LOGIC COUNTS

REASON AND ARGUMENT

VOLUME 3

The titles published in this series are listed at the end of this volume.

Logic Counts

by

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Preface

The time when logic appeared indistinguishable from psychology is over. The time of omnipresent pressure of the Positivistic paradigm for the unification of science and against philosophical "obscurity" is also over. Logic has grown up; it now has an apparatus beyond Aristotle's or even Frege's or Russell's expectations. Philosophy has survived and nowadays seems to be on the offensive. It has returned to deal with the great problems in ontology, epistemology and axiology. Humanities, although opposed to the sciences, still need similar tools, even formal tools. Logic is such a tool. Logic is a tool in different fields in all research.

Perhaps we are still in trouble when defining what exactly logic is. Is it rooted as something ontically real in the real world or rather in the human mind? Does there exist one "true" logic approximated by different systems, or does logic lack this kind of identity? Nevertheless we can quite efficiently make use of logic while leaving those problems open. We can assume instead that logic is what logicians do now and/or what they did in the past. We can even believe that in tracing how logic is used we can come nearer to understanding what its very nature is.

Of course there are different ways of applying logic, just as there are many types of logical theories and many methods of constructing logic. Mathematics or physics have their special needs and it would be unreasonable to suppose that the use of the tools of logic in the humanities should be determined by the needs of those scientific disciplines. Humanities, philosophy included, have their own expectations with respect to logic. The art of using logic depends on our ability to make decisions as to what kind of formalism, if any, should be applied to what kind of problem? The answer to such questions should be determined not only by the type of the problems under consideration, but also, and importantly, by the user's specific insight. This is so especially in the domain of philosophy.

Logic can be employed in philosophy in very subtle ways, underlying analyses and directing argumentation, e.g. *via* different forms of suppositional proofs. Thus one suitable use of logic would amount to submitting arguments to standard logical norms. On the other hand, using CONCEPTUAL logic would also make sense, for instance in an attempt to start philosophising as if assuming NOTHING: logic is useful for making the basic concepts clear. Logic counts truth-values... All well and good, but truth-values are primarily attributed to judgements (or statements) in which concepts usually needing clarification appear. Sometimes quite simple, well known logical tricks can achieve such clarification, sometimes more

sophisticated logical devices must be used, involving mathematical logic, set-theory or algebra. It happens that some philosophers seem to share Humpty Dumpty's attitude towards words. They believe that *when THEY use a word, it means just what THEY choose it to mean - neither more nor less.* It is then a logician's task to take Alice's part and ask whether they *can* do such things with words.

Let us take as an example the case of modal operators, so essential in philosophical analysis. Reviewing different axiomatic systems of modal logic, a philosopher must notice that the same word, referring to the same concept, sometimes means rather *MORE*, sometimes rather *LESS.* The ultimate decision as to which system is to be chosen as a tool for making the modal concepts clear is the philosopher's. Logic just provides tools, helpful in testing the philosopher's choice. Strictly speaking, logic counts truth-values - not meanings: but counting truth-values, logic participates in philosophizing that deals with meanings.

"Calculemus" ? Well, but what about natural language à propos this universal programme? Either one can do without natural language or one must tame this wild, natural entity. Mathematics has chosen the first solution: a great deal of mathematical work can be done within the framework of formalized languages, manipulated as if not motivated by natural language. For the humanities, however. for philosophy, no possibility of abandoning natural language exists. Thus also logical tools must be oriented towards natural language.

Austin said that the first word, although not the last word, belongs to the natural language (as opposed to formalized language).

Is that the case?

In the history of logic, as is well known, codification of a logical system in the terminology of natural language is prior to any formalized system of symbols. In the order of ontogenetic development it could have been otherwise. Perhaps an indexical "formal" sign, obtaining its meaning by convention, could be discovered first and could lead to the realisation that human voice can also be used arbitrarily and intensionally. On the other hand, how can one predict what the LAST word would be? Would it necessarily belong to a formalized systems of signs? Think about computers making progress in the learning of natural language... Thus HERE Austin's point is neither as convincing nor as important as might appear. What is important is the fact that sometimes (e.g. in mathematics) natural language plays a complementary role with respect to formalized language and sometimes (e.g. in philosophy) the latter is auxiliary - even if essential. Human knowledge develops better for the use of formalized languages. Human knowledge is unable to develop except by relying on natural language. Some view the pastures of natural language as *greener.* For others - *greenness* is

associated only with those pastures where formalisms are carefully cultivated. Each side identifies an aspect of truth..

"Calculemus" ? All right. Only remember, that in different formal systems 1+1 sometimes makes 1, sometimes makes 2. Quite similar can be the effects of encoding information in a natural language. As Themerson's interpreter explains,

one apple plus one apple makes two apples;
one drop of water plus one drop of water makes one drop;
one drop of water plus one drop of oil makes two drops,
while one atom bomb plus one atom bomb makes nobody knows what.

Natural language profits when using logical symbolism, but its horizons are always broader. It says, e.g. that the nature of apples, of drops of water, of oil, or of atom bombs defines, *via* context, the meaning of the highly ambiguous operation of addition. So perhaps natural language does have the LAST WORD after all.

* * *

The contents of this volume - LOGIC COUNTS - follows the above ideas.

The two initial papers of Part I bring some general conceptions regarding the scope of possible applications of logic in philosophy. Part II opens with an essay stressing the cognitive values of studying the history of logic. Part III starts with a paper which recalls the first systems of formal pragmatics, leading the reader into topics surrounding natural language.

Each part also contains a more detailed discussion of the questions chosen to illustrate the different roles of the tools of logic.

Thus in Part I the reader will find examples of research in philosophy in the strict sense, supplemented by the study of the formal (algebraic) motivation for several systems of modal logic, as well as examples of the research in axiology.

Analyses of some basic pragmatic concepts and an attempt to bring logic nearer to natural quantification are included in Part III. In this part one finds also investigations into GT-semantics and analyses important for preparing computer implementations of special programmes for reading natural language documents.

Detailed historical studies of Part II, describing the early interrelations of logic and philosophy, link Part I with Part III. The historical import of fundamental logical achievements for neighbouring disciplines is exemplified by Tarski's theory of truth as adopted by a certain logic of norms. On the other hand it is argued that logic, despite its instrumental role, from the very beginning has been used to formulate and solve problems for its own sake as well.

The historical perspective, pointing out the *rationale* of human thinking, is also present in nearly all the papers constituting the volume. In fact, all the authors of the papers collected in this volume have been friendly oriented towards the history of logic, often are they participants of the yearly Cracovian Conferences devoted primarily to the history of logic. The papers witten by E. Anscombe, J. Fall, J.M. Font & V. Verdú, K.-H. Krampitz, A. Łachwa, A. Madarász, E. Metzler, L. Regner, M. Tokarz, J. Waszkiewicz & A. Wojciechowska, T. Weber, and B. Wolniewicz were presented (in this or somewhat different form) during 33rd Conference on the History of Logic, Kraków, 27-29 October 1987. Professor Bocheński's essay is his address to the participants of the 34th Cracovian Conference, 18-20 October 1988.

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Throughout the volume some special themes or aspects recurr. The book aims to give a fresh interpretation of the problems considered and e.g. the dialogical form of argumentation is shown as:

(i) working when philosophers argue questions confronting 'pros' and 'cons',

(ii) present in all human thinking from Greek Antiquity, through European Middle Ages, even influencing the academic model of education,

(iii) underlying new approach towards semantics, namely the Game-Theoretical approach.

In opposition to traditional treatment, priority is given to the concept of *text*, not to that of *sentence*. On the other hand, concepts otherwise definable by formal means, such as the concept of identity, are seen both as existing in historical situations and still vital in contemporary philosophy.

It is also the purpose of this publication to provide the reader with different conventions that can be applied to the topics in question. So the volume contains new philosophico-logical analyses or formal results, but also source studies; looking for new horizons in research is supplemented by a maniére of data referring and recollecting the facts known, just arranging them in a suitable order.

The way in which logic counts is the total sum of all the cases of its manifestation.

Ewa Żarnecka-Biały

Part I

TOWARDS PHILOSOPHY

Jan Srzednicki

LOGICAL CONCERNS OF PHILOSOPHICAL ANALYSIS

For a time it was fashionable to say that analytical philosophers had little time for logic, and possibly this is still a common impression. Typically it was formal logic that was supposed to be so unpopular with these thinkers. There is some justice in this; at least historically the preoccupation of an analytic philosopher seemed to be away from symbolic logic. Even if one respects symbolism, one finds that other pastures are greener, other problems more absorbing. While this type of logic is obviously vast, it still can strike one as too narrow for a lifetime interest and preoccupation; as insufficient for the main interest. But is this tantamount to a negative attitude towards logic as such?

It is the purpose of the present paper to explore this area and to discover what logical questions, if any, exercise the analyst.

I would like to suggest that philosophical analysis is especially sensitive to a particular type of logical problem. This concern relates to basic epistemological questions, sometimes with an admixture of ontology, sometimes touching on other types of problem. I would like to suggest that while the analyst's interest is in a way alternative to the more traditional preoccupations with logic it is not opposed to them, rather it complements them by providing an analysis of the difficult area within which the ultimate justification of the enterprise of logic must lie.

I would like to cite J. L. Austin from "A Plea For Excuses"[1]:

> Certainly then, ordinary language is not the last word; in principle it can everywhere be supplemented, and improved upon, and superseded, only remember it is the first word.

To all intents and purposes Austin could have been replying to a remark I heard Bertrand Russell make in Melbourne. As I remember he protested: - "Why should the ordinary language be the test and criterion of what is acceptable in philosophy or theory. After all it is the creation of uneducated, and untrained minds; minds prone to unreflective thought; minds devoid of theory and critical habit, for whom the sun actually rises above the horizon. Why should we submit our more subtle, more sophisticated, better

thought out result to such a final judgement?". So much for the rather irritated Russell. Yet Austin seems to take the point seriously, and shows the greatest possible respect for the "inherited wisdom and experience of many human generations". Our question is Why should this mean that the ordinary language, rather than e.g. popular views, should have such a central role in philosophical enterprise?

Let us now turn to G. E. Moore in "The Defence of Common Sense"[2]. His conception is roughly as follows: We cannot take seriously the view that basic ordinary judgements, or, as Austin would have it, expressions, are mistaken in principle, especially where we consider the paradigms of the use of language. Moore writes:

> ...Such an expression as 'The Earth has existed for many years past' is the very type of an unambigous expression the meaning of which we all understand. ... The question what is the correct analysis of the proposition meant on any occasion ... is, it seems to me a profoundly difficult question, and one, as I shall presently urge, no one knows the answer. But to hold that we do not know what, in certain respects, is the analysis, of what we understand by such an expression, is an entirely different thing from holding that we do not understand the expression ... So that in explaining that I was using the expressions used in ... in their ordinary sense ... I have done all that is required to make my meaning clear ...

Let us observe that Moore stresses the fact that an expression such as "The Earth has existed for many years past" is an ordinary expression that has perfectly ordinary readily intelligible sense, and also one of which we know that it is true in this very sense. Paradigmatically then this is the central fact vis a vis our conceptual system.

Where the difficulty of analysing such an ordinary expression is concerned, Moore takes as example an expression such as: "This is my hand" and tries to insist that about this we know only two things, and to wit: That we always have to do with a sense datum relevant to the expression in question, and further that when I use such an expression I do not state that my hand is in this sense datum.

According to Moore the difficulty in analysing such a situation consists in that we should *know* whether we should construe the sentence "This is my hand" in such a way that we know whether we have immediate contact with the hand or not. He conceives three possibilities: 1) we directly perceive the surface of our hand; 2) the sense data we are aware of stand in some kind of relation to whatever is the surface of my hand, or 3) in Mill'ean spirit that the material object is, constituted by permanent possibility of sensations. Notably Moore is in no doubt where our contact

with alleged sense data is concerned. One would wish to *protest* that he has disregarded the most obvious possibility that sense data are *our way to perceive objects.* This calls into question the very status of sense-data. One suspects that this was not a perspective available to Moore. There is a related limitation worthy of mention - in this tradition sense-data tended to be seen as primitives of cognition. Yet should we see these as our means of perceiving material objects it is not at all clear that they are relevant to other types of cognition, a question that clamours for an answer.

The main object in presenting these views was to illustrate a type of philosophical concern that is obviously analytic, yet one that shows a preoccupation with the logic of the situation, and where such an attitude is central.

It was often said that this approach is mainly concerned with clarity of expression. H. D. Lewis named an article and a book after his objection here, - "Clarity Is Not Enough"[3], but taking the lead from Austin we could easily reply - yet it is the prime desideratum. If we start without clarity our results are likely to be at the very least suspect, as suspect as our clarity, that is. Rightly then does the analytic philosopher stress this point. The issue is methodologically of central importance. Our capacity to reason, argue, and construct proofs, rests on it. It could be objected that this is not really a point about logic any more than the point that legibility is essential, for written presentation to be effective, is a point about logic.

This reply is not apt. Clarity is not merely presentation, it is also a matter of the kind of grasp that we have of our subject matter. To learn to think clearly is to *learn to think*, not merely to learn to present one's thoughts. This point was very central to the analytic methodology in the classical period of that school, but it needs to be noted that it is a kind of logician's concern. A central concern of logic is to process the material of argument with sufficient precision to obtain theorems. Many aspects of the natural subject matter are routinely disregarded for this purpose, only what can be processed with absolute certainty is retained. Now clarity is a *conditio sine qua non* of such precision. In symbolic logic especially, the precision and clarity is often obtained by making manageably clear-meaning assumptions. If these are fully explained and circumscribed no grey area of sense remains to plague the logician with unexpected problems. Or, given antinomies and the like, so we fervently hope.

This assumption method carries a large price tag. Every assumption lessens our grip on the subject matter in that it is not derived from it, but tends to be imposed on it. I am not claiming that the imposition is likely to be capricious; the contrary is generally sought, but to assume is to forego our capacity to test vs. the subject matter. Therein consists the efficacy of

the tool. For that precise reason the subject matter cannot spring surprises on the logician. - Antinomies, and paradoxes are but products of internal difficulties of the system. If all that we wish to do is to construct an efficient uninterpreted calculus, all is fine. But if then we wish to use this calculus as an epistemic tool, e.g. to further our knowledge of some subject difficulties emerge. This is not the place for rehearsing such problems; they are familiar enough anyway, but it is obvious that logic as a tool of knowledge needs to make contact with its subject-matter, and further that that contact should present the logician with propositions, judgements or whatever that can be assessed with some precision and assurance. We cannot handle adequately material which appears as something that is more or less such and such, possibly being also related in more or less such and such ways to something else that is more or less of this or that nature. Even vagueness, if manageable, must give us more purchase than that. But isn't Moore trying to provide us exactly with expressions that are free from this type of problem? Perhaps the ordinary language should not be regarded as the ultimate arbiter in philosophy, but perhaps philosophical analysis has a good claim to assuming this mantle.

Analytic philosophers, and sometimes Wittgenstein suggest that to clarify a question is often tantamount to answering it. This perhaps represents the metaphysical, or purely philosophical side of the coin. For it assumes that our problem *consists* in making a sure and effective contact with our subject matter. But then all sides of this coin are of importance, and this contact needs to permit us to handle what we are aware of *via our contact*. This means that the material thus obtained must be tractable, and if we wish to check the correctness of our reasoning, tractable by logic. Clarity here will mean more than just the fact that we know we have made contact; it must mean that we also know something of the nature of this contact, and Moore clearly tries to provide just this. For him we know the efficacy of our judgement where its sense, and where its logical value are concerned. There is no gainsaying that in this way logic appears to gain a purchase.

We have then the following picture: - The analyst starts with a fully justified concern for the clarity of basic, and paradigmatic expressions, especially ones that introduce important philosophical concerns. This involves logic in its broad sense. We are not perhaps concerned with the structure of proof, the nature of theorems, or other "technical" problems, but we are concerned with elements essential to the enterprise of logic. Yet this type of preoccupation leads the Analytic man in the direction of shaded areas where the distinctions between logic, ontology and epistemology tend to become problematic. There to satisfy our logical curiosity we may be forced to deal with epistemological, and even ontological questions. It will be clear

that in the cited paragraphs Moore is tending in that very direction. The enterprise is interesting, and of some importance. Even if we reject Moore's eventual results that in itself need not mean that the type of problem considered is not to be solved by the introduction of the specified type of clarity and certitude.

Let us consider an example: - the problem of free will. Accepting determinism, we accept that any event is caused i.e. happens as the result of actual conditions (events etc.) antecedent to it. Assuming, as we also tend to do, that persons are responsible for their actions, we must show that they have genuine choice of actions, i.e. unless we wish to deny the Kantian dictum that must implies can. We can safely assume that most of us wish to retain it together with the other two assumptions, that creates a *quid pro quo* - since any action by any person is an event it is determined by its antecedent conditions, and we have contradicted our second assumption. Libertarians say at this point that acts for which we are responsible are indeterminate, but if so how are they our actions? Whatever I have done before was an event antecedent to the act in question *ergo* irrelevant, but then am I not irrelevant to the act as well? I must be, unless the person I in question lasts only the momentary cross-section of time in which the act occurs. It would be boring to actually try to show that this is absurd. If I am a person enduring in time then this supposed act is but an accident that happened to me, yet for accidents I am not responsible either for I cannot control them.

We speak commonly of the problem of the freedom of the will, but is it sensible to attribute freedom to the will of a person? The will is not a person, how then can it be free or unfree? If we seriously maintain that it is free, aren't we guilty of taking a metaphorical expression in the literal sense? Should we say that only persons can be either free or unfree our problem changes. A person is complex, and endures in time; a person has desires, reason, will, and what have you. If so a will is some person's - in fact nothing other makes sense. Rather than being the person the will is but part of the workings of that person, a part of a whole, as such, it cannot then have freedom *vis a vis* that very whole. It follows that we need to ask whether *a person* is free or not. And here the obvious answer is YES. The choice will depend on what that person is, and there is no problem, for we cannot try, let alone need, to raise the spectrum of deterministic base of the growth of that person as a problem for freedom - we can only refer to a person as a single whole. Theory of persons is another matter and irrelevant to the problem of freedom of the will.

This clarification resolves the problem of free will. It will be clear that if successful it addresses a basic philosophical problem of our contact with the subject matter *via* clarification, but does it address anything

relevant to logic? We have suggested above that if such a move were to show whether and how an expression, or a set of expressions is tractable by logic, then a basic logic service has been performed. Here we saw why the paradox arose, for we saw why the propositions in a set were not properly tractable in terms of the principles ostensively relevant to them. It is also clear that the reasoning utilised in the case encroaches on the areas of epistemology and ontology - we should not be really surprised that this is so with concerns appertaining to the relation between logical aptness, and what we apply our logic to. I have no intention whatever of suggesting that all, or even most philosophical problems are concerned with lack of clarity. But I would maintain that a great many do, and more than we might suspect. Further it needs to be noted that even where the salient element of the question is not concerned with relative clarity, our capacity for handling it might depend on our ability to handle the relative pitfalls of unclarity. This was quite aptly illustrated by our example.

It appears by now that such problems should concern any logician, especially if he has some interest in applying logics, i.e. if that logician looks beyond the pure manipulation of inscriptions. And even there there is a rub. S. Leśniewski[4] constructed a set of systems: Protothetics, Ontology, and Mereology - designed to avoid antinomies and paradoxes in logic, especially in symbolic logic. His idea was that abstraction leads to antinomies and paradoxes. His solution, that we should not calculate inscriptions but objects, their collections and sets. And by *objects* he meant real, possibly material objects.

Accepting Leśniewski's method antinomies can be avoided, or at the very least so it seems. Yet his method rests very heavily on our accurate grasp of the relation between the expressions we use, and the objects (subject matter) we handle in this manner. Any uncertainty would be likely to abort the attempt for then we could not be sure what objects we handle, nor perhaps to what purpose we handle them. I suspect that the problems that appear as e.g. paradoxes in more traditional logic will tend to appear as difficulties in establishing a clear sense of what it is we are saying about what (kind of) object in Leśniewski's alternative logic.

The point that is relevant here, however, is that there is a clear relation between our clarity of paradigmatic grasp, at least where our basic sense of being presented with an object is concerned, and the logical viability of Leśniewski's system. Since this system is a response to problems in other types of logic - the logical relevance of the clarity of this type of grasp is thereby demonstrated.

Moore's ingenious reasoning is intended to delineate a cluster of expressions whose meaning cannot be in question because they are paradigmatic in the sense that they are the model of the very possibility of

any expression whatever making sense. Let us observe that any given expression can be understood, i.e. it's meaning can be grasped either immediately or mediately, as the result of a process, or a reasoning. Clearly any such reasoning must refer back to something immediately graspable on the pain of vicious infinite regress. Unmistakably then not all expression can be mediately intelligible. Consider for example that someone is told for the very first time that some intention was subconscious, and fails to understand. Says he: - 'Either I am conscious, and can have an intention, or I am not and cannot have one, and if I am not conscious of having one, what is the sense in attributing it to me?'. A reply to this very genuine worry could be: 'In some cases, e.g. as the result of extremely strong childhood experience, we can be left with a situation where certain of our decisions are determined in a way such that if we had a certain intention we would decide in exactly the same way, but in fact we are not aware of any reason why we should so decide. - In such cases we speak of subconscious intention'. This explanation is intelligible only because we already understand the expressions utilised in the giving of it. But the chain of explanations of sense must end somewhere, and Moore is searching for this precise point.

What would happen were we to deny Moore the possibility that some paradigms provide us with the desired effect? Let us look e.g. at the views of Eddington. According to him a solid object such as a table is not solid at all, but consists of mostly empty space with some infinitely minute particles swirling about at very great distances from each other. How then are we able to obtain the concept of a solid in the first place?

In fact we obtain the idea of solidity by looking at objects, such as the table, that deny the space they occupy to us and to all other competitors for that occupancy. Not only that, but we gain the idea of such occupancy from our sensory contact with them. We not only assume, but have to assume that in this way we know what we mean by 'solid', and that we are genuinely capable of recognising solid objects. Remove the possibility of ostension, and our grasp of what we mean disappears with it. Eddington seeks to make such knowing theoretical i.e. not accessible without sophisticated theoretical backing. Presumably only particles are really solid in his conception. But then the possibility of us experiencing solidity does not even arise. This is serious, for the concept, together with all related concepts must be *constructed*, but on the basis of what? Sole occupancy of space is the essence of solidity, and that is an empirical situation. How are we to grasp it without ostension?

There is no difficulty in multiplying such examples, and each of them demonstrates the initial plausibility of Moore's view. But that plausibility is not impeccable when we look critically at his embellishments. We saw for instance that Moore tends to accept sense data as ostensible primitives, yet

it would be impossible to learn the language of sense data ostensively. Sense data are theoretical concoctions arrived at by disregarding the situation of ostension. In that situation we always face objects that have properties, and could not even grasp the idea of free floating properties - but for Moore such free floaters have to be seen as combining to form objects. Perhaps this is harsh on Moore, he might have restricted this picture to the attempt at analysis, even if his language suggests the opposite, but others e.g. Russell[5] held such a view unmistakably. It is a matter of some importance that we identify correctly what the paradigms are, and further that what we paradigmatically grasp should not be something that we can only theoretically understand. It is perfectly O.K. to say that we need theory to correctly analyse what we grasp with immediacy provided only that the analysis is neutral where the possibility of immediate grasp of what we mean is concerned - I suspect that this is Moore's intention, but it does not work out that way. At any rate we can assume *at this stage* that the point as intended is well taken.

Still ordinary language cannot be sacrosanct, neither Moore nor Austin even propose that it should be, but they fail to provide any clear idea how, and on what grounds its authority can be questioned. Moore, of course, says that it is the analysis of ordinary expressions that is the problem, yet it is not all that clear how he understands analysis in such a situation.

How is it seriously possible to accept that an expression such as "The Earth existed for many years past" has a transparent common meaning that is quite unassailable *together with the view* that it could be improved upon, and replaced to a good purpose? What is it that we are supposed to be able to improve here? Surely not clear-meaning and unassailably true expression that we are supposed to have to begin with. Yet the move is intended to handle a real problem identified e.g. by Bertrand Russell.

Perhaps we can find some guidance by looking at two other authors. Firstly Gilbert Ryle in "Systematically Misleading Expressions"[6]. Writes Ryle:

> ... I conclude that there is after all sense in which we can properly enquire, and even say "what it really means to say is so and so". For we can ask what is the real form of the fact recorded when this is concealed, or disguised and not fully exhibited by the expression in question. And we can often succeed in stating this fact in a new form of words which does exhibit what the other failed to exhibit.

Ryle's view seems here to coincide with Moore's *vis a vis* the analysis of unassailably clear-meaning expressions. As the quotation shows Ryle himself can perform the operation on some expressions, but are these the right expressions? The other more successful expression must after all have a meaning that we grasp. This will be either immediately or mediately

available. One might be excused in thinking that in the first case the new expression becomes the paradigm, and in the second the paradigm remains hidden. Moore and Ryle seem to rely on a picture. It might appear to us when we say e.g. "This is my hand" that we present as the relevant fact that there is a 3-dimensional object possessed of many properties, and we are in direct contact with it. Yet Moore thinks, we can be certain only that when we perceive our hand we point to a thing such that (from a certain point of view) we understand why certain philosophers suppose that it is part of the actual surface of my hand, and others suppose that it cannot be - for so Moore seems to understand sense-data. To an unbeliever this might appear strange as immediate knowledge since it is all very obviously *thought out.* Moore's picture seems to be of an observer gaining an impression that is seen as in need of correction as soon as we are aware of the real situation of perception. But this is not the sufferer's picture, it is irreducibly the perspective of an observer; if a paradigm of sense is unassailable it is unassailable from the sufferer's perspective, and it is a paradigm for there is no other immediate (sufferer's) perspective on the basis of which it can be corrected.

Exclusions seem to dominate Moore's picture - we are not to think that the transparently meaningful, and true sentence "This is my hand" presents as a fact that there is this 3-dimensional object etc., independent of us both in its character and its existence. Clearly only an observer can conceive of this *vis a vis* the sufferer's paradigm - this is a problem especially if we wish to rely on common sense - common sense unhesitatingly underwrites the very opposite conception - this appears to be the very unassailable clear meaning that Moore tries to identify. Moore hides behind the screen of analysis, and so less obviously does Ryle, but any such evasive action must comply with some requirements. - We need to allow that some expressions are unassailably and transparently meaningful, and further that this is at the base of the possibility of all sensible thought, and discourse. Ryle would have to say that this can take place when the real sense of the paradigmatic expression is hidden from us - not a comfortable position. We are after all searching for indubitably irreducibly meaningful, and significant expressions that because of this very unassailability can carry the possibility of making sense. Combine this with a somewhat unreflective concern for correct analysis, and you can easily be tempted to search for the simple, *assuming* that it will be both *logically and paradigmatically* simple. Logical Atomism could be a child of such a reaction. Yet epistemologically, paradigmatically simple is not the same as logically simple - it is simply what the ultimate sufferer finds unmoveable and unassailable; if that is not logically simple according to our lights there is no helping it. - There can be no outside perspective from which to realise that we are

misled - something that escaped the attention of both Moore and Russell.

Moore's attempt is in a very good sense an attempt to find this unassailable *sufferer's contact* with his world. The very language suggests this. Short of this the role that Moore assigns to his "very type of unproblematic expression" becomes unintelligible. For Moore we need to identify a truth that we know and understand with complete clarity. I mention truth advisedly, for his position is perhaps most plausible construed in this way. A true fact cannot be misleading in itself. But if we have such a paradigm how can that immediately known fact stand in a misleading relation to the very and only way in which we can express it immediately? Disregarding the sufferer's paradigmatic perspective the picture used by Moore, and Ryle makes sense, but it is at odds with that perspective - yet Moore, at least, is searching for that very perspective, without perhaps articulating sufficiently well that it must be irreducible from the sufferer's point of view.

The word 'analysis' as used by Moore covers a multitude of sins. The element of truth in this idea lies in the fact that at point of paradigmatic impact clarity covers a possible range of mental perspectives, at least with respect to the order of examples selected by Moore. We are then forced into a search for simplicity appertaining to both sides of the coin. I do not think that Moore came even close to this, his idea being that the description of sense-data offered is clear in the required sense. Yet this is not even plausible - sense-data are simple *vis a vis* a given situation that *has* to be understood prior to our ability to see sense-data as simple - they can be simple only against the background of this type of situation. *Supra* I have remarked that it would not be possible to learn the language of sense-data directly, and this is the reason for it. Kant saw that the world presents objects to us, and if this is so the paradigmatic simple must consist in the basic awareness that we are faced with an object. That situation is a great deal more primitive than the one envisaged by Moore, and could not in itself provide a full epistemic situation. It is quite important to develop this idea in some detail, but here is not the place for it, and it must remain for the time being as only an indication of an alternative hypothesis. Stanisław Leśniewski based his alternative logic on some such insight. He held that antinomies cannot arise if we simply identify objects and their sets, and collections - if we abandon this assured base, we need abstractions, generalisations, analysis, and we are subject to a host of difficulties. It is of course utopian to believe that we can avoid such problems. If I am right then the assured base needs additions before we can form judgements, or write sentences, and our indubitable base becomes adulterated in the process of becoming intelligible.

In a way Moore is right in his opinion - without analysis we have no articulate concepts of object, and related concepts. But on paradigmatic level we do not articulate, we merely make articulation possible. Moore talks of typical unproblematic expression that we all understand; the language fits my alternative perspective, but then he cites fully articulate judgements (and a bit complex to boot) and moves out of that level to the next one of simple articulation, where, if I am right, we cannot anymore find any paradigms.

Our contact with the world is *via* impressions, whole perceptions are complexes of impressions, and can be described by listing the impression-elements that compose them. Yet the moment we start doing this we leave the realm of immediately presented epistemic wholes, and attempt something analytically more sophisticated. Sense data are the creation of this more sophisticated search. But it has to be remembered that this search presupposes the less sophisticated awareness of objects. In forgetting this Moore falsifies his search for "the very type of unambigous expression that we all understand". Not all of us understand the results of the analytic search, and none of us understand it with immediacy. We enter the realm of metaphysics where the search for the relation between perception and its object is at home. This is illegitimate for Moore's enterprise since he is searching for the fulcrum that would make this very enterprise possible.

Let us have another look at the idea of a paradigm. Using Kant's distinction between noumena and phenomena it will be clear that neither can be grasped in a paradigmatic situation - there *only objects* are accessible. To even attempt Kant's distinction we must raise problems about the relation between the observer and the observed, to both of which we must be able to refer intelligibly prior to the attempt being made. Immediacy is clearly impossible. Immediacy *may not* presuppose any previous grasp of anything, not on this very basic level. What then is the status of sense data? If they are only the simplest possible impressions then they are simply elements of our private consciousness, and it makes no sense to say that we perceive them. Just as it makes no sense to say that we perceive our hunger in the simplest possible situation of just being hungry, so to be hungry is to be in a state of awareness of lack of nourishment, lack of needed food - to perceive this as an object is to engage in introspection. I am suggesting in contradistinction to Moore that our paradigmatic awareness of an object is in similar case. To understand that I have an immediately paradigmatic awareness of an object is the very nature of the unproblematic epistemic situation that gives rise to the paradigmatically unassailable expression that Moore is describing. We do not actually say - "I am now aware of a hand", we express our awareness of the hand instead. The situation is not emotive because the situation we express awareness of is epistemic and constative, albeit on a primitive level. It is precisely because we are not in

any way commenting on the situation, nay because we are **in no position to begin to comment on it**, that our expression is unassailable as the basis of possible contact with our subject matter. In failing to pay attention to the importance of this impossibility Moore has twisted his own correct insight, and robbed it of plausibility in the long run. Clearly both Kant and Wittgenstein of the Tractatus would lean towards such a view.

What then happens to the problem that was acknowledged by Austin, Moore, Ryle and Ayer? Writes Ayer[7]:

> ...what is it that gives an analysis, or a definition, a philosophical character ... I suggest that the answer should refer not to the form of the analysis but rather to the effect of it on us. The common sense propositions which call for philosophical analysis are those which are formulated in such a way that they encourage us to draw false inferences, or ask spurious questions, or make nonsensical assumptions ... and propositions about material things call for it because they encourage belief in a physical world 'behind' the world of phenomena ... and philosophy in one way or another tries to remove all these dangers. I say 'in one or another' because I do not think that all processes of philosophical analysis are of a single form.

In the period of circa ten years between Moore's position from "The Defence of Common Sense", and this statement, the analytic philosopher, it might appear, lost a bit of his self assurance *vis a vis* the efficacy of common sense. Ayer says, very carefully, that common sense expressions "encourage" us to err, not that they are erroneous themselves. Possibly he is not careful enough. It will be clear all the same that clarity is of prime importance in philosophical analysis. - We avoid mystification by clarifying what we are saying, or intend to say. Should we succeed in stating what we are talking about with complete clarity we would cease to be encouraged to make mistakes. In "Philosophical Analysis"[8] Urmson maintains that the main object of both Ryle, and Ayer lies in avoidance of errors suggested by the language. Formerly, Urmson seems to think, the analyst was more concerned with the logic of facts (e.g. Logical Atomism), or with the structure of language, but the centre of gravity has shifted. We could read Urmson as holding that this shift amounted to a radical change of views. Yet it is not at all clear that anything over and above the way of analysing has changed.

Moore and Russell clearly impinged on the realm of metaphysics. They tried to find solutions by reducing the more complex to something as simple as possible - in their view the more dubious complex expressions should be reduced to strings of better established simpler expressions, and these dealing with obvious matters. Moore tackled the analysis of what we tend to say, Russell in the spirit of Leibniz attempted to create a more efficient

language; we could almost call it *lingua mentalis*. Both fail to meet the requirements of the true paradigm. The matter is more obvious where Russell is concerned for he wishes to exchange intuitive language for a theory-based language, but it applies also to Moore. Ryle and Ayer can be seen as making attempts to avoid this very difficulty. - Should we succeed in identifying some expressions as immediately, and intuitively sensible, and yet such that they can lead us into error, we should have found the reason for analysis of the appropriate kind. If further we could assume that these expressions were genuinely paradigmatic we could suppose quite plausibly that we are on the way to a solution. This complicates the theory but does not destroy it. Both philosophers attempt the elimination of spurious expressions, Ayer *inter alia* through his verification principle; Ryle notably in his "The Concept of Mind"[9]. Immediately following the words cited above Ayer states "I do not now think that philosophising consists entirely in providing translations".

Formerly the fashion was reductionism - we tried for instance to replace all statements about states with statements about individuals, and without residue; statements about love were translated without residue into statements about people loving other people etc. etc. Now, it is being suggested we do not replace faulty expressions with strings of O.K. expressions, we eliminate them altogether - the idea is similar but it has been radicalised. The original analyst noticed that his "translations" are inadequate, *vide* e.g. sense-data. There is a dimension gap between objects and sensations that thwarts attempts at saying that objects are logical constructs out of sense-data. Yet the radical "Vernichtungs" reductionist does not avoid stormy waters, and again the requirements of a paradigm are the basic stumbling block.

In thinking that in order to have the ability to think articulately, indeed in order to have the very capacity to operate a language we need to base ourselves on some clear-meaning and totally unambigous expressions, Moore is clearly right. Further he is correct in stating that such expressions must be natural and immediate. I would like to indicate here that these strictures cannot be seen as applying exclusively to empirical, or sensory expressions, for it is clear that various other types of expression e.g. technical, abstract, formal etc., have to have a similar base, and it is not clear that all can be construed with reference to empirical ones. Moore's reasoning is adequate if we read it as stating that all such truths are expressed in words that either have immediate natural meaning, or construed meaning, and then all meaningful expressions start with, and descend from the first type of expression. The question of whether there are expressions, or even whole types of expression that fail in that they cannot be adequately connected up with an appropriate paradigmatic base, must remain a further question at this stage. Nor are we permitted to assume here that

there can be only one type of paradigmatic base, and that it is e.g. empirical. It will be easy to see how such an unwarranted assumption would lead to reductionism, or some form of Verification Principle on unacceptably *a priori* grounds.

The central point here consists in the realisation that without intuitively immediate unproblematic paradigms there could be no secondary, theoretical, abstract etc. expressions, and further that without both the enterprise of articulate thought could not proceed. Given obviously intelligible expressions we may not dispose of them simply because we cannot deduce them from our preferred type of paradigm, there are likely to exist paradigms appropriate to them. How else would they be intelligible? To dispose of them we would need to show that there is no function for them to perform, not merely that we cannot understand how they come to perform the function they do. The second only poses the Kantian question. Unless we can show that there is no job for them, or that they can rest on a base we acknowledge, we *need* to address that Kantian question, and that without *a priori* prejudices.

When Ayer maintains that statements of religion have no literal meaning he is in a strong position for he is attacking secondary expressions remote from natural paradigmatic ones, and what is significantly more, expressions that embody a controversial theory. There is the chance then that they amount to no more than internal signs limited to the task of making moves within that theoretical structure. In such a situation there is always a chance that we deal with no more than a game that is not really interpretable as a hypothesis about the world. With such a game we can have expressions that are purely play-expressions of that game, and as such legitimate (e.g. chess moves) but totally devoid of any epistemic weight. On the other hand when he treats similarly all value statements his position is weak. It is a whole parameter of the world that faces us that there is the possibility of choice, and deliberate action. That in this respect there are what appear to be good, poor and bad choices; obvious and difficult choices; good and bad preferences. This dimension unlike that of religion cannot be simply imposed on the world as we see it from a game that we have constructed, for without it the whole parameter of adequate/inadequate action and reaction becomes unintelligible. Given this, these expressions must have their roots in some epistemically valid paradigms, whatever these are. One can hear objections from the religious camp - religion too, they will say, is a natural dimension - but it is not like the normative parameter at all, it is quite possible to live and react to the world while quite unaware of religion, but not possible to do so without any normative awareness whatever. As a sop to religion we could admit that this does not show that religion must be meaningless in the Ayer'esque sense, only that it

could be. Ryle's removal of the mind however is more problematic, and on a par with Ayer's removal of values. Both come from an implausible conviction that only actually constative expressions can be paradigmatic, that basically we can talk paradigmatically only about material things - this position is based on no positive grounds only on a rather fanatical attachment to some Occamite principle of parsimony. It found its most extreme expression (among the respectable and important ones) in Tadeusz Kotarbiński's[10] Reism.

This type of move cannot be legitimate for in this fashion we adjust elements of the situation we face in the wrong direction. It will be clear that we can talk of objects with some confidence precisely because we possess reasonably obvious paradigms - expressions that cannot be seriously doubted in any respectable sense. From the fact that these paradigms have to do with objects does not follow that all paradigms must. It is an entirely different question what are the paradigms of e.g. quantifying expressions, or of normative expressions. The correct method consists then in approaching the problems in parallel ways, not however identically. We must search directly for "the very type of unambigous expression that we all understand" in each of the ostensively different epistemic areas. The most plausible assumption is that these paradigms are likely to differ as much as the secondary expressions derived from them differ. There are likely to be clear-meaning: value; mental; formal etc. expressions. This presumption is strong enough to prevent us from simply assuming the contrary in any case. - Also this would be the real common-sense stand on the matter. For example it is plausible to say e.g.: that I know immediately that I am the thinking being that is making the point, or that it has positive value to try to discover the truth here, negative to try to conceal it, etc. It will be clear that it is easy to parallel Moore and his "very type of unambigous expression" in other fields, when we do not try to make them empirically constative.

Ryle denies this because of the difficulties of the analysis of expressions concerned with the mind. Analysis means here an attempt at providing ourselves with a completely clear and unambigous picture of the epistemic situation, and one free of paradoxes, contradictions, and unjustified propositions. The main difficulty seems to relate to the fact that we do not at all *perceive* our mind. Yet it should be noted that this situation exists *vis a vis* any putative substance. Should we assume that direct perception of something is the only correct reason for accepting its existence we would direct ourselves to deny the existence of the mind, as others (Mill, Berkeley etc.) did direct themselves from time to time into the denial of the existence of other substances. By rights if we deny one of them on such grounds we should deny all - a result to gladden the heart of any sceptic. This type of move underlies other attempts viz.: reductionism, ordinary and Vernichtungs; Verification Principle; Materialism; Idealism etc.

Basically the motive consists in ascribing real sense only to an expression that encodes something that we have perceived as such. All other expressions appear then as logical constructs out of that set of real-meaning expressions.

To illustrate: - A reductionist attempts e.g. to show that all state-statements are reducible without residue to statements about people. We perceive people; we do not perceive states. The verification principle denies reality to values and spirituals. These could be only logical constructs since we do not perceive them, and yet such constructs are impossible. The behaviourist makes a similar move *vis a vis* mental states. He goes to the point at which only behaviour is perceptible, not the states supposedly responsible for it. Ryle is quite clearly right if he holds that producing logical constructs in this situation mystifies, and is 'systematically' misleading, hence the doctrine of the ghost in a machine.

All that would have been fine if these logical broom people had access to relevant clear-meaning unambigous paradigmatic expressions so placed that the expressions used in these reductions could be construed on their basis. This is not so.

We said above that that we do not perceive sense-data, and further that the sense of an expression such as "sense-datum" is construed on the basis of immediately sensible expressions such as: "thing"; "hand"; "object"; etc. Let us have another look at the concepts of mind and value. Let us assume for the sake of argument, that they are found misleading, and have been eliminated from our language - we should be able to carry on without difficulty, indeed we should be able to breathe easier.

Vis a vis mind, we can now speak with sense of: feelings; impressions; etc. as independent designates of our thought, alternatively we can talk in this fashion of e.g. doing things in certain frames of mind. These designates must then function as independent individuals providing us with a focus of basic, and immediate reference to the elements of our subject matter. But it will be clear that we cannot think that way in any natural and immediate manner. Our immediate way is to refer to ourselves as thinking, feeling, perceiving etc. [Despite reism, Kotarbiński in "Gnosiology"[11] sees it this way]. We would find it well nigh impossible to regard a certain mental content as substantially independent and a focus of primary unfettered reference. A perception, let us say, so placed that the question whose perception it is would not even arise. But if we deny mind, and its equivalents this is the only way in which we can think this subject matter. Our basic reference would be - *"this perception is"*. "This doing of that in such a frame of mind is", and so on. There is no need to argue the problems this would create. In contrast we all understand expressions such as: "I think"; "You perceive" etc.

Moore would of course claim that here we have entered the realm of analysis and have left the paradigmatic base. Where I differ from Moore, and quite certainly from Ryle and Ayer is in seeing clearly that it is precisely: sense-data; acts in a frame of mind; feeling type designations, e.g. "anger" that are the constructs in the case that rest on theory-like abstraction. Were we to accept the removal reduction of e.g. minds or values, we can understand the general move, but we could not grasp the moves left to us as a result of it, remembering always that this is the *situation of the sufferer, not the observer* of his sufferings. If expressions concerning e.g. minds, are systematically misleading, on this showing, they become invincibly misleading as well. If we accept that no value statement has any literal meaning we must say that we pretend to value things. Putting aside the problem of pretending all the time without even knowing what we do as relatively trivial, we should stress that now any ascription of value is necessarily empty and senseless. If so it cannot be used even to pretend anything. An empty sound is an empty sound until some meaning has accrued to it, and when it has that expression is not meaningless anymore. Even if Ayer wishes to eliminate the whole parameter of values he owes us an explanation of how it is possible for us to err. What can we expect of the so called 'emotive' noises ? Nothing after we have accepted Ayer's elimination of real values - for then all such noises are necessarily aimless, and if not then they create values, contrary to our assumption. If Ayer is right the question of value does not ever even arise. How then are we to explain purposeful action - purposeful yet completely aimless? Something is clearly less than clear-meaning, and less than immediately intelligible.

These points were only roughly sketched in, but they are only illustrations. Now if these illustrations are apt, and carry their point, are we to conclude that analysis as a method is bankrupt as well?

The reply is clearly NOT, and this for an obvious reason - for in the criticism above we used the analytic method - I do not share the views of the philosophers I criticised, but I accept their argumentative methods.

I have asked what are the conditions in which sentences and/or expressions can have sense, and especially regarding the type of expression favoured by criticised philosophers. Let us note that I have accepted their statements as, at least, *prima facie* sensible in order to derive from these a set of results that then can be shown to be either acceptable or inacceptable. I did not try to square such results with results that we all accept, I have not tried to reduce this or that tenet to absurdity. In fact I made no attempt to test these views against other views and propositions at all. Instead I have analysed and examined certain presuppositions involved in the making of these statements, and the use of the kind of expression

proposed for a given logic position. I have asked whether the proposed expressions, and propositions are capable of doing the job demanded of them.

I took as the starting point, the tenet that if any expressions are to carry sense, then some expressions must be, what I would like to call clear-meaning. The ground for this is that we must have expressions that carry meaning directly and immediately if we are to have expressions that carry it indirectly and derivatively. Spinoza[1 2] not only noticed this fact but made it the basis of his system. But Spinoza understood the point ontologically. (It is true that the perspective of "is understood through itself" looks epistemological, but this is misleading; the "Is in itself" ontological perspective dominates the system; the epistemic parameter would actually fit best his idea of an attribute.) Our perspective is truly epistemological - whether or not the object of our cognition exists independently or dependently, our thought requires that the basic expression (judgement?) be immediate and transparent.

Starting in this way I asked whether it were true, and possible, that the types of expression accepted by the theorists as basic and clear-meaning could be such. I asked quite searchingly what makes: immediacy, clear-meaning, transparency possible, I asked whether propositions about sense-data could be in this category, and so on. I used these results to ask questions concerning the nature of paradigmatic expressions that would *inter alia* fit the picture of "the very unambigous expression" as specified by Moore.

Subjected to this analysis tenets typically accepted by the authors in question disintegrated. It became clear that simplicity that could produce such a result was neither logical nor yet ontological; the sense of the proposed expressions is accessible only *via* analysis. True paradigms are not found that way, and are recalcitrant where analysis is concerned. The last needs more argument, but this is not the place for it unfortunately. I will only say that we have a truly paradigmatic expression if, and only if, it can be grasped immediately, and that analysis cannot improve on this. But it has to be remembered that we should be able to analyse the situation created by their existence. I.e. an observer can, a sufferer cannot until he puts on the observer's hat. Logical simples, and ontological simples such as Russell's "logical" atoms however are known as the result of analysis *ergo* they are not epistemologically simple, which incidentally marks the difference between logic and epistemology. It is quite plausible to say that the least complex elements of what we are aware of are simple, but that does not mean that they are also the simple epistemic paradigms. They are theoretically simple which is an entirely different matter. It is, unfortunately, fatally easy to confuse the two types of simplicity.

In a basic uncritical epistemic situation we accept without question that we are faced with some object of cognition, and we disregard its

complexity. Even if it were complex, we could not theorise about it in such a situation for the situation *is the fulcrum* we need to start on the road to theory, and reflection. This is the very point that rightly impressed Moore. Once we have a reflective theory we can theorise about that basic paradigmatic situation, *but we cannot change that situation.*

On such grounds I concluded that the investigated theories were wrong. My basis was in fact the analysis of the expressions put up by the authors of these theories, and the analysis of the situations in which they were to saliently figure.

It should be clear that the method adopted above is philosophical analysis, I shall refrain therefore from comparing it in detail with the ways of a metaphysician, and of the typical logician. We need to remember that an adequate argument in philosophy is seldom pure, philosophy is a complex field. For instance a metaphysician will need logic analysis and so on, to do his job properly. A logician can limit himself to the internal affairs of logic, in a way in which a metaphysician and a general philosopher cannot. If he does, however, he leaves the problem of the need for logic to others. It is perfectly proper for him to tackle such problems himself, even if the other stand is proper as well.

The final point that needs to be raised here is that the concern with whether logic is: needed; important; adequate, and concern with how and where it is adequate etc., is a concern with logic. This is not only legitimate but of prime importance. Short of this the game of logic is in danger of seeming just a game. Importantly it is not just a game. I expect we *need* philosophical analysis to work seriously on this aspect of the matter.

NOTES

1. In: *Philosophical Papers*. Clarendon Press 1961.
2. In: *Contemporary British Philosophy*, vol.2, H. Muirhead (ed.), Allen and Unwin 1923.
3. Allen and Unwin, London 1963.
4. Cf. e.g. *S. Leśniewski's Lecture Notes in Logic*, NIPS, Dordrecht 1988.
5. *Our Knowledge of the External World*, London 1914.
6. In: *Logic and Language*, A. Flew (ed.), Oxford 1952.
7. *Does Philosophy Analyse Common Sense*, Proceedings Aristotelian Society. Supp. Vol. XVI, 1937.
8. Oxford University Press 1956.

9. London 1950.

10. *Gnosiology.* Oxford/Wrocław 1956, translated from *Elementy teorii poznania, logiki formalnej i metodologii nauk*, 1st ed. 1929.

11. Op. cit.

12. B. Spinoza, "Ethics". In: *The Chief Works of Benedict de Spinoza*, R.H.M. Elves (transl.), New York 1951.

Jerzy Perzanowski

ONTOLOGIES AND ONTOLOGICS

1. Philosophy, taken from the point of view of its problems and methods, is the collection of distinct philosophical disciplines. In fact, metaphilosophical analysis leads to rather troublesome questions: Are philosophical disciplines methodologically and/or essentially related and connected? Are particular philosophical disciplines scientific? And, if the answer is not definite, to what extent is this so? Do philosophical disciplines form a uniform and organized (at least in its depth) system?

The most important factor in the characterization of any scientific discipline is its problematics. Hence, there are as many philosophical disciplines as there are different and autonomous families of philosophical problems.

Certainly, two philosophical disciplines are particularly distinguished: logic - for methodological reasons and ontology - for essential ones.

Instead of considering the initial question in its full complexity, let's go to its kernel - ontology itself.

Ontology and its parts.

2. Ontology is the theory of what there is, the theory of being. It considers the full ontological universe, all items that are possible, describing and classifying them and searching for the principles of this universe, principles of taking together the plurality of ontic objects, particular beings, into one - the Being.

Thus, two questions govern ontological investigations: what is possible and why? The second question, concerning the being's principles, may be strengthened to the deepest - last in the logical order - question: how that which is given, or rather what there is, is possible? The question about principles of being, i.e., general laws of nature, *plus* the question: what makes possible what there is and renders impossible what there isn't?

Because of its matter and problematics ontology is the most general discursive discipline. It is the general theory of possibility. By the nature of its questions it is also very modal.

3. Ontology has two sides: descriptive - phenomenological, and theoretical - formal.

Hence, it is divided into three parts: onto-ontics (in brief: ontics), ontomethodology and ontologic.

4. Ontics is devoted to the selection of ontological problems and notions, their differentiation, classification and analysis. Doing ontics we construe the conceptual net of a given ontological theory, i.e. its categories. It is also one of the tasks of ontics to state ontological hypotheses, based on the previous analysis of concepts.

Ontics, being a part of ontology, is itself complex. Its further description depends on the general idea of ontology, on accepted classification of ontological concepts. For example, Ingarden has distinguished three parts of ontology: the material ontics, the formal ontics and the existential one. Notice that his ontology is, in our terms, ontics!

5. Ontomethodology concerns ways of doing ontology, methods and types of ontological constructions as well as principles of choice between ontological statements and theories. Examples of such ontomethodic principles are: the principle of non-contradiction, the principle of sufficient reason, and Ockham's razor.

Indication and discussion of the appropriate principles is necessary for sure for any critique of ontological theories, particularly the critique of the logical means used in ontology.

6. Ontologic is a logic of the ontic realm. It is an investigation of ontological connections, concerning particularly logical relations between pieces of ontic information. Also, it is a theory of the fundament of ontic relations.

Ontologic considers the organization of the ontological universe, trying to describe its mechanism. It describes the complexity of the Being, looking for its laws and base - the Logos.

7. Ontics is a purely descriptive and analytical discipline, ontologic is speculative and formal. They are, however, closely connected and interrelated disciplines, affecting one another. The product of ontics is a description, usually complex, of the ontological universe, whereas ontologic supplies different theories of this universe.

Certainly, at present ontic considerations are more common. In ontology we have many descriptions and claims, but not as many theories.

Among Polish ontologists, for instance, Ingarden may be regarded as a typical ontics reasoner, while Leśniewski should be treated as a typical ontologician.

General ontology versus particular ontologies.

8. The ontological question "How is possible?" applies either to the domain of all objects, the full ontological universe: How is what there is possible?; or to some particular, more narrow, universe D: How is D possible?

The last question has two levels. The first, is the question of what makes objects from the universe under consideration possible: How objects from D are possible? The second is what makes the very universe possible: How is D itself possible?

Both the general and the particular ontological question has to be understood as a question concerning realization conditions for the appropriate objects: what determines them? Hence, respectively, as the general questions we have "What makes what there is possible?", or "What makes what there isn't impossible?", in particular "Why is there something rather than nothing?". And as analogous questions obtained by particularization: "What makes objects belonging to D possible?", "What makes D itself possible?", "What makes objects outside D impossible in it?", "What makes D itself impossible?".

9. Ontological questions are deeply modal. To formalize them, as well as to formalize the appropriate ontological theories, we must use ontological modalities, the theory of which clearly differs from the usual theory of logical modalities (cf. [12], [13]).

Logical modalities are connected with the question "What is possible?". They are review modalities useful for collection and comparison, for external description of the ontological universe. Usually, logical modalities take adjectival form: possible, necessary, contingent, etc. In this way, they obviously presuppose that the universe is given.

On the other hand, ontological modalities serve to describe the internal mechanism of this universe, its rules. Usually, they are expressed by nouns: possibility, necessity, contingency, compossibility, exclusion, making possible, making impossible, etc.

Ontological modalities are connected with the proper ontological question: Why and *how* is possible?

10. Particular ontologies, D-ontologies, are ontologies of particular domains D.

Previous remarks concerning general ontology also apply to any particular ontology. Thus, D-ontology should be divided into D-ontics, D-ontomethodology and D-logic.

For example, let's take the physical universe. Now, D is the domain of physical objects: things, events, states, processes, phenomena, fields, etc. Let d denote an arbitrary chosen, but fixed element of D. The question "How d is (physically) possible?" leads us to a family of physical laws, to the appropriate physical theory T. But when we ask "How the theory T - taken as the theory of D - is possible?", and then "How physics itself is possible?", or more generally "How mathematical natural sciences are possible?", we move immediately from physics, through epistemology and metaphysics, to physico-ontology.

By asking general questions concerning the whole physical universe, nature, we start physicoontological investigations as well. What is the source of the order of the universe? Does such an order exist at all? If yes, where? In the realm of reason? In the nature of the world? Then we are let to ask: How the physical world is possible? What are its characteristics and its place in ontological space, the space of all possibilities?

The first questions lead to epistemological investigations, with an idea that in the world and/or in us there is something what makes our, partial - but quite interesting, knowledge of the world possible; the next questions entail immediately the proper question of general ontology, concerning the nature of being.

Physico-ontology, like, to some extent, any particular ontology, depends on the claims of general ontology. This reflects the (logical) priority of ontology.

Physico-ontology, like any ontology, general or particular, has three parts: physico-ontics, physico-(onto)methodology and physico-logic. Physico-logic includes, for example, J. von Neumann and G. Birkhoff's attempt to establish the special logic of the quantum universe (cf. [7]).

11. The most characteristic for ontologies, both the general one and particular ones, is the form of the basic ontological question: "How X is possible?". The question usually expresses the thirst for ontological knowledge, indicating the proper field of ontological research.

This is not so in every case, however. The How-questions are relevance dependent. They can be asked of any well-determined, distinct domain D, but "How pencils are possible?" is well-formed, but quite unreasonable. It is much better to ask "In what way and from what materials are pencils made?".

In regard to some narrow group of concrete objects technological questions are certainly more reasonable than ontological ones. This is not so, however when we pass to broader and more abstract domains.

Even in the most general case of the world the ontological questions "How the world is made?" and "What makes the world possible?" are reasonable, though, perhaps, imprudent. They express human anxiety for knowledge, our desire to understand what surrounds us, to understand ourselves.

Ontomethodological considerations

12. Ontology, as outlined above, is similar to abstract mathematics, or to appropriate disciplines of natural sciences.

Take, for example, topology. General topology is the most general mathematical discipline concerning continuity, whereas particular topologies concern particular types of topological spaces, for instance differential manifolds.

Similarly, ontology is the most general theory of what there is and why - hence the general theory of possibility, whereas particular ontologies concern particular cases.

13. Ontology is similar to mathematics in, amongst others, the necessity of its statements, and their formal, deeply modal, character. On the other hand, ontological formalities are not purely formal, not art for art's sake. On the contrary, they are led by basic, well-formed, ontological intuition.

Ontology is also similar to mathematics in its generality. It considers all possibilities, treating them initially with full parity. Next they are differentiated and evaluated by the appropriate ontological theories - through investigation and comparison.

14. Thus, the character of ontology - general, formal and modal - is the root of ontology's influence and value.

Ontology is interested in all answers. Even the most extraordinary, but consistent, ontological theory is ontologically valuable. No consistent ontological theory is such that it must be renounced. In ontology theories are selected by intuition, whereas in metaphysics and particular sciences and other disciplines of reason some additional criteria can be used.

15. The source of ontology lies in combinatorial and purely possibilistic power of reason. It works with data given in pre-scientific as well as scientific cognition, producing a general picture of the world, expressed by the appropriate ontological theories.

16. Yet, can such a general discipline as ontology be fruitful? Certainly, the best way of answering this question is to try it; *the proof of the pudding is in the eating*. In attempting myself to do such work (cf. [10], [12], [13]), I became deeply convinced that ontology is not only important, but also interesting and fruitful.

Leaving the reader to review the appropriate literature through lack of space, I will only outline the criteria of ontology's fruitfulness. Surely, ontology would be fruitful, if it could delineate its space - the universe of all possibilities, indicate the way in which ontological space is generated, its objects differentiated and combined into complexes; that means, if it were able to outline the mechanism unifying objects into ontological systems (individuals, worlds, the space itself, etc.).

17. Two disciplines are particularly related to ontology: logic and metaphysics.

Logic, which concerns modification rules for information, is the most general theory of information processing. In its kernel it considers truth-connections between verbalized pieces of information. The basis of logic lies in the ontological nature of the world, whereas its source comes from combinatorial, *a priori* power of reason.

As regards their subject matter ontology and metaphysics are the closest disciplines. Both investigate possibility. The first - the possibility of what is possible, the second - the possibility of what is real. Ontology concerns the realm of possibility, metaphysics - the realm of facts, the world.

Metaphysics is the ontology of what is real, the ontology of reality.

Ontology considers being in itself. Metaphysics considers involved beings, inherent in concrets and phenomena.

18. Finally, let us resume the most important claims concerning ontologic. Ontologic belongs to applied logic. It is the set of appropriate logical theories of the ontic realm, which are based on suitable logical calculi, investigating interconnections between pieces of ontic information.

To do ontologic is to do ontology in a formal way. It means, trying to establish formal theories in a way similar to that used in mathematics: formalize what is done by ontic investigation, under control of intuition growing from it.

Types of ontology

19. Ontology should be distinguished from its theories. Some of them, suitably developed, are called ontologies as well. We speak, for example, about the ontology of Aristotle, or the ontology of Leśniewski.

As a matter of fact, quite a lot of ontological theories exist, also ontologies, more or less developed, concerning more or less different subjects. Their classification is indeed the key to description of the contemporary ontology, and then grasping its less developed parts.

Types of ontology are separated according to different principles, which are not so easy to catch. The classification of ontologies according to types, as outlined below, follows its description with respect to the main planes of being.

20. The planes of being are determined by the position of the man in the world.

Man remains on the boundary. In him three worlds come into touch. The objective, physical world - of concrets, things, events, processes, etc. The subjective world of the psychic, including intersubjectivity domain - the world of thoughts. And the world of man's creation, the world of culture, including science, and the distinguished realm of language: the world of signs and other cognitive operators.

The above picture is modified and ordered by ontology, unifying objects from different worlds in one complex domain of being, with several aspects distinguished.

21. There are three main ontic planes: the plane of being, the plane of thought, and the plane of language. The first is the proper ontic plane, the rest are obtained by separation and discrimination of, respectively, the intersubjectivity world and the world of culture.

The basic ontic conviction states that ontic planes and their worlds are ontologically inequivalent, that some of them are primary and basic, whereas others are secondary and derivative. Ontologies are thereby divided into fundamental ontologies - of the primary, and derivative ontologies - of the secondary.

The basic ontological preference manifests itself in the judgment of which ontological plane or domain is primary, which - secondary. Without going into details, we take it for granted that the ontology of a factor unifying all ontological domains - the ontology of being with roots in the ontology of the real, is fundamental.

22. The fundamental problem of any ontology of being is the problem of decomposition of wholes into components - the problem of analysis, and the reverse problem of composition of elements of some multiplicities into wholes, their combinations - the problem of synthesis. It is, in fact, the traditional problem of ONE and MANY.

In thought ontologies the problem ONE-MANY recedes into the background. Instead, thought ontologies are chiefly occupied with the opposition *thought - real*, confronting the world of thoughts with the real world. Now, the basic problems are: the problem of the principles governing modification of information - laws of thought, the problem of psychophysical unity - the mind-body problem, and the problem of cognition understood as the problem of representation.

Changing the questions results in changing the ontological picture of the world. The most important ontological question is now the question concerning possibility of knowledge and its adequacy. Thought ontology in its very heart is the ontology of epistéme.

Ontologies of language, linguistic ontologies, also deal with epistemic questions, though is a rather indirect way. They are occupied with oppositions: *syntax - semantics*, *expression - its referent*, with the basic question of the correspondence *language - the world*, and the subordinate, technical questions of naming and truth.

Thought ontologies see the world through the idealized (and fictitious) cognitive subject, linguistic ontologies see it through (a given) language. Indeed, we come to know thoughts by verbalizing them by means of a language.

Thought and language, however, are only means of getting at knowledge. And nothing else! An operator is only an operator, a medium is only a medium!

We will see this clearly after considering particular types of ontologies of being.

Ontologies of being

23. Ontologies investigate ontological universes - usually very complex items. The question of organization of the universe is thereby the central problem of ontology. In fact, everything in ontology is connected with this problem. The most natural classifications of ontologies into types are also relative to it.

24. Generally ontologies may be divided into static and dynamic ontologies.

Each ontology tries to outline the mechanism of the ontological universe. It can be either static, following some given instructions faithfully, working because of external forces and/or such information; or -

on the contrary - it can be dynamic and independent, working because of some internal sources, because of the nature of being itself.

Thus, the above distinction entails appropriate division of ontologies, according to the picture of the ontological universe provided by them.

25. Next, in connection with the same problem, ontologies of being are to be divided into ontologies of a differentiated and ordered universe, and ontologies of an indifferentiated and/or unordered (chaotic) universe.

Ontologies of the second type are, in a sense, irrationalistic. They claim being's transcendence and inaccessibility. There are two main sorts of them: ontologies of an indifferentiated and unordered universe - of chaos, or magma; and ontologies of an indifferentiated but ordered universe - ontologies of flux.

From the ontological point of view they are rather uninteresting - not to differentiate means to be silent, and unfalsifiable - the inaccessible cannot be approached.

Moreover, they switch epistemic questions to questions concerning the subject alone, reducing any accepted differentiation to subject's activity. In this they are quite similar to some ontologies of thought, or ontologies of language, sharing their splendours and miseries.

26. There are quite a lot of ontologies of differentiated and ordered universe(s), called simply - ontologies of being. They form, in fact, a rather complicated system.

Ontologies of being concentrate on basic ontological questions: What there is and why? What can be, or cannot be and why? What makes possible, or renders impossible what is given, and/or considered?

The fundamental problem of ontologies of being is the problem ONE-MANY, the problem of division and composition, analysis and synthesis together with the question of wholes (complexes) transformations.

27. There are at least seven sorts of being ontologies:

(1) Ontologies of objects and properties, with a variant of existential ontologies;
(2) Ontologies of conditions, relational or functional ones;
(3) Combination ontologies, with variants: combinatorial ontologies, combinators ontologies, and situation ontologies;
(4) Transformation ontologies;
(5) Events, or processes ontologies;
(6) Ontologies of states, and
(7) Mereological ontologies.

Each of the above ontologies has both its ontics and ontologics developed in varying proportion. In further sections of this chapter we will survey them step by step.

28. Any ontology of objects and properties, shortly: object ontology, is based on nominal-, i.e. noun- and adjective-, predication. Objects are apprehended by it as *some:* as predicated, or being the subject of predication. Its aim is to classify and to qualify them.

The basic categorial opposition of this ontology is the pair: *object - property.* Thus, it grasps the universe as containing objects of two types: things and properties. Things are collected according to the similarity.

The object ontology is based upon nominal predication, analysed in terms of *subject/theme - predicate/rheme.* The upshot is throwing the structure of noun predication on the world, with the conviction that appropriate ontic forms correspond to natural forms of nominal predication. The world is thereby apprehended as the picture of the language whereas, on the contrary, it is the language that pictures the world.

However, the correspondence *language forms-ontic forms* is not so exact. It is quite easy to produce well-formed and meaningful expression without reference, for instance 'a son of a childless mother', and next come to the conclusion that such an object, in a sense, exists, i.e., that some object being a son has a rather unusual mother. Such reasoning is certainly fallacious: the fact that an expression is well-formed does not mean that its reference exists. Hence, linguistic predication does not correspond exactly to ontic predication.

On the other hand, the concept of property, on which object ontology is based, is rather obscure, and, because of paradoxes concerning it, difficult to formalize. Object ontology is thus based on property ontologic which is, by the way, natural to develop in terms of combination ontology, in terms of: *simple-complex* and *primary-secondary.*

In addition to that, in object ontology we do not differentiate between properties and functors (determiners).

The object ontology is most popular, anyway. It is connected with Aristotelian traditional logic and school philosophy. The most advanced version of its ontic was developed by A. Meinong (cf. [8]). His ontics, general and clear, was the object of the strong critique by logicians, mainly B. Russell. It has been consequently reformulated in the 70's in various ways. However, the price was serious weakening of Meinong's ontology (cf. Parsons [9], Routley [15], etc.).

Despite many efforts we still do not have satisfactory theory of properties. As an *ersatz* of it a suitable theory of types can be taken.

29. Existential ontology is a variant of object ontology. The expression "S exists" is treated in it as the fundamental form of predication.

In this way metaphysics is stuck together with ontology, and existential statements are credited to have full and primitive information value.

The proposition "S exists", however, is only a digest, assuming previous theory of existence. Such theory is, in fact, difficult to develop directly. It is much easier to develop it in the framework of another, e.g. combination, ontology.

30. In most cases the conceptual part - ontics and formal part - ontologic of object ontologies are developed in proportion. In particular, there are well-developed linguistic theories of property predication: attribution and predication *sensu stricto* (using predicates).

Object ontologies have suitable thought ontologies as their counterparts. Some of them, particularly ontologies of attribution and predication, set-theoretical ontologies and model-theoretical ontologies are better developed and better grounded than their being counterparts. We know quite a lot about properties expressible in language but at a price of special assumptions.

Our knowledge about properties in general and their connections with objects is not so good, however. These connections are analysed in one of the four ways: the logical way, using substitution (conversion) with suitable conditions of property reification; the set-theoretical way, by analysis of membership relation restricted to the class defined by means of the property under consideration; the model-theoretical way, using satisfaction classes of formulas expressing possession of the given property; and last but not least - the mereological way - formalizing the idea that properties are inherent in objects, their parts.

In conclusion, object ontologies are connected with other ontologies, supplementing them.

31. Ontologies of conditions, or conditional ontologies, describe the world in categories of dependency: *to condition - to be conditioned*, or in more developed version: *objects being conditions - the way of conditioning - what is being conditioned.*

If conditioning is treated as relation we have relational ontologies, which in model-theoretical version of A. Tarski play at the present the fundamental role in semantics. If, instead, we apprehend the basic dependence as connection, for definite arguments, of function with its value we obtain functional ontology of G. Frege and A. Church. Functional ontologies are examples of binary ontologies splitting world's objects into two types only.

The further reduction lead to unary ontologies, i.e. ontology of combinators of H. Curry and A. Church, which are a variant of combination ontology.

Certainly, conditional ontologies are closest to mathematics. Hence, they are most formal ontologies, unique in their disproportion between ontics and ontologics, to the advantage of the latter. They seem, however, to be partial ontologies, apprehending only some aspects of the being. It is, indeed, reasonable to consider them as the necessary component of sound ontologies of the world (if any) which, however, should be richer.

32. Mereological ontologies (Leśniewski, Leonard, Goodman et al.) are formal ontologies concentrated on important (fundamental?) ontological relation "to be a part of", pursued in a way similar to general geometrical investigation.

These ontologies are mainly devoted to one aspect of the problem ONE-MANY, the question of universe division. Therefore, they are - in any version - the necessary component of each well-developed ontology, in particular combination ontology. For recent description of mereological ontologies cf. P. Simons [16].

33. Ontologies of events and processes, shortly - process ontologies, are based on verbal predication ("John speaks") or adverbial one ("John is away"). Now, the world is described in terms of events, processes, actions, happenings, whereas the things (if any at all) are apprehended as dynamical beings, involved in them.

It is not clear, however, what is the change and what are its sources. It is easier to answer these questions, I think, if subjects of a change are comprehended as complex objects, whereas the change itself as their transformation, metamorphosis.

Process ontologies were developed, chiefly by A. N. Whitehead (cf. [18]), to grasp and investigate the picture of the world given by contemporary physics. And indeed they derive much benefit from it. It is their connections with science what make them strong, but also, paradoxically enough, it is their weakness. They freely borrow statements, ideas and methods from sciences (physics, chemistry, biology); many times, however, in a rather uncritical way, without taking enough care over their specific ontologies. Instead, they usually use formal means with a rather unclear ontological status, without serious attempts to interpret these means in terms of process ontology, or attempts to modify them. For instance, application of set-theoretical formalism to the universe of events and processes needs caution and ontomethodological reflections.

34. Combination ontology is the ontology of elements and their combinations, ontology of unions and compounds. It considers objects in their

involvement and mutual connections between them. It is closely connected with locative predication, expressed by locative sentences ("John is here", "John is at home", etc.).

Combination ontology considers the universe taken in two aspects - of decomposition and composition, analysis and synthesis, trying to answer basic ontological questions - being its proper questions indeed.

The crux of the combination ontology is the theory of synthesis. In its more advanced versions (above all Leibniz, but also - at not the same scale, however - Wittgenstein) elements are joined together because of their internal factors, determiners, by which they are characterized. These characters (requisits in Leibniz's, internal properties in Wittgenstein's terminologies) are determinents of combinations in which given objects can be involved. In dynamic versions of the combination ontology they are, in addition, active and dynamic, like forces, being real sources of amalgamation. This is what, in the very end, makes the composition possible.

Determiners constitute the form of their objects. The form is the possibility of combinations, what makes them possible.

Combination ontology, as outlined above, is a formal discipline. Because of determiners' theory it is deeply modal, based on a suitable theory of ontological modalities (cf. [10], [12], [13]).

35. The fundamental idea of combination synthesis is that objects are combined according to their traits. They are - in part and secondarily - bond instructions, but - in the first place and primarily - they are sources of arrangement and bond of a resulting combination. This combination can be, in turn, the object of further combinations.

It can be characterized by its structure (the way in which elements are joined together into one) and network (bond system) as well as some secondary combination's effects, i.e. local fields of the complex, its integrity, combination's strata and, smaller or higher, stability.

Similarity of the main ideas of combination ontology to those used in sciences is striking. Particularly, to chemistry - in its bond nets and to set-theory - in the emphasis put in it on collections and operations. Hence, we can say shortly: *combination ontology equals set theory plus chemistry.*

As a matter of fact, combination ontology is very popular in science though implicit; much more popular than in philosophy where it is eclipsed by linguistic ontologies. It is, however, the main ontology of being.

36. Combinatorial ontology is a version of combination ontology, obtained by taking all collections of elements as combinable.

37. Combination ontologies are, in principle, pluralistic: Wholes are set together from many. Multiplicity of elements is primary, wholes are secondary. They are obtained from the elements by ontological synthesis.

On the contrary, transformational ontologies, quite akin to combination ones, usually are monistic. Primary whole (One) is differentiated by transformations and modifications. Plurality is given secondarily as the collection of states of the basic whole, which, in turn, can be involved in new unities. This plurality is thereby obtained by local perturbations, *modi* of the whole, expressing its transformations.

Transformational ontologies are the offspring of Spinoza's thought. At the present they are connected with geometrical interpretation of relativistic physics.

Unfortunately, until now they have no satisfactory ontologics.

Ontologies of thought

38. Most of thought ontologies are counterparts of ontologies of being. They are, in a sense, more free in constructing objects, following some rules of pure a priori, without necessarily entering into what is given and/or the nature of the world.

Usually thought ontologies are tied to appropriate logical calculi and mathematical theories providing them with ontological foundations and natural interpretations. These theories are, in fact, most clear and pure creations of the *a priori* (projection-combinatorial) power of reason.

39. The following sorts of thought ontologies should at least be mentioned:

(1) Ontology of concepts, with important variant: ontology of ideas. It is very close to object ontology.

On the other hand, relations between concepts are usually understood as truth relations. Hence, concept ontologies are closely connected with logic - what, however, cannot replace their own ontologics. Certainly, it is not the same to provide logic with interpretations instead of having an appropriate logic of concepts and relations between them.

At the present we have rather well-developed formal theories of concept's extension, but not concept's intension.

(2) Ontology of multiplicities, devoted to thought collections of objects into one, given by some principles or laws, as well as to relations between them.

Due to common subject they are closely connected with some ontologies of being, chiefly - combination ontology. In particular, multiplicities are more free in definitions and criteria of existence (usually only consistency is needed) than combinations.

Regarding ontologics, ontologics of multiplicities, particularly several set theories, are most advanced ontologics available at the moment.

(3) Ontology of types, or typical ontology, which is a variant of set-theoretical ontology considering universe as divided into types. It is particularly important for extension of set-theoretical approach to essential fragments of concept ontology.

(4) Algebraic ontologies, which are closely connected with function ontologies.

Whereas in previous cases emphasis was put on objects, now it is put on relations between them. The pattern of algebraic ontologies is Boolean ontology of truth-relations obtained by attempts to select laws (equations) characterizing the way in which information is modified. Boolean ontology is a transparent thought ontology; according to Boole himself, it was produced by looking for laws of thought.

(5) Ontology of constructions. It is, on the one hand, the operational counterpart of concepts' ontology. On the other hand, it is the counterpart of multiplicities ontology, hence combination ontology as well. It can be obtained by stressing the operational character of thought creations. Therefore, it can be treated as the best candidate to be the ontology of active mind (Brouwer). The world is pictured by it as full of thought creations, what particularly distinguishes it from its conjugate realistic ontologies.

Constructions' ontologic is quite rich. It contains many formal theories of constructions.

Ontologies of language

40. Of the realm of objectivized creations of man (the world of culture) language is best investigated and described.

In ontology the situation is similar - linguistic ontologies are most popular and advanced. It is so because of the common logicians' and philosophers' attitude to investigate language. This attitude is connected with quite popular conviction that the structure of the language mirrors the structure of the world.

41. Among language ontologies the following should certainly be mentioned here:

(1) Categorial ontologies, resulting from categorial analysis of language, particularly its division of expressions into independent, basic expressions (names, sentences, etc.), and dependent and auxiliary expressions (functors), with subsequent categorial grammar(s). Semantically, they are closely connected with functional and typical ontologies. On the other hand, they are similar to model-theoretic ontologies by apprehending the world as model of the language.

Which one, however?

Categorial ontology has at the present quite advanced ontologic - categorial grammar, chiefly concerned with syntactical categories. The parallel grammar of semantical categories is however still not satisfactory.

On the other hand, the connection *language - world* is not investigated enough, partly because of limitating the language ontologies to language alone. Particularly, consequences of the idea that the language is a part of the world are, in fact, not discussed. The question is quite sophisticated: If, for example, we assume that this inclusion entails the existence of one-to-one mapping between language and some bigger part of the world, we come to the conclusion that the world is infinite!

Categorial investigations frequently lead to opinion that "syntax mirrors ontology" (Bocheński's *dictum*, cf. [2]), strengthened sometimes to the claim that also reversely - "ontology mirrors syntax" (Buszkowski's *dictum*, cf. [3]). However, in the case of categorial ontologies it is not too astonishing. They are after all language ontologies; from the formal point of view, even more - ontologies of language's syntax.

Certainly, categorial ontologies, more generally - linguistic ones, are, in a sense, connected with the proper ontologies of being. They have not to be, however, considerated as ontologies of being. Otherwise, the logico-linguistic research would be the basic investigation of the world, what, however, is not in conformity with the common sense and the usual practice of scientists.

(2) Language ontologies of types, closely connected with their thought counterparts and the categorial ontology as well.

Two types of linguistic ontologies of nominal predication:

(3) Attribution ontologies, and

(4) Predication ontologies.

Both of them go with appropriate grammars different from categorial ones. They are also closely connected with object and function ontologies.

Semantical ontologies of being, after putting emphasis on the language in the semantical connection, become language ontologies as well. In this way we receive, i.e.:

(5) Model-theoretic ontologies of language (cf. particularly R. Carnap's [4] and A. Robinson's [14] approach to model theory);

Or, following Wittgenstein's *Tractatus:*

(6) Ontologies of situations understood as ontologies of semantical correlates of sentences, cf. Suszko [17], Wolniewicz [20] and [21], Barwise and Parry [1].

Ontologies of situations are closely connected with combination ontology, in which they can be defined.

Comparison and conclusion

42. We listed and commented on 18 variants of ontology, what certainly doesn't exhaust the full spectrum of ontologies. On the other hand, the number of reasonably differentiated types of ontologies is undoubtedly smaller.

The classification of ontologies into types has certainly not to be arbitrary. It should both follow ontologies' goals and consider their contents.

We considered previously two such classifications:

First, following opposite descriptions of synthesis mechanism, into

STATIC vs. DYNAMIC ontologies;

and the second, according to three main planes of being, into

BEING vs. THOUGHT vs. LANGUAGE ontologies.

In addition, at least three more natural, self-explaining classifications should also be mentioned:

The third, according to the nature of ontologies' objects, into

MODAL vs. NON-MODAL ontologies.

It is easy to see that the proper ontologies of being are modal.

The fourth, taking into account the way of doing ontology, into

DESCRIPTIVIST vs. CONSTRUCTIVIST ontologies.

They either try to describe or try to construct the ontological universe. Surely, the golden mean is the best. Particularly, ontologies of being should be - in proper proportion - both of this and that kind.

And, the fifth, regarding the role of the language in ontology: *message* vs. *medium*, into

LINGUISTIC vs. EXTRALINGUISTIC ontologies.

Bringing a given ontology into one type we decide, in fact, to what extent the language, including the language of ontology itself, should be taken into account. Moderation is welcome. Certainly, the language is an important but not alone component of the world.

43. Plurality of ontologies is not without a reason.

Namely, we are interested in different aspects of being. Its full picture shows itself, however, only through comparison.

44. The advanced and proportional development of the full spectrum of ontologies, particularly the basic ontologies of the three main types of ontologies of being, would certainly enable us (even more - force us) to discuss the fundamental problems from other philosophical disciplines, including the problem of knowledge and the problem of representation: *pure reason/mind - categories of pure reason/language - the world.*

The ontological foundations of these problems are provided by proportional (and advanced!) development of the appropriate ontologies of thought - language - being, and comparing them.

Also, other sides of the complexity of the world can be investigated by comparison of the advanced ontologies of different types and sorts.

45. Unfortunately, at the present we observe unequal development of particular ontologies.

Ontologies of language and ontologies of thought are more advanced than ontologies of being. Hence, usual tendency to substitute them for ontologies of being. This tendency can be restricted either by critique or by trying to develop ontologies which are neglected.

The latter way is certainly better.

46. Taking globally into account the above 18 sorts of the 3 types of ontologies we can see that they form a system.

First, there are natural "axes" to connect them. Types of factors correspond to types of ontologies: Ontologies of being work with determiners and combinations, ontologies of thought investigate multiplicities, whereas ontologies of language deal with properties expressible in language. However, multiplicities and combinations are collections, whereas determiners and properties are qualities (answering questions: How?, and What (like)?). Hence we have two axes: connecting being and thought ontologies (namely - *collections*), and connecting being and language ontologies (*qualities*). Observe that both axes refer to ontologies of being!

Secondly, among other ontologies, combination ontology is distinguished. It is primary ontology of combinations and elements, hence, the ontology of ontological substratum (substance).

Event and process ontologies as well as ontologies of transformations seem to be superstructures of combination ontology. Indeed, the event is a modification of combination(s), the process - recombination of events. Transformation is the way of modification, i.e. combination's combination. If

events, processes, transformations concern complexes then combination ontology is logically primary in respect to their ontologies.

Properties belong to complexes as a result of the latter's arising, becoming combined. In particular, the properties of real things or facts are secondarily in relation to constitution of them: the colour of my eyes belongs to me secondarily in relation to forming elements into combination being the support of my body. Thanks to the progress of science part of this forming can be now grasped.

On the other hand, language ontologies are ontologies of the language, apprehended - through its grammatical forms - in comparison with the real. Also, ontologies of thought are connected with the world, hence with ontologies of being as well. Thought ontologies are ontologies of the modifications of information, emerging from the world.

47. The listed ontologies don't compete with one another. To the contrary, they are connected and arranged in a system.

They speak about the world not only alone, forming pictures of it, but also together, by way of connecting them.

The plurality of the pictures is not an excess. To know something about the world we must apprehend and confront many of its pictures.

Through the multiplicity of its pictures the world reveals itself.

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Elizabeth Anscombe

TRUTH, SENSE AND ASSERTION, OR: WHAT PLATO SHOULD HAVE TOLD THE SOPHISTS

You will remember how Protagoras brought a kind of paradox to public attention. He contracted with a pupil to teach him advocacy. Half the fee was to be paid at once, the other half only if the pupil won the first case in which he pleaded. Time passed, the pupil brought no case. So Protagoras sued him ...

Protagoras and the other Sophists invented the argument, recorded by Plato, that there cannot be false thinking. "He who thinks what is false thinks what is not; what is not is nothing; so he who thinks what is false isn't thinking anything; so he isn't thinking"[1]. Plato tried to deal with this by introducing *the different*: if you think what is false you are thinking *the different from being*. An argument of Russell's shows this is no good. If you introduce something (call it *d*) such that you explain *not p* as *dp*, you still have to show that *dp* is incompatible with *p*, i.e. that it entails *not p*. You can't explain negation, and so not falsehood either, in any such way.

In Anselm's short book *De Veritate*[2], a dialogue between him and a pupil, the pupil is asked how he explains truth of propositions. Is it their sense? Is it their definition? and so on. No, he says, for all these are the same whether the proposition is true or false. "All I can say", he goes on, "is that a proposition is true when what it says is the case". We are made to understand that truth and rightness are the same.

The master asks now "What was affirmation created for?" and gets the reply "to say that to be the case which is the case". Note two things here:
1) since Anselm says we can make the parallel point about denial, it is fair to him simply to speak of assertion, and
2) assertion is an action, so the explanation of what assertion is for is not a mere repetition of what truth is in a proposition.
A proposition, as Wittgenstein says in the *Tractatus*, "shows how things are if it is true, and says that that is how they are"[3]. A human being uses a proposition himself to say that things are as the proposition says they are.

The pupil, who is no stooge, now asks his master: "What am I to reply if someone says that even a false proposition is doing its job in saying what it says?" - for it has been accepted that a true proposition is doing

what it should in saying what it does say, that being what is the case. If so, someone might say that a false proposition is in a way doing what it should in saying what it does say, and if a proposition is true by doing its job then even a false proposition is true.

The master does not deny this: instead he says there are two ways of being true. One, which he calls 'natural truth', is to be found in anything which is what it is, which is what it is meant to be. In a proposition, this truth consists precisely in saying what it does say, and this character is constant. But there is another way for it to be true, which is what we usually mean by calling it "true", and which Anselm calls 'non-natural truth'. This is the sense of truth in which a proposition is true only when what it says is the case, and this truth is variable and inconstant. (Aquinas took over this distinction from Anselm in the *Summa Theologiae*, though he does not, I think, mention his source.) We should note that unlike Russell and most modern logicians of the earlier part of this century, Anselm thinks that a proposition - a sentence - is variable in truth-value. "I am standing up" is true when I am, and becomes false when I am not standing up.

The twin character of truth and falsehood is strongly stressed by Anselm. "It could not be given to a proposition to say what *is* the case to be the case, if it were not also given to it to say that to be the case when it is not the case". In this discussion we can see a strong similarity to Wittgenstein in the *Tractatus*, who observes that we must remember that the sense of a proposition is 'independent of the facts' - i.e. of its truth-value - and that if we don't remember this "it becomes easy to think of truth and falsehood as equally justified relations between sign and what is signified". Then we'd say that, e.g., 'p' signifies in the true way what 'not p' signifies in the false way. However, he observes that we couldn't communicate with false propositions as hitherto with true ones, so long as we know they are meant to be false. "For if we use 'p' to say that not p, and what we think (*meinen*) is the case, then in the new way of speaking 'p' is true, not false"[4]. It is notorious that Wittgenstein scorned Frege's and Russell's assertion sign (*Urteilstrich*). But this passage shows that it does matter what we are asserting, and this indeed would show which forms are tautologous and which are contradictory.

That 'p' and '~p' can say the same (i.e. that they are exchangeable) is of importance, he says, because it shows that 'not' stands for nothing in the reality. It is interesting that Joan of Arc used the exchangeability to have a code for letters to her generals. Marked with a little cross at the top, they were to be taken in the sense which was contradictory of their normal sense. The blasphemy of marking lies with a cross was a charge made against her in her infamous trial. Alas, neither Anselm nor Wittgenstein were there to tell the court that these were not lies on her part.

I now come to my sub-title: what Plato should have said to the Sophists. They were not being merely silly-clever. No, they were pointing to a genuine problem. We have seen, I hope, that we can speak of what a proposition says just as much as of what a human being says. The human being, in straightforward uses of language, is saying just what the propositions, the sentences which he is using, say. Plato should have told the Sophists: "If you believe a true proposition, then it tells you something - namely, that that is the case, which it says is the case. But a false proposition, even if you believe it, does not tell you anything".

Some notes on this: the grammar of "tells" is different with a proposition as subject from what it is with a personal subject. A person can tell you falsehoods by saying false sentences to you in the process of ordinary communication by means of sentences. But a proposition does not *tell* you a falsehood if it is false and you believe it.

Further: I could hardly say Plato should have said this to the Sophists if he could not have said it in his Greek. But he could have: he could have used the word "didaskein" as I am using "tell". "Didaskein" means "to teach", but it could perfectly well be used here. (Like the German "lehren".)

Third, of course a false proposition (believed or not) can tell you various things - e.g. about the use of words in it. I once made the quite false remark "If you can eat any fish, you can eat any fish". I realised at once that it was false, but I was glad to have said it, because it is a nice illustration of the use of that word "any" with which the English language is (I believe uniquely) blest.

Now, when I say: a false proposition, even believed, tells its believer nothing, I do not mean it may not tell its hearer something in the sense of the last paragraph. I am formulating the truth that there was in what the Sophists argued. The proposition says something. If it is true, it tells its believer something (or, if you like, does so if he didn't already believe that thing): namely, just that thing that it says. But in this restricted application of "tell", it tells its believer nothing if it is false.

Plato would of course have had to argue, given the Sophists' argument about the impossibility of false thinking: "If you argue there can't be false thinking in that manner, you will surely also argue that there cannot be false saying: he who says what is false says what is not, etc.". Given that, he could have said what I am saying he should have said.

An almost final small note: I have not mentioned the virtuous occurrence of false propositions as subordinate clauses within other propositions. Neither did Anselm: indeed at one point he writes as if a false proposition was always failing to do its duty of saying that to be the case which is the case. Or else he meant by an *enuntiatio*, which I have translated

"proposition", only a complete proposition. If he was being careless, the carelessness is easily corrected.

It would remain to discuss what a proposition's saying what is false is. It cannot be: saying nothing. But that, deep question as it is, is beyond the scope of my present contribution.

Finally, it may be said of Protagoras' paradox: the right course for the court would be to say "Go away, and come back only when the pupil has been involved in another case, has won, and you have a dispute about pay". The propositions "The pupil owes the fee" and "He does not owe the fee" require a case other than the one Protagoras brought if they are to have truth-values. We have here an example of what Arthur Prior was fond of showing - namely, that whether a proposition makes sense (which I interpret as: does have a truth-value) can depend on the facts surrounding its production. This is a better solution than what I think Russell would have done: namely, invoked his idea of a "vicious circle fallacy" to dismiss the case.

NOTES

1. See *Theaetetus* 188 D3-E2, 189 A 11-14.
2. Anselm, *De Veritate*, Cap.2.
3. *Logisch-Philosophische Abhandlung* 4.022.
4. Ibid., 4.062.

Peter Geach

IDENTITY OVER TIME

My subject has nothing to do with that identity of which various groups (e.g. women, priests, homosexuals) are said to be in search; I am concerned with the logic of identity. However, the sense of identity that concerns me is one that bears on very practical questions, which have sometimes led to protracted and expensive litigation. Did the same horse run in the Derby and also, under another name, in other races? Was the derby winner Running Rein the same horse as the four-year-old Maccabeus? Again, was the man Castro, who returned to England from Australia, the same man as Roger Tichborne, who had emigrated many years before? Such questions cannot be settled by scrutiny of our own minds or our own linguistic practices; we must investigate what Frege called the *Bedeutung* of the names in question. To settle whether a horse is indeed identical with a certain four-year-old, it may be necessary to inspect the horse's teeth; to settle the identity of a man, we shall need to consider permanent bodily and mental traits. In the statement 'Running Rein is Maccabeus' we have a double mode of presentation (*Art des Gegebenseins*): this is what keeps the statement from being *trivially* true, like 'Maccabeus is Maccabeus'; but the statement does not affirm some *relation between* two modes of presentation; it is true because of what holds good of some *horse*, and its truth must be (had to be) established by investigation of that horse.

I once read a review of a book entitled 'Understanding Identity Statements'. Reviewers, even if sympathetic in intention, may seriously distort an author's meaning; so it will be fairest if I suppress the author's name, not having his own words to consult. If the reviewer was fair, the author fell into a number of serious confusions, ones which others certainly have fallen into; and I shall discuss these confusions on the road to a clear view. As Polonius said in Hamlet:

> Your bait of falsehood takes this carp of truth;
> And thus do we of wisdom and of reach ...
> By indirections find directions out.

Two of the confusions are suggested by the very title:
(1) The main way that identity enters into our discourse is in propositions saying that so-and-so is identical with such-and-such;
(2) This sort of propositions must be understood by seeing how they figure in *assertive* use, to make a *statement*.

In (2) we have a particular case of a sin against Frege's cardinal principle that the same proposition, taken in the same sense, may occur now asserted, now unasserted, and still has a truth-value if it is unasserted. But the general obviousness of Frege's thesis is sometimes obscured as regards some particular class K of statements. Let me use here a term I introduced in a paper in honour of Elizabeth Anscombe, "Kinds of Statement"[1]: I call a (grammatical) statement *normal* if it serves straightforwardly to assert what might otherwise be e.g. merely hypothesized (as in Alice's 'I only said *if* '). The sentence 'Jones is dishonest' could be used to make a normal statement; the sentence 'I do not suggest that Jones is dishonest', said assertorically, is not used to affirm that the speaker is *not suggesting* something, but is precisely used to suggest that Jones *is* dishonest; this is then a non-normal statement.

The Fregean insight is of course applicable only to normal statements; in a perverse desire to blind themselves to the Fregean insight, people will devise theories that some class of statements K are non-normal statements. If his reviewer did not misrepresent him, the author of 'Understanding Identity Statements' devised just such a theory:
(3) Identity statements do not affirm something true or false about the objects mentioned in them.

Whether or not this author got into such a confused way of thinking, others certainly have: for example, P. F. Strawson in *Subject and Predicate in Logic and Grammar*[2]. If we remind ourselves of the long legal battles that have sometimes been needed to get at the truth-value of an identity statement, thesis (3) will not appear attractive.

A further error is to be found in Strawson and others: what Elizabeth Anscombe has called the Fallacy of Being Guided by the Truth. The mistake consists in confining attention to what difference it makes to the structure of our knowledge and beliefs if we *discover* that so-and-so is identical with such-and-such; that is, in considering only *correct* identifications and forgetting that there may be *incorrect* ones. According to an anecdote, a British statesman had to learn that Burma and Bermuda are two different places; Strawson and others fail to consider how *such* discoveries affect a man's state of information.

I return to thesis (1). Here we find neglect of the multifarious ways in which identity and its Siamese twin difference enter into our discourse when no infirmative or negative identity thesis is presented. If a proper

noun is used in one definite naming use, then obviously an identity is presupposed. To be sure, equiform names get used and reused, like trousers in a numerous and indigent family. But in logic there is supposed to be a strict taboo against such use of equiform names, such a taboo as I have heard prevails in Africa against reusing the names of the dead, and in Japan against using a proper name equiform with the Mikado's name. L. J. Cohen indeed has criticized logicians for observing this taboo (in a paper published in the volume *Philosophical Subjects* in honour of P. F. Strawson[3]); I hope few will follow him.

Again, identity and difference are involved in the use of pronouns; I say 'Bill wounded a lion and Tom killed it'; medievals call 'it' a *relativum identitatis*. If I had rather said '... and Tom killed another (lion)', 'another' like 'it' would attach to the antecedent phrase 'a lion', as what the medievals would call *relativum diversitatis*. Again, certain prepositions like 'except' or 'but' express diversity; 'every donkey but Brownie ran in the race' may be paraphrased as 'Brownie was not running in the race but every *other* donkey (in a given region) was running'. We must nourish our thought about identity on a varied, well balanced, diet of examples.

The identity on which a given use of a proper name is founded is often identity over time. It is always, if I am right, identity by a certain criterion, which is built into that use. An intention hardly to keep referring to the same *thing* by a proper name is void for uncertainty, as English lawyers say; we must intend to refer always to the same *A* – and not every grammatically possible noun for which '*A*' stands in will be allowable. We need to have in mind a *criterion* of identity. For example, 'the same man' or 'the same dog' answers to a criterion of identity; but no definite criterion is expressed by 'the same material object', this phrase merely indicates a family of different criteria. If 'animal' is used in a wide sense, even 'the same animal' will not express one definite criterion that suits all animals; we cannot first identify a thing as the same animal and then characterize it by added *Merkmale*. How different is the identity of a man from that of an insect which for some time is a grub, or of an amoeba, or of the jellyfish called 'Portuguese man of war'! (In the last case it may be disputed what is *one* individual of the kind.) People sometimes talk of 'spatiotemporal continuity' as affording an all-embracing criterion of identity for material objects; but a moment's thought refutes this idea. A priest baptizes and names a baby; the atoms that then were in the baby's body may still exist forty years on, though afar and asunder, but this collection of atoms is *not* what the baptismal name was meant to name.

The difficulty of identity over time arises over the logical syntax of assignments of time (*Zeitangaben* in Frege's terminology). We accept that the same man may be slim in 1853 and fat in 1866; though regarding a famous

English law case which historians describe, such an identification was treated as, upon the face of it, unacceptable! But whatever view we take of the logic of *Zeitangaben* (Prior and Quine, for example, give quite different accounts[4]), there is a manifest incoherence in having *two Zeitangaben* for the same proposition, as in 'Running Rein at 2.30 won the race at 3.30'. How then can we construct 'A slim man in 1853 was the same man as the fat man in 1866'? *When* did the identity hold? or is the identity tenseless?

Quine would solve this problem by appeal to an ontology of four-dimensional objects. A man, Robert say, extends a little way in three spatial dimensions and seventy years in time; a segment of Robert occupying the year 1853, Robert-in-1853, has smaller spatial extension than Robert-in-1866; Robert-in-1853 and Robert-in-1866 are different, indeed qualitatively different, slices of the one man Robert.

Quine's ontology is unacceptable to me in this matter; I have spelled out my reasons in the paper 'Some Problems about Time', in the book *Logic Matters*[5]. Like Prior, I regard such phrases as 'Robert in 1853' as syntactically incoherent fragments of a sentence, wrongly extracted from it and treated as complex designations. For me as for Aristotle names are tenseless, *aneu chronou*, and to tag them with *Zeitangaben* is simply nonsensical; *Zeitangaben* can attach only to sentences or predicative parts of sentences. But then to what do they attach in this sentence?

Robert was slim in 1853 and was fat in 1866.

I think the structure may be displayed by this paraphrase:

As regards some man Robert: he was in 1853 slim
and he was in 1866 fat.

The *Zeitangaben* attach to two separate predicative parts of the sentence; *pace* various Oxford philosophers, there is no sense in asking after the reference of 'he', any more than that of the variable 'x' in:

As regards some man x: x was in 1853 slim
and x was in 1866 fat.

Finally, 'man' and 'Robert' are for me two tenseless names, standing in apposition, a shared name and a proper name; for, like many Polish logicians, I recognize names of both sorts.

I end with a problem I cannot yet solve: the problem of what are called phase sortals, like 'baby' and 'caterpillar'. Syntactically, how could shared names of this sort be disqualified as names? And could not *proper* names have corresponding uses, as when a child loses its childish name after a *rite de passage*? If a little girl has a pet caterpillar called 'Epaminondas', *must* she give this name to the corresponding chrysalis and butterfly? I have no

final answer as yet; but I think the Aristotelian insight that names are tenseless should not be lightly abandoned.

NOTES

1. In: *Intention and Intentionality: Essays in Honour of Elizabeth Anscombe.* Worcester Press Ldt, Brighton, Sussex, 1979.
2. Methurn and Co Ltd, London 1974, pp. 51-56.
3. Oxford University Press, 1980; paper entitled "The Individuation of Proper Names".
4. For Prior, cf. e.g. the paper "Thank Goodness That's Over", in: *Papers in Logic and Ethics,* Duckworth, London, 1976. For Quine cf. e.g. *Word and Object,* M.I.T. Press, Boston, Mass., 1960, pp.172-173.
5. Published by Basil Blackwell, Oxford, 1972, pp.309-311.

Josep M. Font, Ventura Verdú

TWO LEVELS OF MODALITY: AN ALGEBRAIC APPROACH*

This paper deals with the study of abstract properties of several closure operators related to several relations of logical consequence in some modal logics, namely those associated with algebraic models. It is an attempt to study modal logics within the framework of the theory of abstract logics. We begin by recalling some terms, constructions and notations, mainly taken from Brown & Suszko (1973).

1. **Definition**: An *abstract logic* is a pair $\mathbf{L} = \langle \mathbf{A}, C \rangle$ or $\langle \mathbf{A}, \mathbf{C} \rangle$ where $\mathbf{A}$ is an abstract algebra, C is a closure operator on A (the carrier of $\mathbf{A}$) and $\mathbf{C}$ is the associated closure system.

Thus an abstract logic is something more than a closure operator: it incorporates the algebraic structure. We hope the readers will find in this paper some reasons for the use of the whole concept.

2. **Definition**: If $\mathbf{L}_1$ and $\mathbf{L}_2$ are two similar abstract logics then a *bilogical morphism* between $\mathbf{L}_1$ and $\mathbf{L}_2$ is an epimorphism f from $\mathbf{A}_1$ onto $\mathbf{A}_2$ which projectively generates $\mathbf{L}_1$ from $\mathbf{L}_2$, that is, which satisfies that $f^{-1}(\mathbf{C}_2)$ is a basis of $\mathbf{C}_1$. The set of all bilogical morphisms between $\mathbf{L}_1$ and $\mathbf{L}_2$ is denoted by $\mathrm{Epi}^*(\mathbf{L}_1, \mathbf{L}_2)$.

When there is a bilogical morphism between two logics, then it establishes an isomorphism between the two closure systems $\mathbf{C}_1$ and $\mathbf{C}_2$ taken as lattices, and moreover for any $a,b \in A_1$, $f(a) = f(b)$ implies $C_1(a) = C_1(b)$. Thus we can say that, in a sense, bilogical morphisms are identifications which respect the algebraic and the logical structure of abstracts logics. The strongest identification consists precisely in identifying all elements with the same closure:

3. **Definition**: For any closure operator C on A, the *associated (natural) relation* is $\Theta = \{(a,b) \in A \times A : C(a) = C(b)\}$.

The relation Θ is always an equivalence relation and so we can form the quotient A/Θ, consider the canonical projection π: $A \longrightarrow A/\Theta$ and define the following set $\mathcal{C}/\Theta = \{T \subseteq A/\Theta : \pi^{-1}(T) \in \mathcal{C}\}$. It is always a closure system, and:

4. Definition: If $\Theta \in Con(\mathcal{A})$, the congruences of the algebra $\mathcal{A}$, then the *(natural) quotient logic* is $\mathbf{L}/\Theta = \langle \mathcal{A}/\Theta, \mathcal{C}/\Theta \rangle$.

In the cases where $\Theta \in Con(\mathcal{A})$ we have that the canonical projection is a bilogical morphism between $\mathbf{L}$ and its quotient $\mathbf{L}/\Theta$. Now a question naturally arises: When do we have that $\Theta \in Con(\mathcal{A})$? Of course a fully general answer is not very interesting, as the quotient A/Θ is always an ordered set (under the relation $a/\Theta \leq b/\Theta$ if and only if $C(b) \subseteq C(a)$!) and so we should specify which kind of ordered algebraic structure and which kind of quotient logic do we want to find in the quotient. As an example of how the method works and as a basis for the rest of the paper we recall here the case of intuitionistic logic, which is very "standard".

5. Definition: Let $\mathcal{A} = \langle A, \wedge, \vee, \rightarrow, \neg \rangle$ be an algebra of type (2,2,2,1), and $\mathbf{L} = \langle \mathcal{A}, C \rangle$ an abstract logic over it. We say that $\mathbf{L}$ is an *intuitionistic logic* if and only if it satisfies the following:

(1) C is algebraic (or finitary);

(2) $C(a \wedge b) = C(a,b)$ for all $a,b \in A$;

(3) $C(X, a \vee b) = C(X,a) \cap C(X,b)$ for all $a,b \in A$ and $X \subseteq A$;

(4) $a \rightarrow b \in C(X)$ iff $b \in C(X,a)$ for all $a,b \in A$ and $X \subseteq A$;

(5) $\neg a \in C(X)$ iff $C(X,a) = A$ for all $a \in A$ and $X \subseteq A$.

Given any algebra $\mathcal{A}$ of type (2,2,2,1) we can define on it a syntactic consequence operator by using axioms and rules in the usual way. If $\mathcal{A}$ is the algebra of formulas, then it is well-known (see for instance Grzegorczyk, 1972) that this gives the least intuitionistic logic over $\mathcal{A}$. The other important group of intuitionistic logics is that of all logics of the form $\langle \mathcal{H}, \mathcal{F} \rangle$, where $\mathcal{H}$ is a Heyting algebra and $\mathcal{F}$ is the closure system of all its filters. In a sense, these are "all" intuitionistic logics, up to a bilogical morphism, as is stated in the following:

6. Theorem: Let $\mathbf{L}$ be an abstract logic over an algebra of type (2,2,2,1). Then the following conditions are equivalent:

(1) $\mathbf{L}$ is an intuitionistic logic;

(2) $\Theta \in Con(\mathcal{A})$, $\mathcal{A}/\Theta$ is a Heyting algebra and $\mathcal{C}/\Theta$ is the set of all its filters; and

(3) There is an $f \in Epi^*(\mathbf{L}, \mathbf{L}')$ with $\mathbf{L}' = \langle \mathcal{H}, \mathcal{F} \rangle$ for some Heyting algebra $\mathcal{H}$.

The method of proof is not difficult: From (1) one checks each one of the statements of (2); from (2), take $\mathbf{L}' = \mathbf{L}/\Theta$ and the canonical projection is the required bilogical morphism; and finally from (3) we prove (1) because $\langle \boldsymbol{H}, F \rangle$ is an intuitionistic logic and all conditions of Definition 5 are preserved under bilogical morphisms.

In this Theorem we see some connections between logics and congruences. These relations can be precisely stated as in the following:

7. **Proposition:** Given an algebra $\boldsymbol{A}$ of type (2,2,2,1), the correspondence which assigns to every closure operator C its associated natural relation Θ is an isomorphism between the lattices of all closure operators C on A such that $\langle \boldsymbol{A}, C \rangle$ is an intuitionistic logic, and of all congruences Θ of $\boldsymbol{A}$ such that $\boldsymbol{A}/\Theta$ is a Heyting algebra.

The constructions and processes used in Theorem 6 and Proposition 7 are very general, and can be applied to a lot of cases; mostly those of a syntactic origin, as the fragments or weakenings of intuitionistic logic, but also to some with a semantic definition, as the four-valued "De Morgan" logics treated in Font & Verdú (1988). It is also worth noting that in Bloom & Brown (1973) there is a result for classical logics similar to our Theorem 6, without part (2); but the method used there is not that of quotient logics, but dual spaces, which is typically Boolean, and so not suitable for generalization.

Now we pass to modal logics. We treat only four systems, although some of the constructions can be applied or generalized to other cases. The systems we treat are the well-known ones *S4* and *S5*, and their intuitionistic counterparts *IM4* and *IM5*. Remark that we take these systems with necessity $\Box$ as the only primitive modal operator, so in the case of the intuitionistic systems there is no longer the classical duality between necessity and possibility. Another more important remark is that all these systems incorporate the *Rule of Necessitation*, but it can be taken in two different senses: as a rule for theorems (if $\vdash \psi$ then $\vdash \Box\psi$) or as a full inference rule, that is, as a rule for all theories ($\psi \vdash \Box\psi$). This distinction is not important when one considers logics as sets of formulas closed under the rules, but of course it is when one considers logics as operators, as is our case. Thus each one of the systems listed above actually defines two different closure operators, which have the same theorems. We will consider two closure operators on every algebra, and the abstract formulation of the modal properties will consist in a relation linking both operators.

Only the case of IM4 will be dealt with in some detail, the other three ones being obtained as extensions of the former. We begin by considering its algebraic models.

8. Definition: An algebra $\mathbf{H} = \langle H,\wedge,\vee,\rightarrow,\neg,I\rangle$ of type $(2,2,2,1,1)$ is a *topological Heyting algebra* (*THA* from now on) iff it satisfies:

(1) The reduct $\mathbf{H}^- = \langle H,\wedge,\vee,\rightarrow,\neg\rangle$ is a Heyting algebra; and

(2) The unary operation $I: A \longrightarrow A$ is an interior operator.

In every THA we can consider the closure system $\mathbf{F}$ of all *filters* of the underlying lattice, and also that $\mathbf{F}^+$ of all *open filters*, that is, the closure system $\mathbf{F}^+ = \{T\in\mathbf{F} : I(T)\subseteq T\}$. Therefore we can associate with each THA three different abstract logics:

9. Notation: If $\mathbf{H}$ is a topological Heyting algebra, $\mathbf{F}$ is the closure system of all filters, and $\mathbf{F}^+$ is that of all open filters, then we put:

$$\mathbf{L}(\mathbf{H}) = \langle \mathbf{H}, \mathbf{F}\rangle \ ; \ \mathbf{L}^-(\mathbf{H}) = \langle \mathbf{H}^-,\mathbf{F}\rangle \ ; \text{ and } \mathbf{L}^*(\mathbf{H}) = \langle \mathbf{H},\mathbf{F}^+\rangle.$$

We already know that $\mathbf{L}^-$ is intuitionistic, and moreover there is an interesting property linking both operators: for any $X\subseteq A$, $\mathbf{F}^+(X) = \mathbf{F}(I(X))$, that is, the open filter generated by any set is the ordinary filter generated by its interior. This was proved for the first time by Font (1984); see also Font (1987) for further algebraic properties of THAs and of their congruence lattices. The two properties just referred to are enough to characterize the logics $\mathbf{L}(\mathbf{H})$ and $\mathbf{L}^*(\mathbf{H})$, as we will see in Theorem 12. In order to be able to state it we need the abstract counterpart of the constructions just performed on THAs.

10. Notation: Let $\mathbf{A} = \langle A,\wedge,\vee,\rightarrow,\neg,I\rangle$ be an algebra of type $(2,2,2,1,1)$. We call $\mathbf{A}^-$ its reduct obtained by dropping the last unary operation, $\mathbf{A}^- = \langle A,\wedge,\vee,\rightarrow,\neg\rangle$.

Let $\mathbf{L} = \langle \mathbf{A},C\rangle$ be an abstract logic over such an algebra. Then we put $\mathbf{L}^- = \langle \mathbf{A}^-,C\rangle$. *Moreover, we consider* $\mathbf{C}^+ = \{T\in\mathbf{C} : I(T)\subseteq T\}$, which is always a closure system, and put $\mathbf{L}^* = \langle \mathbf{A},C^+\rangle$.

With the above notations we are prepared to define a first concept of abstract modal logic, which will be somehow justified *a posteriori:*

11. Definition: An abstract logic $\mathbf{L}$ over an algebra of type $(2,2,2,1,1)$ is an *IM4 logic* iff $\mathbf{L}^-$ is intuitionistic and $C^+ = C\cdot I$.

This definition enables us to find a Characterization Theorem similar to Theorem 6:

12. **Theorem**: The following conditions are equivalent:

(1) $\mathbf{L}$ is an IM4 logic;

(2) $\theta \in Con(\mathcal{A})$, $\mathcal{A}/\theta$ is a THA, and $\mathcal{C}/\theta$ is the set of all its filters; and

(3) There is an $f \in Epi^*(\mathbf{L}, \mathbf{L}(\mathcal{H}))$ for some THA $\mathcal{H}$.

Moreover, we see that IM4 logics are characterized by the logics of their open theories; this is what the following proposition states:

13. **Proposition**: If $\mathbf{L}_1$ and $\mathbf{L}_2$ are two IM4 logics such that $\mathbf{L}_1{}^* = \mathbf{L}_2{}^*$, then $\mathbf{L}_1 = \mathbf{L}_2$.

Now looking back at Definition 11, it is obvious that all the non-modal part of the work has been done by the first condition, while the equality $C^+ = C \cdot I$ concentrates all the modal information. May be it seems to be lacking of a direct modal interpretation, but it can be proved, in some sense, that it is exactly equivalent to the algebraic properties defining an interior operator. Compare the following result with § 2.3.1 of Wójcicki (1988):

14. **Proposition**: The following conditions are equivalent:

(1) $C^+ = C \cdot I$;

(2) $I(C(I(X))) \subseteq C(I(X))$ and $C(X) \subseteq C(I(X))$ for all $X \subseteq A$; and

(3) The mapping $X \longrightarrow C(I(X))$ is a closure operator on A.

Theorem 12 and Proposition 13 do not say much about C^+, which appears there only indirectly, as a by-product of the properties of C. To express its own properties we define a new algebraic structure on the same universe A, and associate with $\mathbf{L}$ another abstract logic $\mathbf{L}^+$:

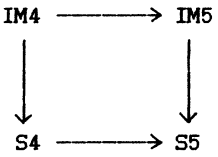
15. **Notation**: Let $\mathcal{A}$ be an algebra of type (2,2,2,1,1) as before. We put $a \vee^+ b = Ia \vee Ib$; $a \rightarrow^+ b = Ia \rightarrow b$; and $\neg^+ a = \neg Ia$. Then the algebra $\mathcal{A}^+ = \langle A, \wedge, \vee^+, \rightarrow^+, \neg^+ \rangle$ is an algebra of type (2,2,2,1) and we put $\mathbf{L}^+ = \langle \mathcal{A}^+, C^+ \rangle$.

We can also take the pair $I(Ia \rightarrow Ib)$ and $I \neg Ia$ instead of $Ia \neg b$ and $\neg Ia$ for $\rightarrow^+$ and $\neg^+$. In any case the properties of C^+ can be summarized by saying that if $\mathbf{L}$ is an IM4 logic then $\mathbf{L}^+$ is an intuitionistic logic. But this property is more important, because of the following result:

16. **Theorem**: An abstract logic $\mathbf{L}$ is an IM4 logic if and only if the logics $\mathbf{L}^-$ and $\mathbf{L}^+$ are both intuitionistic and have the same theorems, that is, they satisfy $C(\emptyset) = C^+(\emptyset)$.

According to this result, IM4 logics have a double intuitionistic character, but this is hardly surprising, because the Gödel translation already showed that the classical S4 has an intuitionistic character.

Theorem 16 reduces in some sense the study of an abstract modal logic to the study of two abstract non-modal logics. This fact makes also possible a unified treatment of the three other systems announced before as extensions of IM4:



As is well-known, vertical extensions are performed by adding for instance the law of the excluded middle $\psi \vee \neg\psi$ to the intuitionistic basis. Horizontal extensions can be performed by adding the axiom scheme $\Box\psi \vee \Box\neg\Box\psi$ to the modal axiomatics, but note that this is excluded-middle written in the new operations: $\psi \vee^{+} (\neg^{+}\psi)$. This fact and previous work with the algebraic models of these systems (see Font, 1984a) lead us to the definitions of abstract logics corresponding to each one of the extensions, and to the respective Characterization Theorems. The situation is summarized in the following table:

System	$\vdash^{-}$	$\vdash^{+}$	Algebraic Models
IM4	intuitionistic	intuitionistic	topological Heyting algebras
IM5	intuitionistic	classical	semisimple THAs
S4	classical	intuitionistic	topological Boolean algebras
S5	classical	classical	monadic Boolean algebras

That is, we define abstract logics of type IM5 (S4, S5) as IM4 logics such that $\vdash^{+}$ ($\vdash^{-}$, both) is classical. And for each class of abstract logics we can prove a result similar to Theorem 12 above, only by putting in it the corresponding class of algebraic models in the place of THAs, wherever they appear.

We end this paper by comparing several approaches to modal logics, and in particular to these four systems. These approaches have in common the property that there is a clear distinction between the modal part and the non-modal part of the formulations they make, and in all of them we find **a key property**, of a metalogical character, which can work both in the non-modal and in the modal levels, giving in the first case classical systems from intuitionistic ones (the vertical extensions of the diagram above), and giving in the second case systems of **type 5** from systems of **type 4** (the horizontal extensions). The following table summarizes our analysis:

Approach	Key Property	$\downarrow$	$\rightarrow$
axiomatization	excluded-middle, Peirce's law...	$\vee,\neg,\rightarrow$	$\vee^+,\neg^+,\rightarrow^+$
abstract logics	being classical	$\mathbf{L}^-$	$\mathbf{L}^+$
algebraic models	semisimplicity	H^-	H
relational models	symmetry	R_I	R_M
higher-level sequents	multiple-conclusion property	level 1	level 2

We have already commented the first two lines. For the third, observe that Boolean algebras are the semisimple Heyting algebras, and that monadic Boolean algebras of Halmos (1955) are the semisimple topological Boolean algebras. But there are also a number of interesting works on relational models for intuitionistic modal logic, beginning with Ono's (1977) paper, and including papers by Fischer-Servi, Sotirov, Božić, Došen, and the recent ones by Yokota. The idea is to put together Kripke models for intuitionistic logic and for modal logic. Thus these models have a set of "possible worlds" with (at least) two accessibility relations, one R_I to define the forcing relation when dealing with non-modal connectives, and the other one R_M for the modal connectives. Models for what we call *IM4* have both relations reflexive and transitive, and the models for the other systems are obtained by adding symmetry, to R_I for the vertical extensions, and to R_M for the horizontal ones. And finally one should note the syntactic formulation of Došen (1985, 1986) using sequents of higher level. Here the key property might be called "multiple conclusion property", and it is to allow more than one formula in the right part of the turnstile. As is well-known, when applied to ordinary sequents (here of level 1) this gives classical logic out from intuitionistic logic. And it happens that the same property, when applied to sequents of level 2 (sequents of sequents) gives the horizontal extensions.

These aesthetical (or philosophical) comments can be summarized by saying that in all these formalizations of modal logic (intuitionistic and classical) we can express in a formal way the **fact** that there are two **levels** or two **kinds** of modality, that of **type 4**, which has an intuitionistic character, and that of **type 5**, which has a classical character. In this sense, we can add more arguments to support Bull's (1965, p.3) and Došen's (1985, p.7) claims that IM5 should be considered "intuitionistically implausible" or "spurious". In any case, it is clear that philosophical discussion about modal logic is still open.

NOTE

* This is a summary of a talk delivered by the first author in the 33rd Conference on the History of Logic held in Kraków on 27-29 October 1987. It contains the results of joint work together with several classical ones which form a basis for further developments. A longer and far more comprehensive paper, containing all proofs of the technical part, has been published in Font & Verdú (1989).

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Bogusław Wolniewicz

ELZENBERG'S LOGIC OF VALUES

1. Values are what our value-judgements refer to, and the passing of judgements is one of our vital activities, like sleeping and breathing. We constantly appraise things as good or bad, pretty or ugly, as noble or base, well-made or misshapen. No wonder that both the act of appraisal and that which it refers to - i.e. the real or spurious values - have been always the source of philosophical reflexion. In systematic form such reflexion is what we call *axiology*.

In Polish philosophy it was Henryk Elzenberg (1887-1967) who reflected upon matters of axiology most deeply and incisively. However, not all would agree to that. To take an example at hand, we have seen recently the announcement of a book called *Man and Moral Values: Studies in the History of Independent Ethical Thought in Poland*.[1] It presents the views of eleven well-known Polish authors who have all dealt with questions of axiology, and they are the following: Twardowski, Kotarbiński, Witwicki, Czeżowski, Ossowska; then Tatarkiewicz, Petrażycki, Znamierowski; and finally Abramowski, Krzywicki and Jan Hempel. Elzenberg is out, *absentia fulgens*. Evidently, the editors have not deemed him a worthy enough representative of "independent ethical thought". It is even possible to guess why, for there is something in common to the eleven authors mentioned, in which Elzenberg was definitely wanting. They all share a confidence in man as the agent of social and cultural progress, with the first eight representing that confidence in its liberal and positivistic variant, and the remaining three representing it in the socialist and Marxist one.

Elzenberg did not share that confidence in man and progress in the least. In a draft of 1952, with the heading "Human Affairs and My Way of Thinking", he wrote:

> What about history? There is in it no trend towards the better, nor any *sense*. In this regard my position is definite. /.../ Eventually the good is always to lose. In society there may well be people benevolent and wise; society itself is always evil and stupid: its aspirations are mean, and its beliefs trite. It is infinitely more evil than the average decent individual.[2]

In that historical pessimism Elzenberg was out of tune with his time, and he was very much aware of his isolation. To the last of his books - a collection of papers covering the whole period of his intellectual activity - he gave the telling title "Trying Communication"[3]. He had his fans, but he certainly had no resonance.

2. In Elzenberg's system of axiology the whole field of inquiry is cut in two: into "formal" and "material" axiology. The former does not pronounce on what is good or evil, fine or foul. It passes no value-judgements. What it puts forward and argues for are merely hypothetical statements of the form "if this is of value, then something else has to be the case".

The field of formal axiology is marked out by three questions: (1) what are values? (2) how do we come to know them? (3) in what way do they affect our souls?. To each of these questions there corresponds a different division of formal axiology. The first leads to a *logic* of values which is to analyse the concept of value in all its ramifications; the second gives rise to an *epistemology* of values investigating the ways value-judgements may be justified; and the third is answered by an *anthropology* of values describing their "dynamics", so to say. For values are forces, even if their efficacy is revealed only indirectly by suitable monitors. These monitors are the sentient and rational human souls.

Material axiology, on the other hand, puts forward propositions of a categorical form: "this is good, that is bad; this is of some value, that has none". Thus it points out definite values, correctly or not.

By their very nature the statements of material axiology are highly controversial. Let us take just one example. Over many years of his life Elzenberg was pondering on questions of nationality, statehood and war. (He had been himself a soldier of the First Brigade, and then a volunteer in the Polish-Soviet war in 1920.) His views, however, were very different here from the presently current ones. They embody a philosophy of pessimistic idealism, akin to that of Joseph Conrad. As Elzenberg put it in 1936: "Such views offend these days the Western mind. But this has not always been so. Whoever sticks to that kind of pessimism is in good and numerous company".

Here are some of the more staggering statements of Elzenberg's material axiology, stemming from the years 1909-1919:

> The tenet which I take as my point of departure is that of the supremacy of ideas over life. /.../ A nation is an idea. /../ So consider what might be the *interests* of an idea. Evidently its power, i.e. that the nation in question be powerful as such. And this means it should have an impact on the life of mankind. /.../ Certainly, both may happen in a purely ideal domain. Thus a nation subjugated, but with poets, artists, scientists, philosophers, possibly also founders of religions,

> and all so great as to impose their bend of mind upon the world (like ancient Greece) will be a powerful one. Consequently, the first necessity for a nation is to have its culture as great as possible.[4]

These words are easily misunderstood, so let us observe what they really amount to. It is a primary interest of any nation to have its culture great, but only in so far as the nation is taken as an idea, not as a flock of people. For the interests of such flocks pertain to quite different things, like food or fun, and culture is concerned then only indirectly, as a kind of stock yielding sometimes nice dividends.

On Elzenberg's view the existence of nations is inseparably connected with the phenomenon of warfare. This is so because - as he put it - "our country ceases to be our country when we are not willing any more to fight for it"[5]. This willingness is to him the simple and basic criterion of there being a nation at all.

Here is another piece of Elzenberg's material axiology, going even more against the grain of modern opinion:

> War! In a battle 20.000 men get killed: everybody is upset and horrified. But none of those men, to be sure, was to live forever: so what is the difference whether they die one after another over a period of some thirty years, or all on one day? There is only a difference to the sensitivity of those who cannot stand the *sight* of a battle-field covered with dead and wounded, but who can stand very well the thought of thousands of people dying daily all over the world in their beds, and whose appetite is not spoiled by that thought. But let Vereschagin paint out for them in colour heaps of skulls and corpses, and they are going to cry and wring their hands. It's all just nerves, not the heart!
>
> But had they lived, they might have accomplished something. -Not at all: most would have lived uselessly. /.../ And so, by dying in combat, they left their writ on the pages of history, where that battle is to stay as one of the most splendid feats mankind is capable of. Thus the ideal interests of man do not suffer a loss here; on the contrary, they gain.
>
> Maybe you are going to say that those twenty thousands dead are twenty thousands of miserables? - Not at all again, as death is by itself no misery, and as life by itself is of no value whatever.[6]

Such statements were put forward by Elzenberg just before the outbreak of the First World War and shortly after its end. Clearly, they are highly disputable, especially in view of the experiences of the Second one and its aftermath. We shall not dwell upon them here, observing merely that corresponding modifications of Elzenberg's views would certainly not be in the direction of an individualistic pacifism which tends to regard war "as a

lunacy and a horrifying violation of human nature"[7]. Let us note that in the words of Elzenberg we find the same spirit as that which spoke some eighty years earlier in those of Marquise de Custine addressed to his French contemporaries:

> When finally our cosmopolitan democracy succeeds in making war abominable to whole populations; when the nations deemed the most civilized enfeeble themselves completely by their political gambols, falling eventually asleep; and when any alliance with such totally egoistic peoples will prove impossible, - then, at least, the sluices of the North will open up, and we shall sustain the ultimate invasion: not any more of ignorant barbarians, but of masters cunning and astute, - much more astute than we are, for taught by our vagaries how to govern.[8]

And with this we leave the controversial field of Elzenberg's material axiology, turning to its formal part.

3. Leibniz had said somewhere: "There are two mazes in which the human mind is most likely to get lost: one is the concept of continuity, the other is that of liberty". This admits of generalization: all concepts are mazes, *viz.* mazes of logical relations between the propositions that involve them.

One such maze is the concept of 'value'. Possibly it is even the same as one of the two mentioned by Leibniz, only entered - so to say - by another door. For it would be in full accord with Elzenberg's position - and with that of Kant too - to adopt the following characteristic: values are what controls the actions of free agents. Thus the concepts of value and of liberty should constitute one conceptual maze, or - which comes to the same - two mazes communicating with each other.

To get a survey of such logical maze the first thing is to fix the ontological category of the concept in question. Thus, in our case, we ask what kind of entities are those 'values' supposed to be. (Ontological categories are the most general classes of entities, the *summa genera*. A term even more general has to cover literally everything: like 'entity' or 'something'. For everything is an entity, just as everything is a something.)

Different ontologies admit different sets of categories. The categories most frequently referred to are those of 'objects', 'properties', and 'relations'; the more exotic ones are those of an 'event', a 'set', a 'function', or a 'situation'. One point, however, is of paramount importance: the categories admitted in one ontology have to be mutually disjoint. Thus if C_1 and C_2 are two categories of the same ontology, their intersection has to be empty:

$$C_1 \neq C_2 \Rightarrow C_1 \cap C_2 = \emptyset.$$

If, for instance, functions are regarded in set theory as a variety of relations, and relations as variety of sets, then only sets are eligible as a category there. In Frege, on the other hand, it was the other way round: sets and relations were regarded as varieties of functions. Hence functions are one of his categories, whereas sets and relations are not.

What is the category of values? Are they a variety of objects, or rather of properties, or perhaps of yet something different? In any case we assume them to be ontologically *homogenous*, i.e., all to fall into one category:

(1) $$V \subset C_i ,$$

where C_i is one of the categories of our ontology.

On the other hand, values are the *goals* of human action and aspirations; they are - at least some of them - what people want and seek:

(2) $$V \cap G \neq \emptyset .$$

Note that we do not exclude the possibility of there being worthless goals of human strivings, nor of there being values nobody is ever striving for. We merely assume that there is something which is at the same time a value and the goal of somebody's striving.

Finally we assume that goals are never objects, always some possible states of affairs, or *situations*. Thus

(3) $$G \subset S .$$

For instance, the goal is never an automobile, but the possession of one; nor America, but going to America or living there; not clear water, but a situation such that clear water comes from our taps, or is found in the Bay of Gdańsk.

It is patent that under the assumptions indicated the category of values must coincide with that of goals:

$$C_v = C_g .$$

Indeed, $V \subset C_v$ and $G \subset C_g$. Hence by (2): $C_v \cap C_g \neq \emptyset$. And as categories are disjoint, the last may hold only under the identity indicated.

All goals are situations, so the same must apply to values, i.e.:

$$V \subset S .$$

An axiology taking all values to be of one category might be called *monocategorial*. Such was that of Elzenberg, though he vacillated whether to make his only category that of situations, or rather that of substances in Aristotle's sense, i.e. of entities which are capable of change without loss of identity (*cf.* "Categories", 4^a10).

4. The main problem of Elzenberg's formal axiology arises in view of the fact that apparently the term 'value' covers two different concepts which it is very hard to tell from one another. For they both fall into the same category of situations, and both indicate goals of human aspirations. For one of these concepts Elzenberg coined the appellation "utilitarian value", for the other that of a "perfective" one. The difference is to be roughly this: a utilitarian value is what gratifies somebody's needs or desires, a perfective value is what makes the world as a whole a more perfect one. Thus utilitarian values are always relative: they are values *to* somebody. Perfective values, on the other hand, are supposed to be absolute.

The first difficulty in trying to refine that rough and ready distinction is that it does not yield a classification. Elzenberg insisted correctly that what we deal with here are not two kinds of values but two quite different concepts of value. Should they be two kinds of values - like two kinds of aeroplanes: the mono- and the bi-planes - then they would be disjoint as sets. But to have utilitarian value (e.g. a market price) does not exclude having the perfective one too (e.g. some aesthetic quality). An axiology admitting utilitarian values only is *naturalistic*. The question is whether any other is conceivable.

5. Elzenberg tried to pin down the difference in question by relating the concept of perfective value to that of *obligation*, and defining it as that *which is as it ought to be*. Values being situations, i.e. entities represented by propositions, his definition may be put down as follows:

$$V^{+}p \quad \text{iff} \quad (Op \text{ and } Fp) ,$$

which we read as: 'it is good that p if and only if it ought to be the case that p and as a matter of fact it is the case that p'. Substituting, e.g., the proposition 'somebody advocates a pessimistic idealism' for p, we obtain the formula: 'it is good that somebody advocates a pessimistic idealism' means 'somebody ought to advocate it, and as a matter of fact somebody does'.

Elzenberg's definition, however, misses its point, for it does not differentiate perfective values from the utilitarian ones. Obligation itself may be taken either in a perfective, or a utilitarian sense. This is readily seen in examples stemming from Elzenberg himself, like 'a judge ought to be impartial' and 'a train ought to be on time'. Granted that we deal here with two different concepts of obligation, it is just as difficult to tell them apart as the two concepts of value themselves.

So Elzenberg tried yet another expedient. For saking a universal criterion he looked now merely for a way of showing that perfective obligation is not an empty concept, i.e., that there are situations to which it conceivably applies, while the other does not. To that purpose he defined

in the first place the utilitarian concept of obligation, looking then for an example in which the phrase 'ought to' could not be interpreted in accordance with the definition adopted. This, he reasoned, would show that we have after all a concept of obligation which is different from the utilitarian one.

His definition reads as follows: 'it ought to be the case that p' means in the utilitarian sense that 'if it were not the case that p, then some of our needs or desires would not be gratified.' Note that the *definiens* is an implication having a propositional variable as its antecedent, and a propositional constant as its consequent. Now take 'd' to be short for the proposition 'all our desires are gratified'. Expressing utilitarian obligation by '$O_u p$' we may put down Elzenberg's definition schematically as:

$$O_u p \quad \text{iff} \quad (\neg p \Rightarrow \neg d) ,$$

with '$\Rightarrow$' indicating strict implication. Thus to be obligatory (or *requisite*) in the utilitarian sense is to be a necessary condition of the situation described by 'd', i.e. of the gratification of all our desires.

As a counter-example Elzenberg substituted in his definition the constant 'd' itself, observing correctly that thereby the *definiens* is turned into a tautology. He was wrong, however, in thinking that this somehow disqualifies the substitution itself. Actually it merely shows that under the definition adopted it is simply a truth of logic that all our needs and desires ought to be gratified.

Does all that mean that the distinction between perfective and utilitarian value is a specious one? Not at all. It merely shows that distinguishing them is a very tricky thing indeed.

NOTES

1. In Polish.
2. Author's translation from Elzenberg's inedited notes, deposited in Archives of the Polish Academy of Science, Warsaw.
3. In Polish. *Próby kontaktu. Eseje i studia krytyczne.* Znak, Kraków, 1966.
4. Archives ..., cf. note 2.
5. Archives ..., cf. note 2.
6. Archives ..., cf. note 2.
7. Elzenberg, *Próby kontaktu ...*, p.50.
8. Author's translation.

Jerzy Szymura

WHEN MAY G. E. MOORE'S DEFINITION OF INTERNAL RELATION BE USED RATIONALLY?

G. E. Moore's definition of the so-called internal relation[1] clarified that meaning of the concept which had been most often employed in the well known controversy over the nature of relations. The author of that definition was right in his claim that the sense he established was metaphysically neutral for its serviceability in expressing each of the three positions to be separated in the dispute. Neohegelians saying that every relation is internal, logical atomists arguing that there are only external relations, and adherents to Aristotelian and Common Sense philosophy maintaining that each of the two kinds of relations have many instances in the world - all of those philosophers, not discarding other understandings, used the expression "internal relation" also in Moore's sense.[2] Nevertheless, his definition is not strictly neutral. This paper is to show that any speech or mental act of classification in which the definition is used is reasonable only on the condition that the speaker or thinker presupposes - consciously or unconsciously - a certain theory of universals. It is not the theory of the abstract universal that Moore upheld but the Neohegelian theory of the "concrete universal" that Moore endeavoured to turn down. Thus Moore's definition of the internal relation is justified only within the absolute monism of his opponents.

Before this conclusion can be derived in section 3, something should be said to recall - in section 1 - the Neohegelian theory of universals as opposed to Moore's point of view and to remind - in section 2 - how Moore understood the internality of relation.

1. Abstract and concrete universals

As Moore's theory of universals is very well known there is no need to present its details here. For the purpose of this paper it is enough to remember that, according to the philosopher: (1) universals, i.e. characteristics being properties or relations, exist independently of the subject predicating them on individuals; (2) universals are eternal and

unchangeable; (3) changes in the world depend on universals' capacity of being connected and disconnected with individuals which may acquire and lose predicable characteristics; and (4) owing to the connections, numerically different individuals may be qualitatively identical.

Neohegelians called this stand a "theory of abstract universals". In their view, to think that one may distillate a universal - say, yellowness or animality - from different individuals in whom it is unchangeably the same in all its instances is to fly in the face of the most patent facts of our experience. People do not even try to find two leaves which would be completely identical with respect to a single feature, e.g. their colour, shape or ribbing. And, as Blanshard said: "To insist on an abstract animality, running like a golden thread through snakes and pigs and Socrates, is to read into the nature of things what is only useful convention"[3]. When one is ready to admit that nothing happens twice in the River of Heraclitus, one should be inclined to acknowledge the same with reference to any "stream of consciousness" where ripples seem much more changeable and volatile than leaves. There is no simple identity to be found in different things experienced through extraspection or introspection.

Nevertheless, we treat some things as if they are identical in some respects. We do it although an alleged identity often turns out to be a difference which has been hidden until subjected to more careful examination. And it is typical to treat things as identical even though they undoubtedly differ from each other under any considered respect, as they do in the case of a snake and Socrates being animals. There is no room for any identity in the world independent of the classifying subject; it is a result of the latter's mental acts of abstraction.

It might be said that identity is simply a neglected difference but that would only be half true. The second half is that it must be something in the virtue of which some differences are neglected whilst others are not. Approaching this point realists are looking for causes *in natura rerum*; some of them, like D.Armstrong, try to support their position with findings in the natural sciences. Opposed to that, Neohegelian absolute monists prefer to talk about a creative articulation of the Universe by knowing subjects who, being themselves ways in which the Absolute exists, enable its appearances to appear. On the one hand, it is only a "feeling" of those subjects - their desire for a harmonic coherence - that decides what is identical and what is different. On the other hand, that which they feel constitutes the contents of Absolute Reality articulated in the subjects.

The above remarks do not explain adequately enough what Bradley and other absolute monists meant by "identity in difference". If this expression stood only for "one concept applied to the different things" - referring to the concept as being simply one and identical, and to the things as being

many and merely different - the expressed theory of universals would be a kind of concept nominalism alone. In opposition to the nominalists, Neohegelians maintained the universal to be real after all. But they thought of the "concrete universal" making "identity in difference".

The concrete universal is a real object of any act of knowing and a real object of the reference of any name.

When we take different views of the coin lying on the table and see its different appearances - circular, elliptical, and rectangular, or bigger and smaller, etc. - it is useless to look for the real coin as hidden behind the variety of the semblances or being one of these semblances. It is the whole of its appearances that is a real coin. The particular coin aspects are identical with regards to the fact that they are appearances of the same thing. It is the coin that appears - preserving its identity - whenever we see it. As one whole, made of many experienced particulars, it is concrete. As a common object, knowing and speaking subjects refer to, it is universal.

Discussing the nature of thing which appears in a series of events occurring in space and time, Bradley said: "The individual is so far from being merely particular (...) it is a true universal. (...) We are accustomed to speak of, and believe in, realities which exist in more than one moment of time or portion of space. Any such reality would be an identity which appears and remains the same under differences; and it therefore would be a real universal".[4]

The coin being a universal does not belong to a different ontological category than the coin lying on the table in front of us. Like water being one and the same concrete liquid poured into seas, lakes, rivers, pools, spoons, etc., the universal coin assumes the shapes - let alone functions - of guineas, zlotys, dollars, etc., each of them made of metal and having - analogically - its own particulars. The u n i v e r s a l coin is the whole of particular coins in the U n i v e r s e and, consequently, the whole of appearances the particular coins make.

Similarly, the mass that physicists talk about is the one whole seen from different points of view in the history of physics, and the triangle is the one and the same thing which has appeared differently in different geometrical systems. In this way, every theoretical concept is identical to its whole historical development, realised by scientists creating its parts.

In the works of Bradley and Bosanquet, the mysterious relation between the universal and its particular turns out to be the transitive relation between a concrete whole and its concrete part. The theory would deserve to be called mereological nominalism if it did not treat particular members of the universal as results of abstraction satisfying the feelings of knowing subjects.

Because the universal is a concrete and individual thing appearing in its parts which are realised by these subjects, a name for the universal is - in fact - a proper name. As it is well known, the gap between those two kinds of names has been - in great part - filled in recently by K.S.Donnellan, S.Kripke, H.Putnam and others. In Kripke's views, both proper names like 'Plato' and 'London' and natural kind names like 'water' and 'metal' are "rigid designators", i.e., terms referring - rigidly - to the same individual in every possible world in which that object of reference exists.[5] Although, in different circumstances - some of them easy to imagine - Plato might have had a different character and metal might have different features from those to which we are accustomed no one other than Plato could have been Plato and no stuff other than metal could be metal. The names 'Plato' and 'metal' refer - respectively - always and only to these changeable but still the same objects of their reference. It is difficult to disagree with these observations. If Neohegelians had used Kripke's term they would have surely used it more extensively and called all universal names "rigid designators". They might have said: Every universal is a "concrete universal", and thus, a "real individual" appearing in various possible ways. Wherever it appears - as a certain coin seen from different viewpoints, or as a certain theoretical concept in different academic heads or in different scientific "paradigms" - it may be christened - e.g. as 'electron' or 'logarithm' - and then called by its "rigid designator". Even the most sophisticated concept is not a "floating idea" but a "real individual" with its proper name and biography.

As nature and natural kinds are results of abstraction, so abstracts are, in Neohegelian views, very natural. In these views, people use an abstract concept in the way they share air, water or food, i.e. by assimilating its parts. Every user of the concept is a contributor to the whole universal which - as containing the other participants quota - transcends that user's particular point of view. In J. W. Scott's words: "That other-ness is essential to knowledge hardly needs to be proved. Knowledge, to be at all, must be of something; and of something, which is not me or just--mine, but is, on the contrary, common to others than just-me, to numerous others in the end, infinitely numerous others. The question is how what is mine can be others'"[6]. For Neohegelians only a whole divided into parts can be "mine and others'" at the same time.

To take a real part in the discourse in which human beings are engaged is to develop a concrete universal. Elements of the discourse are judgements. Judgement - as Bradley maintained - is a developed idea, that is a developed concrete universal. Someone may judge, say, an animal to be capable of moving. In this way the idea of what is in front of the judging person may become more distinct. But every judgement - as Neohegelians claimed - is true only in the particular circumstances. In another situation, e.g., when we

observe a sponge at the bottom of the sea, we notice that the observed animal can not move. Then one may still attempt to look for another criterion of animality, hoping that at least one sense, the absence of photosynthesis, growing up during a part of life, cells covered with thin membranes, presence of hemoglobin or chitin, or something else - will turn out to be the solution.[7] However, no candidate for a property belonging to all and only to animals is good enough; there are always animals deprived of it, and there are some beings possessed of it which do not deserve the name 'animal'. Because - particularly in borderline cases - the question of how to deserve a name cannot be answered by providing any obligatory list of properties the named object must possess, the only way to decide the issue is an appeal to feeling. "Meaning is", as Putnam said, "in part a normative notion."[8] Wine-tasters' feelings determines that a liquid is good enough to be called 'wine'. In the case of the question "to be or not to be an animal" more important is what zoologists feel.

All these illustrations may support the Neohegelian theory of the concrete universal revealing itself - part by part - in the judgements of feeling subjects. Bradley's theory "implies that sameness can exist together with difference" and "throughout different contexts".[9] Much the same thinking has been expressed lately in Putnam's words: "meanings have an identity through time but no essence".[10]

Long before Wittgenstein's *Philosophical Investigations,* Hegel and Neohegelians were fully conscious of the inability to precisely define any concept by Aristotelian *genus proximum* and *differentia specifica.* Although they did not supply such a convincing analysis as Wittgenstein did in the case of 'game', their theory of judgement was - as Wittgenstein's views of meaning - in glaring opposition to the belief in the possibility of establishing necessary and sufficient conditions of whatever.

It may be good - in particular for those readers who follow neopositivists in their critiques of "Hegel's nonsenses" - to finish this section with a remark that Carnap's concept of "partial definition" is also based on assumptions very much resembling the Neohegelian theory of universals. The author of "Testability and Meaning"[11] was forced to admit that theoretical terms never had synonyms to replace them everywhere, and that they could be defined only by different "meaning postulates" for different contexts. Bradley would have accepted this modest solution implied by his thesis that "every judgement in the end is no more than conditional".[12] He would have added: It is a concrete universal that appears in its different "partial definitions", having no essence but keeping its identity owing to the feeling of knowing subjects - the feeling which decides which and how many postulates must be satisfied in order for it to deserve a certain name.

2. Internal relation and necessity

It is interesting to note that the view of universals outlined above does not allow us to consider a standard interpretation of Neohegelians' theory of relations to be adequate. In light of this interpretation - propagated by Moore and Russell[13] - the thesis that "all relations are internal" which absolute monists undoubtedly uttered, is treated as if it states that any relation possesses a property of internality under any circumstances. In accordance with Neohegelians' views it must be said that the term 'relation' like every other universal term designates - "rigidly" in Kripke's sense extended - a concrete universal. It means that the object of its reference, i.e. the relation, observed from one point of view reveals a different character from that revealed from another. These assumptions may help in explaining Bradley's assertion: "Mere internal relations, then, like relations that are merely external, are untenable if they make a claim to ultimate and absolute truth"[14].

In Moore's interpretation, a relation is to be considered as internal for a given object when the object which possesses a so-called relational property consisting in being in this relation, would necessarily be other than it is if it were deprived of this property. Rewriting it in a more formal fashion: *R* is an internal relation of *x* iff

$$P_R(x) \Rightarrow \bigwedge_y \Box [\sim P_R(y) \Rightarrow \sim (x=y)]$$

where "x", "y" are symbols for individual variables, "P_R" designates the relational property which is qualified by a given relation - the latter being marked with an "R", "$\bigwedge$" stands for the universal quantifier, "$\Box$" symbolizes the modal operator "it is necessary that", "=" is a sign of identity, and "$\Rightarrow$", "$\sim$" are standard propositional connectives. In modal semantics terminology: a relation is internal for a given object if that object possesses the property of being in this relation in every possible world.

Moore made use of this concept of internal relation to express his understanding of reality: "It seems quite obvious that in the case of many relational properties which things have, the fact that they have them is *a mere matter of fact*: that the things in question *might* have existed without having them"[15]. In fact, it is difficult to deny, e.g., that my pen which is filled up now might have been the same pen without so much ink inside. And it is also easy to agree that not each of its relational properties is similarly unessential for the pen identity. Accepting the old Aristotelian distinction between essential and accidental properties of an individual and appealing to Common Sense, Moore was astonished "/.../ how any philosopher

could have supposed that it was not true"[16].

In spite of the predominant interpretation of the Neohegelian "theory of internal relations" it is doubtful whether the theory contends - without reservations - that all relations are "internal" in Moore's sense of the word. The contention may be understood and then evaluated in different manners depending on what concept of meaning is applied for names which may replace the individual variables in the foregoing formal definition.

As logical atomists maintained that real names are only so-called logically proper names having univocal reference but no connotation, they deprived named particulars of any character and, therefore, made every characteristic - thus every relation - of these particulars external to them.

Another conception of names which comes into play treats them as definite "clusters of descriptions" or "bundles of universals". This conception that "particulars were bundles of universal concepts" was ascribed to Bradley by H.Hochberg who is to some extent right in his interpretation[17]. Nevertheless, in Bradley's view, the only particular bundle of universals in which these universals are necessarily connected to each other is the Absolute containing all of them. Nothing short of the Universe - nothing i n the Universe - can have any property with no conditions attached. Hence, so far as more or less everyday objects - such as a coin, metal, physical mass or triangle - are concerned, none of them possesses any relational property in all possible worlds. Consequently, Bradley's theory of relations, in reference to things appearing in a human experience - even if it is the experience of a mathematician - ought to be called a "theory of external relations".

Nothing but the theory of name as a "rigid designator" can be coupled with the Neohegelian thesis that all relations are internal so as to endow it with some truth.

Kripke's "rigid designator" refers to Neohegelians' "concrete universal", i.e. to the mereological totality of appearances felt as the same thing under different circumstances. It is important to note that the circumstances can be both those under which people live and those they only imagine in so--called mental experiments. This gives rise to difficult questions: What is possible and what is not possible for x ? In what possible relations would x appear as being still the same x and when would it disappear as changed by possible inopportune conditions? How much alteration is possible before x would be annihilated? Once again the only answer is an appeal to feeling which tells us, e.g., whether Stalin's Palace of Culture in Warsaw would be the same building if it were reshaped in an American style. Bradley's consolation that the feeling could be satisfied with nothing less than a coherent solution does not help very much, because the logical coherence could be explained - in accordance with his theory - only in terms of

psychic feeling as well. Whatever difficulties there are in establishing what is possible, the concrete universal is to be treated as a totality of possibilities which are felt to be possibilities of the same thing.

The "concrete universal" is not the Aristotelian essence of many concrete substances having indispensable attributes. The universal attributes are bundled or disjoined differently in various - possible - appearances of the same total thing. It is neither necessary nor even possible for this totality to have anything that would occur, as one and the same element, inside each of its partial - possible - appearances. But if the whole is to preserve its identity, it is necessary for it to have exactly those parts which it contains. In short, each p o s s i b i l i t y i s n e c e s s a r y for the sum of the possibilities to be the same sum. And, as Bradley said: "The possible must, in part, be really, and that means internally, necessary"[18].

Thus Bradley's conception of relations, as analysed in Moore's terms, turns out to be - in spite of Moore himself - a theory that all relations are, on the one hand, external, and, on the other hand, internal. They are external - i.e. accidental - for a given object because they do not exist in all possible worlds in which the object exists and they are internal - i.e. necessary - for this object, because they are indispensable parts of its whole concept.

3. The concept of internal relation and two theories of universals

As a philosopher of Common Sense, Moore should have been upset by his own manner of speaking about relations which was far from being natural. When we want to decide whether a given relation is internal to its term or not his criterion of internality, proposed in "External and Internal Relations", seems perplexing since it makes us ask such unusual questions about the term as "Could John be the same John if he had not had the children that he has?" or "Could the number 4 still be the number 4 if it did not amount to the sum: 2 + 2 ?" etc. The problem is whether and - if so - when any question of the sort: "Could x be the same x if it had a relational property P_R which x does not posses?" or "Could x be the same x if it did not have a relational property P_R which x possesses?" may be used in a rational discourse.

What would one want to learn if a question of this type were about a concrete individual with all its details, e.g., about John having a wrinkled forehead, a shabby suit, twelve children, an aunt visiting him every two days, an old car and so on without end? As the concrete individual in this sense, John constitutes a complete object in which everything is necessary for the object to be itself. To ask whether John - being of this kind - could be John if he had not had his children is as if to ask whether t h i s could

be t h i s if t h i s had not been t h i s . Therefore, Ryle said: "I cannot satisfy myself that there *is* such a 'predicate' as 'being the particular that it is' or 'not being the particular that it is', and, consequently, I cannot understand what is being said when we are told that if something was not in a certain relation it would not be what it is"[19].

Similar troubles come up when a question of the considered form is about a universal. In Ewing's words: "This formula cannot be applied to relations between abstract universals, e.g. the relation of equality between the pure number 4 and 2 + 2, because we cannot speak of the possibility of an abstract universal being different from what it is ..."[20]. It is difficult to disagree with this opinion. If abstract universals are - *ex definitione* - unchangeable, then the question of how they would change under such-and-such conditions seems to be at least unwise if not contradictory. Who could seriously ask, e.g., whether a triangle might have twice as many sides as a square?

This was the reason why Moore confessed - in a private conversation with Ewing[21] - that his distinction between external and internal relations could not be applied to the relations of abstract universals. As stated above, the distinction cannot be used for relations of concrete individuals as well. In both these cases it is not possible, without giving an impression of absurdity, to ask any question of the sort "Would x be the same x if it did not have a relational property P_R which x has?"

If x stands for a concrete individual that contains all and only those characteristics, and hence all the relational properties which are truly predicable of it, then any alteration in its relations is - *ex definitione* - an annihilation of x. Therefore, the above question when it concerns a concrete individual is a question about a possibility which is excluded by the concept of the concrete used in this question. In other words, it is a useless question, for everyone who understands the words used in this question, to ask if what is necessarily obviously impossible is possible. The same holds true with reference to any question about a possible alteration of an abstract universal being - *ex definitione* - unchangeable. It is not forbidden to ask a question when the answer is very well known to both a speaker and a listener; nevertheless, it cannot be treated as a rational speech act.

Finally, when may Moore's definition of internal relation be used rationally in questions about relations? When does it make sense to ask if x could be the same x if x did not have a characteristic which x in fact has? The answer suggests itself: It makes sense only on the condition that the answer may be unknown to the person who asks. This requirement is complied with when the question is neither about an unrepeatable concrete, nor about an abstract form or idea, but about a concrete universal. It is the

Neohegelian theory of concrete universals which explains satisfactorily why we ask if the concrete universal *x*, which differentiates itself in different contexts, "having /.../ an identity but no essence", could be *x* in a new context deprived of a certain relation. It is a question of whether the concept of *x*, developing itself in the evolution of language, ought to embrace this new situation also in which *x* loses the relation, or not. If Moore's question is not just rhetorical it is a question about a norm - i.e. a rule of language - to follow.

NOTES

1. G. E. Moore, "External and Internal Relations" in *Philosophical Studies*, London 1922, p. 276-309.
2. For other senses of "internality of relations" see A. C. Ewing, *Idealism. A Critical Survey*, London 1934, p. 117-142. Another original interpretation of the term in A. R. Manser, *Bradley's Logic*, Oxford 1983, p. 119-135.
3. B. Blanshard, *The Nature of Thought*, New York 1939, vol.I, p. 602.
4. F. H. Bradley, *The Principles of Logic*, Oxford 1958 (first ed. 1883), vol.I, p. 45.
5. S. Kripke, *Naming and Necessity*, Oxford 1980 (first ed. in *Semantics of Natural Language*, ed. G. Harman and D. Davidson, Reidel 1972), see Index.
6. J. W. Scott, "Symposium: Is the 'Concrete Universal' the true Type of Universality?" in *Proceedings of the Aristotelian Society*. New Series - vol. XX, 1910, p. 129.
7. See L. Koj, "On Defining Meaning Families" in *Studia Logica*, vol. XXV, 1969, p. 141-150.
8. H. Putnam, *Representation and Reality*, Cambridge Mass. 1988, p. 11.
9. H. F. Bradley, *Appearance and Reality*, Oxford 1959 (first ed. 1893), p. 307.
10. H. Putnam, *Representation and Reality*, op.cit., p. 11.
11. R. Carnap, "Testability and Meaning" in *Philosophy of Science*, vol.3, 1936, and vol.4, 1937.
12. F. H. Bradley, *The Principles of Logic*, op.cit., vol.II, p. 639.

13. For instance, see G. E. Moore, "External and Internal Relations", op.cit., p. 278-280; and B. Russell, "The Monistic Theory of Truth" in *Philosophical Essays*, London 1910, p. 160-161. See also N. Rescher, *A Theory of Possibility*, Pittsburg 1975, p. 180-181.

14. F. H. Bradley, "Relations" in *Collected Essays*, Oxford 1935, p. 645.

15. G. E. Moore, "External and Internal Relations", op.cit., p. 289.

16. Ibid., p. 289.

17. H. Hochberg, *Logic, Ontology and Language. Essays on Truth and Reality*, München 1984, p. 14.

18. F. H. Bradley, *Appearance and Reality*, op.cit., p. 348.

19. G. Ryle, "Symposium: 'Internal Relations'" in *Supplementary Proceedings of the Aristotelian Society*, vol.XIV, 1935, p. 163.

20. A. C. Ewing, *Idealism*, op.cit., p. 130-131.

21. Ibid., p. 131, note 1.

Part II

HISTORICAL PERSPECTIVE

Józef M. Bocheński

HISTORY OF LOGIC AND THE CRITERIA OF RATIONALITY

The problem of the relativity of the criteria of rationality is much discussed nowadays in philosophy. The advocates of extreme relativism maintain that those criteria depend exclusively upon the cultural circle - to the extent that, in their opinion, there are no generally accepted criteria.

What should be understood by "Criteria of rationality"? Translating this rather obscure expression into language comprehensible for logicians, I would say that it refers to the rules of acceptance or rejection of statements. If this is the case, the formal logic of each cultural circle must contain them, for logic is, among other things, the formulation of these very criteria.

Hence, the question "Do generally accepted criteria of rationality exist?" may be stated in the following way: "Do the rules of acceptance and rejection of statements appearing in the formal logics of all cultural circles exist?" The answer to that question obviously assumes that each of those cultural circles has its own, specific formal logic...

The problem is of empirical character. Even if the absoluteness of our rules is justified by eidetic insight, it could happen, that people in some cultural circle would be, so to say, "logically blind": they would not accept obvious rules.

The discipline competent in this field is the history of logic, for it examines empirically formal logics developed in various cultural circles. It alone can answer the two questions: (1) Does a specific formal logic exist in the given circle? and (2) if so, are certain rules of acceptance and rejection of statements contained in each of them?

In order to state the second of these questions more precisely, we shall choose the following rules included in normal contemporary logic:

modus ponens - if a conditional and its antecedent are given, its consequent should be accepted;

dictum de omni - what has been stated about all elements of a class, should be stated about each of them;

the principle of coherence - contradictions should be avoided.

What does history of logic answer to our questions?

(1) The answer to the first is affirmative, with certain restrictions. We know of four, or maybe five, different cultural circles, which created specific formal logics. They were: European Antiquity, Middle Ages, the XIXth and XXth century and India. In the last case we have apparently two different logics, appearing successively: the old Nyaya and the Navya-Nyaya.

These logics differ strikingly one from the other. For instance, scholastic logic differs from ancient logic - to mention only one detail - by the great amount of semantic it uses. Indian logic is, contrary to all European logics, intensional - it does not know quantifiers. Mathematical logic differs from all others e.g. by the extensive use of formalism.

The answer to the first question is, therefore, affirmative.

(2) Despite this fact, all the logics known, with no exception, contain our three rules. It is true, that, so far as is known, no precise formulation of the principle of coherence was found up to now in Indian logic; but it is consistently applied in it, just as Aristotle used the principle of identity, which he did not formulate himself.

Therefore, the answer the history of logic gives to the second question is also affirmative - though restricted to the cultural circles in which formal logic known to us, was present.

And so the history of formal logic makes a contribution to the discussion of the relativity of the criteria of rationality. It declares itself against extreme relativism and for the existence of generally accepted criteria of rationality. Of course, it expresses its opinion in the way which characterizes all statements of empirical and historical sciences - not laying any claim to absoluteness. It teaches that, in the present state of knowledge concerning the cultural circles which developed formal logic, we have to admit, that there exist generally accepted rules of acceptance of statements, i.e. criteria of rationality - i.e. top reject extreme relativism.

But history of formal logic and it alone is competent in that field. Everything else may be said to be just speculation.

Jan Waszkiewicz,
Agnieszka Wojciechowska

ON THE ORIGIN OF REDUCTIO AD ABSURDUM

Our particular interest is in the origin of *reductio ad absurdum* method of theorem proving.[1] It is approached from both - the history of mathematics and history of logic side[2]. We proceed by analysing the cultural context of both sciences. Hence, it is necessary to regard some cultural phenomena much older than logic or even mathematics. The considered problem is well established. It has been widely discussed but no definite conclusions have been reached so far. Existing opinions are summed up by A. Szabó[3].

An explanation of the origin (and priority in use) of *reductio ad absurdum* is crucial for the interpretation of the early history of the deductive method in mathematics and also of dialectics in philosophy. As A. Szabó stated (reformulating Proclus' opinion), "mathematics was at least in one respect a branch of dialectics"[4] and it is exactly this respect we are going to investigate.

Proving of theorems, which appeared at the beginning of mathematics in the Greek culture, manifested itself in an argumentation appealing to intuition, to the tangibility of some statements recommended as reliable. The visual or tactile character of such proofs is suggestively shown by the etymology of words such as "to prove" or "theory"[5]. For instance, Greek word "*deiknymi*" originally meant not "to prove" but "to show, to point out, to make known, to explain", i.e., "to point out" both in a figurative and literal sense. Such use of this word is clearly evident in Homer[6]. Some Greek texts and reconstructions from intermediary sources provide evidence of this kind of argumentation and its subsequent development into a logical one. Some fragments of Plato[7] and the oldest fragments of Euclid known as the "theory of the even and the odd" with theorems supposedly proved by *psephophoria* (i.e. by manipulating stones or pebbles)[8], seem to be the most interesting. As Szabó states:

> at some time during pre-Euclidean period, Greek mathematics underwent a remarkable transformation. Visual arguments were no longer accepted as proofs; instead the Greeks sought to 'show' the correctness of their mathematical statements in entirely different manner. I think that this

'transformation' can best be described as anti-empirical and anti-visual[9].

It seems that this statement should be reformulated. Firstly, the specific meaning of "empirical" in Greek culture should be remembered: Greek empiricism was rather weak. As Lange said about Aristotle's "inductive mounting from facts to principles": "At the most, what he does is to adduce a few isolated facts, and immediately spring from these to the most universal principles, to which thenceforward dogmatically adheres in purely deductive treatment"[10].It is true that Aristotle can be considered as a representative of an "anti-empirical period"; still his attitude to empirical evidence resembles the attitude of his predecessors. The method of *tekmairesthai* - extrapolation of observational data beyond the borders of evidence was already used by Anaximander[11]. This shows that either the anti-empirical turn was not so deep as Szabó assumes or it happened much earlier than he supposes (or both, as the authors are inclined to believe)[12].

Moreover, visualization in "*deiknymi*" could have a figurative sense, hence the discussed turn could be deeper: it was oriented against visual character of the proof in the large sense. It was anti-intuitive, and so we call it in the sequel.

It seems reasonable to suppose that this anti-intuitive turn emerged from more complex situations, in which it was not possible to identify "true" with "intuitive" or "obvious". Such situations happen when the statements are true but unintuitive or, conversely, intuitive but false (or leading to false implications). The first occurrences are more typical in matured deductive theories where chains of deductions lead the thought far away from the original intuitions; one can hardly expect to find such situations at the beginning of mathematics or philosophy.

For the period discussed, the second type of situation seems to be more prevalent. It deals with propositions which are intuitive but false what makes the method of *reductio ad absurdum* trustworthy (if not indispensable). Moreover, such a situation shows the necessity of founding proofs on the ground more rigid than appeals to intuition. Thus it leads to more strict rules of dialectics in philosophy, and to deduction in mathematics. That is why the emergence of *reductio ad absurdum* is so important for explaining the genesis of deductive sciences.

According to almost univocal opinion, commensurability of all segments was the first intuitive but false mathematical thesis. The proof of its falsity, i.e., the proof of incommensurability of the side and diagonal of a square, is probably the best known example of a proof *a contrario*[13]. Although attribution of this proof to Pythagoras is probably a result of erroneous interpretation of Proclus[14], still there exists a strong evidence that this proof can be ascribed to rather early Pythagorean achievements.

According to Zeuthen, this proof should be dated in the first half of the 5th c. B.C. It was an early stage of "the even and the odd" theory[15]. Moreover, as Szabó points out, almost half of the theorems in Book IX of Euclid, which deal with this theory, have indirect proofs. The same can be said about the other part of the old Pythagorean mathematics in Book VII[16]. Anyway, it is clear that already at this early stage of mathematics this kind of argumentation had been commonly used. A.Szabó writes that "it is no accident that so many propositions of early Greek mathematics are proved indirectly. Their proofs must have come down to us substantially unchanged. Even in the fifth century mathematics the indirect method must have been the most widely used technique of proof"[17].

The fact that the indirect proof was used in the early Pythagorean mathematics does not decide the problem of the priority of the method. It could well be that its origin is not Pythagorean and not even mathematical. Such are, for instance, the conclusions of Szabó; in quoted works he argues that the method of *reductio ad absurdum* is of Eleatic origin. His argument gave rise to the discussion in which W. Kneale defended traditional opinion on Pythagorean priority and P. Bernays made supposition that the method was probably known to Thales[18]. We will try to support the last opinion with some new arguments, but we should start from solid ground - i.e. from Euclid. According to Proclus, Euclid

> deserved admiration preeminently in the compilation of *Elements of Geometry* on account of the order and of the selection both of the theorems and of the problems made with a view to the elements. For he included not everything which could have been said, but only such things as he could get down as elements. And he used all the various forms of syllogisms, some getting their plausibility from the first principles, some getting out from demonstrative proofs, all being irrefutable and accurate and in harmony with science. In addition to these he used all the dialectical methods, the divisional in the discovery, the definitive in the existential arguments, the demonstrative in the passages from the first principles to the things sought, and the analytic in the converse process from the things sought to the first principles[19].

This quotation (even the terminology used) makes visible the connection between the deductive method of "Elements" and the Eleatic dialectics - the method of philosophical analysis of problems by means of confronting opposite points of view. Originally, those viewpoints were expressed by interlocutors in dialogue (as in Plato's works) and represented the true opinions of authentic persons. The aim of discussion was to gain the consensus bringing both opponents nearer to the objective truth. Later, as

for example in Sophistic debates, the aim was to destroy the adversary and to win the discussion then treated as a game. In this case the hypothesis presented by the winner did not have to be true; it could even contradict the personal experience of the debaters, it could be paradoxical. In written texts, as well as in rhetoric speeches, both viewpoints were represented by the same author. The same is the case of mathematical deduction: the author of a proof answers all probable questions (and objections) of a virtual opponent (a reader of the given mathematical text).

In such a debate model of dialectics (as well as in deduction) some logical questions can be easily inscribed. For example, the *tertium non datur* principle is nothing more than accepting the pattern of discussion.

Also the proof *a contrario* is something natural in this model. It is simply an eristic procedure consisting in a temporary acceptance of the opponent's position in order to demonstrate (to point out, to prove - *deiknymi*) the absurdity of his standpoint. Exactly such is the extensive use of indirect reasoning in Plato's dialogues: as Szabó states, "Plato's dialectic was wholly dependent of indirect proof"[20]. Such a use of this method in philosophical dispute had been parodied in the comic dialogue of Epicharmus already in 500 B.C. or thereabouts.

It is well known that Aristotle describes Zeno of Elea as the founder of dialectics[21], the one who established some rules of dialectical method or gave them the matured shape. As Simplicius writes:

> Zeno was engaged in contrasting one hypothesis with another. (...) These were (...) 'the hypothesis which states that what exists is many' and 'the hypothesis which states that what exists is one'. Zeno then examined the statements which agreed with each of these hypotheses or (...) he checked to see which of these propositions led to a contradiction so that he would be able to reject as false the one which did[22].

The indirect reasoning appears thus to be the very essence of Zeno's method which he put to use in investigations of his famous paradoxes demonstrating the falsehood of some most intuitive images.

It is more than probable that Zeno's activity in 5th c. B.C. paralleled the development of these parts of mathematics in which *reductio ad absurdum* was also used. Thus, although his thought gave great impetus to the way of reasoning of interest for us, the problem of priority is still open.

Let us take another step back: to Zeno's teacher, Parmenides. Szabó points out that in the real life, like in Plato's dialogue "Parmenides", Zeno was a defender of the doctrine of his master. It should be assumed that not only the teaching of Parmenides but also his method was taken over by Zeno. As Szabó writes, "the earliest application of indirect proof which I have

encountered in my studies of Greek language and culture occurs in the didactic poem of Parmenides"[23]. Since this point is crucial for our investigations, let us quote the larger fragment by the same author:

> It was Parmenides who proved his theses by refuting their negations. The discovery of indirect proof was perhaps his greatest and most lasting contribution to philosophy. (...) It probably came about as a result of Parmenides' critique of Milesian cosmogony or, to be more exact, of Anaximenes' cosmogony. Anaximenes held that the world came into being from a primary substance by means of 'rarefaction' and 'condensation'. For example, he maintained that *water* could be reduced to *air*; *water*, before it came into being (i.e. before it was condensed), was *'air'* or *'non-water'*. On the other hand, this line of reasoning could not be applied to the *ón*. As Parmenides realized, it would be self-contradictory to say of the *ón* that, before it came into being, it was *μὴ ón*; this would be inconceivable and hence impossible. *What is* must always have existed; it could not have come into being from *what is not*. (Using the terminology of Zeno and Plato we could describe the situation as follows: the *hypothesis* that what is 'come into being' leads to a contradiction or *adynaton*, for it implies that *what is* was once *non-existent*.)[24].

This summary of Parmenides argumentation Szabó finishes with the conclusion which is not so evident:

> Thus the notion of *logical contradiction* was formulated by Parmenides in the course of his criticism of Milesian cosmogony, and this in turn led him to discover the method of indirect proof.[25]

The above conclusion is doubtful for us, since Szabó's reconstruction can be fruitfully applied to the thought of other philosophers. Using the same argumentation one can give sense and coherence to some propositions of Xenophanes[26], Heraclitus[27], and, especially, Anaximander. The reconstruction of his reasoning given by N. Hartmann is almost identical with the above one[28]. Anyhow, it is easy to see that such attributes of Anaximander's *arché* as infinitude and everlasting, or even its name - *apeiron*, are rooted in indirect reasoning: the opposite (and more intuitive!) possibilities become refuted as causing logical incoherence. As was suggested elsewhere[29], Parmenides and Zeno can be seen as executors of the same program (in the sense of Lakatos) as Anaximander (and hence Thales), and if one needs the continuity in heritage, the name of Xenophanes should be recalled. The continuity of investigations could be accompanied by the continuity of methods, *reductio ad absurdum* included.

Concluding this part we should point out that in mathematics and philosophy alike the method of *reductio ad absurdum* can be traced to very early stages, almost to the beginnings of these sciences.

So, let us change the context and consider problem from the standpoint of the patterns of Greek culture and political life.

As it has been already stated, the deduction, and more generally - dialectics, is rooted in the practice of discussion omnipresent in the Greeks' public life. As we noted elsewhere,

> the problem of the origin of the deductive method (...) leads us to more general issues - the origin of dialectics, the role of dispute, discussion and argument in the Greek culture (especially in social and political life); to the causes of the fact that in the system of *polis* a discourse or a speech became a political instrument, a basis of all authority, a tool of management and control of other people. Briefly speaking, we approach the problem of the 'erosion of power' characteristic for ancient Greece and lasting from the Doric invasion in the 12th century B.C. up to the reign of Alexander the Great, and the sources of *polis* with its democratic system[30].

No convincing evidence of public discussions have been preserved from the time preceding the usage of *reductio ad absurdum* in philosophy and in mathematics. Unfortunately all we have is poetic paraphrases. Among them there are two cases of famous discussions: the litigation between Hesiod and Perses, and the quarrel of Agamemnon and Achilles. In both of them the elements of indirect reasoning in poetical disguise are clearly visible. When Hesiod persuades Perses to be virtuous by showing him the disastrous influence of Hybris (i.e. haughtiness, violence, audacity)[31], the argument is clearly indirect.

The same, although the word "argument" is hardly adequate, can be said about the quarrel of Agamemnon and Achilles. We recall that this quarrel constitutes the main thread of the Iliad[32]. The thesis of Achilles can be stated as follows: "Since my role in Achaian struggle is decisive, then my position should be extraordinary". This means that the rights of Achilles should be respected. Agamemnon is of the opposite opinion, and uses his power to force the adversary to accept his position. To some extent the whole epos can be regarded as the indirect proof of the Achillean thesis: his withdrawal had exactly the same character as temporary acknowledgement of the opponent's position in the debate. Such is the intended purpose: discrediting the opposite hypothesis and hence showing its falsity.

Of course, we do not regard the identity between the constructional pattern of the Iliad[33] and the logical scheme of indirect proof. We only

suggest an analogy which shows that the kind of rhetoric and eristic figure from which the proof *a contrario* could have been derived is natural and was used in various times, situations and cultures. The necessary condition for refining this method of reasoning was discussion. Of course the Greeks' priority consists not in discussion at all, but in making it the pattern of their culture. Consequently, they probably did not invent the proof *a contrario*, but they used it relatively often and with growing consciousness of its nature and value. The priority of Pythagoreans was not in using *reductio ad absurdum* in mathematics, but as already Proclus claimed, in transformation of mathematical studies into "the form of liberal education examining its principles from the beginning and tracking down the theorems immaterially and intellectually"[34]. The priority of Parmenides, Anaximander or Thales is not just in the use of this or other figures of dialectics. We see it in the promotion of discussion to the rank of the main attribute of science, and in extending its domain to *arché*, the principle of universal order.

None of the great Greeks opened an entirely new way, but they widened the paths already used, and were able to point to new destinations for their followers.

At this stage our contribution leaves the problem of priority in use of *reductio ad absurdum* as yet unsolved, but already dissolved in a larger context.

NOTES

1. This is the text of the lecture delivered at the 33rd Conference on the History of Logic, Jagiellonian University, Cracow, October 1987.

2. Our point of view in these more general problems is presented, e.g., in: A. Wojciechowska, "Geneza matematyki i logiki - przykład wspólnych problemów historii obu nauk" [Genesis of mathematics and logic - an example of parallel problems in history of both sciences], *Ruch Filozoficzny* 1987; J. Waszkiewicz, "The influence of cultural background on the development of mathematics", *Organon* 16-17 (1981), pp. 93-113.; J. Waszkiewicz, "Korzenie greckiej matematyki. Studium socjokulturowych uwarunkowań genezy matematyki" [Roots of Greek Geometry. Study of Sociocultural Conditions of the Genesis of Mathematics]. Prace Naukowe OBP Politechniki Wrocławskiej, Wrocław 1988.

3. See A. Szabó, "Greek Dialectics and Euclid's Axiomatics", in: I.Lakatos (ed.), *Problems in the Philosophy of Mathematics. Proceedings of the International Colloquium in the Philosophy of Science*, London 1965, vol.I, North Holland, Amsterdam 1967, pp. 1-27 (lecture and discussion).

4. A. Szabó, *The Beginnings of Greek Mathematics*, Akademiai Kiado and D. Reidel, Budapest - Dordrecht 1978, p. 245.

5. Ibidem, p. 185 ff.

6. *Odyssey* 12,25 and 10,33, *Iliad* 19,322.

7. E.g., Meno 82b-85e.

8. See A. Szabó, op. cit., pp. 192-194, B. L. van der Waerden, *Science Awakening*, P. Noordhoff, Groningen 1954, pp. 108 f. The theory mentioned, with probable Pythagorean origin, was first reconstructed by O. Becker (*Die Lehre vom Geraden und Ungeraden in neunten Buch der Euklideschen Elemente, Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, B3 (1936), pp. 533-553).

9.. Op. cit., p. 195.

10. F. A. Lange, *The History of Materialism*, Harcourt, Brace and Co., New York 1925, vol. I, p. 88.

11. See A. Krokiewicz, *Zarys dziejów filozofii greckiej (od Talesa do Platona)*. [Outline of Greek Philosophy: from Thales to Plato], (in Polish), PAX, Warsaw 1971, p. 77.

12. Since we follow the one reconstruction by Szabó we should mention here other possibilities, such as the hypothesis of M. Foucault (*L'ordre du discours*, Gallimard, Paris 1971) on the decisive role of the Sophists in the turn under discussion.

13. This proof was originally located at the end of the Book X of *Elements* (see A. Szabó, op. cit., p. 213 about the location of this theorem by Heiberg and his followers). Aristotle mentioned this proof in the *Prior Analytics* I.23 (41a26) and I.44 (50a37).

14. Friedlein reads as "irrational" what Thomas reads as "proportional" (see: *Greek Mathematical Works*, ed. by I. Thomas, vol.I, Harvard University Press, Cambridge Mass. 1939).

15. See A. Szabó, op. cit., p. 26, B. L. van der Waerden, op. cit., p. 110.

16. A. Szabó, op. cit., p. 244.

17. Ibidem, p. 247.

18. In discussion quoted in Note 3.

19. We quote I. Thomas' translation of Proclus' *Summary* (see Note 14).

20. Op. cit., p. 219.

21. See Diog. Laert., IX, 5, 25.

22. Simplicius, *Commentary on the Physics of Aristotle*, quoted after A. Szabó, op. cit., p. 248.

23. Op. cit., p. 219.

24. Ibidem, p. 250. The Critique by Parmenides was first of all directed against Anaximenes, but it seems to the authors that Heraklitus was more evidently its subject.

25. Ibidem.

26. See the first seven gnomes in the Fairbanks arrangement (23-26, 14, 15, 11 in Diels' numbering), especially the one on the immobility of the God.

27. See fragments 40, 91, 110, 127 in Diels' ordering (Bywater 16, 41-42, 104, 130a) where one can see traces of *reductio ad absurdum* or at least the transposition of implications.

28. See N. Hartmann, *Platos Logik des Seins*, Berlin 1965, pp. 13-19.

29. J. Waszkiewicz, "Roots of Greek Geometry" (see Note 2).

30. J. Waszkiewicz, "The influence...", p. 100. For the political roots of the peculiarities of the Greek culture see J. P. Vernant, *Les origines de la pensée Grecque*, Presses Universitaires de France, Paris 1962.

31. *Opera et dies*, v. 213.

32. See *Iliad* I, 165-170.

33. The same can be said of the biblical Book of Job and probably many other books of different cultures.

34. Transl. by I. Thomas.

Ewa Żarnecka-Biały

PREMONITION OF MATHEMATICAL LOGIC IN ARISTOTLE'S PRIOR ANALYTICS

Starting with the investigations undertaken by Jan Łukasiewicz, there goes a line of modern interpretations of Aristotle's logic.[1] Although nothing revealing could be stated here, Still some rearrangement and complementation of those interpretations are possible, and this paper attempts to make a kind of synthetic inventory of "Prior Analytics" from a logical point of view.

I assume that one knows what is needed to have a system of logic, a certain kind of language, a certain kind of rules, etc. And now let us ask - a-historically in a way - about the premonition of our routine logical devices in Aristotle. The discussion will be focused on the following topics:

1) The syntactic status of Aristotle's letters A, B,... etc.

2) His idea of forming propositional formulae.

3) Aristotle's strategies in defining the alternative sets of axioms.

4) The effects of using two different conventions: "intralogical" and "metalogical" (language - metalanguage).

We shall concentrate on the problems of assertoric propositions without entering into the specificity of modal logic.

1. Aristotle's variables

When a formalized language is constructed and the rules for producing well formed formulae as well as the rules for system construction are established, then there is no difficulty in deciding whether a given symbol IS a variable or not. What is more, when the contemporary systems are constructed in logic, we are quite often instructed that the given symbols ARE DESIGNED to be logical variables with all the usual ramifications, for instance, when distinguishing bound and free variables. If one wants to make sure whether a given symbol is really a LOGICAL variable, he is to check out how this symbol in the given system can be manipulated. I stress the word

"logical", because logical variables perhaps differ from other kinds of variables.

Similar criteria, although treated in a more liberal way, are to be observed when we look for the early roots of the modern concept of a logical variable. Two simple stereotypes should be cited here. The first idea is that Aristotle uses name-variables symbolized by capital letters A, B, Γ,... M, N, X... etc. According to the second opinion, propositional variables are the invention of Stoic logic and are symbolized there by numerals. Roughly speaking it is really so, but some more detailed comments are needed to make the problem clearer.

Let us start with a digression concerning the origin of the number "zero" that introduced a real revolution in the system of numbers notation. Despite its abstract nature its origin is supposed to be very concrete and empirical. Namely, its earlier denotation was "the empty row of the abacus", where in the course of calculation the rows used to be filled with the appropriate quantities of stones. Was not the situation connected with the origin of the logical variables similar to that? Perhaps it really was.

This is easier to trace with respect to Stoic logic; for instance, in Diogenes Laertios' description of Stoic schemata[2] we find an example where the antecedent and consequent of a conditional referring to Plato (*breathing - if living*) are mentioned as "the first" and "the second". Such an intralinguistic deictic use of numerals could have existed prior to their use for codification of the general undemonstrable schemata and could have inspired the way of formulating those schemata.

When dealing with Aristotle's syllogistic we must not be misled by the fact that the first use of symbolic letters in "Prior Analytics" allows - due to the context - to identify them with name-variables. If we compare their other mention in the "Analytics", the role of capital letters seems rather ambiguous from the syntactical point of view. The same letters appearing in the places where names are to be denoted are also used to refer to the propositions.[3]. On the other hand, those propositions could also have been mentioned via a construction in which two letters appear (cf. 25ª, 26ª). Analogies with the use of letters for identifying points or lines in the geometrical space are clearly seen here and perhaps it goes back to the suspected, even if lost, syllogistic diagrams involved in the definition of the three syllogistic figures.

The interdependence between the variable-like letters and the supposed diagrammatic representation can also be traced in the convention governing the arrangement of letters for defining the three syllogistic figures. Scholastic logic made the student familiar with the use of the letters S,P,M (mnemonic with respect to the grammar) throughout all the figures. Notice, however, that Aristotle's practice is different: he uses the letters A, B, Γ

when describing the first figure, letters M, N, and X for the second, whilst for the third figure the letters Π, P and Σ are reserved. In the course of syllogistic reductions this order disappears, but it is still worth mentioning that it had been introduced to start with. The conception of the specifically "empirical" character of the variable-like letters seems to be well supported by this.

The argumentation presented above does not mean that we should ignore the more sophisticated use of the letters in question. Already at the moment when, for instance, the "second figure" letters M, N, X are translocated into the first one, being the domain of A, B, and Γ letters, they are manipulated rather in a way the very variables ought to be. But we have yet more convincing reasons to believe that, in some of their uses, the letters A, B, etc. play the role of logical variables (of the name syntactical category) exactly in the sense that is appropriate for modern logic.

Namely:

(1) Their use is clearly subordinate to the idea of creating logical patterns for propositional constructions which appear in syllogisms, and writing them down clearly and unambiguously. It is the main reason that inclined Aristotle to introduce grammatical constructions which avoid ambiguity when letters are used, although these constructions, as we learn from philologists, are far from natural for Greek.[4] If Aristotle was so careful about this, he must have recognized the importance of the logical role of his letter symbols.

(2) Let us remember that in modern logic the simple criterion of treating some symbols of the given system as logical variables lies in their being subjected to the operation of substitution (e.g., substitution of any well-formed propositional expressions for propositional variables). Of course it is not to be expected that in Aristotle's system the rule of substitution could be literally articulated, yet there are places in the "Prior Analytics" where purely formal substitution seems to intervene.

It happens in the course of discussing syllogisms with contrary premisses. The whole context is highly sophisticated. Aristotle is eager to know which of the syllogistic moods, already identified as valid, may start with premisses entailing contradiction. At first (Book II, 63^b) he states that there is no place for something like that in the frames of the first figure, but finds such interpretation for some moods in the second figure; e.g. *Camestres* is suitable for such an interpretation (63^b–64^a). This interpretation, where only two terms are taken to be substituted for the three letter-symbols appearing in the premisses, results in the conclusion being an universal negative statement with identical subject and predicate.

I believe that it is a good formal substitution, although only names, not variables, are directly manipulated. The scheme described in Aristotle's text with the help of three letter symbols remained unchanged, but at the intermediate level a formal insight must have arisen, which involved treating letters as variables, in the sense of their being ready for ANY arbitrary interpretation.

On the other hand the deliberate use of symbol-letters relating to the propositional construction also happens to appear in "Prior Analytics", in Book II. For instance in Chapter 2 (53^{b}) we read about the conjunction of two premisses as represented by the letter A.

In some quite recent interpretations variables can be explained away; prior to it they had necessarily to be discovered. It is fascinating to observe, how it has started with in the "Prior Analytics".

2. Aristotle's well formed formulae

Only one feature of Aristotle's regulations will be mentioned here - namely, what kind of expressions are to be treated as well formed formulae. There is no tendency in Aristotle to relate the problem of admissible constructions to the problem of intentional connections between the terms. Well, as far as the affirmative propositions are concerned, nothing like the ghost of artificiality appears. We read, for instance, about all **men** being **animals**, about (some) **horses** being **white**, and so on. One would expect that the principle of the rather natural treatment of the universe of discourse would be observed with respect to the negative propositions as well. Meanwhile we read in the "Analytics", for instance, that **no knowledge is a line**. But what is the sense of such information? Could anybody have expected that some knowledge might eventually be a line? Of course not. So, there must have been some special reason to contrast such terms in this way. I understand it as Aristotle's programme of treating the rules for constructing well-formed formulae as a purely formal procedure. Of course the precise category of well-formed formulae is quite late, but nevertheless Aristotle seems to be near to it.

The problem of what kind of formulae are well-formed concerns also the method of quantification: optional or obligatory. In the context of the later development of traditional syllogistic only two kinds of propositions, distinguished with respect to their so-called "quantity", are legitimate, i.e. either universal or particular ones (denoted by the mnemonic letters "a" and "e" or "i" and "o"). It should be remembered, however, that at the very start of "Prior Analytics" still another kind of propositions is enlisted, namely, the Indesignate propositions.[5]

Cf. An. Pr., I, Chapter 1, 24^a, ([2] - Geach's translation):

> A Proposition is a form of words that affirms or denies something of something. It will be either universal, or particular, or indesignate. By a Universal proposition I mean one saying 'applies to every one' or 'applies to none'; by a Particular proposition I mean one saying 'applies to some' or 'is inapplicable to some' or 'does not apply to every one'; by an Indesignate proposition I mean one that just says something 'applies' or 'does not apply' without any 'universally' or 'in part': e.g. 'There is a single science of contraries' or 'Pleasure is not a good thing'.

In the subsequent chapters Aristotle discusses the effect of having this kind of propositions in syllogisms. For instance in Chapter 4, while describing the first figure moods, Aristotle notices that in the perfect syllogism a particular premiss 'B applies to some C' can be replaced by an indesignate one, 'B of C', still leaving the syllogism valid. "We get the same conclusion whether the premiss is indesignate or particular". (26^a). Also, in discussing a case where there is no (valid) syllogism (B applies to no C,... A applies to some B...), Aristotle observes that the same interpretation (terms: white, horse, swan/cow) can be adopted with respect to the indesignate premiss 'A of B'. It is clear that Aristotle allowed not only quantified propositions in syllogistic reasoning, but that he also knew how to deal with the unquantified ones.

Why is this an essential point in interpreting Aristotle's logical world? From Aristotle's three-fold classification there would lead - one can imagine - quite a straight path towards the introduction of quantifiers and bound or free individual variables, taking $\forall x\, P(x)$, $\exists x\, P(x)$ and $P(x)$ alone, for the Universal, Particular and Indesignate propositions, respectively. Once the "a", "e", "i", "o" schematization was introduced by the scholastic logic, this trace must have been lost.

3. Anticipation of deductive systems methodology

Let us not enter into the details of Aristotle's syllogistic. They are well known and thoroughly commented upon from different philosophical and logical points of view. The fact is that IT IS an axiomatic system of logic. It is a system observing the rules of the kind obligatory for deductive systems; some of these rules are precisely described. Well, Aristotle's syllogistic is not a formalized system in the sense that the logic in Principia Mathematica is, but it aims clearly enough at distinguishing what is assumed at the beginning and what is to be proved. And - what is essential here - all this

formal construction works well. Of course, a contemporary logician may be worried about omitting the rules of propositional logic in Aristotle's syllogistic, though they are used in proofs. But still it is a system of logic, because it is a system, and because a certain logic is its subject. And, on the other hand, one can argue that a sort of basic logic is always presupposed in all deductive constructions, not necessarily to be identified with the propositional logic though entailing it. I mean here, for instance, the very idea of making suppositional proofs, direct as well as indirect. They were commonly used in mathematics as well as in dialectics prior to Aristotle[6]. They were for a philosopher and a logician like air for breathing: one needs it and perhaps is aware of its necessity, but is not necessarily motivated to discuss such elementary and obvious data. More than two thousand years had passed before these ways of deductive behaviour were codified as deduction theorems. And, in fact, if Aristotle had only written the rules of deduction (direct and indirect) down, nothing could be questioned by a puritan mind[7].

So we have Aristotle's logic as an axiomatic system. Some forms of syllogisms are distinguished as the so-called Perfect ones. It is rather inessential whether they would be better reconstructed as axioms or as primitive rules of inference, but the second solution seems more accurate.

Aristotle's attitude towards the problem of axiomatization recommends itself as an up-to-date one. Aristotle, what is quite justified from the ontological as well as from the methodological point of view, aims to choose as Perfect those syllogisms which are "the best", the most evident. The additional desideratum is to have a perfect syllogism for justifying any kind of conclusion: affirmative or negative, universally quantified or particular. The other syllogisms would be "imperfect", eventually subjected to "perfection" via deductive connection with those declared to be perfect.

Now, once having the non-perfect but still valid syllogistic schemes enlisted, Aristotle undertakes a new task. He seeks the possibilities of other axiomatic arrangements, e.g. showing how the system can be based on the second figure syllogisms alone, with the four "perfect" syllogisms of the first figure deduced from them. Or he states that two of those perfect syllogisms (Darii, Ferio) can be deduced from the second figure, where all the moods need for their proofs only two perfect syllogisms (Barbara, Celarent). Thus we are confronted with the evident anticipation of modern investigations in deductive sciences methodology (the problem of the deductive equivalence of different sets of axioms, and the problem of the independence of axioms).

We will accomplish this part of Aristotle's logic inventory with the observation that his procedures included the paradigm of decidability. Namely, the syllogistic moods with two premisses are decidable, because they

are recognized, one by one, as valid or not, and syllogistic schemes with more than two premisses are discribed as reducible to the basic, two-premisses moods (Book I, 42^a, 42^b). Treating his syllogistic as a tool, Aristotle was strongly motivated to make this tool effective.

But it is clear that logic was not only the tool in Aristotle's own eyes, that sometimes he was absorbed by logical game for its own sake - and he was very good in this game.

4. The idea of Cn-theory anticipated

While recognizing only three figures in his system, Aristotle takes it for granted that the order of premisses is inessential. The question whether he was correct in dividing syllogisms in such a way was extensively discussed. Now we understand that there was no error in Aristotle. Still, the problem as to why Aristotle made his classification as he did remains open. I have tried to contribute to this discussion elsewhere [9], referring to the possible semantic interpretation for the moods of all the figures. But there is also another aspect intervening here. Remember that enlisting his moods one by one, as grouped into these three figures, Aristotle does distinguish certain moods only on the basis of the order in which the premisses of the given mood appear. For example, in the second figure, *Cesare* and *Camestres* are treated as different moods. It may be seen as a lack of consequence, but it is not. The idea of such a procedure is deeper and has non-trivial justification. Namely, Aristotle operates here on two different levels.

Firstly, he works on the metasystemic (metalinguistic) level, where the general features of the system that he wants to construct are described. Then there comes the construction itself. Here intra-systemic linguistic convention is needed. On the first level the order of premisses is really inessential. They are not yet needed (and thus on this level there is no place for the fourth figure). In the standard set-theoretical approach (e.g., in Tarski's theory of consequence), it is actually assumed that the elements of the set for which the consequence operation is defined are not ordered. This is just the sort of treatment that we find in Aristotle's basic, general descriptions of the syllogistic figures. On the other hand, in our own practice, while constructing logical calculi - for instance the propositional calculus - we do not identify conjunctions which differ in order of their components. And this was Aristotle's way as well.

* * *

Aristotle the Logician is usually seen via the system he founded. On reflection we learn that the idea of axiomatization was familiar to him, we see how skilful he was in using the direct and indirect method of suppositional proofs. His sophisticated way of constructing counter-models for non-valid syllogistic moods is worthwhile as well.

Let us try to complete this picture. Let us have a glance at Aristotle's logical workshop. One statement will serve us as the starting point. (Prior Analytics, Book I, Chapter 7,29[a]):

> It is evident also that in all the figures, whenever a proper syllogism does not result, if both the terms are affirmative or negative nothing necessary follows at all, but if one is affirmative, the other negative, and if the negative is stated universally, a syllogism always results relating the minor [as predicate] to the major term [as subject], e.g. if A belongs to all or some B, and B belongs to no C : for if the premisses are converted it is necessary that C does not belong to some A.

Notice the logical structure of this statement, bringing justification of the fourth figure (added later) moods: *Fesapo*, *Fresison*. Notice the quantifying expressions, conditionals. Try to reconstruct the table of all the moods necessarily taken into account as the basis for formulating this statement. Aristotle must have undertaken an extended and systematic analysis and generalized afterwards its results as far as the subject matter would permit. The author must have had excellent logical acumen!

Finally, a remark on Aristotle's workshop as a teacher. Many examples are cited in his text: men are mortal, swans are white, etc. As a rule all those examples appear in the context of disproving invalid moods; there are no examples for valid moods - only their construction is described[8]. It would seem that Aristotle wished to avoid the danger of misunderstanding, i.e. of an example being regarded as a proof.

Prolonging the beneficial tradition of quarrels about the importance of Aristotle's ideas, Leon Chwistek once wrote:[9]

> Aristotle's system was a fragment and was not free from fundamental errors. It was but the first step along a long and wearisome path; unfortunately men have regarded Aristotle's work as perfect and for many centuries did not attempt to go beyond it. Even today it is still defended and passionately adhered to, although it is well-known that its study is a useless requirement.

Is such an opinion acceptable? I believe it is only as far as it states that syllogistic was but the first step along the path towards contemporary logic. Aristotle's errors (if any - it depends on what one defines as an

error) were not so great as to call them fundamental. Later many attempts were undertaken to go beyond the text of Prior Analytics. All the centuries of the history of logic are accompanied by different attempts to such effect, although the horizons for those attempts certainly were often limited. The petrified shape of the academic courses in logic in scholastic education is quite another problem.

Last, but not least: is the study of Aristotle's logic a useless requirement? I believe not. The only trouble is that it is more useful to read "Prior Analytics" than to learn SaM-MaP-SaP-style constructions.

For every discipline it is profitable to know and understand its roots. For logic it is even more important: to know its beginnings means to know something essential for an understanding of the very nature of human thinking. Through reading Aristotle, we see how efficient this thinking can be. Re-reading Aristotle, we are still finding something as yet unnoticed.

NOTES

1. Two stages can be distinguished here. Reconstructions initiated by Łukasiewicz take as the point of reference axiomatic propositional calculi with substitution and detachment. Then there comes a new (and nearer to Aristotle's own text) stylistics to interpret Aristotle's system as based on the suppositional rules. Here papers incorporated in Corcoran's monograph [3] are most instructive. Cf. also monographs by P. Thom and G. Patzig [8], [7].

2. Diogenes Laertios, Vitae, VII, 76 ff. Cited after Mates [6].

3. Letter symbols as referring to propositions are extensively used e.g. in Chapter 25, Book I, 41^{b}, 42^{a}. Cf. Łukasiewicz comments in [5].

4. Compare Patzig's remarks, especially p.9 in [7].

5. I guess that Geach's term "indesignate" is more accurate than other translations, for instance Jenkinson's "indefinite" in [1]. Fragments of the "Analytics" in Geach's translation are included in [2], along with the bibliographical information (p.24) prepared for the forthcoming volume in the Claredon Aristotle Series.

6. Compare with Waszkiewicz and Wojciechowska "On the Origin of Reductio ad Absurdum", in this volume.

7. Suppositional interpretations and practice of the proofs of this kind are - as well known - recognized as justified once the deduction

theorems have been proved (A. Tarski, J. Herbrand, 1930). Aristotle's proofs "per impossibile" are usually reconstructed with reference to the so called "strong" (classical) indirect deduction theorem (see [3]). However, the intuitionistic variant of it would be here sufficient if only the theory of oppositions (logical square) were adopted in the appropriate way.

8. Łukasiewicz [5], p.2, points out that Aristotle's only illustration of a valid syllogistic mood appears in Analytica Posteriora, Book II, Chapter15, 98^b ("If all broad-leaved plants are deciduous and all vines are broad-leaved plants, then all vines are deciduous").

9. L. Chwistek, Granice nauki, 1935. English translation "The Limits of Science", London 1946. By "fundamental errors" Chwistek means the nonrecognisition by Aristotle of the consequences of using null class in syllogistic ("The Limits ...", p.9).

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Leopold Regner

IMPOSSIBILIA OF SIGER OF BRABANT

Apart from lectures the main method of teaching in the universities of the Middle Ages was disputation. It was the purpose of the disputations to exercise in upholding theses by appropriate arguments and in solving scientific or philosophical questions.

The objects of disputations in the faculties of arts were questions of grammars, of rhetoric and of logic. But soon the masters of arts began to discuss also philosophical and scientific questions.

The logical disputations are of three kinds: the fallacies (*sophismata*), the *insolubilia*, and the *impossibilia*.

The fallacies are manifold. They may - namely - consist in discordance with rules of reasoning, or in breach of rules of syntax.

An example of syntactical fallacy is the sophism of Albert of Saxony[1]:

Omnes homines sunt asini vel homines et asini sunt asini,

which is a false sentence, if the first *vel* (or) is interpreted as a propositional connective, but a true sentence, if the first *vel* is interpreted as a name-forming functor.

The *insolubile* is almost identical with what is called **antynomy**.

The *impossibile* was a disputation, in which one or several masters of arts have refuted a sentence which was contradictory with any scientific or philosophic principle.

The script "Impossibilia" of Siger of Brabant is a report of disputations of masters of the Faculty of Arts in Paris with one "sophist" (perhaps sophist-errant). This report contains six *impossibilia*, which conserved as manuscripts was edited by Cl. Baeumker (1895)[2].

The author of the "Impossibilia" was taking the principles of Averroismus.

First of all author writes: *Convocatis sapientibus studii Parisiensis proposuit sophista quidam impossibilia multa probare et defendere* (One sophist challenged the masters of the school of Paris and proposed to argue and to defend numerous *impossibilia*).

Impossibilia of Siger of Brabant are following:

1. *Deum non esse* (There is no God).
2. *Omnia, quae nobis apparent sunt simulacra et sicut somnia. Ita quod non simus certi de existentia alicuius rei* (Everything which appears to us is phantom and like a dream. Therefore we are not sure that anything exists).
3. [...] *quod bellum Troianum esset in hoc instanti* ([...] that the Trojan war is at present [at this instant]).
4. *Grave existens superius non prohibitum non descenderet* (A heavy body, which is up high without any support, would not fall).
5. *In humanis actibus non esset actus malus, propter quam malitiam actus ille deberet prohiberi vel aliquis ex eo puniri* (Among human acts no one is wrong, and therefore no malice may be cause to interdict this or to punish for it).
6. *Contingit aliquid simul esse et non esse, et contradictoria de se invicem vel de eodem verificari* (It happens, that anything may exist and at the same time not exist, that contrary predicates can verify each other, or both the same thing).

Each *impossibile* includes three parts. In the first part the sophist evolves arguments for his own statement; in the second the defender (*defendens*) explains the meaning of the examined question and proves the statement, which is contrary to the sentence of the sophist; in the third - the defender refutes the arguments of his opponent.

At the beginning of the first *impossibile* the author warns that the sentence of the sophist is an *impossibile*, since the contrary sentence is a necessary principle, and - what is more - a first one.

Let us confine our reference to the most interesting reasoning we find in the third and in the fourth *impossibilia*.

The third *impossibile*, which states, that the Trojan war is at present [in hoc instanti] brings out difficulties which are implied by the notion, or rather by a peculiar way of apprehending time.

The sophist's reasoning is as follows:

The proposition "that the instant, which is now, is the same as that [of the Trojan war], may be argued as follows: Since the numerous instants can not exist simultaneously, [...] and the instant, at which the Trojan war took place, was not annihilated, therefore it remains as the same as this instant, which is now".[3]

Further reasoning of the sophist is as follows:

The instant, at which the Trojan war took place, was not annihilated and continues to exist. Whenever anything perishes, we may point out the instant in which what perishes loses existence. But we cannot point out, when the present instant disappears for it is not annihilated while existing nor

does it disappear at the nearest time after, since the instant - as a frontier of any part of time - cannot be equally limited by any frontier.

In the second part of *impossibile* the defender says, that this instant now is not the same as any instant during the Trojan war, for it came to an end long ago. The problem requiring a solution is to answer, how this instant, which is now, vanishes.[4]

It is impossible to indicate, when the present instant ceases to exist, since the transition from being to non-being is accomplished at a frontier, which separates the time of being from the time of non-being. It is possible to indicate the point of time, in which any thing ceases to be, only in the case of things, which persist throughout a part of time, but it is impossible in the case of any thing, which does not persist. The present instant is not anything which persists and is divisible.

In the third part of *impossibile* the defender knocks out the bottom of conclusions deduced by sophist and indicates that the annihilation of instances is a consequence of the fluent change of time. This change affects the being only, but not the "substance" of time. Hence the "substance" of time is unique, the motion of time is continuous.[5]

The same question is investigated by Aristotle: Is the now (τὸ νῦν) always the same or not?, and he answers, that in one respect it is the same, and in another, something else, similarly, as Coriscos in Agora and Coriscos at Liceum is the same in respect to substance, not the same in respect to being here and being there.[6]

Saint Thomas Aquinas, who was the contemporaneous with Siger of Brabant and his adversary, does not add any more to what Aristotle said. Thomas says that in the the flowing of time there is only one "now" (*nunc*), which is always the same in respect to substance (*secundum substantiam*).[7]

We ought to note, that the term 'substance' in this context: "the instant 'now' is the same in respect to substance" is void of sense, since we cannot perceive what is the substance of instant or what is the substance of time, or what means the term "the same" or "identical" in the category of attributes. We cannot - for example - say that the whiteness of snow, which fell during the Trojan war, was the same as the whiteness of snow which is falling now, since we cannot say, what the term "the same" means in respect to whiteness.

The problem of identity of the instant "now" interested also other scholastic masters. Among the questions disputed at the University of Cracow in the XVth century was the question: *Utrum hoc ipsum nunc maneat unum et idem toto tempore?* (Did the alone "now" persist as the same throughout time?).[8]

The fourth *impossibile* shows, how helpless is a man, who - restricted by false prejudices and devoid of proper methods of scientific research - initiates an investigation of the phenomena of the material world.

In the reasoning, that a heavy body, which is at some height without any support, would not fall, the sophist refers to the Aristotelian principle, that the body, which has not an inherent source of motion, can be moved only by some outer cause.[9]

The sophist disregards the belief of Aristotle, that the heavy body falls downwards because it is naturally heavy. The heavy body has the inherent cause of falling.

The response of the defender opens doors to other questions or even to insoluble difficulties. The defender takes for granted the supposition of the sophist, that a heavy body cannot fall without an external cause of motion. The defender gives the following solution to the question, how can fall the heavy body, which is at some height without support. Such body's fall is caused by the movement of the environment, such as the surrounding air, but the environment is moved by the heavy body. Therefore the heavy body and the environment push each other.

The defender is referring to Averroes, who illustrates this solution by the example of a boatman, who pushing the boat to make its movement is himself moved by the movement of the boat.[10]

In opposition to this theory Averroes' opponent points out that the heavy body cannot move the environment before it has moved itself.[11]

The defender endeavours to solve this difficulty, but becomes caught up himself in a vicious circle.[12]

Since according to the theory of Averroes the heavy body falls because it is driven by the movement of a non empty environment, the question arises, whether or not the body can fall in empty space and whether - when it can fall - it will fall faster, than in a non empty environment. The defender refers to Aristotle, who asserted, that a vacuum is the same as nothing, i.e. non-being. Therefore movement in vacuum is impossible. Aristotle argued that the speed of movement in an empty space would be infinite, since the body would shift from place to place of vacuum at the same instant - which is impossible. The notion of vacuum accepted by Aristotle is a metaphysical one.[13]

Albert the Great and Thomas Aquinas did not repudiate the opinion of Aristotle, but they had adopted another notion of vacuum. Their notion of vacuum is that of any part of space which is entirely devoid of matter. Albert the Great calls the vacuum by the name of separate space (*spatium separatum*)[14] and Thomas Aquinas (reporting to an Arabian philosopher, Avempace) says that a vacuum has the properties of cosmic space in which

the heavenly bodies travel[15]. This notion of vacuum is almost the same as that in modern physics.

The defender disagrees with Thomas Aquinas, who assumed that the fall of a body in empty space is speedier than that in a non empty environment, and that the difference of speed is caused by the resistance of matter in the environment.

The defender distinguishes between being moved by the agency of one's own movement (*motus per se*) and being moved by the agency of the movement of another thing (*motus per accidens*). Therefore a boatman is moved *with the movement of his boat* (accidentally, *per accidens*), whereas the boat is moved *with its own movement* (*per se*). However the boat is moved *with* its own movement, yet *not by itself*, because it is moved by the boatman. On the contrary the boatman is moved *by himself*, but *not with his own movement.* The defender says: "Therefore we take the side of the Commentator [Averroes], that a heavy body is moved accidentally (*per accidens*), and that its movement must depend on a motive agent which is moved by itself, and without it there would be no movement.[16]

The defender cannot accept the theory of Thomas Aquinas.

Many authors had maintained that science in the Middle Ages has been restricted to the study of Holy Scripture, the writings of Fathers of the Church and of very few philosophers. Such an opinion is not quite right, because we find in the history of science in the Middle Ages, at least as far back as the 11th century, remarkable individuals, such as Gerbert (later Pope Sylvester II), Frederick II and Roger Bacon, who were conscious of the necessity of investigating the natural world by observation and experiment. Numerous scholastic masters were interested not only in questions of philosophy or theology but also in the investigation of natural phenomena. The testimony for that can be found in numerous disputed questions, such as the "Impossibilia" of Siger of Brabant.

We may consider the centuries of the late scholastic period as an early spring of modern natural sciences.

NOTES

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3. Baeumker, *op. cit.*, p.11.
4. *Idem, op. cit.*, p.11.
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6. Aristotle, *Phys.* 219 b 26 *sqq.*
7. Thomas Aquinas, *IV Sent.* d.11, a. 2 ad 2^{am} quaest. ad 2.
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11. *Idem, op. cit.*, p.14.
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13. Aristotle, *op. cit.*, 214 b 28 - 215 a 14; 215 a 24 - 216 a 7.
14. Albertus Magnus, *Phys.* IV, tr. 2 c. 7 (t.II, p.173, col.a, Jammy, ed.). - Baeumker, *op. cit.*, p.17 (note).
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16. Baeumker, *op. cit.*, p.19.

Tomasz Weber

DEFENDING THESES IN MATHEMATICS AT A 19TH CENTURY UNIVERSITY

In 19th century Wrocław University was an average European university, so it may well serve to illustrate the procedures employed to grant university degrees in those days. The procedures differ from those of our times; the aspirants for scientific degrees had to satisfy the requirements which, in some respects are now found to be divergent from the contemporary ones.

The principles of conferment of a doctor's degree at the Wrocław University Department of Philosophy were set by the university statute [8] and Department Regulations [6] including the amendments introduced later on [2].

A candidate for doctor's degree submitted to the Dean the application for doctoral commencement, including the following:

1. Autobiography
2. University certificates
3. Doctoral dissertation with theses

The dissertation was usually hand-written, and up to 1867, similarly as the autobiography, had to be in Latin. The certificates had to explicitly state that the candidate had studied for at least three years. If, according to the documents, the candidate was found to be politically suspicious, the Department was obliged to consult the superintendence office in order to obtain the agreement for doctoral commencement. If the candidate was a foreigner, such an agreement was obligatory. Those who had not been matriculated at Wrocław University prior to doctoral commencement were to do so to be subject to academic jurisdiction.

Having been accepted by the Dean and the Department, the dissertation was handed over to the professor of a given field (mathematics) to be reviewed. On the basis of the review the Department allowed the candidate to take an oral examination.

The **doctoral examination** was organized as a committee examination. According to the regulations [2],[6], it was held at the Department. The candidate was thoroughly examined in mathematics by one or, after 1876, by

two professors of mathematics. Afterwards, the professors of associate disciplines, i.e., astronomy and physics, began examination. That part of examination being over, the other Department members, the professors of philosophy and classic philology, history and natural sciences in particular, were allowed to ask questions. Questions and answers, depending on the field, were in German or in Latin.

In practice, the examination scope gradually became wider, which resulted in the increase of the number of doctoral theses accepted at the Philosophy Department at Wrocław University in 19th century. [4]. In the years 1850-1865, the candidate was expected to pass the following: mathematics as the main subject, physics, philosophy, classic philology, astronomy, and one of natural sciences (botany, zoology, chemistry) or history. In the years 1865-1878 only mathematics (from the year 1876 onwards, two mathematics examinations were to be passed with two mathematics professors), physics, astronomy, philosophy and classic philology remained for the candidate to pass. After 1878, philology was omitted. In 1893, an amendment was issued to supplement the regulations at the Philosophy Department [2], which controlled the selection of examination subjects. According to the amendment, mathematics as the main subject, philosophy and two optional subjects were to be passed. Notwithstanding this, the candidate could take an expanded examination in mathematics, philosophy and one optional (usually physics) examination; the mathematics exam was conducted by two professors. The subjects were passed one after another, so the whole examination took several hours.

The final result of a doctoral examination was determined by voting. Practically, only the examiners and the dean did the voting. Partial exams as well as the whole examination were graded in the following way:

magna cum laude	- very good
cum laude	- good
superato	- satisfactory

The unsatisfactory grade was marked as *non superavit*, or remained unmarked. In some exceptional cases a positive mark was given - *summa cum laude*. If, however, the candidate's knowledge of one of the partial subjects was found unsatisfactory, then the exam could be accepted as passed under condition that the remaining examinations were passed as very good and the knowledge of the unmasked subject was at the secondary school level.

As an illustration let's take the formal record of July 27, 1865, issued after the doctoral examination held for a famous mathematician, Moritz Pasch [9]:

Today, at eight o'clock in the morning, the Philosophy Department was convened to attend an oral examination for Moritz Pasch.

During the astronomy exam, the candidate understood but few questions about theoretical astronomy and partially those about spherical astronomy. He revealed some understanding of these disciplines although they had not been thoroughly analysed, and the exam showed that the candidate deserved no more than *cum laude*.

/-/ Galle

The mathematics exam started with a discussion on the theory of conic sections; this being made in reference to the candidate's dissertation. Both analytic and synthetic elaboration of these curves was found to show a complete knowledge of this problem as well as his thorough study of the discipline. As regards algebra and the theory of determinants which were tackled rather in details, the candidate revealed less efficiency. Furthermore, as regards the issues connected with the latest branches of mathematics - the theory of elliptical functions and indefinite integrals, the answers given clearly indicated, in spite of some minor mistakes here and there, that his over-all comprehension is fully intelligible and correct, and that the concepts used were clear and succinct. Finally, basic principles of analytical mechanics were discussed, as well as their application in the problems of planets; in addition to that the theory of disturbances (Störungen) was discussed which revealed the candidate to possess a good knowledge of application of pure mathematics. Thus, he is hereby given *magna cum laude*.

/-/ Schröter

A few questions on the principles of logic (the law of contradiction and the law of excluded middle) were on the whole answered correctly by the candidate. He also showed solid and wide knowledge of Leibniz's views. So, he deserves to be given *superato*.

/-/ Evenich

After thermic laws, the candidate was examined on thermal radiation, light radiation and drachemistry causes (Drachemie) appearing in phosphorescence and fluorescence. The candidate was visibly nervous and due to that, was a little distracted - he got *cum laude*.

/-/ Frankenheim

In Latin, the candidate was presented Seneca's epist.88: 20-24 and 28, and in Greek, Xenofont's annal. T.3: 1, 2; both texts were translated with joy giving certainty and fluency. The candidate proved that his

specialized studies were not detrimental to his knowledge of grammar derived from school, which he preserved in its entirety. I am giving *cum laude* with regard to the fact that he is not a philologist.

/-/ Haase

As for history, the questions concerned the history of mathematics. There is no mark to be given to him. The whole examination deserves without hesitation to be marked as *magna cum laude*.

/-/ dr W. Junkmann
decan

/-/ Haase /-/ Schröter /-/ Galle

After having successfully passed the doctoral examination, the candidate had his doctoral dissertation printed together with the theses and autobiography. Printing costs were covered by him, and the edition had to comprise 100 copies, since this was the number of copies to be delivered to the university. Some of the copies were sent to different libraries, and the remaining ones came into possession of those Wrocław University professors who showed interest in the dissertation. All this, however, had to take place at least three days before the **public defence of the dissertation and the theses**. The defence itself had to take place no later than three months after the doctoral examination. The date of the defence was announced on a special notice-board by the Dean ("am schwarzen Brett").

To the defence the candidate invited the so-called **opponents**, out of which at least three to be approved of by the Department. They mainly criticized the candidate's thesis, that is, the statements of general or philosophical nature which normally did not have much to do with the contents of the dissertation. After the opponents, the dissertation and the theses could be criticized by any academic teacher or student who prior to that had to report at the dean's office. The professors could expand the range of discussion if it seemed too narrow to them, or if they simply chose to raise its level. It was up to the Department to decide about the final result of the defence.

As an example of theses defended in 19th century Wrocław University let's take Moritz Pasch's theses defended on Aug. 21, 1865 [5]:

1. *Probanda videtur ratio, qui in mechanica tradenda dynamicem cum statice contexunt.*

(It seems that right are those who connect dynamics and statics in lectures on mechanics).

2. *Matheseos historia injuste negligi videtur.*
(It seems that the history of mathematics was unrightly neglected).

3. *Elementorum, quae dicunt, mathematicorum fines ut longius dilatentur, hodienae scientiae status jam nunc necessario postulat.*
(The state of present-day science requires that the range of elementary mathematics teaching be expanded).

4. *Nisi physicarum rerum studia in iis scholis, quae ad universitatem literarum adeundam juvenes praeparant, cum diligentia colantur, ad eum quem spectant, inteligentiae gradum nego in illis discipulos promoveri posse.*
(I maintain that if the schools preparing future university students did not conscientiously nurse physics teaching, graduates of these schools would not attain desirable intelligence level).

The doctoral theses presented by Wrocław mathematicians reflect the philosophical disputes permeating the 19th c. mathematics. In order to defend them successfully the candidate had to be able to polemize and to know the issues connected with them.

If the defence of the theses was successfully completed, then the ceremony of doctoral commencement took place. A newly promoted philosophy doctor took an oath and received the doctorate diploma. From 1867 onwards, the oath contents was as follows:

> *Ego (nomen) juro tibi, Philosophorum Ordinis Decano tuisque successoribus et toti Ordini Philosophorum debitant reverentiam, deinde, me hunc gradum, alibi non esse iteraturum; postremo profiteor, me abhorrere a fanaticis opinionibus, et sancte promitto juroque, me consensum doctrinam ecclesiae christianae in scriptis propheticis et apostolicis traditiam constanter retenturum perpetuoque defensorum. Ita me Deus adjuvet et sacrosanctum ejus Evangelium.* [6]

This oath had been in till 1899, before it was abandoned as being "not always possible to be fulfilled". The doctorate diploma included the information about the title of the dissertation and the doctoral examination mark. The expenses of doctoral commencement were covered by the candidate. They were 100 talers, and after 1871, 450 marks. Half of the fee the

candidate paid at the university cashier's office before the doctoral examination. In case the examination had been failed the money was not returned unless the candidate took a "second session" in half a year's time. For the "second session" he would have had to pay half of what had already been paid before. The remainder of the fee was paid after the examination but before the doctorate diploma conferment. High costs of the doctorate might be ascribed to the fact that "doctor's degree was honoured on equal terms with noble titles" [4] in the society of those days.

As it can be seen from partially investigated further activities of Wrocław doctors, doctorate was a solid token of their high qualifications as future grammar school teachers, and a good prerequisite for the profession as teachers. Scientific career was taken up only by seven of them, they are: R. Sturm, M. Pasch, J. Rosanes, V. Eberhard, F. London, G. Landsberg and E. Steinitz. They constitute only 13% of all those mathematicians who got doctorate diploma in Wrocław in the 2nd half of 19th century.

Habilitation at 19th c. German universities was strictly connected with the work of the candidate as private assistant professor (Privatdozent). It was not possible for one to habilitate oneself without working at the university. The number of private assistant professor 'posts', however, was regulated at the Wrocław University Department of Philosophy for given specialization lectures in consultation with the Ministry. The word 'post' is set in inverted commas because a private assistant professor did not get his salary from the state but lived on the fees paid by the students attending his lectures. It was not a lucrative occupation for a junior worker as there were applications issued by the Department to the Ministry asking for financial support for private assistant professors [1]. After 1875 the living conditions of private assistant professors improved. They could get a state granted fellowship of 1500 marks, and after 1884 - 6000 marks a year. The fellowship was granted for the period of four years, and after 1884 - up to the moment when the private assistant professor got a permanent post as an extraordinary or ordinary professor, or until retirement [2].

The principles controlling habilitation were set by the Wrocław University Statute [8] and Regulations at the Philosophy Department of Royal University in Wrocław [6] including later amendments [2]. According to the principles, habilitation took the following routine:

I. A candidate for private assistant professor submitted at the Dean's office the application for habilitation together with the following documents:

1. Two copies of autobiography written in Latin with special regard being paid to the history of studies.
2. Original of doctorate diploma.
3. Graduation certificates issued by the universities at which the candidate had received education (They were to assert that at least two years had passed after graduation).
4. Intended topics of lectures he intended to give.
5. **Habilitation thesis (dissertation)** on mathematics in manuscript together with the declaration that one is the author of thereof. Up to 1867, the dissertation had to be written in Latin; later it could be written in German [2].
6. Declaration issued by the University Curator stating that there is nothing against the candidate to become a private assistant professor.

The Department accepted the candidate for habilitation by voting. Negative voting results were non-abrogative.

II. The **habilitation commissioner**, that is, a mathematics professor appointed by the Department reviewed the habilitation dissertation and then made the motion to accept the candidate for further habilitation stages.

III. Having been accepted by the Department, the candidate presented three optional topics for the so-called **trial lecture**. It was up to the commissioner to make a choice of the topic. Moreover, the candidate paid 20 talers in gold at the university cashier's office to cover habilitation expenses.

As an example of proposed topics for the trial lecture let's quote the topics submitted at the Department on Aug. 8, 1856, prepared by Heinrich Schröter who, later on, was the Wrocław University professor for many years:

1. *Über die Apolloniusche Berührungsaufgabe und deren verschiedene Lösungen vermittelst der Prinzipien der neueren synthetischen Geometrie.*

   (On Apollonian task of tangents and its various solutions based on principles of new synthetic geometry).

2. *Über die Transformation der Gleichungen der Bewegung in neue Variable und die Zurückführung der dynamischen Probleme auf eine partielle Differentialgleichung zweiten Grades und erster Ordnung, nebst einer Anwendung auf die Behandlung der geradlinigten Bewegung durch elliptische Koordinaten.*
(On presenting motion equations in new variables and on reducing dynamics problems to partial differential equation of second degree and first order together with the application to the treatment of straight-line motion set in eliptic coordinates).

3. *Über die Additionstheorem der elliptischen Funktionen erster und zweiter Gattung.*
(On the theorem of addition for eliptic functions of first and second type).

Professor Kummer wrote down on Schröter's proposal paper the following: "I advise you to choose the second topic". The trial lecture on the selected topic and the colloquium took place at 3 o'clock in the afternoon on Aug. 10, 1855 [1].

IV. The candidate gave the trial lecture in front of the Philosophy Department members. After that he went through the **colloquium** during which he was examined on mathematics by the commissioner. The other members of the Department also had the right to ask questions. The examination was held either in German or Latin; the choice of the language was up to the examiner.

V. After successful completion of IV stage, the candidate had the dissertation, theses and autobiography printed. After having been printed, the materials were distributed among the Department members and others who found interest in the work. Then there was a **public defence of the dissertation and theses**. The date and the place were printed on the cover of the work, close to the names of three opponents and one respondent. It was the task of the latter to help the candidate parry the objections raised by the formers on principles similar to those followed during doctoral defences. The defences consisted in discussing the theses presented by the candidate, which, on the whole, like with doctoral theses, were of general or philosophical nature.

For example, Schröter's theses [7] discussed on Oct. 20, 1855 are:

1. *Mit dem Fortschritte jeder Wissenschaft mehren sich die Thatsachen und scheinen sich zu verwickelt; aber die Erklärung der ihnen zu Grunde liegenden Ursachen wird einfacher.*

   (With the development of each science, facts seem to multiply and complicate; their underlying reasons, however, become more and more easy to explain).

2. *Allgemeinere mathematische Sätze sind immer einfacher zu beweisen als spezielle Fälle.*

   (General mathematical theorems are always easier to prove than the particular cases).

3. *Der Weg der Enfindung ist selten der kürzeste, aber der nützlichste beim Unterricht.*

   (The path of discovery is rarely the shortest, but most useful while teaching).

4. *Physikalische Hypothesen können nur dadurch gerechtfertigt werden, dass die auf theoretischen Wege aus ihnen abgeleiteten Resultate mit den beobachteten Tatsachen übereinstimmen.*

   (Physical hypotheses can only be justified by the fact that their results theoretically derived from them agree with observed facts).

5. *Keine physikalische Theorie hat eine gläzendere Bestätigung gefunden, als die Undulationstheorie des Lichtes.*

   (No physical theory gained so full confirmation as the wave theory of light).

6. *Passend gewählte Bezeichnungen mathematischer Operationen haben oft zu neuen Entdeckungen gefürt.*

   (Appropriately selected definitions of mathematical operations often led to new discoveries).

7. *Die geometrische Methode des Unendlich-Kleine führt in den meissten Fällen schneller zum Ziele, als die analytischen Operationen der Differenzialrechnung.*

   (Geometrical method of infinitely small leads, in the majority of cases, more quickly to the final goal than the analytical operation of differential calculus).

8. *Die Theorie der elliptischen Funktionen enthält noch eine grosse Anzahl verborgener Eigenschaften der Zahlen.*

   (Theory of elliptic functions still incorporates a great number of hidden properties of numbers).

9. *Die Forschritte der neueren Analysis auf die Probleme der Mechanik angewendet, gestatten deren vollständige Auflösung.*

   (Progress of new analysis oriented towards problems of mechanics - it enables their full solution).

VI. After the defence, the Department admitted the candidate to the final stage of habilitation, i.e., to a public lecture known as **inaugural lecture** or **habilitation lecture**. The topic of the lecture had to be accepted by the Department, its date was announced on special posters and in "Schlesische Zeitung".

Only after this stage had been completed successfully, did the dean give the candidate the certificate of private assistant professor ("private dozent") at the Philosophy Department of Wrocław University.

The costs of habilitation are worth noting. According to the regulations, the exemption from the habilitation fee could apply only to the doctors promoted in Wrocław, who were offered the habilitation, to sons of professionally still active, retired or deceased professors, of the University judge, bursar and secretary. Any other candidate was obliged to pay 20 talers in gold before the lecture. This money was equally divided among the Department members. In case the habilitation had been failed, the money was not returned.

The aim of habilitation was to check on scientific and didactic abilities of a candidate for the post of private assistant professor. Scientific qualifications of the habilitation candidate was testified by the dissertation which had to fulfil at least the requirements stipulated for the doctoral thesis. Habilitation colloquium and the trial lecture supplemented the verification of the candidate in respect of his didactic abilities; this was confirmed during the inaugural lecture given in public. If the habilitation candidate was not the Wrocław University graduate, he was to go through the defence of habilitation theses which actually were of the same type as the doctoral ones - this was the confirmation for the Wrocław scientific community about his doctoral qualifications [10].

The 19th century mathematicians got their scientific degrees at a younger age than it is now. An average age of the doctorate candidate was 25 years, and the habilitation candidate 27-28 years. Among other things, this was due to the fact that the candidate had to master a smaller amount of scientific knowledge before achieving his own scientific results than it is now [3], [10].

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Gerhardt Plöchl

BASIC NORM AND METALANGUAGE. HISTORICAL BACKGROUND OF KELSEN'S IDEAS *

The basic norm concept is the central idea of Hans KELSEN's (1891-1973) 'pure theory of law'. According to one of his students - the famous Viennese professor of international law and legal philosophy Alfred VERDROSS (1890-1980) - the pure theory of law - is the "most refined and most successful" theory of law[1]. It should be mentioned that VERDROSS held a strong natural law position - quite contrary to KELSEN. According to another student of KELSEN, the famous Danish philosopher of law Alf ROSS (1899-1979) the pure theory of law is "the most important issue in legal philosophy in this century"[2].

The average continental lawyer may perhaps know less about the philosophical background of this concept than of its importance for legal theory. Saying this I do not refer to Immanuel KANT (1724-1804) and New Kantianism; their influence on KELSEN is widely known. I want to dig out those deep structures of thinking which shape scientific thoughts of every epoch, as has been shown by Michel FOUCAULT (1926-1984) in his inquiry into biology, philology, and economics[3], where he worked out the unity in the fundamental ideas in these sciences. FOUCAULT did not deal with jurisprudence, but his method can show new relations between KELSEN's basic norm concept and the ruling philosophical ideas of his time.

1. 'Vienna Circle' and 'Viennese School of Legal Positivists'

If we ask for the motives of KELSEN's writing, for the intellectual climate in which he conceived his theory, we have only to re-read the preface of the first edition of his *Pure Theory of Law*[4], which unfortunately is not part of the English edition. According to this preface, it was his aim for more than two decades to develop a legal theory "free from any political ideology, free from all elements of natural science". During that time he wanted to bring

legal theory "up to the heights of true science" with the aim "to bring it, as far as possible, near to the ideals of every science: objectivity and exactitude"[5].

Further we must bear in mind that from the very beginning of his reasoning, Hans KELSEN tried to work out strong parallels between physics and legal theory[6]. Knowing this and knowing the preponderant value of logic in KELSEN's thinking[7], the similarity between his ideas and the 'logical positivism' of the 'Vienna Circle' is obvious.

Like the Vienna Circle KELSEN's 'Viennese School of Legal Positivists' had its origin in the last years before World War One[8], and so some similarities in the leading ideas should not surprise us. We can find them from the very beginning: The Yugoslav professor Leonidas PITAMIC (1885-1971) has shown parallels between ideas in KELSEN's first major book[9] and the epistemological position of Ernst MACH (1838-1916), one of the founders of the Vienna Circle[10]. Later Hans KELSEN admitted that PITAMIC had been right[11].

The great time for both of these schools - the Vienna Circle and the Viennese School of Legal Positivists - was the twenties. At that time Moritz SCHLICK (1882-1936) was the central figure of the Vienna Circle. From 1922 until his murder by a student, he was successor to MACH as professor of the philosophy of inductive sciences. At the same time - from 1919 onwards - KELSEN was professor of law at Vienna university. But surprisingly we do not find any reference to the Vienna Circle in the *Pure Theory of Law*. KELSEN mentioned SCHLICK as the founder of logical positivism[12], and in a footnote[13] he discussed some ideas of Karl MENGER (1902-1985) no more.

But if we look at the characteristics of the Vienna Circle, we also find an attitude that is typical for the work of KELSEN: a radical opposition to any kind of metaphysics, a high appreciation of empiricism and a deep consciousness of the importance of formal logic as well as strong feeling for the necessity to have these ideas universally accepted in the whole scientific world.

Logical positivism and its rejection of 'metaphysics' was in the air. The pure theory of law was successful not only because of KELSEN's genius but also because of that intellectual climate, which is always a prerequisite for ideas to be successful. Part of this climate is "science in background[14], which forms the common basis of the sciences and philosophies of a certain period.

In 1921 Ludwig WITTGENSTEIN (1889-1951) published his *Tractatus logico-philosophicus*[15]. Regardless of the differences that arose later, that treatise was the logical basis for the work of the Vienna Circle. In the same year 1921 KELSEN published an essay about the relations between state and law in the light of epistemology. But reading this article one feels

disappointed. The title promises a discussion of leading philosophical thoughts but the article itself is restricted to an attempt to find parallels between the state and God - the state being limited to its legal structure. The essay ends with the claim "that the *theory of state* must stop being a *theology* of state"[16].

The essay was perhaps one of KELSEN's invectives against the theories of Carl SCHMITT, who the year after published a *Political Theology* - but KELSEN did not mention Carl SCHMITT! He argued vaguely against the "German theory of state"[17].

One point in KELSEN's biography seems to be important: when starting his university studies he had profoundly wished to choose philosophy, mathematics and physics. His final decision in favour of law had an economic background - poor professional prospects for philosophers[18]. We can feel this original intention as well as the intellectual climate of Vienna in the twenties[19], when KELSEN formulates his tenet in the preface of the *Pure Theory of Law*: "to bring legal science - this province far from the centre of the intellect - into quicker motion by direct contact with the universal theory of science"[20]. Or when we read - in the article mentioned above about "Scheinprobleme", i.e., "spurious problems" or "pseudo-problems" in the theory of state.

In spite of these endeavours KELSEN was not aware of the most modern ideas in physics[21], and although he underlined the importance of logic, he did not refer to the latest state of logic and of mathematics - and as a non-professional probably he was not able to do so. But exactly because of this lack of contacts to the newest achievements of logic, one is surprised to find roots that - according to the leading idea of FOUCAULT[3] - lie in deeper layers of scientific thinking. We find a strong structural relation between the concept of the basic norm and some - seemingly remote - problems of mathematics, logic and formal semantics. I will start my inquiry into these problems by giving a short sketch of KELSEN's basic norm concept, which is the central idea of his theory.

2. The Basic Norm - A Central Concept in the *Pure Theory of Law*

KELSEN's tenet was to establish "a theory of positive law ... a theory of positive law in general, not of a specific legal order"[22].

Looking for the right position for jurisprudence in a system of sciences, whether in natural science or in social science, KELSEN made out a problem: acts of parliament, judgements, agreements and even murder - all kinds of legal acts are acts of human beings and objects of sensual

perceptions. Thus they are *not* objects of legal theory. All these acts of human beings receive their specific legal contents by *norms*. "The norm functions as a scheme of interpretation"[23]. These norms, which are objects of legal knowledge, aim to regulate human behaviour. The contents of a norm is an 'ought' and as such it is radically different from an 'is'. It is this fundamental distinction between the 'ought' and the 'is' that forms the basis of KANT's critical philosophy.

Although KELSEN had studied the work of KANT as a high school student[24] and although he quoted KANT frequently, he did not do so in this part of his *Pure Theory of Law*. Perhaps KELSEN did not want to deal with philosophy but with an objective basis of that very science that was the object of *his* theories. In his theory, the distinction between the 'ought' and the 'is' is traced back to our consciousness. This distinction - he wrote - "cannot be explained further. We are immediately aware of the difference"[25]. This reference to immediate awareness is quite remarkable. It takes recourse to everyday thinking, to some sort of common sense. This might be enough if we investigate propositions about norms. But it is quite another question whether it is enough to appeal to common sense if we want to prove the validity of norms. This question may remain open, because KELSEN restricted himself to logical relations. For him "the causality of the natural order and freedom under a moral and legal order are not incompatible with each other; even as the natural order and the legal-moral orders are not contradictory, and cannot be contradictory because the one is an order of something that *is* and the others are orders of something that *ought* to be. Incompatibility as a consequence of logical contradiction can exist only between an assertion that something *is* and an assertion that it *is not*, or between an assertion that something *ought* to be and an assertion that it *ought not to be*; but not between an assertion that something *is* and that it *ought not to be*"[26]. A norm does not exist - says KELSEN - a norm is valid. "By the word 'validity' we designate the specific existence of a norm"[27].

As contents of a norm the 'ought' has an objective sense, which must be distinguished carefully from that subjective sense of 'ought' which is the contents of any human act of volition, that is to determine the behaviour of others. But such 'ought' comes to an end with the death of the very person that has wanted. A norm "created" by a legislator, on the other hand, continues to have its meaning and purpose beyond the existence of the historical legislator; it has this sense objectively.

In modern civilized societies legal norms are directed to determine human behaviour. In primitive legal systems they might also have (or have had) intended to regulate the behaviour of animals, plants, or inanimate things. But in every legal system - contrary to moral ones - norms are coercive. Their 'ought' is subjective to sanction by coercive acts.

Additionally, a system of norms that is to be considered as a legal system must on the whole be effective. We have to see, however, that KELSEN did not always hold this view[28].

Trying to interpret a certain human behaviour as a legal one, we will find that a specific norm which qualifies a certain human act as a legal act or as an illegal one "is itself created by an act, which in turn receives its legal character from yet another norm"[29]. This concept of hierarchical order was originally not formulated by KELSEN but by one of his most brilliant students - professor Adolf MERKL (1890-1970), who thus became a "co-founder" of the pure theory of law.[30]

This hierarchical coherence however, in the creation of norms must find, and does find its end in the constitution that is valid in a given time. But what makes the actual constitution valid? To appeal to a command of God means stepping outside the boundaries of science and gives rise to another question: What is the norm by which we ought to obey God? And no appeal to nature as such or to the nature of man can build a bridge that covers the gap between the 'is' and the 'ought'. The origin of the prevailing constitution may lie in the dim light of prehistory. *Logically*, the creation of a norm in a legal system must be based on another norm. KELSEN solved this logical dilemma by introducing the concept of the basic norm[31]. The basic norm is the very centre of his pure theory of law, its "coronation" as VERDROSS put it[32]. It is conceived as follows:

If only a norm - some sort of higher norm - can create the validity of another norm, then there must be an end to this succession of norms somewhere. The last, the supreme norm, which institutes validity and which KELSEN called the basic norm, cannot be enacted by any authority, it must be presupposed logically[33], "if we leave aside God or 'nature'", which in the German version are qualified as some sort of "meta-legal authority"[34]. And "a norm enacted by an authority superior to the legal authority" is called a "meta-legal norm".[35]

We can avoid the - otherwise inevitable - *regressus ad infinitum* only if this last norm is not stated but merely thought of. It is a norm which is thought of as a presupposition "of an effective coercive order, as a system of objectively valid legal norms"[36].

One circumstance is quite remarkable: that Hans KELSEN developed his ideas of a legal theory as pure theory - a theory, that is free from any elements of political ideology or natural science[37] - exactly in those years in which mathematicians discussed the foundations of their science.

Let me try to outline briefly that discussion, the subject of which is the old question of all sceptics:

3. What is Truth?

For more than 2000 years EUCLID (ca 300 B.C.) has been the authority in geometry. For Immanuel KANT Euclidean geometry was one of the reasons for the idea, that synthetic *a priori* judgements are possible. A leading mathematician of his time however, one of the greatest of his profession in all times, Carl Friedrich GAUSS (1777-1855) doubted, that geometry could "be founded completely *a priori*"[38]. He formulated his revolutionary idea in private correspondence, but "fearing the clamour of the Boeotians" he decided not to publish his discovery of non-Euclidean geometry. After him, in the 19th century non-Euclidean geometries were developed, but the decisive step was done by David HILBERT (1862-1943) in 1899 in his *Grundlagen der Geometrie* (i.e. Foundations of Geometry). With the famous opening phrase he made the radical cut that separated geometry from any reality, putting geometry on an axiomatic basis. In a modern textbook on the history of mathematics we can read, that his view was quite abstract from the very beginning. HILBERT does not say, what the term 'point', 'line' or 'plane' denote or what the relations between them are. The formal qualities of the relations between the basic elements of geometry are defined by axioms and in this way the geometric elements and relations are defined implicitly[39].

The radical separation between mathematics and the inductive sciences has become the paradigm of a new scientific method, which can be illustrated by a statement of Albert EINSTEIN (1879-1955), who in 1921 - in a festive lecture on "Geometry and Experience"[40] - coined the famous sentence: "In so far as the propositions of mathematics are related to reality they are not certain and in so far as they are certain, they are not related to reality." He insisted on the fact that without this insight he would have been unable to develop his theory of relativity.[41] But it must be underlined, that James Clerk MAXWELL (1831-1879) had already thought in this way in the 19th century[42].

With amazing precision WITTGENSTEIN formulated this program in one single sentence of his *Philosophical Investigations*[43]: "We must do away with all *explanation*, and description alone must take its place."

In his "Meta-mathematics" David HILBERT wanted to define the term 'prove' exactly and free from all "metaphysical influence"[44]. Parallels to KELSEN and to WITTGENSTEIN cannot be overlooked.

At the time when KELSEN published his *Hauptprobleme* (1910), Bertrand RUSSELL (1872-1970) and Alfred North WHITEHEAD (1861-1947) published their *Principia Mathematica* (1910-1913). Based on the ideas of Gottlob FREGE (1848-1925) this work propounded the logic of mathematics. It was to exert a decisive influence for decades[45], and came to revolutionize 20th century

mathematics to such an extent that it even influenced primary school teaching with its theory of sets.[46]

The founders of 'intuitionism' Henri POINCARÉ (1854-1912), Hermann WEYL (1885-1955) and L.E.J. BROUWER (1881-1966) disturbed the basis of many assumptions in classical mathematics. The idea of an unlimited realisation of mathematical programs - in everyday language, of their 'truth' - had to be given up. Intuitionism replaced 'truth' by 'proof' and as a consequence replaced Aristotelian logic by another one. This can be shown by the phenomenon of double negation.

In classical logic the negation of a 'true' proposition is a 'false' one and *vice versa*. 'True' is equivalent to 'non-false' and consequently a proposition is equivalent to its own double negation.

$$A \Leftrightarrow \neg\neg A$$

If we replace the terms 'true' and 'false' in this scheme by 'provable' and 'disprovable', it is clear that a provable proposition is 'not disprovable'. But the opposite is not true: not every 'not disprovable' proposition is 'provable'. In this type of logic a proposition and its own double negation are *not* equivalent.

Only the following is valid:

$$A \Rightarrow \neg\neg A$$

Consequently HILBERT replaced the call for 'truth' by the criterion of 'consistency'.

But the realisation of his program was decisively limited by Kurt GÖDEL (1906-1978), the famous mathematician of the Vienna Circle. He showed that the consistency of a given mathematical calculus cannot be proved by means of that calculus, even if the given calculus *is* consistent.[47]

Ten years earlier WITTGENSTEIN had formulated similar concepts for the solution of philosophical and linguistic problems in the *Tractatus*, where we read in paragraph 2 about *picture*:

2.173 A picture represents its subject from a position outside it. (Its standpoint is its representational form.) That is why a picture represents its subject correctly or incorrectly.

2.174 A picture cannot, however, place itself outside its representational form.

In paragraph 3 we read about *proposition*:

3.332 No proposition can make a statement about itself, because a propositional sign cannot be contained in itself (that is the whole of the 'theory of types').

And in paragraph 4 we find another statement about proposition:

> 4.442 ...It is quite impossible for a proposition to state that it itself is true...

In the same paragraph 4 we read about *language*:

> 4.121 ...What expresses *itself* in language, *we* cannot express by means of language...

In paragraph 5 he wrote about *function*:

> 5.251 A function cannot be its own argument, whereas an operation can take one of its results as its base.

And lastly we find the same notion about *laws of logic* in paragraph 6:

> 6.123 Clearly the laws of logic cannot in their turn be subject to laws of logic...

These quotations demonstrate that certain problems of truth can be seen and dealt with as problems of self-reference too[48] - and the same can be done with problems of validity.

For the problem of truth GÖDEL gave an exact proof; the results of his theorem are of greatest importance for the theory of science[49], and this should be so in legal theory too.[50]

4. Object Language and Metalanguage

For a mental experiment we will presuppose - as KELSEN told us - a basic norm that constitutes the validity of the commands of a tyrant, a certain Dionysius.

Dionysius establishes a single norm: "My commands shall not be valid." This looks familiar - and not by chance. Formulating the norm of Dionysius I have used the scheme of a classical paradox: the antinomy of the liar or of the Cretian (formulated in the 4th century B.C. by EUBULIDES).

More than 2000 years after the invention of the paradox its solution was found by Alfred TARSKI (1902-1983), a representative of Polish logical mathematics, who was in close contact with the Vienna Circle. And in 1942 both Hans KELSEN and Alfred TARSKI were called to Berkeley.

TARSKI was "the first who in fact stated a correct definition of a true proposition."[51] With his famous essay "The Concept of Truth in Formalized Languages"[52] he became the founder of formal semantics. The history of this essay can show us the drama that very often lies in the competition of scientific ideas.

TARSKI developed his theory of truth exactly when GÖDEL formulated his first theorem. Since the publication was delayed by some years, he added "Historical Notes" to demonstrate the independence of his ideas from those of GÖDEL. Wherever he pursued GÖDEL's ideas further, he said it expressly.

In 1934 - it was the year in which KELSEN published the first edition of his *Pure Theory of Law* - Karl R. POPPER met TARSKI for the first time - in Prague, at a conference organized by the Vienna Circle. The year after, in 1935, they met again in Vienna at Karl MENGER's colloquium. "It was in those days" told POPPER at a symposion in honour of TARSKI in 1971, "that I asked Tarski to explain to me his theory of truth, and he did so in a lecture of twenty minutes on a bench (an unforgotten bench) in the *Volksgarten* in Vienna". And: "I looked upon him as the one man whom I could truly regard as my teacher in philosophy. I have never learned so much from anybody else."[53] What was this famous theory of truth?

Starting from the classical paradoxes TARSKI showed that it is not possible in any complex language to describe its underlying system by means of this language. "What expresses *itself* in language," wrote Wittgenstein,[54] "we cannot express by means of language." Paradoxes are the inevitable result if one tries to do so. This can be avoided only if the description of the semantic system of a language is given in a language, which is different from that of the semantic system to be investigated. While the latter may be referred to as 'object language', the language used to describe it may be called 'metalanguage'. The metalanguage must be "stronger" or "richer" than the object language, because it must contain the whole contents of the object language and additionally all that we need to formulate the semantics of the object language. The object language and the metalanguage belong to different spheres that must not be mixed. But exactly this occurs in antinomies. This can be shown easily if we describe the antinomy of the liar by a scheme followed by Ota WEINBERGER[55]:

The statement in the right rectangle is true	The statement in the left rectangle is false

These two phrases disprove each other because they contain - like the phrases in the antinomy of the liar - propositions about their respective truth (or falsehood).

If we separate these propositions, as is postulated by the theory of the spheres of language, then it is impossible to construct antinomies. A phrase that mixes the spheres of language is wrong in its structure. And since a phrase that is formulated wrongly cannot be true or false, the problem of antinomies does not exist any longer.

In this field there is to be found RUSSELL's antinomy of the set of all sets that do not contain themselves - formulated in 1903. To illustrate this antinomy let us imagine an apprentice in a library who is to compile a catalogue of all books not recorded in the catalogue yet. This chap will find himself in a dilemma: Should this catalogue contain itself and thus eliminate the criterion for being part of itself or should it not contain itself and thus remain incomplete?

To solve this problem RUSSELL conceived his theory of types for which WITTGENSTEIN in his *Tractatus* coined the formulas mentioned above:

> 3.332 No proposition can make a statement about itself, because a propositional sign cannot be contained in itself (that is the whole of the 'theory of types').
>
> 4.442 ...It is quite impossible for a proposition to state that it itself is true...

By introducing metalanguage, TARSKI put truth again on it's throne: by defining it as correspondence with facts.

Neglecting the important difference between formalized and natural languages and between formalized languages of different complexity - and these differences play an important role in TARSKI's treatise - one can demonstrate the performance of the metalanguage concept by using two different natural languages, as POPPER has done using German as the object language:

> The German words *'Das Gras ist grün'* form a statement of the German language.
> On the other hand, we should be able to describe in our (English) metalanguage the fact which the German statement *'Das Gras ist grün'* describes. We can describe this fact in English simply by saying that grass is green.
>
> We can now make a statement in the metalanguage about the *correspondence of a statement of the object language to the facts* as follows. We can make the assertion: *The German statement 'Das Gras ist grün' corresponds to the facts if, and only if, grass is green* (Or: *...only if it is a fact that grass is green*)

This sounds trivial and it looks even more trivial if one uses English as metalanguage, as POPPER has done it:

> The English statement 'Grass is green' corresponds to the facts if, and only if, grass is green.[56]

But one must be aware that the statement between quotation marks is the *name* for a proposition in the object language.[57] On the other hand the proposition without quotation marks, which says that the grass is green, is *not* a metalinguistic name but a metalinguistic description of a certain fact. TARSKI's achievement lies in the discovery that the correspondence between propositions and facts can be formulated only in a (meta)language which can speak about propositions and which can describe facts too. If the system of the metalanguage is not more than equal to the system of the object language, then it is impossible to construct a definition of truth.[58]

A metalanguage of course can also be the object of semantic investigation, but to do this, we need another metalanguage, which is richer or stronger than the first one. This leads to an infinite hierarchy of languages. Here again we find a *regressus ad infinitum* and so we must stop by decision. A metalanguage can be used for the description of a given object language only, if we "posit" that we know the metalanguage.[59]

5. The Analogous Structure of Basic Norm and Metalanguage

We have seen that we can use a certain natural language as object language *and* as metalanguage if we are aware of the logical difference between object language and metalanguage. This can lead us to suppose that TARSKI conceived the idea of metalanguage for exactly the same reasons that had caused KELSEN to introduce the basic norm. GÖDEL showed that no system of propositions can make propositions about its own consistency (cannot prove itself logically) - even *if* it is consistent. And quite in the same way no system of norms can prove its validity, even *if* it is valid.

We can demonstrate this phenomenon by a "normative variation" of the antinomy of the liar:

Do obey the norm in the right rectangle!	Do not obey the norm in the left rectangle!

WITTGENSTEIN wrote about laws of logic, that they cannot be applied to themselves. And that can also be said about systems of law. If we look at the scheme above we find it evident that the paradoxical norm of Dionysius is structurally wrong. A norm cannot decree its own validity.

We can describe a language only by a metalanguage, the knowledge of which we presuppose, and according to KELSEN science can describe equally the validity of a legal system only by means of a norm, the validity of which is presupposed: the basic norm.

The similarity of the two problems and their solutions are hidden by the different names that were found for the new concepts.

TARSKI called the logically presupposed language in which we speak about language 'metalanguage', thus creating a term quite similar to HILBERT's metamathematics, to metalinguistics[60] in the linguistic sciences, or to metaethics[61] in ethics.

But there is an important difference. Metamathematics is part of mathematics (and therefore one prefers the term 'theory of proof'). This is so because mathematics, as a formal science like logic is its own object. "In logic," WITTGENSTEIN wrote[62], "process and result are equivalent. (Hence the absence of surprise)." A metalanguage on the other hand is not part of the object language.

It is useless to discuss the adequacy of terms if they are widely used. But if KELSEN had chosen the term 'metanorm' for this presupposed norm instead of 'basic norm', then it would be evident that his problem had the same structure as TARSKI's.

Such terminology would not have been far from KELSEN's own use, because he called 'meta-legal'[63] all those authorities (like God or "nature") to which validity is traced back in natural law systems.

If it is essential for the concept of a legal order that its norms "must be created by a specific process"[64], that they are "created by a legal authority"[65], then a norm which has not been created in this way, is not a norm but a meta-norm, which "decrees" - logically - the validity of the norm that is described by the person that presupposes the meta-norm for logical reasons.

The term 'basic norm' hides these qualities, because it induces us to think of a fundamental norm, which is the "basis" or foundation of the whole system - be it a constitution[66] or be it a fundamental law as it is understood by Ija PAWŁOWSKA[67] and as it was understood by KANT, when he called the categorical imperative the "fundamental law of the pure practical reason."[68] VERDROSS underlined that KANT's philosophy must be seen as the source of KELSEN's terminology, although he stressed the fact that KELSEN's basic norm "is not a basic norm of natural law as meant by KANT, but the hypothesis of legal thinking that deals with given legal system."[69] But there

is a logical *nexus* between KELSEN's basic norm and KANT's philosophy, although it is to be found in quite another place!

6. Social Contract as Basic Norm

In his "Metaphysics of Morals" KANT discusses the tenet of social contract theories with the result that "the origin of supreme power is *practically inscrutable*." Questions as to which came first, civil government or an actual contract of submission "are either entirely aimless, or even fraught with subtle danger to the state."[70] KANT realised quite clearly that "It is vain to inquire into the historical origin of the political mechanism", because "Savages do not draw up a documentary record of their having themselves submitted to law."[71] And for these reasons he presupposes the validity of laws as a "principle of the practical reason."[72] The introduction of this principle of reason fulfils exactly - and more directly - the task that the basic norm is meant to fulfil.

In believing the social contract to be a historical fact not even the "contractarians" were unanimous. This is quite clear for David HUME (1711-1776), whose works KANT had studied thoroughly.[73] The same can be said about Thomas HOBBES (1588-1679), who under the influence of EUCLID[74] became "founding father" of all geometric philosophies of state.[75] It was the Historical School of Law that lost the understanding of the hypothetical character of the social contract scheme.[76]

I want to underline the fact, that even Jean-Jacques ROUSSEAU (1712-1778) conceded that the clauses of the social contract might have "never been formally enunciated."[77] And in his modern German edition of the "Contrât social" BROCKARD pointed out that Rousseau - at least in this statement - "does not want to give a historical description of how the state has been created; the *contrât social* is to provide the theoretical preconditions for the state."[78] Thomas CORNIDES is more subtle about the subject when he writes, that we cannot see clearly, whether ROUSSEAU claims the social contract to be a fact or not.[79]

To presuppose a norm, which has never been created, only for reasons of logic is not any more convincing or "scientific" than to presuppose for the very same reasons the existence of a contract that has never been concluded.

It is quite remarkable that KELSEN made the social contract a subject in his *Pure Theory of Law* only in the annex on "The Problem of Justice" (which is to be found in the second German, but not in the English edition). Discussing reasons that speak against the principle of justice he conceded that "the principle of self-determination" could be preserved "for the imaginary event of the first establishment of social order."[80]

KELSEN formulated his ideas about the role of history in theoretical constructions quite clearly in his reasoning about federal systems. Referring to the relation between families and family law he wrote: "In the same way, the validity of the order of a single state is based upon the constitution of the federal state, although the latter's creation is later in time than the formerly independent states which only subsequently are gathered together in a federal state. *Historical and normative-logical relations should not be confounded.*"[81] We must concede the same approach to the theorists of the classical era.

7. Differences Between Basic Norm and Metalanguage

The remarks of VERDROSS about the Kantian origin of the basic norm concept show clearly that the term 'basic norm' can cause misunderstanding. Such misunderstandings have actually arisen and were opposed by KELSEN in a footnote concerning Karl MENGER, a footnote which cannot be found in the English version. According to KELSEN, Karl Menger had denied that concrete norms can logically be derived from the basic norm.[82] And this argument, says KELSEN - although undoubtedly true - does not interfere with the pure theory of law, which "asserts that from the basic norm only the validity of a norm can be derived and not its contents."[83] This is typical Kelsen's argumentation. Karl MENGER - then member of the Vienna Circle - had used the term 'basic norm' without any reference to KELSEN and he had distinguished this use clearly from the relations between norms of law. There can be no doubt that the term 'metanorm' instead of 'basic norm' would have better expressed the meta-legal character of a norm that is only presupposed for reasons of logic.

But wrong interpretations of the term 'basic norm' may contribute to an understanding of the important difference between this term and the term 'metalanguage'. And the analogies between the two concepts should not camouflage these differences.

TARSKI showed that a metalanguage must be *richer* or *stronger* than the object language to be described. There is a similar understanding of the basic norm, if one argues that concrete legal norms cannot be derived logically from the basic norm, which in KELSEN's understanding or interpretation of MENGER was done by Karl MENGER. But contrary to a metalanguage, which must contain all propositions of the object language (and their interpretations too), the basic norm does *not* similarly contain all the terms of a legal order, the validity of which is established - logically - by this basic norm. In the hierarchy of norms the basic norm is *superior* to the legal order, the validity of which is to be established. In this

understanding the basic norm is *stronger* than the corresponding legal system, but it is *not richer*. It is even *poorer* than this legal order as far as its contents are concerned, because the basic norm contains only one commandment: the "historically first constitution" is to be obeyed.

This term means the constitution "whose validity ... cannot be traced back to a positive norm created by a legal authority."[84] If a historical first constitution cannot be found, or in other words "if the constitution of the legal community is ... created ... by custom", then "a basic norm must be presupposed which institutes ... the fact of a qualified custom as law-creating fact."[85]

8. The Basic Norm as Operational Term

In spite of all these differences between basic norm and metalanguage, their formal equivalence is amazing. A legal system consists not only of relations of validity. Its concrete contents must be described by means of language and so the problem of validity - like the problem of truth - becomes a problem of language.[86]

Validity cannot be defined *per se* or "by its very nature". It is defined - implicitly - as the result of the "rules of procedure" that are elements of a meta-theory.

The similarities to the case of geometry are obvious. Since Euclidean geometry has been dethroned, we have not been able to define 'point' or 'line' or 'plane' by their contents - let us say concretely or "ontologically". These basic terms of geometry have different meanings in different axiomatically defined geometries. And equally, in pure theory of law "validity" is not defined as such, but introduced as a theoretical postulate, if a system of *ought-phrases* is to be interpreted as a legal order.

The theoretical arbitrariness of the basic norm is similar to the axioms of geometry. And this arbitrariness can be shown by the description of international law in KELSEN's *Pure Theory of Law*.

KELSEN refutes the dualistic construction with its independence between national and international law and adheres to two different monistic constructions that are equally logical: the first based on the primacy of national law, the second on the primacy of international law. "Both systems are equally correct, and equally justified. It is impossible to decide between them on the basis of the science of law. This science can do no more than describe them both, and state that one or the other reference system must be accepted if the relationship between national and international law is to be determined."[87]

The choice cannot be justified; any (seeming) justification is unmasked as a fallacy by the pure theory of law. Pure theory of law - as a theory - is "indifferent" to political argument.[88]

The separation from real life could not be more radical. In the natural sciences we find an interesting parallel, which goes back to POINCARÉ. If it were found that real space is different from Euclidean geometry, then - according to POINCARÉ - there could be two different consequences: either the description of physical space by using non-Euclidean geometry *or* a reformulation of the laws of physics. POINCARÉ's prophecy that physicists would prefer the second alternative has not become true as has been demonstrated above by reference to EINSTEIN.[89] But introducing the basic norm, KELSEN did for law what HILBERT had done for mathematics - he caused a revolution.

Loss of relation to reality and gain of logical certainty are strongly connected to each other. The 'normative variation' of EINSTEIN's statement would say: in so far as a proof of validity is logically certain it has no relation to an existing legal system, and proving the validity of an existing legal system is not logically certain.[90]

But the loss of reality should not mislead us to overlook the methodological progress in KELSEN's pure theory of law as well as in HILBERT's *Foundations of Geometry*. In his *Hauptprobleme* KELSEN made a remarkable statement:

> Considering its formal character and using a metaphor not quite accurate in all points, jurisprudence might be qualified as the geometry of the legal phenomenon as a total.[91]

But notwithstanding this beautiful phrase, KELSEN does not seem to have realized that his pure theory of law was to such a great extent constructed *more geometrico*.[92]

9. Final remarks

I tried to depict the context in which KELSEN's pure theory of law should be seen. And the surprising result of this inquiry is, KELSEN established a theory of his science according to the same principles that in his time could be found in the latest developments of logic and mathematics, although he apparently did not know them. But it is those ideas which seem to prove one fact: if logical relationships of legal norms are to be described - and such description must be distinguished from empirical descriptions of existing norms like mathematics from the description of empirical phenomena - then legal theory must go the way which logic and mathematics have gone since

the beginning of this century. And this might well mean, that a contact with the universal theory of science, which KELSEN had wanted, will be impossible without the instruments of formal logic, which KELSEN did *not* use.

Perhaps his method was not adequate already in those years, in which he wrote his *Pure Theory of Law* [93], years in which TARSKI wrote about the necessity of "a knowledge of the principles of modern formal logic" and of "certain purely mathematical concepts and methods ... although in a modest degree", referring to which he hoped "to convince the reader that these methods already are necessary tools even for the investigation of purely philosophical problems."[94]

His remarks lead back to the beginnings of occidental academic thinking, to the motto which is said to have been put above the entrance of PLATO's academy: *Medeis ageometretos eisito* - Who has not learnt geometry must not enter.

NOTES

* The substance of this essay was expressed in a lecture held on June 13th, 1984 for the *Wiener Juristische Gesellschaft.* A more explicative version was given in lectures at Katowice University (in German, at a conference in Bielsko-Biała) and at the Jagiellonian University at Kraków (in English) on the 5th and 6th of May, 1987 - resp. This version was published in *Rechtstheorie* 1987, 77 ("Grundnorm und Metasprache - Strukturparallelen von Reiner Rechtslehre und formaler Semantik"). A very condensed version of the main concepts with additional references to Ludwig WITTGENSTEIN was reported at the 1987 Wittgenstein-Symposium in Kirchberg/Austria as "Struktur-analoge Konstruktion von Geltung und Wahrheit durch Grundnorm und Metasprache", Reports of the 12th International Wittgenstein-Symposium 1987 (1988) 39. The English version, which is given in this essay follows the German version in *Rechtstheorie*, but contains additional references to WITTGENSTEIN and some remarks on James Clerk MAXWELL. German and French Literature is - as far as available to the author - referred to in English editions.

The concept of KELSEN is referred to as 'pure theory of law' to indicate his theory, and as *'Pure Theory of Law'* to indicate his book on the subject. In notes *'Pure Theory of Law'* is used for references to the English version (translation from the Second - Revised and Enlarged - German Edition by Knight, 1970); *'Reine Rechtslehre'* is used for references to the German version (2nd edition 1960; reprint 1976), when the English edition does not contain the relevant statements.

1. VERDROSS, *Abendländische Rechtsphilosophie - Ihre Grundlagen und Hauptprobleme in geschichtlicher Schau*² (1963) 190
2. ROSS, *Om ret og retfaerdighed* (1953) quoted from LOSANO's (Italian) introduction to KELSEN, *La dottrina pura del diritto*³ (1975) XXXVII
3. FOUCAULT, *Les mots et les choses* (1966)
4. *Reine Rechtslehre* III
5. *Reine Rechtslehre* III (Preface to the 1st edition)
6. MÉTALL, *Hans Kelsen - Leben und Werk* (1969) 9
7. But compare note 93!
8. MÉTALL (Note 6) 29
9. *Hauptprobleme der Staatsrechtslehre entwickelt aus der Lehre vom Rechtssatz* (1910, 2nd ed. 1923 reprint 1960), of which there is no English translation, but there is a Polish one by Przeorski: *Podstawowe zagadnienia nauki prawa państwowego. W rozwinięciu nauki o normie prawnej* (Wilno 1935)
10. PITAMIC, "Denkökonomische Voraussetzungen der Rechtswissenschaft", *ÖZÖR* 3 (1918) 339 with reference to MACH, *Erkenntnis und Irrtum*² (1906)
11. KELSEN, *Das Problem der Souveränität und die Theorie des Völkerrechts - Beitrag zu einer reinen Rechtslehre* (1920) 99
12. *Reine Rechtslehre* 17 FN *
13. Compare Note 25 and paragraph 7 (Note 82).
14. VOLLMER, *Evolutionäre Erkenntnistheorie - Angeborene Erkenntnisstrukturen im Kontext von Biologie, Psychologie, Linguistik, Philosophie und Wissenschaftstheorie*³ (1981) 109, and especially his statement that the theory of Copernicus missed this "exterior consistence", and that therefore a scientific revolution was necessary to make the heliocentric system acceptable.
15. Quotations refer to the new translation into English by Pears - McGuinness (1963)
16. "Das Verhältnis von Staat und Recht im Lichte der Erkenntniskritik", *ZÖR* 2 (1921) 453 (510; stress in the original)
17. SCHMITT, *Poetische Theologie - Vier Kapitel zur Lehre von der Souveränität* (1922, 2nd ed 1934). Compare: PRISCHING, "Hans Kelsen und Carl Schmitt - Zur Konfrontation zweier staatstheoretischer Modelle", in: WEINBERGER - KRAWIETZ, *Reine Rechtslehre im Spiegel ihrer Fortsetzer und Kritiker* (1988)

18. MÉTALL (Note 6) 4

19. Compare TORRANCE, *Juridical Law and Physical Law. Toward a Realist Foundation for Human Law* (1982) 18: "In our day this view has been somewhat reinforced by the influence of the Viennese school of legal positivists, not unlike the way in which English philosophical positivists have come under the impact of the so called 'Vienna Circle', both of which derive from a Neo-Kantian and Machian conception of scientific knowledge and its formalisation through regulative principles and conventions for which no claims are made as to their bearing upon objective structures in being."

20. *Reine Rechtslehre* IV (Preface to the 1st edition)

21. KELSEN was very occupied by the problem of causality - but as described in terms of classical physics and without any reference to quantum mechanics.

22. *Pure Theory of Law* 1

23. Ib 4

24. MÉTALL (Note 6) 4

25. *Pure Theory of Law* 5; in emphasizing the distinction between the 'ought' and the 'is', KELSEN is explicitly in opposition to the opinion of SCHLICK, that the statement, a behaviour corresponds with a norm, was a perception of facts. For KELSEN a norm was not - as SCHLICK thought - a definition; it did not declare what the designated was, but what should be.

26. *Pure Theory of Law* 98, 99

27. Ib 10

28. So in: *Souveränität* (Note 11) 96 as arbitrary decision, which can be based only on the "economy of decision"; compare notes 10 and 11.

29. *Pure Theory of Law* 4

30. "Möglichkeiten und Grenzen der Rechtslehre Hans Kelsens", *JBL* 1984, 126 (129), with reference to a review of the 2nd edition of the *Pure Theory of Law* by ENGISCH, *ZfgesStRW* 75 (1963) 610 and to KELSEN, "Adolf Merkl zu seinem 70. Geburstag am 23.3.1960", *ÖZÖR* 10 (1960) 313; in the 2nd ed of *Hauptprobleme* (Note 9), XV MERKL had been acknowledged by KELSEN.

31. *Reine Rechtslehre* 47, 196; compare the remarks concerning the term 'basic norm' in note 87.

32. VERDROSS (Note 1) 192

33. For KELSEN's scrouples about the presupposition of a basic norm by science - which are then removed by the exact distinction between 'enacting' and 'presupposing' a norm - compare *Reine Rechtslehre* 208 FN *.

34. *Pure Theory of Law* 200; *Reine Rechtslehre* 203

35. *Pure Theory of Law* 205

36. *Pure Theory of Law* 204

37. *Reine Rechtslehre* III (Preface to the 1st edition)

38. GAUSS to BESSEL January 1st, 1829, quoted from BECKER, *Die Grundlagen der Mathematik in geschichtlicher Entwicklung* (1975) 178

39. BECKER (Note 38) 203

40. EINSTEIN, *Geometrie und Erfahrung* (1921) 3

41. Ib 6; translation by PLÖCHL

42. Compare TORRANCE, "Das Verhältnis zwischen christlichem Glauben und moderner Naturwissenschaft - Die geistesgeschichtliche Bedeutung von James Clerk Maxwell", *ibw-Journal*, Sonderbeilage zu Heft 2, Februar 1982

43. WITTGENSTEIN, *Philosophische Untersuchungen - Philosophical Investigations* - Translated by Anscombe (1958) 47e (§ 109)

44. Compare LORENZEN, *Metamathematik* (1962)

45. Compare FREY, *Einführung in die philosophischen Grundlagen der Mathematik* (1967) and STEGMÜLLER, *Strömungen der Gegenwartsphilosophie* (1969)

46. FUCHS, *Formel und Phantasie. Eine Weltgeschichte der Mathematik* (1979) 169

47. GÖDEL, "Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I", in: *MonatshMathPhys* 38 (1931) 173: compare BECKER (Note 38) 383

48. For these aspects of our subject compare PLÖCHL, "Strukturanaloge Konstruktion" (Note *)

49. Compare NAGEL - NEWMAN, *Gödel's Proof* (1958)

50. TORRANCE (Note 19) 17 shows that we had to know since Gödel that no system of legal norms can be complete and consistent - except a tautological system without relevance for real life.

51. STEGMÜLLER, *Das Wahrheitsproblem und die Idee der Semantik - Eine Einführung in die Theorien von A.Tarski und R. Carnap*^{2} (Reprint 1972) 17

52. TARSKI, "Der Wahrheitsbegriff in den formalisierten Sprachen", *Studia Philosophica - Commentarii Societatis Philosophicae Polonorum* (1936 printed as 1935) 261, republished as "The Concept of Truth in Formalized Languages" in TARSKI, *Logic, Semantic, Metamathematics* (1956) 152. In a footnote to this English version TARSKI added a "Bibliographical note", in which he wrote: "This article was presented (by J. Łukasiewicz) to the Warsaw Scientific Society on 21 March 1931. For reasons beyond the author's control, publication was delayed by two years. The article appeared in Polish" in: TARSKI, O pojęciu prawdy w odniesieniu do sformalizowanych nauk dedukcyjnych, *Ruch Filozoficzny* 12 (1930-1) 210-11

53. POPPER, *Objective Knowledge - An Evolutionary Approach* (1972, Reprint with corrections 1974) 322

54. *Tractatus* 4.121

55. WEINBERGER, *Rechtslogik - Versuch einer Anwendung moderner Logik auf das juristische Denken* (1970) 65

56. POPPER, (Note 53) 315

57. TARSKI, (Note 52) uses the example: "The snow is white corresponds to the facts if, and only if, snow is white." According to LYONS, *Language and Linguistics* (1981), this example is deceivingly simple (quoted from German edition 163)

58. TARSKI, (Note 52) 274: "*It is impossible to establish the semantics of a language in this way if the order of the language of its morphology is at most equal to that of the language itself*"

59. WEINBERGER, (Note 55) 64

60. Meaning a) the same as 'metalanguage' or b) the study of languages in their cultural context in the structuralistic linguistics after Leonard BLOOMFIELD (1887-1949), LYONS, *Language* (Note 57, quoted from German edition 287)

61. Compare ALBERT, "Ethik und Meta-Ethik. Das Dilemma der analytischen Moralphilosophie", *ArchfPhil* 11 (1961) 28, republished in ALBERT - TOPITSCH, *Werturteilsstreit* (1979) 472

62. *Tractatus* 6.1261

63. *Reine Rechtslehre* 203, 205; *Pure Theory of Law* 202

64. *Pure Theory of Law* 198

65. Ib 200

66. So the constitution of the Federal Republic of Germany, which is called "Grundgesetz" (= "Basic Law")

67. PAWŁOWSKA, "Kontradiktion und Inkompatibilität in einem ethischen System", *Studia Filozoficzne* 3 (1961) 137, republished in ALBERT - TOPITSCH (Note 61) 518

68. KANT, *The Critique of Practical Reason* I/1 § 7 (302 in the English edition published by Benton[26], 1984)

69. VERDROSS (Note 1) 192

70. KANT, *The Metaphysics of Morals, The Science of Right* § 49; 439 in the English edition (Note 68)

71. Ib 450

72. Ib 439

73. HUME, "Of the Original Contract", in: BARKER, *Social Contract. Essays by Locke, Hume, Rousseau* (1948) 209; compare WATKINS, "Social Contract", in: *Encyclopaedia Britannica* vol 20 (1967) 739 and STEIN, *Legal Evolution - The story of an idea* (1980) 12

74. Compare: WOLF, *Die neue Wissenshaft des Thomas Hobbes - Zu den Grundlagen der politischen Philosophie der Neuzeit* (1969) and MEYER-TASCH in his preface to: HOBBES, *Leviathan oder Wesen, Form und Gewalt des kirchlichen und bürgerlichen Staates* (1965) XIII

75. RÖD, "Geometrischer Geist und Naturrecht - Methodengeschichtliche Untersuchungen zur Staatsphilosophie im 17. und 18. Jahrhundert", *AbhBayerAkWiss/Phil-HistKl* NF 70 (1970) 10

76. WATKINS (Note 73) 739; for the detaching of the ideas of the 18th century by the "Age of History" compare FOUCAULT (Note 3; 269 in the German edition)

77. ROUSSEAU, *Du contrât social, ou principes du droit politique* (1762) 1st Book 6th Chapter; English translation in: BARKER (Note 73) 255

78. BROCKARD in: ROUSSEAU, *Gesellschaftsvertrag* (1977 Reclam Universal-Bibliothek Nr. 1769) 157, Note 25 (stress in the original)

79. CORNIDES; "Die Denkmöglichkeit einer 'realistischen' Theorie vom Gesellschaftsvertrag (contrât social)", in: *Reformen des Rechts. Festschrift zur 200-Jahr-Feier der rechtswissenschaftlichen Fakultät der Universität Graz* (1979) 625 (627). CORNIDES pretends that social contracts can be regarded logically as facts. This is also - as he shows (626) - the opinion of John LOCKE (1632-1704). CORNIDES based his "realistic" theory of the social contract concept on the theory of games. For the importance of the theory of games for jurisprudence

compare PLÖCHL, "Die Regeln des Spiels und das Spiel mit der Regel", in WINKLER-FS (1989) 281 and "Was sind und was sollen die Regeln?", in: WINKLER-FS (in print)

80. *Reine Rechtslehre* 389

81. *Pure Theory of Law* 339 (Stress by PLÖCHL)

82. MENGER, *Moral, Wille und Weltgestaltung - Grundlegung zur Logik der Sitten* (1934) 21; the term 'basic norm' is used in this sense too by PAWŁOWSKA (Note 67) 519.

83. *Reine Rechtslehre* 201 (Note *)

84. *Pure Theory of Law* 200

85. Ib 226

86. To ethics as a problem of language in meta-ethics compare ALBERT (Note 61)

87. *Pure Theory of Law* 346; without using the term 'basic norm' KELSEN had expressed these ideas in his essay "Reichsgesetz und Landesgesetz nach österreichischer Verfassung" *AÖR* 1916 (sometimes referred to as 1914 or 1918, because these editions of *AÖR* were published in one volume for reasons of war) 202, 390. LOSANO's statement (Note 2, XXVII) that the first remark concerning a *"norma fondamentale"* can be found in this essay of KELSEN's is wrong. There might have been a misunderstanding of KELSEN's preface to the 2nd edition of his *Hauptprobleme* (Note 9, XV), in which he wrote that he had explicated the concept of the basic norm in that essay, although without the later (!) developed distinction between constitution in the sense of logic of law and constitution as part of positive law. It seems, that KELSEN wanted to secure his priority against VERDROSS because he continued: "The concept of the basic norm as constitution in terms of logic has been brought essentially further by *Alfred Verdross* in his essay 'Zum Problem der Rechtsunterworfenheit des Gesetzgebers' (*JBl* 1916, 471, 483) by using it as a hypothesis in relation to the material of positive law analogously to the hypothesis in the natural sciences." In his essay VERDROSS uses the term 'Grund-ordnung' (= basic order) but without priority to the relevant constitution. The validity of (the historically first) constitution is logically "presupposed" by VERDROSS.

88. *Pure Theory of Law* 347

89. Compare Note 40 and 41.

90. Compare TORRANCE (Note 46)

91. KELSEN (Note 9) 93

92. For *mos geometricus* compare RÖD (Note 75)

93. For problems which ensue from the multiple use of terms by KELSEN compare LEIMINGER, *Die Problematik der Reinen Rechtslehre* (1967)

94. TARSKI (Note 52) 154

Part III

LOGIC AND NATURAL LANGUAGE

Marek Tokarz

EARLY SYSTEMS OF FORMAL PRAGMATICS

The present paper is devoted to pragmatics, so I think it should be stressed at the very beginning that I myself simply do not know what exactly pragmatics is, and still worse - I am afraid nobody does. In fact, the dispute on the aims, methods, scope and limits of pragmatics started as early as in the late thirties, and continues up to now. The discussion is far from being concluded, and that is just why you can still find there in the literature opinions like this one, formulated by professional pragmaticists in the eighties - fifty years after:

> Pragmatics is one of those words (...) that give the impression that something quite specific and technical is being talked about when often in fact it has no clear meaning. (cf. Searle et al., 1980)

Well, of course, those who want to be offered a purely technical and operational definition, would not be satisfied at all with either, say: *pragmatics is the study of language use*, which is too wide and obviously too vague, or - and even less so - with Montague's one: *pragmatics is the study of indexical expressions*, which, in turn, is too narrow. But it might be doubted if such a definition is really necessary: we can all do quite well without being presented with any precise definition of the term 'logic', to give just one example. It will be much better then to follow Stalnaker's (1970) advice and define pragmatics by simply pointing at its branches, so as to obtain, e.g.: *pragmatics is the study of*

- *speech acts*
- *deixis*
- *presupposition*
- *implicature*
- *discourse structure.*

Each of these problems is of great importance, and is deeply analysed in modern linguistic theories, but the very first systems offered by logicians, and precise enough by logical standards, were all located in the second area: deixis. By the first systems I mean the ones invented in the

period of approximately five years, from 1968 to 1973, namely the following: Richard Montague's (1968), Dana Scott's (1970 - lectured in 1968), Robert Stalnaker's (1970), David Lewis's (1970), M.J. Cresswell's (1973), and David Kaplan's, whose manuscripts circulated in the USA starting from the mid seventies.

All those works are, of course, different in some respects, but similar in many others, in particular in that, as Montague and Scott have both explicitly pointed out, they are all following the lines of Bar-Hillel's program for logical pragmatics, formulated in 1954. According to Bar-Hillel's conception, pragmatics is no less, and no more than the study of deixis, and consequently, the central categories which should be - according to his conception - discussed in a pragmatic theory are:

- *utterance*
- *context of use*
- *context dependence*,

and, of course, as it is *logical* pragmatics that we are talking about, *truth*, *entailment*, etc.

Let me briefly sketch the ways in which these notions were investigated and the means they were handled with.

Utterance

As to the utterances, there are two typical approaches we can observe and distinguish in the history of logical pragmatics. The first comes perhaps from Tarski's, and is represented by, among others,

Carnap (40s) ——————— Martin (50s) ————— Cresswell (70s)

The other approach to be noted is that of, say,

Bar-Hillel (50s) ———— Montague (60s) ———— Kaplan (70s).

In the first approach, utterances are simply physical objects, of material or acoustic nature. An abstract sentence in this view is just a set of those objects similar, in some sense, to a given standard pattern. Such an idea can easily be realized in a formal system and in fact the job is done, by Cresswell for example, but it seems not to have much promise because of some essential difficulties which emerge in it. For example, two different oral productions cannot be uttered in the same time, and thus we cannot consider two different sentences as being produced in the same circumstances, which, of course, is extremely counterintuitive. For this, and for some other reasons, the second approach, in which an utterance is more

like an event, not an object, is more promising. Thus Bar-Hillel, and then Montague, Stalnaker, Kaplan and others consider an utterance to be a pairing of a sentence, taken as an abstract entity, *and* concrete circumstances. We would thus have

utterance = sentence-in-context.

Context

As to the next notion, one may say that the history of all the modern formal pragmatics could quite adequately be lectured on as the history of the attempts to define technically the notion of the context of use. Bar-Hillel, who was the first trying to, was not able to find any precise explication. Here is a typical passage from his *Indexical expressions*:

> I have left the central concept of this paper, namely *pragmatic context* in a rather thorough vagueness, and this for the very simple reason that I see no clear way to reduce this vagueness at the moment. From a technical point of view, it seems to me preferable to replace the contexts by *context descriptions*.

Bar-Hillel would never be more specific than that. In Montague's theory the context becomes a primitive notion, called index, and the part played by the notion in his system is close to that of the possible world in Kripke's semantics. According to Dana Scott, and also to David Kaplan, the context of use (or index, or else point of reference) is a complex entity, including the speaker, his position in the universe, time *and* a possible world. In 1973, Cresswell observed that when searching for models of the natural language, we should not restrict ourselves to the four coordinates: world, speaker, time and place, and that in fact there is no finite list of such aspects, fixed in advance, to be taken into account. In his theory the context is defined to be any property of utterances.

Context-dependence

Now, what the context-dependence is like? Let us consider the example:

(1) Bill loves Kate.

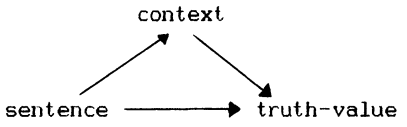
that is, the sequence of the three words: 'Bill'+'loves'+'Kate'. Suppose that *a* is the referent of the word 'Bill', that is, *a* is Bill in fact, *b* the referent of 'Kate', and R the referent of 'love'; R is then the relation such that

x R *y* iff *x* loves *y*.

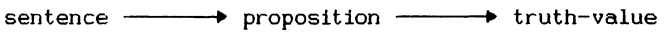
From the semantic point of view (1) means something like $\langle a,b\rangle \in R$, and that is final.

And now, how about the truth-value of (1)? Suppose that Bill fell in love with Kate on May 23, 1987, and had been loving her for exactly five days, that is, till May 28. Is this sentence true or not? The only correct answer is that it depends on the circumstances, which is to say, on the context of use. It would be true if uttered in the period of the five days, from May 23 to May 28; otherwise it would not.

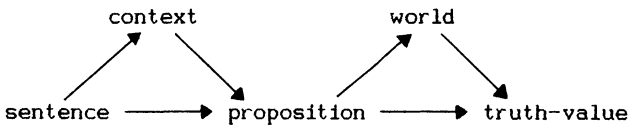
And that is just what Bar-Hillel has pointed out: sentences in general are neither true nor false. It is utterances only, not sentences, that do have a truth value. The first formal exposition of Bar-Hillel's idea is to be found in Montague's and Scott's works of the late sixties. In their systems the meaning, or *intension*, of a sentence is always a function from contexts into truth-values, which can be schematically conceived in a diagram:



(the truth-value of a sentence is not given directly; a go-between is needed: the context of use, I mean). As early as in 1970 Robert Stalnaker observed that Montague and Scott's scheme was inadequate, for the reasons which will soon become clear. According to Stalnaker's idea, a sentence determines a proposition, and the proposition determines a truth-value:



but that is not done directly, either. To determine the proposition we must be given the context of use, and to determine the truth-value a world is needed:



I am now turning to a bit more detailed, but still not very pedantic, presentation of Scott's, Stalnaker's, and Kaplan's pragmatic systems, as examples of theories accounting for the above phenomena.

Dana Scott

The language $\mathbf{L}$ investigated in Scott's system is just the usual one, with modality and descriptions; the only non-standard feature is the following: where D is the set of possible individuals, $\mathbf{L}$ is assumed to contain a constant $\bar{a}$ for every $a \in D$. By an *interpretation* is understood (roughly, let me stress it) any triple

$$\mathcal{A} = \langle D, I, A \rangle,$$

where D is a set of possible individuals; I is the set of so-called indices, or points of reference, or simply contexts, as we take it; A is an indexed family of sets such that for every $i \in I$,

$$A_i \subseteq D.$$

A_i is interpreted to be the totality of all the individuals existing in circumstances i.

Indices should be thought of as being complex entities composed of four coordinates, namely

$$i = \langle w, t, p, a \rangle$$

where: w - is a possible world,
t - is a moment of time,
p - is a position, and
a - is an agent, say, the speaker.

Sentences are always interpreted in relation to an index, exactly as in Montague's theory. Intuitively, for example, the sentence 'I am hungry' uttered in circumstances $\langle w, t, p, a \rangle$ means that, in the world w, the speaker a is hungry at the moment t. The denotation (or *extension*) of an expression α at some index i is denoted by $|\alpha|_i$; if α is a sentence, $|\alpha|_i$ is a truth-value. Hence $|\alpha|$ is always a function whose domain is I; this function is said to be the *intension* of α. Let me quote some clauses of Scott's recursive definition of extension:

$|\bar{a}|_i = a$ for all $a \in D$, $i \in I$; hence $|\bar{a}|$ is a constant function;

$|\forall x\, \alpha(x)|_i = 1$ iff $|\alpha(\bar{a})|_i = 1$ for all $a \in D$;

$|\Box\alpha|_i = 1$ iff $|\alpha|_j = 1$ for all $j \in I$.

The main ideas of Montague's early papers are quite similar.

Robert Stalnaker

Stalnaker's objections to Montague's approach, and to Scott's as well, go as follows. Consider

(2) I am here now.
(3) Marek Tokarz is in Cracow on October 28, 1987.
(4) □ I am here now.

Clearly, (2) cannot be uttered falsely: I am certainly always where I am. But on the other hand, (2), when produced in concrete circumstances, means something like (3). It is immediately seen that (3) is contingent, and so must be (2); hence (4) is false, because it says of some contingent proposition to be necessary. But under a simpler Montague's analysis, (4) is a tautology, for (2) is true at every point of reference. Therefore Montague's theory needs corrections.

Formally taken, the interpretation in Stalnaker's sense would be a triple, say,

$$\langle S,C,W\rangle,$$

where C is a set of contexts, W - a set of possible worlds, and S - a function whose arguments are sentences of the language; where α is such a sentence, $S(\alpha)$ is itself a function, from contexts into propositions this time, that is,

$$S(\alpha) : C \longrightarrow \{0,1\}^{W}.$$

$S(\alpha)$ is said to be an *interpreted sentence*; for $c \in C$, $S(\alpha)(c)$ is a proposition, that is, a function from possible worlds into truth-values; thus for $w \in W$,

$$S(\alpha)(c)(w) \in \{0,1\}.$$

Stalnaker's paper is purely philosophical and programmatic, with no formulas or other formalisms at all, but the very idea of his has been worked out with mathematical precision by Cresswell and Kaplan, some three years later. Let us have a look, for example, at how the analysis of 'I am here now' has been carried out in Kaplan's excellent work.

David Kaplan

There are two types of terms in his language: individual and positional ones. The symbols are, among others:

I, Here, Now, □, Located.

'I' is an individual term, 'Here' is a positional term, 'Now' and '□' are sentential connectives, and 'Located' is a designated two-place predicate symbol. (Never mind the other symbols, predicates, tenses, etc.) Where a is an individual term and p a positional one, 'Located(a,p)' is to be interpreted this way: the object a is situated at the location p. Hence, in Kaplan's language, 'I am here now' becomes

(5) Now(Located(I,Here)),

and 'It is necessary that I am here now' becomes

(6) □Now(Located(I,Here)).

Extensions of all the expressions depend on: the context (c), the world (w), and on the time (t); the denotation (or extension) of α at c, w, t is denoted by $|\alpha|_{cwt}$. Every context c itself determines some person, namely the agent of c, denoted by $c(A)$, the position $c(P)$ of the context, the world and the time of c, $c(W)$ and $c(T)$, respectively.

Where R is a predicate of the language, the interpretation of R, denoted by $J(R)$, is always a two-place function from worlds and times such that $J(R)(w,t)$ is a relation, whose number of arguments and their types (positional or individual, I mean) are suitable for the given R. As to the special predicate 'Located' it is assumed that, for every context c,

(*) $\langle c(A), c(P)\rangle \in J(\text{Located})(c(W), c(T))$,

which is quite intuitive, I hope: in the time and world of the given context, the agent of the context is always situated at the location under question.

The denotations of 'I' and 'Here' are obvious: $|I|_{cwt} = c(A)$; $|\text{Here}|_{cwt} = c(P)$. The relation of satisfaction, $\models_{cwt}$, in the case of the two sentential connectives is defined by:

$\models_{cwt} \text{Now}(\alpha)$ iff $\models_{cwc(T)} \alpha$

$\models_{cwt} \Box\alpha$ iff for every possible world u, $\models_{cut} \alpha$.

A sentence α is *true in the context* c iff $\models_{cc(W)c(T)} \alpha$; α is *valid*, in symbols $\models \alpha$, if true in every context.

Valid sentences form a system, called LD (for: the Logic of Demonstratives), of which (5) is a theorem. Let me prove it:

$\models$ Now(Located(I,Here)) iff

$\forall c$ $[\models_{cc(W)c(T)}$ Now(Located(I,Here))] iff

$\forall c$ $[\models_{cc(W)c(T)}$ Located(I,Here)] iff

$\forall c$ $[\langle |I|_{cc(W)c(T)}, |Here|_{cc(W)c(T)}\rangle \in J(\text{Located})(c(W),c(T))$ iff

$\forall c$ $[\langle c(A),c(P)\rangle \in J(\text{Located})(c(W),c(T))]$,

and this has just been assumed in (*) to hold true.

But (5), being a tautology, and thus analytic, is not necessary: (6) is not valid in LD. Hence, the so called Gödel's necessitation rule:

$$\alpha \vdash \Box\alpha$$

(from α to infer that necessarily α) is not permissible in Kaplan's logic, which formally solves Stalnaker's problem of the sentences that are both contingent and logically true, or, in other words, analytic but not necessary.

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Barbara Stanosz

DEDUCTION AND THE CONCEPT OF ASSERTION*

The pre-theoretical, intuitive concept of assertion of a sentence and the closely related concept of belief are not among the clearest. The difference between them is easily observed. The assertion of a sentence α of a language L presupposes the knowledge of this language and, with the qualifications that will be mentioned later, the disposition to give an affirmative answer to a *yes-or-no* question belonging to L, made up of α and a suitable interrogative device. Believing what a given sentence of a language L says does not presuppose the knowledge of L. The sentence 'N believes that the press lies' may be true even if N does not know English and hence is not disposed to give an affirmative answer to the question 'Does the press lie?' The truth of this sentence presupposes at the very most that N knows a certain language L' and is disposed to give an affirmative answer to the question belonging to L' and being a translation of the English quotation 'Does the press lie?' into L'.

Neither the assertion of a sentence nor believing what the sentence says can be identified, however (when these concepts are taken in their intuitive meanings), with the suitable disposition to linguistic behavior. Otherwise we would not have the notion of a lie, nor could we reasonably suspect anybody of avoiding an answer to the question or of falling to understand it. We are inclined to treat the linguistic behavior of a given person as the evidence of his/her assertion of a corresponding sentence, or as an expression of his/her belief, only if we assume that the person wishes to answer our question, that he/she does it sincerely and that he/she understands the question the way we do - which clearly indicates that asserting a sentence or having a belief is in fact something different from having a disposition to corresponding linguistic behavior. What is it then?

There are two different views of the matter. According to one, to assert a sentence or to believe what the sentence says is to have a certain mental attitude towards that sentence or (in the case of belief) towards a translation of that sentence into some other language, an attitude we

recognize by introspection and render accessible to others, according to our needs and abilities, by suitable linguistic behavior. This behavior - giving affirmative answers to yes-or-no questions in particular - is then only a way of expressing our beliefs and assertions. The only conclusive way to determine whether we have a given belief and thereby assert a corresponding sentence is by appealing to introspection. This is the subjectivistic concept of belief or assertion. According to the other view, asserting a sentence or believing what the sentence says consists in having a certain behavioral disposition depending on the content of that sentence. It is not a disposition to linguistic behavior, though, but rather a disposition to actions which - in light of the knowledge and in view of the interests (hierarchy of values) of the person in question - are profitable if and only if that sentence is true. In other words: if we are inclined to behave in a way which in the light of our knowledge fits our interests if a given sentence is true and does not fit our interests if it is false, we assert this sentence (we have the corresponding belief); and conversely. Asserting a sentence or believing what the sentence says is assumed to be identical to having a disposition to such actions; linguistic behavior is only a reflection of that disposition, often consciously or unconsciously distorted. This is the behavioristic concept of belief or assertion.

The behavioristic conception can be reproached for being unpractical and unrealistic. First because it assumes that to determine whether someone asserts a certain sentence (has a corresponding belief) one has to recognize that person's hierarchy of values, which is a rather hopeless task, especially when one consistently gives up the method of questions and answers. Second, because it also requires a prior determination of the set of other sentences asserted by this individual (the set of his/her other beliefs), for dispositions to actions are, as a rule, dependent on a conjunction of many beliefs. Third, because in many cases asserting a sentence seems to be behaviorally neutral at least in the sense that circumstances in which dispositions to certain actions could be manifested are not likely to occur, or even are certain not to occur. It applies, for instance, to sentences about events that took place a long time ago. Finally, because this conception assumes, unrealistically, that people always act rationally, i.e. in a way that, in the light of their beliefs, is most conductive to the fulfilment of their needs.

For all these reasons, to ascribe assertion of a certain sentence to anyone on the basis of his/her extralinguistic behavior is a very difficult, if not practically unfeasible task. It is evident that the behavioristic conception of assertion and belief fits computers, whose internal states and external stimuli may be freely manipulated, in far greater degree than real people. This argument, however, only indicates that the conception has an

idealizing character: it directly applies only to ideal bearers of beliefs, observed in perfectly controllable conditions. Real bearers of beliefs, and situations in which they may be exposed to observation, differ from the former in many respects; yet they differ to various degrees and for reasons we are able to identify. This allows us to regard the conception at issue as applicable, with some qualifications, to real people exposed to observation in normal conditions.

Idealization involved in the behavioristic conception of assertion and belief is an expedient not only permissible but notoriously applied in science. On the other hand, the subjectivistic conception is, for the purposes of scientific inquiry, entirely useless. This is so because of the role it ascribes to introspection. Science uses only intersubjective criteria of acceptance of theorems, which must refer to extraspective data. The subjectivistic concepts of assertion and belief, separated from linguistic behavior, are of no use for science. And if one identifies these concepts with the concepts of dispositions to corresponding linguistic behavior (consisting in answering suitable *yes-or-no* questions), i.e. if one assumes that generally people tell the truth and understand questions addressed to them, the concepts admittedly gain the status of intersubjective ones but have a very limited range of applications. At best one can use them to ascribe and deny beliefs (assertions of sentences); there seems to be no way of changing them into comparative concepts, which would enable one to talk about different degrees of belief (assertion) and to assign them numerical values, that is to measure beliefs and assertions; whereas all important scientific problems that could be solved through the explication of these concepts require taking them as quantitative ones.

Within the frame of the behavioristic conception this can be done: one can compare beliefs (assertions of sentences) with respect to their strength and assign them numbers, without violating any basic intuitions connected with these concepts. K. Ajdukiewicz conceived such a procedure and applied it to solve the perplexing problem of the rationality of non-deductive inferences. He says:

> If a person X is ready to undertake some activity D if and only if this activity brings him at least a profit Z in the case of a sentence A being true, and at most a loss S when A is false, then we assign the measure
>
> $$P_{\varkappa}(A) = \frac{S}{Z + S}$$
>
> to the degree of certainty with which this person accepts the sentence A.[1]

The degree of assertion of a sentence defined in this way is included in the interval [0,1] and is greater the higher the risk S/Z connected with the action that will yield a profit not smaller than Z if the sentence is true, and a loss not bigger than S if it is false. In other words, a disposition to action connected with greater risk indicates a higher degree of assertion of a sentence. Full assertion, i.e. assertion the degree of which is equal to 1, corresponds to readiness to act with unlimited risk; null assertion corresponds to readiness to actions which do not involve any risk.

These dependencies conform to our intuitions, which are clearest in the case of making bets: being sure or almost sure that a given sentence is true, we are inclined to bet a lot on it even if the expected prize is not large, and conversely: when our degree of belief is low, we are not inclined to bet a lot even if the expected prize is large.[2] Extrapolation of the dependencies on all situations in which we face the decision to undertake action connected with some risk, seems fully justified. It presupposes, of course, the possibility of measuring all losses and profits in certain units; these need not, however, be absolute units, but so-called units of utility.

The previously formulated behavioristic notion of assertion, according to which 'x asserts α' means the same as 'x is disposed to undertake any action that is profitable for him if and only if α is true', coincides with Ajdukiewicz's concept of full assertion. Thus, Ajdukiewicz's concept of assertion is more general. Moreover, it involves a simplification consisting in neglecting the factor of x's background knowledge (the rest of his beliefs); it is tacitly assumed that background knowledge relevant to behavior is the same in the case of all observed persons and the observer himself.

Accepting the behavioristic concept of assertion - in the version suggested by Ajdukiewicz or any other - one has to ignore some intuitions connected with its pre-theoretical counterpart. In particular, there is no longer any basis for the distinction mentioned before, between believing and asserting a sentence. This is a consequence of eliminating linguistic behavior from the criteria of believing and asserting sentences. If the only admissible criterion is extralinguistic behavior then asserting a sentence α of a language L does not presuppose knowledge of L, so it does not differ from having the corresponding belief. Moreover, asserting any sentence and having the corresponding belief does not presuppose knowledge of any language that this sentence could be translated into. According to the behavioristic conception, it is allowed in some situations to ascribe both beliefs and assertions of sentences (of any human language) to dogs or horses; these categories become applicable to all beings capable of discriminative behavior.

This effacement of the difference between concepts of assertion and belief, and extension of their application range, does not severely violate our intuitions. First of all, the distinction mentioned above is itself an explication of the relation between these concepts and - as any explication - sharpens the borderline that is originally, i.e. in linguistic experience, not very clear. Besides, the range of application of the concepts of assertion and belief is more exactly defined by this distinction than linguistic experience warrants: statements ascribing beliefs and assertions of sentences of a human language to animals are, as a matter of fact, neither exceptional nor plainly metaphorical.

However, there are some consequences of the behavioristic concept of assertion that clash with our deeply established intuitions; accepting them requires revising some of our strong convictions concerning, among other things, the interpretation of the basic discipline of theoretical knowledge, i.e. the logic of deduction. These consequences are stated below in points 1-3.

1. Let us notice that if we ascribe to someone the assertion of a sentence α on the basis of his extralinguistic behavior then, if a sentence β is logically equivalent to α, we must also ascribe to him asserting β (with the same degree of certainty): indeed, the same activities of x that are profitable for him if and only if α is true, are profitable for x if and only if β is true. And if x is ready to undertake a certain activity if and only if it will bring him at least a profit Z in the case of α's being true and at most a loss S in the case of α's being false, then thereby x is ready to undertake a certain activity if and only if it will bring him at least a profit Z in the case of β's being true and at most a loss S in the case of β's being false. Briefly speaking, if α and β are logically equivalent and $P_{\varkappa}(\alpha) = m$ then

$$P_{\varkappa}(\beta) = m.$$

Thus, asserting any sentence with a given degree of certainty is a necessary and sufficient condition of asserting, with the same degree of certainty, each sentence that is logically equivalent to it. In other words: if α and β are logically equivalent sentences, it is not possible for anyone to assert α without asserting β or to assert them with different degrees of certainty.

2. If α^* is a logically true sentence then for any α, the conjunction α & α^* is logically equivalent to α. Therefore, if $P_{\varkappa}(\alpha) = m$ and α^* is a logical truth then

$$P_{\varkappa}(\alpha \ \& \ \alpha^*) = m.$$

Thus, whoever asserts a sentence α with a given degree of certainty, also asserts, with the same degree of certainty, the conjunction of α and any logical truth. On the other hand, it should be taken for granted that whoever asserts the conjunction of two sentences with a given degree of certainty, also asserts, with at least the same degree of certainty, each component of the conjunction. Hence, if $P_x(\alpha) = m$ and α^* is a logically true sentence then:

$$P_x(\alpha^*) \geqslant m.$$

Thus, anybody who asserts any sentence with a given degree of certainty, also asserts, with at least the same degree of certainty, all logically true sentences. In particular, asserting any sentence with absolute certainty is a sufficient condition for asserting all logical truths with absolute certainty.

3. If β is a logical consequence of α then the conditional $\alpha \rightarrow \beta$ is a logical truth. Hence the conditional is asserted by anyone who asserts at least one sentence. On the other hand, it should be taken for granted that if anyone asserts a conditional with absolute certainty and asserts its antecedent with a given degree of certainty, he also asserts its consequent with at least the same degree of certainty. Thus, if β logically follows from α and x asserts any sentence with absolute certainty, and $P_x(\alpha) = m$, then

$$P_x(\beta) \geqslant m.$$

Thus, anyone who asserts a sentence with absolute certainty, also asserts, with absolute certainty, all logical consequences of that sentence; moreover, he asserts, with at least the same degree of certainty, all logical consequences of any sentence asserted by him with a given degree of certainty.

The conclusions reached in points 1 - 3 above seem implausible, because they do not fit subjectivistic preconceptions permeating the common notion of assertion. As long as we treat asserting a sentence as a psychological state, accessible in an act of introspection and manifested in linguistic behavior, it is obvious that one can assert a given sentence without asserting a sentence logically equivalent to it, and *a fortiori* - without asserting its logical consequences. And we cannot say, in accordance with these preconceptions, that assertion of all logically true sentences may be ascribed to anyone merely on the basis of his asserting at least one sentence with absolute certainty. Thus, replacing the pre-theoretical notion of assertion by behavioristic concept changes significantly the set of assumptions concerning assertion and belief that we used to count as true.

These changes are not limited to the so-called common-sense assumptions; they also affect some standard theoretical presuppositions,

namely those of the theory of inference. The following definition presents the generally accepted concept of inference:

> Inference is a mental process by which, on the strength of a more or less categorical acceptance of premises, we arrive at the acceptance of the conclusion which we previously either did not accept at all or accepted less categorically, the degree of acceptance of the conclusion being not higher than the degree of certainty of acceptance of the premises.[3]

Among inferences defined in this way deductive inferences are distinguished as those whose conclusions logically follow from their premises. Formal logic in a narrow sense of the term, is defined as the theory of such inferences.

In the light of our considerations it can be readily observed that the behavioristic concept of assertion cannot be reconciled with the concept of deductive inference. Since the requirement of full assertion of at least one sentence is always obviously satisfied, and therefore asserting any sentence means asserting, with at least the same degree of certainty, all logical consequences of that sentence, there is no place for a mental process that would lead from the assertion (acceptance) of a sentence to the assertion (acceptance) of any of its logical consequences. From the point of view of the behavioristic conception of assertion the term "deductive inference" turns out to be empty.

One might be tempted to reply by saying "So much the worse for the behavioristic concept of assertion!" Yet reflection leads to doubt in this matter and reveals incoherence of our intuition. We notice a connection between the consequences of the conception in question and the frequently repeated thesis that deduction does not broaden our knowledge: the conclusions include only what is included in the premises. We are aware of a peculiar compulsion to assert sentences that logically follow from the sentences we have asserted: the lack of choice seems to show that the acceptance of those sentences has already taken place, along with the acceptance of the sentence they follow from. The teacher of logic may recall his strong impression that by teaching his subject he only helps his pupils verbalize something they have always known. The student of this discipline may recount in the same way his own sense of the triviality of its truths.

But if deductive inferences do not exist, what is the logic of deduction about? Is it a theory without a subject matter?

It seems reasonable to regard it as a special part of the theory of language, understood the way it has been construed in linguistics since N.Chomsky and in analytic philosophy at least since R.Carnap. The task of the theory of language is a reconstruction of finite mechanisms making it

possible for the speaker to construct and to interpret infinitely many sentences. This is to say that the theory should be an explanatory description of human linguistic competence. The ability to recognize the logical truth of a given language and to identify the logical consequences of any of its sentences is an important component of that competence. This is the subject matter of the logic of deduction: its various systems pretend, more or less successfully, to be a theoretical representation of the structure of the logical component of human linguistic competence.

That users of a language often do not immediately recognize the logical properties and relations of sentences of the language (and that in some cases the recognition of those properties and relations requires special abilities and training) is one of the aspects of linguistic performance that distinguish it from linguistic competence. Logical errors can be treated as a phenomenon whose sources are no different from those of grammatical errors: limitation of memory, shifts of attention and various natural disturbances of "data processing." That the correcting of these types of errors does not require any additional factual information but only a repetition of "data processing" in more suitable conditions, seems to be a good intuitive argument for the suggested interpretation of the logic of deduction.

NOTES & REFERENCES

* First version of this paper has been published in Polish, *Studia Filozoficzne* **8-9** (237-38), 1987.

1. Kazimierz Ajdukiewicz, "The problem of the rationality of non-deductive types of inference", in: K.Ajdukiewicz, *The Scientific World-Perspective and Other Essays*, ed. by J.Giedymin, D.Reidel Publishers: Dodrecht, Holland / Boston, USA; p. 241.

2. Professor Quine made me aware that this is so only if the sentence involved is decidable (or, more exactly, only if we believe that it can be decided); in the opposite case we can make bets in an irresponsible way.

3. Kazimierz Ajdukiewicz, *Pragmatic Logic*, translated by Olgierd Wojtasiewicz, D.Reidel Publishers: Dodrecht, Holland / Boston, USA; PWN Warsaw, Poland, 1974; p. 107.

Helmut Metzler

METHODOLOGICAL INTERDEPENDENCIES BETWEEN CONCEPTUALIZATION AND OPERATIONALIZATION IN EMPIRICAL SOCIAL SCIENCES

We can still find discussions on operationalization in the empirical social sciences, although Hempel has already given interesting answers and valuations in his general methodological considerations from 1952, which allow us to answer a number of questions debated today. Hempel also critically named a number of inaccuracies in the empirical social sciences, which are still notorious. The following should make a contribution to find answers to the present problems, using a different approach than that chosen by Hempel.

Hempel introduced the concept of operationalization with respect to operationalism. Therefore he was concerned with the concept of operational definition which he declared in principle to be an empirical rule of interpretation. Disregarding the enlargement of the concept of definition in a version of Robinson's doubtful understanding, Hempel's reflections are based on the discussion of empirical verification of theories. Arrangements of methodological questions in such topics of the philosophy of science and of epistemology are widespread. This contribution is concerned with the process of scientific or, more generally, of mental labour. We let ourselves be guided by the idea that, if we introduce 'methodological procedure' as a methodological basic concept, it is legitimate to build a class of abstraction concerning this concept. Hence, such an approach is orientated towards the process. What then is the consequence for the understanding of operationalization? It will not be of interest if operationalization is regarded from the point of view of being a process of verification or of material transformation. We consider only the structural aspect. This means we deal with operationalization as a special relation, and that the question of realization of the *relata* of this relation should be dealt with on a second level.

Let us begin by clarifying the meaning of the two terms central to the topic discussed. Conceptualization is a term which, when used in psychology,

has two basic meanings. When describing psychic phenomena, conceptualization means, on a concrete level, that pictorial, fixed schemata are produced on an abstract level and that information is put into abstract systems of reference. With regard to methodology, which involves the second basic meaning, conceptualization is a methodological procedure whereby concepts or sets of propositions are formed. Alternatively, operationalization is used as a methodological procedure which has a complementary or enriching function with respect to conceptualization. Operationalization transforms each set of given results of conceptualization considered, into rules of operation, and these rules can be applied materially. At this point, a comparison can be drawn with current theoretical work in logic as regards the relation between conceptualization and formalization. As in the case of formalization, the results of conceptualization are mapped onto sets of formulae which can be transformed by means of formal systems of rules thus enabling the logical structures and consequences contained in these results to be uncovered. Likewise, operationalizations indicate transformations through which the results of the conceptualization are mapped onto real contingencies that offer the possibility of practical application. Operationalization as a procedure of transformation, or more specifically, of the material realization of the rules of operation given in the operationalization in the form of practical action, offers an empirical enrichment or an empirical test of the results of the conceptualization given in the operationalization. From a methodological point of view, the investigation into operationalization and conceptualization as a methodological procedure, must be carried out on the same level upon which, in the past, formalization was researched and methodologically elaborated. After having first considered the conceptual definitions of conceptualization and operationalization, let us take a closer look at their content.

Conceptualization is the treatment of cognition that occurs in the process of scientific work. It begins with an initial abstractum which is conceptual or propositional. If one takes an initial abstractum, this can be given by a name or by a combination of terms whose corresponding concept has little or no structure. Based on theoretical and empirical investigations and constructions, the conceptualization involves a progression from the initial abstract to the concrete, that is to the structurally enriched concept. Thus development proceeds, according to the law of the progression, from the abstract to the concrete, brought out in dialectical materialism. In this respect, conceptualization is normally a historical process, shaped by several socially determined subjects, meaning groups of workers and not individuals. The structure of this process has been brought out and explained from various perspectives; e.g., by Lakatos (1976). Since this process unfolds as an interpersonally (socially) and not individually shaped process, it has

scarcely been investigated from the view-point of methodological procedure or of theoretical science. Exceptions to this are the historical analyses of the development of concepts, as for example by Lakatos. From a methodological point of view, it is useful to consider Hegel's *The Science of Logic* and the exemplifications in his *Phenomenology of the Mind*. The methodological transformation of the said law of cognition is shown in practice in Marx's *Capital*. In each science, the question of conceptualization has been approached in different ways. In psychology, for example, it is considered in connection with the formation of constructs, meaning it involves arguments concerning a constructivistic procedure rather than ones which are operationalistic and one-sidedly empirical. In the sense of concept formation, different methodological techniques, especially definitions, are employed to facilitate the conceptualization, and include partial definitions and explications. In the most simple case, a conceptual content is developed by means of a system of propositions, a procedure which is especially transparent when it is constructed in deductive systems. This is the case for the basic concepts contained in the system of axioms, as well as for the concepts that are introduced by definitions within the framework of deductions. Originally, conceptualizations were developed, peculiarly to mental work, according to a division of labour. They were made even more precise through the debate of opinions. The connection to empirical experience resulted through competition with every-day thinking on reality, that is, through every-day comprehension in action and observation. This everyday interaction was subject to methodological criticism that took different forms. Firstly, it is important to mention the criticism of cognition. It was undertaken as a criticism of results and the way these are produced, and led to the further development of measurement. Subsequently, there followed a methodological criticism of action, which prompted the development of the experiment and the unfolding of experimental techniques, which again was closely connected with the development of the technique of measurement. From the interaction between these two directions of criticism, operationalism emerged as a methodological procedure. It was developed both in respect of concepts and propositions. Here, work with operational definitions is important, along with other forms of transformations which realize propositions through actions.

In order to bring out the interactions between conceptualization and operationalization, it is possible to choose from several approaches. A logical-methodological one uses the linguistic formulation of operationalizations and conceptualizations. To begin with, let us restrict our investigations to the operationalization of concepts. In this case the method of procedure can be analysed to advantage as a special variant, over-generalized in operationalism, of operationalizing by means of an operational

definition, so that the interdependencies between operationalization and conceptualization can be uncovered. The understanding of operationalization relevant to operationalism, can be reconstructed in two steps. (1) Operationalization means the transformation of a concept that has been employed descriptively, into an operative concept, in so far as a 'definiens' of an operational definition is associated with it. (2) If we presume that the descriptive concept is expressed in a definition, the operationalization then means the transition of a descriptively defined concept into an operationally defined one. This shows clearly that in the transition from a descriptive to an operational definition, the 'definiendum' must express a fundamental change in the relevant concept. In practice, this change is often of far-reaching importance in mental work. We shall consider first the border-line case in which this change may be ignored, using the following simplified example as an illustration. Descriptive definition: - perceptible green is uncovered light of a wave length of 530 nm. Operational definition: perceptible green is to be obtained if so far as uncovered light of a wave length of 530 nm is produced. In this case, it is obvious that the 'definiens' of the operational definition only differs from that of the descriptive definition by a piece of information, added in the form of the phrase 'is produced', which indicated the rule of creation. Otherwise, the instruction to the result that is produced by the application of the rule of creation, consists of a text that is word for word the same as that in the 'definiens' of the descriptive definition. Because of this concordance, we can talk of a complete operationalization of the descriptive concept, and the change of the concept in the transition from one of these definitions to the other, need not play a part in our considerations. If, however, there is no clear correlation between the descriptive definition and its mapping onto the expression of the result in the operational 'definiens', the descriptive concept will only be partially operationalized in the operational definition. This has a feed-back upon the conceptualization, for not the desired descriptive concept, but only a limited one will be empirically enriched or tested in this manner. Also, from the enrichment or the testing of a partially defined concept, it is impossible to infer similar effects in completely defined concepts.

As an illustration of this point let us consider the well known operational definition: "Intelligence is what the ..... test measures." The gap could be filled by the names of various tests, 'Raven', 'HAWIE', 'IS' or 'LP'. If we ignore the shortened form of the expression in the operational definition above, it is possible to determine principally the incompleteness of the operational comprehension of intelligence in relation to the descriptive, conceptual content of intelligence. Even if relations exist between the two tests permitting a reciprocal validation, it is evident both from the

constructions and from practical experience with tests, that intelligence is defined in different ways and in each case only partially. Because of this we want to take into consideration the transition in the use of the term. This refers to the change of a concept in respect of its mapping, in so far as, in the case of a descriptive concept, we can talk of a 'definiendum' and in the case of an operationalized concept, of a concept which is to be operationalized. Therefore we express it symbolically as Opd instead of Dfd. This also does justice to the fact that operationalization can be applied to concepts as well as to propositions. If the descriptive concept is not given by an explicit definition, or one which carries even less rigour, the operationalization feeds back upon the conceptualization and always in such a way that the transition from the Opd to the Dfd cannot be achieved through a simple equality.

If it is assumed in this connection that the Dfd is embedded not in a given (linguistic) system but in everyday thinking, there is a danger that it is fuzzy if not vague. In this case, operationalization is only one step - even though it is an important one - on the way to the formation of concepts. In the operationalization, unlike the definition, it is not the intention to determine in which language it should be used, but rather to carry out the transformation into a rule of operation. Therefore, in the case of a translation into a definition, an additional operation is necessary, transfering the original Opd in the form of a Dfd in a language. The operationalization of propositions (hypotheses, etc.) must take more complicated conditions into consideration, for in this case there is no simple procedure with which the logical unfolding of propositions could oppose the Ops.

Bearing in mind the cases presented, it is apparent, as Hempel also notes, that a definition of operationalization in the form of an operational definition, is insufficient. Furthermore, it is to be noted that not every concept can be subjected to an operationalization in the sense of the enriching of an empirical content. Consequently, we should not seek to operationalize all concepts. This point is recognized today in methodological considerations on the subject of operationalization and is included in discussions.

The understanding of operationalization from the point of view of an operational definition is a narrow one. This is directly due to its origin in the philosophy of operationalism. This narrowness can be overcome with the following definition. Operationalization is the transformation of *descriptive* concepts or propositions into those with an *operative* form, in such a way that they can be used in practical applications. Thus we acknowledge the practical application, not only in that given descriptive terms are operationalized, but also because propositions are to be transformed into a

practical psychological intervention, for example into a therapy. In the case of the latter, it is at the same time apparent that the narrowness of the received opinion, namely that operationalizations serve to "combine the theoretical level with the level of observation", is overcome. Operative concepts should not only be understood as concepts on the level of observation, but also as such concepts that permit a multifaceted application. Operationalization is not only intended to serve the verification of cognition, but also to ensure that the descriptive cognition is made *applicable*. Similarly, the narrowness with regard to operational *definitions* is also eliminated, in so far as descriptive propositions cannot be operationalized in the form of definitions, and their operationalization cannot be reduced to the simple combination of operationally defined concepts.

In terms of an intellectual act, operationalization can be divided into three parts, namely that which is to be operationalized (Opd), that which operationalizes (Ops) and the transformation (Opt) of Opd into Ops.

One can speak of operationalization if a transition from propositions or concepts as mappings or mental constructs, with a corresponding real or fictive object (class of objects), is undertaken in order to apply these propositions/concepts in practice, i.e., materially, and when the transition is carried out according to methodological rules. These rules can either be applied in a conscious way or as an internalized methodological procedure, not conceptually reflected by the user. They represent the content of Opt and can be described in their schematic structure, for example as a rule whereby a descriptive definition of a concept is mapped onto an operational definition of a concept, and *vice versa*. When dealing with real cases, the schemata themselves must be adapted to these cases. Opd can be of varying complexity; thus, for example, an Opd of an interpersonal perception is of a much higher complexity than that of the simple perception of colours. In accordance with this, the related descriptive concepts/propositions which concern connections between these perceptions and other facts, meaning those concerning the dynamic in these perceptions, are also of varying complexity, providing that a simplification of the Opd is not undertaken at the same time. If we want to take various possible variants into consideration, it is advisable, in our above mentioned definition of operationalization, to understand the Ops not simply as an operative concept or operative proposition, but to consider it in a wider sense as a rule or directive of operation.

The varying degree of complexity of the Ops can be graded in terms of this rule of operation as follows. In the most simple case, this rule consists of the statement of an elemental operation to be carried out. Next is the rule concerning the application of many elemental operations that are

connected to each other. Such combinations of elemental operations that are to be represented, are to be called modules, which in their most simple forms are molecular operations, but without a formulation of hierarchies. If modules are combined with elemental operations or with other modules, or combined with both elemental operations and other modules, they give rise to produce further levels of complexity which for their part can also be subsequently differentiated, to see if hierarchies are formed or not. At this point, it is appropriate to compare the understanding of "operationalization" as it is used in psychology and in modern physics. Let us take a process of measurement to be an operationalization of measurable quantities. In quantum physics we find that the process of measurement is described in such a way that it involves a connection of various empirical and mental operations of information processing. At this point it should also be emphasized that currently these mental operations can be realized objectively with the help of computers which have a closed technical system of measurement. Also, it must be pointed out in this case, that these mental operations have not somehow sprung up out of the subjective arbitrariness of the investigator. On the other hand, the actual fact that computers are used is not of primary importance. Indeed, the description of the Ops in the form of a rule of measurement, this is of secondary importance. In this regard, there still exists in psychology the tendency to think that operationalization is successful if one has found elemental operations of measurement as Ops, and these are to be connected through simple arithmetic operations. More demanding mental operations are then evaluated as far as possible, not as an integral part of the operationalization, but rather as a subsequent step in the processing of the information. Such an understanding is found especially when an operational definition is used. In the variants of the understanding of the term operationalization, carried out by V. Gadenne, we can also find such cases in which mental connections are included. From our point of view, it is absolutely necessary to advocate in general for psychology, the extended understanding of operationalization, so that demanding, mental operations that are connected with empirical operations, can also appear in the Ops. Only thus can a restriction of operationalism, with respect to the topic of operationalization, be overcome. However, one further point needs to be added to the information we give on the rule of operation, Ops. It must be possible to carry out this rule at varying degrees of complexity. There must be either a finite number of steps, or, on the other hand, a criterion of interruption must be known, which, according to the requirements of the products, can determine the point at which the rule of operation is to be switched off.

Proof of whether the operationalization proceeds in the direction worked out in each case is always discovered only after the rule for

operation Ops has been transformed into its appropriate practical action. The rule for operation only presents a schema for action. In general, it stands relative to processes that may be repeated many times and take the form of realized operationalizations. In fact, operationalization is over only when the Ops can be actually put into practice. The transition from the cognition that has been made applicable (as Ops) to its actual application, uncovers existing insufficiencies in the mental transformation, so that from this point onward, corrections may be made to the Ops. Just as the operationalization only ends with transition into a practical action, so according to a rule, the result of the mental transformation is also defined, not as something that has been operationalized, but rather as something which operationalizes (Ops).

If we look closely at the relation 'Operationalization', it will be seen to consist of three parts. A subject which is constitutive for the transformation to be carried out, both in its cognition and its actions, determines which descriptive expression and which operative expression are to be associated. According to its respective purposes, the subject will concern itself with the operationalization of expressions describing material or mental phenomena, and with respect to these, choose the appropriate operation-schemata. Here we see the narrow relationship between formalization and operationalization. With respect to special points of view the first can be understood in investigations of systems of concepts and systems of propositions as a special case of the latter.

Modern representations of the relation of inference through axioms and rules, can be understood as operationalizations - taking the term in a wide sense - for the expression of mental phenomena. With regard to current efforts to transfer comprehensively human abilities onto machines, similar possibilities on the conceptual side should also be tested. Differences in the operationalization can be determined by material circumstances or by levels of complexity. In order to clarify the mentioned differences in complexity, we shall consider three examples of operationalization:

1. 'Perceptible Green' has already been operationalized. The structure is simple, because it is restricted to only one empirical operation with regard to secondary circumstances. We will complete our exposition by saying what Opt is; it is given by the phrase 'is to be obtained'.

2. In contrast to this, an operationalization of intelligence using an intelligence test, as for example that from Hamburg-Wechsler, is extended by a series of mathematical operations. Empirical operations occur here in forms that vary considerably; this is indeed the case for the 'probands' as well as for the test organizer. In addition, the test organizer carries out mathematical operations and also uses mental

tools which themselves have been created through a complicated system of operations in the elaboration of the test. The complexity of these operations is obviously much greater than that of the first example. However, one cannot say that 'It is intelligence that the test measures'. In fact, when this operationalization is finished, as mentioned above, one should go back from the Ops and ask oneself what is the difference between the Opd that has been mapped into this Ops, and the descriptive concept 'intelligence'. V. Gadenne characterizes examples like this as 'partial definitions'. However, from the standpoint of Frege's logic, such incomplete definitions are unacceptable as solutions. From this, we can conclude that we should map onto the restricted Ops a concept as a subdivision of that concept which originally formed the basis of the Opd. The rule of operation which lays down the measurement of intelligence using the HAWIE test, including the subsequent interpretation of the result, is therefore the Ops of a special section of intelligence that renders distinct the Opd mapped onto it. In practice, the subsequent Opd is not given in the form of a single, individual concept with a specialized name. The experienced diagnostician always thinks the restriction of the Opd as given. Consequently, test-batteries are used in the case of demanding tests of performance capacity.

3. Let us consider the third degree of difficulty, as, for example, contained in the task of the operationalization of the concept of interpersonal perception. Here, two particularities are striking. The complexity of the operationalization is increased by the fact that the psychic phenomenon must be expressed not only in the case of the individual, but also in relation to another individual or several others who are able to perceive, and are as a rule integrated into action. Since the descriptive concept of interpersonal perception has only been partially explained, the operationalization demands and also sometimes even inspires that the descriptive concept be understood in a more differentiated form. In this case, operationalization can be carried out, together with explication of the concept or of the Dfd, in a positive feed-back. This feed-back should not be misleading. The more exact definition of the descriptive concept in this way, is not a part of the operationalization. The same holds true for the necessity to restrict the Opd to the available Ops, as mentioned above in connection with the partial operationalization of the concept of intelligence. Both these cases of the feed-back of the formation of a particular type Ops upon the Opd, can be seen as adhering to the view with which M. Irle agrees, that operationalization is a formidable process of achieving cognition.

As we could recognize before, there exist interdependencies between operationalization and conceptualization. In scientific practice, only one aspect of this is often taken into account. On the one hand, demanding operationalizations are carried out, whereby little attention is paid to the conceptual differentiation. On the other hand we find that the conceptualization is carried out with great sophistication, but the operationalization is neglected or is rejected as being non-realizable. These conflicting tendencies are often clothed in ideologies through the contrasting of quantitative and qualitative orientations in scientific research. What is needed to improve the effectiveness of the scientific process, is for operationalization and conceptualization to be carried out in a balanced relation to each other, with reciprocal positive feed-back.

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Jarosław Fall

GAME-THEORETICAL SEMANTICS APPLIED TO DEFINITE DESCRIPTIONS AND ANAPHORA*

0. Historical Remarks

Game-Theoretical Semantics (GTS) - which is a variant of a truth-value semantics employing some elements of mathematical game-theory - was introduced by Jaakko Hintikka in the middle of the seventies. In his paper (1976) he claimed that the semantical games in his sense "are at one at the same time designed as language-games in Wittgenstein's sense; they are games in the precise sense of mathematical game-theory; they offer a handy tool for systematizing logical theory; and they give rise (in my judgment) to an extremely promising approach to the semantics of natural language".

Hintikka was not the first among logicians who has applied game-theory. In the early sixties Ehrenfeucht, Henkin, Lorenz and Lorenzen employed game-theoretical ideas for the analysis of some formal languages. There is nevertheless a novelty in Hintikka's approach worth pointing out. It consists in a proposal of using game-theoretical concepts not only for the analysis of formal but also of *natural* languages.

Game-Theoretical Semantics has been so far studied mainly by researchers from Scandinavia. Their names are apart of Hintikka himself also Carlson and Saarinen. However, Hintikka's closest collaborator over last few years does not come from that region. His name is Jack Kulas. In 1985 they both published a book which seems to be the main achievement in the field of natural language analysis by means of GTS. The book has presented an application of GTS for a unified treatment of anaphora and definite descriptions, the topics of a great concern in many current linguistic, logical, psychological and AI (Artificial Intelligence) studies.

1. Introduction

The basic ideas of GTS are explained most easily in terms of a formal first-order language $\mathbf{L}$. One is given for the purposes of a truth-definition a model $\mathbf{M}$ consisting of a domain of individuals $do(\mathbf{M})$ and an interpretation of all the primitive predicates of $\mathbf{L}$ on $do(\mathbf{M})$. This interpretation determines the truth-values of all atomic sentences in every language $\mathbf{L}(\mathbf{I})$ that is an extension of $\mathbf{L}$ obtained by adding to it a finite set $\mathbf{I}$ of names of members of $do(\mathbf{M})$.

A game-theoretical truth-definition for compound sentences works its way from the outside in, defining in effect a semantical analysis of the sentence in question. This analysis is obtained in GTS by associating with each sentence $\mathbf{S}$ of $\mathbf{L}$ a two-person zero-sum (semantical) game $G(\mathbf{S})$ between Nature and Myself. Myself wins if the play ends with a true atomic sentence in $\mathbf{L}(\mathbf{I})$. If $G(\mathbf{S})$ ends up with a false sentence, Nature wins and Myself loses.

The play can be thought of as an attempt on the part of Myself to verify $\mathbf{S}$ and on the part of Nature to falsify $\mathbf{S}$. The sentence $\mathbf{S}$ is true if Myself can verify it no matter what Nature does. That means that there exists in $G(\mathbf{S})$ a winning strategy for Myself. The sentence $\mathbf{S}$ is false if there exists a winning strategy in $G(\mathbf{S})$ for Nature. The basic rules of the games $G(\mathbf{S})$ are as follows:

($G.\ \vee$) The game $G(\mathbf{S}_1 \vee \mathbf{S}_2)$ begins with a choice by Myself of a disjunct $\mathbf{S}_i$ (i = 1 or 2). The rest is played as in $G(\mathbf{S}_i)$.

($G.\ \wedge$) The game $G(\mathbf{S}_1 \wedge \mathbf{S}_2)$ begins with a choice by Nature of a conjunct $\mathbf{S}_i$ (i = 1 or 2). The rest is played as in $G(\mathbf{S}_i)$.

($G.\ \exists$) The game $G(\exists x\, \mathbf{S}[x])$ begins with a choice by Myself of an individual from $do(\mathbf{M})$. Then the game is continued as in $G(\mathbf{S}[b])$, where b is a name of the chosen element and $\mathbf{S}[b]$ is the outcome of replacing every free occurrence of x in $\mathbf{S}[x]$ by b.

($G.\ \forall$) The game $G(\forall x\, \mathbf{S}[x])$ is analogous to $G(\exists x\, \mathbf{S}[x])$, except that the individual is chosen by Nature.

($G.\ \sim$) The game $G(\sim \mathbf{S})$ begins by a switch of roles by the two players as defined by these game rules and the rules of winning and losing. Then the game is continued as in $G(\mathbf{S})$.

It is clear that each semantical game $G(\mathbf{S})$ will come to an end after a finite number of moves which is no greater than the number of logical symbols in $\mathbf{S}$. Hence the rules for winning and losing are applicable to every semantical game.

It may be also easily proved that the above game-theoretical truth-definition is equivalent to the usual Tarski-type one.[1]

Among the above rules the last one consists a special case because it changes "behaviour" of the competitors in a semantical game. After application of that rule Myself changes its status to a falsifier and Nature - to a verifier. It means that after (one) application of $(G. \sim)$ Myself wins if a semantical game ends with a false sentence and Nature wins when a game ends with a true sentence. The strategies of Myself and Nature after $(G. \sim)$ are also reversed.

A simpler way of treating a sentence $\sim S$ in a semantical game is to play a (local) game $G(S)$. When it is over the result of $G(\sim S)$ is obtained by taking the reverse of $G(S)$ result.

2. Subgames

GTS makes possible some extensions when compared with the usual first-order languages. One of such important extensions is the idea of a *subgame*.[2] It consists in allowing a semantical game to be divided into several subgames. A subgame is sometimes played with the usual roles of the two players reversed depending on a rule in force. Such a rule may also specify which of the strategies used by the two players are available to which of the players in later subgames.

The rules for conditionals are of the above kind. It will be enough here to say that the semantical game $G(S_1 \supset S_2)$ should in principle go as follows. First, a subgame is played with the *antecedent* S_1, but with the roles of the two players interchanged. If Myself wins this subgame, Myself wins the entire game. If Nature wins, the game is continued to the second subgame, played with the *consequent* S_2, and with the usual roles for Myself and Nature (conversly to the previous subgame). What is important here is that in this second subgame Myself has access to the strategy Nature used in the first subgame.[3] Examples of applying this rule for natural language consequence expressions will be given later, when discussing anaphoric definite descriptions (cf. **5.3.**).

3. GTS for natural languages

One of the possibilities of extending GTS to natural languages is easily obtained when we have a translation of natural language expressions into first-order logical notation. Then all the rules formulated above for formal language might be used in semantical games on natural language.

Though the above translation is what many linguists and philosophers are trying to do, Hintikka & Kulas propose another and in their opinion better motivated extension of GTS towards natural languages. It consists in formulating a number of GTS rules directly for an unspecified fragment of English, without assuming a prior translation into first-order notation. It means that a semantical game $G(S)$ based on exactly the same rules as for formal languages (see above) is associated to each sentence of that fragment of English. The principles of truth-definition for a sentence **S** are also similar to those for formal languages (there must be a model in which the basic vocabulary has been defined). Finally, some game rules such as (*G. and*), (*G. or*), (*G. not*) are practically analogous to the corresponding formal rules (*G.* $\wedge$), (*G.* $\vee$), (*G.* $\sim$) respectively.

Quantifier rules have to be different. Individual names chosen by the players must substitute entire quantifier phrases (*some X who Y, every X which Y, an X where Y, each X whose Y, any X which Y*, etc.). what causes that some additional changes are also needed. Here is a special case of (*G.some*) rule:

(*G. some*) If the game has reached a sentence of the form

X - *some* Y *who* Z - W

then Myself chooses a person from the domain of the model with respect to which the game is played. If the name of the individual is *b*, the game is continued as in

X - ***b*** - W, ***b*** *is a* Y, *and* ***b*** Z.

The rule (*G. a(n)*) is analogous, except that *a(n)* replaces *some*.
The rule (*G. every*) in its special case is like that of (*G. some*) with the differences as follows:

(i) The input and output sentences are respectively

X - *every* Y *who* Z - W

and

X - ***b*** - W *if* ***b*** *is a* Y *and* ***b*** Z

(ii) The choice of ***b*** is made by Nature.

The rules (*G. each*) and (*G. any*) may be formulated alike.

In order to be more precise a few additional comments are needed in this point:[4]

a). The choice of an individual as in the above rules is to be made from an appropriate subset of the domain. Sometimes this subset is indicated by the relative pronoun occurring in the quantifier phrase (*who* - subset of persons in the model, *where* - locations in space, *when* - locations in time, etc.).

b). The above rules are correct only for nominative. They should be appropriately reformulated for other cases.

c). Only the singular is taken into account. The plural case should be dealt with separately.

d). The order of the constituent phrases in the output sentences above is free, i.e. can be chosen by Myself. This is a characteristic feature of the discussed rules but not necessarily of other GTS rules for English.

The subgame idea (cf. 2.) enables to perform semantical games not only on sentences but also on some discourses. In the simplest case of successive asserting utterances the discourse may be treated as conjunction of several sentences. Then consecutive (sub)games played on these sentences may be treated as parts of one "supergame". However, for more complex cases discourse semantics would need considerable redefinition.

4. Ordering principles

From a point of view of GTS there are many similarities between formal and natural languages some of which has been already discussed. There are also significant differences. The main of the latter is syntactically determined order of game rule application in formal languages *vs.* a lack of such determinacy in natural language. Hence it is clear that in the case of natural language usual game rules must be complemented by some additional ordering principles. GTS makes use of two kinds of ordering rules: general and special ones. The former depend only on the syntax of the sentence in question; the latter - also on lexical items occurring in that sentence. Special ordering principles are stronger than the general ones.

The most important (general) ordering principles are as follows:

(*O. LR*) In one and the same clause, game rules are applied from left to right.

(*O. comm*) A game rule must not be applied to an ingredient of a lower clause if a game rule applies to an ingredient of a higher one.

For a sentence in a labelled-tree form a node N_1 is said to be in a higher clause than N_2, if the S-node most immediately dominating N_1 also dominates N_2, but not *vice versa*.

Among **special** ordering principles there are the following:

(*O. any*) which says that (*G. any*) has a priority over (*G. not*), (*G. or*), modal (nonepistemic) rules, and (*G.* cond),

(*O. each*) which says that (*G. each*) has a priority over propositional and quantificational rules.

The force of ordering principles is first of all to indicate scopes of different quantification phrases which cannot be explicitly given by means of natural language. Here are three examples of sentences and their "logical forms" according to GTS ordering principles:

Some boy loves every girl.

$$\exists x\{(x \text{ is a boy}) \wedge \forall y[(y \text{ is a girl}) \supset (x \text{ loves } y)]\}$$

Every girl is loved by some boy.

$$\forall y\{(y \text{ is a girl}) \supset \exists x\,[x \text{ is a boy} \wedge (x \text{ loves } y)]\}$$

That someone will some day beat him never occurs to any real champion.

$$\forall x\{(x \text{ is a real champion}) \supset \sim\exists t[\text{it occurs to } x \text{ at time } t \text{ that}$$
$$\exists z\,\exists u\,((u \text{ is a future day}) \wedge (z \text{ beats } x \text{ on day } u))]\}$$

For formal languages it is syntactically determined which sentence is an atomic one and what are truth-conditions for it. Contrary, GTS for natural languages needs apart of game rules and ordering principles also a proper lexical theory which would state when a sentence might be treated as an atomic one. However, even without such a theory a number of observations concerning behaviour of definite descriptions and anaphora may be made.

5. Definite descriptions and anaphora

There is not enough space here to go with the topics under consideration into deep details. We will concentrate our discussion on the most important features of GTS approach to definite descriptions. However, it should be noted that what will be said about anaphoric *the*-phrases does generally concern also anaphoric pronouns.

5.1. Russellian definite descriptions

Though criticised from different points of view Russellian treatment of definite descriptions consists a well motivated starting point for GTS approach to them. While having given his definition of definite descriptions Russell had obviously in mind primarily those occurrences of English *the*-phrases which are context-independent and in which the speaker is attributing or presupposing uniqueness (*the highest mountain of the world, the best-selling brand of vodka in Poland in 1986, the prettiest woman at someone's own home*).

5.1.1. Limitations of Russellian treatment of definite descriptions

There are at least two different uses of *the*-phrases for which Russellian treatment is dissatisfying. They are *anaphoric* and *generic the*-phrases:

ANAPHORIC

If Bill owns a donkey, he beats *the donkey (it)*.
If you are accosted by a stranger, don't talk to *the man (him)*.
Nobody stole your diamonds, unless *the thief* scaled a slippery 50-foot wall.
Some man is capable of falling into love with any woman, at least if *the woman* is blond.

GENERIC

The tiger is a dangerous animal.
In the United States *the president* now has far greater powers than were enjoyed by *the president* in the nineteenth century.

Russell's treatment fails also in the case of Bach-Peters sentences which cannot be translated into iota-notation because of crossing references occurring in them. For similar reasons these sentences appeared a serious problem for generative syntax. In GTS the general solution for Russellian *the*-phrases succeeds also in the case of Bach-Peters sentences. This solution is given by the following game rule:

(*G.* Russellian *the*) When a game has reached a sentence of the form

X - *the* Y *who* Z - W

an individual, say ***b*** , is chosen by Myself, whereupon a different individual, say ***d*** , is chosen by Nature. The game is then continued with respect to

X - ***b*** - W, ***b*** *is a* Y, ***b*** Z, *but* ***d*** *is not a* Y *who* Z.

The choices are made from the appropriate "category", which in the case of *who* is the set of persons.

5.1.2. Bach-Peters Paradox

As far as Bach-Peters sentences are concerned, eg.:

(1) The boy who was fooling her kissed the girl who loved him.

applying the rule (*G.* Russellian *the*) to (1) yields a sentence of the following form:

(2) Harry kissed the girl who loved him, Harry is a boy, Harry was fooling her, but Dick is not a boy who was fooling her.

A second application of the same rule yields:

(3) Harry kissed Harriet, Harry is a boy, Harry was fooling her, Dick is not a boy who was fooling her, Harriet is a girl, Harriet loved him, but Margaret is not a girl who loved him.

The limitations of Russell's notation are overcome in GTS, as shown above.[5] It is because GTS proceeds from outside in, and hence can take into account context-dependencies as in (1).

5.1.3. Primary vs. secondary occurrence of definite descriptions

The following example:

(4) George knew that the author of "Waverlay" is Scott.

has two possible readings

(5) ιx [x authored "Waverlay" $\wedge$ George knew that (x = Scott)]

and

(6) George knew that [ιx (x authored "Waverlay" $\wedge$ x = Scott)].

These two readings result from taking the definite description to have (according to Russell) a *primary* occurrence in (5) and a *secondary* occurrence in (6).

GTS offers an explanation **why** the difference takes place. The answer is that it is caused by different order of applying the rule (*G.* Russellian *the*) in relation to other game rules. In GTS it may be also specified **when** the above difference occurs, i.e., what are the ordering principles connected with the rule (*G.* Russellian *the*).

5.1.4. Strawson's bold king of France

In connection with the example:

(7) The present king of France is bold.

Strawson criticized Russell's treatment of the existential force of definite descriptions. Instead he applied the notion of presupposing which in Hintikka & Kulas's (1985:43) opinion is rather a **discourse** (**not** sentence) concept. What they propose is that "the rules and especially the ordering principles of **discourse semantics** will be such that if the uniqueness and existence components of a definite description are not satisfied, that failure will normally have stopped the semantical game (played on a segment of discourse) before the rule (*G.* Russellian *the*) is applied".

However, it makes only a slight difference, in my opinion, to allow for a failure **during** application of the above rule. It would mean that *presupposing* suffice for a use of a definite description but only *existing* individuals can be successfully referred to. Such a failure could be interpreted in two ways, depending on a definition of rules for winning and losing. Firstly, a sentence with an invalid definite description could be treated as *false* (there is no winning strategy for Myself). Secondly, it could be treated as *"senseless"* (there is no winning strategy for Nature either).

5.1.5. Referential and attributive use of definite descriptions

All the above discussion was concerned with attributive aspect of definite descriptions. Here is an example of a referential use of them:

If someone says at the party,

(8) The man standing next to the hostess is a famous writer.

and it turns out that the man is standing next to the hostess's twin sister, (8) still conveys some true information, even though in Russell's treatment (8) should refer to someone else, who is actually standing next to the **real hostess**.

After taking into account (a possible world of) perceptual individuals rather than descriptively identified ones and assuming that the intended force of the referential use is as follows:

(9) The man who - I believe - stands next to the hostess is a famous writer.

GTS offers a proper meaning of (8), i.e., a semantical game played on (9) with (*G.* Russellian *the*) to be applied first.

5.2. Anaphoric *the*-phrases

In **5.1.1.** a few examples of anaphoric and general uses of *the*-phrases were offered. For the both groups Russell's theory of definite descriptions does not give satisfactory results. Some restrictions on the ranges of quantifiers to a contextually given part of the universe of discourse are needed. But such restrictions cannot be fixed. GTS offers a very dynamic way of restricting those ranges depending on the stage reached by a semantical game at the time when a given *the*-phrase is treated in the game. This way is inscribed in the following rule to be applied in a case when an anaphoric *the*-phrase has been found in a semantical game:

(*G.* anaphoric *the*) When a semantical game has reached a sentence of the form

X - *the* Y *who* Z - W

then an individual, say ***b***, may be chosen from a set **I** of individuals by Myself, whereupon Nature chooses a different individual, say ***d***, from the same set **I**. The game is then continued with respect to

X - ***b*** - W, ***b*** *is a(n)* Y, *and* ***b*** Z, *but* ***d*** *is not a(n)* Y *who* Z

Here **I** is the set of all individuals chosen by either player earlier in the game. If **I** = $\{b\}$, then the game is continued with respect to

X - ***b*** - W, ***b*** *is a(n)* Y, *and* ***b*** Z.

In connection with the above rule some comments similar to those presented in **3.** for (*G. some*) would be needed here. However, we neglect doing that because of a lack of space.

5.3. Application of (*G.* anaphoric *the*)

We have chosen for presentation an example which shows how the game rule (*G.* anaphoric *the*) is dealt with and, at the same time, how the idea of a subgame (cf. **2.**) may be applied.

(10) Nobody stole your diamonds, unless *the thief* scaled a slippery 50-foot wall.

The first subgame is played on *Nobody stole your diamonds*. In it, Nature chooses an individual, say Gregory. Myself wins if the sentence *Gregory didn't steal your diamonds* is true. If false, players go on to play another subgame on

(11) *The thief* scaled a slippery 50-foot wall.

In this second subgame, the players "remember" Nature's earlier strategy, i.e., have Gregory in **I**. Hence an application of (*G.* anaphoric *the*) will yield:

(12) Gregory scaled a slippery 50-foot wall, and Gregory is a thief.

This is true if and only if Gregory scaled a slippery 50-foot wall, which yields the right meaning to (10).

5.4. The anaphoric use of definite descriptions as a semantical phenomenon

In this point we will show on more and more complex examples (taken from Hintikka & Kulas 1985) that a general approach to definite descriptions (and to anaphora) cannot be restricted to syntax only, as it is usually made in most of linguistic theories (especially in generative linguistics). Both of these phenomena are of semantic nature and only when semantic factors are taken into account one may receive a satisfactory theory. First of all, in many cases it is difficult to find a purely syntactically determined *head* (or *antecedent*) of an anaphoric phrase. GTS offers a proper treatment even in such cases. They include among others:

- **epithetic *the*-phrases**

  Harry borrowed ten dollars from me, but *the bastard* never paid me back.

- **counterepithetic *the*-phrases**

  An old fisherman walked toward the beach. *The fisherman* was thinking of the day ahead.

- ***the*-phrases with implied "antecedent"**

  A couple was sitting on a bench. *The man* stood up and *she* followed his example.

The last sentence shows apparent similarity between anaphoric pronouns and anaphoric definite descriptions.

> When a plain virgin of forty-five falls in love for the first time and gets her first taste of sex, God help *the man*.
>
> Surely there is night life in Tallahassee. Unfortunately, this weekend *the lady* is in Tampa.

6. A general GTS framework for definite descriptions

Though a lot of details (some of great importance) have been left out of this paper, the above (simplified) game rule (*G.* anaphoric *the*) conveys enough clues for how anaphoric *the*-phrases are dealt with in GTS.

6.1. Anaphoric *the*-phrases – the fundamental case

It has to be emphasized here that anaphoric uses of English *the*-phrases are considered in GTS as the *fundamental case* and other uses are treated as pragmatically determined variants of that case. Anaphoric *the*-phrases appear apparently more frequently than other ones. It seems also that when the anaphoric interpretation of *the*-phrases is possible, it excludes the Russellian reading:

> You want to see Mr. Lowell? Well, *the president* is in Washington, conferring with Mr. Roosevelt. (Reputedly said by a Harvard secretary in the early Thirties to a visitor who wanted to see A.L.Lowell).

6.2. Deixis and perceptually identified objects

Deixis, e.g., if an animal trainer is in trouble I shout:

> Look out for *the tiger*.

may be explained in a way similar to the one used for Donnellan's referential occurrences of *the*-phrases (cf. **5.1.5.**). It means that the semantical game should be played over perceptually identified objects.

6.3. The Russellian use

The Russellian definite descriptions are possible only in the case when the fundamental (anaphoric) use of *the*-phrases is excluded. What a hearer of a sentence as:

> The author of "Waverlay" is a Scot.

does is to apply Davidson's (1973) *principle of charity*. Since there is no nonempty set $\mathbf{I}^6$ available, the hearer interprets the *the*-phrase by making the next most obvious choice, i.e., setting the set $\mathbf{I}$ equal to *the whole domain of discourse* (or the relevant category). Because such a decision is expected by (and obvious for) any listener or hearer of discourse the above interpretation of a *the*-phrase may be treated as application of Davidson's *principle of charity*.

6.4. The generic use

The tiger is a dangerous animal.

When a sentence like the one above is uttered the Russellian interpretation is obviously not possible (more than one tiger exists). If no other case of the ones discussed previously takes place, *a transcendental* (or *pragmatic*) *deduction* is employed in order to uncover the force of a generic *the*-phrase in discourse. This pragmatic deduction is something as invoking an "axiom of choice". The choice in the above example is made in the range of a biological species called *tiger*. The tiger is chosen there solely for its representativeness. Hence the force of the last example is an assertion of what is true of a typical tiger.[7]

NOTES

* This paper was thought of as an introductory part of the project 'GTS and Anaphora'. As such it was prepared for presentation during the 33rd Conference on the History of Logic (Kraków, Poland, October 1987). It was read in its shortened form during that conference. No doubt, the paper is not fully original. It was based mainly on Hintikka & Kulas (1985). For those who do not know that book the paper may serve as a stimulus for reading the whole book. For the others it will be perhaps tolerable to read the paper *pro memoria*.

1. for the proof cf. Hintikka & Kulas (1985 : 6-7)

2. Hintikka & Kulas (1979), Carlson (1983)

3. for the details cf. Hintikka & Kulas (1985 : 10-11)

4. cf. Hintikka & Kulas (1985 : 12-14)

5. Some problems with semantics of (3), namely with pronouns, may be resolved as follows. The rule (*G.* Russellian *the*) should be supplemented by a **natural** demand that only individual ***b*** chosen by Myself (and not ***d*** chosen by Nature) enters the set **I** - cf. the rule (*G.* anaphoric *the*), below. This demand is compatible with a proposal of "depronominalization possibility" (in phrases 'X' and 'W' from the rule (*G.* Russellian *the*) as soon as an individual ***b*** is chosen by Myself - cf. Hintikka & Saarinen (1975).

6. cf. point **5.2.** - (*G.* anaphoric *the*)

7. for more detailed discussion cf. Hintikka & Kulas (1985 : Part II).

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ON LOGICAL ANALYSIS OF ORDINARY SENTENCES

In natural language we meet sentences of quite different grammatical and logical structure. Sentential logic and the logic of quantifiers investigate some "basic structures" of language expressions. In the last few decades logicians have created a lot of formal systems to extend the expressive power and potential of formal languages in accordance with the multiplicity of forms in the natural language. In the main it has been by constructing ideal languages, i.e. under assumptions which are not fulfilled by natural language or by real language communication. Many problems arise if we try to apply logical analysis to philosophy or other sciences which use natural language.

This paper is concerned with some of such problems. The aim is to suggest a way natural language expressions can be analysed more adequately. For this purpose I shall restrict myself to expressions (basic expressions) which sentential logic and the logic of quantifiers are concerned with. Such expressions are of the form: "Socrates is (is not) a man", "Socrates is (is not) mortal", "All (some) Greeks are (are not) men", "All (some) Greeks are (are not) mortal", etc.

Traditional Aristotelian logic offers four formal structures to express ordinary propositions:

A: All A's are B's

E: No A is (a) B

I: Some A is (a) B

O: Some A is not (a) B

In this logic it is not possible to express relations. Singular names and categories are excluded. The only *term category* allowed is that of general names. Thus, all ordinary propositions we want to include into logical consideration we have to translate into one of two given forms and all terms occurring in these propositions have to be understood as general names. Here I see the main problem. Take for example the sentences:

(1) "All men are mammals" and (2) "All men are mortal"

(3) "Some man is a mammal" and (4) "Some man is mortal".

If we translate these sentences into Aristotelian forms we get:

(1') "All A's are B's" and (2') "All A's are C's"

(3') "Some A is (a) B" and (4') "Some A is (a) C",

where A, B, and C are general names which stand for "man", "mammal", and "mortal" respectively.

For (1) and (3) this translation seems to be adequate. But I cannot agree that (2) and (4) are to be translated by the same structure. In languages which use articles we can see the difference between (3) and (4) directly. If we omit the "a" in (3) or add it in (4) we get expressions which are grammatically not correct. Of course this cannot be a general criterion because some languages like Russian or Polish do not have articles. Another grammatical difference is that in (1) and (3) there is a noun in the predicate position whereas in (2) and (4) there are adjectives. Only nouns (general names) occur in (1') - (4'), and it is an open question how to translate adjectives in an adequate way into general names.

Grammatical criteria for distinguishing between subject expressions and predicate expressions were already discussed by P. Strawson (1952). We have to agree that the grammatical structure of a sentence cannot be a general guide for explaining its logical structure. But differences in grammar indicate that there could be logical differences too.

And, indeed, words like "man" and "mortal" play a very different role in our language. We see this in the different way subject expressions ("man") and predicate expressions ("mortal") have to be introduced (defined). We can define a subject expression by referring to another, more general subject expression and special features. Predicate expressions cannot be defined in general, but only for a class of subject expressions. If we use the predicates "true" and "false" in logic, we know at the very least since Tarski's famous paper appeared that these predicates are defined for sentences only.

And if we want to give some "ontological explanation" we could say that subject expressions have to *name things* (objects in the widest sense), whereas predicate expressions do not name at all, or we have to accept properties as some kind of abstract entities. They have to *express properties or relations*.

Some of the problems mentioned above are solved by Fregean Logic. Relation can be expressed. We have singular names and can permit categories.

But the translation problem arises, too. Frege's *Begriffsschrift* is based on the notions of "functor" and "argument". Elementary sentences like

(5) "Socrates is a man" and (6) "Socrates is mortal"

are analysed as consisting of two parts - the singular name "Socrates" and the function expressions "is a man" and "is mortal" where the function expressions are understood by Frege as having a free place for receiving the argument.

Now we meet a problem similar to that mentioned with respect to Aristotelian logic - (5) and (6) have to be analysed in the same way and expressed by (5') P(a) and (6') Q(a) where a, P(...), and Q(...) stand for "Socrates", "is a man", and "is mortal" respectively.

The copula "is" which we have in Aristotelian and most natural languages disappears in Frege's understanding of elementary propositions. Many authors emphasize that the word "is" is ambiguous (see: Wessel 1976). Nevertheless the omission of the copula (the predication sign) is a disadvantage. It restricts the expressiveness of formal language in comparison with the natural one. The only way to deny P(a) is $\sim P(a)$. But if we have the copula (a sign for the predicate operation itself) within a sentence we can negate it in two ways - by negating the whole sentence and by negating the predicate operation.

The introduction of quantifiers is a great advantage of Fregean logic. By means of quantifiers it offers more opportunities for forming sentences and is better suited to the analysis of ordinary propositions than the Aristotelian one. But there are disadvantages too. The language of classical quantification theory introduces individual constants, individual variables, and predicate constants. To handle the propositions (1) - (4) in quantification theory they have to be translated into:

(1'') $(\forall x)(P(x) \supset Q(x))$ and (2'') $(\forall x)(P(x) \supset R(x))$

(3'') $(\exists x)(P(x) \wedge Q(x))$ and (4'') $(\exists x)(P(x) \wedge R(x))$

where P, Q, and R stand for the *predicates* "man", "mammal", and "mortal" respectively. It means that all three terms have to be understood as predicates. For "mortal" there is no problem. But how to translate the general names "man" and "mammal" into predicate expressions? On the other hand (1) - (4) refer to (all/some) men and only to men. But the formal expressions (the "translations") speak about *all individuals* of some universe of discourse.

What I have stated gives rise to the view that Aristotelian structures as well as Fregean ones do not supply an adequate translation for (1) - (6). I want to suggest the thesis that a quantification theory applicable to

natural language has to be similar to it as closely as possible with respect to the available term categories. In natural language we have singular names and general names (subject terms) but also terms for expressing properties and relations (predicate terms). They are used in a different way. In my opinion it is a quite natural approach to set up a quantification theory in which we have formal expressions for all these term categories.

The revised system I want to suggest in this paper is based on the classical calculus and on some ideas on predication and term theory developed by A. Sinoviev and H. Wessel. Let us call this system QT. The language of QT contains the following symbols:

1. $a, b, c, a_1, b_1, \ldots$ – singular subject term constants
2. $x, y, z, x_1, y_1, \ldots$ – individual variables
3. $A, B, C, A_1, B_1, \ldots$ – general subject term constants
4. $P, Q, R, P_1, Q_1, \ldots$ – predicate constants
5. $\sim, \wedge, \vee, \supset$ – usual functors

6a. $\forall, \exists$; 6b. Δ, ∇ – two kinds of quantifiers

7. $\leftarrow, \tilde{\leftarrow}$ – predication operators
8. $\blacktriangleright$ – relation of term inclusion
9. "..." – term forming functor which makes the name of α from the term α

With the exception of elementary formulae the formation rules are as usual.

Elementary formulae of QT are built in the following way:

a. If $\alpha_1, \alpha_2, \ldots, \alpha_n$ $(n \geq 1)$ are subject term constants or individual variables and Π is an n-place predicate constant then $(\alpha_1, \alpha_2, \ldots, \alpha_n \leftarrow \Pi)$ and $(\alpha_1, \alpha_2, \ldots, \alpha_n \tilde{\leftarrow} \Pi)$ are elementary formulae of QT. "$(a \leftarrow P)$" is read as "a is P" or "P is predicated of a" and "$(a \tilde{\leftarrow} P)$" as "a is not P" or "P is denied of a".

b. If α and β are subject term constants then "α" $\blacktriangleright$ "β" (the term α includes the term β by meaning) is an elementary formula of QT. "a" $\blacktriangleright$ "A" is understood as 'all that is named by "a" can be named by "A"', too. It is read as "a is an A".[1]

Formulae with the quantifiers $\forall$ and $\exists$:

The quantifiers $\forall$ and $\exists$ bind individual variables only. Formulae with these quantifiers are interpreted in the framework of a substitutional quantification, i.e.

"$(\forall x)$ H is true" is interpreted as "every substitution instance of H is true" (For x singular names of a given language are substituted in all places where x appears in H.)

"(∃x) H is true" is interpreted as "some substitution instance of H is true" (see: Barcan Marcus 1962)

All axioms and rules of the classical quantification theory are valid in this interpretation. But this is not the case if the quantifiers ∀ and ∃ are replaced by Δ and ∇.

Formulae with the quantifiers Δ and ∇:

The quantifiers Δ and ∇ "bind" general subject term constants only. These constants function like restricted variables in quantification systems (see: Hailperin 1957).

A semantical explanation can be given in the following way:

"(ΔA) H is true" is interpreted as "H[α/A] is true for every singular or general subject term α of a given language which includes the general term A", i.e., for that "α" ▶ "A" where H[α/A] is built from H by replacing A by α.

"(∇A) H is true" is interpreted as "there is a singular subject term α which includes the term A and for which H[α/A] is true".

A system which can be interpreted in this way is that suggested by Sinoviev & Wessel (1975, p. 330).

A1: $(\forall\beta)A \vdash A$

A2: $A \vdash (\exists\beta)A$

A3: $(\forall\beta)A \wedge (\exists\beta)B \vdash (\exists\beta)(A \wedge B)$

A4: $(\forall\beta)(A \vee B) \vdash (\forall\beta)A \vee (\exists\beta)B$

A5: $(\forall\beta)A \vdash \sim(\exists\beta)\sim A$

A6: $\sim(\exists\beta)\sim A \vdash (\forall\beta)A$

A7: $(\exists\beta)(\mathfrak{E}'t') \ldots (\mathfrak{E}^n t^n)(\mathfrak{E}\beta)A \vdash (\forall\beta)(\mathfrak{E}'t') \ldots (\mathfrak{E}^n t^n)(\mathfrak{E}\beta)A$
where $\mathfrak{E}$ is either ∀ or ∃ and $n \geq 0$.

R1: $A \vdash B \Rightarrow (\forall\beta)A \vdash (\forall\beta)B$

R2: $A \vdash B \Rightarrow (\exists\beta)A \vdash (\exists\beta)B$

The quantifiers ∀ and ∃ of this system can be read in our language as ∀ and ∃ but also as Δ and ∇ respectively. But some theorems of classical quantification theory do not hold in this system. For example

(a) $(\forall x)H \vdash H[y/x]$

(b) $(\forall x)H \vdash H[a/x]$

(c) $H[a] \vdash (\exists x)H[x/a]$ are not valid in the Sinoviev & Wessel system.

But on the other hand all theorems of the Sinoviev & Wessel system are valid in classical quantification theory.

In our interpretation the formulae

(a') $(\Delta A)H \vdash H[B/A]$

(b') $(\nabla A)H \vdash H[a/A]$

(c') $H[a] \vdash (\nabla A)H[A/a]$

hold only with restrictions, i.e., if the conditions "B" $\blacktriangleright$ "A" and "a" $\blacktriangleright$ "A" are fulfilled.

Therefore let us take the following quantification schemes as an extension of the Sinoviev & Wessel system modified by replacing $\forall$ by Δ and $\exists$ by ∇:

A8: $(\Delta\beta)H \wedge$ "α" $\blacktriangleright$ "β" $\vdash (\Delta\alpha)H[\alpha/\beta]$

A9: $(\Delta\beta)H \wedge$ "α" $\blacktriangleright$ "β" $\vdash H[\alpha/\beta]$

A10: $H[\alpha] \wedge$ "α" $\blacktriangleright$ "β" $\vdash (\nabla\beta)H[\beta/\alpha]$

A11: $(\nabla\alpha)H \wedge$ "α" $\blacktriangleright$ "β" $\vdash (\nabla\beta)H[\beta/\alpha]$

The translation of natural language inferences into formal language of quantification theory becomes more adequate now. Take for instance: "From 'All men are mortal' and 'Socrates is a man' follows 'Socrates is mortal'" and "From 'All men are mortal' and 'All Greeks are men' follows 'All Greeks are mortal'". This will be translated in the following way:

(d) $(\Delta A)(A \leftarrow P) \wedge$ "a" $\blacktriangleright$ "A" $\vdash (a \leftarrow P)$

(e) $(\Delta A)(A \leftarrow P) \wedge$ "B" $\blacktriangleright$ "A" $\vdash (\Delta B)(B \leftarrow P)$

where a, A, B, and P stand for the *terms* "Socrates", "man", "Greek", and "mortal" respectively.

Both (d) and (e) are theorems in the system thus outlined. We get them by substitution in axioms (A9) and (A8).

Last I shall try to translate the reading of $\forall$ and $\exists$ into that of Δ and ∇. For this purpose let us postulate a special general subject term constant I which has the following feature: for every singular subject term constant α "α" includes "I", i.e., "α" $\blacktriangleright$ "I" is an axiom. The restriction mentioned with respect to (a') - (c') are fulfilled for I by this axiom and can be left out. Now $(\forall x)(x \leftarrow P)$ can be read as $(\Delta I)(I \leftarrow P)$. And if we replace A by I in (a) - (c) these formulae hold without restrictions. In my opinion it would be quite natural to call I "individual".

NOTE

1. Constants and variables are used in the way Frege (1879) uses Latin and German letters.
The features of the "inner negation" $\tilde{\in}$ are described in detail by Sinoviev & Wessel (1975) and Wessel (1984). The case of inner negation in quantified expressions will be left out here.

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GAME THEORETICAL SEMANTICS WITH VALUE-GAPS AND DISCOURSE ANALYSIS

The logical and linguistic researches of the seventies and the eighties show that in the fields of both grammatics and logic we have left behind the notion that the sentence is the unit up to which analysis can go. The analysis of units greater than the sentence, that is, of discourses (texts), with the most varied of purposes and approaches is usually (but not solely) carried out within the so-called dynamic semantic or pragmatic frame theories. The question common to these analysis may perhaps be put thus: when do we say that a discourse is coherent? How can we show (demonstrate) the rule-following manner in which an utterance within a discourse follows or follows from another? What, therefore, are the rules of a well constructed discourse? From among the various possible answers - like Hans Kamp's discourse representation theory or the situation semantics of Barwise and Perry - in this paper I shall sketch a modified version of the so called dialogue-game theory. The basic idea for this comes from the notion of L. Carlson according to which the criterion for a discourse's coherence is that it can be embedded into a well-formed dialogue-game. This dialogue-game concept is a sort of generalization on the conception of game theoretical semantics.[1]

To expound the conception of dialogue-game chosen to be the framework for discourse analysis, it is necessary to examine the following questions:

1) The basic ideas of game theoretical semantics.
2) The rule-system of Hintikka's dialogue-games.
   The role of interrogative models in the description of dialogues.
3) The possibility and advantages of uniting a value-gap semantical approach and Hintikka's results.
4) Suggestions for new dialogue-game rules within the value-gap interrogative frame-theory.

1. The Basic Ideas of Game Theoretical Semantics [2]

Jaakko Hintikka began the elaboration of game theoretical semantics (GTS hereon) from the beginning of the seventies. The introduction of GTS was primarily motivated by dissatisfaction with the existing logical semantics. The two main cases wherein the Tarski-type truth condition semantics proved to be insufficient are the following:

(i.) When context-independence cannot be ensured. (One of the greatest virtues of the Hintikka-school is that for them, the representation of context-dependence is the paradigm rather than the exception.)

(ii.) When the lack of atomic sentences makes the recursive definition of complex expressions impossible. (The mathematically relevant question of infinitely deep languages.) This application plays no role in the reconstruction of natural languages. (For the literature on this, see [4].)

The objective is to elaborate a theory of meaning which is not solely based on a static truth-condition conception. The philosophical groundwork for this is provided by Kant's transcendental argument and Wittgenstein's concept of "language-game". From the former the emphasis is on the Kantian idea that our conception of existence must be based on the **human activity** whereby we come to know the existence of individuals. The true basis for the logic of existence and universality (which is realized in quantification theory) lies in the human activities of **seeking** and **finding**. It is for this that Hintikka borrows Wittgenstein's term of a "language-game". (A language-game is a meaning-constitutive activity.) Game theoretical semantics expounds the rule-governed activities of seeking and finding for the purpose of reaching relevant propositions. The conceptual framework for this process of verification is the **game** against a "malicious nature" bent of preventing this experiment. Thus, it is a two-person game between Myself and Nature.

The applicability and reliability of the theory is based on mathematical game-theory and especially on the ideas of Johann von Neumann.

Now, in order to understand the basic ideas the simplest procedure is to apply some rules of GTS to an interpreted first-order language. Then all atomic sentences of the language are either true or false, determined in the usual manner by the interpretation.

In GTS, the definition of the truth-conditions for complex sentences occurs in two-person games. Hintikka calls the players "Myself" and "Nature". These are zero-sum semantical games. To each sentence **S** of a first-order language we attach a game **G(S)**. The game goes on according to definite rules depending on the structure of **S**.

With the help of the rules we try to verify the sentence on the side of M (Myself) and falsify it on the side of N (Nature). This naturally yields the following rules:

G(A) If **A** is an atomic sentence, M wins **G(A)** and N looses if **A** is true in model μ. If **A** is false in μ, N wins **G(A)** and M looses.

G(&) **G(S_1&S_2)**: N chooses between the members S_1 and S_2. If N chose S_1 the game continues with **G(S_1)**.

G(∨) The rule is the same, but the right of choice goes to M.

G(~) In **G(~S)** the players play **G(S)** with inversed roles.

G(∃) **G(∃x.Sx)** begins with the choice of M. M chooses an individual from the range **D(μ)**, denoted by **c**, and the game continues with **G(Sc)**.

G(∀) **G(∀x.Sx)** is the same as before, but the right of choice belongs to N.

(We don't necessarily have to accept Hintikka's terminology. It is customary to call the players of a two-person game "proponent" and "opponent". This terminology is more exact, too, for it may happen during the course of a game that the players change roles: the previous opponent - Nature - actually becomes the proponent; the same goes for Myself.)

The game ends when, after a finite number of moves the players reach an atomic sentence. Then **G(A)** tells who won and who lost.

We can extend the rules over modal operators using possible-world semantics. We tread the "□" operator analogously with the universal, and the "◇" operator analogously with the existential quantifier.

In the case of a **G(□)** game N chooses an alternative world to **G(□A)** and the game continues in this world with **G(A)**.

G(◇) is the same, but the right of the choice of world goes to M.

That is, the game is not carried on within a single model, but within a totality of models on which the adequate alternative-relation is defined.

Once **G(S)** is defined, the concept of truth can also be defined game theoretically as follows:

> A sentence **S** is true under the given interpretation if and only if M has a winning strategy in the attached **G(S)**, and **S** is false if N has a winning strategy.

As to the concept of **strategy**: a strategy is a rule (a function) which tells what move a player should make in each possible situation during the game. Once the strategy of the players is determined, the whole course and result of the game is also determined.

A **winning strategy** is one which leads the player to winning the game regardless of how the other one plays.

From the above said it can be seen how close a connection there is between Tarski-type semantics and GTS's definition of truth. First of all, both are based on the fact that the truth values of atomic sentences are given. Secondly, in the case of non-atomic sentences an adequation can be set up between the Tarski-type rules of evaluation and the rules for the moves of GTS. That is, we may state that GTS is also a truth conditional semantics.

(For logicians GTS is hardly a novelty. It is merely a different systematization and generalization of such ideas as the Skolem-function, the Henkin-quantifiers, the interpretation of the Gödel-function, etc. Rather, the novelty is in its application to natural languages; e.g. in the analysis of "any", the treatment of anaphors and that of non-standard quantifier-words, the analysis of "if-then" words, etc.)

The **difference** between Tarski- and Hintikka-type theories consists in that while in the former case truth-condition definitions work from the inside out, the definitions of GTS work from the outside in. (This means that Tarski builds from atomic sentences as if it were possible to construct the truth of complex sentences from that of atomic ones. As opposed to this GTS reduces in a definite manner the truth of complex sentences to that of the simpler ones. Following Barbara Partee, this difference is often expressed by the terms "bottom up", "top down".)

This difference has serious consequences as concerns "tactics" and provides advantages for GTS in certain complicated cases. The advantage appears especially in that GTS is not committed to the principle of compositionality, it does not presuppose that principle the way the Tarski-type theory based on Frege does. This liberty towards the principle of compositionality renders possible the analysis of numerous phenomena of natural language as for example the already mentioned treatment of anaphors, the treatment of branching quantifiers and generally that of context-dependence (due to the possibility of step-by-step evaluation) etc.

Such tools of GTS as the ordering principles, which substitute for the marking out of the range of quantification or the idea of **subgame**, according to which the player can continue his second game depending on the strategy he used in the first, make the **unified treatment of sentence-semantics and discourse-semantics** possible. Among others, such a unified theory is the theory of dialogue games.

2. Hintikka's Theory of Dialogue-Games and the Basic Ideas of the Interrogative Model

Hintikka's dialogue-game conception develops organically from his game theoretical semantics.[3] For the system of rules and the arsenal of GTS naturally lend themselves to the examination of such questions as: how can we attain new information? This goal can be realized within the information-seeking dialogue-game. Hintikka's dialogue is basically one of questions and answers. Maintaining the spirit of Socratic dialogues the players endeavour to verify their own theses by using the premises obtained from the other player by means of questions.[4]

Naturally, the player also has to defend the statements he made in response to his partner's questions. Within a dialogue game each player can play the role of both interrogator and interrogated.

The interpretation of the concept of "dialogue":

1) There are two players. Let us call them White and Black (or any which way we like).
2) The following moves can be discerned:[5]
   - opening move,
   - deductive move,
   - interrogative move,
   - assertoric move,
   - definitory move.

Technically, the dialogue is conducted by the players' constructing so-called semantic tables. This operates according to the principles of Beth's construction.

Both players construct a Beth-table. In the opening moves each gives his own thesis with a single formula. The player writes his own thesis in the right hand column of the opening table and that of his opponent in the left hand column. (It is a matter of agreement, depending on the nature of the dialogue, which player should make the opening move.) The first move usually belongs to the "white" player. In the case of each move each player can decide upon the kind of move (deductive, assertoric, etc.) he will make in accordance, of course, with the rules for the given type of move.

These rules are:

Rules for the deductive move: essentially these are identical with the construction rules for Beth-tables, being their finite applications. The (finite) number and the order of the application of the rules is arbitrary.

The rule for the interrogative move: the treatment of questions in dialogue game presupposes Hintikka's analysis of questions and his interrogative model conception.[6]

The basic idea of the interrogative model - the form of which is the two-person game - is very simple. The seeker (interrogator) tries to draw a conclusion **C** from the premisses **T**. As opposed to the purely deductive process the difference is that the seeker puts questions (which the author calls "oracles" in this case) to his partner and uses the answers obtained as additional premisses.

In Hintikka's interrogative model the interrogative components are attached to a given first-order language. A model μ is given to the language in the usual way. We understand the premisses **T** as being true in μ.

Hintikka distinguishes between two kinds of question: the propositional question and the wh-question. The adequate presuppositions for both types of question must also be formulated.

The form of a propositional question is: "X or Y?" (where **X** and **Y** are sentence-variables). The presupposition of a propositional question is $S_1 \vee S_2 \vee \ldots \vee S_n$. A special case of the propositional question is: "(we ask if) A?". The presupposition of this is $A \vee \sim A$. (Later we shall see the problems of the dialogue-game rule related to this type of question and the special role of the premiss "$A \vee \sim A$" within the interrogative model.)

The form of a wh-question is: "Who, what, etc. possesses a property **A**?" E.g.: "Who lives in this house?" The presupposition of this question is: "There is someone who lives in this house." Generally: the presupposition of a wh-question is: $\exists x.S(x)$. And the possible answer is: $S(b)$. (In our example: "Peter lives in this house.") [7]

The game rules related to the two kinds of question are:

Only such questions may be asked, whose presuppositions occur on the interrogating player's table among the premisses. The answer, on the other hand, must connect up with the premisses of the answering player. An important rule is that, previously to the interrogative move, the player about to make that move must give the subtable pair T_1, T_2 to which he refers his questions T_1 belonging to the questioning, T_2 to the answering player.

There are two possible cases: 1. The answer given to the question is inscribed into the corresponding column of the corresponding table, or 2. The player declines to answer the question because he does not know the answer (or because of strategic considerations). In this case, according to Hintikka's rule, the **negation of the question's presupposition** must be

attached to the answering player's premisses. Now, this is a drastic stipulation against the answering player, for he can thus easily be forced into contradiction.

Let us, for example, take the following situation. Suppose $\forall x.A(x)$ is true in μ. How could the seeking, questioning player prove this? He takes the extra premiss: $\forall x.A(x) \vee \exists x.\sim A(x)$. The following trick can be used. One half of the disjunctive subtable (i.e. $\forall x.A(x)$) can be grounded. As to the other half, the questioner asks: which individual **x** satisfies $\exists x.\sim A(x)$? Let us suppose the partner's answer is **b**. Then the questioner can close the table by asking "$A(b) \vee \sim A(b)$?" As $\forall x.A(x)$ is true in μ, the partner answers $A(b)$. This contradicts his former answer.[8] If, on the other hand, the partner replies "no answer" to the above question, then the negation of the question's presupposition must be attached to his thesis, and this is an explicit contradiction: $A(b) \& \sim A(b)$.

As Walton pointed out, the above rule clears the way for an aggressive questioner. The game is "burdened" with the questioner's "goodwill", for the rejection (ignorance) of the answer forces the player into making false statements if he has to deny **A**'s presupposition when it happens to be true. This stipulation causes trouble not only in the case of the special sentences of the form "$A \vee \sim A$", but also in the case of an ordinary "$A \vee B$" question. If, for example, we were to ask: "Is the crossing sign painted on the road white or grey?", it would be a natural requirement to be able to answer "I don't know" or "It is neither white, nor grey". The two answers are certainly not equivalent, yet, according to Hintikka's rule, "no answer" to the above question is identical with accepting "neither **A** nor **B**". The question is, therefore, whether the rule giving rise to the anomaly has any positive role within the interrogative model?

One of the important features of the interrogative model is that at the outset of the game there are two kinds of formula sets on the table of the beginning player with the codes **T** and **RA**. **T** contains the theoretical premisses while the elements of the set **RA** are of the form $S_i \vee \sim S_i$ on the side of the questioner. Their role consists in making it possible to enter new individuals into the game, to be able to ask new questions.[9]

The significance of this consideration is beyond doubt, but the objections arising from our natural linguistic behaviour against the maintenance of the rules of table construction are also beyond doubt. Therefore, the search for alternatives to the Hintikka-rule cannot be dismissed.

Before making a suggestion for the alternative solution, however, another problem needs to be answered, namely: what should the rules be for **winning** and **loosing** in the case of a dialogue-game?

From the purpose of the dialogue-game, from the fact that it is a search for information, it follows that each player endeavours to reach the end of the game, to close his table, first. The criterion for closing the table is formulated within the above mentioned rules for table construction.

On the other hand, it follows from the character of the dialogue-game that it is impossible to determine who wins or who looses the game merely from the final state.[10] For in the information seeking dialogue the question is not "who has the last word".

If, for example, a player has reached his goal and managed to close his table, his opponent may still continue the game. If, during the course of this, he asks another question, he might, thereby, force his partner to join the dialogue again.

It is part of the "information-seeking" nature of the dialogue game that the goals of the two players do not necessarily oppose each other. The dialogue-game does not operate with perfect information and is not a zero-sum game. It is possible that both players have faith in their respective theses and these theses are compatible with one another. When, however, the players wish to refute each other's theses - a situation which Hintikka calls a **dispute** - the refutation occurs during the **course of the game** and not at its outset (e.g. when one of the players denies the other's presupposition).

After constructing the semantic table revealing the dialogue's structure, the following can be said: (1.) **The player who is able to verify his thesis first wins the game.** (If he closes his table while his opponent fails to do so.) (2.) If a player cannot give a full answer to a question, he looses and the other one wins. (For the presupposition of the question is inscribed into the left hand side of the questioner's table and together with the negation of the presupposition - which is also an answer! - a contradiction arises.)

As to strategic considerations, we can say the following:

The best strategy for the player to make the opening move is to agree with his partner's statements if they are true and disagree with them (in case of a dispute) if their truth value is the opposite. If the players are, for example, scientists, this attitude (i.e. that of scientific objectivity) is not only a "moral" imperative, but a strategic one too.

The above manner of choosing the best strategy can be made clear with the help of the concept of interpretation. As Hintikka's dialogues are not merely "agreement-seeking" but truth-seeking dialogues, the definition of the truth condition of statements cannot be avoided. GTS always presupposes an interpreted language. Let us take the set of all the legitimate interpretations of a given language. At each move of the game the set of those interpretations (possible worlds) in which the formulae of a subtable's left hand column are all true while those of its right hand column are all

false is determined for both players. When the case is that the common part of the corresponding sets of the pair of tables is not empty and contains the specified world, these sets serve to explain why telling the truth is the best strategy.

3. The Possibility of Joining GTS and the Value-Gap Conception

3.1. Why does the necessity of introducing a semantic value-gap into GTS arise?

How does GTS behave in the case of sentences neither true nor false?

Why should have a winning strategy to sentences of the "bridge is ruminating" type?

Is it possible to treat "denial of information" in a manner different from the one we have seen in the preceding section?

I shall try to answer these questions in the present section.

First, therefore, a word on motivation: Taking into account of a semantic value-gap is motivated by two distinct (though convergent) considerations:

(i.) On sentence-level everything that motivates partiality. Usually "logical omniscience" and the equivalence of logically true propositions (i.e. that the meaning of all logically true propositions is the same) are mentioned as the pitfalls of possible-world semantics. Let us, for example take the following two formulae and propositions:

(a) **Pab** $\vee \sim$ **Pab**
(b) **Pba** $\vee \sim$ **Pba**
(c) Peter loves Budapest or doesn't love Budapest.
(d) Budapest loves Peter or doesn't love Peter.

Here (a), (b) and (c), (d) are logically equivalent to each other. If we wish to express this in game theoretical terms, we may say that whichever interpretation we choose for the given language, M has a winning strategy. The causes mentioned here supply arguments in favour of introducing the value-gap at the static, sentence level.

(ii.) The value-gap sources of dialogue games show new characteristics due to the nature of the games. Within the information seeking dialogue the answer provides the questioner with new information, though not always. The term playing the central role in game theoretical semantics is "information set". While game theoretical semantics operates with perfect information (both players know the situation at each move), in dialogue games one has to

deal with imperfect information. It may happen that a player does not know (or withholds) the answer to a certain question. It seems clear, that the true-false dichotomy is not the solution in this case: the falsehood of the answer or ignorance of it are two different states. Thus, in the case of dialogues, partiality is in a way a natural requirement.

It is not indifferent, however, which motivation we stress. The latter one - which treats lack of information as the source for the value-gap - and its implications may differ considerably from the truth-value semantical approach. E.g., we may also say, that the value-gap arising due to lack of information can be modelled by the impossibility to complete the game. Later on we shall see that a different approach is also possible.

Let us now turn back to the possible connection between sentence-level partial semantics and GTS.

3.2. We can briefly sum up the value-gap conception as follows. If a term **t** denotes an object **u** which is not an element of some **w** possible world's universe of quantification, then we may say that **t** has no factual value in **w**. We accept that in extensional contexts the value-gap is passed on from the functor's input to its output. We allow the values of extensional functors to be partial functions (such functions, that is, which are not defined for certain objects.)

Generally: we speak of a semantic value-gap when a well-formed expression (whatever syntactic type it belongs to) has no extension in some possible world.

By the introduction of the semantic value-gap we extend classical first-order logic as follows:[11]

Grammar: We add the symbols **I** and **NON** to the set of logical constants. The new syntactic rules are:

If **x** is a variable and **A** is a formula, then **I x.A** is a term.

If **A** is a formula, **NON(A)** is also a formula.

Semantics: Under an interpretation of the extended first-order language we customarily understand a pair **(U,ρ)**, where **U ≠ 0** and the function **ρ** assigns factual values to the language's non-logical constants.

It is reasonable to denote the lack of semantic value with "Θ" in the case of terms and with "2" in the case of sentences. We have also allowed partial predicates, which means, that for example in the case of one-argument predicates the predicates interpretation divides the universe of discourse into three rather than just two parts. The third part contains those individuals on which the predicate is not defined. In the case of many-argument predicates the same goes for U^n.

The rules for factual values: (By $|A|$ we denote the factual value of the expression **A** according to the **v** evaluation of the variables.)

(i) If **x** is a variable, then $|x|_v = v(x)$;

(ii) If **c** is a constant term then $|c|_v = \rho(c) \in U$;

(iii) If **p** is a constant sentence-parameter, then $|p|_v = \rho(p) \in \{0,1,2\}$;

(iv) If **p** is an n-argument predicate constant and $t_1,\ldots,t_n$ are terms, then

$$|Pt_1\ldots t_n|_v = \begin{cases} 2, \text{ if one of } |t_1|_v,\ldots,|t_n|_v \text{ is } \Theta \\ 1, \text{ if } |t_1|_v,\ldots,|t_n|_v \in \rho(p) \\ 0 \text{ otherwise.} \end{cases}$$

(v) If t_1 and t_2 are terms, then

$$|(t_1=t_2)|_v = \begin{cases} 2, \text{ if } |t_1|_v \text{ or } |t_2|_v = \Theta \\ 1, \text{ if } |t_1|_v = |t_2|_v \neq \Theta \\ 0 \text{ otherwise.} \end{cases}$$

(vi) If **A** is a formula, **x** is a variable and there is only one such $u_o \in U$ that in the case of $|x|_v = u_o$, $|A|_v = 1$, then $|\mathbf{I}\,x.A|_v = u_o$. In all other cases $|\mathbf{I}\,x.A|_v = \Theta$.

(vii) If **A** is a formula, then

$$|\mathbf{NON}(A)|_v = \begin{cases} 0, \text{ if } |A|_v = 1 \\ 1 \text{ otherwise (that is, if } |A|_v = 0 \text{ or } 1 \end{cases}$$

$$|{\sim}A|_v = \begin{cases} 2, \text{ if } |A|_v = 2 \\ 0, \text{ if } |A|_v = 1 \\ 1 \text{ otherwise} \end{cases}$$

(viii) If **A**, **B** are formulas, then

$$|A \supset B|_v = \begin{cases} 2, \text{ if } |A|_v \text{ or } |B|_v = 2, \\ 0, \text{ if } |A|_v = 1 \text{ and } |B|_v = 0, \\ 1 \text{ otherwise.} \end{cases}$$

(ix) If **A** is a formula and **x** is a variable, then

$$|\forall x.A|_v = \begin{cases} 2, \text{ if } |A|_v = 2 \text{ for all values of } x \\ 1, \text{ if } |A|_v = 0 \text{ for at least one value of } x, \\ 1 \text{ otherwise} \end{cases}$$

We can see, that with the exception of **NON** the rules ensure the inheritance of the value-gap. The introduction of the operator **NON** requires some explanation. What makes the introduction of this new logical constant necessary is the need to be able to express statements of such form as "it is not true, that **p**". What we mean by such an expression cannot be expressed with the usual negation ("~"). To see an example: suppose that Peter has no

brother. In this case the statement "Peter's brother plays sports" has no true value, therefore

$$|\sim \text{(Peter's brother plays sports)}|_v = 2,$$

but

$$|\mathbf{NON}\text{(Peter's brother plays sports)}| = 1$$

and

$$|\mathbf{NON}\text{(Peter's brother does not play sports)}| = 1.$$

We can introduce the following contextual definition for "it is true, that **p**":

$$\mathbf{Ver(p)} =_{df} \sim \mathbf{NON(p)}.$$

Furthermore: $\mathbf{NON(p)} \,\&\, \mathbf{NON}(\sim \mathbf{p})$ express that **p** is without truth value, that is $|p|=2$.

In addition to the usual central semantic concepts we must introduce that of **irrefutability**. We call a formula **A** irrefutable if **A** does not take the value false in any one interpretation. Of course, this does not mean any more that it is always true (as it can be without truth-value gap). That is, within the value-gap theory "valid" and "irrefutable" do not coincide.

3.3 Let us now look at the modification of the GTS framework. Let the interpreted language be a Ruzsa-style first-order interpreted language with value-gap.

Let us extend the concept of two-person zero-sum game by adding to the outputs 0,1, the output represented by 2. We modify the rules of winning and loosing, in fact, generally we modify their concepts. For according to the **G(T)** rule: **S** is true if Myself has a winning strategy and false if Nature has one to the **G(S)** in question. This rule is not sufficient for judging sentences neither true nor false. My suggestion is the following: let us introduce the concepts of **strong** and **weak winning strategy**.

Let the **strong winning strategy** work on functors not inheriting the value-gap (e.g. the **NON** functor and the intensional functors) on the one hand and in those cases when at the end of the game we get either true or false sentences (i.e. the sentence reached has no value-gap) on the other hand.

On Myself's part let the **weak winning strategy** mean that whatever moves Nature makes, the atomic sentence produced at the end of the game is **not false** (it is either true or has a value-gap) if it is defended by Myself, and **not true** if Nature is defending it.

The taking into account of the semantic value-gap modifies the game rule for universal quantification. The defence of a formula of the form "$\forall x.A(x)$" requires demonstrating that **A(x)** is not false under any evaluation of the variable **x** and true under at least one evaluation of **x**. Consequently the fair continuation of the game is with the formula "$\mathbf{A(c)} \,\&\, \mathbf{NON}(\sim \mathbf{A(b)})$", where the proponent chooses the individual called **c** and the opponent the one called **b**. According to the game rule for conjunction it is now the opponent

who chooses which of the two members of the conjunction the proponent has to defend. (The game rule for existential quantification remains unaltered.)

As we can notice, here the functor **NON** can also occur in the semantic game, therefore we now have to deal with the according game rules as well. If **A** is an atomic formula then the values of **NON(A)** and **NON(~A)** are directly determined by the interpretation. If **A** is a complex formula then there is a formula **B** which does not begin with a negation-sign and **~A ⇔ B**; so **NON(~A)** can be reduced to **NON(B)**. Further rules of reduction:

NON(A&B) ⇒ NON(A) ∨ NON(B)

NON(A∨B) ⇒ (NON(A) & NON(B)) ∨ (A & NON(B∨~B)) ∨ (B & NON(A∨~A))

NON(A∨~A) ⇒ NON(A) & NON(~A)

NON(∀x.A(x)) ⇒ ~A(c) ∨ NON(A(b)∨~A(b)); where the proponent chooses **c** and the opponent **b**,

NON(∃x.A(x)) ⇒ NON(A(a)); the opponent chooses **a**

NON(NON(A)) ⇒ ~NON(A).

If we employ descriptions also as terms, we take these into account within the atomic formulae. Let **P** be an atomic predicate (one of) whose argument(s) is a description of the form "**Ix.A(x)**". The game rule for the formula of the form "**P(Ix.A(x))**" is as follows. The proponent chooses an individual called **c**, the opponent one called **b** and the game is continued with the conjunction "**P(c) & A(c) & (A(b) ⊃ b=c)**". Thus in the next move it is the opponent who decides which member of this three-member conjunction he requires the proponent to defend.

As the adequation between the Tarski-type truth conditional semantic and the original GTS can be shown, so can it be shown between the rules of value-gap semantics and those of the modified (value-gap) GTS.

Overall, we may say about the truth condition of a sentence **S** (in the given interpretation of a given language):

The value of **S** is 1, if the proponent has a strong winning strategy.

The value of **S** is 0 if the opponent has a strong winning strategy.

Otherwise the value of **S** is 2, that is, when the proponent of the sentence in question has a weak winning strategy.

4. Dialogue with a Value-Gap

4.1. The application (extension) of value-gap GTS to dialogue-games or dialogue-analyses: my suggestions for the modification of the dialogue game rules are as follows.

Within the value-gap interrogative model we allow that the sentences, and thus the presuppositions of questions have no truth values.

We have seen the drastic consequence: if the opponent declines to answer the given question he is forced to accept the negation of the presupposition and so may have to lie or fall into contradiction.

The new rule for the interrogative move: if the opponent declines to answer the given question, let the negation of the questions presupposition have the value 2. This entails that according to the rules for the evaluation of negation, the presupposition itself also has the value 2.

This rule does not burden the game with the goodwill of the questioner, that is, does not clear the way before an aggressive questioner, for he cannot reach his goal by vileful questions (will not win). For example, in the case of the question "**A?**" "no answer" means that the presupposition "$A \vee \sim A$" has a value gap and therefore so does its negation, that is: $|A \& \sim A| = 2$.

The new rule for declining to answer strengthens the dynamic character of the game since a later move influences the qualification of an earlier one. That is, **the value of the presupposition stipulated by the questioner must be made answer-dependent**. (Naturally, if we were to use the two-value "**NON**" functor for the negation of the presupposition, Hintikka's drastic solution we have criticised would be restored. However, there is nothing to force us to take this step when constructing our system of rules.)

At the same time, by understanding classic tautologies as **irrefutable**, the system leaves open the possibility for the questioner to enter new individuals into the game with the help of the additional premisses of the form $S_i \vee \sim S_i$ and raise new questions.[12]

That much is surely clear from the aforesaid, that complementing Hintikka-style interrogative models with a Ruzsa-style value-gap theory provides finer tools for the treatment of dialogues and the elimination of the problems mentioned.

Up to now we have analysed the possibility of linking GTS rules with the static kind of value-gap truth rules. That is, it was based on taking into account those sources of value-gaps which appear on sentence-level and either contaminate or do not contaminate the whole game (dialogue). We have treated the case of lack of information also within this static value-gap semantics.

4.2. We cannot within the framework of this paper undertake to examine the relation between the value-gap (dialogue) game-rules and a dynamic system of rules which presupposes a dynamic semantics. The most we can say is that the solution lies in the direction where "lack of information" is taken into account as a value-gap source. Dynamic treatment would mean that, for example, the following questions would have to be answered.

How some information which was acquired at a later stage - and appeared at an earlier move as lack of information perhaps - can modify the course and end of the dialogue. In other words, how can a later move influence the earlier game. (We have referred to this in connection with a single game-rule.) The answer for the question would probably have to be connected up with the basic idea elaborated within the framework of discourse representation: "blocking".[13] This would consist in introducing a rule according to which in the case of lack of information the given player's given table should block but without contaminating the whole course of the game and furthermore, it should be possible to return to the block when acquiring information as a later move.

The need for the representation of context-change is not alien from the goals and possibilities of GTS. Examples of analysing context-changes have been given by V. Rantala's urn-models (which are partial models).[14]

The treatment of context-dependence is among the basic aims of GTS and the problems arising from it, such as the game-rules for anaphors can indeed be treated well with the help of GTS tools.[15]

However, the problem of partially ordered quantifiers also belongs here[16], which can be treated in a relatively natural way with the help of game theoretical tools. The fact that the problems of context-dependence are or can be solved within the framework of GTS is not surprising as this was precisely what it set out to do.

Yet, the tools of GTS have been worked out by the Hintikka-school against a background of truth-value semantics which does not allow for value-gaps.

It follows from our arguments here, however, that the analysis of questions and, relatedly, that of dialogues makes it essential to allow partiality, and not only as it is **ab ovo** present within GTS, but rather the way we have tried, that is trough re-thinking the rules of evaluation of sentences and the concept of strategy and determining the implications of these. The elaboration of further particulars now is not a theoretical, but a practical problem.

On the other hand, the problem of the dynamic treatment of lack of information requires further theoretical examination and elucidation, which again cannot be carried out within this paper. Yet, I believe that the combination of GTS with the value-gap semantical theory might contribute to the solution of this problem too.

Appendix: Partiality and Monotonicity

The value-gap semantics examined in 3.2 (which originates from Imre Ruzsa) is of an ontological nature: the lack of semantic value is caused by the fact that a name has no actual meaning or that a function is not defined for one of its possible arguments. A conception more common within foreign literature is that the truth value-gap is caused by lack of information. We evaluate a sentence to 2 if we are ignorant of its actual truth value. The value-gap may be extinguished by an increase in information. This epistemic-type conception of value-gap does not accept the general law of the value-gap's inheritance (within extensional contexts). If, for example, one of the members of a conjunction is false, then according to this conception the conjunction is false even if the other member has the value 2 - for regardless of whether the member with the unknown truth value would prove to be true or false, the conjunction is sure to be false as one of its members is known to be so.

Here the so called principle of **monotonicity** steps into the place of the general law of the value-gap's inheritance. This states that if the value of a function's some argument changes from indefinite (2) to definite (1 or 0) then the function-value can also only change in this direction. If we understand the value 2 as "lower" than the values 0 or 1, then the principle of monotonicity can briefly be expressed thus: **the function-value cannot decrease with the increase of the argument-value.** The principle of monotonicity is also extended to range over quantification: if some **F(a)** value grows then the values of **∀x.F(x)** and **∃x.F(x)** cannot decrease either[17] the matrices of monotonous conjunction and alternation are the following:

&	1	0	2
1	1	0	2
0	0	0	0
2	2	0	2

v	1	0	2
1	1	1	1
0	1	0	2
2	1	2	2

The matrices of negation (**~**) and **NON** remain unchanged:

	~	NON
1	0	0
0	1	1
2	2	1

If we understand the value 0 to be lower than the value 1, we see that **NON** is not monotonous: if the argument value changes from 2 to 1, the function value changes from 1 to 0. We can define the following value-gapless functions with the help of **NON**:

$\mathbf{Ver(A)} =_{df} \sim \mathbf{NON(A)}$;

$\mathbf{Deg(A)} =_{df} \mathbf{NON(A)} \,\&\, \mathbf{NON}(\sim \mathbf{A})$;

$\mathbf{Stat(A)} =_{df} \sim \mathbf{Deg(A)}$.

The matrices of these are:

A	Ver(A)	Deg(A)	Stat(A)
1	1	0	1
0	0	0	1
2	0	1	0

The rule for calculating the value of monotonous universal quantification:

$$|\forall x.A(x)| = \begin{cases} 1, & \text{if } |A(c)|=1 \text{ for all c's} \\ 0, & \text{if } |A(c)|=0 \text{ for at least one c} \\ 2 & \text{otherwise.} \end{cases}$$

We define the existential quantifier in the usual way, so

$$|\exists x.A(x)| = \begin{cases} 1, & \text{if } |A(c)|=1 \text{ for at least one c} \\ 0, & \text{if } |A(c)|=0 \text{ for all c's} \\ 2 & \text{otherwise.} \end{cases}$$

With the help of monotonous functions and **NON** a number of non-monotonous functions can also be defined. Eg., $\mathbf{NON(A)} \vee \mathbf{B}$ gives a special non-monotonous conditional. whose matrice is as follows:

⇒	1	0	2
1	1	0	2
0	1	1	1
2	1	1	1

As Johan van Benthem has shown[18], Ruzsa-type universal quantification ($\forall_R$) can also be defined with the help of monotonous quantification and the **NON** functor. The rule for calculating Ruzsa-type quantification, as a reminder:

$$|\forall_R x.A(x)| = \begin{cases} 0, & \text{if } |A(c)|=0 \text{ for at least one c} \\ 2, & \text{if } |A(c)|=2 \text{ for all c's} \\ 1 & \text{otherwise.} \end{cases}$$

Expressing $\forall_R$ with monotonous quantification:

$$\forall_R x.A(x) \Leftrightarrow \forall x(Stat(A(x)) \supset Ver(A(x))) \,\&\, \exists x.A(x).$$

To prove their equivalence it is enough to show that the two formulas take the value 1 and the value 0 in the same cases.

Let us suppose that the value of the right hand formula is 1. Then, on the one hand, $|\exists x.A(x)|=1$, therefore for some c: $|A(c)|=1$. Furthermore $|\forall x.(Stat(A(x)) \supset Ver(A(x)))|=1$, therefore in all cases when $|A(c)|\#2$ it holds that $|A(c)|=1$. The text in bold type shows that it is in this case and only in this case that the value of the left hand formula is also 1.

Now let us suppose that the value of the right hand formula is 0. In this case, either $|\exists x.A(x)|=0$, that is, $|A(c)|=0$ for all c's or $|\forall x.(Stat(A(x)) \supset Ver(A(x)))|=0$, that is, for some c $|Stat(A(c))|=1$ and $|Ver(A(c))|=0$, that is, for some c $|A(c)|\#2$ and $|A(c)|\#1$, i.e.. for some c $|A(c)|=0$. Both cases are sufficient for the left hand formula of the equation to have the value 0.

However, the monotonous quantifier can also be expressed with the Ruzsa-type quantifier and other operations. Beside the **NON** functor, the "1" sentence function too has an important role in the expression. Its matrice is:

A	⊥A
0	0
1	2
2	2

Within Ruzsa-type value-gap logics this can be defined - among else - with the help of the descriptor:

$$\perp A =_{df} \sim \exists_R y\ (Ix\ (x=y\ \&\ \sim A) = y)$$

The expression of monotonous quantification in Ruzsa-type value-gap logic is as follows:

$$\forall x.A(x) \Leftrightarrow \forall_R x.Ver(A(x))\ \&\ \sim\perp\ (\exists_R x.Deg(A(x))\ \&\ \forall_R x.A(x))$$

In classical logic, in case of a finite quantificational range, universal quantification can be expressed with a conjunction:

$$\forall x.A(x) \Leftrightarrow A(c_1)\ \&\ \ldots\ \&\ A(c_n).$$

This same relation holds in the case of value-gap monotonous quantification, but is not fulfilled for Ruzsa-type value-gap quantification. (The reason Ruzsa gives for the difference is that for him universal quantification is not a "generalised conjunction" but an extensional operator which forms sentences from one-argument predicates. According to the general rule about the value-gap, the result of the operation can be a value-gap if and only if the predicate acting the role of the operation's argument has no factual value, that is, yields a sentence without truth value if used over any argument.)

In my opinion, this phenomenon supports giving advantage to monotonous quantification. In game theoretical semantics and especially in dialogue-games the employment of monotonous quantification also seems more advantageous than that of the Ruzsa-type value-gap quantification. In the case of monotonous quantification the rule for conjunction and the universal quantifier can be formulated uniformly: Nature wins, if there can be found a sentence with which the game can end only with a false atomic sentence; Myself has a strong winning strategy if in the case of choosing any member sentence the game ends with a true atomic sentence and has a week winning strategy (does not loose) if choosing any member-sentence the game cannot end with a false atomic sentence.

Finally, we have to note that the fact, that each of the two kinds of value-gap quantification can be expressed with the other one (plus other operations) does not degrade their difference to a purely technical level. The point is a different conception of the semantic value-gap and therefore of the word **all**. The difference in sense still remains even if we are able to express the one - with the help of thechnical implements - with the other.

NOTES

1. See L.Carlson [2].
2. For details on this, see [4], [14], [15].
3. See [3], [7], [9].
4. See Walton [21].
5. See Hintikka [7].
6. The basic works on the analysis of questions are: Hintikka [6] and [8]. Ferenc Kiefer has made critical remarks on the Hintikka-type analysis of questions, partly concerning simplified classification and partly emphasizing the relevance of pragmatical approaches as opposed to semantics. See [12].
7. See e.g. [8]. Before giving the presupposition of the question, first the epistemic nature of the question-answer relation must be elucidated. To take Hintikka's example for wh-questions:

   Who killed Julius Caesar?

   This question requires the fulfillment of the following epistemic command:

   Bring to the surface that "I know, who killed J.C.".

   The state which the questioner desires the answerer to bring to the surface is called the question's **desiderate**. The desiderate of the above question is:

I know who killed Julius Caesar.

If **K** denotes the operator **knows** and **a** denotes **Myself**, the logical structure of this is:

x.Ka (x killed Julius Caesar).

If we leave the epistemic operator out of the desiderate we get the question's presupposition:

x. (x killed Julius Caesar).

Further, see Hintikka [6].

8. Further see Hintikka [8].

9. *Ibid.*

10. See [7].

11. The logic with value-gap set forth here is the work of Imre Ruzsa.

12. Indeed the fact that "$S_i \vee S_i$" is irrefutable is not enough to be able to enter new individuals. Yet any arbitrary new individual name **a** can be introduced by taking the formula "**Ver(a=a)**".

13. See H.Kamp [10], Polos [13], Kálmán [11].

14. See Rantala [19], Madarászné [14].

15. Hintikka & Kulas [5].

16. See [4], [14].

17. See [1].

18. I set forth J. van Benthem's definition and proof on the basis of his letters to me.

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THE SEMANTIC AND FORMAL CONNECTIONS BETWEEN TEXT COMPONENTS

1. Introduction

The textological problems discussed in the present paper are of a certain relevance to natural language processing by automata and particularly to automated retrieval of information inherent in natural language texts. The investigations connected with those problems have been undertaken be the author within the frame of the research project *Anaphora* (cf. [4], [5], [6]) and of the authors doctoral dissertation (cf. [1], [2]).

2. Formal units

By the term *formal unit* we shall mean an expression of a written language singled out according to its formal rules within a definite text or within some other linguistic object as a definite whole. For example, all words, sentences and paragraphs of the present text, as well as the whole of it, or all labels included by it are formal units. On the contrary the lines of which this text is made up can not be regarded as formal units, as long as they do not form sentences or labels.

3. The assumption of correctness

In the most simple case a natural language text can put on the form of a sentence or of a sequence of subsequent sentences. In reading a multisentential text we are prone to subdivide it mentally, in the natural way, into a number of smaller units each composed of a number of sentences. The units of this kind refer to various topics spoken of in the given text. Such a subdivision made by the reader may either correspond with that one intended by the author, or not. When a text has been partitioned by its author into various multisentential units, it may occur that each of the units in question was meant by him not only as a purely formal but also as a semantic unit. In other words it may occur that every formal unit singled out by the author within the frame of the whole text has been conceived of

by him, at the same time, as a semantic whole. The belief that it is really the case shall be denoted in this paper by the term *the assumption of correctness.*

4. SF-units

The assumption of correctness seems to be fully justified in the case of the scientific as well as of the statutory texts. When the other texts, e.g. the literary ones are concerned it may turn out to be misleading. There is no reason of giving up that assumption in the present paper because the main topic of our discussion here shall be the texts of statutes. Therefore, all texts discussed below are to be regarded as wholes composed of the units which are formal and semantic at the same time. We shall call them in short *the SF-units.* This term shall be used also to denote the whole of a given text.

5. Text components

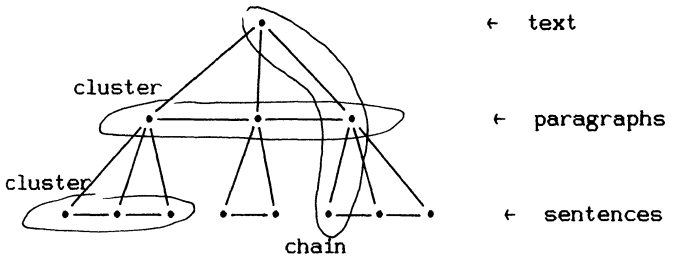
By the term *text component* we shall denote below the following SF-units: (1) the SF-units endowed with labels, (2) sentences and (3) multi-sentential SF-units having no labels. So, for example, none of the words used in this text has the status of a component in spite of the fact that each of them is to be regarded as a SF-unit in the above sense.

6. Formal structure

A set the elements of which are a definite text conceived of as a whole and all of its components is partly ordered by two relations, in particular by the relation of "being a proper part of" and by the relation of succession in a definite text. Some of the subsets of the above set are linearly ordered by the former of the above relations while the other ones are ordered in the same way by the latter. The subsets ordered by the relation of "being a proper part of" shall be called below *chains* while those ordered by the relation of succession shall be called *clusters* (see pict.1).

It is to be stressed that the component by which a definite cluster begins is by no means the successor of the component by which the foregoing cluster ends. Therefore, for example, the last sentence of the foregoing paragraph and the first sentence of this one can not be regarded as links of the relation of succession. Nor are they necessary in any semantic relation, to each other. Therefore, no text that includes multisentential components can be regarded as a sequence of sentences. A text of this kind can be conceived of as a system of its components, having the structure that has

been described above. The formal structure characteristic of such systems we shall call *the hierarchically-linear structure.*



Pict.1. The formal structure of the text.

7. Semantic structure

The array of the semantic connections holding between text components is of a much complicated nature because sentences, as well as multisentential components of the text, are by no means autonomous. The whole of the semantic connections overlaid on the hierarchically-linear structure of a given text we shall call its *semantic structure* (see pict.2).

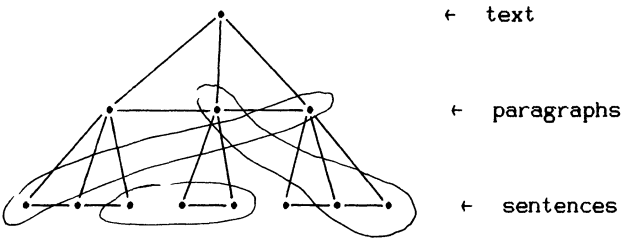
8. Natural connections

The sense to be assigned to a definite component C is in a way determined by the sense of the components of a lower degree forming the sequence included by this component (i.e. forming the corresponding cluster). Apart from that the sense of this component C is also subject to processes of complementation taking place within the frame of each of the components of the higher degree in which the aforementioned component C is included. Therefore the sense assigned to each component is subject of mutual determination and complementation taking place within the corresponding clusters and chains. The semantic connections of this kind can be regarded as natural. Thus they shall be denoted below by the term *natural connections.*

9. Cross connections

Sometimes, however, the sense of a component, located in a definite chain and in a definite cluster of a given text, is also dependent of the sense of another component, located elsewhere within the same text, i.e. placed neither

in the same chain nor in the same cluster. Moreover it may occur that the sense of a definite component of a given text is not definitely constituted within that very text, because it is also dependent on the sense of some component of another text. So, it is to be distinguished between what is denoted in the present paper by the term natural connections, i.e. semantic connections holding between components belonging each to the same chain or cluster, and what might be called *cross-connections*, i.e. connections which may hold between the components not satisfying this requirement (see pict.2). The intra- and inter-textual cross-connections can also be distinguished.



Pict.2. Examples of the cross-connections.

10. Connectors

Each set of natural language texts, e.g. the set of texts forming a given data base, can be conceived of as a formal whole. Such a linguistic object we shall call *aggregation*, or *texts aggregation*. The natural, as well as the cross-connections of semantic dependency leaves often at the surface level of the corresponding aggregation a trace, which enables us to identify the components connected by that dependency without reconstructing the meanings of the connected components. Such traces we shall call *connectors*. A connector of the semantic dependency may be located within the component which has been made dependent on the other component or components. As examples of the connectors of this kind we can regard the conjunctions "thus", "therefore", "moreover" etc. Here belong also, among other things, the phrases forming the cross-reference clauses (cf. [4], [5], [6]). Connectors can also occur outside of the components which are dependent in particular within the component or components on which the other components have been made dependent. An example of a connector located in this way is a phrase by which a semantic convention, e.g. a semantic definition, or an explanation of meaning, is expressed. Consider, for example, the following utterance:

> The provisions of the present chapter are to be applied under conditions defined in art.7 §2, 3 and 5, art.8 and art.9 §2

The above clause signalizes a strong semantic connection holding between this quoted sentence and the component "the present chapter" as well as the components defined by the phrase "in art.7 §2, 3 and 5, art.8 and art.9 §2". This last phrase singling out the labels of definite components or their paraphrases we shall call *address phrase* (cf. [1], [2]). In our opinion most of the connectors, in particular the conjunctions, the cross-references clauses and the address phrases, are capable of being identified and *comprehended* by automata.

11. Comprehension

A statement that a definite system (e.g. a certain human being or a certain technical device) comprehends a given utterance, cannot be sensibly made without singling out, at least implicitly, the operation or the set of operations with regard of which the comprehension is meant to occur in the given case. By these operations we mean here the performing of which by the aforementioned system has been made possible by its act of comprehension (cf. [3]). Therefore, a distinction must be made between the comprehension of a definite connector located within a given component C with regard of the operation of identifying the components linked by it and the comprehension of the same connector with regard of some other operation or operations (e.g. of the operation of translating the corresponding text to a definite language). The former of these cases of comprehension is the subject of investigation in the research projects mentioned at the outset of the present paper.

12. Conclusion

As yet most of the effort of the *natural language processing* theorists has been directed to investigating the connections which function within isolated sentences. It is clear, however, that the operations performed on natural language texts should take account not only of the properties of isolated sentences and of multisentential components but also of those of the much larger units. There may be no doubt that in many cases the comprehension of definite texts or even of their definite components cannot be achieved without carrying out an analysis of both the natural and the cross-connections, functioning within the corresponding aggregation of texts. So, the investigation of all of the aforementioned connections may prove significant for a progress in the *Natural Language Understanding* technics.

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