg(\mathrm{O}x \wedge \mathrm{O}\neg x)$, $\mathrm{O}(x_1 \wedge x_2) \equiv (\mathrm{O}x_1 \wedge \mathrm{O}x_2)$, $\mathrm{O}(x \vee \neg x)$, and corresponding to part (2) of 3.7, the derivation rule that if Ox is a thesis and $x \supset y$ a tautological implication, then Oy is a thesis. However that is not our purpose at the moment. Logic does not consist only of soundness and completeness theorems for syntactic calculi. The important point that emerges from observation 3.7 is that the relation of delivery provides a well-behaved means for overcoming contradictions implicit in a body of legislation without modifying the propositions contained in the legislation. We suggest that judges working with inconsistent legislation do sometimes approximate some such process as this.

The attractions of delivery should not, however, blind us to an important limitation. Delivery based on a fixed relation $\leq$ over A cannot serve, *alone*, as a general means of overcoming all contradictions but must, in some cases, be used in association with a notion of relevance. Delivery is not a device for dispensing with the notion of relevance, but rather an instrument for resolving contradictions in situations where a concept of relevance, no matter how refined, is of little avail – that is, as mentioned at the beginning of this section, situations where a verdict x and its negation $\neg x$ are both implied quite directly by a portion of the code that is relevant to cases of the kind in hand. In other situations, however, the notion of relevance, no matter how vague and unformalized it might be, is indispensible. For example, imagine a code A that contains half a dozen laws $a_1, \ldots, a_n$ on the rights and duties of the citizen when arrested by the police, all of the most fundamental nature (say, for simplicity, each maximal in the hierarchy); and at the other end of the scale half a dozen laws $b_1, \ldots, b_m$ on the presence of domestic pets in public places, so that $b_j < a_i$ for all j, i. Imagine also that there is a contradiction in the subset $\{a_1, \ldots, a_n\}$. What does delivery tell us? The set A does *not* deliver any of the regulations $b_1, \ldots, b_m$; for whenever $B \subseteq A$ and $b_j \in \mathrm{Cn}(B)$ then, assuming that $b_j \notin \mathrm{Cn}(\emptyset)$, there is a set $C \subseteq A$, namely $\{a_1, \ldots, a_n\}$, such that $\neg b_j \in \mathrm{Cn}(C)$ and $C \not\prec B$. This runs counter to common sense, which tells us that we should, as far as possible, be able to continue with our daily regulations on pets and suchlike even when there are conflicts and contradictions in other, admittedly more important, areas of the law. We thus need to apply the delivery that is determined by a given ordering after making an initial judgment of relevance. When faced with a specific problem about a domestic pet, we must *first* determine in some way the laws of the code A that can be considered as relevant to the case in hand; and *then*, if these themselves from an inconsistent subset A_0, apply delivery to A_0, ignoring contradictions outside A_0. It may be possible in some cases to carry out the work of the discrimination of relevance by varying the underlying relation $\leq$, but that is a separate question which we leave for subsequent study.

We return now to the formal structure of delivery itself. The range of propositions delivered by a hierarchy $(A, \leq)$ increases with the strength of the relation $\leq$. Put formally, it is easy to verify that if A is a code of regulations and $\leq_0, \leq$ are partial orderings of A, then if $\leq_0$ is included in $\leq$ we have $N_0 \subseteq N$ and thus $\{x/\prec_0 : A \Rightarrow x\} \subseteq \{x/\prec : A \Rightarrow x]$. It is therefore natural to ask under what conditions it is possible for a judge to deliver a given proposition by *extending* a given partial ordering. We have the following necessary and sufficient condition, whose formulation and proof are analogous to those of observation 2.7.

OBSERVATION 3.8. Let $(A, \leq_0)$ be any hierarchy of regulations, let $B \subseteq A$, and let x be any proposition. Then the following conditions are equivalent:

(1) There is a partial ordering $\leq$ of A with $\leq_0 \subseteq \leq$ and $\prec : B \rightarrow x$,

(2) $\leq_0 : B \rightarrow x$.

Proof. Suppose first that $\leq_0 : B \not\rightarrow x$. Then either $x \notin \mathrm{Cn}(B)$ or there is a $C \subseteq A$ with $\neg x \in \mathrm{Cn}(C)$ and $B \leq_0 C$. If $x \notin \mathrm{Cn}(B)$ then $\prec : B \not\rightarrow x$ no matter how the relation $\leq$ over A is chosen. If on the other hand there is such a C, then since $B \leq_0 C$ and $\leq_0$ over A is included in $\leq$, we have $B \leq C$ and so by observation 1.3, $C \not\prec B$, and so $\prec : B \not\rightarrow x$.

Suppose for the converse that $\leq_0 : B \rightarrow x$. Define ∇_B as in the proof of observation 2.7, and define $\leq$ to be $[(A - \nabla_B) \times \nabla_B] \cup \leq_0$ as in 2.7. Then $\leq$ is a partial order that extends $\leq_0$. It remains to show that $\prec : B \rightarrow x$. Since $\leq_0 : B \rightarrow x$ we have $x \in \mathrm{Cn}(B)$. Suppose $C \subseteq A$ and $\neg x \in \mathrm{Cn}(C)$; we need to show $C \prec B$. Since $\leq_0 : B \rightarrow x$ we have $B \nleq_0 C$, so there is a $c \in C$ such that $c \notin \nabla_B$. That is, $c \in C - \nabla_B \subseteq A - \nabla_B$ and so by the definition of $\leq$ we have $c \leq b$ and indeed $c < b$ for all $b \in B$. Moreover since $c \in C$, $C \neq \varnothing$. Thus $C \prec B$ and the proof is complete.

The construction that we used to prove observation 3.8 is rather radical from a practical point of view. Imagine a judge working with a hierarchy $(A, \leq_0)$ such that $\leq_0 : B_1 \rightarrow x$ but also $\leq_0 : B_2 \rightarrow \neg x$ and who wishes to extend $\leq_0$ to a relation $\leq$ under which A delivers a unique verdict. He would seldom extend $\leq_0$ in such a sweeping way as that described in the proof of 3.8; he may well do less when less will do, seeking a *minimal* extension of $\leq_0$ that achieves a delivery. Now observation 3.8 does not give us an explicit characterization of any such minimal extension of $\leq_0$; but since A is finite, it guarantees that one exists.

We end this section by drawing attention to some further properties of the set N, as lemmas for future use.

OBSERVATION 3.9. Let $(A, \leq)$ be any hierarchy of regulations, with N the set of all normal elements of A. Then for all $B \subseteq A$,

(1) If $B \subseteq N$ then $\{x / \prec : B \rightarrow x\} = \mathrm{Cn}(B)$

(2) If $B \nsubseteq N$ then $\{x / \prec : B \rightarrow x\} = \varnothing$.

Proof. For (1), if $\prec : B \rightarrow x$ then of course $x \in \mathrm{Cn}(B)$. Suppose for the converse that $x \in \mathrm{Cn}(B)$ and $B \subseteq N$. Then $x \in \mathrm{Cn}(N)$ and so by the fundamental theorem $\prec:N\rightarrow x$. Hence whenever $C\subseteq A$ and $\neg x\in\mathrm{Cn}(C)$ then $C\prec N$ and so since $B\subseteq N$, $C\prec B$.

For (2), let x be any proposition. If $\prec:B\rightarrow x$ then for every inconsistent $C\subseteq A$, since $\neg x\in\mathrm{Cn}(C)$ we have $C\prec B$ and so by the definition of N, $B\subseteq N$.

OBSERVATION 3.10. $N=\{a\in A/\prec:\{a\}\rightarrow a\}$.

Proof. Immediate from 3.9.

OBSERVATION 3.11. $\mathrm{Cn}(N)\cap A=\{a\in A/\prec:A\Rightarrow a\}$ and is the unique element of $A\perp[A-(\mathrm{Cn}(N)\cap A]$.

Proof. The first part is immediate from the fundamental theorem. The second part follows from observation 2.4 and general properties of consequence operations.

4. RELATIONS BETWEEN DEROGATION AND DELIVERY

The purpose of this section is to lay bare the relative powers of derogation and delivery. As in the preceding section, *we shall assume* that our code A of regulations is non-empty and finite, and that Cn includes tautological implication and is non-trivial.

Derogation versus Delivery

Our first and most surprising result is that every relation of delivery is equipowerful with the selection of a remainder from some derogation.

OBSERVATION 4.1. Let $(A, \leq)$ be any hierarchy of regulations. Then there is a set Y and a remainder $B\in(A\perp Y)$ such that for every proposition x, $\prec:A\Rightarrow x$ iff $x\in\mathrm{Cn}(B)$.

Proof. Put $B=\mathrm{Cn}(N)\cap A$ and put $Y=A-B=A-\mathrm{Cn}(N)$. Then by 3.11 B is the unique element of $A\perp Y$. Moreover, by the fundamental theorem 3.6 we have that for every proposition x, $\prec:A\Rightarrow x$ iff $x\in\mathrm{Cn}(N)$ $=\mathrm{Cn}(\mathrm{Cn}(N)\cap A)=\mathrm{Cn}(B)$ by general properties of consequence operations, and we are done.

We have a partial converse of 4.1: for *inconsistent* hierarchies, every selection of a remainder from a derogation of a non-empty set is equipowerful with the delivery determined by some partial ordering.

OBSERVATION 4.2. Let A be a code of regulations, inconsistent under a consequence operation Cn, and let Y be any non-empty set of propositions. Then for every $B \in (A \perp Y)$ there is a partial ordering $\leq$ of A such that for every proposition x, $x \in \mathrm{Cn}(B)$ iff $\prec : A \Rightarrow x$.

Proof. We use the same basic construction as for 2.6. Given $B \in (A \perp Y)$ let $\leq$ be the relation $[(A - B) \times B] \cup I$ where I is the identity relation. Clearly $\leq$ partially orders A. By the fundamental theorem 3.6 it suffices to show that $B = N$.

First, to show $B \subseteq N$, let $b \in B$. We need to show that for every inconsistent $C \subseteq A$, there is a $c \in C$ with $c < b$. Since $B \in (A \perp Y)$ and Y is non-empty, B is consistent, so whenever C is inconsistent we have $C \nsubseteq B$ and so there is a $c \in C$ with $c \in (A - B)$, and so $c < b$ by the definition of $\leq$.

For the converse implication $N \subseteq B$, let $d \in N$. Since A is inconsistent, we have by the definition of N that there is an $a \in A$ with $a < d$, which by the definition of $\leq$ implies $d \in B$, and we are done.

We note that observation 4.2 does not extend to consistent codes A. For if A is consistent under Cn, then for every partial ordering $\leq$ of A, delivery collapses to Cn itself, that is, $\prec : A \Rightarrow x$ iff $x \in \mathrm{Cn}(A)$ for every proposition x. On the other hand, if Y is chosen to have a non-empty intersection with $\mathrm{Cn}(A)$, then of course for all $B \in (A \perp Y)$, $\mathrm{Cn}(B) \subset \mathrm{Cn}(A)$.

Relative Delivery

These results suggest the question of whether there is some other concept, similar in formulation and spirit to delivery, but sufficiently more general to be equipowerful with selection of a remainder from a derogation even for consistent codes. We shall bring this paper to a close by defining and describing such a notion.

Let $(A, \leq)$ be any hierarchy of regulations, Cn a consequence operation satisfying the conditions mentioned at the beginning of this section, and Y a set of propositions. For $B \subseteq A$ and propositions x we say that B *indicates* x, *relative to* Y, and write $\leq(Y) : B \rightarrow x$, iff $x \in \mathrm{Cn}(B)$ and for every non-empty $C \subseteq A$, if $\mathrm{Cn}(C) \cap Y \neq \varnothing$ then $B \nleq C$. We say that B *determines* x, *relative to* Y, and write $\prec(Y) : B \rightarrow x$, iff $x \in \mathrm{Cn}(B)$ and for every non-empty $C \subseteq A$, if $\mathrm{Cn}(C) \cap Y \neq \varnothing$ then $C \prec B$. Finally, we say that A *delivers* x, *relative to* Y, and write $\prec(Y) : A \Rightarrow x$, iff $\prec(Y) : B \rightarrow x$ for some $B \subseteq A$.

Note that in the special case where $Y = \varnothing$, and more generally when $\mathrm{Cn}(A) \cap Y = \varnothing$, then these notions collapse into Cn itself: $\leq(Y) : B \rightarrow x$ iff

$\prec(Y){:}B\to x$ iff $x\in \mathrm{Cn}(B)$, and also $\prec(Y){:}A\Rightarrow x$ iff $x\in \mathrm{Cn}(A)$. And in the special case where $\leq$ over A is identity, then $\prec(Y){:}B\to x$ iff $x\in \mathrm{Cn}(B)$ and either $\mathrm{Cn}(A)\cap Y=\varnothing$ or $B=\varnothing$. Note also that when $B\subseteq A$ is inconsistent, then if Y is non-empty, B does not determine anything at all relative to Y. We begin by setting out the basic relations between relative delivery and delivery *tout court*.

OBSERVATION 4.3. For non-empty Y, $\prec(Y){:}B\to x$ implies $\prec{:}B\to x$ but not always conversely.

Proof. Suppose $\prec(Y){:}B\to x$. Then $x\in \mathrm{Cn}(B)$. Let $C\subseteq A$ and suppose $\neg x\in \mathrm{Cn}(C)$; we need to show that $C\prec B$. Since $x\in \mathrm{Cn}(B)$ and $\neg x\in \mathrm{Cn}(C)$ we have $x\wedge \neg x\in \mathrm{Cn}(B\cup C)$, and so since Y is non-empty we have $\mathrm{Cn}(B\cup C)\cap Y\neq\varnothing$, and moreover since $B\cup C$ is inconsistent, $B\cup C\neq\varnothing$. Thus since $\prec(Y){:}B\to x$ we have $B\cup C\prec B$. Since A is finite, B is finite, and thus it follows that $C\prec B$ and we are done. There are small finite counterexamples to the converse.

OBSERVATION 4.4. Let w be any proposition. Then for every $B\subseteq A$ and every proposition x, $\prec(w\wedge\neg w){:}B\to x$ iff $\prec{:}B\to x$.

Proof. We already have the left-to-right implication by 4.3. For the converse, suppose that $\prec{:}B\to x$. Then $x\in \mathrm{Cn}(B)$ and for all $C\subseteq A$, if $\neg x\in \mathrm{Cn}(C)$ then $C\prec B$. Now suppose $D\subseteq A$ and $\mathrm{Cn}(D)\cap\{w\wedge\neg w\}\neq\varnothing$, that is, suppose $w\wedge\neg w\in \mathrm{Cn}(D)$; we need to show $D\prec B$. Since $w\wedge\neg w\in \mathrm{Cn}(D)$ and Cn includes tautological implication, we have $\neg x\in \mathrm{Cn}(D)$, so $D\prec B$ and we are done.

Thus for each non-empty Y, determination and delivery relative to Y are subrelations of determination and delivery *tout court*, and the two coincide when Y is chosen to be $\{w\wedge\neg w\}$.

The general behaviour of relative determination and delivery is very similar to that of their plain counterparts. Every one of the observations 3.1–3.11 has an analogue, sometimes formulated under the condition that Y is non-empty, for the relative concept. The proofs are in general similar, so we shall merely state for the record the analogues without going through the details of the verifications once again. The reader is advised to skip through to observation 4.5, coming back to 3.1(Y) – 3.11(Y) as needed or interested. Observation 3.1 carries over without condition:

OBSERVATION 3.1(Y). Whenever $\prec(Y){:}B\to x$ then $\leq(Y){:}B\to x$, but not in general conversely.

Observations 3.2 – 3.4 have analogues under the condition that Y is non-empty:

OBSERVATION 3.2(Y). If Y is non-empty then not both $\leq(Y):B \rightarrow x$ and $\leq(Y):B \rightarrow \neg x$.

OBSERVATION 3.3(Y). If Y is non-empty then not both $\prec(Y):B \rightarrow x$ and $\leq(Y):D \rightarrow \neg x$, where $B, D \subseteq A$.

OBSERVATION 3.4(Y). If Y is non-empty, then no two sets of regulations from the same hierarchy determine, relative to Y, incompatible verdicts.

Preparing for an analogue of the fundamental theorem, we say that an element $a \in A$ is *normal relative to* Y iff whenever C is a non-empty subset of A such that $\mathrm{Cn}(C) \cap Y \neq \varnothing$, then there is a $c \in C$ with $c < a$. We put N_Y to be the set of all elements of A that are normal relative to Y.

OBSERVATION 3.5(Y). If Y is non-empty then N_Y is consistent.

FUNDAMENTAL THEOREM 3.6(Y). Let $(A, \leq)$ be any hierarchy of regulations and Y any set of propositions. Then the following conditions are equivalent: $\prec(Y):A \Rightarrow x$, $\prec(Y):N_Y \rightarrow x$, $x \in \mathrm{Cn}(N_Y)$.

OBSERVATION 3.7(Y). Let $(A, \leq)$ be any hierarchy of regulations, and Y any set of propositions. Then:

(1) If Y is non-empty then for no x do we have both $\prec(Y):A \Rightarrow x$ and $\prec(Y):A \Rightarrow \neg x$.
(2) Whenever $\prec(Y):A \Rightarrow x$ and $y \in \mathrm{Cn}(x)$ then $\prec(Y):A \Rightarrow y$.
(3) $\prec(Y):A \Rightarrow (x_1 \wedge x_2)$ iff $\prec(Y):A \Rightarrow x_1$ and $\prec(Y):A \Rightarrow x_2$.
(4) For all x, $\prec(Y):A \Rightarrow (x \vee \neg x)$, and indeed more generally, whenever $y \in \mathrm{Cn}(\varnothing)$ then $\prec(Y):A \Rightarrow y$.

OBSERVATION 3.8(Y). $\leq_0(Y):B \rightarrow x$ iff there is a partial ordering $\leq$ of A that extends $\leq_0$ such that $\prec(Y):B \rightarrow x$.

OBSERVATION 3.9(Y). Let $(A, \leq)$ be any hierarchy of regulations, Y a set of propositions, and N_Y the set of all elements of A normal relative to Y.

Then for all $B \subseteq A$:

(1) If $B \subseteq N_Y$ then $\{x/\prec(Y):B \rightarrow x\} = \mathrm{Cn}(B)$

(2) If $B \nsubseteq N_Y$ then $\{x/\prec(Y):B \rightarrow x\} = \varnothing$.

OBSERVATION 3.10(Y). $N_Y = \{a \in A/\prec(Y):\{a\} \rightarrow a\}$.

OBSERVATION 3.11(Y). $\mathrm{Cn}(N_Y) \cap A = \{a \in A/\prec(Y):A \Rightarrow a\}$ and is the unique element of $A \perp [A - (\mathrm{Cn}(N_Y) \cap A)]$.

We are now in a position to state the precise relationship between relative delivery and derogation.

OBSERVATION 4.5. Let $(A, \leq)$ be a hierarchy of regulations and let X be any set of propositions. Then there is a set Y and a remainder $B \in (A \perp Y)$ such that for every proposition w, $\prec(X):A \Rightarrow w$ iff $w \in \mathrm{Cn}(B)$.

Proof. Put $B = \mathrm{Cn}(N_X) \cap A$ and put $Y = A - B$. Then by 3.11(Y), B is the unique element of $A \perp Y$. Moreover by the fundamental theorem 3.6(Y), we have that for every proposition w, $\prec(X):A \Rightarrow w$ iff $w \in \mathrm{Cn}(N_X) = \mathrm{Cn}(B)$.

OBSERVATION 4.6. Let A be any code of regulations (consistent or not) and Y any set of propositions (empty or not). Then for every remainder $B \in (A \perp Y)$ there is a partial ordering $\leq$ of A and a set X such that for all w, $w \in \mathrm{Cn}(B)$ iff $\prec(X):A \Rightarrow w$.

Proof. This can be seen as a converse to 4.5. Its proof is similar in style to that of 4.2, but as the requirements of the inconsistency of A and non-emptiness of Y are here relaxed, we give the details in full. Let $B \in (A \perp Y)$. We put $\leq$ as in 2.6 and 4.2 to be $[(A - B) \times B] \cup I$, and we choose X to be Y itself. We need to show that for all w, $w \in \mathrm{Cn}(B)$ iff $\prec(Y):A \Rightarrow w$. By the fundamental theorem 3.6(Y) it suffices to show that $B = N_Y$.

First, to show $B \subseteq N_Y$, let $b \in B$. Let C be a non-empty subset of A with $\mathrm{Cn}(C) \cap Y \neq \varnothing$. We need to show that there is a $c \in C$ with $c < b$. Since $B \in (A \perp Y)$ we have $\mathrm{Cn}(B) \cap Y = \varnothing$, so since $\mathrm{Cn}(C) \cap Y \neq \varnothing$ we have $C \nsubseteq B$, so there is a $c \in C$ with $c \in (A - B)$, so by the definition of $\leq$, $c < b$ and we are done.

For the converse inclusion $N_Y \subseteq B$ we have two cases to consider. If $\mathrm{Cn}(A) \cap Y = \varnothing$ then clearly $N_Y = A$ and $B = A$ and so $N_Y = B$. Suppose on the other hand that $\mathrm{Cn}(A) \cap Y \neq \varnothing$, and let $d \in N_Y$. Since A is non-empty and $\mathrm{Cn}(A) \cap Y \neq \varnothing$ we have by the definition of N_Y that there is an $a \in A$ with

$a < d$, which by the definition of $\leq$ implies that $d \in B$; so that $N_Y \subseteq B$ and the proof is complete.

The relationship between derogation on the one hand, and delivery and relative delivery on the other, can thus be summarized as follows. There is an exact correspondence between the process of relative delivery modulo a set X and the composite process of selecting a remainder from a derogation (4.5, 4.6). For inconsistent codes there is an exact correspondence between delivery and the selection of a remainder from the derogation of a non-empty set (4.1, 4.2). By combining 4.5 and 4.2 we also obtain a further relationship between delivery and relative delivery, which can be seen as a partial converse to 4.4.

OBSERVATION 4.7. Let $(A, \leq_0)$ be any inconsistent hierarchy of regulations, and let X be any non-empty set of propositions. Then there is a partial order $\leq$ over A such that for all w, $\prec_0(X): A \Rightarrow w$ iff $\prec: A \Rightarrow w$.

Proof. Given $(A, \leq_0)$ and X, *choose* B and Y as in the proof of 4.5 (taking $\leq_0$ in place of the $\leq$ of 4.5). Then for all w, $\prec_0(X): A \Rightarrow w$ iff $w \in \mathrm{Cn}(B)$. Moreover, Y is not empty. For if Y is empty then $B = A$, so that $\mathrm{Cn}(N_X^0) \supseteq A$, and so N_X^0 is inconsistent, which contradicts 3.5(Y) and the hypothesized non-emptiness of X. Since Y is non-empty, and A is inconsistent, we may apply observation 4.2 to choose a partial order $\leq$ of A such that for every w, $w \in \mathrm{Cn}(B)$ iff $\prec: A \Rightarrow w$. Thus we have $\prec_0(X): A \Rightarrow w$ iff $\prec: A \Rightarrow w$ and the proof is complete.

The question arises whether in observation 4.7 we can always choose the relation $\leq$ in such a way as to bear an interesting connection with the given relation $\leq_0$. We leave this question open, making here only the negative observation that it is not always possible to choose $\leq$ as an extension of $\leq_0$, as can be shown by suitably manipulating the example given at the end of section 2.

5. FURTHER APPLICATIONS

In this paper we have been thinking of A as a set of regulations or rules of some legal system. But in our formal development, the only condition that we have imposed on A is that it be a set of propositions (non-empty and finite from section 3 onwards), and that the accompanying consequence operation Cn be compact over A, non-trivial, and include classical tautological implication. Thus the same concepts and techniques may be taken up in other areas, wherever problems akin to inconsistency and derogation arise. In particular, the notion of delivery makes sense, and may perhaps also be useful, in the following areas.

(1) The study of codes of morality and also other systems of rules (tennis, bridge, duelling, etiquette, . . .). Here the problem of contradiction arises, as does too that of derogation. Such codes and systems may not be promulgated and enforced by a state, nor accompanied by special officers given powers of derogation, as is usually the case for law, except such areas as international law and primitive law; but each man who reflects on morals and tinkers with the ideas around him to fashion his own code, can be seen as his own promulgator and derogator.

(2) Nothing in our treatment of delivery requires that A actually be a system of norms or contents of norms. We can also choose A to be a set of descriptive or theoretical or even mathematical propositions. The notion of delivery is thus of interest for the philosophy of science, in that it studies the logical subtleties of eliminating an unwanted consequence or component of a theory, and of living with an inconsistent theory. In particular, it shows how one can sometimes get along fairly well, and quite rationally, with a theory that is known to be falsified or inconsistent but which lacks a better substitute.

(3) Finally, the concept of delivery may be of use for the theory of information-processing. Imagine a computer continually being fed an ever-increasing amount of information about air, train, and bus timetables. Suppose that at some point the computer is accidentally fed inconsistent information about the Omsk–Tomsk air link. How would we like it to behave? Not to stop dead and refuse to operate until its memory banks are consistent; nor to print out everything that its information implies. Rather, *first* to signal the occurrence of trouble; *second* to try to isolate the area of the contradiction and proceed if necessary as normal in all areas to which it considers the trouble area irrelevant (under a warning that error could still possibly occur); and *third*, if necessary, continue to handle even the inconsistent segment using the concept of delivery based on a relation of relative security (but under a warning of likelihood of error), while waiting for the repair team to determine the exact cause of the contradiction and make the right excision.

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NOTE

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NON - KRIPKEAN DEONTIC LOGIC

1. INTRODUCTION

Deontic logic has as its defining characteristic that it treats the word 'ought' as a *logical word*. Although one might undertake such a project as a purely formal exercise, there is a much deeper philosophical motivation. It is accepted among moral philosophers that the word 'ought' must be interpreted in a way which makes certain inferences valid. One of the most notable of these is:

$$\frac{\text{It ought to be the case that } \alpha}{\text{It is logically possible that } \alpha}$$

which is a very weak version of the slogan that "ought implies can".

Now if the meaning of 'ought' is crucial to the determination of the validity of any inference then 'ought' functions as a logical word with respect to that inference. This is simply an application of the proper definition of the phrase 'logical word'. Having seen this, our task is now simply to give the details of the logic. We must first provide an appropriate syntactical category for ought, say that of a unary sentence operator, written 'O'. We may then define deontic logic, L_D, as the weakest logic which makes all of the central inferences valid.

Of course this programme is beset on every side with difficulties. On the one hand it is by no means pellucid which are the inferences, the validity of which must be preserved. On the other, there is often room for debate on the question of whether or not some proposed candidate for deontic logic really is the weakest such logic which preserves the core validities. It seems to us that much of the literature is addressed to the first controversy. We intend this essay as a contribution to the second.

From its infancy, deontic logic has been an avid consumer of results in alethic modal logic. So much so that deontic logic has come to be viewed as simply *applied* alethic modal logic. The approach holds its attractions for both the deontic logician and the 'straight' modal logician. On one side, the modal logician is gratified that his work *has* philosophical applications. Being able to interpret his abstract semantic structures forestalls the

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objection that his symbols are bloodless, and his formalism without soul. On the side of deontic logic, many problems and puzzles appear already to have been solved through the efforts of modal logicians. There are, laid out for his inspection, a very large number of logics of the sort required by his interpretation. The relations between these logics have been thoroughly investigated and it is clear what will be gained or lost in adopting one logic over another. Thus the most favoured client role of deontic logic in recent years.

Upon reflexion, it seems almost inevitable that deontic logic should have taken this course. For were they not invited to this new methodological feast by its principal host? And one's principal guests should prove one's least dyspeptic. The *Accipite et Manducate* is in the final paragraph of Kripke's 1963 paper [Kripke 1963]:

> If we were to drop the condition that R be reflexive, this would be equivalent to abandoning the modal axiom $\Box A \supset A$. In this way we could obtain systems of the type required for deontic logic

We shall argue that the wiser course had been to adapt the cuisine and decline the feast. We begin our critique with a brisk survey of the usual relational semantics for modal logic.

2. KRIPKE SEMANTICS: THE COLLATION

The language of modal propositional logic $PC(\Box)$ results from the language of classical propositional logic by the addition of a new unary sentence operator '$\Box$'. Thus it is stipulated that $\Box\alpha$ is well-formed whenever α is.

Sentences of the language are evaluated by objects called models which may be described as follows:

A frame is a pair (U, R) where U is a non-empty set and $\mathrm{R} \subseteq U^2$ a binary relation. *Truth* is defined for the atomic sentences of the language (the set of which is called At) relative to members of U (often called informally, possible worlds). More specifically we get a model $\mathscr{m}$ on a frame whenever we give a function $\mathrm{V}: At \to 2^U$ which associates with every atomic sentence a set of worlds (informally: the set of worlds in which the sentence is true).

V is extended (uniquely) to a function $\|.\|^{\mathscr{m}}$ which evaluates all sentences of the language by means of the *truth-conditions*. These are the obvious counterparts of the usual truth-table definitions for the classical operators, e.g.

$$\|\alpha \wedge \beta\|^{\mathscr{m}} = \|\alpha\|^{\mathscr{m}} \cap \|\beta\|^{\mathscr{m}}$$

For the modal operator we have $\models^{m}_{u} \Box\alpha \Leftrightarrow \forall\, v \in U : u\mathrm{R}v \Rightarrow \models^{m}_{v} \alpha$ where "$\models^{m}_{u} \alpha$" abbreviates "$u \in \| \alpha \|^{m}$".

In words: α is necessarily true in u if α is true in all the alternatives of u. We ensure that particular axioms and rules will hold, by placing restrictions on the frame relation R. Some principles, however, hold even when the relation is left completely unencumbered in particular

[RM] $\vdash \alpha \rightarrow \beta \Rightarrow \vdash \Box\alpha \rightarrow \Box\beta$;

[K] $\Box p \wedge \Box q \rightarrow \Box(p \wedge q)$.

3. PARADOXES – THE AETIOLOGY OF DYSPEPSIA

Deontic logic, from its birth, has served as a rich source of paradoxes. By this we mean that it has yielded results in the form of theorems and valid inferences which run counter to our intuitions about the intended interpretation of 'ought'. If we apply the standard modal semantics then, as we have noted above, we are automatically committed to the deontic readings of [K] and [RM]. The latter in concert with certain obvious additional deontic principles, has allowed the derivation of many counter-intuitive theses, far too many to survey here. We can see however that [RM] by itself suffices to generate at least one class of the 'derived obligation' paradoxes.

On the deontic interpretation the principle reflects the fact that logically necessary conditions of sentences which ought to be the case also ought to be the case. This seems right and indeed useful in moral philosophy, for by means of this axiom we may persuade moral agents that they are committed to the logical consequences of their moral principles. Without at least this much, moral philosophy would be a very curious endeavour. The problem with this is that if we agree that:

(S) We feed the starving poor

ought to true, then we seem to invite the consequence that the sentence

($\exists$S) There are starving poor

which is logically implied by (S) ought also to be true!

Examples of this sort have led to a certain amount of squirming and reformulating on the part of deontic logicians. It seems obvious that we do want [RM] and just as obvious that we don't want "it ought to be the case that ($\exists$S)". The most appealing way out of this messy situation has been to adopt some notion of *conditional* as opposed to *absolute* 'ought'. Thus we don't say that we ought to feed the starving poor no matter what. For, as we have seen, in circumstances of universal affluence this absolute obligation requires that we maintain a rota of peckish paupers. What we do say is something along the lines of: "We ought to feed the starving poor provided that there are starving poor".

Of course, a good deal of work is required in a formulation of the theory of conditional ought sentences. Paradoxes lurk here as well. Some of these are discussed in [van Fraassen 1972]. In spite of this it can still be maintained that the deontic logician is essentially an applied modal logician. The grounds for such an assertion are twofold. First the study of conditional modalities lies within the province of modal logic (even if that province is largely *terra incognita*) and secondly that deontic logicians operate under the constraint that the theory of absolute oughts (which arise by conditionalizing on a tautology) be one of the familiar modal logics in deontic clothing. This is the position of e.g. van Fraassen in the paper cited above and of von Wright as well.

So much is to be found in a not very thorough inspection of the literature of deontic logic. The consensus seems to be that if we are careful we can have what we want and need (viz. [RM]) without becoming embroiled in the most obvious paradoxes at least. Presumably nobody can be sure that he has managed to dodge all the paradoxes that flow from [RM]. We wish to draw attention to the other modal principle, [K], which is fundamental on the usual modal semantics.

In the terminology which we have introduced elsewhere [K] is an *aggregation* principle. It is in fact the strongest one for any reasonable modal logic. [K] says that any finite number of necessities can be aggregated to produce one. On this account we call it the principle of *complete aggregation*. At first glance such a principle seems at least compatible with, and possibly required by the deontic interpretation. Unfortunately, a new infestation of paradoxes is just over the horizon; these we shall call the paradoxes of complete aggregation.

The problem with [K] is that it is so strong that it collapses deontically significant distinctions between modal sentences. Certain of these distinctions may strike the uninitiated as curiously mandarin but others will be apparent unto the palanquin bearers or ordinary philosophy. The most central of the distinctions obliterated by [K] is that between

[D] $\Box p \rightarrow \neg \Box \neg p$ (alternatively $\Box p \rightarrow \Diamond p$)

and

[Con] $\neg \Box \bot$

From a deontic logical point of view this collapse is the most galling of the paradoxes of complete aggregation. At a somewhat more abstract level we lose the distinction between:

[D*] $\Box(\Box p \rightarrow \Diamond p)$ and

[D′] $\Box\Box p \rightarrow \Box\Diamond p$.

Other deontically feasible distinctions are dissolved by [K] but we focus upon these.

4. DEONTIC LOGIC, DEONTIC DISTINCTIONS AND COMPLETE AGGREGATION

Two major intuitions have informed deontic logic as well as much of the development of moral philosophy. The first of these is the Kantian doctrine most often expressed in the slogan "ought implies can". The second is the intuition that if it ought to be the case that p, then it is false that it ought to be the case that $\neg p$. These two dicta have frequently been taken to be synonymous with the two claims: (1) Nobody is under an obligation to bring about an impossible state of affairs, and (2) there can be no genuine conflicts of obligation, i.e., no irresolvable moral dilemmas. In [Lemmon 1965], E. J. Lemmon argues that these two pairs of aphorisms are, in fact, distinct. Central to his argument is the necessity for distinguishing between a claim that something ought to be the case and a claim that something is obligatory. If to say x is obligatory is to say that someone is obliged to do x, then not only can there be no conflicts of obligation but further there can be no unfulfilled obligations. This seems not to be a deontic sense (moral sense) of obligatory. On the other hand, in the sense of 'obligatory' which is akin to the moral 'ought', there can be genuine conflicts. Curiously, Lemmon supposed that his counterexamples to the no-conflicts principle were therefore also counterexamples to the Kantian principle, for he regarded the two as equivalent.

We agree that ought and obligation (in its two senses) are to be distinguished. What we wish to call into question is Lemmon's larger claim that the following two are equivalent:

(a) "that cases of real moral conflict... cannot occur"

and

(b) "that what we ought (are under an obligation etc.) to do we always can do"

It is incontrovertible that the logic which Lemmon adopted *makes* the two principles equivalent; what we deny is that such a logic is suitable for the analysis of "ought".

It is useful to examine Lemmon's axiomatization of his deontic logic. He uses:

[K′] $\Box(p \to q) \to (\Box p \to \Box q)$

[RM] $\vdash \alpha \to \beta \Rightarrow \vdash \Box\alpha \to \Box\beta$

As the fundamental modal principles (i.e. those which must belong to *any* set of principles for deontic logic). From these Lemmon derived:

[K*] $(\Box p \wedge \Box q) \leftrightarrow \Box(p \wedge q)$

which he regarded as a deontic truism. The presence of this biconditional forces the collapse of the distinction between:

[D] $\Box p \to \neg\Box\neg p$

and

[Con] $\neg\Box\bot$

which are the formal counterparts of the principles (a) and (b) respectively. In fact if [RM] is accepted the weaker [K] $\Box p \wedge \Box q \to \Box(p \wedge q)$ is sufficient to guarantee the collapse, as we noted above.

If the basic modal logic were axiomatized with the principles [K] and [RM] rather than [K*] one would be better aware of a choice between remaining the strong aggregation principle [K] and being able to choose between [D] and [Con] while saving what genuinely *is* a deontic truism, [RM].

Stripped to the bone the Lemmon argument may be schematized: [K] is a truism, therefore since [D] is false, so also is [Con]. It would seem more sensible in this context to view ourselves as considering the various trade-offs involved in subscribing to any of the three principles. In such circumstances one might well find that [Con] respects very deeply held intuitions while persuasive counterexamples can be found for [D]. In the face of this, it is [K] which must go to the wall.

This cannot quite be an end to the matter. In the face of widespread support for (a) among the community of moral philosophers, we must say a little more about the counterexamples to [D]. These come generally from one of two sources. First, conflicting obligations can arise from some simple accepted rule, as when someone makes a number of promises not all of which can be fulfilled. Second one may be lead into a quandary by conflicting principles as when an obligation to save unnecessary suffering conflicts with an obligation to tell the truth. Beyond these quite commonplace sorts of cases, there are situations in which the conflicts bite much deeper. These are the conflicts which arise when we subscribe simultaneously to two (or more) large scale theories of conduct; perhaps one of religion and one of politics. A particularly plausible example of such an occurrence may be reconstructed from any of the numerous "refutations" of utilitarianism. These usually take the form of noticing that according to utilitarian theory a certain act which is morally repugnant (the act varies from refutation to refutation; often it is one by which the innocent are made to suffer) is in some circumstances not only permitted, but obligatory. So much the worse for utilitarianism. Such an argument may now be juxtaposed to a "refutation" of some deontological theory. In order to refute one of these, we simply observe that a certain action is always forbidden even though we might easily be able to imagine circumstances in which performing that act is "the only right thing to do". The moral here is clearly that one theory, either utilitarian or deontological, cannot possibly evaluate 'ought' sentences in a satisfactory way.

The moral philosopher, in these situations, is often blissfully if unwarrantedly optimistic. The conflicts are merely apparent; they are *prima-facie*; the problem will resolve itself as we learn more of the circumstances. This is not an argument, let alone a demonstration. In fact there are *no* demonstrations in moral philosophy that absolute obligations cannot conflict; certainly the existence of a distinction (which we grant) between these and *prima-facie* obligations does not, by any stretch of the imagination, prove that the 'no conflict' position is correct. It is mere stipulation to insist that of two apparently conflicting obligations one will finally emerge as absolute and override the other, *prima-facie*, one. As Russell remarks: "the method of postulating what we want has many advantages; these are the same as the advantages of theft over honest toil".

Until there is a convincing argument against the possibility of moral conflict (in the light of ordinary moral experience could any such argument be convincing?) we must keep an open mind. With seemly restraint we insist only that moral theory at least recognize their possibility.

Suppose then that we wish to capture formally some features of general moral reasoning (as opposed to say utilitarian or intuitionist reasoning). We would certainly want to include among our deontic principles the law of moral consistency [Con] $\neg\Box\bot$, because that principle cuts across all moral theories. Were one to adopt as well, the principle [K], of complete aggregation, one would then be committed to the view that if both α and β ought to be the case then α and β are consistent. This clearly flies in the face of our resolution to keep an open mind concerning conflicting obligations. To put the matter brutally, there can be no deontic *logic* which takes as a primitive law, the principle [K]. The best that can be done, if we subscribe to [K], is to formalize certain particular ethical theories – namely those which do not allow moral conflicts.

We have seen that adherents to the 'no-conflicts of obligation' view in effect must postulate (at least) a kind of unity in the deontic realm. If we think of obligations as devolving from moral principles then there must be only one such principle. Put into a theoretic context, one cannot be guided in assigning truth-values to ought-sentences by more than one theory (in the sense that if two are used they cannot be distinguished by giving for some sentence α, differing truth values to: it ought to be the case that α). The arguments of Lemmon, van Fraassen and others (including ourselves) go to show that monism of this stripe is, at best, an unrealistic approach to moral life in the real world. At worst it is wicked.

5. SEMANTIC STRUCTURES FOR DEONTIC LOGIC

We have argued above that the programme of deontic logic is incompatible with the standard account of modal logic because of the paradoxes of complete aggregation. We shall now outline what we take to be the leading candidates for the right semantic approach to deontic logic. These turn out to be *generalizations* rather than specializations of the standard apparatus of modal logic.

Perhaps the most direct approach to the semantics of deontic logic is to avoid [K] by making the 'ought' operator ambiguous. This matches the diagnosis of conflicts of obligation as the result of employing two or more distinct theories to evaluate ought sentences. Thus we might commit ourselves to several moral theories at once. Alternatively, it might be the case that our moral and political (and also perhaps religious) views compete in some cases in the evaluation of oughts. This situation can be represented formally by a relational structure of the form:

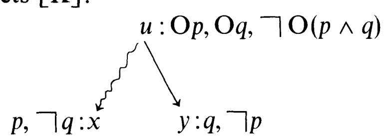
$$^{n}\mathscr{F} = (U, \mathrm{R}_1, \ldots, \mathrm{R}_n)$$

where U is a non-empty set (informally a set of possible worlds or cases or realizable states or whatever) and every R_i ($1 \leq i \leq n$) is a binary relation defined on U. Here we have taken over the notion of an 'alternativeness' or 'accessibility' relation from modal logic but we allow two or more notions of accessibility to operate simultaneously.

If we now employ the truth-condition:

$$\models^{m}_{u} O\alpha \Leftrightarrow \forall x : uR_1 x \Rightarrow \models^{m}_{x} \alpha \text{ or} \ldots \text{or} \; \forall x : uR_n x \Rightarrow \models^{m}_{x} \alpha$$

it is easy to see that [K] will not be valid in the class of such generalized structures. The following diagram gives an example of the simplest model which rejects [K]:



In this $U = \{u, x, y\}$, $R_1 = \{\langle u, x \rangle\}$, $R_2 = \{\langle u, y \rangle\}$, $V(p) = \{x\}$, $V(q) = \{y\}$.

A somewhat more general semantic analysis is possible, one which in fact has the above account as a special case. In our first generalization we allow oughts to be evaluated by many theories but all these theories are required to be *coherent*. We shall have more to say about this below but for now we notice that each of the 'access' relations in a structure ${}^n\mathscr{F}$ behaves just like an ordinary relation from a structure for a modal logic. As an alternative, we may allow oughts to be evaluated by a single 'theory', but permit that theory to be somewhat *incoherent*. Sometimes a theory of this kind may be represented as a collection of theories each of which is individually coherent but such a representation does not hold in general. To some extent this new approach seems a more realistic account of the type of 'ought' evaluating procedures employed by actual moral agents. These agents *may* find themselves in dilemmas because they wish to subscribe simultaneously to competing accounts of the evaluation of oughts but we can also allow that the best theory available to them can't resolve conflicting oughts. In this case the possibility of conflict is far more deeply entrenched. In contrast with the initial approach in which any of the 'underlying' theories, if employed by itself, would resolve every conflict, here there just is *no* available procedure for conflict resolution.

The structures appropriate to the second approach have the form:

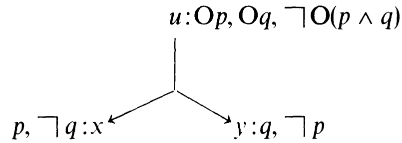
$$\mathscr{F}^n = (U, R^{n+1})$$

with U as before and R^{n+1}, the accessibility relation of the structure an $n+1$-ary relation defined on U. In this account we have changed to a more general concept of accessibility.

For 'ought' sentences we employ the truth-condition:

$$\models^{m}_{u} O\alpha \Leftrightarrow \forall x_1, \ldots, x_n : uR^{n+1}x_1 \ldots x_n \Rightarrow \models^{m}_{x_1} \alpha \text{ or} \ldots \text{or } \models^{m}_{x_n} \alpha.$$

On this account the simplest [K]-rejecting model has the form:



where $U = \{u, x, y\}$, $R^3 = \{\langle u, x, y\rangle\}$, $V(p) = \{x\}, V(q) = \{y\}$.

6. DEONTIC LOGIC

The two semantic approaches which avoid the paradoxes of complete aggregation may be employed in the representation of two sorts of deontic logic. These are distinguished by the degree to which they take seriously the possibility of moral conflict (although of course both take it more seriously than applied standard modal logic). So far, we have left things at a relatively informal level but we shall now take up an expository style which is tempered with precision.

We take the core principles of deontic logic to be:

[RM] $\vdash \alpha \rightarrow \beta \Rightarrow \vdash O\alpha \rightarrow O\beta$

[Con] $\vdash \neg O \bot$

The 'O' in these is to be read as an absolute ought, leaving us free to use the conditional ought to dodge paradoxes of the 'derived obligation' sort which might otherwise follow from [RM].

We shall also employ the principle

[RN] $\vdash \alpha \Rightarrow \vdash O\alpha$

although we do not regard this as a central principle. Its function is to allow us to employ a more streamlined semantics than would otherwise be possible. Should [RN] be thought genuinely counter-intuitive, rather than just odd, our subscription to it may be terminated. The semantic representation of the resulting logic(s) must then be complicated along lines familiar from the analysis of so-called non-normal modal logics.

Also lying outside the core are various kinds of principles which allow the aggregation of oughts. The strongest of these is of course [K] which may be recovered in either of the two semantic approaches by imposing sufficiently strong restrictions on the underlying structures. For example, on the 'many theories' semantics

$$\forall x, y : x\mathrm{R}_1 y \Leftrightarrow \ldots \Leftrightarrow x\mathrm{R}_n y$$

and on the incoherence approach

$$\forall x_1, \ldots, x_n, u : u\mathrm{R}x_1 \ldots x_n \Rightarrow x_1 = \ldots = x_n$$

will validate [K]. Of course, in this case there would be simpler semantics, including that of Kripkean modal logic, which would serve for the resulting deontic logic.

Less stringent restrictions return aggregation principles which are weaker than [K]. These are of two distinct kinds depending upon the choice of semantics. On the many theories view, we might restrict ourselves to structures employing at most some fixed number, n, of access relations. If $n = 2$, then we validate:

$$[{}^2\mathrm{K}] \qquad \mathrm{O}p \wedge \mathrm{O}q \wedge \mathrm{O}r \rightarrow (\mathrm{O}(p \wedge q) \vee \mathrm{O}(p \wedge r) \vee \mathrm{O}(q \wedge r))$$

In words if three sentences ought individually to be the case, then some pair of sentences drawn from the three ought jointly to be the case.

Alternatively, if we impose on the single access relation semantics the condition that the relation have arity no greater than 3, then we validate the weaker principle:

$$[\mathrm{K}_2] \qquad \mathrm{O}p \wedge \mathrm{O}q \wedge \mathrm{O}r \rightarrow \mathrm{O}((p \wedge q) \vee (p \wedge r) \vee (q \wedge r))$$

By allowing more relations than two (although imposing some definite limit) or allowing relations of arity $\leqq n$ for fixed $n > 3$, we commit ourselves to definite schemes of 'ought' aggregation weaker than those displayed above but of essentially the same pattern. We always get the corresponding n-relations principle by distributing the initial O in the consequent of the one $n+1$-ary relation principle over the disjuncts. As we progressively weaken the aggregation principle the major change is that we require an ever larger number of antecedent oughts.

If we impose no limit on either the number of relations or the arity of our relation then no principle of aggregation will be validated.

To return to our core principles, no tinkering with the semantics is required to obtain [RM]. That rule automatically preserves validity in the entire class of both sorts of structures. The situation is different for [Con] – the law of moral consistency. To obtain the validity of this we require in either case a simple existence condition. These are respectively:

$$\forall x \exists y : xR_1 y \text{ or} \ldots \text{or } xR_n y$$

and

$$\forall x \exists y_1, \ldots, y_n : xR^{n+1} y_1 \ldots y_n$$

But neither of these conditions will validate

[D] $\quad O p \rightarrow \neg O \neg p$

which should not be surprising since one of our aims is to restore the distinction between [D] and [Con] which is collapsed by [K]. What is a bit surprising is that while the condition

$$\forall x \exists y : xR_1 y \;\&\ldots\&\; xR_n y$$

validates [D] in the n-relations semantics, there is no condition (at least no condition that we can write down in a first-order language) which will 'exactly' validate [D] on the one n-ary relation approach.

The technical details of this result together with the usual completeness proofs and discussions of the fate of other principles in the new semantics are to be found in [Jennings & Schotch A].

7. CONTACT WITH OTHER AREAS

The present project in deontic logic is of obvious interest to modal logic since it represents one useful direction in which the latter may be generalized. Somewhat less obvious, but at least as interesting, is that the style of semantic representation which we employ may be used in an approach to the problem of constructing a theory of inference from inconsistent premise sets.

The basic idea which we only sketch here (for details see [Jennings & Schotch A]) is that we may "read off" from the inferences of the form:

$$\Box\alpha_1, \ldots, \Box\alpha_n \vdash_K \Box\beta$$

ordinary inferences:

$$\alpha_1, \ldots, \alpha_n \vdash \beta$$

provided that '$\vdash_K$' is the notion of logical consequence of the weakest K normal (i.e. ordinary) modal logic. More precisely we have a connexion between '$\vdash_K$' and '$\vdash$'

$$\Box\alpha_1\ldots,\Box\alpha_n \vdash_K \Box\beta \Leftrightarrow \alpha_1,\ldots,\alpha_n \vdash \beta$$

If we now give to the '$\Box$' formulae a 'deontic' interpretation of the sort we advocate we shall obtain a similar connexion but with the classical '$\vdash$' replaced by some new (generalized) notion of consequence. As an example, suppose we adopt the single $n+1$-ary (n fixed) accessibility relation semantics for '$\Box$'. If we call the weakest 'modal' logic under this interpretation K_n and its associated notion of consequence '$\vdash_{K_n}$' then we may define a new consequence relation '$[\vdash_n$' by

$$\text{“}\alpha_1,\ldots,\alpha_n [\vdash_n \beta\text{”} = \text{“}\Box\alpha_1,\ldots,\Box\alpha_n \vdash_{K_n} \Box\beta\text{”}$$

The properties of this new relationship may be derived in a straightforward way from the properties of the K_n-'$\Box$' and $\vdash_{K_n}$.

The most interesting property of '$[\vdash$' is that $\alpha, \neg\alpha \not[\vdash \beta$. This is because in K_n we are allowed to distribute these formulae within the scope of '$\Box$' over n-tuples of worlds rather than (as in modal logic) collecting them together in a single world.

For each choice of $n+1$ we tolerate different degrees of incoherence in our premise sets. These levels are measured by the number of subsets into which we must divide the premise set in order that each subset be (classically) consistent.

The many relations semantics may also be used to generate a generalized notion of consequence similar to the one described above.

CONCLUSION

A logic sufficiently rich in distinction to serve as a basis for a deontic logic must be aggregatively weaker than any of the Kripkean logics usually studied under the heading 'modal logic'. For this reason deontic logic considered as the study of moral argument cannot be regarded as an *application* of modal logic. Indeed, genuinely deontic logics enjoy such a generality of syntax and semantics that if anything, alethic modal logic as studied since Kripke might reasonably be regarded as a highly specific application of deontic logic properly conceived. This is the specific conclusion of this report.

There is a more general conclusion which offers counsel of hope for deontic logicians. This is that deontic logics can be formulated which do not compromise our abilities to make morally vital distinctions and which do not require the abandonment of first order methods.

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PART III

DEONTIC LOGIC AND TENSE LOGIC

RICHMOND H. THOMASON

DEONTIC LOGIC AS FOUNDED ON TENSE LOGIC[1]

1. CONDITIONALITY AND OBLIGATION

Most of the recent work in deontic logic has concentrated on problems concerning "conditional obligation". I've felt for a long time that this has been a mistake. A proper theory of conditional obligation, like one of "conditional quantification", will be the product of two separate components: a theory of the conditional, and a theory of obligation.

The intuition behind this view is simply that 'if' is univocal;[2] in particular, it doesn't take on different semantic interpretations when it interacts with various modals. To dramatize this point, consider the following two sentences.

(1.1) If John has promised to give up smoking then he ought to give up smoking.

(1.2) If John has promised to give up smoking then he will give up smoking.

The content of 'if...then' must be the same in (1.1) and (1.2). Otherwise, how will we be able to account for examples like the following ones?

(1.3) If John has promised to give up smoking then either he ought to give up smoking or he will be released from his promise.

Besides, it seems to me that many of the puzzles that have been put forward as pertaining to conditional obligation are really specialized versions of problems concerning the conditional: well known problems discussed by Goodman, and revived in the recent literature by persons like (David) Lewis and Stalnaker. These problems are perceived more clearly in this general context.

I suspect that concentration on questions such as this has helped to create the impression that deontic logic is irrelevant to moral philosophy.[3] If there is such an impression it's unfortunate, because there are several foundational questions in deontic logic that bear directly on philosophical issues. And the attempt to solve them is a combined philosophical and logical enterprise. In this paper I'll discuss one such question, relating to the interaction of time and obligation.

R. Hilpinen (ed.), New Studies in Deontic Logic, 165–176.

2. TIME AND OBLIGATION; SOME EXAMPLES

I want to claim that deontic logic requires a foundation in tense logic; the notion of obligation is so dependent on temporal considerations that a logical theory of obligation presupposes an appropriate logical theory of tense. I don't mean just that obligations are time-dependent. Of course they vary from time to time, and can also involve reference to certain times. My obligation to write you a letter can spring into existence at various times, and whenever it does it will be an obligation to write you within a (perhaps vague) period of time.

This isn't sufficient to make my point. There are similar interactions between probability and belief, for instance, and time. The probability that I will write you a letter, and your belief that I will write you a letter, will vary with time just like my obligation to write you a letter. But I would not claim that this shows the logic of probability and belief to presuppose tense logic in the same way that the logic of obligation does.

The difference is that tense is vital in working out the notion of logical consequence for formal languages containing the deontic operator O. To make this point I'll consider two examples of principles that have been endorsed in some formal treatments. The question is whether we're justified in holding these principles to be valid; the point is that this question raises temporal considerations.

Example 1: $\mathrm{P}A \supset \mathrm{OP}A$. This has been endorsed by Fitch, though it seems implausible on the face of it. Let's try to construct a counterexample. Suppose that George, a high school senior, has been asked to provide transportation to a picnic. He obtains permission to use the family car for the purpose if he wishes, his driver's license is valid, and there is nothing to prohibit him from providing transportation. He is free to say "yes" or "no" to the request.

Let Q stand for 'George provides transportation'. Then, since George is free to provide transportation and free not to provide it, both

(2.1) $\mathrm{P}Q$

and

(2.2) $\mathrm{P}\sim Q$

are true. On the other hand, George is permitted to say "yes", and if he does this will then establish an obligation. So

(2.3) $\mathrm{PO}Q$

also is true. But the conjunction of (2.2) and (2.3) is inconsistent with $P \sim Q \wedge OP \sim Q$, an instance of Fitch's principle.

But is (2.3) a proper formalization of 'George is permitted to be obliged to provide transportation'? It's crucial to the example that George hasn't yet committed himself; otherwise (2.2) wouldn't be true.

Let's correct this by putting in tense operators. A single future tense operator, F, will do for the purpose; there are important differences between this operator and the English future tense, but they don't affect the present point. There are two times involved in the example: a time before George has committed himself – call it Monday – and the time of the picnic – call it Saturday. What George is deliberating on Monday is this.[4]

(2.4) George provides transportation on Saturday.

Revising our previous formalization let Q represent (2.4). Now, consider OQ. It is false on Monday, and will remain false until George promises to provide transportation, at which point it will become true. Thus $P \sim Q$, which is logically equivalent to the negation of OQ, is true on Monday and will remain true until George promises to provide transportation, at which point it will become false.

According to our example George is permitted on Monday to promise to provide transportation; but he hasn't yet promised. The promising, if it occurs, will take place at a later time.[5] In consequence of the promising an obligation will arise, and I assume as a normative principle that if it were not permissible for this obligation to arise the promising would not be permissible. So on Monday $PFOQ$ must be true. But there is no contradiction between $P \sim Q$, $OP \sim Q$, and $PFOQ$.

To refute Fitch's principle, then, we would have to show that in this example George is permitted on Monday to be obliged on Monday to provide transportation on Saturday. Now, however, direct intuitions about what is true and false are not of much use. Under the assumption that 'Monday' represents an instantaneous time (I'm imagining, if you like, that we're dealing with a time on Monday so close to midnight that no further human action can take effect before Tuesday), there will be nothing George can do on Monday to change the fact that $\sim OQ$ is true. So POQ belongs to a genuine class of sentences PA, where the truth value of A is beyond the control of human agency. Not surprisingly, the truth values of such sentences are often problematic; in inquiring about them we are dealing with cases that would be abnormal as utterances. (In one form, the example in question is even ungrammatical: 'George may must provide transportation'.)

At this point, theoretical considerations become important in deciding questions of truth value. I will argue that on one important use of the deontic operators, anyway, Fitch's principle holds and $\mathrm{PO}Q$ is false in this example.

The validity of $\mathrm{PO}A \equiv \mathrm{O}A$ has not been accepted by many deontic logicians. So it's worth noting that the same considerations that make this principle seem defensible are also needed in order to maintain accepted principles. Take, for example $\mathrm{OO}A \supset \mathrm{O}A$. We can easily construct examples like the following, which seem to be counterexamples to the principle until temporal considerations are introduced.

At 3:00 I promise my wife to get a can of dogfood at the corner store by 4:00. If I fulfill my promise to her I ought to pay for the dogfood by 4:00. So, supposing that my promise is binding, it ought to be the case that I ought to pay for the dogfood by 4:00.

But now, suppose that I start to the corner store just in time to get the dogfood, but in a moment of moral weakness I spend all my money on bubble gum and comic books. Say that it's 3:45 just as I complete my purchase. At this moment, since I haven't taken the dogfood out of the store I'm not obliged to pay for it by 4:00. And, since I don't have money enough to pay for it, I won't take it out of the store by 4:00 and so am not obliged to pay for it by 4:00. On the other hand, the promise I made to my wife is still in force at 3:45. So it looks as if the following are simultaneously true.

(2.5) I ought to get the dogfood by 4:00.
(2.6) It ought to be the case that I ought to pay for the dogfood by 4:00.
(2.7) It is not the case that I ought to pay for the dogfood.

These can be formalized as follows, letting Q stand for 'I get the dogfood by 4:00' and R for 'I pay for the dogfood by 4:00'.

(2.5) $\mathrm{OF}Q$
(2.6) $\mathrm{OOF}R$
(2.7) $\sim \mathrm{OF}R$

(2.5) holds in virtue of a promise I have made. (2.6) holds as a consequence of (2.5), since fulfilling (2.5) will give rise to an obligation to pay for the dogfood. And (2.7) holds because of the fact that, being a law-abiding person, I won't take the dogfood out of the store since my previous actions have placed me in the position of being unable to buy it. Since I won't take it out, I'll have no obligation to pay for it.

But again, something is wrong here; things can't be this simple. For one thing, (2.5) is not a straightforward, unqualified truth at 3:45. If I have a fit of remorse at just this moment I may suddenly ask myself what I ought to do. Under these circumstances, it is out of place to take an elevated moral tone and offer "You ought to get the dogfood" as advice. Since I *can't* get the dogfood by 4:00 (without stealing it, which would be worse), this counsel is merely a way of upbraiding me for my past actions. Really, it isn't counsel at all, since it doesn't help me to choose what to do.

On the other hand, if I were to say to myself

(2.8) I ought to tell my wife I won't get the dogfood by 4:00,

this might well be sound, useful advice. A disinterested observer might even conclude that this is what I ought to do under the circumstances. But (2.8) calls (2.5) into question, since by a rule of inference.

$$\frac{\mathrm{O}A \qquad \mathrm{O}B}{\mathrm{O}(A \wedge B)}$$

that is generally accepted in deontic logic we can infer

(2.9) I ought to get the dogfood by 4:00 and tell my wife that I won't get it by 4:00.

Does this mean that (2.5) is false, after all, at 3:45? This will depend on how we decide to account for its misplacedness. At least, though, the example makes it clear that there is a certain tension between (2.5) and (2.8); as judgments about what I ought to do, they don't fit together in the same context.

3. COUNSEL VS. JUDGMENT

At this point I'll simply blurt out the solution of this difficulty that seems best to me: (2.5) and (2.8) aren't both true at 3:45 *in the same respect*. Something besides their mere location in time is relevant to determining the truth-values of (2.5) and (2.8); these values also depend on a background consisting of certain alternatives presumed to be open to the agent. Relative to the choices actually open to me at 3:45, (2.8) is true and (2.5) is false. Relative to a wider selection of choices that were open to me at 3:30, (2.5) is true and (2.8) is false.

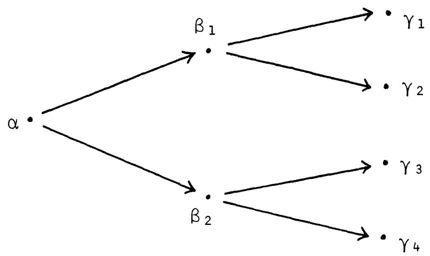
Put it another way: I want to distinguish between two ways in which the truth values of deontic sentences are time-dependent. First, these values are time-dependent in the same, familiar way that the truth values of all tensed sentences are time-dependent. Second, their truth values are dependent on a set of choices or future options that varies as a function of time. If you think of deontic operators as analogous to quantifiers ranging over options, this dependency on context is a familiar phenomenon. The set of people involved in 'Everyone came to the party', for instance, will depend on context; usually (but not always) it will be the set of people who were at the party in question.

The distinction I have in mind can be clarified by fixing on a particular time and letting the choices vary that are presumed open. At 3:45, consider the alternatives actually open to me at 3:45; let these be presumed open. Here I am looking for *practical advice* and, as I've said, (2.8) is true and (2.5) is false. But there are circumstances, even circumstances that might obtain at 3:45, in which it is appropriate to presume a different set of choices open. For instance, this may hold of someone who is not deliberating along with me but is *judging* me. To a recording angel who is making up a list of my misdemeanors, it will be clear that one has occurred at 3:45. Relative to a set of choices that *might have been* open to me at 3:45, and indeed ought to have been open to me, (2.5) is true at 3:45. In this sense, since I won't get the dogfood by 4:00, an obligation is violated at 3:45, and so I get another black mark.

I don't think this dependence on a set of presumed choices is a feature that belongs uniquely to obligation. Desire, for instance, has exactly the same characteristic. I was recently in a situation where an unpleasantly large needle had to be stuck into one or another of my hands. Being right-handed, I wanted it in the left hand. But in another respect, I didn't want the needle in either hand.

4. THE UNDERLYING TENSE LOGIC

If deontic logic is to be developed along these lines, not just any tense logic can serve as its foundation. We need a tense logic that provides for choices among alternative possible futures. Arthur Prior's work in indeterministic tense logic provides the beginning of such a theory. Here, you consider cases in which time isn't linearly ordered, but can branch toward the future. For discrete temporal orderings this yields treelike structures such as the following finite example, consisting of just seven instants.



At α there are four possible future courses of events or *scenarios*. At β_1 and β_2 there are two scenarios. The fact that instants β_2 and β_2 are incomparable signifies that realization of β_1 will exclude those alternatives in which β_2 is realized. This theory of time therefore allows certain future alternatives that formerly were open to become extinguished with the flow of time – which is just what I've argued is required by deontic logic.

Some technical terminology. A *temporal model structure* is a pair $\langle \mathscr{K}, < \rangle$, where $\mathscr{K}$ is a nonempty set and $<$ a relation on $\mathscr{K}$. For our purposes, the most important condition on these structures is that for all $\alpha, \beta, \gamma \in \mathscr{K}$, if $\beta < \alpha$ and $\gamma < \alpha$ then $\beta < \gamma$ or $\gamma < \beta$ or $\beta = \gamma$. A *history* h on a model structure is a maximal chain on that structure: a linear pathway through the structure that skips no instants. $\mathscr{H}_\alpha$ is the set of all histories containing α. Since time branches only towards the future, $\mathscr{H}_\alpha$ represents the set of scenarios open at α.

It's surprisingly difficult to define satisfaction for the future tense when time is permitted to be nonlinear in this way. In other words, it's hard to define the truth-value

$$\mathrm{V}_\alpha(\mathrm{F}A)$$

given by a valuation V to a formula FA at an instant α. I have a way that seems good to me of doing this – the method is described in detail and defended in an article in *Theoria* 36 (1970). Let me just sketch it briefly here.

First you define the truth-value $\mathrm{V}^{\mathrm{h}}_\alpha(A)$ given to A by V at an instant α, *relative to a history* h *in* $\mathscr{H}_\alpha$. It's defined by induction on the complexity of A in the usual way, the critical clause of the definition being:

$$\mathrm{V}^{\mathrm{h}}_\alpha(\mathrm{F}A) = \mathrm{T} \text{ iff } \mathrm{V}^{\mathrm{h}}_\beta(A) = \mathrm{T} \text{ for some } \beta \in \mathrm{h} \text{ such that } \alpha < \beta.$$

You can think of $\mathrm{V}^{\mathrm{h}}_\alpha(A)$ as the truth-value taken by A at α, *provided* that the scenario h represents what will happen after α. It gives the truth-value of A

assuming that a certain course of events will come to pass. This is Prior's "Ockhamist" theory of the future tense.

But if there is more than one course of events h in $\mathscr{H}_\alpha$, $V^h_\alpha(A)$ will not reflect the unvarnished, unqualified truth-value $V_\alpha(A)$ taken by a formula A at α. To get at this, I introduce truth-value gaps using van Fraassen's method. The definition is this.

$$V_\alpha(A) = T \quad \text{iff} \quad V^h_\alpha(A) = T \text{ for all } h \in \mathscr{H}_\alpha.$$

$$V_\alpha(A) = F \quad \text{iff} \quad V^h_\alpha(A) = F \text{ for all } h \in \mathscr{H}_\alpha.$$

$V_\alpha(A)$ is underfined otherwise.

Take a particular future-tense formula, say FQ. According to this definition of satisfaction, FQ will be true at α if Q comes out true sooner or later, no matter what future course of events follows α. FQ will be false at α if Q is always and invariably false along all the future courses of events following α. But if Q is true at least once along some future course of events following α, and is invariably false along another future course of events, FQ has no truth-value at α. Future contingencies are neither true nor false.

5. THE DEONTIC OPERATORS

The intuitive discussion at the beginning of this paper was supposed to communicate an idea of what sort of deontic logic would be desirable, and the preceding account of tense logic to lay a foundation for the logic of obligation. We now have enough background to begin to do deontic logic. We will add a deontic operator O to our tense-theoretic language and try to give a semantic interpretation of the expanded language.

One thing that emerged from the example at the beginning of this paper was a distinction between statements of obligation that arise in contexts of *deliberation* and those that arise in other contexts, for instance contexts of *judgment* or *wishing*.

As a warmup, let's define satisfaction just for deliberative contexts. This is a good place to begin because other contexts are more difficult to handle. Here, we suppose that we're given a model structure $\langle \mathscr{K}, <, \mathscr{S} \rangle$, where $\langle \mathscr{K}, < \rangle$ is a temporal model structure and $\mathscr{S}$ is a relation between instants and histories such that if $\alpha \mathscr{S} h$ then $h \in \mathscr{H}_\alpha$. The semantic determinant $\mathscr{S}$ will be used to interpret the deontic operator. Intuitively, the meaning of $\alpha \mathscr{S} h$ is that h represents a future course of events at instant α that would be an acceptable moral choice.

The definition of $V^{h}_{\alpha}(A)$, the truth value of A at α relative to the scenario h, is defined inductively. The clause for the deontic operator is as follows.

$$V^{h}_{\alpha}(OA) = T \quad \text{iff} \quad V^{g}_{\alpha}(A) = T \text{ for all g such that } \alpha \mathscr{K} g.$$

$V_{\alpha}(A)$ is then defined just as before, using van Fraassen's schema.

Let me pause to mention some consequences of this definition. First, whenever A is true, OA also is true: A implies OA. I don't find this result unwelcome; it seems to me to be a natural outcome of the deliberative context, in which one always chooses between possible futures open at the time of deliberation. The reasons behind the implication can be made clearer by bringing an operator L for inevitability into the tense-theoretic language. The semantic rule for L is:

$$V^{h}_{\alpha}(LA) = T \quad \text{iff} \quad V^{g}_{\alpha}(A) = T \text{ for all } g \in \mathscr{H}_{\alpha}.$$

LA is true at α if A's truth at α is independent of scenarios for α. It follows at once that A implies LA; whatever is presently true is presently inevitable. But LA implies OA in deliberative contexts, since here, as we have said, obligations are ascertained by choosing among possible futures open at the time of deliberation. By transitivity of implication, then, A implies OA.

Second, if we rule out moral blind alleys by requiring that for all α there is at least one h such that $\alpha \mathscr{S}$ h, any formula having the form $OA = \sim L \sim A$ is valid. Although the principle that 'ought' implies 'can' seems to be a reasonable consequence when O is read deliberatively, it does not hold on other readings.

Third, this gives O an S5-like structure; for instance, all formulas having the form $\sim OA \supset O \sim OA$ are valid. Several deontic logicians have challenged the validity of such formulas, but it seems to me that the counterexamples they have offered all turn on a confusion of deontic and temporal relations. Here is another illustration of the claim I made earlier, that a theory of tense is needed in order to determine the notion of consequence for deontic logic.

Now, what about the interpretation of O in nondeliberative contexts? Translated into technical terms, my conclusion at the beginning of the paper was that we can't speak simply of the truth-value $V^{h}_{\alpha}(A)$ that V gives to A at α, relative to h. We have to define

$$^{\beta}V^{h}_{\alpha}(A), \text{ for } \beta \leq \alpha,$$

festooning V with symbols on almost every side. This is the truth-value of A at α, relative to h and also relative to a context of future possibilities given by the instant β, where β is either α itself or in the past of α.

It would be nice philosophically if we could define this more general kind of satisfaction in terms of deliberative satisfaction, but I can't see any way to do this. Instead, we must complicate the semantic determinant $\mathscr{S}$ so that it does several things for us. First, $\mathscr{S}$ has to transport us from α to a (not necessarily unique) instant which, from the point of view of β, *ought to have been realized* instead of α. Second, it tells us which future courses of events for these instances are deontically acceptable. Here, then, $\mathscr{S}$ is a pair consisting of two functions $\mathscr{S}_1$ and $\mathscr{S}_2$. $\mathscr{S}_1(\alpha, \beta)$ will be a set of instants. For each $\gamma \in \mathscr{S}_1(\alpha, \beta)$, $\mathscr{S}_2(\alpha, \beta, \gamma)$ will be a set of histories in γ. The crucial clause in the definition of satisfaction is then carried out as follows.

$$
\begin{aligned}
{}^{\beta}\mathrm{V}_{\alpha}^{\mathrm{h}}(\mathrm{O}A) &= \mathrm{T} \text{ iff for all } \gamma \in \mathscr{S}_1(\alpha, \beta), \\
{}^{\gamma}\mathrm{V}_{\gamma}^{\mathrm{g}}(A) &= \mathrm{T} \text{ for all } \mathrm{g} \in \mathscr{S}_1(\alpha, \beta, \gamma).
\end{aligned}
$$

In evaluating the truth of OA at α this definition, in effect, substitutes other situations for α: OA is evaluated by considering the deontically acceptable future courses of events of the γ's. Something like an act of imagination takes place here. We "bracket off" a piece of recent history that has led up to the actual instant α in which we find ourselves and suppose, for purposes of determining the truth-value of OA, that we occupy any of certain other instants having different histories from α.

For instance, suppose that I've drawn to a stop behind a line of cars at a red light and, looking in my rear view mirror, I see a car coming up behind me too fast for there to be a chance of its stopping in time. There is a sense in which the driver of the car ought not to hit me. While I'm waiting there with nothing better to do, I make this judgment in the following way. First, I go back a few seconds to a point in time at which the driver still had a chance of stopping safely. This, as I've said, is an act of imagination: wishful imagination, perhaps. Then I consider a variety of alternative scenarios stretching ahead of this instant, all scenarios in which he drives as he ought. In general there is more than one of these. Here, for instance, there is one in which he drives as he ought and scratches his left ear, and another in which he drives as he ought and scratches his right ear instead. All of these scenarios are, of course, might-have-beens, since he didn't in fact drive as he ought. Along each of these scenarios, then, I choose a particular instant to serve as an alternative for the one in which I unhappily find myself. The most natural way of doing this in our example is to use the metric properties of time and take instants along the other scenarios in which clocks show the same time they do at the instant in which I find myself. Then I note that in all of these alternative instants the driver ought not to hit me, where this 'ought' is construed deliberatively.

There are certain conditions that are reasonable requirements to put on $\mathscr{S}_1$: for instance that if $\gamma \in \mathscr{S}_1(\alpha, \beta)$ then $\beta < \gamma$, and that all the members of $\mathscr{S}_1(\alpha_1, \beta)$ are pairwise incomparable. Also, that $\mathscr{S}_2(\alpha, \alpha) = \{\alpha\}$; this yields deliberative obligation as a special case. As for $\mathscr{S}_2$, I can see no reason not to require that $\mathscr{S}_2(\alpha, \beta, \gamma)$ be independent of both α and β. If this is right, then $\mathscr{S}_2$ can be construed as the relation $\mathscr{S}$ of deliberative obligation.

Contrasting the resulting theory of logical consequence with the one we get in deliberative contexts, we find that neither A implies $\mathrm{O}A$ nor that $\mathrm{O}A \supset {\sim}\mathrm{L}{\sim}A$ is valid. (However, as David Kaplan pointed out to me, $\mathrm{O}A \supset \mathrm{P}{\sim}\mathrm{L}{\sim}\mathrm{F}A$ – 'ought' implies 'could have' – remains valid.) These results are reasonable for contexts of judgement. "That bank is being robbed, but it oughtn't to be robbed." "That car is going to hit me, but it oughtn't to be about to hit me." And "I ought to return this car I've borrowed, but I can't," said in a situation where I've driven the car so far away that there is no chance of my returning it in time to keep a promise.

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NOTES

[1] This paper was first presented at the Temple University Conference on Deviant Semantics, December, 1970. (The theory is deviant, I suppose, but I hope not perverted.) An earlier draft was written in January, 1971, and distributed privately. Major revisions were made in 1974, and minor ones in 1980. This work was supported under NSF Grant GS-2517.

[2] [Added in 1980.] I am now persuaded that this formulation is overstated. The argument shows only that there are sentences of English involving 'if' and 'ought' that derive their meanings from 'if' and 'ought' in a compositional way. It does not show, however, that there is no 'if-ought' conditional connective in English. I had supposed that such a connective would be unnecessary, given the many ambiguities that can be shown to accompany the independent 'if' and 'ought'. Several considerations have convinced me that this was wrong. (The simplest of these is that for any fixed scope combination of 'if' and 'ought' the dominance principle turns out to be valid. But dominance is invalid. See Chapter 1 of R. Jeffrey, *The Logic of Decision*, New York. 1965.) This means that, on reflection, the language we use to talk about conditional obligation turns out to be outrageously complicated. It also raises the problem of how the meaning of 'if-ought' can be predictable from the meaning of 'if' and 'ought'.

[3] This is a tendency, not a blanket charge; I know of many papers in deontic logic that are exceptions. In general, contemporary deontic logicians have done more to make their work philosophically relevant than moral philosophers have done to keep up with this logical work.

[4] I want to think of (2.4) as true at any time if and only if 'George provides transportation' is true on Saturday. In English, if one wanted to express the thought expressed by (2.4) before Saturday, future tense would be required; after Saturday, past tense is required. Even on Saturday, (2.4) would be unnatural. This has no bearing on the point I wish to make here.
[5] My use of days of the week may be misleading here. It's important for the argument to choose time intervals fine enough so that none of the sentences we're concerned with change truth value during these intervals. So grant me the assumption (which loses me no generality) that none of our sentences changes in truth value during any day of the week.

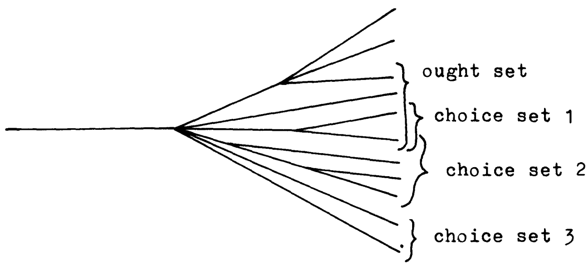
RICHMOND H. THOMASON

DEONTIC LOGIC AND THE ROLE OF FREEDOM IN MORAL DELIBERATION[1]

I want to present a model of deontic logic that relates *oughts* and *choices* to *alternative possible futures*, or *scenarios*. After presenting the model I'll try to show how it can be related to some interesting twists that deliberative reasoning can take. What I will do is rough and very tentative, but maybe it suggests that deontic logic can be related to some interesting questions of moral philosophy.

The basic idea is to represent what one ought to do by a set of scenarios (the ought set – this will be the set of scenarios in which one's oughts are fulfilled) and what one chooses to bring about by another set of scenarios (the choice set – this is the set such that you will [*ceterus paribus*] adjust your actions so that some member of the set should be realized – you devote yourself to realizing some member or other of the set).

So we get the following sort of picture.



Choice set 1 represents someone who attends to his duty, in the sense that he chooses to do as he ought.

Choice set 3 represents someone who transgresses; he chooses to do as he ought not.

Choice set 2 represents someone who consciously neglects his duty, in that he does not choose to bring about what he ought to. An example is someone who has promised not to lose at poker but chooses to play, knowing that this may result in his losing.

[Some notes. First, as I have represented things it doesn't follow that you will do what you choose to do. You may be unlucky, or have planned badly,

R. Hilpinen (ed.), New Studies in Deontic Logic, 177–186.

or have a fit of weakness of will. Secondly, the ought set is an idealization; it represents what we really ought to do (as if an oracle that knew everything relevant to our decision were to give us perfect advice about what we ought to do). Ignorance creates many complications with this idealization that I will ignore here.]

To show how this apparatus can be used in semantic interpretation let me take a simple formal language with truth functional connectives and a connective O, for 'ought to be the case'. It won't have any way of talking about choice, but will have a connective $\Box$ for 'is settled' – I'll explain this in a minute. The language also has a future tense connective, F.

We begin with a definition of $V^h_t(A)$, the truth value of a formula A at a time t, relative to a scenario h. [Note: when I talk of a "time" I don't mean a clock time, but something like a possible world; in this sense of 'time', every time has a unique past.] This truth value is defined recursively, according to the complexity of A: for example, $V^h_t(\sim A) = T$ if and only if $V^h_t(A) = F$ and $V^h_t(FA) = T$ if and only if $V^h_{t'}(A) = T$ for some t′ along h such that $t < t'$. [Another note: you may or may not sympathize with my preference for saying that one should also define an absolute $V^h_t(A)$ that doesn't depend on any choice of a scenario h. I have a way of doing this that I like, but it isn't relevant to the points I want to make in this paper.]

To interpret formulas of the sort OA we assume that each time t is equipped with a nonempty set P_t of scenarios passing through t – the ought set of t. We then stipulate that $V^h_t(OA) = T$ if and only if $V^g_t(A) = T$ for all $g \in P_t$.

To take an example, let A correspond to 'I will give you \$50 tomorrow' and suppose that I owe you the money, have promised to give it to you, and can do so – there are scenarios ahead of me on which I pay you the money tomorrow. Then I would want to say that OA is true – I ought to give you the money tomorrow – and so, every scenario in my ought set is such that on that scenario I *do* give you the money tomorrow. It is compatible with this situation that there also are scenarios ahead of me on which, through no fault of mine, this ought is cancelled. For instance, it may be a matter of chance whether I will be hit by a car later today and spend tomorrow in a coma in some hospital. At a time after the accident, on a scenario on which there *is* such an accident, there will be no scenarios on which I give you \$50 tomorrow, and so my ought set at the time won't contain any such scenarios. This example shows that blame needn't always be associated with failure to fulfill an ought; it also hints at how a theory such as this might go on to say something about excuses and "*prima facie* obligation". It is characteristic of such a theory (and might lead some to entertain alternative theories) that when I ought pay you $50 tomorrow it will then also be true that I ought not to have an accident that will prevent me from paying you; and that I ought not to have a heart attack; and that I ought not to have a death in the family that will call me out of town suddenly. What makes *these* implausible is that such matters are, to a large extent, out of my control, and when I promise to pay you $50 tomorrow it seems I am not promising (or even obligating myself) not to have a heart attack beforehand. The theory I'm proposing denies this; I want to say that oughts are *risky*, and if I ought to pay you $50 tomorrow I ought not to have a heart attack that will prevent me from paying you. Nevertheless, I agree, it would be peculiar to say 'I ought not to have a heart attack' in these circumstances – but that isn't because it's false.

Returning to the interpretation of our little language, what about $\Box$? Well, let H_t be the set of *all* scenarios passing through t; then $V_t^h(\Box A) = T$ if and only if $V_t^g(A) = T$ for all $g \in H_t$. If $\Diamond A$ is $\sim \Box \sim A$, then $V_t^h(\Diamond A) = T$ if and only if $V_t^g(A) = T$ for some $g \in H_t$.

I want to claim that $\Diamond$ corresponds to one important use of 'can'. I call this a "use" because 'can' also has other uses, the most important of these being an epistemic use, as in 'Plato's Atlantis story can have a basis in fact, but it can also be a complete fabrication'. I refrain from calling this a different "sense" because it could be regarded as having the same sense as the temporal 'can' (this sense being something like existential quantification over a contextually determined set of possible cases), the differences between the two being treated as pragmatic. At any rate, I take the temporal use of 'can' to have these properties: (1) if we say a thing can happen, its subsequent happening shows that what we said was true; (2) the fact that in circumstances similar to ours a certain thing has happened is *prima facie* evidence for the claim that it can happen (supposing it to be in question whether it can happen).

The interpretation I gave above for OA, similarly, corresponds only to one use of 'ought' – a use appropriate for deliberation or advising. There is another use of 'ought' in English to pass judgment; I will characterize this use as involving a certain amount of "bracketing" or "iffiness". (An example of this use: I say 'You oughtn't to be here' to someone who ought to have left town yesterday.) Here, in giving OA an interpretation you imagine that at some point in the past things had gone more as they should than in fact they did. This gives you a set S of counterfactual moments, perhaps very like the moment in which we find ourselves, but differing in some respects – e.g., in the promise-keeping record of some person you are judging. Then $V^h_t(OA) = T$ if and only if $V^g_{t'}(OA) = T$ for all $g \in P_{t'}$, for all $t' \in S$. Maybe we should speak here of $V^S_t(OA)$, dropping the irrelevant 'h' in favor of the 'S' required by judgmental 'ought'. Then we obtain the deliberative as a special case of the judgmental 'ought' – the case in which $S = \{t\}$.

Let me illustrate this with an example. Say that I borrow from you some notes that you need for a lecture you're giving at 2:00, and promise to have them back in time for your lecture. I take them to my home, half an hour away from where the lecture is to occur, and sit down to spend time. Up to 1:30 I ought (on the deliberative reading) to return the notes by 2:00, and we can even measure the increasing strength and urgency of the ought as time wears on toward 1:30, by the time available before it will become impossible for me to fulfill the ought. Or perhaps it should be measured by the increasing chanciness my fulfilling the ought, since if I start off with the note at the last minute, only to get a flat tire or discover that I can't find a parking space, I can justly be accused of negligence. At 1:30 or thereabouts, this deliberative ought is replaced by a judgmental ought (good only for condemnation, since by this time it's impossible for me to return the notes before the lecture), and also gives way to a deliberative reparational ought of some sort – perhaps I ought to call you and provide some warning, or perhaps I ought to take the notes directly to the lecture. As time goes on, if I ignore them these reparational oughts will in turn be replaced by further condemnations. It is part of all this that at no time is OA true on the deliberative reading when $\Diamond A$ is false. In fact, according to this theory 'ought', on the deliberative reading at least, always implies 'can'.

This theory arises fairly naturally, I think, out of difficulties in early deontic logic that were relevant, in some sense, to ethical concerns – questions relating to reparational oughts and such. And I would claim that it provides solutions to these. But rather than discuss this claim I would like to consider how the model provides a useful tool for conceptualizing certain interesting questions in moral philosophy.

At this point I will assume that this theory not only is correct as far as it goes as a theory of what we really ought to do (say, of the advice an oracle would give us, that shared our values and had perfect information about the relevant facts) but that it constitutes the meaning of 'ought' accurately enough to also explain – as a first approximation, anyway – our *judgments* about oughts. I will confine myself to deliberative judgments, judgments that OA is true on the deliberative reading. In this case the theory suggests that a judgment that OA is true will be accompanied by a judgment that $\Diamond A$ is true.

But the relation between the two judgments is more complex than just this would suggest. It isn't as if we start out with a bunch of "absolutely possible" scenarios, together with a set X of such scenarios "absolutely compatible" with my doing what I ought, and then determine what I really ought to do by intersecting X with a set of "real possibilities".

To illustrate this, take a case where we'd judge that if we can do A we ought to do B, but if we can't do A we ought to *refrain* from doing B. I go backpacking with a friend in winter. When we're a day away from the nearest people the weather turns bad and my friend breaks a leg. If I judge that I can get back with help before he dies of exposure I ought to leave him, and leave him right away; if I judge that I can't get back with help before he dies of exposure, I ought not to leave him. To decide what I ought to do in such case, I first have to decide what can be done and the difference between an ought and an ought-not can hang on a knife's edge.

This connection between what we judge we ought to do and what we judge we can do leaves open a certain temptation: to adjust our opinion about what we can do in order to obtain the easiest or most pleasant ought.

Such examples haven't been much discussed in the literature of deontic logic; but they are suggested by points Larry Powers makes in his 'Some deontic logicians' (*Noûs* **1**(1967), pp. 380–400; in this connection see especially p. 396) and explicitly mentioned by Pat Greenspan on p. 267 of her 'Conditional oughts and hypothetical imperatives' (*Journal of philosophy* **72** (1975), pp. 259–276). Here is such a case: a person who is afraid of flying should attend a meeting on a certain date, and the only way to get there is by flying. Though he has agreed to attend he begins to have second thoughts, and reasons as follows. "In the past I've never been able to do things like this; even though I tried to get on the airplane I didn't manage to. Very likely, then, I won't be able to go this time. And if I can't attend the meeting, I ought to let the organizers know right away that I won't be there; after all, this will make things much easier for them than if they find out at the last minute." So he concludes he ought to write a letter saying he won't attend, and – doing as he judges he ought – writes the letter and tries to forget about the whole thing.

I think everyone will recognize this as not at all an uncommon form of reasoning. I want to say it's fallacious – not in that it leads from true premises to a false conclusion, but in that it trades on a misrepresentation of the background conditions of an ought. This becomes clear when we realize that the letter writer must be aware, if he looks the matter in the face, that he *can* attend the meeting. But by assuming to be *settled* that he won't attend the meeting he makes things easy on himself, and yet in other respects he is able to pursue a pattern of deliberation that is formally correct, and yields the conclusion that he *ought* to excuse himself from attending. Often, in fact, it is reasonable to take certain things as settled; e.g., if my car has a history of stalling a few minutes after I start it, it may be reasonable for me to take it as settled that I won't be able to get somewhere in half an hour. What is unreasonable, though, is to do the same for my *own actions*, where there is evidence that other people in similar circumstances can perform the action in question. [I'm not entirely satisfied with this formulation, but it's the best I can do at present by way of stating the principle I have in mind. It does lead, and I mean it to lead, to a heroic standard of moral deliberation.]

These examples can be put in a slightly different light, which makes them more puzzling and raises difficult questions of interpretation; you can treat them as paradoxical instances of the Prisoner's Dilemma, where the "payoffs" are moral evaluations of a player's behavior. (It is not the form of payoff that makes these cases paradoxical, it is rather the fact that a player plays against himself.)

Take an academic example. Let's suppose I've indebted myself to a journal, and have promised to referee papers for them, so that when they send me a paper and ask me to referee it there seems to be no question that I ought to do so. And of course I ought to do it in a reasonable amount of time. But, as I think about it, it occurs to me that I occasionally have a block about refereeing; I accept a job and then get all tangled up with other commitments, keep making false starts at the job and then putting it aside, and in these cases the consequences have been terrible for the author and embarrasing for the journal. Looking at my conduct in these cases I see that I have earned a big moral debit. So, I reason, perhaps I ought – all things considered – to avoid the risk by sending the paper back unread right away. I recognize that this course would be reprehensible, but it avoids a risk of being even more reprehensible. So I reach a judgment that I ought to turn the job down.

The relation of this to the Prisoner's Dilemma should be clear. However, it's an essential precondition of the Prisoner's Dilemma that you have *two* prisoners who are kept from communicating with each other. If they are able to communicate it clearly will be in their interest to cooperate, to agree to hold out, and to reassure one another that they are committed to the agreement. In case the prisoners are never separated, even for interrogation, the dilemma loses its force entirely.

But in the case of the unwilling referee only *one* person is involved who plays the game with a future stage of himself. Surely, if we can trust anyone we can trust ourselves and so such reasoning ought to be impossible. Clearly, what makes it possible in this case is the fact that though I recognize my commitment to help the journal, I haven't thrown myself into it heart and soul. If I were wholly committed, for instance to making money by building and selling furniture, I wouldn't be likely to refuse an offer of $1,000 for a foot-stool I can make in one day, with the provision that I have to pay a penalty of $500 if I don't complete the job in a week. Self-doubt over whether I will actually get myself to carry through with the job won't enter, unless I'm an extraordinary furniture maker.

I suspect that people who reflect on these examples will divide according to their intuitions and inclinations into two camps, which I will label the *existentialist* and the *utilitarian* camps. (These labels are suggestive, but please don't take them too seriously.) The existentialists would feel that the relevant difference between the referee and the furniture maker is that the former's situation gives rise to a certain temptation, while the utilitarians would say that it consists in a difference in the *probability of reneging.*

Since the utilitarians would liken the reasoning in this case to that in a two-person case in which someone's doing as he ought is in doubt, let's consider such a case. First, though, I should explain that the theory of deontic logic I presented earlier was a one-person theory. Deontic logicians have had a habit of speaking as if there were a single O, and the formalization of 'John ought to apologize to Jane will be O*Pa* (where *Pu* formalizes '... apologizes to Jane') and that of 'Bill ought to apologize to Jane' will be O*Pb*. Semantically this would mean one and the same ought set, serving for everyone. This is wrong; everyone should have his own ought set, and the formalizations of our two sentences should be $O_a Pa$ and $O_b Pb$. Or on a more sophisticated theory we could adopt Richard Montague's theory of infinitive clauses in English ('ought' anyway, though not 'should' involves such clauses) and say that 'ought' expresses a relation between an individual and a property. [Note: I don't mean to suggest that there is no impersonal 'ought'. I believe there is such an 'ought' too, and that it's involved in examples like 'Someone ought to call a doctor', and 'You ought to be forced to pay for that damage' – at least on the most natural understandings of these examples.]

To see why a single ought set won't do, suppose for the moment that I'm *editing* the journal, and I have a referee who ought to report on a paper in a week. I am bound by the policies of the journal to return a decision to an author within a week of receiving a report. So it ought to be that if I receive the report in a week, I return a decision within two weeks from now. If my ought set and the referee's were the same, it would follow (by distribution of O over the conditional) that I ought (now) to return a decision to the author within two weeks from now. But ought I to? Certainly not! The referee may be late, in which case I would have more than two weeks. And until I receive his report, my ought to communicate a decision is not activated. [I owe this type of example to correspondence with Mr. E. Loevinsohn.]

Now, to begin with, I certainly should not assume in my deliberation about what I ought to do that others will act as they should. Even if they appear to have the best intentions of carrying through a project, if it is *my* responsibility (at least in part) that the project be completed, or if my fulfilling of my oughts depends on the completion of the project, it would be negligent of me not to take into account any likelihood of backsliding. It's easy enough to make use of this to construct two-person examples of deliberative reasoning – quite sound reasoning – that resemble the example of the referee. Suppose that I have young children and oughtn't to leave them alone when I leave the house. On the other hand, I've agreed to represent my department at an important meeting with the Dean, and so I arrange for a baby sitter to stay with the children at the time of the meeting. Shortly before the meeting – too late to arrange for another sitter – I discover that the sitter is unreliable and there is a substantial chance he won't show up despite his promises. In this case maybe I ought to call my department and renege so that they can send someone else to the meeting.

In the case of the reluctant referee, it is as if I and the baby sitter are one; it is mistrust of my own future conduct that leads me to renege. What is peculiar about this, and (on my view) makes the reasoning unsound and condemns the reasoner is that it is the point of my deliberation to determine my conduct; I am deliberating to shape what I will do. Among the things I can do (in the example of the referee, this was supposed not to be in doubt) is to commit myself to refereeing the paper and carry it through. If I find there is *prima facie* reason to believe I ought to accept the job and carry it out, I shouldn't use probabilistic doubts about my carrying it out to block a judgment that I ought to accept. This is so even if from the editor's point of view (or from the point of view of any dispassionate calculator of expected utility) it would be best, all things considered, for me not to take on the job and return the paper immediately.

Having revealed here my existential leanings, let me try to formulate the principle involved. A deliberating agent should not allow the hypothesis that he ought to follow a certain course of action to be influenced by considerations about the probability that he will not choose to follow this course of action. [Again I am not entirely happy about this formulation. Development of a theory of deliberation and choice might help.]

In conclusion I'll try to say a little more about existentialism. I've identified two patterns of deliberative reasoning – patterns that I would want to condemn as displaying a kind of moral weakness but which also present a real temptation to agents – a temptation to which, I would think, many of us occasionally succumb.

After doing this much I reread a good deal of Sartre's *Being and Nothingness* to see if these patterns – which after all are patterns of bad faith – could provide me with any interesting things to say about Sartre's views. In a way, the answer is that it doesn't. Much of the book is concentrated on such a heavy metaphysical plane, if these are the right words for it, that I can't manage to take seriously. But many of his examples, and the attitudes he takes towards them, are very relevant to the cases I've discussed. (I should acknowledge that these examples served as the inspiration of many of my own.) I suppose that I am less interested in existentialism itself than in the tension that arises when decision theoretic models of practical reasoning are confronted with the vivid and all too plausible picture of the human condition that existentialism provides.

In the fear of flying case, I would say that self-deception must be involved. The person has to be aware, on some level, that he can attend the meeting. And his assumption that he *can't* attend can be considered a denial of his freedom. In order to present this denial to himself, the agent treats himself like an object in the world. Focusing on his own past, his own traits, he generalizes on these and arrives at a prediction of his own behavior.

In the case of the reluctant referee, the agent recognizes that his future commitment to a course of action is unreliable. He may not follow through on promises if he makes them. True enough, at least on my view. You may have noticed that, like Sartre, I am inclined to regard our future possibilities as very rich; I feel (though I have no way of proving it) that we are replete with possibility. Ahead of all of us I'd expect there to be scenarios in which we totally deny our past and throw over our present commitments – perhaps to take up an entirely new life. People like us have done so, most of us feel we can do so. So far, then, the reluctant referee is right; he can't, in this sense, entirely trust himself.

Where he goes wrong, I suppose, is in persuading himself that he is doing as he ought in refusing to take on the commitment. He knows he may, after all, not see it through. And if he were reasoning about someone else it would be legitimate for him to use this as a reason to renege on an obligation. But it's *his own* commitment to *this very* project that he is deciding, and he knows in some sense that he *can* set himself to do it and carry it through. His lapse, then, consists in treating himself not as a thing in the world, but as another person.

I haven't said anything helpful about the problem of setting limits to what is possible, that arises in cases where we suspect some compulsion is at work and we may have every reason not to trust ourselves. Here, of course, Sartre is an extremist, and I would never agree with many of the things he says. But I do feel that a kind of extremism in this regard is a salutory thing – at least in the case of *deliberative* oughts. If in the deliberative case we bear in mind our freedom, refuse to lose sight of the fact that we *can* follow through certain courses of action, it is a way of resisting the very real temptation that is offered by patterns of reasoning such as these. I'm not sure it would be a better world if everyone did this, in the sense that things would *go* better in it. (It seems to follow that in such a world, for instance, many referees would behave abominably along many scenarios, maybe even worse than they have done in the past in our world; and many authors would be hurt, and editors frustrated. And so forth.) But at least it would be a world in which people would have fewer resources to excuse themselves to themselves for their own backsliding.

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NOTE

[1] This is a preliminary draft, without references and other baggage, intended only for presentation as a talk. Written in 1976. Presented at a meeting of the American Philosophical Association in 1977.

LENNART ÅQVIST AND JAAP HOEPELMAN

SOME THEOREMS ABOUT A "TREE" SYSTEM OF DEONTIC TENSE LOGIC*

CONTENTS

1. *Introduction*

The purpose of this paper is to develop and to investigate some properties of a system of so called *deontic tense logic*, i.e. a logic which combines tense operators, or temporal modalities, with operators expressing obligation and permission. See sections 3–4 below. We call the proposed system DARB, where "D" suggests "deontic" and "ARB" the Latin *arbor* (meaning "tree"). DARB is interpreted semantically or model-theoretically by means of certain set-theoretical structures successively built up in a chain:

R. Hilpinen (ed.), New Studies in Deontic Logic, 187–221.

"integral temporal frames", "trees based on integral temporal frames", "enriched trees" and "DARB-models"; naturally, this leads up to a well defined conception of *validity* in the system DARB. See sections 5–9 below. The DARB-models are closely related to the models used by Brian F. Chellas (in his work on imperatives) in Chellas (1969) ch. IV sect. 3 and in Chellas (1971) sect. 6 as well as to the models used in Åqvist's work (1977) on action sentences.

The rest of the paper, sections 10–17, is devoted to an inquiry into the proof theory of the system DARB. In sect. 10 we attempt an axiomatic formulation of DARB (leading up to a notion of DARB-*provability*), which in sect. 17 is seen to be semantically sound or correct relatively to the model theory previously presented; the question of completeness is left open altogether. In the remaining sections 11–16, THEOREMS 1–7, we state and prove various results on the axiomatic formulation DARB, which shed light on the system both in relation to other work done in deontic-imperative logic and tense logic and as far as detailed matters of special interest are concerned.

We have also found it useful to incorporate into the paper a fresh discussion of the so called Paradox of Contrary-to-Duty Imperatives, which, as argued in Åqvist (1966) and (1969), highlights the need for a successful combination of deontic and temporal logic. See sect. 2. Incidentally, we note with pleasure that Spohn (1975) p. 251 stresses the very same need in his critical discussion of the Hansson (1969) system DSDL3 of dyadic deontic logic.

With all respect for the precursors of deontic tense logic, notably von Wright (1963) and Rescher (1966), we are anxious to emphasize that the present shape of the area is essentially due to one single contribution, viz. Chellas (1969). For that reason we wish to point out the following differences between the Chellas (1969) system of imperative tense logic and DARB:

I. Both systems contain a pair of operators expressing *historical necessity* and possibility, but they are interpreted differently; see Chellas (1969) ch. IV sect. 3 and below sect. 9. The Chellas (1969) operator of historical necessity (possibility) turns out to be slightly stronger (weaker) than its counterpart in DARB; see sect. 11 below, THEOREM 1, proof of (vi). The Chellas (1969) historical modalities are definable in DARB (see D1 and D2 of sect. 4 below) and, conversely, the DARB modalities are readily definable in Chellas's system. In our opinion, the historical modalities of DARB more naturally suggest themselves than those of Chellas (1969); perhaps this explains why a counterpart to the DARB axiom schema A28 is missing in Chellas's own list of valid schemata.

II. Chellas's system has two primitive monadic imperative operators, ! and ¡; there seems to be no way of defining these operators in terms of the remaining logical symbols of the system. By way of contrast, the object language of DARB contains, in addition to the *monadic* deontic operators SHALL and MAY that correspond respectively to ! and ¡, a "prohairetic" (preference-logical) *selection* operator $\triangle$ expressing "optimality" or "admissibility". These greater expressive resources of DARB lead to two types of definition not available in Chellas: first, we are able to define DARB-analogues of the constants Q of Kanger (1957) and S of Anderson (1956), in terms of which, secondly, we lay down definitional axiom schemata governing SHALL and MAY. See below sect. 4, D9 and D10, and sect. 10, A41 and A42.

III. The presence in DARB of $\triangle$ enables us to generalize the schemata just mentioned into definitions of two *dyadic* deontic operators, Shall and May, closely akin to those studied in Hansson (1969). See A39 and A40 of sect. 10 below. No account of dyadic deontic, or imperative, modalities is forthcoming in the system of Chellas (1969).

In view of these differences between the two systems we have found it worth while to embark on the present inquiry into DARB. Among the results reached, those stated in THEOREM 6.1 (collapse of the SHALL operator on a certain assumption) and THEOREM 7 (equivalence of three forms of conditional obligation on the same assumption) appear to be most important and novel. In turn, these results rest on THEOREMS 2 and 3, of which the former is indeed the basic one.

2. *The Paradox of Contrary-to-Duty Imperatives revisited*

According to THEOREM 6.1 of sect. 15 below we have the result that all instances of the following schema are provable in DARB:

T45^{+}. SHALL $A \leftrightarrow A$, provided that the formula A contains no occurrences of the operators $\triangle$, $\boxed{F}$, $\langle F \rangle$ and $\oplus$;

where $\triangle$ is our preference-logical "optimality"-operator and where $\boxed{F}$, $\langle F \rangle$ and $\oplus$ are the *future tense* operators of the system DARB. By T45^{+}, then, the SHALL modality is *vacuous* or *collapses into its argument* when the latter contains no operators of the kind stated in the proviso. Obviously, this suggests that in the system DARB the SHALL modality functions as a genuine deontic or imperative operator only and especially when it is prefixed to a future tensed sentence, i.e. one involving $\boxed{F}$, $\langle F \rangle$ or $\oplus$ in some essential way (disregarding $\triangle$ for the time being). Now, this feature of DARB seems in fact to reflect a rather common intuition. For instance, Spohn (1975) p. 249 f. decides that

> ... I suggest applying Hansson's semantics, only to formulas in which the obligatory state of affairs lies in the future relative to the time the obligation is in force;...

And on p. 250 he asserts:

> ... it is a widespread metaethical view that all ethical judgements, part of which the obligation operator is meant to explicate, are essentially concerned with guiding human action and thus are looking to the future. Dewey, for instance, says that "morality is largely concerned with controlling human nature", and Stevenson is quite explicit about the point in saying "that ethical judgements look mainly to *future actions*". Similar views could be found in many other places.

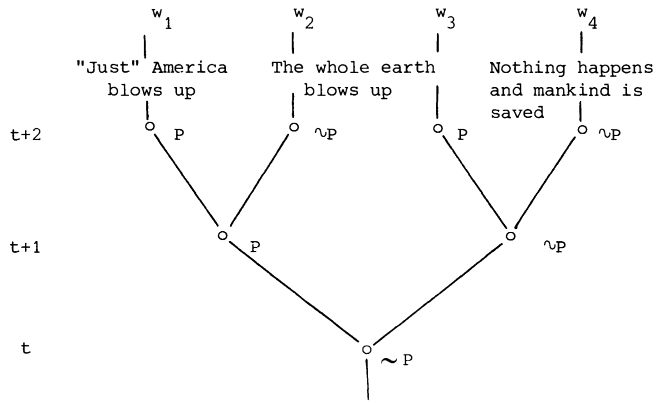
Again, in Rescher (1966) we find the following statement:

> Our view of commands rejects this conception (*sc.* that a logician is entitled to construct imperatives in all persons and in all tenses), taking them to be inescapably oriented towards the present and/or future.... The command 'Do A whenever P is the case!!' should be construed as '*Now and henceforward*: Do A whenever P is the case!!'.

Bearing such considerations in mind, let us now turn our attention to a familiar puzzle in denotic logic, viz. the so called Paradox of Contrary-to-Duty Imperatives. The discussion of this puzzle apparently originated with Chisholm (1963); as is clear from Åqvist (1960) p. 148 n.2, it had already been stated and dealt with in the fifties by T. Dahlquist in Uppsala. Here, we shall consider a version of the difficulty which goes back to Åqvist (1969) and owes some inspiration to Powers (1967).

Suppose that John is operating a devilish machine having just one button B, let there be three successive times of operation t, $t+1$ and $t+2$, let P be the statement that John presses button B, let $\sim P$ be the statement that John does not press button B, and let the destiny of mankind depend on John's pressing B, or not pressing B, as well as upon the order in which this is done, in the way appearing from the figure on the following page.

This diagram shows a tree structure with four paths w_1, w_2, w_3 and w_4, representing different "possible worlds" or "possible courses-of-events" in which John successively does not press button B and/or presses it as pictured by the drawing. As a result of John's activity "just" America blows up after $t+2$ in the world w_1, the whole earth blows up after $t+2$ in the worlds w_2 and w_3, whereas nothing happens in w_4 and mankind is saved.



Let us agree that, among $w_1, \ldots, w_4$, w_4 is the best or optimal course-of-events, then comes w_1, and that w_2 and w_3 are equally bad and indeed the worst possibilities that could be realized.

Now, the following situation seems to be a perfectly possible one:

I. It shall (ought to) be that John does not press button B at $t+1$ (because this is required by the optimal course-of-events w_4).

However, as a matter of fact, it so happens that

II. John presses button B at $t+1$.

III. If John presses button B at $t+1$, then he shall (ought to) press it at $t+2$ "again" (because this is required by w_1, which is the best course-of-events among those open to him after he has violated the primary duty in I, viz. w_1 and w_2)

IV. It shall (ought to) be that if John does not press button B at $t+1$, then he does not press it at $t+2$ "either" (because, if he were to do so, he would realize the bad course w_3 instead of the good w_4)

Now, the "paradox" arises as a result of an attempt to formalize the sentences I–IV by means of the following quartet of formulae:

(1) $\text{SHALL} \sim p$
(2) p
(3) $p \rightarrow \text{SHALL } q$
(4) $\text{SHALL}(\sim p \rightarrow \sim q)$

where p formalizes "John presses button B at $t + 1$", q "John presses button B at $t + 2$", and where $\sim$ represents negation and $\rightarrow$ material implication.

But, in the standard monadic deontic logic of SHALL, we easily deduce SHALL q from (2) and (3) as well as SHALL $\sim q$ from (1) and (4). This is a contradiction, although I–IV are perfectly consistent intuitively. As remarked by Hansson (1969) p. 385, (1)–(4) are no good as a formalization. He also observes that manipulating the relative positions of SHALL and $\rightarrow$ in (3) and/or (4) is no good either, for reasons bound up with the intuitive non-redundancy of the set of statements I–IV; see also Åqvist (1967) p. 365. Finally, it remains obscure to us how the puzzle is explained by taking III to involve a *dyadic* deontic operator Shall(__/...), i.e. by formalizing III as Shall (q/p); see Åqvist (1967) p. 366 note 5.

Let us now consider what to say about the difficulty on the basis of the system DARB and its deontic-tense-logical resources. First of all, we still let P represent the *temporally unqualified* statement that John presses button B (*simpliciter*, cf. our figure above). Also, the operator $\oplus$ is read as "it will be the case at the next instant that" and the operator $\ominus$ as "it was the case at the last instant that". Again, as a guide to the formalization of I–IV in DARB, we recall that the SHALL modality should operate on a future tensed sentence as its argument (in order to have genuine imperative force). We then translate I–IV into the following DARB-formulae, respectively:

Ia.	SHALL $\oplus \sim P$
IIa.	$\oplus P$
IIIa.	$\oplus P \rightarrow \oplus$ SHALL $\oplus P$
IVa.	SHALL$(\oplus \sim P \rightarrow \oplus \oplus \sim P)$

From IIa and IIIa we can deduce $\oplus$ SHALL $\oplus P$, and from Ia and IVa we obtain SHALL $\oplus \oplus \sim P$. But these conclusions are perfectly compatible with one another in DARB. Note in particular that the compound operators $\oplus$ SHALL ("it will be the case at the next instant that it shall be the case that") and SHALL $\oplus$ ("it shall (now) be that it will be the case at the next instant that") are logically independent of each other in DARB; this agrees nicely with the contention made in Åqvist (1966) pp. 245–249.

The satisfiability or consistency in DARB of the set $\{$Ia, ..., IVa$\}$ can in fact be said to be established by the model exhibited in our tree diagram above. Appealing to the truth conditions laid down in sect. 9 below, we verify that Ia, ..., IVa are all true in the worlds w_1 and w_2 at the time t. Of special importance is, of course, our interpretation of the SHALL operator (see clause (17) of sect. 9), which is indeed suggested clearly enough by our intuitive example as it stands. The quartet Ia, ..., IVa gives an adequate description, we claim, of John's predicament in w_1 or w_2 at the time t.

It is also possible to view John's situation from the time point $t + 1$ in the less than fully perfect worlds w_1 and w_2. We then obtain a reasonable description *via* the following quartet of DARB-formulae:

Ib. $\ominus$ SHALL $\oplus \sim P$
IIb. P
IIIb. $P \rightarrow$ SHALL $\oplus P$
IVb. $\ominus$ SHALL $(\oplus \sim P \rightarrow \oplus \oplus \sim P)$

Although SHALL $\oplus P$ follows from IIb and IIIb and $\ominus$ SHALL $\oplus \oplus \sim P$ from Ib and IVb, there is no inconsistency. It is important not to confuse the latter form with $\ominus \oplus$ SHALL $\oplus \sim P$, which is equivalent in DARB to the plain SHALL $\oplus \sim P$ and indeed inconsistent with SHALL $\oplus P$.

I. SYNTAX AND MODEL THEORETICAL SEMANTICS OF DARB

3. *Languages of DARB*

A *language* L *of the "tree" system* DARB *of deontic tense logic* or, simply, a DARB-language, is a structure made up of the following disjoint *basic syntactic categories*:

(i) An at most denumerable set Prop_L of *proposition letters*. Syntactic or metalinguistic notation: P.

(ii) *Propositional constants*: $\top$ and $\bot$ for, respectively, tautologyhood and contradictoriness ("absurdity").

(iii) *Boolean sentential connectives*: $\sim$, &, $\vee$, $\rightarrow$ for, respectively, negation, conjunction, disjunction and material implication (in terms of which the symbol $\leftrightarrow$ for material equivalence is defined in the usual way).

(iv) *Classical Priorean future* and *past tense operators* (temporal modalities): $\boxed{F}$, $\langle F \rangle$, $\boxed{P}$ and $\langle P \rangle$, to be read respectively as: "it will always be the case that", "it will at least once be the case that", "it has always been the case that" and "it has at least once been the case that".

(v) *Tense operators* (temporal modalities) *for the next* and *the last moment* (in discrete time): $\oplus$ and $\ominus$, to be read respectively as "it will be the case at the next instant that" and "it was the case at the last instant that".

(vi) Temporally dependent *modal operators of historical necessity* and *possibility*: $\Box$ and $\Diamond$, to be read as "it is necessary (possible) on the basis of the past that".

(vii) A *prohairetic* (preference-logical) *selection operator* $\triangle$ expressing "optimality" or "admissibility".

(viii) Two pairs of *deontic* or *imperative operators*: Shall, May and SHALL, MAY; where the first pair expresses a kind of *relativized* or *conditional obligation* (*permissibility*) and the second pair a matching kind of *absolute* or *unconditional obligation* (*permissibility*).

For any DARB-language L, the set $\mathrm{WFF}_{\mathrm{L}}$ of *wffs of* L, i.e. *well formed formulae* or *sentences of* L, is then defined as the smallest set S such that

(a) all proposition letters in $\mathrm{Prop}_{\mathrm{L}}$ as well as $\top$ and $\bot$ (i.e. all *atomic* L-wffs) are in S,

(b) if A is in S, then so are $\sim A$, $\boxed{\mathrm{F}}A$, $\langle\mathrm{F}\rangle A$, $\boxed{\mathrm{P}}A$, $\langle\mathrm{P}\rangle A$, $\oplus A$, $\ominus A$, $\Box A$, $\Diamond A$, $\triangle A$, SHALL A and MAY A, and

(c) if A, B are in S, then so are $(A \,\&\, B)$, $(A \vee B)$, $(A \rightarrow B)$, Shall(A/B) and May (A/B).

REMARK. It appears from this recursive definition of $\mathrm{WFF}_{\mathrm{L}}$ that &, $\vee$, $\rightarrow$, Shall, and May are two-place (dyadic) sentential (formula-making) operators, whereas all the remaining connectives and operators are one-place (monadic).

4. *Definitional abbreviations and notational conventions*

First of all, we define two modalities of what may be called *historical necessity* and *possibility in the sense of* Chellas (1969):

D1. $\boxdot A =_{\mathrm{df}} \ominus \Box \oplus A$

D2. $\langle\cdot\rangle A =_{\mathrm{df}} \ominus \Diamond \oplus A$

Secondly, following Chellas (1969) p. 74, we define what is known as the *Diodorean* non-past and non-future temporal modalities:

D3. $\boxed{\mathrm{f}}A = \mathrm{df} \quad A \,\&\, \boxed{\mathrm{F}}A$

D4. $\langle\mathrm{f}\rangle A = \mathrm{df} \quad A \vee \langle\mathrm{F}\rangle A$

D5. $\boxed{\mathrm{p}}A = \mathrm{df} \quad A \,\&\, \boxed{\mathrm{P}}A$

D6. $\langle\mathrm{p}\rangle A = \mathrm{df} \quad A \vee \langle\mathrm{P}\rangle A$

Thirdly, we introduce the so called *omnitemporal* modalities (see e.g. Åqvist (1979)):

D7. $\boxed{u} A =$ df $\boxed{P} A \,\&\, A \,\&\, \boxed{F} A$
D8. $\Diamond_u A =$ df $\boxed{P} A \vee A \vee \Diamond_F A$

Finally, we define DARB-analogues of the constants Q of Kanger (1957) and S of Anderson (1956):

D9. $Q =$ df $\triangle \top$
D10. $S =$ df $\sim \triangle \top$

As to conventions for dropping parentheses, we agree that $\rightarrow$ and $\leftrightarrow$ make a greater break then & and $\vee$ and that outer parentheses are mostly omitted around wffs.

5. *Integral temporal frames*

By an *integral temporal frame* we shall understand a structure $F = \langle T, <, >, 0, +1, -1 \rangle$ where

(i) $T \neq \varnothing$ (a non-empty set of *times* or moments)

(ii) $<, > \subseteq T \times T$ are two binary relations on T satisfying the following conditions:

(a) For all t, t' in T: $t < t'$ iff $t' > t$ ($<, >$ are the *converses* of each other in T);

(b) for each t in T: there is a t' in T with $t < t'$, and there is a t'' in T with $t > t''$ ($<, >$ are *serial* in T);

(c) $<$ and $>$ are *strict linear orderings* on T in the sense of being *irreflexive*, *transitive* and (weakly) *connected* in T;

(iii) $O \in T$ and $+1$, -1 are functions from T into T such that the following conditions are fulfilled:

(a) T = the smallest set S such that (1) $O \in S$, and (2) if $t \in S$, then so are $t+1$ and $t-1$;

(b) for each t in T: $t - 1 < t < t + 1$;

(c) for each t in T: $(t+1) - 1 = t = (t-1) + 1$.

REMARK. The time-set T in an integral temporal frame is readily seen to have the structure of the signed integers. Thus, $T = \{\ldots, -1, 0, +1, \ldots\}$, where $+1 = 0 + 1$, $+2 = +1 + 1 = (0+1) + 1$ etc. and $-1 = 0 - 1$, $-2 =$

$-1-1=(0-1)-1$ etc. Like Chellas (1969) and (1971), then, we regard time as discrete, linear and infinitely open-ended. We do so at least for our present purposes.

6. *Trees based on integral temporal frames*

By a *tree based on an integral temporal frame* or, simply, a *tree* we shall mean a triple $\mathscr{T} = \langle F, Q, W \rangle$ where

(i) $F = \langle T, <, >, 0, +1, -1 \rangle$ is an integral temporal frame;

(ii) $Q \neq \varnothing$ (a non-empty set, disjoint from T, of *points* or *concrete situations* in the tree $\mathscr{T}$);

(iii) $W \subseteq Q^T$ (i.e. W is a set of functions from T into Q, called the *possible courses of events* or the *possible histories* or the *possible worlds* or the *paths* in the tree $\mathscr{T}$); W is to be such that

(a) for each w in W and each t in T: $w(t)$ is in Q;

(b) for each q in Q there is *exactly one* t in T such that for *at least one* w in W: $q = w(t)$; (this condition entails that Q is included in, and in fact identical to, the union of the ranges of the functions w, for w in W);

(c) for any w, w' in W and for each t in T: if $w(t) = w'(t)$, then for all t' in T with $t' < t, w(t') = w'(t')$;

(d) for any w, w' in W with $w \neq w'$ there is exactly one t in T such that

(1) $w(t) = w'(t)$, and

(2) for all t' in T with $t' > t$: $w(t') \neq w'(t')$.

REMARK 1. Let $\mathscr{T} = \langle F, Q, W \rangle$ be a tree. For any w, w′ in W and for any t in T we say that *w is historically identical to w' at t*, in symbols: $w \underset{t}{\approx} w'$, iff $w(t) = w'(t)$; also, we say that *w is historically identical to w' up to* (but not necessarily including) t, in symbols: $w \underset{t}{\sim} w'$, iff for all t' in T with $t' < t$: $w(t') = w'(t')$. Armed with these two notions, we can now reformulate condition (iii)(c) above as the following

AXIOM OF HISTORICAL IDENTITY: For any w, w' in W and for any t in T: if $w \underset{t}{\approx} w'$, then $w \underset{t}{\sim} w'$.

This axiom asserts, then, that if two paths (courses of events, worlds) are historically *at* a given time, then they are historically identical *up to* that time in the sense of being historically identical *at* all times preceding it. We may also say that the force of the Axiom of Historical Identity is to exclude "backwards branching" or "branching to the left".

Exercises. Prove that $w \underset{t}{\sim} w'$ iff $w \underset{t-1}{\approx} w'$! Prove that, for each t in T, $\underset{t}{\approx}$ and $\underset{t}{\sim}$ are equivalence relations on W!

REMARK 2. Let $\mathcal{T} = \langle F, Q, W \rangle$ be a tree. For any w, w' in W and any t in T we say that *t is the time of branching for w and w'*, in symbols: $t = \text{br}(w, w')$, iff

(1) $w \underset{t}{\approx} w'$, and

(2) for all t' in T with $t' > t$: $w(t') \neq w'(t')$.

Condition (iii) (d) in our definition of a tree can now be reformulated as the following

AXIOM OF BRANCHING: For any w, w' in W with $w \neq w'$ there is exactly one t in T such that $t = \text{br}(w, w')$.

7. *Enriched trees*

By an *enriched tree* we shall mean any structure $\Gamma = \langle \mathcal{T}, \text{opt} \rangle$ where

(i) $\mathcal{T} = \langle \langle T, <, >, 0, +1, -1 \rangle, Q, W \rangle$ is a tree based on an integral temporal frame, and

(ii) opt is a function: $\mathcal{P}W \longrightarrow \mathcal{P}W$, i.e. from the power set of W into itself, which is to be a *choice function* in the sense that for any $X, Y \subseteq W$:

(a) $\text{opt}(X) \subseteq X$,
(b) $\text{opt}(X) = \varnothing$ only if $X = \varnothing$,
(c) $\text{opt}(X) \cap Y \subseteq \text{opt}(X \cap Y)$, and
(d) if $\text{opt}(X) \cap Y \neq \varnothing$, then $\text{opt}(X \cap Y) \subseteq \text{opt}(X) \cap Y$.

8. *DARB-models*

Let L be any DARB-language. By a DARB-*model for* L we shall understand a pair $M = \langle \Gamma, V \rangle$ such that

(i) $\Gamma = \langle \langle \langle T, <, >, 0, +1, -1 \rangle, Q, W \rangle, \text{opt} \rangle$ is an enriched tree, and

(ii) V is a *valuation of the proposition letters of* L in the sense of a *ternary* function such that, for any P in Prop_L, any w in W and any t in T,

$$V(P, w, t)$$

is defined and is either 1 (truth) or 0 (falsity). In other words, then, V: $\text{Prop}_L \times W \times T \longrightarrow \{1, 0\}$. Furthermore, we assume V to satisfy the following condition:

(C) For each P in Prop_L, for each w, w' in W and for each t in T: if $w \underset{t}{\approx} w'$ (i.e. if $w(t) = w'(t)$), then $V(P, w, t) = V(P, w', t)$.

In regard to (C), see also Chellas (1969), p. 92 n.3, and Chellas (1971), p. 123.

9. *Truth conditions*

Let L be any DARB-language, let

$$M = \langle\langle\langle\langle T, <, >, 0, +1, -1\rangle, Q, W\rangle, \text{opt}\rangle, V\rangle$$

be any DARB-model for L, let w be in W and let t be in T. We now want to define what it means for any L-wff A to be *true in* the world w *at* the time t *in* M, in symbols: $\models^{M}_{w,t} A$. As usual, the definition will be recursive on the length (complexity) of A and runs as follows:

(1) $\models^{M}_{w,t} P$ iff $V(P, w, t) = 1$ (for any P in Prop_L)

(2) $\models^{M}_{w,t} \top$

(3) it is not the case that $\models^{M}_{w,t} \bot$

(4) $\models^{M}_{w,t} \sim B$ iff it is not the case that $\models^{M}_{w,t} B$

(5) $\models^{M}_{w,t} (B \& C)$ iff $\models^{M}_{w,t} B$ and $\models^{M}_{w,t} C$

The truth conditions for L-wffs having $\vee$ and $\rightarrow$ as their main connective are then perfectly obvious.

(6) $\models^{M}_{w,t} \boxed{F} B$ iff $\models^{M}_{w,t'} B$ for all t' in T with $t < t'$

(7) $\models^{M}_{w,t} \Diamond\!\!\!\!\!\!\;\text{F}\; B$ iff $\models^{M}_{w,t'} B$ for some t' in T with $t < t'$

(8) $\models^{M}_{w,t} \boxed{P} B$ iff $\models^{M}_{w,t} B$ for all t' in T with $t > t'$

(9) $\models^{M}_{w,t} \langle P \rangle B$ iff $\models^{M}_{w,t} B$ for some t' in T with $t > t'$

(10) $\models^{M}_{w,t} \oplus B$ iff $\models^{M}_{w,t+1} B$

(11) $\models^{M}_{w,t} \ominus B$ iff $\models^{M}_{w,t-1} B$

(12) $\models^{M}_{w,t} \Box B$ iff $\models^{M}_{w,t} B$ for each w' in W such that $w \underset{t}{\approx} w'$

(13) $\models^{M}_{w,t} \Diamond B$ iff $\models^{M}_{w,t} B$ for some w' in W such that $w \underset{t}{\approx} w'$

(14) $\models^{M}_{w,t} \triangle B$ iff $w \in \text{opt}(\| B \|^{M}_{w,t})$; where

$\| B \|^{M}_{w,t}$ is defined as $\{w' \in W : w \underset{t}{\approx} w' \text{ and} \models^{M}_{w',t} B\}$.

(15) $\models^{M}_{w,t}$ Shall (B/C) iff $\text{opt}(\| C \|^{M}_{w,t}) \subseteq \| B \|^{M}_{w,t}$

(16) $\models^{M}_{w,t}$ May (B/C) iff $\text{opt}(\| C \|^{M}_{w,t}) \cap \| B \|^{M}_{w,t} \neq \varnothing$

(17) $\models^{M}_{w,t}$ SHALL B iff $\text{opt}([w] \underset{t}{\approx}) \subseteq \| B \|^{M}_{w,t}$

(18) $\models^{M}_{w,t}$ MAY B iff $\text{opt}([w] \underset{t}{\approx}) \cap \| B \|^{M}_{w,t} \neq \varnothing$

In (17) and (18) $[w] \underset{t}{\approx}$ is the equivalence-class of w in W under $\underset{t}{\approx}$, i.e. $\{w' \in W : w \underset{t}{\approx} w'\}$.

An L-wff A is DARB-*valid* just in case $\models^{M}_{w,t} A$ for each DARB-model M, for each w in W and for each t in T. We write "$\models A$" to indicate that A is DARB-valid.

II. ON THE PROOF THEORY OF DARB

10. *An attempted axiomatic formulation of DARB*

In this section we lay down certain axiom schemata, all instances of which can be seen to be DARB-valid, as well as certain rules of inference which preserve DARB-validity. The schemata and rules then determine an axiomatic system DARB which is semantically *sound* (or *correct*) in the sense that all DARB-provable wffs are DARB-valid; see Theorem 8, section 17 below. The question as to whether, conversely, all DARB-valid wffs are DARB-provable, so that our proposed axiomatics is also semantically *complete*, will have to be left open in the present study. Furthermore, the presented axiom system is perhaps not entirely free from redundancies.

RULES OF INFERENCE

R1. $\dfrac{A, A \to B}{B}$ R3. $\dfrac{A}{\boxed{P}A}$

R2. $\dfrac{A}{\boxed{F}A}$ R4. $\dfrac{A}{\Box A}$

AXIOM SCHEMATA

A0. All tautologies

A1. $\langle F\rangle A \leftrightarrow \sim \boxed{F} \sim A$

A2. $\langle P\rangle A \leftrightarrow \sim \boxed{P} \sim A$

A3. $\boxed{F}(A \to B) \to (\boxed{F}A \to \boxed{F}B)$

A4. $\boxed{P}(A \to B) \to (\boxed{P}A \to \boxed{P}B)$

A5. $\langle F\rangle \boxed{P}A \to A$

A6. $\langle P\rangle \boxed{F}A \to A$

A7. $\boxed{F}A \to \langle F\rangle A$

A8. $\boxed{P}A \to \langle P\rangle A$

A9. $\boxed{F}A \to \boxed{F}\boxed{F}A$

A10. $\boxed{P}A \to \boxed{P}\boxed{P}A$

A11. $\boxed{P}A \,\&\, A \,\&\, \boxed{F}A \to \boxed{F}\boxed{P}A$

A12. $\boxed{P}A \,\&\, A \,\&\, \boxed{F}A \to \boxed{P}\boxed{F}A$

A13. $\boxed{F}A \to \oplus A$

A14. $\boxed{P}A \to \ominus A$

A15. $\oplus A \leftrightarrow \sim \oplus \sim A$

A16. $\ominus A \leftrightarrow \sim \ominus \sim A$

A17. $\oplus (A \rightarrow B) \rightarrow (\oplus A \rightarrow \oplus B)$

A18. $\ominus (A \rightarrow B) \rightarrow (\ominus A \rightarrow \ominus B)$

A19. $\oplus \ominus A \rightarrow A$

A20. $\ominus \oplus A \rightarrow A$

A21. $\oplus A \,\&\, \boxed{\mathrm{F}} (A \rightarrow \oplus A) \rightarrow \boxed{\mathrm{F}} A$

A21.1 $A \,\&\, \boxed{\mathrm{F}} A \rightarrow \ominus \boxed{\mathrm{F}} A$

A22. $\ominus A \,\&\, \boxed{\mathrm{P}} (A \rightarrow \ominus A) \rightarrow \boxed{\mathrm{P}} A$

A22.1 $A \,\&\, \boxed{\mathrm{P}} A \rightarrow \oplus \boxed{\mathrm{P}} A$

A23. $\Diamond A \leftrightarrow \sim \Box \sim A$

A24. $\Box (A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$

A25. $\Box A \rightarrow A$

A26. $\Box A \rightarrow \Box \Box A$

A27. $\Diamond \Box A \rightarrow A$

A28. $A \rightarrow \Box A$, provided that A is a proposition letter.

A29. $\boxed{\mathrm{P}} \Box A \rightarrow \Box \boxed{\mathrm{P}} A$

A30. $\langle\mathrm{P}\rangle \Box A \rightarrow \Box \langle\mathrm{P}\rangle A$

A31. $\ominus \Box A \rightarrow \Box \ominus A$

A32. $\Box \boxed{\mathrm{F}} A \rightarrow \boxed{\mathrm{F}} \Box A$

A33. $\Box \oplus A \rightarrow \oplus \Box A$

A34. $\Box (A \leftrightarrow B) \rightarrow \Box (\triangle A \leftrightarrow \triangle B)$

A35. $\triangle A \rightarrow A$

A36. $\Diamond A \rightarrow \Diamond \triangle A$

A37. $\triangle A \,\&\, B \rightarrow \triangle (A \,\&\, B)$

A38. $\Diamond (\triangle A \,\&\, B) \rightarrow \Box (\triangle (A \,\&\, B) \rightarrow \triangle A \,\&\, B)$

A39. $\mathrm{Shall}(A/B) \leftrightarrow \Box (\triangle B \rightarrow A)$

A40. $\mathrm{May}(A/B) \leftrightarrow \Diamond (\triangle B \,\&\, A)$

A41. SHALL $A \leftrightarrow \Box (\triangle \top \rightarrow A)$

A42. MAY $A \leftrightarrow \Diamond (\triangle \top \,\&\, A)$

The above axiom schemata A0–A42 and rules of inference R1–R4 determine the following notion of DARB-*provability* (or DARB-

thesishood): the set of DARB-*provable* L-wffs is the smallest set S such that (i) every instance of A0–A42 is in S, and (ii) S is closed under the rules R1–R4. We write "$\vdash A$" to indicate that A is DARB-provable.

11. *General features of the axiomatic DARB*

In the sequel we shall refer to the axiom system just presented in the preceding section as "the axiomatic DARB" or simply as "DARB".

THEOREM 1.

(i) DARB contains the *minimal tense logic* K_t, due to E. J. Lemmon in 1965; see Prior (1967), Appendix A, § 4.1, p. 176. Relevant operators: $\boxed{F}$, $\langle F\rangle$, $\boxed{P}$ and $\langle P\rangle$.

(ii) DARB contains the Scott (1965) *system for linear, non-ending, non-beginning time*; see Prior (1967), Appendix A, § 5.5, p. 177. Relevant operators: $\boxed{F}$, $\langle F\rangle$, $\boxed{P}$ and $\langle P\rangle$.

(iii) DARB contains the just mentioned Scott (1965) system *supplemented by his postulates for the next and the last moment, in discrete time*, given on p. 67 of Prior (1967) and on p. 73 of Chellas (1969). Relevant operators: $\boxed{F}$, $\langle F\rangle$, $\boxed{P}$, $\langle P\rangle$, $\oplus$ and $\ominus$.

(iv) All instances of the following two theorem schemata are DARB-provable:

T0. $\boxed{P}(\boxed{F}A \rightarrow A) \rightarrow (\boxed{F}A \rightarrow \boxed{P}A)$

T0.1 $\boxed{u}(\boxed{F}A \rightarrow A) \rightarrow (\boxed{F}A \rightarrow \boxed{P}A)$

The weaker of these two schemata, T0.1, was proved by Bull (1968) to express the assumption that the structure of time is a discrete, or integral, ordering; see also Rescher & Urquhart (1971), pp. 95–96. In fact, Bull uses a more complex schema than T0.1 and credits Prior with the simplification. See Bull (1968), p. 27 n.3.

(v) DARB contains the modal system S5 for the historical modalities $\Box$ and $\Diamond$, supplemented with the additional axiom A28.

(vi) DARB has as axioms for $\Box$ analogues of schemata (36)–(40) stated on p. 89 of Chellas (1969), viz. A29–A33. Relevant operators: $\Box$, $\boxed{P}$, $\langle P\rangle$, $\ominus$, $\boxed{F}$ and $\oplus$.

(vii) DARB has as axioms for $\triangle$ analogues of schemata A13–A16.2 (for the $*$ operator) given on p. 13 of Åqvist (1973), viz. A34–A38.

(viii) DARB has axioms for Shall, May, SHALL and MAY amounting to definitions of these operators in terms of $\triangle$, $\Box$ and $\Diamond$, viz. A39–A42.

(ix) DARB contains the system DSDL3 of Hansson (1969), p. 396 f., as allegedly axiomatized by Spohn (1975), p. 239 f. Relevant operators: Shall and May.

(x) DARB contains the system KDE4 for the operators SHALL and MAY; see Lemmon & Scott (1977), sect. 4, and Chellas (1969), p. 24.

Proof.

Ad (i): See Prior (1967), Appendix A, § 4.1, p. 176; observe that R2 and R3 are primitive rules of inference of DARB and that A3–A6 are axioms of DARB. The definitions Df.*F* and Df.*P* in K_t are DARB-provable in the form of equivalences on the basis of A1 and A2.

Ad (ii): By definition, the Scott (1965) system is obtained from K_t by addition of schemata A7–A12 as axioms; see the reference made in (ii).

Ad (iii): We note that the Scott (1965) postulates for the next and the last moment in discrete time are, or are easily proved from, A13–A20, A21 and A22; see the reference made in (iii).

REMARK. Being unable to prove the DARB-valid schemata A21.1 and A22.1 in the remainder of DARB, we have stipulated them axiomatically. This may well turn out to be a redundancy.

Ad (iv): With respect to T0 we have

1.	$(\boxed{F}A \to A) \to (\boxed{F}A \to A \,\&\, \boxed{F}A)$	A0
2.	$A \,\&\, \boxed{F}A \to \ominus \boxed{F}A$	A21.1
3.	$(\boxed{F}A \to A) \to (\boxed{F}A \to \ominus \boxed{F}A)$	1, 2, A0, R1
4.	$\boxed{P}(\boxed{F}A \to A) \to \boxed{P}(\boxed{F}A \to \ominus \boxed{F}A)$	from 3 by $\dfrac{A \to B}{\boxed{P}A \to \boxed{P}B}$, which is a derived rule already in K_t
5.	$(\boxed{F}A \to A) \,\&\, \boxed{F}A \to \ominus \boxed{F}A$	3, A0, R1
6.	$((\boxed{F}A \to A) \,\&\, \boxed{P}(\boxed{F}A \to A) \,\&\, \boxed{F}A) \to$ $\to \ominus \boxed{F}A \,\&\, \boxed{P}(\boxed{F}A \to \ominus \boxed{F}A)$	4, 5, A0, R1
7.	$\ominus \boxed{F}A \,\&\, \boxed{P}(\boxed{F}A \to \ominus \boxed{F}A) \to \boxed{P}\boxed{F}A$	A22
8.	$\boxed{P}\boxed{F}A \to \boxed{P}A$	left to the reader; use A6, A8 and A10 *inter alia*
9.	$((\boxed{F}A \to A) \,\&\, \boxed{P}(\boxed{F}A \to A) \,\&\, \boxed{F}A) \to \boxed{P}A$	6, 7, 8, A0, R1
10.	$\boxed{P}(\boxed{F}A \to A) \to (\boxed{F}A \to \boxed{P}A)$	9, D5, A0, R1

Here, 10 = T0 = Q.E.D.

Again, we have in the case of T0.1:

1. $\boxed{u}(\boxed{F}A \to A) \to \boxed{P}(\boxed{F}A \to A)$ — A0, D5, D7
2. $\boxed{P}(\boxed{F}A \to A) \to (\boxed{F}A \to \boxed{P}A)$ — T0
3. $\boxed{u}(\boxed{F}A \to A) \to (\boxed{F}A \to \boxed{P}A)$ — 1, 2, A0, R1

where 3 = T0.1 = Q.E.D.

Ad (v): The point is obvious by virtue of the facts that A23–A28 are axioms of DARB and that R4 is a primitive rule of inference of DARB.

Ad (vi): See Chellas (1969), p. 89. Note that $\Box(\Diamond)$ is weaker (stronger) than the Chellas historical modality $\boxdot(\dot{\Diamond})$; using D1, D2, A20 and A33, we easily prove in DARB the schemata:

T1. $\boxdot A \to \Box A$

T2. $\Diamond A \to \dot{\Diamond} A$

the converses of which are not DARB-provable.

Ad (vii): See Åqvist (1973), p. 13.

Ad (viii): No comment needed.

Ad (ix): The Spohn (1975) calculus which he claims to be sound (correct) and complete with respect to Hansson's purely semantical system DSDL3 of dyadic deontic logic consists, in addition to A0 and R1, of the following schemata:

T3. $\text{Shall}(A/A)$

T4. $\Diamond A \to \text{May}(\top/A)$ (by paraphrase into DARB)

T5. $\text{Shall}(B \,\&\, C/A) \leftrightarrow \text{Shall}(B/A) \,\&\, \text{Shall}(C/A)$

T6. $\Box(A \leftrightarrow A') \,\&\, \Box(B \leftrightarrow B') \to (\text{Shall}(B/A) \leftrightarrow \text{Shall}(B'/A')$ (by paraphrase into DARB)

T7. $\text{May}(B/A) \to (\text{Shall}(C/A \,\&\, B) \leftrightarrow \text{Shall}(B \to C/A))$

DARB-*proof of* T3:

1. $\Box(\triangle A \to A)$ — A35, R4
2. $\Box(\triangle A \to A) \leftrightarrow \text{Shall}(A/A)$ — A39
3. $\text{Shall}(A/A)$ — 1, 2, A0, R1

where 3 = T3 = Q.E.D.

DARB-*proof of T*4:

1. $\Diamond A \rightarrow \Diamond \triangle A$	A36
2. $\triangle A \rightarrow \triangle A \,\&\, \top$	A0
3. $\Diamond \triangle A \rightarrow \Diamond(\triangle A \,\&\, \top)$	$2, \dfrac{A \rightarrow B}{\Diamond A \rightarrow \Diamond B}$
4. $\Diamond A \rightarrow \Diamond(\triangle A \,\&\, \top)$	1, 3, A0, R1
5. $\Diamond(\triangle A \,\&\, \top) \leftrightarrow \mathrm{May}(\top/A)$	A40
6. $\Diamond A \rightarrow \mathrm{May}(\top/A)$	4, 5, A0, R1

Here, 6 = T4 = Q.E.D.

Given S5 for $\Box$, the DARB-proofs of T5 and T6 are also elementary and need not be reproduced here; the former appeals essentially to A39 and A0, the latter uses *inter alia* A34 and A39.

DARB-*proof of* T7:

1. $\Diamond(\triangle A \,\&\, B) \rightarrow \Box(\triangle(A \,\&\, B) \leftrightarrow \triangle A \,\&\, B)$	A37, R4, A38, S5 for $\Box$
2. $\Box(\triangle(A \,\&\, B) \leftrightarrow \triangle A \,\&\, B) \rightarrow$ $\rightarrow \Box((\triangle(A \,\&\, B) \rightarrow C) \leftrightarrow (\triangle A \,\&\, B \rightarrow C))$	$\mathrm{A0}, \dfrac{A \rightarrow B}{\Box A \rightarrow \Box B}$
3. $\Diamond(\triangle A \,\&\, B) \rightarrow (\Box(\triangle(A \,\&\, B) \rightarrow C) \leftrightarrow$ $\leftrightarrow \Box(\triangle A \rightarrow (B \rightarrow C)))$	1, 2, S5 for $\Box$
4. $3 \leftrightarrow \mathrm{T7}$	A39, A40, A0, R1
5. T7	3, 4, A0, R1

So, 5 = T7 = Q.E.D.

Ad (x): On the propositional logic level Chellas's version of the system KDE4 for SHALL and MAY (actually, his ! and ¡) consists, in addition to A0 and R1, of the following rule of procedure and axiom schemata (see Chellas (1969), ch.I sect.4, pp. 13–25):

R5. $\dfrac{A}{\mathrm{SHALL}\ A}$

T8. $\mathrm{MAY}\ A \leftrightarrow \sim \mathrm{SHALL} \sim A$

T9. $\mathrm{SHALL}\ (A \rightarrow B) \rightarrow (\mathrm{SHALL}\ A \rightarrow \mathrm{SHALL}\ B)$

T10. $\mathrm{SHALL}\ A \rightarrow \mathrm{MAY}\ A$

T11. $\mathrm{SHALL}\ A \rightarrow \mathrm{SHALL\ SHALL}\ A$

T12. $\mathrm{MAY}\ A \rightarrow \mathrm{SHALL\ MAY}\ A$

Derivation in DARB *of* R5:

1. A	assumed to be DARB-provable
2. $\Box A$	1, R4
3. $\Box A \rightarrow \Box(\triangle \top \rightarrow A)$	A0, S5 for $\Box$
4. $\Box(\triangle \top \rightarrow A)$	2, 3, R1
5. SHALL A	4, A41, A0, R1

Here, 5 is our desired result.

The DARB-proofs of T8 and T9 appeal to A42, A41 and the fact that we have S5 for $\Diamond$ and $\Box$; they are easy and left to the reader.

DARB-*proof of* T10:

1. $\Diamond \top$	S5 for $\Diamond$
2. $\Diamond \triangle \top$	1, A36, R1
3. $\Diamond \triangle \top \rightarrow (\Box(\triangle \top \rightarrow A) \rightarrow \Diamond(\triangle \top \& A))$	S5 for $\Diamond$ and $\Box$
4. $(\Box(\triangle \top \rightarrow A) \rightarrow \Diamond(\triangle \top \& A))$	2, 3, R1
5. SHALL $A \rightarrow$ MAY A	4, A41, A42, A0, R1

Here, 5 = T10 = Q.E.D.

DARB-*proof of* T11:

1. $\Box(\Box(\triangle \top \rightarrow A) \rightarrow (\triangle \top \rightarrow \Box(\triangle \top \rightarrow A)))$	A0, R4
2. $\Box \Box(\triangle \top \rightarrow A) \rightarrow \Box(\triangle \top \rightarrow \Box(\triangle \top \rightarrow$ $\rightarrow A))$	1, A24, R1
3. SHALL $A \rightarrow$ SHALL SHALL A	A26, A0, R1, A41

where 3 = T11 = Q.E.D.

DARB-*proof of* T12:

1. $\Box(\Diamond(\triangle \top \& A) \rightarrow (\triangle \top \rightarrow \Diamond(\triangle \top \& A)))$	A0, R4
2. $\Box \Diamond(\triangle \top \& A) \rightarrow \Box(\triangle \top \rightarrow \Diamond(\triangle \top \& A))$	1, A24, R1
3. $\Diamond(\triangle \top \& A) \rightarrow \Box(\triangle \top \rightarrow \Diamond(\triangle \top \& A))$	2, S5 for $\Box$
4. MAY $A \rightarrow$ SHALL MAY A	3, A41, A42, A0, R1

Here 4 = T12 = Q.E.D.

The proof of THEOREM 1 is complete.

12. *On the logic of the historical modalities* $\Box$ *and* $\Diamond$

DEFINITION. Let $A \in \text{WFF}_L$. We say that A is *non-prohairetic* iff A does not contain any occurrences of the operator $\triangle$. And we say that A is *non-future* iff A does not contain any occurrences of the operators $\boxed{F}$, $\Diamond\!\!\!\!\text{F}$ or $\oplus$.

THEOREM 2. Any instance of the schema

T13. $A \to \Box A$

is DARB-provable, *provided that* A is non-prohairetic and non-future (i.e. provided that A contains no occurrences of any of the operators $\triangle$, $\boxed{F}$, $\Diamond\!\!\!\!\text{F}$ and $\oplus$).

Proof. By induction on the length of A.

Basis.

Case A = P, for some proposition letter P. Clearly, we have that P is non-prohairetic and non-future. The desired result to the effect that $\vdash A \to \Box A$ is of course immediate by virtue of A28.

Case $A = \top$. Here we have

1.	$\top$	A0
2.	$\Box\top$	1, R4
3.	$\top \to \Box\top$	2, A0 $((A \to (B \to A)))$, R1

whence our desired result.

Case $A = \bot$. By virtue of A0 we obtain that $\bot \to B$ and, in particular, $\bot \to \Box\bot$ are DARB-provable. So the case goes through unproblematically.

Induction Step.

Case $A = {\sim} B$, for some B in WFF_L such that B is non-prohairetic and non-future. Then, we have

1.	$B \to \Box B$	DARB-provable by the inductive hypothesis
2.	$\Diamond B \to \Diamond\Box B$	1, S5 for $\Diamond$
3.	$\Diamond\Box B \to B$	A27
4.	$\Diamond B \to B$	2, 3, A0, R1

5. $\sim B \rightarrow \sim \Diamond B$ 4, A0, R1
6. $\sim \Diamond B \rightarrow \Box \sim B$ S5 for $\Diamond$ and $\Box$
7. $\sim B \rightarrow \Box \sim B$ 5, 6, A0, R1

Here, 7 = Q.E.D. and the case is clinched.

Case $A = (B \& C)$, for some B, C in WFF_L such that both B and C are non-prohairetic and non-future. We get:

1. $B \rightarrow \Box B$
2. $C \rightarrow \Box C$ } DARB-provable by the inductive hypothesis
3. $(B \& C) \rightarrow (\Box B \& \Box C)$ 1, 2, A0, R1
4. $\Box B \& \Box C \rightarrow \Box(B \& C)$ S5 for $\Box$
5. $(B \& C) \rightarrow \Box(B \& C)$ 3, 4, A0, R1

where 5 = Q.E.D.

Case $A = (B \vee C)$, for some B, C in WFF_L with B, C non-prohairetic and non-future. We then obtain:

1. $B \rightarrow \Box B$
2. $C \rightarrow \Box C$ } DARB-provable by the inductive hypothesis
3. $(B \vee C) \rightarrow (\Box B \vee \Box C)$ 1, 2, A0, R1
4. $(\Box B \vee \Box C) \rightarrow \Box(B \vee C)$ S5 for $\Box$
5. $(B \vee C) \rightarrow \Box(B \vee C)$ 3, 4, A0, R1

where 5 = Q.E.D.

Case $A = (B \rightarrow C)$, for some B, C in WFF_L with B, C non-prohairetic and non-future. Then:

1. $B \rightarrow \Box B$
2. $C \rightarrow \Box C$ } DARB-provable by the hypothesis of induction
3. $\sim B \rightarrow \Box \sim B$ from 1 by *Case* $A = \sim B$
4. $(\sim B \vee C) \rightarrow (\Box \sim B \vee \Box C)$ 2, 3, A0, R1
5. $(\sim B \vee C) \rightarrow \Box(\sim B \vee C)$ 4, S5 for $\Box$, A0, R1
6. $(B \rightarrow C) \rightarrow \Box(B \rightarrow C)$ S5 for $\Box$, 5

Here, 6 = Q.E.D.

Case $A = \boxed{P}B$, for some B in WFF_L such that B is non-prohairetic and non-future. Here we have

1. $B \to \Box B$ DARB-provable by the hypothesis of induction
2. $\boxed{P}(B \to \Box B)$ 1, R3
3. $\boxed{P}B \to \boxed{P}\Box B$ 2, A4, R1
4. $\boxed{P}\Box B \to \Box \boxed{P}B$ A29
5. $\boxed{P}B \to \Box \boxed{P}B$ 3, 4, A0, R1

where 5 = Q.E.D.

Case $A = \langle P \rangle B$, for some B in WFF_L such that B is non-prohairetic and non-future. The case is settled as follows:

1. $B \to \Box B$ DARB-provable by the inductive hypothesis
2. $\boxed{P}(B \to \Box B)$ 1, R3
3. $\langle P \rangle B \to \langle P \rangle \Box B$ 2, K_t for $\langle P \rangle$, R1
4. $\langle P \rangle \Box B \to \Box \langle P \rangle B$ A30
5. $\langle P \rangle B \to \Box \langle P \rangle B$ 3, 4, A0, R1

where 5 = Q.E.D.

Case $A = \ominus B$, for some B in WFF_L with B non-prohairetic and non-future. Then:

1. $B \to \Box B$ DARB-provable by the inductive hypothesis
2. $\ominus(B \to \Box B)$ 1, R3, A14, A0, R1
3. $\ominus B \to \ominus \Box B$ 2, A18, R1
4. $\ominus \Box B \to \Box \ominus B$ A31
5. $\ominus B \to \Box \ominus B$ 3, 4, A0, R1

Here, 5 = Q.E.D.

Case $A = \Box B$, for some B in WFF_L with B as usual. The desired result, to the effect that $\vdash \Box B \to \Box\Box B$, is immediate by A26.

Case $A = \Diamond B$, for some B in WFF_L with B as usual. The desired result, $\vdash \Diamond B \to \Box \Diamond B$ (the characteristic reduction thesis of S5), readily follows from A26 and A27 plus familiar S5 principles.

Case $A = \text{Shall}(B/C)$, for some B, C in WFF_L such that both B and C are non-prohairetic and non-future.

1. $\Box(\triangle C \rightarrow B) \rightarrow \Box\Box(\triangle C \rightarrow B)$ A26
2. $\text{Shall}(B/C) \rightarrow \Box\,\text{Shall}(B/C)$ A39, S5 for $\Box$, 1

as desired.

Case $A = \text{May}(B/C)$, for some B, C in WFF_L with B and C as usual. We then have

1. $\Diamond(\triangle C \& B) \rightarrow \Box\Diamond(\triangle C \& B)$ cf. *Case* $A = \Diamond B$
2. $\text{May}(B/C) \rightarrow \Box\,\text{May}(B/C)$ A40, S5 for $\Box$, 1

as desired.

Case $A = \text{SHALL } B$, for some B in WFF_L with B as usual. The desired result, $\vdash \text{SHALL } B \rightarrow \Box\text{SHALL } B$, is obtained as in *Case* $A = \text{Shall}\,(B/C)$, where we take C as $\top$ and appeal to A26 and A41.

Case $A = \text{MAY } B$, for some B in WFF_L with B as usual. The desired result, $\vdash \text{MAY } B \rightarrow \Box\ \text{MAY } B$, is proved as in *Case* $A = \text{May}(B/C)$, where we take C as $\top$ and make use of the characteristic reduction thesis of S5 and A42.

The induction is complete, and so is the proof of THEOREM 2.

13. *On the logic of the prohairetic operator* $\triangle$

THEOREM 3. Any instances of the schema

T14. $A \rightarrow (\triangle \top \leftrightarrow \triangle A)$

is DARB-provable, *provided that* A is non-prohairetic and non-future.

Proof. We start by giving an unconditional proof of the schema

T14a. $A \rightarrow (\triangle \top \rightarrow \triangle A)$ (without proviso, then)

as follows:

1. $\triangle \top \& A \rightarrow \triangle(\top \& A)$ A37
2. $\Box(\triangle(\top \& A) \leftrightarrow \triangle A)$ A0 $((\top \& A) \leftrightarrow A)$, R4, A34, R1
3. $\triangle(\top \& A) \leftrightarrow \triangle A$ 2, A25, R1
4. $\triangle \top \& A \rightarrow \triangle A$ 1, 3, A0, R1
5. $A \rightarrow (\triangle \top \rightarrow \triangle A)$ 4, A0, R1

Here, $5 = \text{T14a} = \text{Q.E.D.}$

Next, we prove:

T14b. $A \rightarrow (\triangle A \rightarrow \triangle \mathrm{T})$, provided that A is non-prohairetic and non-future

1. $\Diamond \mathrm{T}$	S5 for $\Diamond$
2. $\Diamond \triangle \mathrm{T}$	1, A36, R1
3. $A \rightarrow \Box A$	by THEOREM 2, T13, where A is assumed to satisfy the proviso
4. $A \rightarrow (\Box A \,\&\, \Diamond \triangle \mathrm{T})$	2, 3, A0, R1
5. $(\Box A \,\&\, \Diamond \triangle \mathrm{T}) \rightarrow \Diamond(\triangle \mathrm{T} \,\&\, A)$	S5 for $\Box$ and $\Diamond$
6. $A \rightarrow \Diamond(\triangle \mathrm{T} \,\&\, A)$	4, 5, A0, R1
7. $\Diamond(\triangle \mathrm{T} \,\&\, A) \rightarrow \Box(\triangle(\mathrm{T} \,\&\, A) \rightarrow \triangle \mathrm{T} \,\&\, A)$	A38
8. $A \rightarrow \Box(\triangle(\mathrm{T} \,\&\, A) \rightarrow \triangle \mathrm{T} \,\&\, A)$	6, 7, A0, R1
9. $A \rightarrow (\triangle(\mathrm{T} \,\&\, A) \rightarrow \triangle \mathrm{T} \,\&\, A)$	8, A25, A0, R1
10. $\triangle(\mathrm{T} \,\&\, A) \leftrightarrow \triangle A$	line 3 in the proof of T14a above
11. $A \rightarrow (\triangle A \rightarrow \triangle \mathrm{T})$	9, 10, A0, R1

Here, 11 = Q.E.D., where A is assumed to be non-prohairetic and non-future.

T14a and T14b together yield our desired result T14 under the proviso that A is non-prohairetic and non-future. This completes the proof of THEOREM 3.

THEOREM 4. All instances of the following schemata are DARB-provable:

T15. $\Box(A \rightarrow B) \,\&\, \Diamond(A \,\&\, \triangle B) \rightarrow \Box(\triangle A \leftrightarrow \mathrm{A} \,\&\, \triangle B)$

T15 is a syntactic version of the so called Arrow's Axiom; see Hansson (1968), p. 444.

T16. $\Diamond(A \vee B) \rightarrow \Box(\triangle A \leftrightarrow A \,\&\, \triangle(A \vee B)) \vee$
$\vee\, \Box(\triangle B \leftrightarrow B \,\&\, \triangle(A \vee B))$

As for T16, cf. K_2 on p. 48 of Åqvist (1971).

T17. $\triangle(A \vee B) \rightarrow \triangle A \vee \triangle B$

As for T17, cf. Hansson (1968), Lemma 1, p. 446.

Proof.
Ad T15:

1. $\Box(A \to B) \to \Box(A \leftrightarrow A \& B)$ — S5 for $\Box$
2. $\Box(A \leftrightarrow A \& B) \to \Box(\triangle A \leftrightarrow \triangle(A \& B))$ — A34
3. $\Diamond(A \& \triangle B) \to \Box(\triangle(A \& B) \leftrightarrow A \& \triangle B)$ — A37, A38, S5 for $\Box$
4. $\Box(A \to B) \& \Diamond(A \& \triangle B) \to \Box(\triangle A \leftrightarrow$
   $\leftrightarrow \triangle(A \& B) \& \Box(\triangle(A \& B) \leftrightarrow A \& \triangle B)$ — 1, 2, 3, A0, R1
5. $\Box(A \to B) \& \Diamond(A \& \triangle B) \to \Box(\triangle A \leftrightarrow$
   $\leftrightarrow A \& \triangle B)$ — 4, S5 for $\Box$

Here, 5 = Q.E.D.

Ad T16:

1. $\Diamond(A \vee B) \to \Diamond \triangle(A \vee B)$ — A36
2. $\Diamond \triangle(A \vee B) \to \Diamond((\triangle(A \vee B) \& A) \vee$
   $\vee (\triangle(A \vee B) \& B))$ — A35, S5 for $\Diamond$
3. $\Diamond(A \vee B) \to (\Diamond(\triangle(A \vee B) \& A) \vee$
   $\vee \Diamond(\triangle(A \vee B) \& B))$ — 1, 2, S5 for $\Diamond$
4. $\Box(A \to A \vee B) \to (\Diamond(\triangle(A \vee B) \& A) \to$
   $\to \Box(\triangle A \leftrightarrow A \& \triangle(A \vee B)))$ — T15, S5 for $\Diamond$
5. $\Box(B \to A \vee B) \to (\Diamond(\triangle(A \vee B) \& B) \to$
   $\to \Box(\triangle B \leftrightarrow B \& \triangle(A \vee B)))$ — T15, S5 for $\Diamond$
6. $\Box(A \to A \vee B)$ — A0, R4
7. $\Box(B \to A \vee B)$ — A0, R4
8. $\Diamond(\triangle(A \vee B) \& A) \vee \Diamond(\triangle(A \vee B) \& B)$
   $\to \Box(\triangle A \leftrightarrow A \& \triangle(A \vee B)) \vee \Box(\triangle B \leftrightarrow$
   $\leftrightarrow B \& \triangle(A \vee B))$ — 4, 5, 6, 7, A0, R1
9. $\Diamond(A \vee B) \to \Box(\triangle A \leftrightarrow A \& \triangle(A \vee B)) \vee$
   $\vee \Box(\triangle B \leftrightarrow B \& \triangle(A \vee B))$ — 3, 8, A0, R1

where 9 = T16 = Q.E.D.

Ad T17:

1. $\triangle(A \vee B) \& A \to \triangle((A \vee B) \& A)$ — A37

2. $\triangle((A \vee B) \& A) \leftrightarrow \triangle A$	A0 $((A \vee B) \& A \leftrightarrow A)$, R4, A34, A25, R1
3. $\triangle(A \vee B) \& A \rightarrow \triangle A$	1, 2, A0, R1
4. $\triangle(A \vee B) \& B \rightarrow \triangle B$	similar to the proof of line 3; use A0, R4, A34, A25, R1
5. $(\triangle(A \vee B) \& A) \vee (\triangle(A \vee B) \& B) \rightarrow$ $\rightarrow \triangle A \vee \triangle B$	3, 4, A0, R1
6. $\triangle(A \vee B) \rightarrow (\triangle(A \vee B) \& A) \vee$ $\vee (\triangle(A \vee B) \& B)$	A35, A0, R1
7. $\triangle(A \vee B) \rightarrow \triangle A \vee \triangle B$	5, 6, A0, R1

Here, 7 = T17 = Q.E.D.
The proof of THEOREM 4 is complete.

14. *On the logic of the dyadic deontic operators* Shall *and* May

THEOREM 5. All instances of the following schemata are DARB-provable (cf. Hansson (1969) pp. 396–398, Åqvist (1971) pp. 46–47, and Kutschera (1974) p. 156):

T18.	Shall $(A/B) \leftrightarrow \sim$ May$(\sim A/B)$; May$(A/B) \leftrightarrow \sim$ Shall$(\sim A/B)$
T19.	$\Diamond B \rightarrow$ (Shall$(A/B) \rightarrow$ May(A/B))
T20.	Shall$(A \rightarrow B/C) \rightarrow$ (Shall$(A/C) \rightarrow$ Shall(B/C))
T21.	$\Box A \rightarrow$ Shall(A/B)
T22.	$\sim \Diamond B \rightarrow$ Shall(A/B)
T23.	$\Box(A \rightarrow B) \rightarrow$ (Shall$(A/C) \rightarrow$ Shall(B/C))
T24.	$\Box(B \rightarrow A) \rightarrow$ Shall(A/B)
T25.	$\Diamond B \rightarrow$ May(B/B); May$(\top/\top)$
T26.	Shall$(A/B \& C) \rightarrow$ Shall$(B \rightarrow A/C)$
T27.	Shall$(A \vee B/C) \& \sim$ Shall$(A/C) \rightarrow$ Shall$(B/\sim A \& C)$
T28.	Shall$(A \vee B/C) \leftrightarrow$ Shall$(A/C) \vee$ Shall$(B/\sim A \& C)$
T29.	Shall$(A/B) \&$ Shall$(A/C) \rightarrow$ Shall$(A/B \vee C)$
T30	Shall$(A/B \vee C) \rightarrow$ Shall$(A/B) \vee$ Shall(A/C)
T31.	Shall$(B/A) \&$ Shall$(C/A \& B) \rightarrow$ Shall$(B \& C/A)$
T32.	Shall$(A \& B/C) \rightarrow$ Shall$(A/B \& C)$
T33.	Shall$(A/B) \&$ May$(C/B) \rightarrow$ Shall$(A/B \& C)$

T34. $\text{Shall}(A/B) \,\&\, \text{May}(\sim A/B \,\&\, C) \rightarrow \text{Shall}(\sim C/B)$

(As to T34, see Åqvist (1971) p. 51.)

Fragmentary proof. Most of the derivations are left to the reader in the case of this THEOREM; just consider a few non-trivial or non-familiar examples.

Ad T29:

1.	$(\triangle B \rightarrow A) \,\&\, (\triangle C \rightarrow A) \rightarrow (\triangle B \vee \triangle C \rightarrow A)$	A0
2.	$\triangle(B \vee C) \rightarrow \triangle B \vee \triangle C$	T17 of THM 4
3.	$(\triangle B \rightarrow A) \,\&\, (\triangle C \rightarrow A) \rightarrow (\triangle(B \vee C) \rightarrow A)$	1, 2, A0, R1
4.	$\Box(\triangle B \rightarrow A) \,\&\, \Box(\triangle C \rightarrow A) \rightarrow$ $\rightarrow \Box(\triangle(B \vee C) \rightarrow A)$	3, S5 for $\Box$
5.	Shall (A/B) & Shall $(A/C) \rightarrow$ $\rightarrow \text{Shall}(A/B \vee C)$	4, A39, A0, R1

Here, 5 = T29 = Q.E.D.

Ad T30:

1.	$\sim \Diamond(B \vee C) \rightarrow \sim \Diamond B$	S5 for $\Box$ and $\Diamond$
2.	$\sim \Diamond B \rightarrow \sim \Diamond \triangle B$	A35, S5 for $\Box$ and $\Diamond$
3.	$\sim \Diamond \triangle B \rightarrow \Box(\triangle B \rightarrow A)$	S5 for $\Box$ and $\Diamond$
4.	$\sim \Diamond(B \vee C) \rightarrow (\Box(\triangle(B \vee C) \rightarrow A) \rightarrow$ $\rightarrow \Box(\triangle B \rightarrow A) \vee \Box(\triangle C \rightarrow A))$	1, 2, 3, A0, R1
5.	$\sim \Diamond(B \vee C) \rightarrow$ T30	4, A39, A0, R1
6.	$\Diamond(B \vee C) \rightarrow \Box(\triangle B \leftrightarrow \triangle(B \vee C) \,\&\, B) \vee$ $\vee \Box(\triangle C \leftrightarrow \triangle(B \vee C) \,\&\, C)$	T16 of THM 4,S5 for $\Box$
7.	$\Box(\triangle(B \vee C) \rightarrow A) \rightarrow$ $\rightarrow \Box(\triangle(B \vee C) \,\&\, B \rightarrow A) \,\&$ $\& \,\Box(\triangle(B \vee C) \,\&\, C \rightarrow A)$	S5 for $\Box$
8.	$\Diamond(B \vee C) \rightarrow (\Box(\triangle(B \vee C) \rightarrow A) \rightarrow$ $\rightarrow$(consequent of 7) & (consequent of 6))	6, 7, A0, R1
9.	(consequent of 7) & (consequent of 6) $\rightarrow$ $\rightarrow \Box(\triangle B \rightarrow A) \vee \Box(\triangle C \rightarrow A)$	S5 for $\Box$

10.	$\Diamond(B \vee C) \to (\Box(\triangle(B \vee C) \to A) \to$ $\to \Box(\triangle B \to A) \vee \Box(\triangle C \to A))$	8, 9, A0, R1
11.	$\Diamond(B \vee C) \to$ T30	10, A39, A0, R1
12.	T30	5, 11, A0, R1

So, 12 = T30 = Q.E.D.

15. *On the logic of the monadic deontic operators* SHALL *and* MAY

THEOREM 6. All instances of the following schemata are DARB-provable (cf. Chellas (1969), ch. IV, sects. 4–5, pp. 85–93):

T35.	$\Box A \to$ SHALL A (cf. T21 of THM 5)	
T36.	SHALL $A \leftrightarrow \Box$ SHALL A	
T37.	MAY $A \leftrightarrow \Box$ MAY A	
T38.	$\Box A \leftrightarrow$ SHALL $\Box A$	
T39.	$\Diamond A \leftrightarrow$ SHALL $\Diamond A$	
T40.	SHALL $A \to \Diamond A$	
T41.	$\Box A \vee \Box \sim A \to$ (SHALL $A \leftrightarrow A$)	
T42.	$\Box \boxed{P} A \vee \Box \sim \boxed{P} A$	provided that A is *unmodalized* (i.e. does not contain any modalities, temporal, historical, prohairetic or deontic)
T43.	$\Box \langle P \rangle A \vee \Box \sim \langle P \rangle A$	
T44.	$\Box \ominus A \vee \Box \sim \ominus A$	
T45.	SHALL $\boxed{P} A \leftrightarrow \boxed{P} A$	
T46.	SHALL $\langle P \rangle A \leftrightarrow \langle P \rangle A$	
T47.	SHALL $\ominus A \leftrightarrow \ominus A$	

Proof. The task of obtaining DARB-proofs of T35–T40 is essentially an exercise in S5 (plus A36 in the cases of T38–T40), which can be left to the reader.

*Ad T*41:

1.	$A \to$ (SHALL $A \to A$)	A0
2.	$\Box A \to$ (SHALL $A \to A$)	1, A25, A0, R1
3.	$\Box A \to \Box(\triangle \top \to A)$	S5 for $\Box$
4.	$\Box A \to (A \to \Box(\triangle \top \to A))$	3, A0, R1

5.	$\Box A \rightarrow (A \rightarrow \text{SHALL } A)$	4, A41, A0, R1
6.	$\sim A \rightarrow (A \rightarrow \text{SHALL } A)$	A0
7.	$\Box \sim A \rightarrow (A \rightarrow \text{SHALL } A)$	6, A25, A0, R1
8.	$\Box \sim A \,\&\, \Box(\triangle\top \rightarrow A) \rightarrow \Box \sim \triangle\top$	S5 for $\Box$
9.	$\sim \Box \sim \triangle\top$	$\vdash \Diamond\top$, A36, A23, A0, R1
10.	$\Box \sim A \rightarrow \sim \Box(\triangle\top \rightarrow A)$	8, 9, A0, R1
11.	$\Box \sim A \rightarrow (\Box(\triangle\top \rightarrow A) \rightarrow A)$	10, A0, R1
12.	$\Box \sim A \rightarrow (\text{SHALL } A \rightarrow A)$	11, A41, A0, R1
13.	$\Box A \vee \Box \sim A \rightarrow (\text{SHALL } A \leftrightarrow A)$	2, 5, 7, 12, A0, R1

Here, 13 = T41 = Q.E.D.

Ad T42:

1.	$\boxed{P} A \rightarrow \Box \boxed{P} A$	since A is unmodalized by assumption, $\boxed{P}$ A is non-prohairetic and non-future; hence THM 2 (T13) is applicable
2.	$\sim \boxed{P} A \rightarrow \Box \sim \boxed{P} A$	THM 2 (T13) is applicable for the same reason
3.	$\boxed{P} A \vee \sim \boxed{P} A \rightarrow \Box \boxed{P} A \vee \Box \sim \boxed{P} A$	1, 2, A0, R1
4.	$\Box \boxed{P} A \vee \Box \sim \boxed{P} A$	3, A0, R1

Here, 4 = T42 = Q.E.D., where A is assumed to be unmodalized.
The cases of T43 and T44 are handled in a perfectly analogous fashion.

Ad T45:

1.	$\Box \boxed{P} A \vee \Box \sim \boxed{P} A \rightarrow (\text{SHALL } \boxed{P} A \leftrightarrow \boxed{P} A)$	T41
2.	$\Box \boxed{P} A \vee \Box \sim \boxed{P} A$, where A is unmodalized	T42
3.	$\text{SHALL } \boxed{P} A \leftrightarrow \boxed{P} A$	1, 2, R1

where 3 = T45 = Q.E.D. and A is unmodalized.

The DARB-proofs of T46 and T47 are obtained in a parallel manner. This completes our proof of THEOREM 6.

Addendum. It is reasonably clear that schemata T42–T47 are not strong or general enough, which is in turn due to the fact that their proviso, to the effect that A be unmodalized, is unnecessarily restrictive. Indeed, this appears from our DARB-proof of T42–T44, especially lines 1 and 2. As far as T42–T47 are concerned, then, we draw attention to the following stronger and more general result:

THEOREM 6.1. All instances of these schemata are DARB-provable:

$T42^{+}$.	$\Box A \vee \Box \sim A$	provided that A is non-prohairetic and non-future
$T45^{+}$.	SHALL $A \leftrightarrow A$	

Proof.
Ad $T42^{+}$:

1.	$A \rightarrow \Box A$, where A satisfies the proviso	by THEOREM 2
2.	$\sim A \rightarrow \Box \sim A$, where A satisfies the proviso	by THEOREM 2 (*Case* $A = \sim B$)
3.	$A \vee \sim A \rightarrow \Box A \vee \Box \sim A$	1, 2, A0, R1
4.	$\Box A \vee \Box \sim A$	3, A0, R1

where $4 = T42^{+}$ = Q.E.D. and A is assumed to satisfy the proviso.

Ad $T45^{+}$:

1. SHALL $A \leftrightarrow A$, where A is non-prohairetic and non-future
$T42^{+}$, T41, R1

where $1 = T45^{+}$ = Q.E.D.

Schemata T42–T44 can then be viewed as special cases of $T42^{+}$; similarly, T45–T47 are special cases of $T45^{+}$.

16. *Results in the combined logic of* Shall *and* SHALL

THEOREM 7. All instances of the following schemata are DARB-provable:

T48.	Shall$(A/B) \leftrightarrow (B \rightarrow$ SHALL $A)$, provided that B is non-prohairetic and non-future
T49.	Shall$(A/B) \leftrightarrow$ SHALL$(B \rightarrow A)$, with the same proviso
T50.	$(B \rightarrow$ SHALL $A) \leftrightarrow$ SHALL$(B \rightarrow A)$, with the same proviso
T51.	$\boxed{f}$ Shall$(A/B) \leftrightarrow \boxed{f}$ $(B \rightarrow$ SHALL $A)$, with the same proviso

T52. $\boxed{f}\ \text{Shall}(A/B) \leftrightarrow \boxed{f}\ \text{SHALL}(B \rightarrow A)$, with the same proviso

T53. $\boxed{f}\ (B \rightarrow \text{SHALL}\ A) \leftrightarrow \boxed{f}\ \text{SHALL}(B \rightarrow A)$, with the same proviso

Proof. Throughout the following deductions we assume B to be non-prohairetic and non-future so that the proviso is satisfied.

Ad T48:

1. $(B \rightarrow (\triangle B \leftrightarrow \triangle \top)) \rightarrow ((\triangle B \rightarrow A) \rightarrow$ $\rightarrow (B \rightarrow (\triangle \top \rightarrow A)))$	A0
2. $(B \rightarrow (\triangle B \leftrightarrow \triangle \top))$	THEOREM 3 (T14)
3. $(\triangle B \rightarrow A) \rightarrow (B \rightarrow (\triangle \top \rightarrow A))$	1, 2, R1
4. $\Box(\triangle B \rightarrow A) \rightarrow (\Box B \rightarrow \Box(\triangle \top \rightarrow A))$	3, S5 for $\Box$
5. $B \rightarrow \Box B$	THEOREM 2 (T13)
6. $\Box(\triangle B \rightarrow A) \rightarrow (B \rightarrow \Box(\triangle \top \rightarrow A))$	4, 5, A0, R1
7. $\text{Shall}(A/B) \rightarrow (B \rightarrow \text{SHALL}\ A)$	6, A39, A41, A0, R1
8. $(B \rightarrow (\triangle B \leftrightarrow \triangle \top)) \rightarrow ((B \rightarrow (\triangle \top \rightarrow A)) \rightarrow$ $\rightarrow (B \rightarrow (\triangle B \rightarrow A)))$	A0
9. $(B \rightarrow (\triangle \top \rightarrow A)) \rightarrow (B \rightarrow (\triangle B \rightarrow A))$	2, 8, R1
10. $(B \rightarrow (\triangle \top \rightarrow A)) \rightarrow (\triangle B \rightarrow A)$	9, A35, A0, R1
11. $\Box(B \rightarrow (\triangle \top \rightarrow A)) \rightarrow \Box(\triangle B \rightarrow A)$	10, S5 for $\Box$
12. $\Box \sim B \vee \Box(\triangle \top \rightarrow A) \rightarrow$ $\rightarrow \Box(B \rightarrow (\triangle \top \rightarrow A))$	S5 for $\Box$
13. $\sim B \vee \Box(\triangle \top \rightarrow A) \rightarrow$ $\rightarrow \Box \sim B \vee \Box(\triangle \top \rightarrow A)$	THEOREM 2/*Case* $A = \sim B$/, A0, R1
14. $\sim B \vee \Box(\triangle \top \rightarrow A) \rightarrow \Box(\triangle B \rightarrow A)$	11, 12, 13, A0, R1
15. $(B \rightarrow \text{SHALL}\ A) \rightarrow \text{Shall}(A/B)$	14, A39, A41, A0, R1
16. $\text{Shall}(A/B) \leftrightarrow (B \rightarrow \text{SHALL}\ A)$	7, 15, A0, R1

Here, 16 = T48 = Q.E.D., where B is assumed to be non-prohairetic and non-future.

Ad T49:

1. $B \rightarrow (\triangle \top \leftrightarrow \triangle B)$	THEOREM 3 (T14)
2. $(B \rightarrow (\triangle \top \leftrightarrow \triangle B)) \rightarrow ((B \rightarrow (\triangle B \rightarrow A)) \leftrightarrow$ $\leftrightarrow (B \rightarrow (\triangle \top \rightarrow A)))$	A0

3. $(B \to (\triangle B \to A)) \leftrightarrow (B \to (\triangle \top \to A))$ 1, 2, R1
4. $(\triangle B \to A) \leftrightarrow (B \to (\triangle \top \to A))$ 3, A35, A0, R1
5. $\Box(\triangle B \to A) \leftrightarrow \Box(\triangle \top \to (B \to A))$ 4, S5 for $\Box$
6. Shall $(A/B) \leftrightarrow$ SHALL$(B \to A)$ 5, A39, A41, A0, R1

Here, 6 = T49 = Q.E.D., where B is assumed to be non-prohairetic and non-future.

Ad T50:

1. $(B \to$ SHALL $A) \leftrightarrow$ SHALL$(B \to A)$ T48, T49, A0, R1

where 1 = T50 = Q.E.D. and B satisfies the proviso.

Ad T51:

1. $\boxed{F}$ Shall$(A/B) \leftrightarrow \boxed{F}(B \to$ SHALL $A)$ T48, K_t for $\boxed{F}$
2. $\boxed{f}$ Shall$(A/B) \leftrightarrow \boxed{f}(B \to$ SHALL $A)$ T48, 1, A0, R1, D3

Here, 2 = T51 = Q.E.D. and B satisfies the proviso.

Ad T52 and T53: The DARB-proofs are analogous to the one just given – appeal to T49 in the case of T52 and to T50 in that of T53!

This completes the proof of THEOREM 7.

17. *The semantical soundness of DARB*

THEOREM 8. All DARB-provable wffs are DARB-valid (in other words: for all A in $\mathrm{WFF_L}$, if $\vdash A$ then $\models A$).

Outline of proof. We first observe that axiom schemata A0–A42 are all DARB-valid and that the rules R1–R4 all preserve DARB-validity. We then readily verify by induction on the length of proof that for all A in $\mathrm{WFF_L}$, if $\vdash A$ then $\models A$. Hence the desired result.

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NOTES

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Added in proof: Also, we wish here to draw the reader's attention to a quite recent contribution to modal and deontic tense logic, viz. J. A. van Eck's doctoral dissertation *A System of Temporally Relative Modal and Deontic Predicate Logic and Its Philosophical Applications*, Groningen, 1981. In his fascinating work van Eck presents a (quantificational) system akin to ours; among other things, he proves and applies an analogue of our THEOREM 7, which asserts the equivalence of three familiar forms of conditional obligation on a certain assumption.

PART IV

HISTORY OF DEONTIC LOGIC

SIMO KNUUTTILA

THE EMERGENCE OF DEONTIC LOGIC IN THE FOURTEENTH CENTURY

1. In their introduction to deontic logic Dagfinn Føllesdal and Risto Hilpinen write that most of the contemporary discussion of deontic logic has been stimulated by G. H. von Wright's article 'Deontic Logic', *Mind* **60** (1951), pp. 1–15.[1] They also observe that the history of deontic logic goes farther back. Ernst Mally is presented as the first philosopher who attempted to build a formal theory of normative concepts. His monograph *Grundgesetze des Sollens: Elemente der Logik des Willens* (Leuschner and Lubensky, Graz 1926) is mentioned as the starting point of the logical study of the normative use of language. In this paper I want to push the beginning of deontic logic farther back. I will not discuss its possible predecessors in the seventeenth and eighteenth centuries.[2] Instead, I will show that in the fourteenth century we find a logic of norms in which the relations between deontic notions are treated analogously with the interdependencies of modal notions, some rules of consistency of normative systems are defined, and the deontic consequences are discussed and compared with modal consequences. It is no more news that in the late medieval philosophy modal logic was studied in a way which offers striking similarities with the modern understanding of modality as it is codified, *e.g.*, in the so-called possible worlds semantics.[3] In many medieval tractates on modal logic there are notes concerning the analogies between the concepts of necessity and knowledge, and in some recent works those remarks pertaining to epistemic logic have been discussed.[4] However, as far as I know, the fourteenth century discussion of the similarities and dissimilarities between the logical behaviour of modal and deontic concepts has not been studied. In this paper I will make some notes on this topic.

Some years ago Professor Hubert Hubien called my attention to the question 9 of the first *Quodlibet* of Robert Holcot (Ms. *London*, British Library, Royal 10 C VI, *fol.* 152ra–152rb). In this question Robert Holcot discusses certain arguments which are based on a supposed analogy between modal logic and deontic logic. The late medieval modal logic for the sentences *de dicto* was essentially based on the following rules of inference:

$$(1)\qquad \frac{p \supset q}{\Box p \supset \Box q}$$

R. Hilpinen (ed.), New Studies in Deontic Logic, 225–248.

and

$$(2) \qquad \frac{p \supset q}{\Diamond p \supset \Diamond q}.$$[5]

In the question mentioned above, Robert Holcot discusses the problem whether (1) is to be accepted when we instead of necessity are speaking about obligation (i.e., instead of '$\Box$' we write 'O'). So the following rule of inference is put under discussion

$$(3) \qquad \frac{p \supset q}{\mathrm{O}p \supset \mathrm{O}q}.$$

Correspondingly one could ask about permission, whether the following rule of inference is to be accepted (writing permission 'P' for possibility '$\Diamond$')

$$(4) \qquad \frac{p \supset q}{\mathrm{P}p \supset \mathrm{P}q}.$$

The question of Holcot mentioned above is rather sketchy, to be sure. Fortunately, there is an extensive discussion of (3) and (4) in the question 'Utrum aliquis in casu possit ex precepto obligari ad aliquid quod est contra conscientiam suam', which is partly printed as the first question of the *Determinationes magistri Roberti Holkot* published at Lyon in 1518. This question is not written by Holcot, however, and as V. Doucet has shown, it is in fact the opening question of the *Commentary on the Sentences* by Roger Rosetus. Almost nothing is known of the life of Rosetus. He lectured on the *Sentences* after 1332. One copy of his Commentary was made at Norwich in 1337.[6] The manuscript of the *Commentary on the Sentences* by Rosetus I have used in this paper is Ms. *Oxford*, Oriel College 15, *fol.* 235r–279v. The printed version is very unreliable.

One could ask why the discussion of the logic of norms comparable with the modern deontic logic started only in the fourteenth century – why not earlier? If the connection between modal logic and deontic logic is essential, then one partial answer easily suggests itself. The modern type of modal logic was created in the early fourteenth century. And it seems that for some fourteenth century modal logicians it was natural to ask whether there are other notions which might have logical properties similar to those of basic modal notions. The fourteenth-century writer called Pseudo-Scotus gives the following list of words, which are interesting in this respect: *verum, falsum, per se, scitum, dubium, opinatum, apparens, notum, volitum,* and *dilectum.*[7] We have already seen that there were others who could have added words like *obligatum, licitum,* and *illicitum* to this list.[8] The step from modal notions to analogous concepts can be illustrated by a more recent example, too. G. H. von Wright says that his work on deontic logic got its decisive impetus from observations of some analogies between modal and deontic notions, i.e., between necessity and obligation, possibility and permission, and impossibility and prohibition, respectively.[9] It is obvious, however, that these *prima facie* analogies are realized only if the logician is familiar with a normative system. Even this condition is fulfilled in the fourteenth century. It is a generally known fact that in the late medieval theology there was a strong tendency to interpret ethics as a normative system. This, as such, was nothing new. Of course there had been legal systems earlier and also normative ethics. However, it may be that in the late medieval thought there were reasons for studying the formal nature of normative systems at a level of reflexion deeper than that of any earlier investigation.

In his studies on medieval political thought Walter Ullmann has stressed the fact that the Aristotelian avalanche in the thirteenth century brought the natural man into the scope of interest. In the hierocratic doctrines of early medieval thought an individual man was seen as a member of *Corpus Christi*; he was *renatus* because of the baptism and as such he did not belong any more to the natural humanity. In theocratic-descending theories of government *fidelis christianus* was characterized as *subditus.* He was subordinated to the laws of the Christian order and his virtue was *obedientia.*

In the Aristotelian theory of politics the citizen is not simply a *subditus* who has to obey superior authority. Although ideas different from the model of the descending power were already practiced in the feudal system as well as in communal movements, Ullmann says that it is impossible to exaggerate the significance of the emergence of the Aristotelian concept of citizen while considering the background of the new political theories developed in the latter part of the thirteenth century. The new concept of citizen, fully spelt out by Thomas Aquinas, denied the totalitarian point of view. To be a citizen is one of the roles the man is provided with; it is not identical with the role of being a Christian, and thus the spectre of splitting up man's activities began to be discernible.[10]

I think that the degree of autonomy admitted to man *qua* man in the thirteenth century had the effect that if the Christian ethics was understood in a normative sense, it was possible to consider it as a consciously chosen guidance for one's life. While the early medieval consciousness was *a priori* normatively oriented, it seems that in the late medieval period there were thinkers to whom the acceptance of the Christian system of norms was a kind of *Existenzmöglichkeit*. From this point of view it was only natural to study the logical features of normative systems more intensely and explicitly than was usual in the early medieval times.

2. The concept of informative norm includes the possibility of fulfilling the prescription as well as that of violating it. If a principle concerning the behaviour of some people cannot be violated by those people, it cannot be taken as a norm. Neither is it a norm, if it cannot be fulfilled. In a system of deontic logic it is possible to accept for technical reasons "norms" with logically determined contents, but they are not real norms, because they do not tell what ought to be the case.[11]

The discussion of deontic logic in the late medieval philosophy was connected with a tendency to understand ethics as a system of norms in the sense of informative norms just mentioned. The rise of Aristotelianism in the thirteenth century created an atmosphere in which it was possible to speak about different roles of man and hence of having the Christian morality as one's ethics. However, it is interesting that the Aristotelian ethics is not normative. In fact the consciously normative interpretation of ethics was partially motivated by a specific criticism of certain Aristotelian thinking habits which Thomas Aquinas and many other thinkers had adopted in the thirteenth century. Although the philosophical ideas of the fourteenth century discussion of deontic logic can be understood without any reference to this background, I will explain it to some extent in order to elucidate the emergence of deontic logic as historical event. I start with a short note on the contemporary natural law thinking.

After the Second World War several natural law theories grew up in Continental legal thought, especially in the German Federal Republic. Two questions have played an important role in the discussion: Where does the validity of legal rules come from, and when is a rule a binding one?[12] According to the contemporary natural-law views, the positive legal order is valid in virtue of higher norms, i.e., the natural law. "The nature of things" (*Natur der Sache*) has especially been in the focus of attention in the new discussion. It has been thought that natural law can be derived from the necessary structure of things themselves. *Natur der Sache* tells us, *e.g.*, what is needed in every pledging in order it to be a pledging, or more principally, what is needed in every human life in order it to be an ordinary human life.

It has been claimed that positive law lives on natural law which also is a standard of appraising the justness of positive law. Thus it seems that the nature of things points out how things ought to be and so 'ought' can be derived from 'is'. I don't comment the details of this doctrine; it is enough to state here that *Sollen* derived from *Sein* is normative only if it is already presupposed that things should be as they are presented in the descriptions of their alleged natural states. It is possible to derive all kinds of 'oughts' from the nature of things, if these 'oughts' are understood as necessary constituents of the things in question. As such they are not norms, of course; a description of the necessary conditions of a thing becomes a norm only if one has to see to it that there is such a thing. It seems that if the nature of things in the discussion mentioned is taken as an ontological basis of a normative natural law, then normative 'oughts' are confused with other kinds of 'oughts'.[13]

In the discussion just mentioned Aristotle and Thomas Aquinas are often quoted. However, I think that the attempt to try to find direct analogies between the contemporary natural-law-thinking and the Aristotelian type of ethics is not a very happy one. I will not argue, as Hans Kelsen does, that there is no objective pattern of the good man or society in Aristotle.[14] I think that there is. My point is that the style of Aristotle's and Aquinas' ethics is not normative at all. It is true that there are in Aristotle and Thomas Aquinas 'oughts' embedded in the nature of things, but even when the good man and the good society are discussed, these 'oughts' are usually not presented as norms. So it is historically misleading to read the 'oughts' in the ethics of these writers as norms without giving the matter any further thought. In most cases they are nothing but teleological descriptions. They can be norms, to be sure, but as norms they are addressed to persons who hardly can be made convinced of their objective derivability. I will return to this point after having sketched certain features of Aristotle's practical philosophy.

According to Aristotle, every species in the world has its own way of being and the members of the species strive to actualize their characteristic properties as well as possible. This means that the basic explanation for very different phenomena in the world consists in referring to the natural ends of things. If a stone is in the air, it ought not to be there, because its natural place is down. If a thing does not behave in accordance with its end, the deviation is always caused by an external hindrance. This state of affairs is nicely illustrated by Aristotle in *Post. An.* 94b36–95a3, where he writes: "Necessity is of two kinds. It may work in accordance with a thing's natural tendency, or by constraint and in opposition to it; as, for instance, by necessity a stone is borne both upwards and downwards, but not by the same necessity." (Transl. by Mure.)

Let us consider the nature of the necessity which works in accordance with a thing's natural tendency (*kata fysin kai tēn hormēn*). According to Aristotle, if there are no accidental hindrances, natural things are necessarily either expressing their perfect mode of being or they are moving toward such a state. In some places Aristotle refers to hypothetical necessity as the type of necessity found in Nature.[15] In the examples given of hypothetical necessities the physical structure of things as well as their normal behaviour are understood as necessary conditions for their typical perfect mode of being (*telos*). Hypothetical necessities can thus be characterized as teleological descriptions. Teleological 'oughts' tell what Nature does when not disturbed.

It has been thought that this teleological way of looking at natural things already implies the inference from what is to what ought to be in a sense of 'natural norms'.[16] However, we must keep teleological 'oughts' separate from any normative ideas, especially because Aristotle says that the deviation in nature is always caused by an external cause. So Aristotle does not usually connect the end and its constituents with any normative ideas – they are tools for describing and explaining things in the world.

In nature the teleological 'oughts' spelled out vis-à-vis the natural perfection are always fulfilled, if there are no external hindrances. The human beings do not make any exception, even if in their case the possibility of realizing the perfect state of the species depends on education; men do not fulfil their natural end naturally (*EN* 1103a23–26). But those who have had an education sufficient to make them fulfil the real end of human life, necessarily do so if there is no external hindrance (see *EN* 1113a29–33 together with 1147a26–28). So the teleological 'oughts', by which the perfect mode of being a man is delineated, are no norms for good men. Their conduct is described by them.

According to Aristotle's political science the state is a natural association and as such it has a natural end, a perfect form (*Pol.* 1253a7–18, cf. *EN* 1135a5). In a good state the constitution as well as the laws make it possible for the citizens to live a perfect human life (*Pol.* 1252a1–6, 1280b39, *EN* 1103b3–6). So it is possible to estimate in an objective manner how good or bad the constitution and the laws of a state are (cf. *Pol.* 1288b10–1289a25). In Aristotle's political theory one could try to find norms addressed to the reader in two places: the positive law could be understood as a system of norms and, secondly, the descriptions of the good political actions could be understood as norms. Aristotle does not follow either of these courses. According to him, the good men to whom he is speaking realize the perfect form of human life in any case. (For example, see *EN* 1105b5–9).

When Aristotle introduces the notion of lawful acts in Chapters 1 and 2 of the fifth book of his *Nicomachean Ethics*, he says that what is *lawful* is *just* in the wide sense of the word. It is so because Aristotle here understands the function of the law vis-à-vis its ideal end, which is the human perfection. The human perfection (justice in the wide sense) can be described by teleological 'oughts', and the ideal task of the law is to prescribe this kind of 'oughts'. But such 'oughts' are no norms for the good men, because they only describe their conduct. In fact the life of good men is actually an instance of the end aimed at by the laws (see also *EN* 1144a13–20). (In *EN* 1134a28–34 Aristotle qualifies this ideal characterization of laws be remarking that in certain kinds of states lawful is just in an analogous sense only.) So the laws, if they are laws in the real sense, i.e., if they contribute to the human perfection, cannot be norms for good men. Good men cannot act badly (see, *e.g.*, *EN* 1100b33–1101a8). Good men, of course, also try to keep the state as good as possible, as the end of the state and that of an individual are the same (*EN* 1094b7–11).

When Aristotle uses 'oughts' in his political theory, they usually occur as partial descriptions of the human perfection[17], which in its turn is actualized by the good men in any case. For bad people the laws are norms, because they can violate them. However, for them teleological 'oughts' presented as norms cannot be teleological 'oughts' because they, according to Aristotle, have perverted ideas about the end of human beings (see, e.g., *EN* 1113a15–1113b2). So the derivation of normative 'oughts' from the nature of things would necessarily remain obscure to those people whose conduct must be manipulated by positive laws. (Cf. *EN* 1180a4–5.)

Similar ideas feature in Thomas Aquinas' political thought. While discussing different concepts of law he gives a general definition as follows: "The law is nothing else than an ordinance of reason for the common good, made by him who has care of the community and promulgated" (*S. th.* II-I, *q.* 90, *a.* 4*c*). Ordinance of reason for the common good is something that good men are fulfilling anyway; for bad people it is something they can violate, if the force does not prevent them. So the laws, as far as they are derived from natural ends, are describing the behaviour of good men – for them the "oughts" are not norms, however, because they cannot violate them (see *S. th.* II-I, *q.* 96, *a.* 5, "*voluntas bonorum consonat legi*"). For bad people the laws are norms, but they are not the proper audience to listen how the norms are based on teleological 'oughts' because they do not accept the right end. When the law is interpreted as a system of requirements of the human perfection, it has no normative meaning to those people who are already perfect. To others the good laws can be norms, but there is no teleological philosophy of law for them. In *S. th.* II-I, *q.* 95, *a.* 1*c* Thomas Aquinas writes:

Since some are found to be depraved and prone to vice, and not easily amenable to words, it was necessary for such to be restrained from evil by force and fear, in order that, at least they might desist from evil-doing and leave others in peace, and that they themselves, by being habituated in this way, might be brought to do willingly, what hitherto they did from fear, and thus become virtuous.[18]

Let us now turn to another subject which is more interesting for our present purposes. According to Aristotle, the place of man in the great chain of being can be stated in an objective way, corresponding to the conditions of his perfectibility. Because the specific nature of man is characterized by reason, the perfectibility of man consists in excellence in the exercise of reason (*EN* 1097b22–1098a18). This excellence is divided into two kinds, intellectual virtues and virtues of character (*EN* 1103a1–10), I don't comment here the well-known discrepancy between Aristotle's accounts of the theoretical and practical life; I will speak only of the latter.

According to Aristotle it is typical of good and perfect men that there is neither anything too much nor too little in their desires (*EN* 1106a26–1107a6). This means that the dynamic part of the soul is provided with dispositions of behaving on different areas of life so that the whole of life will be a harmonious and perfect totality of fulfilled functions of man as a rational and social being (see *EN* 1097a25–1097b6 together with 1098a18–19, 1106b21–24, and 1109a28–30). Such dispositions are virtues of character and they grow up in education, in which the irrational impulses of the dynamic part of the soul are habituated to listen to the voice of practical reason (*EN* 1106b36–1107a2), which in every situation says which of the virtues in that situation constitutes the perfect life and how one should actualize that virtue (*EN* 1112a18–1113a14, 1142a31–1142b33). The perfection of man presupposes virtuous dispositions of character as well as virtuous dispositions of reason. Both of them are products of education (*EN* 1103a14–1105b18, 1142a11–23).

A good man knows the real end of men and the dynamic part of his soul is directed toward it, because every single desire is ready to function in accordance with the voice of reason. In a concrete situation the practical reason calculates by using, e.g., the practical syllogism, which is the mode of behaving contributing to the perfect life (*EN* 1112b15–24). When the practical reason has stopped its calculation, the action necessarily begins, if there is no external hindrance (*EN* 1147a26–28, cf. *Met.* 1048a6–20). Here is another place where one *prima facie* could hope to find a norm in Aristotle. Is it possible to understand the ultimate statement of the practical reason as a norm which should be fulfilled? The answer is negative, because the good man necessarily chooses to behave in accordance with the dictation of the practical reason. Between thought and action Aristotle puts in his theory a box in which the thought is changed into action. This box is called *prohairesis* (lat. *electio*, choice). It is not a choice among alternatives; they must already have been eliminated by the practical reason. The Aristotelian *prohairesis* is only a kind of transformer (see *EN* 1111b5–1112a17, 1113a2–14, 1139a31–33). Good men behave in accordance with practical reason as examples of the human perfection; it cannot be thought that good men would not follow the voice of the right reason. Vicious men, on the other hand, can behave rationally with respect to their perverted ideas about the end. Because the end is the object of the dynamic part of the soul and it cannot be changed after education, the good man cannot become bad neither the bad one good (*EN* 1114a21). The only group which, according to Aristotle, seems to behave against the end result of the calculative reason is that of incontinent people. Even in their case the voice of reason is not a norm, i.e., something which can be fulfilled or violated. For Aristotle argues that the right reason is not actual in the mind of an incontinent man at the moment of his behaving in a deviant way (*EN* 1147a10–1147b19). Aristotle's conceptual model is that of teleological 'oughts'; he explains why they are not actualized in an incontinent man, and a conscious violation of the rule of reason seems to be beyond the purview of his way of thinking.

The main lines of the Aristotelian ethics are accepted as such by Thomas Aquinas. The theological virtues and the supranatural end are added into it in order to make it more perfect, not in order to deny it (see, e.g., *De virtutibus in communi*). Like Aristotle Thomas Aquinas says that a virtuous man cannot do anything wrong (see, e.g., *De virtutibus in communi a.* 6, *ad* 1); he is an example of what it is to be a perfect human being and correspondingly the 'oughts' of the practical reason cannot be norms to him. In order to make place for the Christian doctrine of sin Thomas Aquinas is interested in the class of incontinent people. But as in Aristotle it remains problematic whether they really violate the right reason, because it is not actual at the moment of their behaving otherwise due to some irrational impulses (*S. th.* II-II, *q.* 156, *a.* 1).

According to Thomas Aquinas the material cause of the choice is the will and the formal cause of it is the reason. This means that the will, i.e. the general tendency of the dynamic part of the soul directed towards the general end, gets a specific form through the calculative operation of practical reason. It is thus clear *per definitionem* that the will, when it is willing, cannot will anything else than that what is put on it as its form by the reason (see *S. th.* II-I, *q.* 1, *q.* 13, *a.* 1).

Thomas Aquinas tries an easy way out of the apparent rational determinism of this Aristotelian psychology of human action. He says that an act of will, if its object is not the ultimate happiness as such, could always be different. This means, e.g., that in certain types of situations people can will different things, depending on their characters; even the same man can will different things in similar situations under different mental or physical conditions. According to the statistical interpretation of modal notions, accepted by Thomas Aquinas for natural order, necessity is connected with the idea of universality and omnitemporality.[19] If something always happens under certain circumstances, then it necessarily happens in such cases. If we are speaking about a type of volition possible in certain kinds of situations, it certainly is not actual in all of them – even if we are speaking about one and the same person only. So such a generic act of will is not necessary. (See *De malo q.* 6, *a. un.*) This is not, however, what we usually mean by the freedom of will which requires that instead of willing *a* at a given moment, we could have willed *b* at that very moment. This idea of an alternative act of will cannot be found in Thomas Aquinas. The actual act of will gets its form from the practical reason. If the reason leaves alternatives to the will, the latter cannot make any choice among them. It has no reason for choosing this or that.[20]

So we see that in Thomas Aquinas the moral 'oughts' are principally understood as constituents of the perfect human life. As far as ethics is a science, 'oughts' are described as virtual dispositions, not as norms (see *S. th.* II-II). In concrete situations that which ought to be done is a dictate of the practical reason, and the virtuous man cannot violate it. The incontinent man behaves otherwise, but his action is understood in terms of the weakness of will rather than as a voluntary violation of the voice of reason.[21]

3. One of the points of the Aristotelian philosophy attacked by the Franciscan thinkers in the thirteenth century was its alleged determinism. It has been shown that in the second part of the thirteenth century almost every Franciscan thinker repeated the claim that in the Aristotelian philosophy the will is moved by the reason and so it could not will otherwise at the moment it is willing something. In this criticism repeated by Walter of Bruges, John Peckham, Peter Olivi *et al.* it is emphasized that the will is free only if, with respect to any act of will, there is an alternative act of will.[22] In its most developed form we find this criticism in John Duns Scotus, who explicitly denied the possibility of defending the freedom of the will in terms of the statistical interpretation of modality.[23]

According to the new theory, at any moment the will starts to will something there is a real possibility of its willing otherwise. Because the will must have a cognitive object, this criticism implies that at any moment there can be several understandable alternatives of behaviour. In the Aristotelian model the practical reason excludes the alternatives, but here it only recommends one of the alternatives and the will makes a choice between real alternatives. Now we can see how the idea of a normative ethics arises. Even if the 'oughts' of the reason were teleologically connected with a certain idea of human perfection, the will could at any moment direct itself towards other possibilities. One does not make one's choices with respect to a fixed end which is established by the unchangeable totality of one's mental dispositions. It is possible to will something which is not teleologically rational vis-à-vis the end one had in mind. In such cases that end is voluntarily given up, too. When a teleological 'ought' is fulfilled as a norm, the end is at the same time accepted as a normative description of what kind of man one should be. Contrary to the Aristotelian theory, the end now belongs to the objects of the choice. And consequently every particular act of will at the same time implies willing to be a certain kind of man.[24]

When the conduct of man was essentially understood in terms of free choice of ends and means, it became impossible to understand ethics as a description of functions of a character which cannot change its constitution. 'Right' and 'wrong' cannot describe *a priori* the behaviour of people, if there are no unchangeable characters. 'Right' and 'wrong' must then refer to individual acts of will, not to persons. This means that an aretological style of ethics must be changed into a normative one.

If it belongs to the essence of man that he can freely choose what kind of man he wills to be, then it is not possible to derive from his essence what he ought to be. One could think that it is left to man to choose what right and wrong is to be for him. According to Duns Scotus, William Ockham, and other fourteenth century 'voluntarists' this is in a certain sense the case, although they believed that man should obey the divine law in order to save his eternal soul. The new understanding of human condition, not unrelated to individualistic ideas of modern ages, had the effect that the concept of following a rule became the key notion of a consistent way of life. Historically the question about the normgiver is of secondary importance. The change of ethical paradigms happens between the idea of personal identity based on essence and exemplified in action and the idea of personal identity sought after by following a rule. The significance of the latter concept for late medieval thinkers explains their interest in deontic theory.

4. In the fourteenth century deontic theory the following equivalences analogous to those between modal concepts were used:

(5) $\sim O \sim p \equiv Pp$
(6) $\sim P \sim p \equiv Op$
(7) $\sim Op \equiv P \sim p$
(8) $\sim Pp \equiv O \sim p$
(9) $Op \equiv F \sim p$
(10) $Fp \equiv O \sim p.$

O stands here for obligation, lat. *obligatum*, *P* for permission, lat. *licitum*, and *F* for prohibition, lat. *illicitum*. I have not been able to find any explicit table of these equivalences, but they are supposed to be known in the texts of William Ockham, Robert Holcot, and Roger Rosetus mentioned in this paper.[25] It is true, but perhaps only a trivial point, that some of the equivalences just mentioned were used already in the early medieval thought.[26] More interesting is that there were some twelfth century writers who in a certain way anticipated the later habit of treating deontic concepts as a kind of modal concepts. According to one definition of modal notions, often repeated by Peter Abelard, necessity is identified with what nature demands, possibility with what nature allows, and impossibility with what nature forbids.[27]

In the fourteenth century the most discussed problem pertaining to the logic of norms was that presented by William Ockham as the question whether God can command men to hate Him. The question was theoretically interesting because it offered an extreme case for considering the rationality of a system of norms. According to Ockham, God can add into the divine law an obligation to the effect that all obligations must be violated. Such a rule, if it is given at the same level as the others, makes the system of norms irrational, because then no rule can be fulfilled, without violating the others. God can make it impossible for man to act meritoriously by making the divine law irrational.[28] This problem was vividly discussed by the subsequent authors, and the tract on deontic theory by Roger Rosetus I am going to discuss below consists of consistency rules for a satisfiable normative system, together with possible counterarguments and their refutations.

In the opening question of his *Commentary on the Sentences* Roger Rosetus discusses the problem whether somebody can be obligated to something against his conscience. The first article of the question contains a detailed discussion of the terms "*maximum*" and "*minimum*". Tractates *De maximo et minimo* belonged to the fourteenth century trend to create analytical languages for the scientific treatment of phenomena. It was thought that certain conceptual algorithms and the vocabularies associated with them provided tools for the conceptual mastery of problems which arose in different areas of scientific thought. As John Murdoch has shown, these analytical languages were interdisciplinary – for example, there was a tendency to apply *prima facie* physical analytical languages into theological questions.[29] The first article of the question mentioned is an example of this interdisciplinary praxis. It was often copied as a separate tract *De maximo et minimo*. In fact it is an extensive answer to the ethical problem whether there is, as regards the intensity, a maximal performance of the act prescribed which would conform to the prescript or a minimal performance which would not so conform. I don't comment this discussion. Instead I will make some remarks on the second article, the title of which is "Whether whatever can be rationally obligated to somebody, if it is permitted and not against the salvation of the soul" (*op. cit., fol.* 249rb–256rb): Rosetus discusses the question from the point of view of the divine law (*secundum legem statutam Dei*); when we have this system of norms, which are the precepts that can be added into the system so that it still is rational? According to Rosetus, the rationality of a system of norms means that fulfilling the norms does not yield any contradictories. A contradiction in question would be, if someone by fulfilling a norm would violate it or another norm of the system (see, e.g., *fol.* 251va).

Before presenting his rules of rationality Rosetus makes a distinction between the different ways in which something can be permitted. In the first case something is permitted in such a way that it is also permitted to will it according to the divine law. In the second case something is permitted in such a way that it is not permitted to will it according to the divine law. An example of the first mode: it is permitted to give alms *ad honorem Dei* and it is permitted to will it. One of the examples of the second case runs as follows: it is permitted to kill one's father in certain circumstances, but nobody can will it in a permitted way according to the divine law. Correspondingly, it is permitted not to give money to a person who necessarily needs it for his life, when one does not have any, but it is not permitted to will such a state of affairs. We see that in Rosetus the permission may concern a commission or an omission on the one hand and the will to commit or omit on the other hand. In the first type of permission an omission or a commission as well as the will to omit or commit are permitted. The second type of permission is such that an omission or a commission is permitted but it is forbidden to will them (see *fol.* 249rb).

It is not immediately clear what Rosetus means by omissions and commissions which are permitted in the second way, i.e. only *licita de se*. However, his later examples suggest the following interpretation. It is trivially true that involuntary omissions and commissions are *licita de se*. If a will operator is added into the scope of permission, the resulting idiom is often false. Insofar as voluntary commissions and omissions are *licita de se*, they are permitted also in the first way, if the will operator in the scope of permission can be iterated so that the resulting idiom is true. Otherwise they are *licita de se* only. This kind of classification is used by Rosetus when he proves that it is not rational to prescribe things which are *licita de se* only. It would follow that one cannot intend to fulfil his duties. (*fol.* 255ra).

After having drawn the distinction between different modes of permission Rosetus presents, as he says, five conclusions. The first and the second of these rules define general conditions of consistency for a system of norms; three others are more theological ones. The rules run as follows:

(1) Every precept by which I am obligated to something permitted which is in my power and which I am allowed to will according to the divine law without any precept, is a rational precept according to the divine law.

This first rule is qualified by a note: it must be possible to fulfil the obligation so that observing it does not result in a great disaster, e.g. death.

(2) No such precept is rational according to the divine law, by which I am obligated to something permitted which I cannot

will in a permitted way according to the divine law except when I am so obligated.

(3) Not everything which is [not] against the salvation of the soul can be prescribed according to the divine law. ("not" is missing from the manuscript I have used.)

(4) Not everything which is for the salvation of somebody's soul can be prescribed to that person according to the divine law.

(5) Whatever is permitted in the second way can, by the absolute power of God (*de potentia Dei absoluta*), become something I can will in a permitted way (*fol.* 249rb–249va).

The rest of the article discusses several possible objections to these rules. I do not comment on every objection and every answer given by Rosetus. I only mention some general ideas pertaining to deontic logic. The first objection to the first rule runs as follows. If you are at your devotations (in a supererogatory manner) and your prelate orders you to do something which is permitted but not supererogatory, then the precept is irrational, because your supererogatory action is meritorious with respect to your eternal life and the new norm would prevent you from acting in such a highly meritorious way (*fol.* 249va). The basic idea in Rosetus' answer is that the value of supererogatory acts, which are permitted but meritorious if committed, must not be thought to be greater than that of fulfilling obligations. Then the points one gets from works of supererogation never can compensate the dismerit caused by an eventual omission of an obligation because of acting in a supererogatory way. Otherwise one could avoid fulfilling his duties by acts of supererogation (*fol.* 250ra–251ra).

Another objection based on the idea of supererogation easily suggests itself. A supererogatory act may be permitted, but one could ask whether it is possible to command one to act in a supererogatory way. Rosetus does not discuss this question, but Uthred of Boldon, another Englishman from the mid-fourteenth century, spends a lot of ink in considering whether works of supererogation are counterexamples to the claim that if something can be done in a meritorious way, God can obligate one to do it. According to this writer, God can command men to perform supererogatory actions, but it must be understood that obligations of this kind refer to two different states of affairs. There are now acts which are supererogatory, but when they are prescribed, they cannot be any more supererogatory.[30]

Next Rosetus discusses another objection to the first rule. It is based on an acceptance of (4) as a rule of deontic inference. The opponent says that if you sleep, you omit to act in a meritorious way. If the antecedent is a permitted in the first way, the consequent is so permitted, too. The consequent cannot be obligated, however, because such an obligation could be violated in a meritorious way only (*fol.* 249vb).

The example is in many ways tricky, as Rosetus remarks in his answer. It is related to Ockham's question whether God can command men to hate Him. There is a separate discussion of this problem later (*fol.* 255va–256rb) and here Rosetus only mentions certain difficulties, which rise when one tries to defend the rationality of the norm in question. The discussion is somewhat half-hearted, however, because Rosetus thinks that he is not bound to defend the rationality of such a norm. His main point is namely that the alleged norm is introduced incorrectly in the example, because (4) has been accepted there without qualification (*fol.* 251rb). The same point is repeated in the discussion of the third counterexample, which is a variant of the second one (see *fol.* 251va).

Now we have come to the most interesting point of the deontic theory of Roger Rosetus. In the question mentioned above Robert Holcot gives certain reasons for the acceptance of the following rule of deontic logic:

$$(3) \qquad \frac{p \supset q}{\mathrm{O}p \supset \mathrm{O}q.}$$

He writes: "Supposing that *A* is the antecedent and *B* the consequent and Socrates is obligated to *A* and not to *B*, if Socrates is not obligated to *B*, it is permitted to him to omit *B*. If he omits *B*, it follows that he omits *A*." Thus it seems that if (3) is not accepted, then while behaving in a permitted way one *ipso facto* will violate an obligation. Correspondingly one could argue for the form

$$(4) \qquad \frac{p \supset q}{\mathrm{P}p \supset \mathrm{P}q.}$$

The argument of Robert Holcot is of the same type by which the validity of the basic rules of inference of modal logic *de dicto* were traditionally elucidated. Aristotle presents it in *An. Pr.* 34a5–12 and in *Met* 1047b14–30. I quote Buridan's version: "If the antecedent is possible, then it is possible to be as it signifies and if this is assumed actually to be, then it should be as the consequent signifies. It follows that the consequent is possible and not impossible. The same holds of necessity, because it is impossible that the antecedent is actual without the consequent". (*Op. cit.*, *lib.* I, *cap.* 8, *concl.* 5).

Rosetus formulates several examples by which he wants to show that the rules (3) and (4) cannot be accepted in deontic theory. All of the examples present cases in which it is permitted or obligatory to will to behave as the antecedent says and it is forbidden to will to behave as the consequent says. Much use is made of the sentence: "If one repents of his sin, he is guilty of a sin". According to the divine law it is obligatory to will to repent of one's sins, but it is forbidden to will to be guilty of a sin. Examples of this kind show that there are obligations which can be rationally fulfilled only in cases in which some norms have been already violated.

In his *Commentary on the Sentences* William Ockham says: "Whoever wants something efficaciously, wants everything also without which in his opinion the desired object cannot be obtained at all" (*Sent.* I, *d.* 1, *q.* 6). Rosetus makes use of the same concept of the efficacious will (see *fol.* 255rb).[31] Because being guilty of a sin is a necessary condition of repenting one's sins, the latter cannot be efficaciously willed without willing the former except when one is already in a sin. This shows that (3) and (4) can be defended by using Holcot's argument only if there are no conditional norms, by which, e.g., one's conduct after having violated some norms is regulated. It is exactly this class of norms which has caused special difficulties in modern deontic logic. After Roderick Chisholm's paper 'Contrary-to-duty Imperatives and Deontic Logic', *Analysis* **24** (1963), pp. 33–36, they are called contrary-to-duty imperatives. It is a special merit of Roger Rosetus (or perhaps one of his predecessors) to notice that this type of obligation prevents one from accepting the rules of inference of modal logic in deontic logic without qualifications.

According to Rosetus, these are not the only problematic cases, however. It seems that sometimes the antecedent is permitted and it is permitted to will it while the consequent's permitted in such a way that it is not permitted to will it. This means that under certain circumstances it is permitted to behave as the consequent says, although it is not permitted to will to behave in that way. If you are in such a situation, then you can will to behave as the antecedent says, so that willing it efficaciously does not imply that something forbidden is willed, too. Let us consider the example in which you omit acting in a meritorious way, if you sleep. It is forbidden to will the consequent, but if you will the antecedent only when you are tired enough, i.e. you are bound to omit acting in a meritorious way in any case, then it does not follow that by willing the antecedent efficaciously you are *eo ipso* willing the consequent, too. So there is a second class of conditional obligations and permissions, and they regulate the conduct in such situations in which no rule has been violated but which cannot be willed without violating the rules. *Mutatis mutandis* this group of obligations and permissions could serve as Rosetus' answer to the so-called paradox of the Good Samaritan.

Some of Rosetus' examples had already been known for generations. However, it was only in his century when it became usual to be interested in the logical properties of the deontic concepts used in the problematic cases. For example, Thomas Aquinas mentions in several places that a man can be *perplexus ex suppositione*, i.e., he can be in a situation in which it is forbidden to do something as well as to omit it. But instead of considering general principles of deontic thought Aquinas is satisfied with giving practical advice. Most of his examples of being perplexed are of the same type. A man has committed a forbidden act, and he has to do something next. Doing it in a sinful state is forbidden, however. The solution is that the man should repent of his sin.[32] Peter Abelard's *Scito teipsum* is one of the few early works in which attention is paid to the theoretical issues concerning obligations and permissions.

In the beginning of his *Scito teipsum* Abelard discusses the question whether it is possible that the antecedent is permitted or obligatory while the consequent is forbidden. He says that such obligations or permissions are irrational.[33] Later there is an argument, in which Abelard seems to accept (3) as a legal principle. Abelard discusses the following case. A judge knows that an innocent is accused by his enemies. The judge cannot rebut false witnesses by convincing reasons. If the man is punished on the basis of the witnesses' testimony, an innocent man is punished. Abelard says that because the judge is obligated to do as the antecedent says, "thus he ought to punish him who ought not to be punished". "Through the compulsion of the law" the judge must behave against his conscience.[34] When Rosetus treats the same example (*fol.* 254vb), he says that the argument is wrong just because it is based on (3).

Before I discuss Rosetus' definition of conditional obligation in more detail, I shortly mention some other fourteenth century remarks about (3). Gregory of Rimini, who commented on the *Sentences* in Paris in 1342–43, says that (3) does not hold for inferences "as of now". This type of inference was usually characterized in the fourteenth century so that the conjunction $p \& \sim q$ is false now, while $\Diamond(p \& \sim q)$ is true. So, according to Gregory of Rimini, it may be obligatory that p, and although p can now be done only if q is also done, q may be forbidden. This is so because p implies q "as of now" but not *simpliciter*. This is how Gregory of Rimini analyses cases of being perplexed.[35] One easily gets the impression that he accepted (3) for simple inferences, but I have not found him saying so.

Many of the examples discussed by Rosetus are treated by his older countryman Robert Holcot, too. Although Holcot presents the argument for (3) quoted above, it is questionable whether he considers (3) and (4) as first principles of deontic theory. In the short question mentioned above he seems to use (3) as a criterion for judging whether an alleged antecedent can be obligatory *simpliciter*. In Holcot's *Commentary on the Sentences* conditional promises, commitments, and obligations are treated in many places.[36] It remains to be studied whether he had a developed theory of those norms which do not obey (3).

I have already mentioned Uthred of Boldon. He tried to solve problems, which Rosetus treats in terms of conditional obligation, by applying to them rules which were developed for logical analysis of the verbs "begins" and "ceases".[37] This is another example of the interdisciplinary use of analytical languages mentioned in the beginning of this section.

One could think now that the general formula of Rosetus' conditional obligation would be something like

(11) $\Box(p \supset Oq)$,

where p is forbidden or something permitted in such a way that it is not permitted to will it. I don't consider here the historical problem of the nature of the logical consequence Rosetus is using. I use in (11) strict implication, because Rosetus usually says in these contexts that the consequent necessarily follows from the antecedent (see, e.g., *fol.* 251va). Anyhow, (11) is not the candidate Rosetus accepted as the formula of conditional obligation. The reason appears when we look at his discussion of the fourth argument against the first rule. This counterargument runs as follows. Suppose that it is obligatory to will to repent, if one is guilty of a sin. Otherwise it is forbidden. Socrates is not guilty of any sin and in spite of that he wills to repent. One could ask whether he should repent or not. If he should, he is fulfilling his obligation, but then he should not repent. If he should not, he violates a norm and he ought to see that he repents. So if he should not repent, he should repent and *vice versa* (*fol.* 249vb).

Rosetus says that in order to avoid difficulties of this kind certain restrictions are needed in conditional obligations. According to him, the intention of the normgiver is that Socrates ought to will to repent of his sins only when he is guilty of a sin, different from the one that he wills to repent when he has not sinned before his repenting. So Socrates while willing to repent in this way violates the intention of the normgiver, although he *secundum formam verborum* seems to fulfil his obligation (*fol.* 251vb). In order to express the intention of the normgiver one apparently should add to the formula (11) a qualification to the effect that the condition must be fulfilled in such a way that the conditional obligation is not fulfilled *ipso facto*. It seems that Rosetus would have been happier with the following formula

(12) $\Box(p \supset Oq) \,\&\, \sim \Box(p \supset q)$.

This happens to be almost the same as the definition of the new deontic operator Q introduced recently by G. H. von Wright in order to solve difficulties of other attempts to define conditional obligation. According to von Wright, "Q(q/p)" can be read as follows: "Assuming that it is in the agent's power to produce "p", then by producing this he becomes "obligated" to produce also "q", unless "q" is something which is of necessity there as soon as "p" is there."[38]

Rosetus is not satisfied with (12), however. Something is still needed. In order to show this he discusses a variant of a well-known medieval sophism. According to it, it is obligatory for Socrates to cross a bridge if and only if he says something true. Socrates says: "I shall not cross the bridge." If this is true, then the condition is fulfilled but the obligation cannot be fulfilled. Rosetus says that in this case the obligation is rational only if it is restricted in such a way that Socrates' saying the truth does not make it impossible that Socrates will cross the bridge (*fol.* 252ra). When we add this restriction to (12), the final formula of conditional obligation runs as follows:

(13) $\Box(p \supset Oq) \,\&\, \sim \Box(p \supset q) \,\&\, \Diamond(p \,\&\, q)$.

The last example belongs in variant forms to the standard problems of medieval discussion of semantical paradoxes.[39] A treatment of other similar problems is added by Rosetus to the discussion of the first argument against the second rule. In this counterexample the following case is presented. Suppose that Socrates is guilty of a sin *a*. He goes then to the priest and says against his conscience that he is guilty of a sin different from *a*. According to the divine law it is forbidden to lie. It seems, however, that Socrates can *bona conscientia* say so. Thus it seems that something forbidden can be rationally prescribed without referring to the absolute power of God, i.e., without changing the permissions (*fol.* 252ra).

While discussing this and other related cases Rosetus introduces several rules of supposition, by means of which certain types of paradoxes can be avoided. The basic idea is that a part cannot stand for the whole of which it is a part (*pars non potest supponere pro toto cuius est pars*). If Socrates says:

"This sentence is false," referring to the sentence he utters, he lies because "false" in that sentence cannot stand for that sentence. And if Socrates while guilty of one sin only says: "I am guilty of two sins," he lies because "sin" in that sentence cannot refer to that sentence (*fol.* 253r).

The ideas of Rosetus' tract are not exhausted by this paper. Exciting comments are made on rules (2)–(5), too. I think, however, that this suffices to show that the fourteenth century discussion of the principles of deontic logic is worthy of further study.

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NOTES

[1] See R. Hilpinen (ed.), *Deontic Logic: Introductory and Systematic Readings*, D. Reidel Publishing Company, Dordrecht 1971, p. 1.

[2] See A. N. Prior, *Papers in Logic and Ethics* (ed. P. T. Geach and A. J. P. Kenny), Duckworth, London 1976, pp. 9–13, A. N. Prior, *Formal Logic*, Oxford University Press, Oxford 1955, pp. 215–216, H. Schepers, 'Leibniz' Disputationen "De conditionibus": Ansätze zu einer juristischen Aussagenlogik', *Akten des II. Internationalen Leibniz-Kongresses, Hannover* 17–22. *Juli* 1972, Studia Leibnitiana Supplementa XV, Franz Steiner, Wiesbaden 1975, vol. IV, pp. 1 – 17.

[3] See, e.g., my papers 'Medieval Modal Logic', forthcoming in A. Kenny, N. Kretzmann, J. Pinborg (eds.). *The Cambridge History of Later Medieval Philosophy*, and 'Time and Modality in Scholasticism', in S. Knuuttila (ed.), *Reforging the Great Chain of Being* (Synthese Historical Library), D. Reidel Publishing Company, Dordrecht 1981, pp. 163–257.

[4] See *Guillelmi de Ockham Summa logicae*, Ph. Boehner, G. Gàl, S. Brown (eds.), Editiones Instituti Franciscani Universitatis S. Bonaventurae, St. Bonaventure, N.Y., 1974, III-1, *cap.* 30, pp. 435–439, "Pseudo-Scotus", *In librum primum Priorum Analyticorum Aristotelis questiones*, in *Ioannis Duns Scoti Opera omnia I*, Wadding ed., Lyon 1639, q. 36, pp. 328–9, *Iohannis Buridani Tractatus de consequentiis*, H. Hubien (ed.), (Philosophes médiévaux, XVI), Publications Universitaires, Louvain 1976, IV, *cap.* 1, *concl.* 3, p. 114. See also Ernest A. Moody, *Studies in Medieval Philosophy, Science, and Logic. Collected Papers* 1933–1969, University of California Press, Berkeley, Los Angeles, London 1975, pp. 321–370, H. Weidemann, 'Ansätze zu einer Logik des Wissens bei Walter Burleigh', *Archiv für Geschichte der Philosophie* **62** (1980), pp. 32–45.

[5] For example, see Buridan, *op. cit.*, *lib.* II, *cap.* 7, *concl.* 12.

[6] See V. Doucet, 'Le studium franciscain de Norwich en 1337', *Archivum franciscanum historicum* **46** (1953), pp. 89–93. For Rosetus, see also W. J. Courtenay, *Adam Wodeham: An Introduction to his Life and Writings* (Studies in Medieval and Reformation Thought, XXI), E. J. Brill, Leiden 1978, pp. 120–1.

[7] *Op. cit.*, pp. 309–10.

[8] In the mid-fourteenth century Richard Billingham wrote a popular tract on the logical analysis of sentences, which goes under the title *Speculum puerorum* (ed. by A. Maierù in *Studi*

Medievali, Serie Terza, **10** (1969), pp. 338–397). Concepts of this kind are there discussed as *termini officiales*, and in some manuscripts the verbs *debeo* and *teneor* are mentioned as members of the group. What Richard Billingham calls *termini officiales* are called *termini modales* in another treatise of this type, falsely attributed to Billingham in its *explicit*. Deontic concepts are not mentioned, but the list of modal concepts is long: *scire, nescire, ymaginari, intelligere, dubitare, oppinare, cogitare, desiderare, appetere, velle, affirmare, negare, agnoscere, credere, hesitare, significare, apparere, possibile, impossibile,contingens, necessarium, per se*. See L. M. De Rijk, 'Another "Speculum puerorum" attributed to Richard Billingham', *Medioevo* **1** (1975), p. 216.

[9] See G. H. von Wright, 'Deontic Logic and the Theory of Conditions' in Hilpinen (ed.), *op. cit.* p. 159.

[10] W. Ullmann, *Medieval Political Thought*, Penguin Books, Harmondsworth 1975, pp. 164–185, see also W. Ullmann, *The Individual and Society in the Middle Ages*, Johns Hopkins University Press, Baltimore 1966, Methuen & Co., London 1967.

[11] Cf. O. Weinberger, 'The Concept of Non-Satisfaction and Deontic Logic', *Ratio* **14** (1972), pp. 16–35.

[12] For this discussion see A. Kaufmann (ed.), *Die ontologische Begründung des Rechts*, Wissenschaftliche Buchgesellschaft, Darmstadt 1965; for its critical evaluation see also A. Aarnio, *Legal Point of View*, Yleisen oikeustieteen laitoksen julkaisuja **3**, Helsinki 1978, pp. 186–225.

[13] Cf. G. H. von Wright, *The Varieties of Goodness* (International Library of Philosophy and Scientific Method), Routledge & Kegan Paul, London 1963, pp. 160–1. Von Wright calls "oughts" in sentences like "If you want to *p*, you ought to do *q*" *technical norms*. He says that they can be used, for example, to inform or remind a person of the existence of a causal tie between an act and a state of affairs, which may be described as an end of the action. Speaking thus is normally not an act of commanding. It may be that "Sollen" derived from "Sein" in the contemporary natural-law-thinking could be characterized as a kind of technical norm.

[14] See H. Kelsen, *Essays in Legal and Moral Philosophy*, selected and introduced by O. Weinberger (Synthese Library), D. Reidel Publishing Company, Dordrecht 1973, pp. 114–153.

[15] See, e.g., *Phys.* 199b34–200b8, *De part. an.* 639b11–640a8, 642a2–b4; ch. *Met.* 1072b12. For the connection between teleological and modal concepts in Aristotle, see J. Hintikka with U. Remes and S. Knuuttila, *Aristotle on Modality and Determinism* (Acta Philosophica Fennica, Vol. **29**, 1), North-Holland Publishing Company, Amsterdam 1977, pp. 26–28, 34–39.

[16] See, e.g., R. G. Mulgan, *Aristotle's Political Theory*, Clarendon Press, Oxford 1977, p. 19.

[17] There are many ways of expressing "ought" in Greek. Instead of entering into philological details I only refer to the manner in which Aristotle uses the words *dei* and *to deon* in his practical philosophy; e.g., *EN* 1094a 18–24, 1103b22, 1106b16–23, 1115a12–14, 1115b11–13, 1144a13–20, 1170b17–19, 1180a15. Cf. also the distinction in *Soph. el.* 165b34–38.

[18] Quoted from the translation of *Summa theologiae* by the Fathers of the English Dominican Province, published by Burns, Oates, and Washbourne, London 1912–1936. While discussing the question "*Utrum conscientia liget*" (*De veritate*, *q.* 17, *a.* 3) Thomas Aquinas says that the necessity which can be imposed upon the will is *necessitas conditionata, scilicet ex finis suppositione*. These conditional necessities are of the following type: *necessarium sit hoc eligere, si hoc bonum debeat consequi*. Aquinas characterizes the laws given by the rulers in this way. It clearly follows that when the laws are in accordance with the natural ends, they only describe how good men necessarily behave.

[19] See S. Knuuttila, 'The "Statistical" Interpretation of Modality in Averroes and Thomas Aquinas', *Ajatus* **37** (1977), pp. 79–98, and the works mentioned in note 3 above.

[20] What I have said above concerns *libertas specificationis*. A similar statistical theory was presented by Siger of Brabant in more radical terms. See *Quaestiones magistri Sigeri super librum De causis*, A. Marlasca (ed.), (Philosophes médiévaux, XII), Publications Universitaires Louvain 1972, pp. 101–102. It seems that Thomas Aquinas wrote the question 6 of *De malo* in order to explain that a theory of action based on the Aristotelian psychology does not yield determinism. Besides the statistical and suspicious *libertas specificationis* Aquinas discusses *libertas exercitii*, too. I don't believe that this concept is more helpful, because it does not mean that alternative acts of will were possible at the same moment. It means only that sometimes no choice is made because the will has stopped functioning.

[21] See also W. Kluxen, *Philosophische Ethik bei Thomas von Aquin* (Walberger Studien der Albertus-Magnus-Akademie 2), Matthias-Grünewald-Verlag, Mainz 1964, pp. 218–241. According to Kluxen, the philosophical ethics of Thomas Aquinas consists of the theory of virtues, whereas the concept of duty connected with the idea of absolute imperatives is never discussed by Aquinas (p. 227).

[22] For this criticism see E. Stadter, *Psychologie und Metaphysik der menschlichen Freiheit. Die ideengeschichtliche Entwicklung zwischen Bonaventura und Duns Scotus* (Veröffentlichungen des Grabmann-Institutes, Neue Folge 12), Ferdinand Schöningh, München, Paderborn, Wien 1971.

[23] See especially *Tractatus de primo principio* IV, *concl.* 4 (the English translation in John Duns Scotus, *A Treatise on God as First Principle*, transl. by A. B. Wolter (Forum Books), Franciscan Herald Press, Chicago 1966, p. 84), and S. Knuuttila, 'Time and Modality in Scholasticism' (see note 3 above), pp. 219–230.

[24] See Stadter, *op. cit.* pp. 51–54, 67–68, 84–85, 126–132, 176–181, 204–206, 211–212, 229–237, 247–248, 254–255, 313–316, C. Balić, 'Une question inédite de J. Duns Scot sur la volonté', *Recherches de théologie ancienne et médiévale* **3** (1931, pp. 191–208, John Duns Scotus, *Lect.* I, *d.* 39, *q.* 1–5, *n.* 45–61, *Ioannis Duns Scoti Opera omnia* XVII, editio Vaticana, Civitas Vaticana 1966. William Ockham, *Quodl.* I, *q.* 16, Guillèlmus de Occam, *Quotlibeta septem*, Strasbourg 1491, republished by Editions de la Bibliothèque S. J., Louvain 1962.

[25] It seems to follow that if an act is neither obligatory nor forbidden, it is morally indifferent. See Ockham, *In sententiarum* III. *Opera plurima* IV, Lyon 1495 (republished by the Gregg Press Limited, London 1962), *q.* 12, YY. The medieval theory of supererogatory actions brought it about, however, that some qualifications were often added. So Duns Scotus writes in his *Quodl. q.* 18, *a.* 2 that a neutral or indifferent act may be imputed to the agent in whose power it lies as somewhat blameworthy, for he could have acted in a praiseworthy fashion. He also uses a weaker expression to the effect that "at least it is credited to him as unpraiseworthy". The English translation from John Duns Scotus, *God and Creatures. The Quodlibetal Questions*, transl. by F. Alluntis and A. B. Wolter, Princeton University Press, Princeton and London 1975, pp. 369–387.

[26] See, e.g., *Peter Abelard's Ethics*, ed. with introduction and notes by D. E. Luscombe (Oxford Medieval Texts), Clarendon Press, Oxford 1971, pp. 18, 16–23, 22, 14–25 *et passim*. For some notes on the early history of the word *obligatio*, see H.-P. Schramm, 'Zur Geschichte des Wortes "Obligatio" von der Antike bis Thomas Aquinas', *Archiv für Begriffsgeschichte* **9** (1967), pp. 119–147.

[27] See Peter Abelard, *Dialectica*, L. M. de Rijk ed., (Wijsgerige teksten en studies 1), Van Gorcum, Assen 1956, pp. 193, 34, 296, 37–39, 204, 11–12, 205, 30, 385, 1–8, and my paper 'Time and Modality in Scholasticism', pp. 182, 241.

[28] See Ockham, *Quodl.* III, *q.* 13. Ockham's ideas are discussed in I. Boh, 'An Examination of Ockham's Aretetic Logic', *Archiv für Geschichte der Philosophie* **45**, (1963), pp. 259–268. For a critical review of Boh's article see M. Tweedale, 'I. Boh, An Examination of Ockham's Aretetic Logic', *The Journal of Symbolic Logic* **34** (1969), p. 499.

[29] See J. E. Murdoch, 'From Social into Intellectual Factors: an Aspect of the Unitary Character of Late Medieval Learning' in J. E. Murdoch and E. D. Sylla (eds.), *The Cultural Context of Medieval Learning*, D. Reidel Publishing Company, Dordrecht 1975, pp. 271–348.

[30] Uthred of Boldon, *Quaestiones seu Determinationes*, *q.* 5, *a.* 1–2, Ms. *Fribourg*, *Convent des Cordeliers* 26, *fol.* 112vb–117va. For Uthred of Boldon, see D. Trapp, 'Augustinian Theology of the Fourteenth Century', *Augustiniana* **6** (1956), pp. 201–213, and Courtenay, *op. cit.*, pp. 93–4. I am indebted to Miss A. I. Lehtinen for calling my attention to this Ms. A similar case is mentioned by J. Feinberg in his book *Doing & Deserving. Essays in the Theory of Responsibility*, Princeton University Press, Princeton 1970, p. 5.

[31] Some authors remarked that when *velle* is used without qualification, it is not permitted to infer that if someone wants the antecedent, he wants the consequent. See, e.g., Walter Burley, *De puritate artis logicae tractatus longior. With a Revised Edition of the Tractatus Brevior*, Ph. Boehner (ed.), The Franciscan Institute, St. Bonaventure, N.Y., 1955, p. 206, 34–207, 2:... haec enim consequentia est bona: Ego sum in luto cum 100, ergo ego sum in luto. Et tamen non sequitur: Vellem esse in luto cum 100 libris, igitur vellem esse in luto; non enim oportet quod volens antecedens velit consequens. See also p. 87, 25–31, and Pseudo-Scotus, *op. cit.*, p. 310.

[32] See, e.g., *S. th.* II-1, *q.* 19, *a.* 6, *ad* 3, III, *q.* 64, *a.* 6, *ad* 3, *q.* 82, *a.* 10, *ad* 2, *De veritate*, *q.* 17, *a.* 4, *ad* 8, *Sent.* II, *d.* 39, *a.* 3, *ad* 5; IV, *d.* 24, *q.* 1, *a.* 3, *q.* 5, *ad* 1.

[33] See Luscombe's edition of Abelard's *Ethics*, p. 20, 1–11. Abelard entitled his book of ethics *Scito teipsum* (*Know Thyself*).

[34] *Ibid.*, p. 38, 22–40, 5, 40, 15–6.

[35] Gregory of Rimini, *Super primum et secundum sententiarum*, Venice 1522, reprint: The Franciscan Institute, St. Bonaventure, N. Y., 1955, II, *f.* 126 J.

[36] For example, see the discussions in the question "*Utrum voluntas creata in utendo et fruendo sit libera libertate contradictionis*" which occurs as the third question of the first book in the early printed versions of Holcot's *Commentary on the Sentences*. (Ms. *London*, British Library, Royal 10 C VI, *fol.* 20ra ff.).

[37] See note 30 above.

[38] G. H. von Wright, *op. cit.*, p. 169.

[39] For late medieval treatises on semantical paradoxes, see P. V. Spade, *The Mediaeval Liar: A Catalogue of the Insolubilia-Literature*, Pontifical Institute of Mediaeval Studies, Toronto 1975, E. J. Ashworth, 'Will Socrates Cross the Bridge? A Problem in Medieval Logic', *Franciscan Studies* **36** (1976), pp. 75–84, A. Maierù, 'Il problema della verità nelle opere di Guglielmo Heytesbury', *Studi medievali*, Serie Terza **7** (1966), pp. 40–74, A. N. Prior, *Papers in Logic and Ethics* (see note 2 above), pp. 130–146, 202–214, M. L. Roure, 'La problématique des propositions insolubles au XIII[e] siècle et au début du XIV[e], suivie de l'édition des traités de William Shyreswood, Walter Burleigh et Thomas Bradwardine', *Archives d'histoire doctrinale et littéraire du moyen âge* **37** (1970), pp. 205–326, John Buridan, *Sophisms on Meaning and Truth*, translated with an introduction by T. K. Scott, Appleton Century Crofts, New York 1966.

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