COMMENTS AND CRITICISM

ESSENTIAL PROPERTIES AND REDUCTION

IT has recently been argued by Gilbert Harman that "the claim that numbers have . . . essential properties is incompatible with the familiar idea that number theory can be reduced to set theory in various ways." * I think his argument is wrong and I try to show why in what follows.

Harman makes his case with reference to the familiar distinction between modality *de dicto* and modality *de re*. An example of a statement that would be held by many philosophers to be necessary *de re* but not *de dicto* is that the number of planets is composite (i.e., not prime). Harman reminds us of the infinity of different ways of mapping the natural numbers into sets in order to obtain different reductions, and then argues as follows:

> A particular set will be a composite number given certain reductions but not others. Apart from one or another of these reductions, we cannot say that a particular set is or is not a composite number. If *de re* necessity is in question, no set is necessarily a composite number. Being a composite number is not an essential property of any set. Therefore, if numbers can be identified with sets and *de re* necessity is in question, no number is necessarily a composite number. Being a composite number is not an essential property of any number (184).

An instructive parody of this reasoning is easily obtained: simply delete each occurrence of the word 'composite'. The resulting conclusion is too bizarre to sustain respect for the argument.

But where does the argument fail? It fails by not providing for properties of objects a treatment parallel to that given the objects themselves. It is recognized by Harman that to effect a reduction we must, with some arbitrariness, select a specific domain of sets to identify with the natural numbers. Doing this amounts to introducing certain defined symbols into our set theory: to certain sets we give new names—the names of numbers. To complete the reduction, however, we must be able to do the same thing for the properties of numbers. Each sentence of number theory, whether atomic or not, must be ultimately translatable into a sentence of set theory containing no defined symbols and provability must be preserved by the translation. If we cannot do this, the reduction will fall short of Harman's requirement that it "allow the full develop-

* "A Nonessential Property," this JOURNAL, LXVII, 6 (March 26, 1970): 183–185, p. 184. Subsequent references are to page numbers of this article.

ment of the theory of numbers" (184). Now as a matter of fact the way in which 'composite' comes to be defined will depend on our definitions of the numbers themselves, and hence we have at the outset a similarly broad range of possibilities. (Of course the point holds the other way too: if we insist at the outset that 'composite' is to be defined in a certain way and if it is possible to define it in this way and still produce a reduction, then we will be unable to choose certain ways of defining the numbers.) The very same remark holds true for any other predicate of numbers and for the various function symbols as well (since these can be regarded as predicates of certain n-tuples). Therefore Harman's argument would seem to have the consequence that *all* properties of natural numbers are nonessential. This should not, of course, be seen as a *reductio ad absurdum;* indeed some may regard it as a further charm of the argument.

Returning to the key point, we see that a given property of numbers must ultimately be definable in terms of properties of sets. Thus let "τ_1" and "τ_2" be two reductions of number theory to set theory, and let '$\tau_1(9)$' and '$\tau_2(9)$' denote the sets identified with 9 under the two reductions. Assume now that these sets are different. The number-theoretic sentence "9 is composite" will be expressible under the two reductions as a purely set-theoretic sentence about $\tau_1(9)$ on the one hand and $\tau_2(9)$ on the other. These sentences may be assumed to differ and may be written as '$P_1(\tau_1(9))$' and '$P_2(\tau_2(9))$', respectively. Thus each sentence will attribute a certain property to a certain set. It would be very interesting to see an argument to the effect that $\tau_1(9)$ did not have the property of necessarily being P_1, or that $\tau_2(9)$ did not have the property of necessarily being P_2. If either of these or a similar claim could be supported, then there would indeed be cause to wonder whether 9 is in fact necessarily composite. Without this type of argument, however, no such sensational conclusion would appear to be justified. (It should be emphasized that such a conclusion does *not* follow either from the fact that 9 is not necessarily $\tau_1(9)$ or from the fact that 9 is not necessarily a set that has property P_1.)

It seems to me that Harman has attempted to put forward this sort of argument, but has not succeeded. He argues that no set is necessarily composite. So, in particular, $\tau_1(9)$ is not necessarily composite. But, as I have tried to point out, these sentences make sense only in connection with a definition of 'composite' in purely set-theoretic terms. If 'having P_1' is chosen as the definition, it emerges that $\tau_1(9)$ is composite. If instead we choose 'having P_2', it may follow that $\tau_1(9)$ is *not* composite.[1] But none of this shows the kind of thing that is required, for example that $\tau_1(9)$ does not have P_1 necessarily, and hence the argument does not work.

All the properties of sets are "there" in advance of any reductions, and a given property either holds necessarily of a given set or else it does not. The fact that a given property of *numbers* must be treated differently in differing reductions of number theory to set theory parallels the different ways of treating the numbers themselves and is completely beside the point.

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NOTES AND NEWS

The following is a report prepared by Rudolf Carnap a few weeks before he died, for an APA committee on imprisoned philosophers in Mexico. The editors of the JOURNAL are honored to have the opportunity to publish it.

A. Some Preliminary Remarks.

When I was in Mexico in August and September 1963, I (along with Feigl, who came there for the International Congress of Philosophy) became acquainted with the first two of the following three philosophers, and soon we became good friends:

1. Rafael *Ruiz* Harrell, Professor at the Law School at the University of Mexico, who also teaches in the Department of Philosophy. He is especially interested in the philosophical foundations of the sciences and of jurisprudence.
2. Nicolás *Molina* Flores, Professor at the Preparatoria (between high school and university). He planned an anthology of articles by logical empiricists. He also corresponded about this plan with Feigl and Hempel.
3. Eli *de Gortari,* Professor of philosophy at the University of Mexico. In the fall of 1969 I and the other signers of the letter in the *New York Times* received a letter from him (I did not know him personally). He listed eight books published by him, chiefly on the philosophical foundations of science and on dialectical logic. He wrote this letter from the jail (the Preventive Jail of Lecumberri).

Molina and de Gortari are in their fifties. Both of them were arrested in September 1968 by policemen who entered their houses by force, without a formal warrant for arrest. When I was in Mexico City in January, 1970, I was in close contact with Ruiz, and he gave me information about the imprisoned philosophers. I proposed to give him a sum of money for

1 Such a choice, if it were made in conjunction with the rest of reduction τ_1, would in some cases prevent the result from itself being a reduction.